

# The effect of a neighboring metal layer on the high-frequency characteristics of a thin magnetic stripe

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The spin-wave spectrum of a ferromagnetic stripe placed above a metallic layer with finite conductivity is studied by using the magnetostatic Green’s function formalism. It is shown that the frequency and linewidth of the resonances are uniquely determined by complex, mode-dependent demagnetization factors. The formalism developed is used to analyze the resonance characteristics of the magnetic stripe as a function of its width and separation from the metallic layer. © 2008 American Institute of Physics. [DOI: 10.1063/1.2937211]

## I. INTRODUCTION

The resonance spectrum of spin waves in thin, finite ferromagnetic (FM) patterns has been intensively studied over the last decade.<sup>1-7</sup> Such studies are particularly important in view of the potential application of FM elements in magnetic data storage and microwave devices.<sup>8-11</sup> In many cases, however, magnetic elements do not stand alone, but are (partially) surrounded by metallic conductors. One, for instance, can think of microwave transmission lines with FM cores<sup>9-11</sup> or inductive techniques utilizing coplanar waveguides for probing magnetization dynamics in thin films.<sup>12,13</sup> Nevertheless, the effect of neighboring metallic layers on the spin-wave spectra of confined magnetic samples has not been investigated, although the former’s influence on the dispersion relation of spin waves in infinite films is well known.<sup>14,15</sup>

This paper presents an analysis of the spin-wave spectrum of a magnetic stripe placed above a metal layer. The analysis is carried out by using the thin-film approximation of the magnetostatic Green’s function formalism.<sup>4,16</sup> It is shown that the presence of the metal ground increases the overall effective anisotropy of the magnetic stripe, shifting the spin-wave resonances to higher frequencies. Our results agree with experimental data obtained from thin Permalloy (NiFe) stripes built on top of a dielectric/metal substrate. Furthermore, we show that the finite conductivity of the metal ground yields a mode-dependent extrinsic damping constant, in addition to the intrinsic damping constant of the film. Finally, by using the formalism developed, we study the dependence of the resonance frequency and damping of spin-wave modes on the stripe-ground plane separation and stripe width.

## II. THEORY

Consider the structure shown in Fig. 1, consisting of a FM rectangular stripe placed above a metallic ground plane (infinitely extended in the  $x$ - $z$  plane) covered by an isolating (e.g., dielectric) layer with the thickness  $d$ . The width ( $w$ ),

thickness ( $t$ ), and length ( $L$ ) of the magnetic stripe are defined along the  $x$ -,  $y$ -, and  $z$ -directions, respectively. For simplicity, we assume uniformity along the  $z$ -direction.

For a FM stripe magnetized along its length ( $z$ -direction) the ac-magnetization  $\mathbf{m} = m_x \hat{x} + m_y \hat{y}$  gives rise to an ac-demagnetization field  $\mathbf{h}_m = h_{m,x} \hat{x} + h_{m,y} \hat{y}$  inside the stripe, where

$$\mathbf{h}_m(\mathbf{r}) = -\nabla \int_S \nabla' \mathcal{G}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{m}(\mathbf{r}') dS. \tag{1}$$

Here  $\mathbf{r} = (x, y)$ ,  $\nabla = (\partial_x, \partial_y)$ , and  $S$  is the cross section of the magnetic strip in the  $x$ - $y$  plane. In the above equation  $\mathcal{G}$  is Green’s function for the magnetic potential in two dimensions in the presence of a conductive ground plane and is given by (see Appendix)

$$\mathcal{G}(\mathbf{r}, \mathbf{r}') = \mathcal{G}_0(\mathbf{r}, \mathbf{r}') + F(\eta) + F(\eta^*),$$

$$\mathcal{G}_0(\mathbf{r}, \mathbf{r}') = -\frac{1}{4\pi} \ln[(x-x')^2 + (y-y')^2],$$

$$F(\eta) = -\frac{1}{4\pi} \ln \eta + \frac{\pi}{2\beta\eta} [\mathbf{H}_1(\beta\eta) - Y_1(\beta\eta)] - \frac{1}{\beta^2 \eta^2},$$

$$\eta = (x-x') + i[y+y' + 2(d+t/2)], \tag{2}$$

where  $\beta = \sqrt{i\omega\mu_0\sigma}$ , with  $\omega$  the (angular) frequency,  $\mu_0$  the vacuum permeability, and  $\sigma$  the electrical conductivity of the ground layer. In the above equation,  $\mathbf{H}_1$  and  $Y_1$  are the Struve

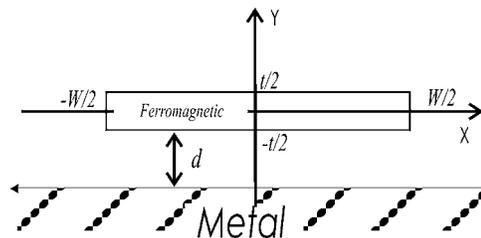


FIG. 1. Cross section of a FM stripe with a width  $w$  and thickness  $t$ , placed at a distance  $d$  above a metallic ground plane.

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and Bessel functions of the first kind, respectively. Note that, in the limit of a perfectly conducting ground plane ( $\sigma \rightarrow \infty$ ), Eq. (2) is reduced to

$$\mathcal{G}_{\sigma \rightarrow \infty}(\mathbf{r}, \mathbf{r}') = -\frac{1}{4\pi} \ln[(x-x')^2 + (y-y')^2] - \frac{1}{4\pi} \ln\{(x-x')^2 + [y+y'+2(d+t/2)]^2\}, \quad (3)$$

which is the free space field of a magnetic line charge at  $x', y'$ , plus the field of its image (with the same sign) at  $x', -y'-2(d+t/2)$ . Therefore, a perfect ground actually increases the demagnetization field inside the stripe. As we shall see later, this conclusion remains valid for imperfect grounds where  $\sigma$  is finite.

The excitation spectrum of the magnetic stripe can be determined by solving Eq. (1) together with the linear-response equation,<sup>17</sup>

$$\begin{aligned} \mathbf{m}(\mathbf{r}) &= \bar{\chi} \cdot \mathbf{h}_m(\mathbf{r}), \\ \bar{\chi} &= \frac{\omega_M}{\Omega_H^2 - \omega^2} \begin{bmatrix} \Omega_H & i\omega \\ -i\omega & \Omega_H \end{bmatrix}, \\ \Omega_H &= \omega_H + i\alpha\omega, \\ \omega_M &= \gamma M, \omega_H = \gamma H_a, \end{aligned} \quad (4)$$

where  $\gamma$  is the gyromagnetic constant,  $M$  is the saturation magnetization,  $H_a$  is the static (dc) anisotropy field along the  $z$ -direction, and  $\alpha$  is the Gilbert damping constant. For a thin FM strip ( $t \ll W$ ), the variation of the magnetization along the film thickness ( $y$ -direction) can be neglected. Hence, one can formulate the problem in terms of the average magnetization vector,<sup>4</sup>

$$\bar{\mathbf{m}}(x) = \frac{1}{t} \int_{-t/2}^{t/2} \mathbf{m}(x, y) dy. \quad (5)$$

Upon combining Eqs. (1) and (4), and performing averaging over the film thickness  $t$ , one arrives at the one-dimensional matrix integral equation,

$$\bar{\mathbf{Q}} \cdot \bar{\mathbf{m}}(x) + \int_{-W/2}^{W/2} \bar{\mathbf{G}}(x, x') \cdot \bar{\mathbf{m}}(x') = 0, \quad (6)$$

where

$$\bar{\mathbf{Q}} = \bar{\chi}^{-1} = \frac{1}{\omega_M} \begin{bmatrix} \Omega_H & -i\omega \\ i\omega & \Omega_H \end{bmatrix}, \quad (7)$$

and  $\bar{\mathbf{G}}$  is matrix Green's function,

$$\bar{\mathbf{G}}(x, x') = \begin{bmatrix} \delta(x-x') - g_1(x, x') & ig_2(x, x') \\ ig_2(x, x') & g_1(x, x') \end{bmatrix}, \quad (8)$$

where  $\delta(x-x')$  is the Dirac delta function. The functions  $g_1$  and  $g_2$  are symmetric, i.e.,  $g_2(x, x') = g_2(x', x)$  and  $g_1(x, x') = g_1(x', x)$  so that  $\bar{\mathbf{G}}_{ij}(x, x') = \bar{\mathbf{G}}_{ji}(x', x)$  for  $i, j = 1, 2$ . (The expressions for  $g_1$  and  $g_2$  are quite complicated and, therefore, are given in the Appendix.)

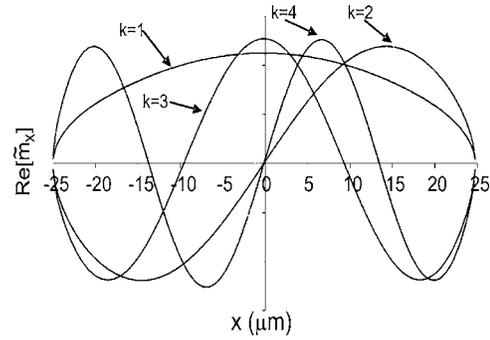


FIG. 2. The (real part of the) in-plane magnetization  $\bar{m}_x$  for the first four eigenmodes ( $k=1, 2, 3, 4$ ) of a  $50\text{-}\mu\text{m}$ -wide and  $0.1\text{-}\mu\text{m}$ -thick Permalloy stripe, placed  $1\text{ }\mu\text{m}$  above a conducting ground plane. The conductivity of the metal ground is  $\sigma = 3.3 \times 10^7$  S/m. The corresponding eigenvalues were  $\lambda_1 = 0.003\,96 + 0.000\,23i$ ,  $\lambda_2 = 0.008\,77 + 0.000\,67i$ ,  $\lambda_3 = 0.012\,78 + 0.001\,18i$ , and  $\lambda_4 = 0.016\,33 + 0.001\,67i$ . The results were obtained by the numerical solution of Eqs. (6)–(9).

Equation (6) possesses nontrivial solutions only for certain frequencies, i.e., the magnetostatic resonance frequencies of the stripe. However, before trying to solve Eq. (6) let us first consider the eigenvalue problem,

$$\begin{aligned} \int_{-W/2}^{W/2} \bar{\mathbf{G}}(x, x') \cdot \Psi_k(x') &= \lambda_k \Psi_k(x), \\ \Psi_k(x) &= \begin{bmatrix} \psi_k^1(x) \\ \psi_k^2(x) \end{bmatrix}. \end{aligned} \quad (9)$$

Because of the special form of  $\bar{\mathbf{G}}$  [Eq. (8)], it can be shown that if an eigenvalue  $\lambda_k$  and eigenfunction  $\Psi_k(x)$  satisfy Eq. (9), then so do the combination

$$\bar{\lambda}_k = 1 - \lambda_k, \bar{\Psi}_k(x) = \begin{bmatrix} \psi_k^2(x) \\ -\psi_k^1(x) \end{bmatrix}. \quad (10)$$

Returning to Eq. (6), we next try a solution of the type  $\bar{\mathbf{m}}(x) = A_k \Psi_k(x) + B_k \bar{\Psi}_k(x)$ , where  $A_k, B_k$  are constants. This yields the equation

$$\begin{bmatrix} \Omega_H + \lambda_k \omega_M & -i\omega \\ i\omega & \Omega_H + (1 - \lambda_k) \omega_M \end{bmatrix} \cdot \begin{bmatrix} A_k \\ B_k \end{bmatrix} = 0, \quad (11)$$

which allows a nonzero solution only for

$$\omega^2 = \omega_k^2 = [\Omega_H + (1 - \lambda_k) \omega_M] (\Omega_H + \lambda_k \omega_M), \quad (12)$$

i.e., the spin-wave resonance frequencies of the stripe. Note that  $\lambda_k$  and  $1 - \lambda_k$  can be interpreted as mode-dependent demagnetization factors in the  $x$ - and  $y$ -directions, respectively.<sup>16</sup>

The solutions of Eq. (6) correspond to standing spin waves caused by the reflection of the waves by the lateral edges of the stripe. Figure 2 shows the distribution of the in-plane magnetization  $\bar{m}_x(x)$  for the first four modes ( $k=1, 2, 3, 4$ ) of a  $50\text{-}\mu\text{m}$ -wide and  $0.1\text{-}\mu\text{m}$ -thick Permalloy stripe, placed  $1\text{ }\mu\text{m}$  above a conducting ground plane. Like in the case of a free magnetic stripe,<sup>1</sup> the distribution of  $\bar{m}_x(x)$  for different modes roughly resembles standing sinusoidal waves arising from imposing the quantization condi-

tion  $k_x = k\pi/w$  on spin waves propagating with a wave number  $k_x$  along the  $x$ -direction in an unbounded magnetic film.

Although expression (12) for the resonance frequency is identical to that of a free magnetic stripe, i.e., in the absence of a ground plane<sup>4,16</sup> one should bear in mind that  $\vec{G}$  and, therefore, its eigenvalues  $\lambda_k$  depend on the stripe-ground plane distance  $d$ , the conductivity  $\sigma$  of the ground, and even the frequency  $\omega$  (through the parameter  $\beta$ ). The dependence of  $\lambda_k$  on the frequency  $\omega$  implies that Eq. (12) should be viewed as a self-consistent equation for the resonance frequencies  $\omega_k$ . Furthermore, because the components of  $\vec{G}$  are complex quantities,  $\lambda_k$  are complex as well. Note that the imaginary part of  $\lambda_k = \lambda'_k + i\lambda''_k$  is, in fact, induced by the finite conductivity of the ground. A perfect ground conductor would yield a real matrix  $\vec{G}$  and, through symmetry properties of  $\vec{G}$ , real values of  $\lambda_k$ .

The imaginary part of  $\lambda_k$  contributes to the imaginary part of the resonance frequency  $\omega_k = \omega'_k + i\omega''_k$ . If both  $\lambda''_k$  and the Gilbert damping constant  $\alpha$  (representing the intrinsic magnetic relaxation loss of the stripe) are small, then it follows from Eq. (12) that

$$(\omega'_k)^2 \approx [\omega_H + (1 - \lambda'_k)\omega_M](\omega_H + \lambda'_k\omega_M), \quad (13)$$

$$\omega''_k \approx \left(\omega_H + \frac{\omega_M}{2}\right)\alpha + \frac{(1 - 2\lambda'_k)\omega_M^2}{2\omega'_k}\lambda''_k. \quad (14)$$

Note that in an actual experiment  $\omega'_k$  corresponds to the central frequency of the resonance observed while the resonance linewidth is given by  $\Delta\omega_k \approx 2\omega''_k$ .<sup>16</sup> Thus, the Ohmic losses in the ground conductor cause additional absorption of electromagnetic energy, increasing the magnetostatic resonance linewidth by an amount proportional to  $\lambda''_k$ . In analogy with the intrinsic magnetic loss, the extra loss induced by the ground plane can be expressed in terms of the mode-dependent, extrinsic damping constant

$$\alpha_k^{\text{ex}} = \frac{(1 - 2\lambda'_k)\omega_M^2}{\omega'_k(2\omega_H + \omega_M)}\lambda''_k. \quad (15)$$

The overall damping constant for any given mode is then simply  $\alpha + \alpha_k^{\text{ex}}$ .

To obtain an experimental verification of the model presented, we measured the resonance frequency of the lowest mode ( $k=1$ ) Permalloy (NiFe) stripes with a width of 100  $\mu\text{m}$  and thicknesses of 100 and 200 nm. The stripes were built on top of a 2- $\mu\text{m}$ -thick aluminum layer with a conductivity of  $\sigma = 3.3 \times 10^7$  S/m, covered by a 1- $\mu\text{m}$ -thick SiO<sub>2</sub> layer for isolation. A second SiO<sub>2</sub> layer was next deposited to cover the stripes, followed by the fabrication of microstrip probe lines on top of the structure. High-frequency impedance measurement of the microstrip lines was performed by using a vector network analyzer. Magnetic parameters (saturation magnetization, intrinsic magnetocrystalline anisotropy) of the Permalloy film were found from  $M$ - $H$  loop measurements performed by using a Princeton Micromag 2900 AGM. Based on the values obtained ( $M = 1.15$  T,  $H_a = 25$  Oe), we calculated the first spin-wave reso-

TABLE I. Measured and calculated values of the frequency of the first spin-wave resonance of a 100- $\mu\text{m}$ -wide Permalloy stripe. The (vertical) distance between the stripes and the aluminum ground plane was 1  $\mu\text{m}$ . For the calculation we used a saturation magnetization of  $M = 1.15$  T and an internal anisotropy of  $H_a = 25$  Oe, as obtained from  $M$ - $H$  loop measurements on nonpatterned magnetic films. An experimentally obtained gyromagnetic ratio of  $\gamma \approx 1.84 \times 10^{11}$  rad/s T was used in the simulations. For comparison, results of the calculation for free magnetic stripes are also included.

Film thickness (nm)	Measured frequency (GHz)	Calculated frequency (GHz)	
		Strip above Al ground	Free stripe
100	2.15–2.38 (16 devices)	2.19	1.92
200	2.75–2.90 (8 devices)	2.69	2.23

nance frequency of the stripes. The results, given in Table I, are in good agreement with the experimental values.

### III. NUMERICAL EXPERIMENTS

In this section, we use the formalism discussed in Sec. II to numerically study the dependence of the magnetostatic resonance spectra of FM stripes on their width and distance from a conducting ground plane. For our simulations we use a saturation magnetization of  $M = 1$  T and an intrinsic (magnetocrystalline) anisotropy field of  $H_a = 5$  Oe (these are the most common values reported for Permalloy films in the literature). The electrical conductivity of the ground plane is taken to be  $\sigma = 3.3 \times 10^7$  S/m.

Figure 3 shows the resonance frequency  $\omega'_k/2\pi$  and the extrinsic damping constant  $\alpha_k^{\text{ex}}$  ( $k=1,2,3,4$ ) of the first four spin-wave resonances calculated as function of the ground plane separation  $d$ , for a 50- $\mu\text{m}$ -wide and 0.1- $\mu\text{m}$ -thick FM stripe. The resonance frequency increases if the ground plane is brought closer to the stripe by reducing  $d$ . This is because, as mentioned in Sec. II, the ground plane strengthens the ac-demagnetization field induced by the magnetization distribution on the stripe. This leads to larger values of the demagnetization coefficient  $\lambda_k$  and, in turn, higher resonance frequencies. In fact, it can be shown that if the magnetic stripe is directly put on top of a perfect ground ( $d=0$ ), then the resulting demagnetization factors are twice larger than that of a free magnetic stripe. Reducing the distance  $d$  also leads to an increase in the effective extrinsic damping constant  $\alpha_k^{\text{ex}}$  which is also shown in Fig. 3. This, of course, is due to the increase in the flow of eddy currents in the ground plane caused by the ac magnetic field around the stripe.

By increasing the distance  $d$ , the resonance frequencies saturate to their “free space” values, as one would have expected. The saturation occurs earlier for modes with higher numbers. This result can be understood by noting that the magnetic field accompanying a spin-wave decays exponentially outside the magnetic film in the  $y$ -direction (Fig. 1). A ground plane at a distance larger than this decay length is not “felt” by the stripe. For spin waves propagating with a wave number  $k_x$  in an unbounded film, the decay length is given by  $\xi = 1/k_x$ . Since the eigenmodes of the stripe roughly correspond to standing waves formed by imposing the quantization condition  $k_x = k\pi/w$  (see previous section), the decay length corresponding to the  $k$ th resonance is  $\xi_k \approx w/k\pi$ .

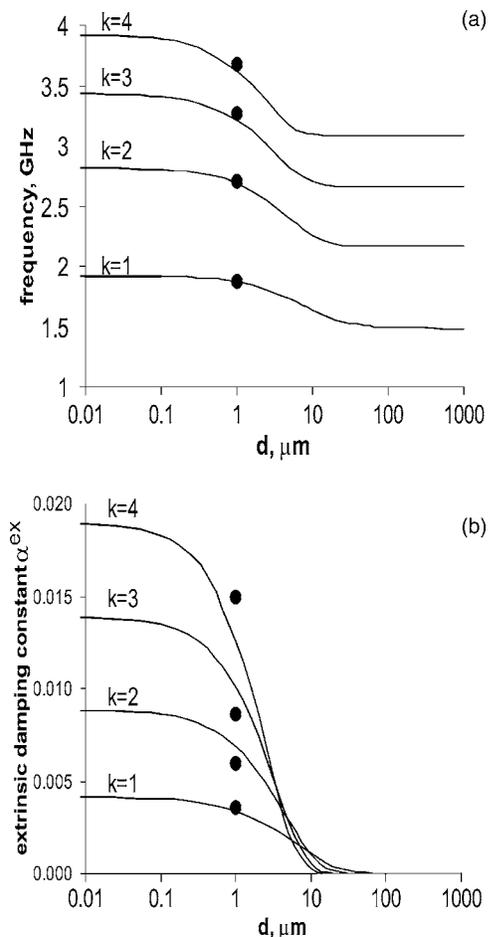


FIG. 3. Resonance frequency  $\omega'_k/2\pi$  (a) and extrinsic damping constant  $\alpha_k^{\text{ex}}$  (b) vs distance  $d$  for the first four spin-wave modes ( $k=1, 2, 3, 4$ ). The circles correspond to the results obtained from ANSOFT HFSS, a commercial three-dimensional full-wave electromagnetic simulator. The saturation magnetization of the stripe is  $M=1$  T and the internal anisotropy is  $H_a=5$  Oe. The width and the thickness of the stripe are  $w=50 \mu\text{m}$  and  $t=0.1 \mu\text{m}$ , respectively. The conductivity of the metal ground is  $\sigma=3.3 \times 10^7$  S/m.

Thus, fields associated with higher modes decay faster in the vertical direction and start to behave as free-stripe modes at smaller values of  $d$ . To provide an independent verification of our method we also calculated the resonance frequency and linewidth of the stripe of Fig. 2 for  $d=1 \mu\text{m}$  using ANSOFT HFSS, a commercial full-wave electromagnetic simulator. The results are shown as circles in Fig. 3.

Figure 4 presents the dependence of the first spin-wave resonance frequency  $\omega'_1/2\pi$  and its associated extrinsic damping constant  $\alpha_1^{\text{ex}}$ , on the width of a FM strip. The most interesting feature of this figure concerns the behavior of  $\alpha_1^{\text{ex}}$ . For the particular configuration considered, the value of  $\alpha_1^{\text{ex}}$  reaches a maximum at  $w \approx 8 \mu\text{m}$  and becomes zero as  $w \rightarrow 0$  or  $w \rightarrow \infty$ . This result can be qualitatively understood as follows. The extrinsic damping of spin waves is caused by the electric currents induced on the surface of the ground conductor. Those currents flow in the  $z$ -direction with a sheet density equal to the tangential magnetic field  $H_x$  on the conductor surface. The total loss induced in the conductor is  $P_l=(1/2)R_s|H_x|^2$ , where  $R_s=\text{Re}(Z_s)$  is the surface resistance of the conductor [see Eq. (A10)]. As spin waves are bound to the magnetic film, the magnetic field decays exponentially

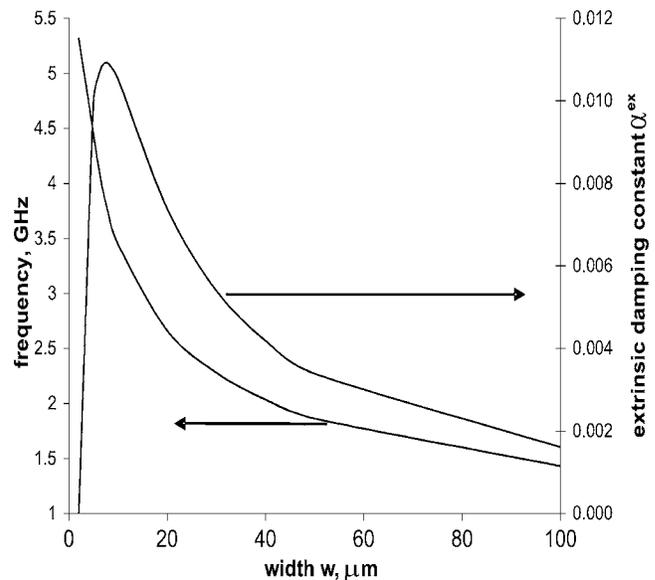


FIG. 4. Resonance frequency  $\omega'_k/2\pi$  and extrinsic damping constant  $\alpha_k^{\text{ex}}$  vs the width  $w$  for the first resonance mode ( $k=1$ ) of a magnetic strip of thickness  $t=0.1 \mu\text{m}$ , placed above a conductive ground plane at the distance  $d=1 \mu\text{m}$ .

inside the dielectric layer as it reaches the ground surface. Thus, for the lowest mode,  $H_x \propto \exp(-d/\xi_1) = \exp(-\pi d/w)$ . On the other hand, the surface resistance of the conductor is given by  $R_s \sim (\omega'_1 \mu_0 / 2\sigma)^{1/2}$ . The observed maximum in  $\alpha_1^{\text{ex}}$  as function of  $W$  is due to the competition between  $R_s$  and  $H_x$ . Increasing the stripe width leads to a lower resonance frequency and, therefore, a smaller  $R_s$ . On the other hand, it results in larger surface currents due to a larger magnetic field  $H_x$  reaching the surface of the ground conductor. Increasing the distance  $d$  leads to less rapid rise in  $H_x$  as a function of  $w$ , shifting the maximum observed to higher values of the width.

#### IV. CONCLUSION

The effect of the neighboring metal layer on the magnetic characteristics of a FM strip is analyzed by using magnetostatic Green's function formalism. It is shown that the metallic layer strengthens the demagnetizing field inside the FM strip and, consequently, increases the frequency of the spin-wave resonances. The finite conductivity of the metal layer leads to extra broadening of the resonance peaks, which is described in terms of a mode-dependent extrinsic damping constant. The extrinsic magnetic damping increases by reducing the distance between the metal ground and the FM stripe. However, it shows a more complicated behavior as function of the width of the stripe, reaching a maximum for a particular value of the stripe width.

#### ACKNOWLEDGMENTS

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## APPENDIX

In what follows, we derive Green's function  $\mathcal{G}$  for the magnetic potential in the presence of a ground conductor. In the regions of space filled with media with zero electric conductivity (e.g., air, dielectrics, nonconductive magnetic materials), one can employ the magnetostatic approximation and represent the magnetic field  $\mathbf{h}$  as

$$\mathbf{h} = \nabla\varphi, \quad (\text{A1})$$

where  $\varphi$  is the magnetic potential. This approximation is justified if electromagnetic propagation effects are negligible, i.e., if the dimension of the structure to be studied is far less than that of the electromagnetic wavelength. From Maxwell's equation  $\nabla \cdot \mathbf{b} = 0$ , where  $\mathbf{b} = \mu_0(\mathbf{h} + \mathbf{m})$  is the magnetic induction, one then arrives at the equation

$$\nabla^2\varphi = -\nabla \cdot \mathbf{m}. \quad (\text{A2})$$

The magnetostatic approximation loses its validity inside the conductive ground layer. Therefore, we solve Eq. (A2) outside the ground conductor only, taking into account the latter by imposing appropriate boundary conditions on  $\varphi$  at its surface. To obtain those conditions, we first note that the time-dependent variation of the magnetic field induces an electric field which, assuming uniformity along the stripe, only has a component  $e_z$  in the  $z$ -direction. Using Faraday's law and Eq. (A1) just above the conducting plane (inside the dielectric layer), we have

$$\frac{\partial e_z}{\partial x} = i\omega\mu_0 h_y = i\omega\mu_0 \frac{\partial\varphi}{\partial y}. \quad (\text{A3})$$

Furthermore, the tangential components of the electric and magnetic fields are continuous across the dielectric-ground plane interface and are related by

$$e_z = -Z_S h_x, \quad (\text{A4})$$

where  $Z_S$  is the surface impedance of the conductor.<sup>18</sup> Combining Eqs. (A1), (A3), and (A5), we arrive at the boundary condition at the metal surface,

$$\frac{\partial^2\varphi}{\partial x^2} = -\frac{i\omega\mu_0}{Z_S} \frac{\partial\varphi}{\partial y}, \quad y = -d - t/2. \quad (\text{A5})$$

For any distribution of magnetization  $\mathbf{m}$ , the solution of Eq. (2) can be expressed in terms of Green's function  $\mathcal{G}$  satisfying the equation

$$\nabla^2\mathcal{G}(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}'). \quad (\text{A6})$$

The function  $\mathcal{G}$  can be evaluated by applying a Fourier transform in the  $x$ -direction,

$$\Gamma(y, y'; k_x) = \int_{-\infty}^{\infty} \mathcal{G}(x, y; 0, y') \exp(ik_x x) dx, \quad (\text{A7})$$

which results in

$$\frac{\partial^2\Gamma}{\partial y^2} - k_x^2\Gamma = -\delta(y - y'). \quad (\text{A8})$$

The boundary condition for  $\Gamma$  reads

$$k_x^2\Gamma = \frac{i\omega\mu_0}{Z_S} \frac{\partial\Gamma}{\partial y}, \quad y = -d - t/2, \quad (\text{A9})$$

where the surface impedance in the Fourier domain is given by

$$Z_S = \frac{i\omega\mu_0}{\sqrt{k_x^2 + i\omega\mu_0\sigma}}. \quad (\text{A10})$$

Solving Eqs. (A8)–(A10) yields

$$\Gamma(y, y'; k_x) = (2|k_x|)^{-1} \{ \exp[-|k_x|(y - y')] + R \exp[-|k_x|(y + y' + 2d)] \}, \quad (\text{A11})$$

where

$$R = \frac{\sqrt{k_x^2 + i\omega\mu_0\sigma} - |k_x|}{\sqrt{k_x^2 + i\omega\mu_0\sigma} + |k_x|}. \quad (\text{A12})$$

Green's function (4) in the space domain is finally determined by applying the inverse Fourier transform,

$$\mathcal{G}(\mathbf{r}, \mathbf{r}') = \int_{-\infty}^{\infty} \Gamma(y, y'; k_x) \exp[-ik_x(x - x')] \frac{dk_x}{2\pi}. \quad (\text{A13})$$

The one-dimensional matrix Green's function  $\bar{\bar{\mathbf{G}}}$  in Eq. (8) is computed from

$$\bar{\bar{\mathbf{G}}}(\mathbf{x}, \mathbf{x}') = \frac{1}{t} \int_{-t/2}^{t/2} \int_{-t/2}^{t/2} \nabla' \nabla' \mathcal{G}(r, r') dy dy' \quad (\text{A14})$$

which results in the elements

$$\begin{aligned} g_1(x, x') &= \frac{1}{t} \int_{-t/2}^{t/2} \int_{-t/2}^{t/2} \frac{\partial}{\partial y} \frac{\partial}{\partial y'} \mathcal{G}(r, r') dy dy' \\ &= \frac{1}{t} \left\{ \frac{1}{2\pi} \ln \left[ \frac{(x - x')^2}{(x - x')^2 + t^2} \right] + U(x - x') \right. \\ &\quad \left. + V(x - x') \right\} \\ g_2(x, x') &= \frac{1}{t} \int_{-t/2}^{t/2} \int_{-t/2}^{t/2} \frac{\partial}{\partial x} \frac{\partial}{\partial y'} \mathcal{G}(r, r') dy dy' \\ &= \frac{1}{t} [U(x - x') - V(x - x')], \end{aligned} \quad (\text{A15})$$

where

$$\begin{aligned} U(x - x') &= F(\eta_1) + F(\eta_2) - 2F(\eta_3), \\ V(x - x') &= F(\eta_1^*) + F(\eta_2^*) - 2F(\eta_3^*), \\ \eta_1 &= x - x' + i2(d + t), \\ \eta_2 &= x - x' + i2d, \\ \eta_3 &= x - x' + i2(d + t/2). \end{aligned} \quad (\text{A16})$$

The function  $F$  is defined by Eq. (2).

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