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DOI
10.1109/CCTA49430.2022.9966122

Publication date
2022

Document Version
Final published version

Published in
Proceedings 2022 IEEE Conference on Control Technology and Applications, CCTA 2022

2022 IEEE Conference on Control Technology and Applications (CCTA)
August 22-25, 2022. Trieste, Italy

Steady-state nonlinearity of open-loop reset systems

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***Abstract*— In this paper, we introduce a new representation for open-loop reset systems. We show that at steady-state a reset integrator can be modelled as a parallel interconnection of the base-linear system and piece-wise constant nonlinearity. For sinusoidal input signals, this nonlinearity takes a form of a square wave. Subsequently, we show how the behaviour of a general open-loop reset system is related to the nonlinearity of a reset integrator. The proposed approach simplifies the analysis of reset elements in the frequency domain and provides new insights into the behaviour of reset control systems.**

I. Introduction

A reset element is a linear time-invariant system whose states, or a subset of states, reset to values defined by a reset law if certain conditions are satisfied [1]. It has been proven that reset systems can overcome limitations of linear controllers [2], [3]. Examples of applications of reset in various fields like process control or networked systems can be found in textbooks [1], [4], [5]. Moreover, reset elements have been successfully applied to control precision positioning systems [6], [7], [8], [9], [10], [11], [12].

One of the reasons why the reset control systems draw so much attention, especially for industrial applications, is the fact that they can be designed using a modified frequency-domain loop-shaping procedure. The steady-state behaviour of nonlinear systems may be, in certain cases, described using the *describing functions* (DF) [13]. For reset system the DF were first derived in [14], and later extended to the *Higher-order Sinusoidal-input Describing function* (HOSIDF) in [15].

With HOSIDF, the nonlinearity of a system is represented by describing harmonics of the output signal to a sinusoidal input at a certain frequency. While this method can be used for tuning the controllers, the influence of the higher harmonics can not be easily interpreted.

In this paper, we introduce a new representation of the steady-state behaviour of reset systems with sinusoidal inputs. First, we show that a reset integrator can be modelled at steady state as a parallel interconnection of the base-linear systems and piece-wise constant non-linearity. For sinusoidal input signals, this nonlinear component of the output takes a form of a square wave.

Any general open-loop reset system can be represented as a feedback system built around a reset integrator. Using this fact, we show how the nonlinearity of any reset element in open-loop is related to the behaviour of a reset integrator.

*This work was not supported by any organization

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The representation introduced in this paper provides a clear interpretation of HOSIDF of open-loop reset systems.

II. Background

A. Notation

I and 0 denote here square identity matrix and zero matrix of appropriate size respectively. $J_{m,n}$ denotes a matrix of ones.

B. Reset control systems

Consider a reset element

$$R: \begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r u_r(t), & \text{if } u_r \neq 0 \\ x_r(t^+) = A_\rho x_r(t), & \text{if } u_r = 0\,, \\ y_r = C_r x_r(t) + D_r u_r(t) \end{cases} \quad (1)$$

where $x_r(t^+) = \lim_{\epsilon \to 0^+} x(t+\epsilon)$, $x_r \in \mathbb{R}^m$ is the state of R, $u_r \in \mathbb{R}^1$ is the input of R, $y_r \in \mathbb{R}^1$ is the output of R and A_r, B_r, A_ρ, C_r, D_r are constant matrices of appropriate dimensions.

The linear system described with (A_r, B_r, C_r, D_r) is referred to by the term *Base linear system* (BLS) and describes dynamics of R in absence of reset.

The *linear reset law* $x_r(t^+) = A_\rho x_r(t)$ describes the change of state that occurs at *reset instants* $t_k, k = 1, 2, \ldots,$ that is when the *reset condition* $u_r = 0$ is satisfied. Alternative reset laws and conditions [5] are not considered in this work.

Since reset systems are a special case of hybrid systems, pathological behaviours like beating, deadlock and Zeno behaviour may occur [16]. In practice, existence and uniqueness of the solution are assured by time-regularization [17], [18]. Time-regularization is a modification of reset system, such that reset instants happen only if a minimum time between resets $\Delta_m > 0$ has lapsed. Any discrete-time implementation inherently features time regularization with Δ_m equal to the sampling time [9]. In remainder of this paper it is assumed that solutions of R are well defined [1].

C. Reset elements

For illustration purpose, let introduce the *First order reset element* (FORE) and the *Second order reset element* (SORE). FORE is a reset element (1) with

$$A_r = -\omega_r, \qquad B_r = \omega_r, \qquad C_r = 1, \qquad D_r = 0.$$

The state-space matrices for SORE are

$$A_r = \begin{bmatrix} 0 & 1 \\ -\omega_r^2 & -2\beta_r\omega_r \end{bmatrix}, \qquad B_r = \begin{bmatrix} 0 \\ \omega_r^2 \end{bmatrix},$$
$$C_r = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad D_r = 0.$$

Here, we choose $\omega_r = 100$, $\beta_r = 0.1$ and $A_\rho = 0$.

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III. Nonlinearity of Reset Systems

To analyse the steady-state behaviour of a system, we require it to be *uniformly convergent* [19], [20]. In this way, we guarantee the existence of a steady-state solution for a system driven by periodic inputs. This condition is satisfied by (1) for a class of sinusoidal inputs if $|\lambda(A_\rho e^{A_r\delta})| < 1, \forall\delta \in \mathbb{R}^+$ [14]. In [21], this property has been studied in closed-loop, related to the H_β condition [22] and proven for a wider class of input signals.

Theorem 3.1 (Reset integrator): The steady-state state response of uniformly convergent reset integrator, i.e. reset system (1) with $A_r = 0, B_r = I, C_r = I, D_r = 0$, to an input signal $u_r(t) = a\sin(\omega t), a \in \mathbb{R}^{m\times 1}$ is given by

$$x_r(t) = x_{bls}(t) + q_i(t), \tag{2}$$

where x_{bls} denotes the steady-state response of the base-linear system (A_r, B_r, C_r, D_r) and q_i is a square wave in phase with u_r, with a mean value

$$\bar{q}_i = \frac{-a}{\omega} I \tag{3}$$

and amplitude

$$\hat{q}_i = (I - A_\rho)(I + A_\rho)^{-1}\frac{a}{\omega}. \tag{4}$$

Proof: Consider dynamics of $q \triangleq x_r - x_{bls}$

$$\begin{cases} \dot{q}(t) = A_r q(t), & \text{if } u_r \neq 0 \\ q(t^+) = A_\rho q(t) + (A_\rho - I)x_{bls}(t), & \text{if } u_r = 0. \end{cases} \tag{5}$$

It is clear that for a reset integrator $A_r = 0$ with any input u_r, $q(t)$ is piecewise constant, and jumps only at the reset instants.

Between consecutive reset instants t_k, t_{k+1} we have

$$q(t) = q(t_k^+) + \int_{t_k}^{t} A_r q(\tau)\,\mathrm{d}\tau, \text{ for } t \in (t_k, t_{k+1}). \tag{6}$$

For an input signal $u_r(t) = a\sin(\omega t)$, we have reset instants $t_k = k\frac{\pi}{\omega}, k \in \mathbb{N}$. Signal q specific for this case is denoted with subscript $_i$. From the second equation of (5)

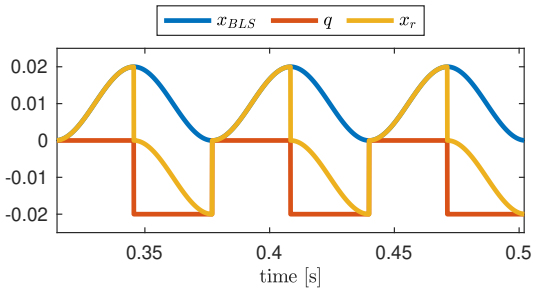
$$\begin{aligned} q_i(t_k^+) &= A_\rho q_i(t_{k-1}^+) + (A_\rho - I)\int_{t_{k-1}}^{t_k} B_r u_r(\tau)\,\mathrm{d}\tau \\ &= \begin{cases} A_\rho q_i(t_{2n}^+), & \text{if } k = 2n+1 \\ A_\rho q_i(t_{2n+1}^+) + (A_\rho - I)\frac{2}{\omega}a, & \text{if } k = 2n. \end{cases} \end{aligned} \tag{7}$$

At the steady state, we have

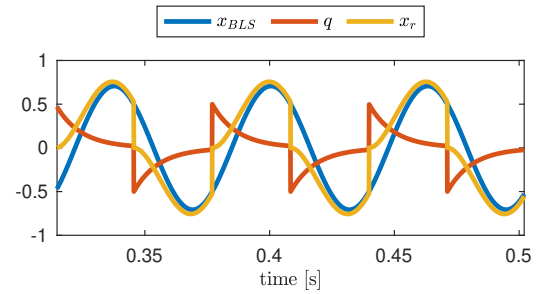
$$\begin{aligned} q_i(t_{2n}^+) &= q_i(t_{2n+2}^+) \\ &= A_\rho q_i(t_{2n+1}^+) + (A_\rho - I)\frac{2}{\omega}a \\ &= A_\rho^2 q_i(t_{2n}^+) + (A_\rho - I)\frac{2}{\omega}a. \end{aligned} \tag{8}$$

After solving for $q_i(t_{2n}^+)$ we have

$$q_i(t_{2n}^+) = (I - A_\rho^2)^{-1}(A_\rho - I)\frac{2}{\omega}a = -(I + A_\rho)^{-1}\frac{2}{\omega}a. \tag{9}$$



(a) Integrator ($A_r = 0, B_r = 1, C_r = 1, D_r = 0, A_\rho = 0$)



(b) FORE ($A_r = -100, B_r = 100, C_r = 1, D_r = 0, A_\rho = 0$)

Fig. 1: Steady-state time-responses of reset elements to $u_r(t) = \sin(100t)$ as a sum of linear and nonlinear contributions.

For $q_i(t_{2n+1}^+)$ we have

$$q_i(t_{2n+1}^+) = -A_\rho(I + A_\rho)^{-1}\frac{2}{\omega}a. \tag{10}$$

Taking all this into consideration we have

$$q_i(t) = \begin{cases} -(I + A_\rho)^{-1}\frac{2}{\omega}a, & \text{for } t \in [t_{2n}, t_{2n+1}) \\ -A_\rho(I + A_\rho)^{-1}\frac{2}{\omega}a, & \text{for } t \in [t_{2n+1}, t_{2n+2}). \end{cases} \tag{11}$$

The mean value $\bar{q}_i$ and the peak amplitude $\hat{q}_i$ are given by

$$\bar{q}_i = \frac{q(t_{2n}^+) + q(t_{2n+1}^+)}{2} = \frac{-a}{\omega} I \tag{12}$$

$$\hat{q}_i = \frac{q(t_{2n+1}^+) - q(t_{2n}^+)}{2} = (I - A_\rho)(I + A_\rho)^{-1}\frac{a}{\omega}. \tag{13}$$

So we can also write

$$q_i(t) = \begin{cases} \bar{q}_i + \hat{q}_i, & \text{for } t \in [t_{2n}, t_{2n+1}) \\ \bar{q}_i - \hat{q}_i, & \text{for } t \in [t_{2n+1}, t_{2n+2}). \end{cases} \tag{14}$$

■

Remark 1: Signal $q(t)$ represents nonlinearity added to the base linear system by the reset actions.

Steady-state response of a reset integrator, divided into the linear and nonlinear components, is presented in Fig. 1a.

Theorem 3.2 (Reset state-space system): The steady-state state response of uniformly convergent reset system (1) to an input signal $u_r(t) = b\sin(\omega t), b \in \mathbb{R}$ is given by

$$x_r = x_{bls} + q = x_{bls} + T_q \circledast q_i, \tag{15}$$

$$T_q(s) = \frac{x_r(s)}{q_i(s)} = Q(sI - A_r)^{-1}s \tag{16}$$

where x_{bls} denotes the steady-state response of the base-linear system (A_r, B_r, C_r, D_r) and q_i is the square wave introduced in Theorem 3.1, representing the nonlinearity in a reset integrator with the same number of states and reset matrix as in (1), that is driven by input $a\sin(\omega t), a = bJ_{m,1}$.

The scaling of the magnitude of the nonlinearity $Q \in \mathbb{R}^{m\times m}$ is given by

$$\min_{Q} \left\| e^{A_r t}(I - A_\rho)x_{bls}(t_k) - e^{A_r t}A_\rho Q q^*(t_k) + Q q^*(t) \right\|_2^2, \tag{17}$$

for $t \in (t_k, t_{k+1})$, where t_k, t_{k+1} denote subsequent reset instants.

Proof: Any state-space reset system can be represented in a block diagram form as a feedback system with a reset integrator and feedback gain A_r, see Fig. 2a. Using the Theorem 3.1, the reset integrator can be represented as a parallel interconnection of its base-linear system and a block generating the signal related to resets (Fig. 2b).

Reset instants of considered elements are depending only on the input signal. In consequence, reset instants for a reset integrator considered in Theorem 3.1 and a general reset state-space system are the same, if both systems are driven by sine waves of the same frequency and phase. However, in the state-space system, the signal entering the integrator e_r is shifted with respect to the u_r. This leads to the change of the magnitude of the square wave, which is represented by the scaling matrix Q.

The complete new representation of a reset element is shown in Fig. 2c. It can be seen that the state of a reset element consists of the response of the base linear system x_{bls} and a contribution due to the nonlinearity q_i. This contribution is denoted by q and defined in (5). By analysing the block diagrams, we have

$$x_r = A_r \frac{I}{s} x_r + B_r \frac{I}{s} u_r + q, \tag{18}$$

$$x_r = x_{bls} + q = x_{bls} + T_q \circledast q_i, \tag{19}$$

$$T_q(s) = \frac{x_r(s)}{q_i(s)} = Q(sI - A_r)^{-1}s \tag{20}$$

The linear transfer function T_q defines the magnitude and shape of the nonlinear contribution.

To find the amplitude of the nonlinear contribution we consider evolution of a reset system states between reset instants. We define $q(t) \triangleq Q q^*(t)$, where q^* can be interpreted as a response of T_q with $Q = I$ to the square wave signal q_i.

Using that $x_r = x_{bls} + q$, for $t \in (t_k, t_{k+1})$ we have

$$e^{A_r(t-t_k)}x_{bls}(t_k) + q(t) = e^{A_r(t-t_k)}x_r(t_k^+) \tag{21}$$

$$e^{A_r(t-t_k)}x_{bls}(t_k) + Q q^*(t) = e^{A_r(t-t_k)}A_\rho(x_{bls}(t_k) + Q q^*(t_k)) \tag{22}$$

$$0 = e^{A_r t}(I - A_\rho)x_{bls}(t_k) - e^{A_r t}A_\rho Q q^*(t_k) + Q q^*(t). \tag{23}$$

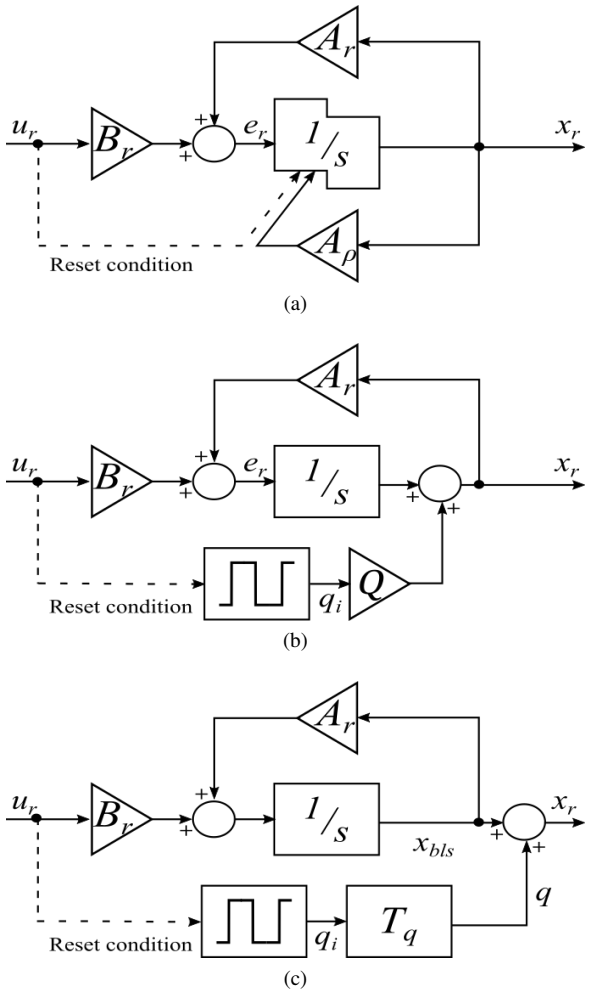


Fig. 2: Block diagram representation of a reset state-space system. a) standard form, b) reset integrator as a sum of linear and nonlinear components, c) entire state-space element as a sum of linear and nonlinear components. Dotted lines indicate signals triggering resets.

In consequence, Q can be found as a solution for a minimization problem

$$\min_{Q} \left\| e^{A_r t}(I - A_\rho)x_{bls}(t_k) - e^{A_r t}A_\rho Q q^*(t_k) + Q q^*(t) \right\|_2^2. \tag{24}$$

For first-order reset elements ($m = 1$), it is sufficient to consider only the before and after-reset state. From (5) we have

$$A_\rho\left(x_{bls}(t_k) + q(t_k)\right) = x_{bls}(t_k) + q(t_k^+), \tag{25}$$

$$(A_\rho - I)x_{bls}(t_k) = Q\left(q^*(t_k^+) - A_\rho q^*(t_k)\right), \tag{26}$$

$$Q = (A_\rho - I)x_{bls}(t_k)\left(q^*(t_k^+) - A_\rho q^*(t_k)\right)^{-1}. \tag{27}$$

To complete the calculation of Q we need to find $x_{bls}(t_k)$ and $q^*(t)$.

The response of the base-linear system at a reset instant is given by

$$x_{bls}(t_k) = e^{A_r t} x_{bls}(0) + \int_0^t e^{A_r(t-\tau)} B_r a \sin(\omega\tau)\,\mathrm{d}\tau = \begin{cases} (A_r^2 + \omega^2 I)^{-1} B_r a\omega, & \text{for } k = 2n+1 \\ -(A_r^2 + \omega^2 I)^{-1} B_r a\omega, & \text{for } k = 2n+2. \end{cases} \tag{28}$$

As we mentioned earlier, q^* can be interpreted as a response of T_q with $Q = I$ to q_i. T_q can be represented in the state-space form by the quadruple (ABCD-matrices) (A_r, A_r, I, I). Note, that T_q has a direct feedthrough term. We calculate response of a linear system T_q to a square wave q_i

$$q^*(t) = e^{A_r t} q^*(0) + \int_0^t e^{A_r(t-\tau)} A_r q_i(\tau)\,\mathrm{d}\tau + q_i(t). \tag{29}$$

At the steady state, the term related to $q^*(0)$ disappears. q_i consists of a constant and a variable component. The contribution due to the constant component is

$$\int_0^t e^{A_r(t-\tau)} A_r \bar{q}_i\,\mathrm{d}\tau = (e^{A_r t} - I)\bar{q}_i \approx -\bar{q}_i \text{ (for large } t). \tag{30}$$

Consider now $t \in [t_{2n}, t_{2n+1})$. The varying part $q_i(t) - \bar{q}_i$ is a zero-mean square wave with peak amplitude $\hat{q}_i$. We have

$$\int_0^{t_{2n}} e^{A_r(t-\tau)} A_r (q_i(\tau) - \bar{q}_i)\,\mathrm{d}\tau = 0 \tag{31}$$

The response for $t \in [t_{2n}, t_{2n+1})$ can be calculated as a response to step with magnitude $\hat{q}_i$, because the constant components $\bar{q}_i$ cancel out. For this, we define the state of T_q to be x_q, such that $q(t)^* = I x_q(t) + I q_i(t)$. The initial condition is $x_q(t_{2n})$. At the steady state we have $x_q(t_{2n}) = -x_q(t_{2n+1})$. By comparing these values we get

$$-e^{A_r \frac{\pi}{\omega}} x_q(t_{2n+1}) + (e^{A_r \frac{\pi}{\omega}} - I)\hat{q} = x_q(t_{2n+1}) \tag{32}$$

$$(e^{A_r \frac{\pi}{\omega}} + I) x_q(t_{2n+1}) = (e^{A_r \frac{\pi}{\omega}} - I)\hat{q}_i \tag{33}$$

$$x_q(t_{2n+1}) = (e^{A_r \frac{\pi}{\omega}} + I)^{-1}(e^{A_r \frac{\pi}{\omega}} - I)\hat{q}_i \tag{34}$$

$$q^*(t_{2n+1}) = \left((e^{A_r \frac{\pi}{\omega}} + I)^{-1}(e^{A_r \frac{\pi}{\omega}} - I) + I\right)\hat{q}_i \tag{35}$$

Following the step response logic, the after reset value $q^*(t_{2n+1}^+)$ is given by

$$q^*(t_{2n+1}^+) = x_q(t_{2n+1}) - I\hat{q}_i. \tag{36}$$

■

To illustrate the Theorem 3.2, Fig. 1b shows a steady-state response of a FORE. The FORE can be represented as a feedback system around the reset integrator, whose response is presented in Fig. 1a. In consequence, the nonlinear components q presented in both figures are related by T_q.

The following corollaries are a consequence of the fact that an ideal square wave can be represented as an infinite sum of sinusoidal waves

$$q_i(t) = \bar{q}_i + \hat{q}_i \frac{4}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t)\ldots\right). \tag{37}$$

Corollary 1: The k-th harmonic of the nonlinear contribution q in the reset system (1) is given by

$$q_k(\omega) = \begin{cases} \frac{4}{k\pi} C_r Q (jk\omega I - A_r)^{-1} jk\omega \hat{q}_i J_{m,1} & \text{for odd } k \\ 0 & \text{for even } k. \end{cases} \tag{38}$$

Corollary 2: The Higher-order Sinusoidal-input Describing function (HOSIDF) of the reset system (1) is given by

$$H_k(\omega) = \begin{cases} C_r (j\omega I - A_r)^{-1} B_r + D_r + q_1(\omega) & \text{for } k = 1 \\ q_k(\omega) & \text{for } k \geq 2 \end{cases} \tag{39}$$

Corollaries 1 and 2 give a clear insight into the behaviour of HOSIDF for reset elements.

Remark 2: For a reset integrator, the influence of a higher-order harmonic on the behaviour of the element is inversely proportional to the order of the harmonic.

Nonlinearities of a general reset system and a corresponding reset integrator are related by a linear transfer function T_q, which depends on the state matrix A_r of the base linear system of the system and the reset matrix A_ρ. In consequence, the influence of a particular harmonic can be amplified or diminished.

Remark 3: For a uniformly convergent reset element, the sum of HOSIDF is always finite, as they compose the nonlinearity q.

IV. Examples

In this section we will illustrate the equivalence of the introduced representation of reset systems with the standard models. To show that the introduced methodology represents the steady-state behaviour of a reset system correctly, we compare the HOSIDF derived in Corollaries 1 and 2 with ones derived using the original method introduced by [14] and [15].

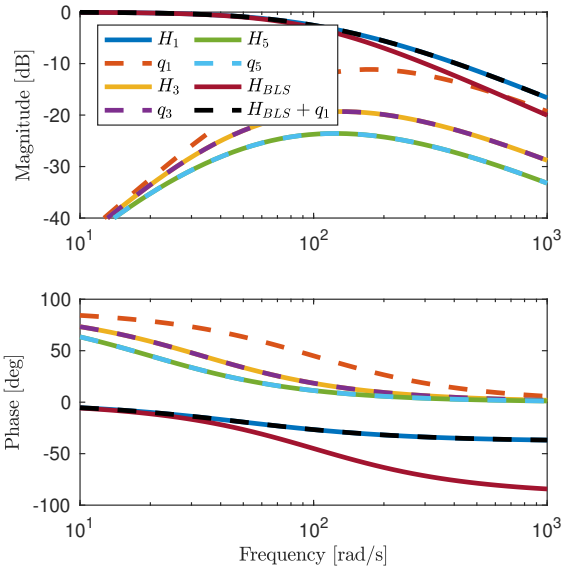


Fig. 3: HOSIDF H_j and harmonics of the nonlinearity q_j for the FORE element.

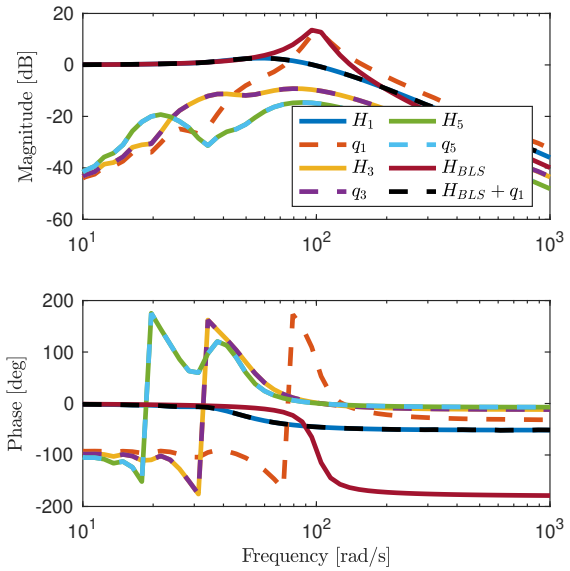


Fig. 4: HOSIDF H_j and harmonics of the nonlinearity for the SORE element.

Figures 3 and 4 compare the HOSIDF and the harmonics of the nonlinear component q of FORE and SORE. The differences between the 1st-order describing function, 1st harmonic of the nonlinear contribution and the transfer function of the base-linear system can be clearly seen. The 1st-order HOSIDF is recreated by summing the transfer function of the BLS and the 1st harmonic of q. As expected, the higher-order harmonics of q and HOSIDF are equivalent.

V. Conclusion

In this paper, we introduced a new representation for the steady-state responses of open-loop reset systems. It provides an intuitive explanation for HOSIDF of reset systems and highlights the influence of nonlinearity on the behaviour of the reset system. The insights from this work can be applied for the shaping of nonlinearities of reset elements. While this paper considers only reset elements in open-loop, the same approach may be taken in the case of closed-loop systems if appropriate assumptions about the reset instants are made [15].

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