

Nonlocal resistance oscillations near the superconducting transition

L. I. Glazman

*Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota,
116 Church Street S.E., Minneapolis, Minnesota 55455*

F. W. J. Hekking*

Department of Applied Physics, Delft University of Technology, 2628 CJ Delft, The Netherlands

A. Zyuzin

Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221

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It is shown that the current response to an electric field is strongly nonlocal slightly above the superconducting transition temperature T_c . The length scale for nonlocality is defined by the correlation length for superconducting fluctuations $\xi(T)$ and diverges at T_c . This makes it possible to observe the nonlocal part of the conductance in a multiterminal measurement on a sample of size $2\pi R \sim \xi(T)$. The local part, originating from the Aslamazov-Larkin correction to the conductivity, exceeds usual mesoscopic interference effects near T_c . We use a simple approach (the time-dependent Ginzburg-Landau equation) to calculate the nonlocal resistances for a ring geometry. We predict that the ratio of voltages measured by two different sets of probes attached to the same ring should oscillate as a function of the flux with a period equal to the "superconducting" flux quantum. This is strikingly different from the known Aharonov-Bohm effect for rings made of a "dirty" normal metal, where such a ratio should be flux independent.

I. INTRODUCTION

The effects of fluctuations of the superconducting order parameter in the vicinity of the transition temperature have been studied intensively, both experimentally and theoretically, for more than two decades.^{1,2} The usual way of an experimental investigation of these fluctuations consists of measurements of the excess conductivity σ' , determined as the difference between the linear extrapolation of $\sigma(T)$ down from high temperatures, and the actual value of the conductivity $\sigma(T)$ in the vicinity of the transition to the superconducting state. Applied to a macroscopic sample,³ these measurements give information about the magnitude of the fluctuations, but not about their spatial structure. The correlation length $\xi(T)$ of the fluctuations diverges at the transition temperature. Thus we can expect an increasing nonlocality of the conductance, while the temperature T approaches the critical value T_c : Current density at a given point \mathbf{r} depends on electric fields existing on distances $\sim \xi(T)$ from this point.

Experimentally, the manifestation of a nonlocal conductance should be expected if contacts to a sample are separated by distances of the order of $\xi(T)$. This determines the main requirement for sample fabrication. Recent developments of patterning techniques allow for the preparation of networks from superconducting metals out of building blocks with a submicron size.^{4,5} Reference 6 demonstrates the possibility to produce a one-dimensional wire with an interprobe distance, which is comparable with $\xi(T)$ at temperatures separated reasonably well from T_c . We believe that manufacturing of

samples with similar sizes and suitable for multiterminal measurements is also possible now.

In this paper we analyze the nonlocal paraconductance for a ring threaded by a magnetic flux. We propose a multiterminal measurement that allows to single out the *nonlocal* part of the conductance. Voltages V_α , V_β measured by two different sets of probes attached to the ring oscillate due to the changing flux. These oscillations contain a "usual" part corresponding to the modulation of the *local* conductivity (the well-known Aharonov-Bohm effect⁷) as well as a new nonlocal part. The modulation of the local conductivity obviously does not affect the ratio V_β/V_α , that will be determined solely by the sample geometry (see Fig. 1) and Kirchof's rule. The nonlocal part of the conductivity, however, makes this ratio oscillate, because this is the part that violates Kirchof's rule. Nonlocality and resulting deviations from Kirchof's rule

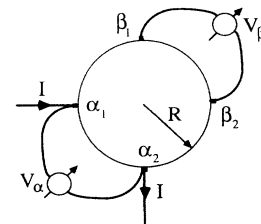


FIG. 1. Four-terminal setup for a ring of radius R . Two current probes are connected to the ring at angles α_1 and α_2 , injecting a current I ; this leads to a voltage difference V_α between these probes. The voltage difference V_β is measured between voltage probes at angles β_1 and β_2 .

depend on the ratio $R/\xi(T)$ that changes with temperature (R is the radius of the ring). We calculate the oscillatory parts of the paraconductance and of the ratio V_β/V_α , in the limits of both large and small $2\pi R/\xi(T)$. Observation of the proposed oscillations would give a clear manifestation of the nonlocal paraconductance.

Our calculations account for the order-parameter fluctuations in the lowest order of perturbation theory, giving the nonlocal contribution to the paraconductance. In the next section, we show that this contribution originates only from the Aslamazov-Larkin (AL) part of the correction to the conductivity,⁸ corresponding to the most divergent diagram in the second order of perturbation theory. The first-order correction, i.e., the well-known Maki-Thompson (MT) part of the paraconductivity,⁹ gives only a local enhancement of the conductivity.

The AL contribution to the paraconductance can be easily accounted for in the framework of the time-dependent Ginzburg-Landau (TDGL) theory.¹ This approach is especially convenient for our problem, as it simplifies obtaining results in spatially-confined and multiterminal geometries. In Sec. III we present the calculation of the nonlocal fluctuation conductivity for a ring, along the same lines as the treatment given in Chapter 7 of Ref. 1. Next we discuss the four-terminal setup in Sec. IV, followed by a calculation of the voltage ratio in Sec. V. There we apply the obtained formulas to a current-biased ring placed in a magnetic field. We calculate voltages V_α and V_β measured by two sets of probes, and the ratio V_β/V_α . The flux dependence of this ratio demonstrates the paraconductance nonlocality. At temperatures near T_c , when $\xi(T) \gtrsim 2\pi R$ (Sec. V A), the paraconductance correction is proportional to $[T - T_c^*]^{-1}$, where T_c^* is the transition temperature suppressed by the magnetic flux (Little-Parks effect, see Ref. 1). In this case the local and nonlocal parts of the AL correction are of the same order of magnitude. Both parts oscillate when the flux is changed. Away from T_c (Sec. V B), if the temperature-dependent coherence length is small enough such that $\xi(T) \lesssim 2\pi R$, the nonlocal correction still has a periodic flux dependence, but of exponentially small amplitude, $\sim \exp[-2\pi R/\xi(T)]$. We discuss our results in Sec. VI. In particular, we analyze there the possibility to distinguish the predicted oscillations from the mesoscopic fluctuations occurring in normal rings due to the coherent motion of free electrons.¹⁰

II. FLUCTUATION CONDUCTIVITY IN ONE-DIMENSIONAL SUPERCONDUCTORS

It is well-known^{1,2} that the normal conductivity σ_{norm} of a superconductor at a temperature T above T_c is enhanced by fluctuations of the order parameter. In a microscopic theory,¹¹ a systematic perturbation expansion up to second order in the fluctuation propagator yields two different contributions to this excess conductivity σ' .

Among all possible *second-order* contributions there is one that is most divergent at T_c . This term is known as the Aslamazov-Larkin (AL) correction,⁸ which describes the conductivity of a fluctuating pair of electrons. Intro-

ducing the reduced temperature $\tau = (T - T_c)/T_c$, we can write the AL correction for the one-dimensional conductivity¹² of a wire as²

$$\sigma'_{\text{AL}} = \frac{\pi}{16} \frac{e^2 \xi(0)}{\hbar} \tau^{-3/2}, \quad (1)$$

where $\xi(0)$ is the zero-temperature coherence length.¹³

The *first-order* contributions are known as the Maki-Thompson (MT) correction,⁹ which describes the additional scattering of electrons by a superconducting fluctuation. In one and two dimensions, this contribution is divergent. The singularity can be removed by introducing a low-energy cutoff, which corresponds physically to the presence of a pair-breaking interaction. In one dimension, the MT term is given by²

$$\sigma'_{\text{MT}} = 4\sigma'_{\text{AL}} \frac{\tau}{\delta} \frac{1}{1 + (\tau/\delta)^{1/2}}, \quad (2)$$

where the pair-breaking parameter δ can be expressed in terms of the phase-breaking length L_ϕ :

$$\delta = \frac{\xi^2(0)}{L_\phi^2}. \quad (3)$$

An important difference between the AL and MT contributions, which is due to their different origin, is the length scale on which they are nonlocal. This is easy to understand by analyzing the corresponding contributions diagrammatically. The AL part is given by the diagram⁸ of Fig. 2(a); a typical diagram for the MT part⁹ is shown in Fig. 2(b). We see that the field vertex and the current vertex in Fig. 2(a) are linked via two fluctuation propagators only. The typical length scale of these ($\xi(T)$) determines the nonlocality of the current response to an applied electrical field—the AL correction describes the excess current carried by superconducting pairs of a

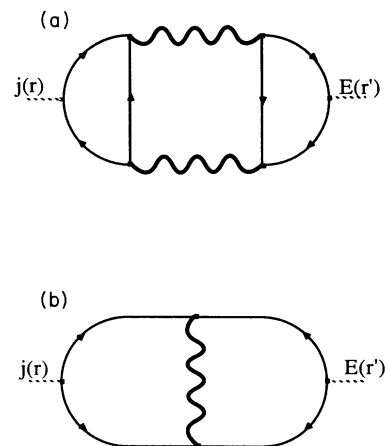


FIG. 2. Conductivity diagrams for superconducting fluctuations. Bold lines denote (impurity-averaged) electron propagators, a wavy line the fluctuation propagator. The AL diagram (a) connects current and field vertex [$J(r)$ and $E(r')$, respectively] by fluctuation propagators only, leading to nonlocality over $\xi(T)$. The MT diagram (b) connects these vertices by impurity-averaged electron propagators, leading to nonlocality over elastic mean free path l_e .

characteristic size $\xi(T)$. The MT correction describes the excess current carried by normal electrons, which are scattered by a fluctuating pair, see Fig. 2(b). However, normal electrons are scattered by impurities as well. As a result, averaged over impurity positions, the electron propagators that correspond to the lines connecting the field and the current vertices in Fig. 2(b) decay on the length scale given by the elastic mean free path l_e , which is much shorter than $\xi(T)$ in the vicinity of the transition temperature. Thus in a sample of a size that is comparable with $\xi(T)$, the fluctuation conductivity, averaged over impurities, will consist of a local contribution due to MT and a nonlocal contribution due to AL. By comparing Eqs. (2) and (1) we see that the AL correction gives the largest contribution to the conductivity close to T_c , provided there is enough pair-breaking, such that $\tau/\delta < 1$, whereas away from T_c the MT correction becomes more important. On the other hand, close to T_c the superconducting fluctuations become strong, and the perturbative results given above will cease to be valid. A lower bound to the allowed values of $T - T_c$ can be obtained by requiring that the larger correction (σ'_{AL}) still be smaller than the normal conductivity σ_{norm} . Substituting for σ_{norm} the well-known Drude conductivity,¹⁴ we obtain the requirement

$$\frac{\sigma'_{AL}}{\sigma_{\text{norm}}} = \frac{3\pi^3}{8} \frac{\xi(0)}{l_e} \frac{1}{k_F^2 S} \tau^{-3/2} < 1. \quad (4)$$

Here S is the cross-sectional area of the one-dimensional wire.

Since we will be interested in the nonlocal AL correction in the rest of this paper, we thus will mainly consider temperatures that are close to T_c , such that the sample size $2\pi R$ is smaller or, at least, does not exceed greatly the coherence length $\xi(T)$.

Making R smaller, one can get eventually into the regime at which $2\pi R \sim L_\phi$. In this limit, the mesoscopic part of the correction to the conductance δG_{meso} (i.e., the part representing interference between normal electrons) inevitably brings nonlocality into the conductance. An estimate δG_{meso} at $L_\phi \lesssim 2\pi R$ is¹⁵

$$\delta G_{\text{meso}} \simeq \frac{\sqrt{3}}{\pi} \frac{e^2}{\hbar} \left[\frac{L_\phi}{2\pi R} \right]^{3/2} \left[1 + \frac{9}{2\pi} \frac{1}{\delta} \right]^{-1/2}. \quad (5)$$

In order to exclude consideration of these fluctuations, it is enough to assume that there is a sufficiently wide interval between $\xi(T)$ and L_ϕ ,

$$\xi(T) \gtrsim 2\pi R \gg L_\phi. \quad (6)$$

This is the most strongly formulated requirement on R , that allows simply to neglect the *overall* mesoscopic correction to the sample conductance, while the nonlocal AL correction is large. In Sec. VI, we will show that this requirement can be relaxed, and that even for $2\pi R \sim L_\phi \gtrsim \xi(T)$, the AL nonlocal term is the leading one in the nonlocal resistance, although it is exponentially small in the parameter $2\pi R / \xi(T)$.

Condition (6) simultaneously means that the magnitude

of the AL correction is larger than the MT one. So, in the range of temperatures

$$\left[\frac{3\pi^3}{8} \frac{\xi(0)}{l_e} \frac{1}{k_F^2 S} \right]^{2/3} < \tau < \frac{\xi^2(0)}{L_\phi^2}, \quad (7)$$

we can neglect the MT contribution to the excess conductivity and also satisfy the condition for small fluctuations.

We finally note that the AL correction can be accounted for in a phenomenological theory as well, by adding an equation of motion for the order parameter to the static Ginzburg-Landau equation.¹⁶ The resulting time dependent Ginzburg-Landau (TDGL) theory is the formalism we will use in the remainder of this paper.

III. NONLOCAL FLUCTUATION CONDUCTIVITY FOR A SUPERCONDUCTING RING

The nonlocal relation between current density and electric field replacing the usual Ohm's law, can be expressed by the equation

$$J_i(\mathbf{r}) = \int d\mathbf{r}' \sigma_{ij}(\mathbf{r}, \mathbf{r}') E_j(\mathbf{r}'), \quad (8)$$

where $\sigma_{ij}(\mathbf{r}, \mathbf{r}')$ is the nonlocal conductivity tensor.

As we discussed in the previous section, we will be interested in the small nonlocal correction σ' to the normal conductivity σ_{norm} , due to fluctuations of the order parameter ψ . Since the normal conductivity is a local, isotropic quantity on the lengthscales we are dealing with, we can write the total conductivity tensor as

$$\sigma_{ij}(\mathbf{r}, \mathbf{r}') = \sigma_{\text{norm}} \delta_{ij} \delta(\mathbf{r} - \mathbf{r}') + \sigma'_{ij}(\mathbf{r}, \mathbf{r}'). \quad (9)$$

We calculate the correction σ' with the help of the Kubo formula

$$\sigma'_{ij}(\mathbf{r}, \mathbf{r}') = \lim_{\omega \rightarrow 0} \frac{1}{k_B T} \int dt \cos(\omega t) \langle J_i(\mathbf{r}, 0) J_j(\mathbf{r}', t) \rangle, \quad (10)$$

which relates the elements of the conductivity tensor to the correlation function of the components of the operator for the supercurrent $\mathbf{J}(\mathbf{r})$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{m^*} \left[\psi^* \left[\frac{\nabla}{i} - \frac{2\pi \mathbf{A}}{\Phi_0} \right] \psi - \psi \left[\frac{\nabla}{i} + \frac{2\pi \mathbf{A}}{\Phi_0} \right] \psi^* \right]. \quad (11)$$

Here, e is the electron charge, m^* its effective mass, \mathbf{A} is the vector potential, and Φ_0 the flux quantum.

Consider a superconducting ring of radius R , kept at a temperature T above the transition temperature T_c . A magnetic field H is applied, perpendicular to the ring, which corresponds to a flux $\Phi = \pi R^2 H$ through the ring. Positions along the ring are measured by an angle ϕ , which is equivalent to the formal change of variable

$$\mathbf{r} \rightarrow R \phi \mathbf{e}_\phi, \quad (12)$$

where \mathbf{e}_ϕ is the unit vector in the angular direction. We expand the order parameter ψ in terms of eigenfunctions ψ_n :

$$\psi = \sum_n c_n(t) \psi_n(\phi), \quad (13)$$

which satisfy the linearized TDGL equation² if we are far enough above T_c , corresponding to the temperature range (7),

$$\frac{\hbar^2}{2m^*R^2} \left[\left(\frac{\partial}{i\partial\phi} - \frac{\Phi}{\Phi_0} \right)^2 + \frac{R^2}{\xi^2(\tau)} \right] \psi_n = \epsilon_n \psi_n. \quad (14)$$

The vector potential has been chosen to be $\mathbf{A} = (1/2)HR\mathbf{e}_\phi$. The temperature-dependent coherence length is given by $\xi(\tau) = \xi(0)\tau^{-1/2}$, where $\xi(0)$ is the zero-temperature coherence length¹³ and $\tau \equiv (T - T_c)/T_c$.

The solution of Eq. (14) reads

$$\begin{aligned} \psi_n(\phi) &= \frac{1}{\sqrt{2\pi R}} e^{in\phi}, \\ \epsilon_n &= \frac{\hbar^2}{2m^*R^2} \left[\left(n - \frac{\Phi}{\Phi_0} \right)^2 + \frac{R^2}{\xi^2(\tau)} \right], \end{aligned} \quad (15)$$

The normalization of ψ_n has been chosen such that $|\psi|^2$ has the dimension of a density.

The order parameter ψ satisfies the linearized TDGL

equation,² which leads to an equation for the time-dependent coefficients $c_n(t)$ and hence for the correlator $\langle c_n^*(0)c_n(t) \rangle$:

$$\begin{aligned} \frac{\partial}{\partial t} \langle c_n^*(0)c_n(t) \rangle \\ = -\frac{1}{t_0} \left[1 + \frac{\xi^2(\tau)}{R^2} \left(n - \frac{\Phi}{\Phi_0} \right)^2 \right] \langle c_n^*(0)c_n(t) \rangle. \end{aligned} \quad (16)$$

The temperature-dependent relaxation time $t_0 \equiv \pi\hbar/8k_B(T - T_c)$, the n th mode decays with time

$$t_n = t_0 / [1 + (\xi^2(\tau)/R^2)(n - \Phi/\Phi_0)^2].$$

The modes decay more rapidly with increasing mode number (energy).

Equation (16) is solved under the boundary condition $|c_n(0)|^2 = k_B T / \epsilon_n$, imposed by the thermal character of the fluctuations. The solution reads

$$\langle c_n^*(0)c_n(t) \rangle = \frac{k_B T}{\epsilon_n} \exp(-t/t_n). \quad (17)$$

Substituting the expansion (13) in the expression (11) for the current, we can calculate the current-current correlator appearing in the Kubo formula (10)

$$\langle J(\phi, 0)J(\phi', t) \rangle = \left(\frac{e\hbar}{m^*R^2} \right)^2 \sum_{n, m, n', m'} (2n + m - 2\Phi/\Phi_0)(2n' + m' - 2\Phi/\Phi_0) e^{im\phi + im'\phi'} \langle c_n^*(0)c_{n+m}(0)c_{n'}^*(t)c_{n'+m'}(t) \rangle. \quad (18)$$

Since the problem is one dimensional, we dropped the subscripts corresponding to spatial components. The coefficients c_n are statistically independent, therefore the only nonzero contribution from the correlator in the right-hand side of Eq. (18) is obtained for $n' = n + m, m' = -m$. Using Eq. (17), we find in the zero-frequency limit

$$\sigma'(\phi, \phi') = \frac{1}{2\pi R} \sum_m e^{im(\phi - \phi')} \sigma'_m, \quad (19)$$

where

$$\sigma'_m = \frac{e^2}{2\hbar} \frac{1}{\tau} \frac{R}{m^2} \left[\sum_n \frac{1}{1 + (\xi(\tau)/R)^2 (n - \Phi/\Phi_0)^2} - \sum_n \frac{2}{2 + (\xi(\tau)/R)^2 \{ (n - \Phi/\Phi_0)^2 + (n + m - \Phi/\Phi_0)^2 \}} \right]. \quad (20)$$

The coefficients σ'_m depend periodically on the flux Φ/Φ_0 .

It is a peculiarity of one dimensionality that the component $\sigma(\phi, \phi')$ of the nonlocal conductivity tensor has the dimension of conductance Ω^{-1} . However, its Fourier components σ'_m have the dimension $\text{cm}\Omega^{-1}$ of a true one-dimensional conductivity if the Fourier decomposition of ϕ -dependent quantities on the ring is defined by

$$f(\phi) = \frac{1}{2\pi R} \sum_{m=-\infty}^{\infty} f_m e^{im\phi}; \quad f_n = R \int_0^{2\pi} d\phi f(\phi) e^{-in\phi}, \quad (21)$$

as we anticipated in Eq. (19).

We proceed by splitting up the quadratic factors in Eq. (20) into linear terms of the form $1/(z - n)$. The sum over n can be performed by noting that

$$\sum_{n=-\infty}^{\infty} \frac{1}{z - n} = \pi \cot \pi z. \quad (22)$$

We find

$$\sigma'_m = \frac{e^2}{2\hbar} \frac{1}{\tau} \frac{1}{m^2} \frac{R^3}{\xi^2(\tau)} \left[\frac{\pi(\xi(\tau)/R) \sinh 2\pi R / \xi(\tau)}{\cosh 2\pi R / \xi(\tau) - \cos 2\pi(\Phi/\Phi_0)} - \frac{2\pi}{[m^2 + 4R^2/\xi^2(\tau)]^{1/2}} \frac{\sinh \pi[m^2 + 4R^2/\xi^2(\tau)]^{1/2}}{\cosh \pi[m^2 + 4R^2/\xi^2(\tau)]^{1/2} - \cos \pi(m - 2\Phi/\Phi_0)} \right]. \quad (23)$$

In the next section we will express the voltage along the ring in terms of the conductivity Fourier components σ'_m .

IV. VOLTAGE IN A FOUR-TERMINAL GEOMETRY

Consider the geometry depicted in Fig. 1. Two current probes are connected to the ring, such that a current I is injected into the ring at angle α_1 and taken out at α_2 . We want to calculate the resulting voltage distribution $V(\phi)$ along the ring. We consider the current and voltage probes, introduced in this section, to be noninvasive, and neglect their influence on the fluctuating order parameter. The induced total current (that includes both the normal part and the part caused by superconducting fluctuations) on the ring $J_t(\phi)$ satisfies the continuity equation

$$\frac{\partial J_t(\phi)}{\partial \phi} = I[\delta(\phi - \alpha_1) - \delta(\phi - \alpha_2)]. \quad (24)$$

This equation expresses current conservation at the nodes α_1 and α_2 . We solve it by substituting the Fourier decomposition of the current

$$J_t(\phi) = \frac{1}{2\pi R} \sum_m e^{im\phi} J_m,$$

which yields a relation for the Fourier coefficients J_m

$$J_m = \frac{IR}{im} (e^{-im\alpha_1} - e^{-im\alpha_2}), \quad (25)$$

valid for all $m \neq 0$. Using the nonlocal relation between current and electric field $E(\mathbf{r}) = -\partial V(\mathbf{r})/\partial \mathbf{r}$,

$$J_t(\phi) = - \int_0^{2\pi} d\phi' \sigma(\phi - \phi') \frac{\partial V(\phi')}{\partial \phi'},$$

we find the Fourier coefficients V_m

$$V_m = \frac{IR^2}{m^2 \sigma_m} (e^{-im\alpha_1} - e^{-im\alpha_2}) \quad (26)$$

of the voltage $V(\phi)$ along the ring, which can now be calculated by summing the corresponding Fourier series. Here, σ_m are the Fourier coefficients of the *total* nonlocal conductivity, given by

$$\sigma_m = \sigma_{\text{norm}} + \sigma'_m. \quad (27)$$

The voltage is determined up to a constant, corresponding to the term V_m with $m=0$, which is unimportant if voltage *differences* are considered.

Suppose that, in addition to the current probes, two voltage probes are attached to the ring at angles β_1 and β_2 (see Fig. 1). The voltage, measured between these probes can be calculated, using (26)

$$V_\beta \equiv V(\beta_1) - V(\beta_2) = - \frac{IR}{2\pi} \sum_{m \neq 0} \frac{(e^{im\beta_2} - e^{im\beta_1})(e^{-im\alpha_1} - e^{-im\alpha_2})}{m^2 \sigma_m}. \quad (28)$$

Since we assume that the correction σ' is small, we expand

$$\frac{1}{\sigma_m} = \frac{1}{\sigma_{\text{norm}}} - \frac{\sigma'_m}{\sigma_{\text{norm}}^2}.$$

We rewrite the voltage difference (28) as

$$V_\beta = - \frac{IR}{\pi \sigma_{\text{norm}}} \sum_{m=1}^{\infty} \frac{1}{m^2} \left[1 - \frac{\sigma'_m}{\sigma_{\text{norm}}} \right] \times [\cos m(\beta_2 - \alpha_1) + \cos m(\beta_1 - \alpha_2) - \cos m(\beta_1 - \alpha_1) - \cos m(\beta_2 - \alpha_2)]. \quad (29)$$

For completeness, we also give the voltage between the nodes α_1 and α_2 ,

$$V_\alpha \equiv V(\alpha_1) - V(\alpha_2) = \frac{2IR}{\pi \sigma_{\text{norm}}} \sum_{m=1}^{\infty} \frac{1}{m^2} \left[1 - \frac{\sigma'_m}{\sigma_{\text{norm}}} \right] \times [1 - \cos m(\alpha_2 - \alpha_1)]. \quad (30)$$

In the next sections, we will discuss the behavior of (29) and (30), as well as their ratio, both near T_c and in the limiting case $\xi(\tau)/2\pi R \lesssim 1$.

V. CALCULATION OF THE VOLTAGE RATIO

A. Nonlocal corrections near T_c

At temperatures T near T_c we can expand the result (23) in powers of the small parameter $2\pi R/\xi(\tau)$. Keep-

ing terms up to second order, we approximate the first term in brackets as

$$\frac{\pi(\xi(\tau)/R) \sinh(2\pi R/\xi(\tau))}{\cosh(2\pi R/\xi(\tau)) - \cos(2\pi(\Phi/\Phi_0))} \approx \frac{2\pi^2 + (4\pi^4/3)(R/\xi(\tau))^2}{1 - \cos(2\pi\Phi/\Phi_0) + 2\pi^2(R/\xi(\tau))^2}. \quad (31)$$

It diverges whenever T approaches a temperature T_c^* , given by

$$T_c^*(\Phi) = T_c \left[1 - \frac{\xi^2(0)}{2\pi^2 R^2} [1 - \cos(2\pi\Phi/\Phi_0)] \right], \quad (32)$$

which is the transition temperature, suppressed by the magnetic flux. The second term in (23) is nondivergent, so close to $T_c^*(\Phi)$ we approximate it by putting $R/\xi(\tau) = 0$. We find for σ'_m

$$\sigma'_m \approx \frac{e^2}{2\hbar} \frac{R}{m^2} \frac{T_c}{T - T_c^*(\Phi)} \left[1 + \frac{2\pi^2 R^2}{3\xi^2(0)} \frac{T - T_c}{T_c} \right] - \frac{e^2}{2\hbar} \frac{R}{m^3} \frac{2\pi R^2}{\xi^2(0)} \frac{\sinh(\pi m)}{\cosh(\pi m) - \cos\pi[(m - 2\Phi/\Phi_0)]}. \quad (33)$$

The first term in this result has a simple $(1/m^2)$ dependence on m , therefore the summation over m occurring in Eqs. (29) and (30) can be done immediately for this part.²⁰ The sum over m corresponding to the nondivergent part converges rapidly (as $1/m^5$), so it can be easily estimated by taking only a few terms into account.

We will discuss a symmetric set up (see also Fig. 1): The current probes are attached to the ring at angles $\alpha_1 = 0$ and $\alpha_2 = \pi/2$; the voltage probes at angles $\beta_2 = \pi$ and $\beta_1 = 3\pi/2$. Substituting these values for the angles in Eq. (29) and performing the summation over m as indicated above, we find for the voltage difference V_β

$$V_\beta \approx \frac{I\mathcal{R}_n}{16} \left\{ 1 - \frac{7\pi}{192} \frac{e^2}{\hbar} \mathcal{R}_n \frac{T - T_c}{T_c^*}(\Phi) \left[1 + \frac{2\pi^2 R^2}{3\xi^2(0)} \frac{T - T_c}{T_c} \right] + \frac{8}{\pi^2} \frac{e^2}{\hbar} \mathcal{R}_n \frac{R^2}{\xi^2(0)} \left[\frac{\sinh\pi}{\cosh\pi + \cos(2\pi\Phi/\Phi_0)} - 0.06 \right] \right\}. \quad (34)$$

We introduced here the normal resistance of the ring $\mathcal{R}_n = 2\pi R / \sigma_{\text{norm}}$.

The first term is due to the normal conductivity, which is local and therefore contributes the constant term $I\mathcal{R}_n/16$, which is determined only by the geometry: It is the voltage drop over 1/4 of the ring, through which 1/4 of the total current flows. The term that diverges for $T \rightarrow T_c^*(\Phi)$ reflects the main nonlocal contribution. It depends both on the temperature and on the flux via $T_c^*(\Phi)$. At $T = T_c$, this term is of the same order of the nondivergent term. The latter is obtained approximately by extracting the term $m = 1$ out of the sum over m , and performing the remaining part numerically, which is almost independent of the flux and gives the constant factor 0.06 in (34). As can be seen, the $m = 1$ term accounts for about 90% of the complete nondivergent part, due to the rapid convergence of the corresponding series. The divergent term is a small correction to the normal part as long as $[T - T_c^*(\Phi)]/T_c > e^2 \mathcal{R}_n / \hbar$, which also implies that it is still more important than the nondivergent term. Along the same lines we calculated the voltage difference V_α . This difference has the same structure of the divergent and nondivergent corrections, but the corresponding numerical prefactors are different.

Next we will study the ratio of both voltage differences

$$\frac{V_\beta}{V_\alpha} \approx \frac{1}{3} - \frac{\pi}{144} \frac{e^2}{\hbar} \mathcal{R}_n \frac{T - T_c}{T_c^*}(\Phi) \left[1 + \frac{2\pi^2 R^2}{3\xi^2(0)} \frac{T - T_c}{T_c} \right] + \frac{16}{9\pi^2} \frac{e^2}{\hbar} \mathcal{R}_n \frac{R^2}{\xi^2(0)} \frac{\sinh\pi}{\cosh\pi + \cos(2\pi\Phi/\Phi_0)}, \quad (35)$$

where we kept only the first term of the nondivergent part. This equation clearly demonstrates the nonlocality of the paraconductance. The voltage V_β is smaller than the geometry-determined value $V_\alpha/3$, and oscillates when the flux is varied. In the absence of superconducting fluctuations, voltages $V(\alpha_1) - V(\beta_1)$ and $V(\beta_2) - V(\alpha_2)$ would be also equal to $V_\alpha/3$. Due to the nonlocality, these voltages are larger than $V_\alpha/3$ and oscillate with the changing flux as well. The phases of oscillations for these voltages and for V_β are *opposite* to each other.

We believe that the main part of the correction [the most divergent term in (35)] can be extracted from a set of measurements of the ratio (35) as a function of T at a constant flux. In order to extract the smaller nondivergent part, we propose to keep the *two-terminal resistance* $\mathcal{R}_\alpha = V_\alpha/I$ constant instead of a constant temperature.

The ratio (35), expressed in terms \mathcal{R}_α and flux Φ , takes the form

$$\frac{V_\beta}{V_\alpha} = -\frac{1}{9} + \frac{64}{27} \frac{\mathcal{R}_\alpha}{\mathcal{R}_n} + \frac{16}{27\pi^2} \frac{e^2}{\hbar} \mathcal{R}_n \frac{R^2}{\xi^2(0)} \frac{\sinh\pi}{\cosh\pi + \cos(2\pi\Phi/\Phi_0)}, \quad (36)$$

which is convenient for a measurement of the nondivergent part as a function of flux.

B. Nonlocal corrections away from T_c

Away from T_c , in the limit $\xi(T)/(2\pi R) \lesssim 1$, we can approximate (23) by

$$\sigma'_m = \frac{e^2}{2\hbar} \frac{R}{m^2} \frac{\pi R}{\xi(0)} \sqrt{1/\tau} \left[1 - \frac{1}{[1 + (\xi^2(\tau)m^2/4R^2)]^{1/2}} + 2e^{-2\pi R/\xi(\tau)} \cos(2\pi\Phi/\Phi_0) \right]. \quad (37)$$

We kept only the leading flux-dependent term, which has an amplitude that decreases exponentially as $\exp[-2\pi R/\xi(\tau)]$ with increasing $T - T_c$. Since we are away from T_c , it is instructive at this point to check whether the local flux-dependent part of the MT correction is still smaller than the *nonlocal* oscillating part of the AL correction, since the conditions discussed in Sec. II were related to the flux-independent values. In order to do this, we calculate the MT term for a ring, following the standard diagrammatic calculation of Ref. 9:

$$\sigma_{\text{MT}} = \frac{e^2}{2\hbar} \frac{\pi}{2} \frac{1}{\tau - \delta} \left[\frac{L_\phi \sinh(2\pi R/L_\phi)}{\cosh(2\pi R/L_\phi) - \cos(2\pi\Phi/\Phi_0)} - \frac{\xi(\tau) \sinh(2\pi R/\xi(\tau))}{\cosh(2\pi R/\xi(\tau)) - \cos(2\pi\Phi/\Phi_0)} \right]. \quad (38)$$

In the case $\tau/\delta \lesssim 1$, we can calculate the voltage difference V_β in the symmetric setup discussed above, using (28), (37), and (38):

$$V_\beta = \frac{I\mathcal{R}_n}{16} \left\{ 1 - \frac{1}{32} \frac{e^2}{\hbar} \mathcal{R}_n \frac{\xi(0)}{R} \left[\frac{1}{\tau} \right]^{3/2} \left[1 + 4 \frac{\tau}{\delta} \right] - \frac{7\pi^2}{96} \frac{e^2}{\hbar} \frac{R}{\xi(0)} \frac{R}{\xi(0)} \mathcal{R}_n \sqrt{1/\tau} e^{-2\pi R/\xi(\tau)} \cos(2\pi\Phi/\Phi_0) \left[1 + \frac{3}{2\pi^2} \frac{1}{\delta} \right] \right\}. \quad (39)$$

The second term in brackets corresponds to the sum of the usual flux-independent AL and MT corrections.¹⁷ Note that the MT part behaves as $4\tau/\delta$ times the AL part, as given by Eq. (2). In addition we find an oscillating term, due to the nonlocal AL part and the flux-dependent MT part. The latter arises because the fluctuation propagator itself depends periodically on flux in a ring geometry. However, as long as it is local, it does not contribute to the flux dependence of the voltage ratio. This holds also in the opposite limit, $\tau/\delta > 1$, i.e., even in the case, when the main oscillatory part of V_α , V_β , originates from the MT term. The ratio of these voltages oscillates only due to the nonlocal part of the AL correction

$$\frac{V_\beta}{V_\alpha} = \frac{1}{3} - \frac{\pi^2}{72} \frac{e^2}{\hbar} \mathcal{R}_n \frac{R}{\xi(0)} \sqrt{1/\tau} e^{-2\pi R/\xi(\tau)} \cos(2\pi\Phi/\Phi_0). \quad (40)$$

Until now we discussed oscillations of only the impurity-averaged quantities. However, for rings with perimeter $2\pi R \sim L_\phi > \xi(T)$, mesoscopic fluctuations may become important. In the next section, we establish requirements under which the nonlocality of paraconductance remains to be the main source of oscillations in the ratio (40).

VI. CONCLUDING REMARKS

Now that we found the precise form of the corrections due to superconducting fluctuations, we would like to revisit the comparison of the magnitude (40) with the magnitude of the usual mesoscopic fluctuations, which occur as a consequence of the lack of averaging on length scales of the order of L_ϕ .¹⁸ Using Eqs. (5) and (39) we require

$$\frac{\delta G_{\text{meso}}}{\delta G_{\text{AL}}} = \frac{(e^2/\hbar)(L_\phi/(2\pi R))^{3/2}}{(e^2/\hbar)(R/\xi(0))\sqrt{\tau} \exp[-2\pi R/\xi(\tau)]} \leq 1, \quad (41)$$

which leads to the inequality

$$\frac{2\pi R}{\xi(\tau)} \leq \ln \left[\frac{2\pi R^2}{\xi^2(0)} \frac{\xi(\tau)}{2\pi R} \left[\frac{2\pi R}{L_\phi} \right]^{3/2} \right]. \quad (42)$$

We can satisfy (42), even if L_ϕ exceeds $\xi(T)$, since the AL nonlocal correction scales with the perimeter, whereas the mesoscopic fluctuations are limited to the value of e^2/\hbar .

Supposing the logarithm in the right-hand side of (42) to be large, one can treat this inequality iteratively. After the first iteration, one has

$$\frac{2\pi R}{\xi(\tau)} \leq \ln \left[\frac{2\pi R^2}{\xi^2(0)} \left[\frac{2\pi R}{L_\phi} \right]^{3/2} \right]. \quad (43)$$

Taking Al as a possible material to make the ring, we substitute typical numbers, taken from Refs. 6 and 19: $\xi(0) = 0.175 \mu\text{m}$, $L_\phi = 1.5 \mu\text{m}$; the radius R can be $0.5 \mu\text{m}$. Supposing the logarithm in the right-hand side of (42) to be large, and solving (42) iteratively, we find that $\xi(\tau) \geq 0.84 \mu\text{m}$, or equivalently that the reduced temperature $\tau \leq 0.04$, which is experimentally feasible. An estimate of V_β/V_α according (40) with the above parameters and with $\mathcal{R}_n \approx 100 \Omega$, shows that the oscillating part of this ratio has an amplitude $\sim 10^{-4}$. Thus we are able to relax the stronger requirement (6) to include the regime where $\xi(\tau) \leq 2\pi R \approx L_\phi$.

In conclusion we studied the response of a superconducting ring to an electric field. Slightly above the superconducting transition temperature T_c , this response is strongly nonlocal. The length scale for nonlocality is defined by the correlation length for superconducting fluctuations $\xi(\tau)$ and diverges at T_c . The nonlocal part of the conductance can be obtained in a multiterminal measurement on a ring of size $2\pi R$ of order or somewhat exceeding $\xi(\tau)$. This part originates from the Aslamazov-Larkin correction to the conductivity, and near T_c , exceeds known mesoscopic interference effects. We use the time-dependent Ginzburg-Landau equation to calculate the nonlocal resistances in the simplest case, when the influence of leads on these fluctuations are not

taken into account. The latter simplification cannot change our main prediction: Due to the nonlocality, the *ratio* of voltages measured by two different sets of probes attached to the ring should oscillate as a function of the flux quantum.

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*Present address: Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-7500 Karlsruhe, Germany.

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In order to return to usual three-dimensional conductivity, one should divide this quantity by the wire cross-sectional area. We use the above definition of one-dimensional conductivity further throughout this paper.

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