



RESEARCH REPORT H-73-1

WAVE TRANSMISSION THROUGH ROCK STRUCTURES

Hydraulic Model Investigation

by

G. H. Keulegan

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February 1973

Sponsored by Office, Chief of Engineers, U. S. Army

Conducted by U. S. Army Engineer Waterways Experiment Station
Hydraulics Laboratory
Vicksburg, Mississippi

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FOREWORD

This report, which deals with the transmission and reflection characteristics of structures constructed from rocks of uniform size, was prepared as a part of the Civil Works Investigation, Engineering Study 833, sponsored by the Office, Chief of Engineers.

The experiments relating to the transmission of waves through structures constructed of rocks and the reflection therefrom were conducted by Mr. Yin Ben Dai during the spring and summer months of 1969 at the U. S. Army Engineer Waterways Experiment Station (WES). The tests were supervised by Dr. G. H. Keulegan.

This portion of Engineering Study 833 is being conducted under the general supervision of Mr. E. P. Fortson, Jr., former Chief of the Hydraulics Laboratory, and Mr. H. B. Simmons, present Chief, and Mr. R. Y. Hudson, former Chief of the Wave Dynamics Branch, and Dr. R. W. Whalin, present Chief.

Directors of WES during the course of this investigation and the preparation and publication of this report were COL Levi A. Brown, CE, and COL Ernest D. Peixotto, CE. Technical Directors were Messrs. J. B. Tiffany and F. R. Brown.

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NOTATION

a	Wave amplitude in the structures
a_i	Amplitude of incident waves
a_r	Amplitude of reflected waves from structures
a_t	Amplitude of transmitted wave past rear or back face
a_0	Initial wave amplitude in the structure at the entering face
a_1	Final wave amplitude in the structure at the existing face
a_{11}	Maximum wave amplitude in the approaches to a structure
a_{12}	Minimum wave amplitude in the approaches to a structure
A	Cross-sectional area of rock structure
A_o	Cross-sectional area of voids in the structure
B	Width of structure
B_1, B_2	Specialized widths
C	Constant relating to resistance of rocks, equation 27
C_f	Coefficient of resistance of rock porous structures; $C_f = \kappa u d / \nu$
C_o	Constant relating to coefficient of permeability and grain size, equation 1
C_1	Constant in relation $C_f = C_1 (u d / \nu)^{-1}$
C_2	Constant in relation $C_f = C_2 (u d / \nu)^{-n}$ $n < 1$
d	Size of rocks
e	Loss of energy per unit volume of structure and per unit time ($ML^{-1}T^{-3}$)
F_x	Resistive force per unit volume ($ML^{-2}T^{-2}$), equation 18
g	Constant of gravity (LT^{-2})
G	Weight of rocks
h	Water-surface elevation measured from undisturbed water level
H	Undisturbed water depth

k	Wave number $2\pi/\lambda$, (L^{-1})
K	Constant of permeability (L/T), $K' = K/\epsilon$
K'	K/ϵ
K_0	A constant in power formula of coefficient of transmission
K_2	A dimensional coefficient in equation 28
L	Length of structure
m	Exponent appearing in the transition formula, equation 42
M	Numerical constant
n	Exponent relating to resistance law, equation 35
N	Numerical constant
p_1, p_2	Pressure
Q	Discharge (L^3T^{-1})
R	Reynolds number, ud/ν
t	Time
T	Period of waves
u	Seepage velocity through porous rock structure, $u = \epsilon u_0$
u_0	Mean particle velocity in the voids
V	Volume of structure
W	Energy flux across a vertical section per unit width of structure (MLT^{-2})
x	Horizontal distance
y	Vertical distance
α	Logarithmic decrement, equation 9
β', β_0'	Constants relating to γ as shown in equations 45, 46, and 47
γ	Constant appearing in the wave transmission formula, equations 43, 52, 59 (bis), and 69
γ_s	Specific weight of rocks
ΔE	Energy loss per unit width of structure and of length Δx (MLT^{-2})
ΔH	Fall of water surface
Δ_1, Δ_2	Fractional increments
ϵ	Porosity of rock structures
θ	Temperature
κ	Coefficient of energy loss, equation 20
λ	Wavelength

- ν Kinematic viscosity (L^2T^{-1})
 ρ Density of water (ML^{-3})
 σ Period number $2\pi/T$, (T^{-1})
 ω Wave velocity of propagation

CONVERSION FACTORS, BRITISH TO METRIC UNITS OF MEASUREMENT

British units of measurement used in this report can be converted to metric units as follows:

<u>Multiply</u>	<u>By</u>	<u>To Obtain</u>
inches	2.54	centimeters
feet	0.3048	meters
cubic feet	0.02831685	cubic meters
pounds per cubic foot	16.0185	kilograms per cubic meter
feet per second per second	0.3048	meters per second per second
square feet per second	0.092903	square meters per second
cubic feet per second	0.02831685	cubic meters per second
Fahrenheit degrees	5/9	Celsius or Kelvin degrees*

* To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: $C = (5/9)(F - 32)$. To obtain Kelvin (K) readings, use: $K = (5/9)(F - 32) + 273.15$.

SUMMARY

Application of the energy method leads to a theoretical expression of the coefficient of transmission through a prismatic structure, constructed from rocks of uniform size, for long waves traversing the structure. When the dissipation in the voids is due partly to turbulent forces, the transmissions are expressed by a power formula. The formula is modified by a parametric factor to respond to the conditions met with in the ordinary model studies of the shorter waves. The factor form is determined from observations made with waves of varying periods. The relation of the adapted formula to the resistance of the structures for steady flow is shown. With the reflections not being amenable to an analytical treatment, test results are consolidated in a single curve, irrespective of the rock sizes, with the porosity remaining constant.

WAVE TRANSMISSION THROUGH ROCK STRUCTURES

Hydraulic Model Investigation

PART I: INTRODUCTION

1. An important problem in the model studies of the rubble-mound breakwaters and jetties is the estimation of scale effects in the observed values of the wave transmissions through these structures and also the reflections from them. The ascertainment of scale effects is one of the items in the problems relating to Engineering Study 833. For a given initial wave approaching a structure, the magnitude of the wave transmitted will be dependent on the dissipation in the structure during passage and also on the reflection from it. If the dissipation is due solely to turbulence in voids of the model rocks and to the degree that losses are quadratic, then flow manifestations are not influenced by scale effects. This applies to transmissions and reflections alike, and there is complete similarity between a model and the prototype. If, on the other hand, dissipation is controlled by viscosity, as is often the case in laboratory models, it becomes necessary to formulate scale-effect laws in order to pass from model to prototype.

2. Owing to the complexity of the flow pattern in the model assemblies of rubble-mound breakwaters and jetties, arising from the general shapes of the structures and the presence of the rocks of different sizes, it is better that the problem of scale effects be studied in a simple structure consisting of uniform rocks of one size, with the frontal and the rear or back faces vertical. As it may be necessary to resort to large structures to complete the study, this simplicity of the structures would be economically advantageous.

3. The structures examined in this study were prismatic, with the frontal and rear or back faces vertical, and tests were conducted in a wave flume. Several rock structures were constructed with varying lengths using limestone or granite. This study is a continuation of the one reported by Kamel,¹ who also used similar structures. As the

structures considered by Kamel were constructed using spheres and cubes of uniform sizes, it was thought desirable to repeat the investigation using rocks. The results of the two studies will be compared to determine if any difference in the scale-effect laws exists due to rock shapes. A comparison of the results is possible since both studies were carried out in the same wave flume, using the same water depth and the same sequence of wave periods. The comparison of results will be carried out on the basis of theoretical expressions derived for the coefficient of wave transmission. There is, however, a divergence between the form developed here and the one given by Kamel. This fact has necessitated a recomputation of the transmission data in the report of Kamel relating to cubes.

PART II: THEORETICAL TREATMENTS

Small-Amplitude Long Waves in Porous Structures

4. Consider a structure consisting of rocks resting on a rigid surface with waves traversing it (fig. 1). Take two vertical sections

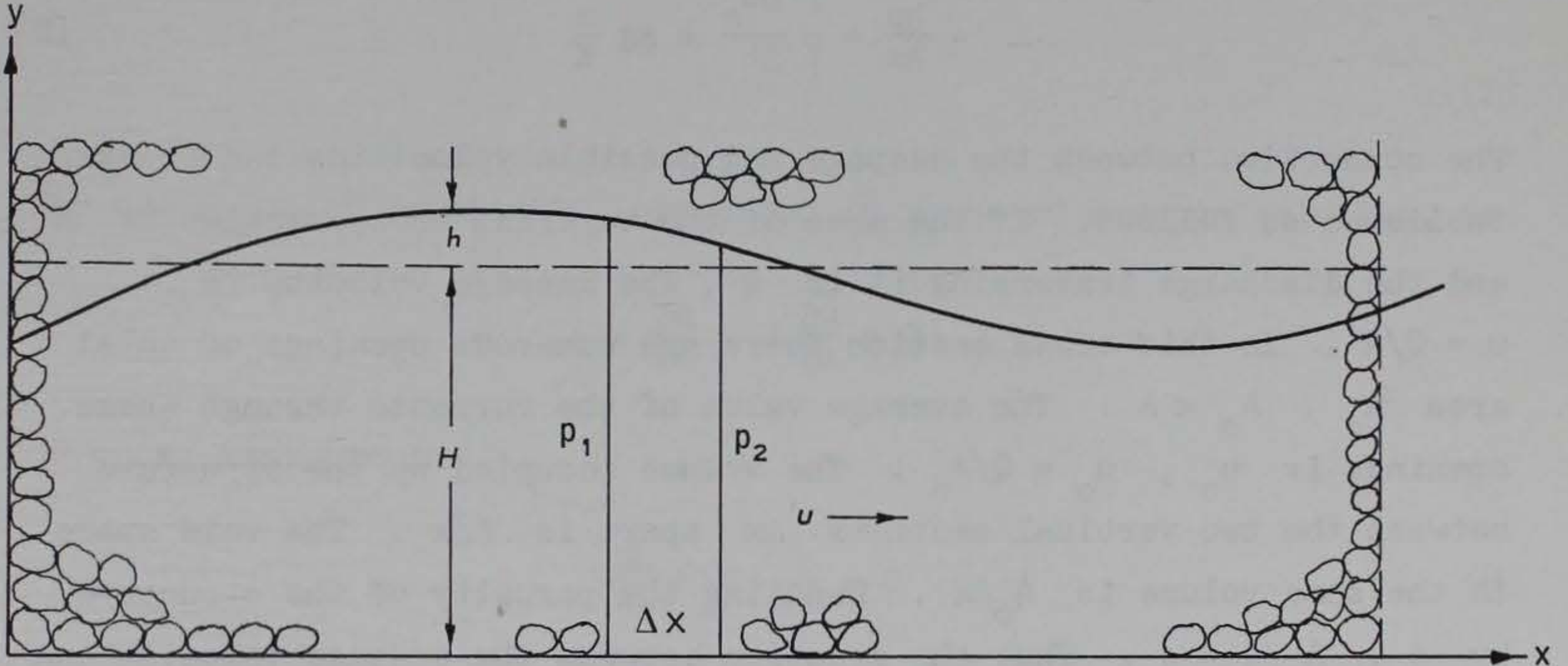


Fig. 1. Waves in rock structure

Δx apart. Let the pressure difference $p_1 - p_2$ be broken up into two parts Δp_1 and Δp_2 , such that the first accounts for the instantaneous acceleration of water particles in the voids of the structure and the second, for the resistance of flow through the rocks

$$\Delta p_1 = \rho \frac{\partial u_0}{\partial t} \Delta x$$

where u_0 is the average particle velocity and next, using Darcy's law,

$$\Delta p_2 = \rho g \frac{u}{K} \Delta x$$

where u is the seepage velocity and K is the permeability as defined by Jacob.² Note that K has the dimensions of velocity. In the Darcy regime, K is independent of velocity and using dimensional analysis

$$K = \frac{C_0 g d^2}{\nu} \quad (1)$$

where d is the rock size, ν is the kinematic viscosity, and C_0 is a numeric varying with porosity. Writing $p_1 - p_2 = -\Delta p$, then

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial u_0}{\partial t} + \rho g \frac{u}{K} \quad (2)$$

The connection between the seepage and possible velocities can be established as follows. If the area of the vertical cross section is A and the discharge traversing it is Q , the seepage velocity is $u = Q/A$. In this cross section there are numerous openings of total area A_0 , $A_0 < A$. The average value of the currents through these openings is u_0 , $u_0 = Q/A_0$. The volume occupied by the structure between the two vertical sections Δx apart is $A\Delta x$. The void space in the same volume is $A_0\Delta x$. Denoting the porosity of the structure by ϵ , $A_0/A = \epsilon$. Thus the relation between the seepage velocity and the particle velocity through the pores is

$$u = \epsilon u_0 \quad (3)$$

We shall suppose that the vertical acceleration is negligible and the wave surface is open to the atmosphere. Thus

$$\frac{1}{\rho} \frac{\partial p}{\partial y} = -g, \quad p = \rho g(H + h - y) \quad (4)$$

where y denotes the elevation of a point measured from the bottom, H is the mean depth of water, and h is the elevation of the wavy surface from the undisturbed level of waters in the structure. The capillarity effect is ignored and the resistance to the motion of air above the water surface and in the pores of the structure is neglected. From equation 4

$$\frac{\partial p}{\partial x} = \rho g \frac{\partial h}{\partial x} \quad (5)$$

Introducing this in equation 2 and using equation 3 results in the dynamic equation, applicable to a permeable and porous structure,

$$\frac{\partial u_o}{\partial t} + g \frac{u_o}{K'} = -g \frac{\partial h}{\partial x} \quad (6)$$

where

$$K' = \frac{K}{\epsilon} \quad (7)$$

The condition of continuity, expressed in terms of seepage velocity, is

$$\epsilon \frac{\partial h}{\partial t} + H \frac{\partial u}{\partial x} = 0$$

or using equation 3

$$\frac{\partial h}{\partial t} + H \frac{\partial u_o}{\partial x} = 0 \quad (8)$$

In this it is supposed that u_o is independent of y . This would be a natural assumption if the wavelength is large in comparison with the depth of water.

5. As the solution of equations 6 and 8, assume

$$h = a_o e^{-\alpha k x} \cos (kx - \sigma t) \quad (9)$$

where

$$k = \frac{2\pi}{\lambda} \quad \text{and} \quad \sigma = \frac{2\pi}{T}$$

λ being the wavelength and T the period of the waves. The waves are progressive waves moving in the positive direction of x and amplitudes decrease exponentially. Substituting in equation 8 and solving for u_o

$$u_o = A e^{-\alpha k x} \cos (kx - \sigma t) + B e^{-\alpha k x} \sin (kx - \sigma t) \quad (10)$$

where A and B are constant, having the dimensions of velocity. In terms of the wave amplitude

$$a_0 \sigma = (1 + \alpha^2) k H A \quad (11)$$

and

$$B = \alpha A$$

Substituting in equation 6 h from equation 9 and u_0 from equation 10, and setting the coefficients of $\sin(kx - \sigma t)$ and $\cos(kx - \sigma t)$ equal to zero, it is found that

$$-a_0 k g + \sigma A + \frac{g B}{K'} = 0 \quad (12)$$

and

$$-a_0 k g \alpha - \sigma B + \frac{g A}{K'} = 0 \quad (13)$$

Multiplying equation 12 by α , adding the resulting equation to equation 13, and using equation 11 we find that

$$\alpha = \frac{\sigma}{2k^2_{HK'}} = \frac{\epsilon \lambda^2}{4\pi H T K} \quad (14)$$

Multiplying equation 12 by α and eliminating K' between this resulting equation and equation 11, we find that

$$\frac{\sigma^2}{k^2} = g H (1 - \alpha^2)$$

or

$$\frac{\lambda}{T} = \sqrt{g H} \left(1 - \frac{\alpha^2}{2} \right) \quad (15)$$

It must be emphasized that λ appearing in equations 14 and 15 refers

to the wavelength in the structure. Equation 15 shows that the waves travel at a slower rate in a porous structure.

6. Subsequently, there will be the need for a criterion to show when the losses are purely from a viscous origin. The criterion will be a specific value of the Reynolds number formed on the basis of maximum seepage velocity and rock size. The velocity would be the greatest at the entrance face of the structure, that is at $x = 0$. From equation 10

$$u_o = A \cos \sigma t + B \sin \sigma t$$

Thus,

$$u_{o\max} = \sqrt{A^2 + B^2} = A(1 + \alpha^2)^{1/2} \quad (16)$$

Introducing A from equation 11 and noting that $u = \epsilon u_o$, the maximum seepage velocity is

$$u_{\max} = \frac{\epsilon a_o \sigma}{kH} \left(1 + \frac{1}{2} \alpha^2 \right)$$

Ignoring α , the desired velocity to form the Reynolds number is

$$u_{\max} = \frac{\epsilon a_o \lambda}{HT}$$

and the Reynolds number itself is

$$R = \frac{\epsilon a_o \lambda d}{HT \nu} \quad (17)$$

7. Biesel³ in an interesting analysis has examined the damping of surface waves of short periods in a porous structure. The analysis applies to a flow lying in the Darcy regime. When the analysis is projected to the range of long waves, the results are found to be consistent with the ones shown above. This comparison, showing the steps of the analysis, will be omitted here.

Application of Energy Method

8. The attenuation of waves in porous structures as discussed above is for the case where the losses are solely from viscous forces. This restricts the application of the formula of damping to the rare cases where the grain sizes are small and the wave periods are large. In the actual cases of the laboratory model studies, dissipation would be partly due to turbulence, and the dynamic equation will be of a form differing from that given by equation 6. If e denotes the loss of energy in the void spaces per unit volume of the structure and per unit time, then the liquid contained in a unit volume of the structure will experience the resistive force

$$F_x = \frac{e}{u} \quad (18)$$

and the dynamic equation becomes

$$\frac{\partial u_o}{\partial t} + \frac{e}{\rho u} = -g \frac{\partial h}{\partial x} \quad (19)$$

9. Resorting to dimensional analysis, as

$$e = (ML^{-1}T^{-3})$$

one may introduce the relation

$$e = \kappa \frac{\nu \rho u^2}{d^2} \quad (20)$$

where κ would be a function of the Reynolds number ud/ν , of porosity ϵ , and of the manner of rock placement. It is this last factor that causes some difficulty in predicting the value of κ for rocks. One may refer to κ as the coefficient of energy loss. Another expression for the loss would be, in terms of permeability coefficient,

$$e = \frac{\rho g u^2}{K} \quad (21)$$

Comparing with equation 20

$$\kappa = \frac{gd^2}{\nu K} \quad (22)$$

which expresses the relation between the coefficient of loss and the coefficient of permeability. With a steady current passing through a prismatic structure of length L , let the head difference of waters be ΔH and u the seepage velocity (fig. 2). Here

$$e = \rho g u \frac{\Delta H}{L} \quad (23)$$

Comparing with equation 20

$$\kappa = C_f \frac{ud}{\nu} \quad (24)$$

and

$$C_f = \frac{gd}{u^2} \frac{\Delta H}{L}$$

The latter constitutes the basis for the experimental determination of energy loss coefficient κ . In comparison with loss of head in a closed conduit, C_f will be referred to as the coefficient of resistance.

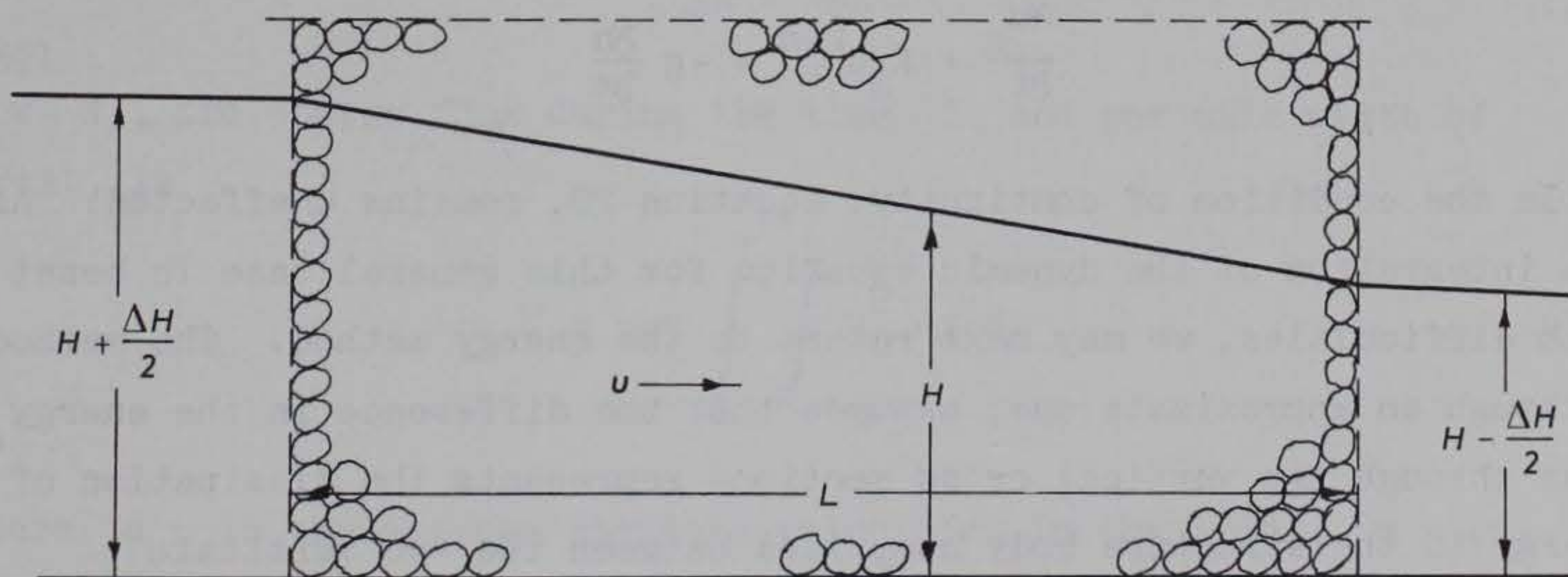


Fig. 2. Steady flow in rock structure

10. Experiments suggest that

$$C_f = C \left(\frac{ud}{\nu} \right)^{-n} \quad (25)$$

where the values of the dimensionless constants C and n depend on the Reynolds number. The exponent n attains the value 1 when the Reynolds number is small. This is for the region where the loss of energy is due mainly to viscous dissipation. At the other extreme where the Reynolds number is large, n becomes nil and the loss is due to turbulence alone. In general, since

$$\kappa = C \left(\frac{ud}{\nu} \right)^{1-n} \quad (26)$$

then

$$\frac{e}{\rho u} = C \left(\frac{ud}{\nu} \right)^{-n} \frac{u^2}{d}$$

or

$$\frac{e}{\rho u} = K_2 u^{2-n} ; \quad K_2 = C \frac{\nu^n}{d^{1+n}} \quad (27)$$

Accordingly, the dynamic equation of motion, equation 19, reduces to

$$\frac{\partial u_o}{\partial t} + K_2 u^{2-n} = -g \frac{\partial h}{\partial x} \quad (28)$$

while the condition of continuity, equation 20, remains unaffected. As the integration of the dynamic equation for this general case is beset with difficulties, we may next resort to the energy method. The method, although an approximate one, assumes that the difference in the energy flux through two vertical cross sections represents the dissipation of energy in the structure body comprised between the two vertical sections.

11. We shall apply the method first to the case where the flow

Reynolds number is very small. In the rock structure contained in a tranche of thickness Δx and of unit width, the loss of energy during time T , the period of the progressive waves, is

$$\Delta E = \int_0^T \int_0^H e \, dy \, dt \, \Delta x \quad (29)$$

As the Reynolds number is very small, $n = 1$ and equation 26 makes κ a constant, say

$$\kappa = C_1 \quad (30)$$

with the result that

$$e = \frac{C_1 \nu \rho}{d^2} u^2$$

Accordingly

$$\Delta E = \frac{\rho C_1 \nu \Delta x}{d^2} \int_0^T \int_0^H u^2 \, dy \, dt \quad (31)$$

This loss represents the difference of energy fluxes ΔW entering and leaving the tranche, that is,

$$\Delta W = -\Delta E \quad (32)$$

Now W , the energy flux during the time T and per unit width of crest, is

$$W = \rho \omega \epsilon \int_0^T \int_0^H u_o^2 \, dy \, dt \quad (33)$$

where u_o is the average particle velocities in the voids, $u = \epsilon u_o$, ϵ is the porosity, and ω is the velocity of propagation of the progressive waves. From equations 31 and 33

$$\Delta E = \frac{C_1 \nu W}{\omega d^2} \Delta x$$

Introducing this in equation 32

$$\frac{dW}{W} = - \frac{\epsilon C_1 \nu T}{d^2 \lambda} dx$$

Remembering that W is proportional to a^2 , a being the wave amplitude,

$$\frac{da}{a} = - \frac{\epsilon}{2} \frac{C_1 \nu T}{d^2 \lambda} dx$$

Hence

$$a = a_0 e^{-\alpha kx}$$

$$\alpha = \frac{\epsilon C_1 \nu T}{4\pi d^2} \quad (34)$$

12. In equations 14 and 34 we have two separate expressions for the logarithmic decrement α . Since these are established by two different methods, the question is whether they are actually equivalent to each other. Confining attention to regimes of flow where the dissipative forces are mainly from viscosity, κ in equation 22 can be replaced by C_1 ,

$$C_1 = \frac{gd^2}{\nu K}$$

and introducing this in equation 34

$$\alpha = \frac{\epsilon}{4\pi} \frac{gT}{K} = \frac{\epsilon}{4\pi} \frac{TgH}{KH}$$

Since in the shallow-water waves $gH = \lambda^2/T^2$

$$\alpha = \frac{\epsilon \lambda^2}{4\pi KHT}$$

This shows that the logarithmic decrements in equations 14 and 34 are equivalent to each other.

Long Waves of Finite Amplitude

13. The law of damping given by equation 34 can have only a very limited application. In the situations we will be interested in, the dissipation within the structures would be brought about by turbulence, in the major part, so that the coefficient of resistance is

$$C_f = C_2 \left(\frac{ud}{v} \right)^{-n} \quad (35)$$

and the corresponding energy loss coefficient is

$$\kappa = C_2 \left(\frac{ud}{v} \right)^{1-n}$$

Using equation 29, the loss of energy in a tranche of thickness Δx and of unit width during time T would be

$$\Delta E = \rho C_2 \frac{v^n}{d^{1+n}} \int_0^H \int_0^T u^2 |u|^{1-n} dy dt \Delta x \quad (36)$$

In lieu of equation 32 it is appropriate to write

$$\frac{dW}{W} = - \frac{\Delta E}{W} \quad (37)$$

where W has the same meaning as in equation 33.

$$\frac{dW}{W} = - \frac{C_2 \epsilon v^n dx}{\omega d^{1+n}} M \quad (38)$$

where

$$M = \int_0^T u^2 |u|^{1-n} dt \div \int_0^T u^2 dt$$

Taking the velocities as

$$u = \omega \frac{h}{H}, \quad h = a \sin (kx - \sigma t)$$

$$M = N \left(\frac{\omega}{H} \right)^{1-n} a^{1-n} \quad (39)$$

where

$$N = \int_0^{\pi/2} \sin^{3-n} \theta d\theta \div \int_0^{\pi/2} \sin^2 \theta d\theta$$

Thus N varies slightly with n . For $n = 0.00, 0.25, 0.50, 0.75$, and 1.00 , the corresponding values of N are $0.849, 0.915, 0.955$, and 1.00 , respectively. Combining equations 37, 38, and 39 and remembering that W is proportional to the square of wave amplitude

$$\frac{da}{a} = - \frac{1}{2} \frac{N \epsilon C_2 v^n}{d^{1+n}} \omega^{-n} \left(\frac{a}{H} \right)^{1-n} dx$$

or

$$a^{n-2} da = - \frac{1}{2} \frac{N \epsilon C_2 v^n}{d^{1+n}} \left(\frac{\lambda}{T} \right)^{-n} \lambda H^{n-1} \frac{dx}{\lambda}$$

Integrating and denoting the initial amplitudes at $x = 0$ by a_0 ,

$$\left(\frac{a}{a_0} \right)^{n-1} = 1 + \frac{1-n}{2} N C_2 \epsilon \left(\frac{vT}{d\lambda} \right)^n \frac{\lambda}{d} \left(\frac{a_0}{H} \right)^{1-n} \frac{x}{\lambda} \quad (40)$$

If the structure is of length L and a_1 is the amplitude of the waves at the terminating end of the structure

$$\left(\frac{a_1}{a_0} \right)^{n-1} = 1 + \frac{1-n}{2} N C_2 \epsilon \left(\frac{vT}{d\lambda} \right)^n \frac{\lambda}{d} \left(\frac{a_0}{H} \right)^{1-n} \frac{L}{\lambda} \quad (41)$$

It will be recalled that C_2 , a dimensionless constant, depends on the porosity of the structure, the shape of the rocks, and the manner of placement.

Transmission of Waves of Moderate Length

14. The expression developed in the above equation is not expected to apply generally when the waves approaching a rock structure are not shallow-water waves. One difficulty in the analysis is the possibility that in the latter case there would be a change in the pattern of the particle velocities as the waves enter a rock structure. These changes presumably would depend on the dimensionless quantity $\sqrt{gH} T/\lambda$. In a previous work relating to screens,⁴ the author had developed a relation which when modified to apply to a rock structure takes the form

$$\left(\frac{a_0}{a_1}\right)^{1-n} = 1 + \frac{1-n}{2} NC_2 K_0 \epsilon \left(\frac{vT}{d\lambda}\right)^n \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2}\right)^{1-n} \left(\frac{a_0}{H}\right)^{1-n} \frac{L}{\lambda}$$

Here K_0 is a numerical constant whose value should depend on n and $gH(T^2/\lambda^2)$. For establishing this value, resort must be made to experiments.

15. It would be advantageous, however, if after dropping K_0 the last expression is written in the form

$$\left(\frac{a_0}{a_1}\right)^{1-n} = 1 + \frac{1-n}{2} NC_2 \epsilon \left(\frac{vT}{d\lambda}\right)^n \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2}\right)^m \left(\frac{a_0}{H}\right)^{1-n} \frac{L}{\lambda} \quad (42)$$

In the case of very long waves this expression reduces to equation 40, a relation developed theoretically. A new constant m is introduced. Its value is to be determined experimentally from the transmission coefficients when the waves approaching the structures have varying periods.

16. The determination of m can be carried out in the following manner. Equation 42 can be written

$$\left(\frac{a_0}{a_1}\right)^{1-n} = 1 + \gamma \left(\frac{a_0}{H}\right)^{1-n} \frac{L}{\lambda} \quad (43)$$

where

$$\gamma = \frac{1-n}{2} N C_2 \epsilon \left(\frac{\sqrt{T}}{d\lambda} \right)^n \frac{\lambda}{d} \left(g_H \frac{T^2}{\lambda^2} \right)^m \quad (44)$$

Write

$$\beta' = \frac{\gamma}{\left(\frac{\sqrt{T}}{d\lambda} \right)^n \frac{\lambda}{d}} \quad (45)$$

Now,

$$\beta' = \frac{1-n}{2} N \epsilon C_2 \left(g_H \frac{T^2}{\lambda^2} \right)^m \quad (46)$$

and

$$\beta'_0 = \frac{1-n}{2} N \epsilon C_2 \quad (47)$$

Then,

$$\frac{\beta'}{\beta'_0} = \left(g_H \frac{T^2}{\lambda^2} \right)^m \quad (48)$$

Details of the computation steps will now be given. Equation 43 implies that the transmission data observed for incident waves of various amplitudes approaching structures of varying lengths, constructed using rocks of the same size, will yield the constant γ . It is understood that the waves used all have the same wavelength λ and the same period T . The proper value of n is inferred from the resistance characteristics of the rocks. If resistance measurements are not carried out, the value of n can be ascertained by trials. Next, β' is determined through equation 45. This is plotted against $g_H(T^2/\lambda^2)$, and the intersection of the curve with the abscissa line $g_H(T^2/\lambda^2) = 1$ determines β'_0 . Forming the ratio β'/β'_0 , a plotting made in accordance with equation 48 should yield the value of m .

17. The amplitudes a_0 and a_1 appear in the above-derived formulas of damping. The first of these is the amplitude of the waves at the start of the structure and the second, at the end. These are

not the same as a_i and a_t , the amplitude of the incident waves approaching the structure and the amplitude of the transmitted waves moving away from the structure, respectively. Ordinarily, in the experimental studies of the transmissions, the amplitudes a_i and a_t are observed, not a_0 and a_1 ; thus, it is desired to indicate their relation with a_0 and a_1 . Unfortunately, this can be done only in a rough manner. Relying on the law of transmission for long waves moving in a constant-depth channel with sudden changes in the cross section

$$\frac{a_0}{a_i} = \frac{2B_1}{B_1 + B_2}, \quad B_2 = \epsilon B_1$$

Here B_1 is the width of the channel in which the initial waves are moving, and B_2 is the effective width in the rock structure. It is also assumed that the wave velocity in the rock structure differs only slightly from the wave velocity in the area of approach. Some error is involved in this assumption if the damping is very severe. Hence

$$\frac{a_0}{a_i} = \frac{2}{1 + \epsilon}$$

At the end of the structure

$$\frac{a_t}{a_1} = \frac{2B_1}{B_1 + B_2}, \quad B_1 = \epsilon B_2$$

where now B_1 represents the effective width of the rock structure relating to the water and B_2 is the width of channel where the transmitted waves are moving. Thus

$$\frac{a_t}{a_1} = \frac{2\epsilon}{1 + \epsilon}$$

The experimental data that we shall be considering subsequently relate to rock structures with porosities close to $\epsilon = 0.5$. With this value, we have

$$\frac{a_0}{a_i} = \frac{4}{3} \quad \text{and} \quad \frac{a_1}{a_t} = \frac{3}{2} \quad (49)$$

Substituting these in the formula of transmission, equation 43,

$$\left(\frac{a_i}{a_t}\right)^{1-n} = \left(\frac{9}{8}\right)^{1-n} + \left(\frac{3}{2}\right)^{1-n} \gamma \left(\frac{a_i}{H}\right)^{1-n} \frac{L}{\lambda} \quad (50)$$

This gives, when n equals $1/3$,

$$\left(\frac{a_i}{a_t}\right)^{1-n} = 1.08 + 1.32\gamma \left(\frac{a_i}{H}\right)^{1-n} \frac{L}{\lambda} \quad (51)$$

This may be replaced, entailing a small error, by

$$\left(\frac{a_i}{a_t}\right)^{1-n} = 1 + \gamma \left(\frac{a_i}{H}\right)^{1-n} \frac{L}{\lambda} \quad (52)$$

$$\gamma = \frac{1-n}{2} NC_2 \epsilon \left(\frac{\nu T}{d\lambda}\right)^n \frac{\lambda}{d} \left(g_H \frac{T^2}{\lambda^2}\right)^m \quad (53)$$

which will be utilized in the subsequent examination of the transmission data.

PART III: EXPERIMENTAL PROCEDURE AND DATA

Selection of Structures

18. The experiments on the wave transmission through and reflection by rock structures were conducted in a wave flume 1 ft* wide and approximately 100 ft long. The wave generator consisted of a plate hinged at the channel bottom, 2.9 ft below the top edge of the flume. The inclined bottom plate in front of the generator rose for a distance of 9.0 ft to meet the experimental part of the flume, which is about 90 ft long with a depth of 1.9 ft. A 13-ft-long beach of permeable material was utilized as a wave absorber. This assured the absence of reflection from the closed end of the wave flume.

19. The rock structures studied for wave transmission and reflection characteristics were constructed from uniform size rocks with vertical faces front and back and were as wide as the wave flume width. Four different rock sizes were chosen with the dimensions

1/2 to 3/8 in.
3/4 to 5/8 in.
1 to 7/8 in.
1-1/2 to 1-1/4 in.

The last numbers give the sieve mesh size retaining the rocks. The first numbers were chosen to represent the sizes of rock, as

$d = 0.042$ ft
 $= 0.062$ ft
 $= 0.083$ ft
 $= 0.125$ ft

With a given rock, four structure lengths were considered: $L = 0.25$, 0.50 , 1.00 , and 2.00 ft. To form the rock structures, two end screens were inserted into the flume and were firmly held in place at proper distances of the value L . The space between the end screens was

* A table of factors for converting British units of measurement to metric units is presented on page xi.

filled with rocks to a height of 1.50 ft. The webbing of the screens was sparse and the wires so small that the screens could not have any effect on the measured reflections and transmissions. The porosities of the structures used were not measured in place. Instead, porosities were measured by filling a 1-cu-ft box to one-third depth with rocks similar to the ones used in the structures and measuring the amount of water required to cover the rocks. These measurements were supervised by Mr. Donald Davidson of the Wave Dynamics Branch. The reported porosities are:

<u>Rock Size, in.</u>	<u>Porosity, ϵ</u>
1/2 to 3/8	0.465
3/4 to 5/8	0.465
1 to 7/8	0.455
1-1/2 to 1-1/4	0.467

We considered the placing of the rocks as unconsolidated with a mean porosity of $\epsilon = 0.46$. This was taken to represent also the porosity of the structures in the tests.

Measurements of Amplitudes

20. Wave heights of the incident waves and those of the reflections by the structures were measured by means of a Saginaw screw carrying a single wave rod. The wave height of the transmitted wave was measured by means of a single, fixed wave rod placed about 1 ft behind the structure. The corresponding amplitudes were obtained by dividing measured wave heights by 2. The moving wave rod of the Saginaw screw gives the maximum and minimum surface displacements $2a_{11}$ and $2a_{12}$ in the approaches to the structures. Assuming that the waves are sinusoidal and the amplitudes small,

$$a_i = \frac{1}{2} (a_{11} + a_{12})$$

$$a_r = \frac{1}{2} (a_{11} - a_{12})$$

Knowing well that the waves are not purely sinusoidal nor are the wave

heights always small, a certain error is introduced in the above equations of a_i and a_r . However, the determinations of a_i and a_r in the tests were made according to this method.

Summary of Experimental Data on Wave
Transmissions and Reflections

21. The transmissions and the reflections relating to the experimental structures were observed with waves having periods T equal to 0.71, 1.00, 1.41, and 1.94 sec. In all the tests, the water depth H was kept constant at the 1-ft value and the wavelengths corresponding to the four periods mentioned were $\lambda = 2.54, 4.53, 7.20$, and 10.40 ft, respectively. The transmission and reflection coefficients relating to the rock structures of this study are summarized in tables 1-16, which show the dependence of the ratios a_t/a_i and a_r/a_i on the relative wave height a_i/H and the relative structure length L/λ . The individual tables refer to tests wherein the structure length was varied and the wave period T was kept constant. The values of the ratios entered in the tables for the selected values of a_i/H are the values read from curves incorporating every observation of the tests. Fig. 3 is an example of the quantity a_t/a_i , computed from the observed values of a_t and a_i , plotted against $2a_i/H$. We read from the curve, drawn through observed points, the values of a_t/a_i corresponding to a few selected values of a_i/H . Fig. 4 is an example of the ratio a_r/a_i , as computed from observed values of a_r and a_i , plotted against $2a_i/H$. The best approximations to the observed values are straight lines. From the lines we read the values of a_r/a_i corresponding to selected values of a_i/H which are shown in the tables. In fig. 5, another graphic representation of the same data found in fig. 4, horizontal lines have been drawn through the data points to represent approximately the means of all the observed values of a_r/a_i for a given L/λ . The ordinate values of the horizontal lines will be referred to as mean values and these are entered in the last column of tables 1-16. These differ but little from the means of the entries in column 5 of the tables.

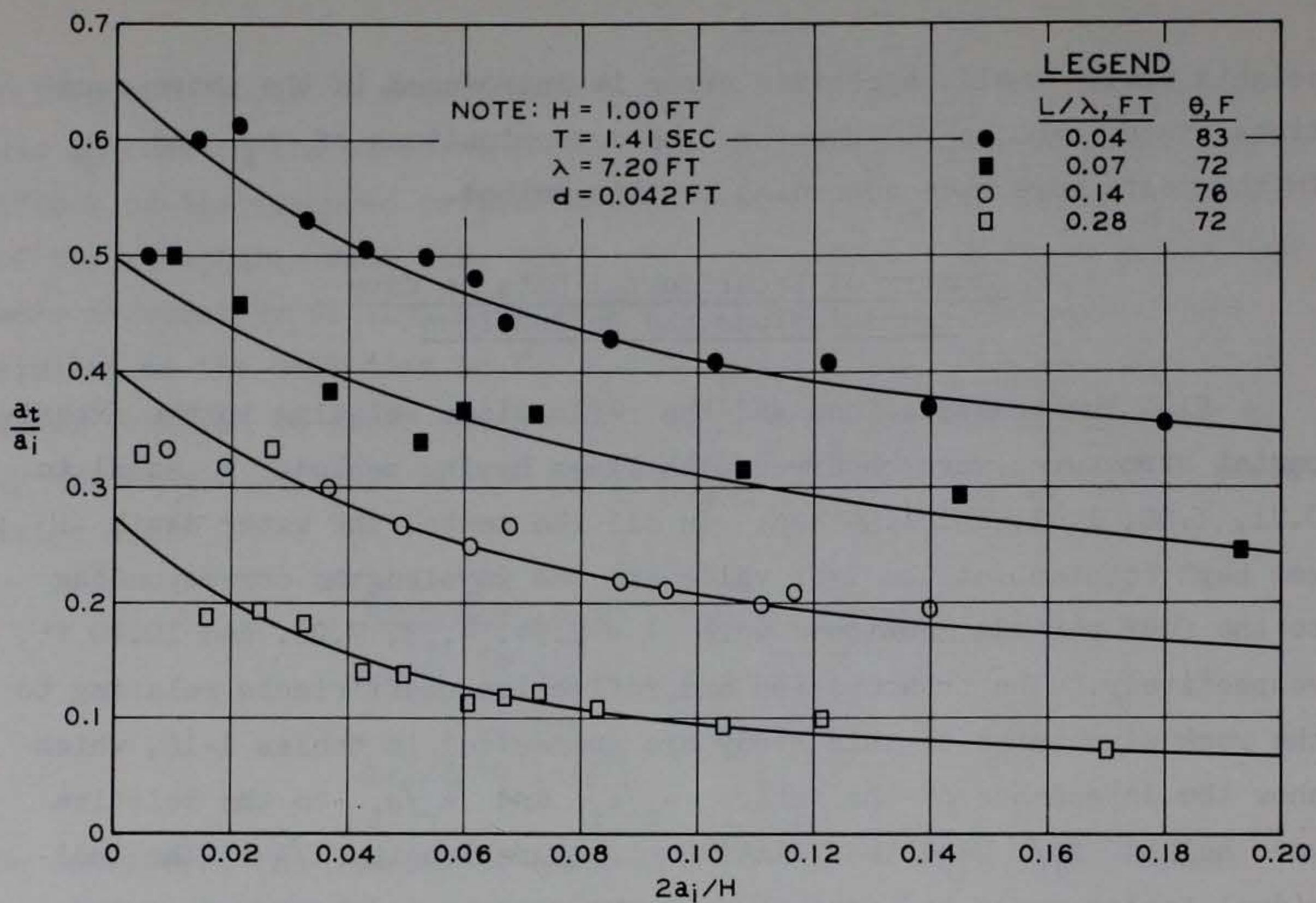


Fig. 3. An example of reduction of observed data of transmission

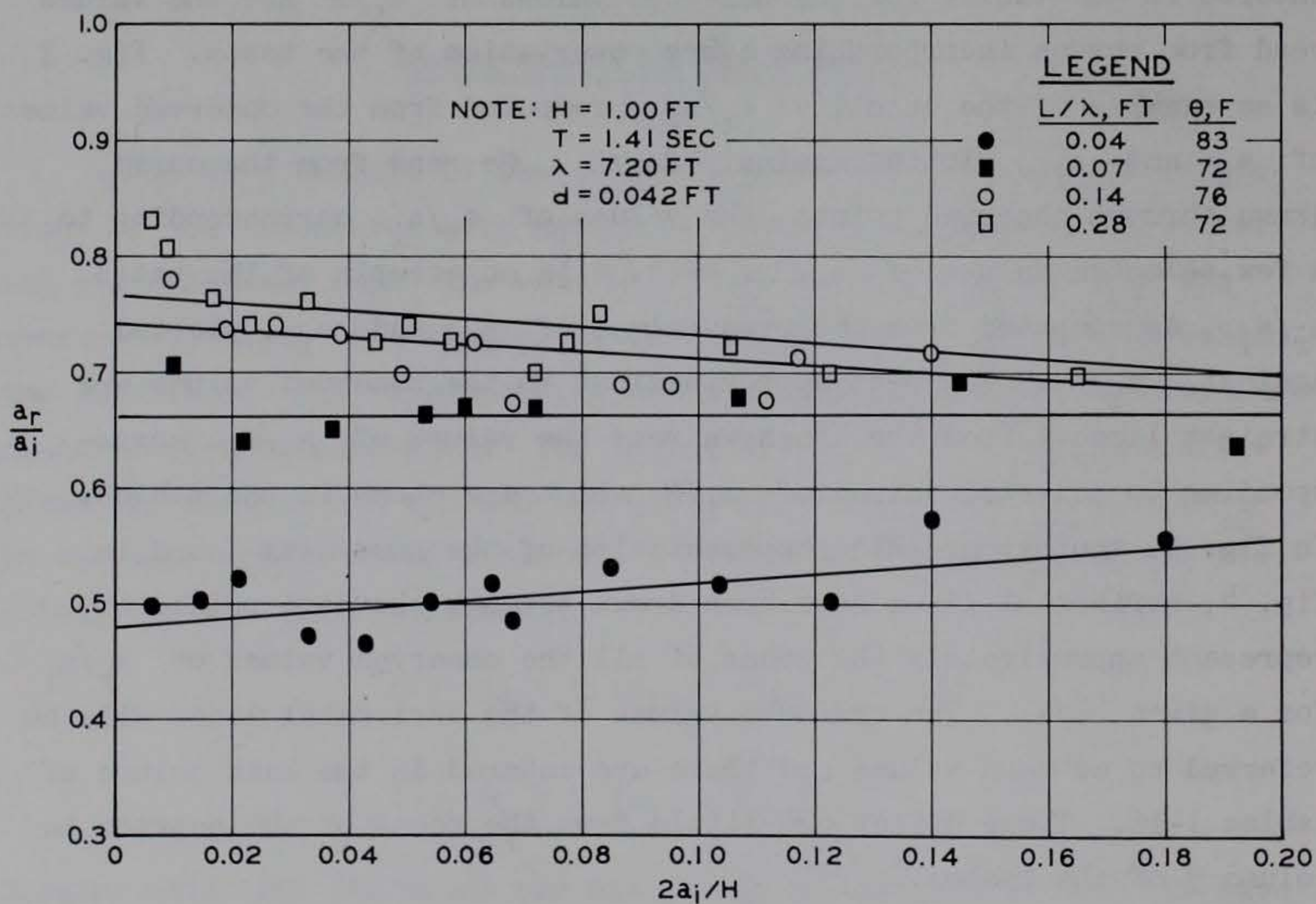


Fig. 4. An example of reduction of observed data of reflection

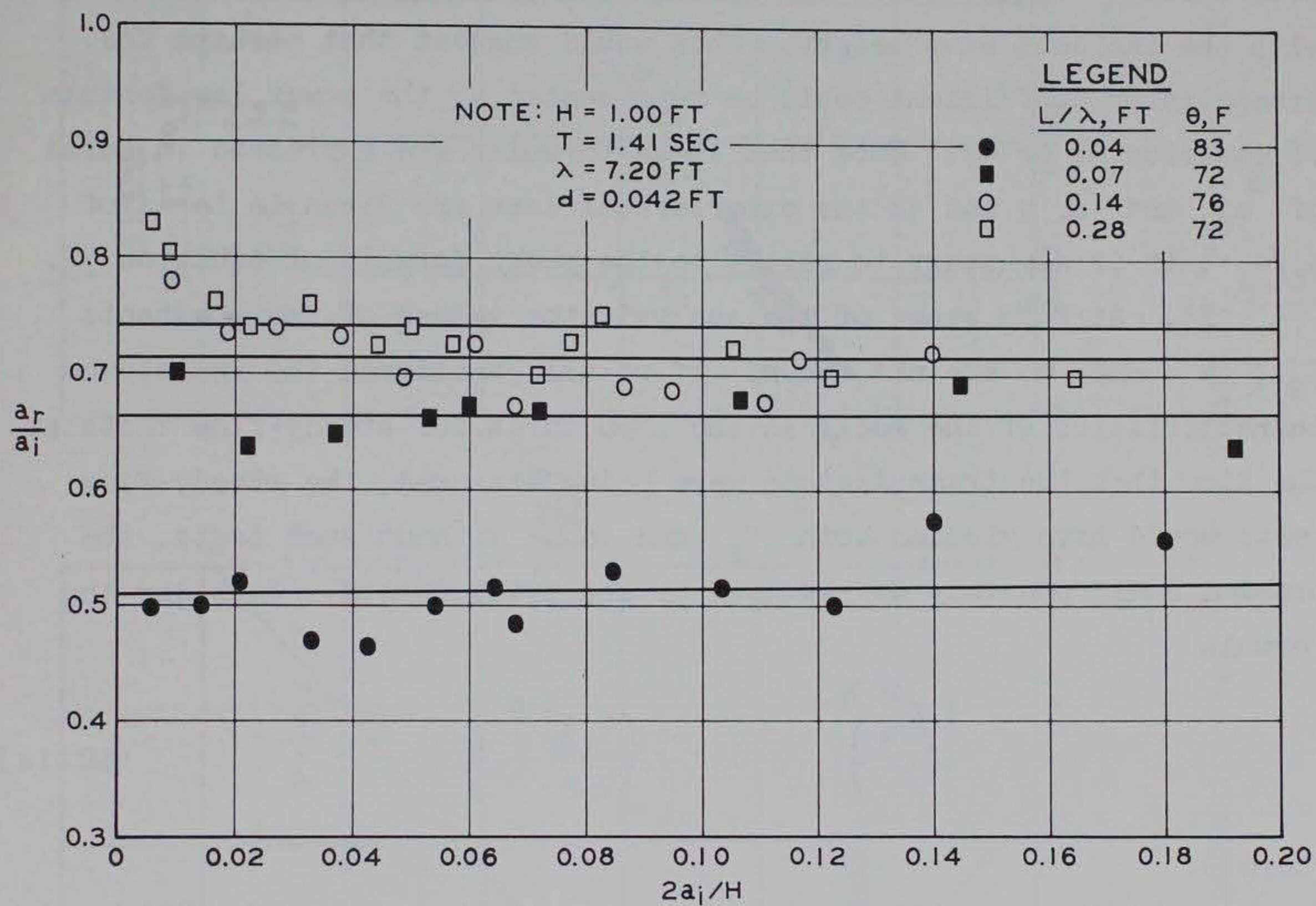


Fig. 5. An example of averaging of observed data of reflection

PART IV: ANALYSIS OF EXPERIMENTAL RESULTS

Wave Transmission

22. According to equation 34 the transmission coefficient a_t/a_i would have been independent of a_i/H if the dissipation realized in the rock voids during the passage of waves was brought about solely by viscous forces. Fig. 4 shows that the transmission coefficient decreases with the incident wave height. This would suggest that perhaps the transmission coefficient could be represented by the power law formula of equation 41 or 42. Note that these formulas are expressed in terms of a_0 and a_1 ; and as the experimental data are given in terms of a_t/a_i , it is necessary to resort to the power formula of equation 52.

23. At this stage of the analysis the values of the constants C_2 , n , and m are not known. If we had considered the resistance characteristics of the rocks in the structures for steady-flow tests at the time that the transmissions were being measured, the steady-flow tests would have yielded both C_2 and n . Without such tests, the problem could possibly be resolved by the trial method. Consider the formula

$$\left(\frac{a_i}{a_t}\right)^{1-n} = 1 + \gamma \left(\frac{a_i}{H}\right)^{1-n} \frac{L}{\lambda} \quad (52 \text{ bis})$$

where

$$\gamma = \frac{1-n}{2} NC_2 \epsilon \left(\frac{\nu T}{d\lambda}\right)^n \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2}\right)^m \quad (53 \text{ bis})$$

Equation 52 (bis) implies that with n properly chosen, γ would be constant in the situation where T , λ , d , ν , and H are fixed and L and a_i are varied. For different trials, one may take n to have one of the values: $1/2$, $1/3$, or $1/4$. The graphs of figs. 6-21 are obtained by letting $n = 1/2$. In the majority of the graphs, the alignment of the points is linear or nearly linear as was desired. One exception is that shown in fig. 18. These are graphs of the observations

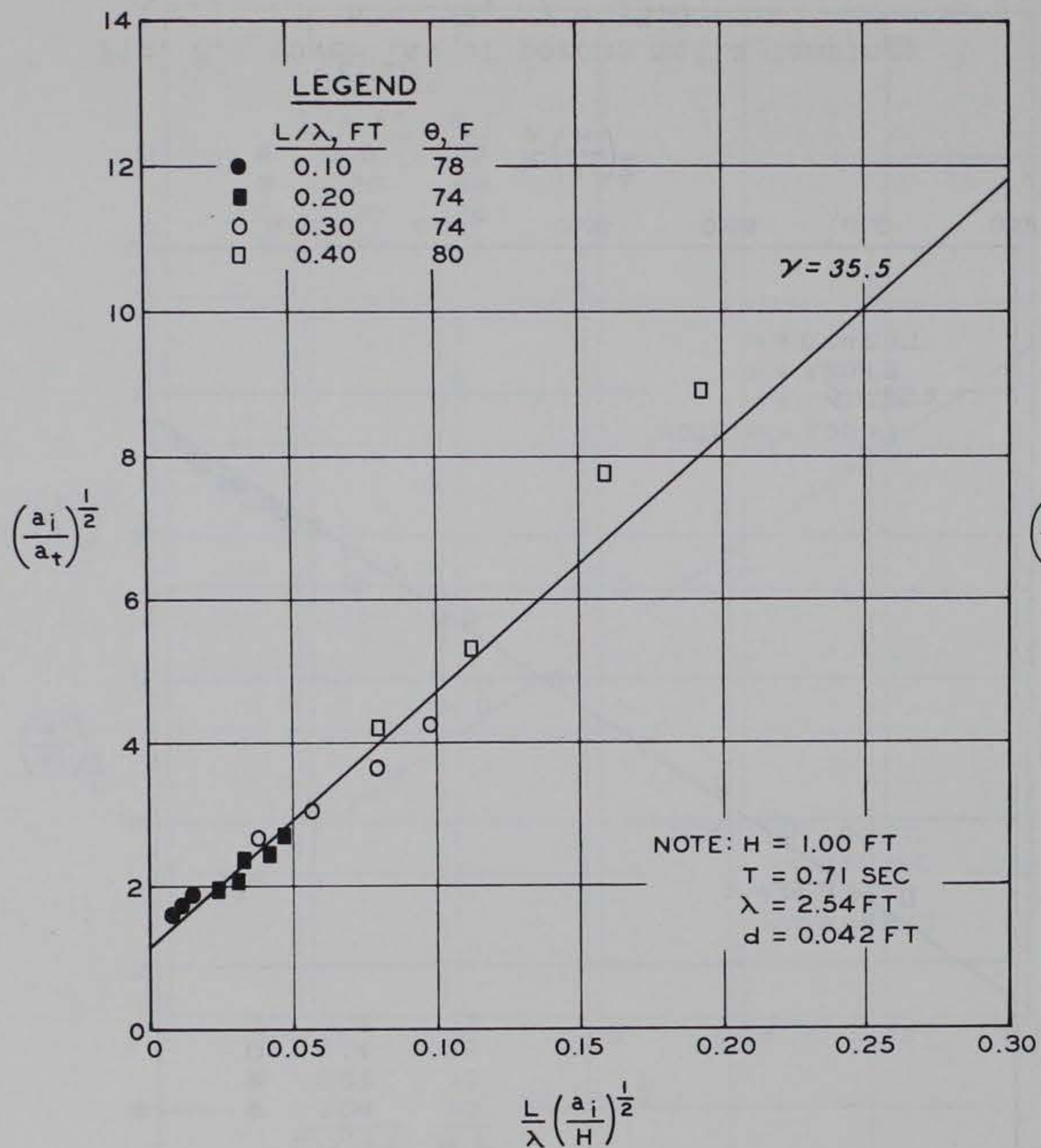


Fig. 6. Power law of porous media damping;
 $n = 1/2, \gamma = 35.5$

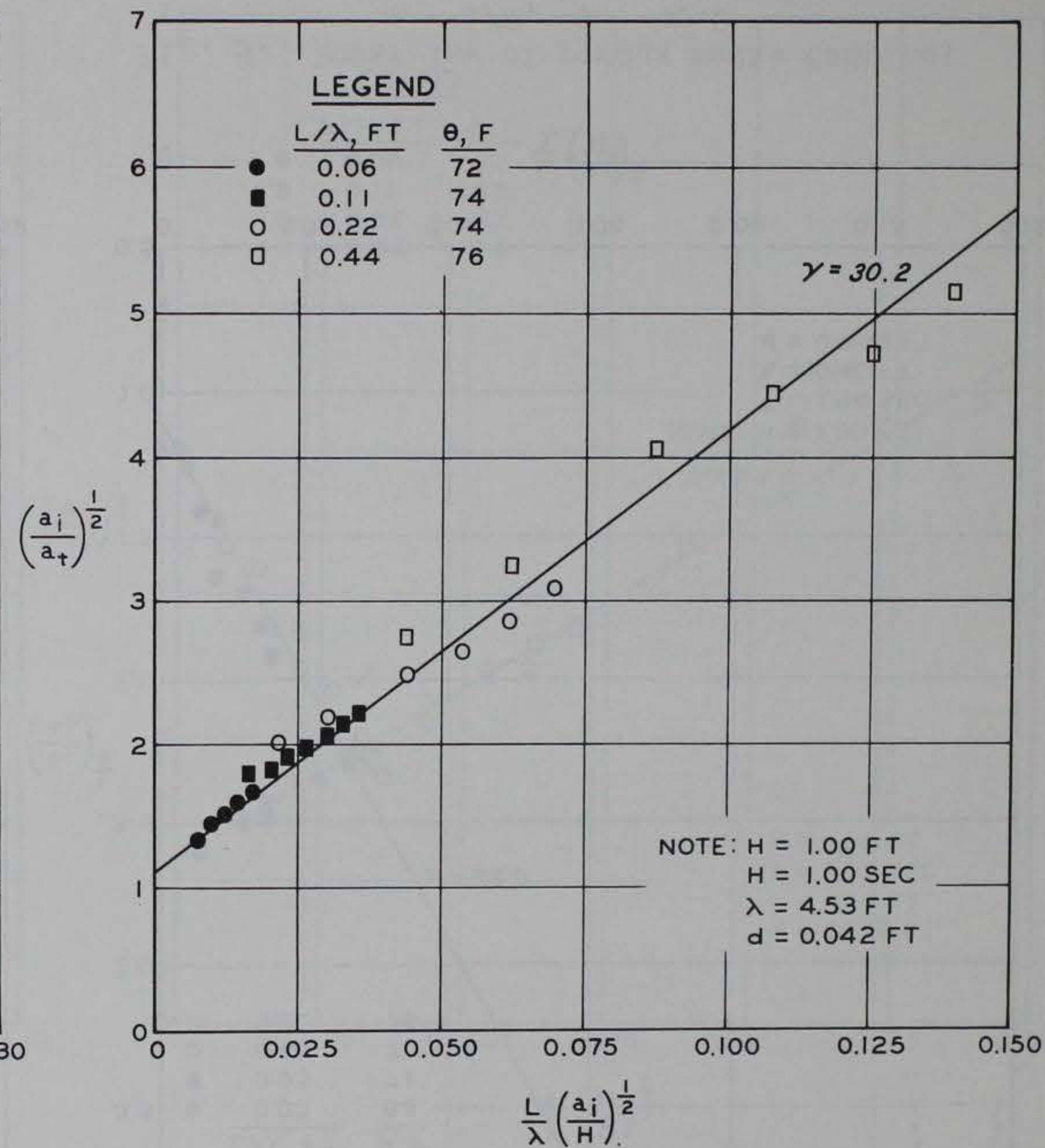


Fig. 7. Power law of porous media damping;
 $n = 1/2, \gamma = 30.2$

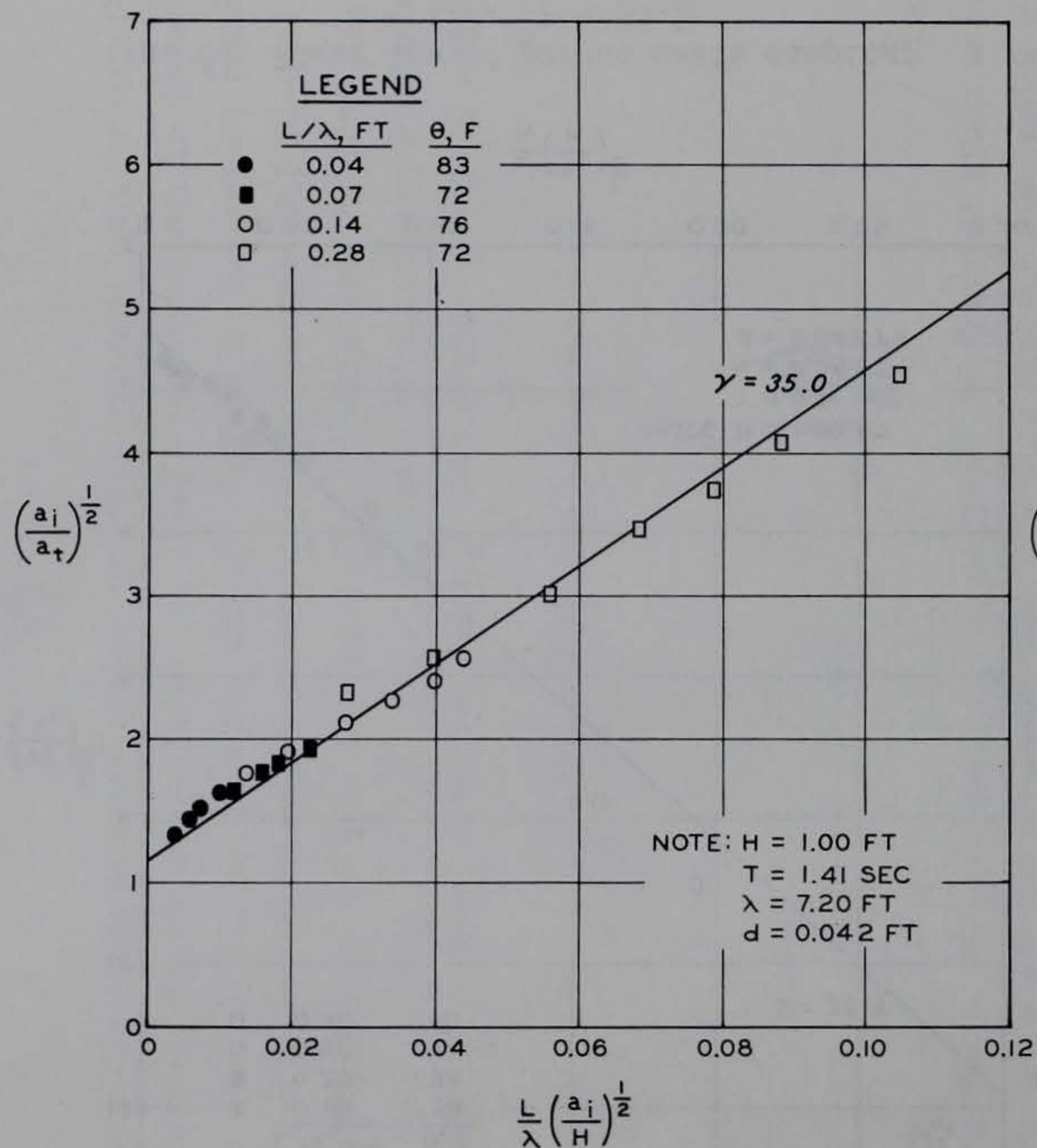


Fig. 8. Power law of porous media damping;
 $n = 1/2, \gamma = 35.0$

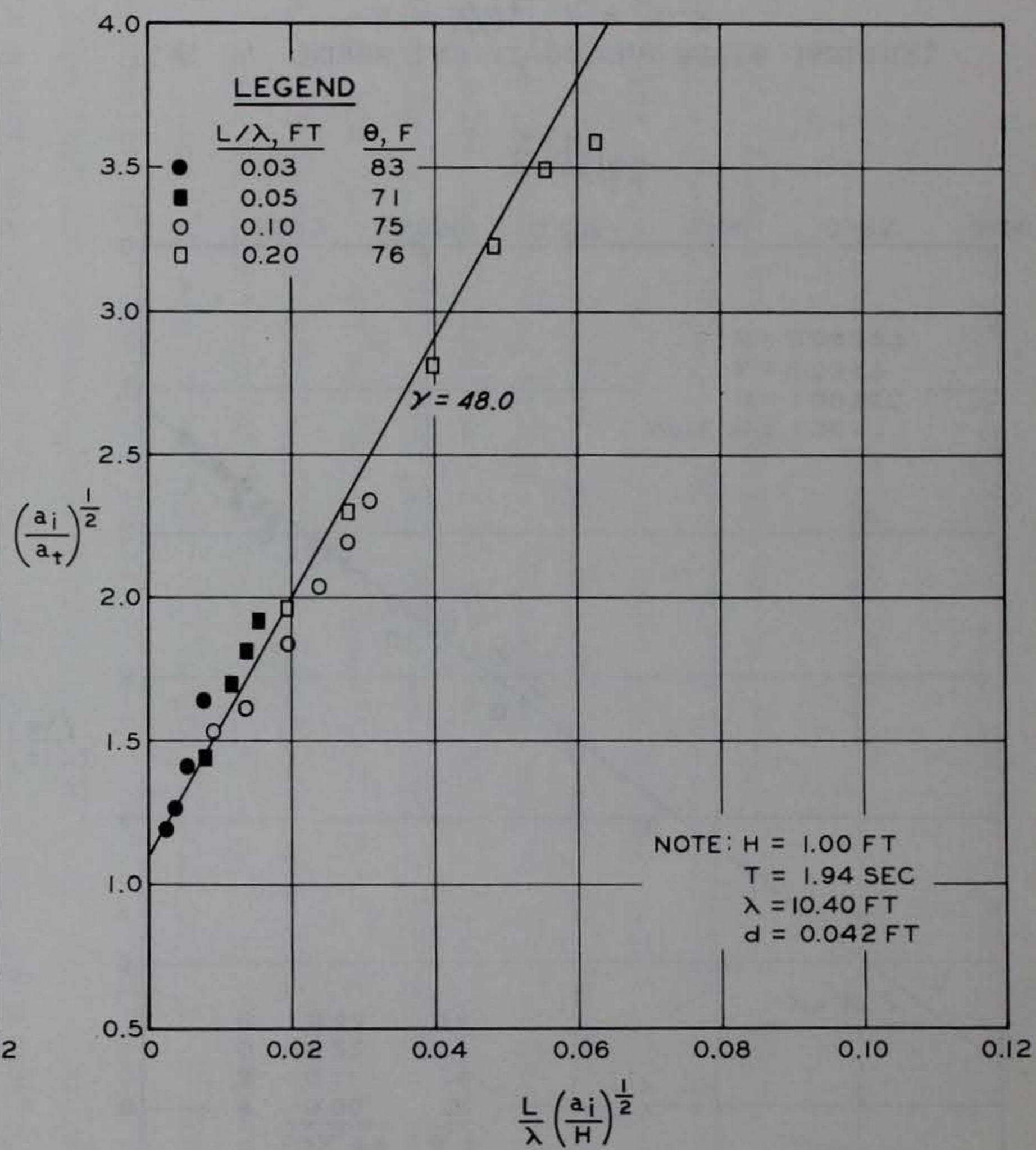


Fig. 9. Power law of porous media damping;
 $n = 1/2, \gamma = 48.0$

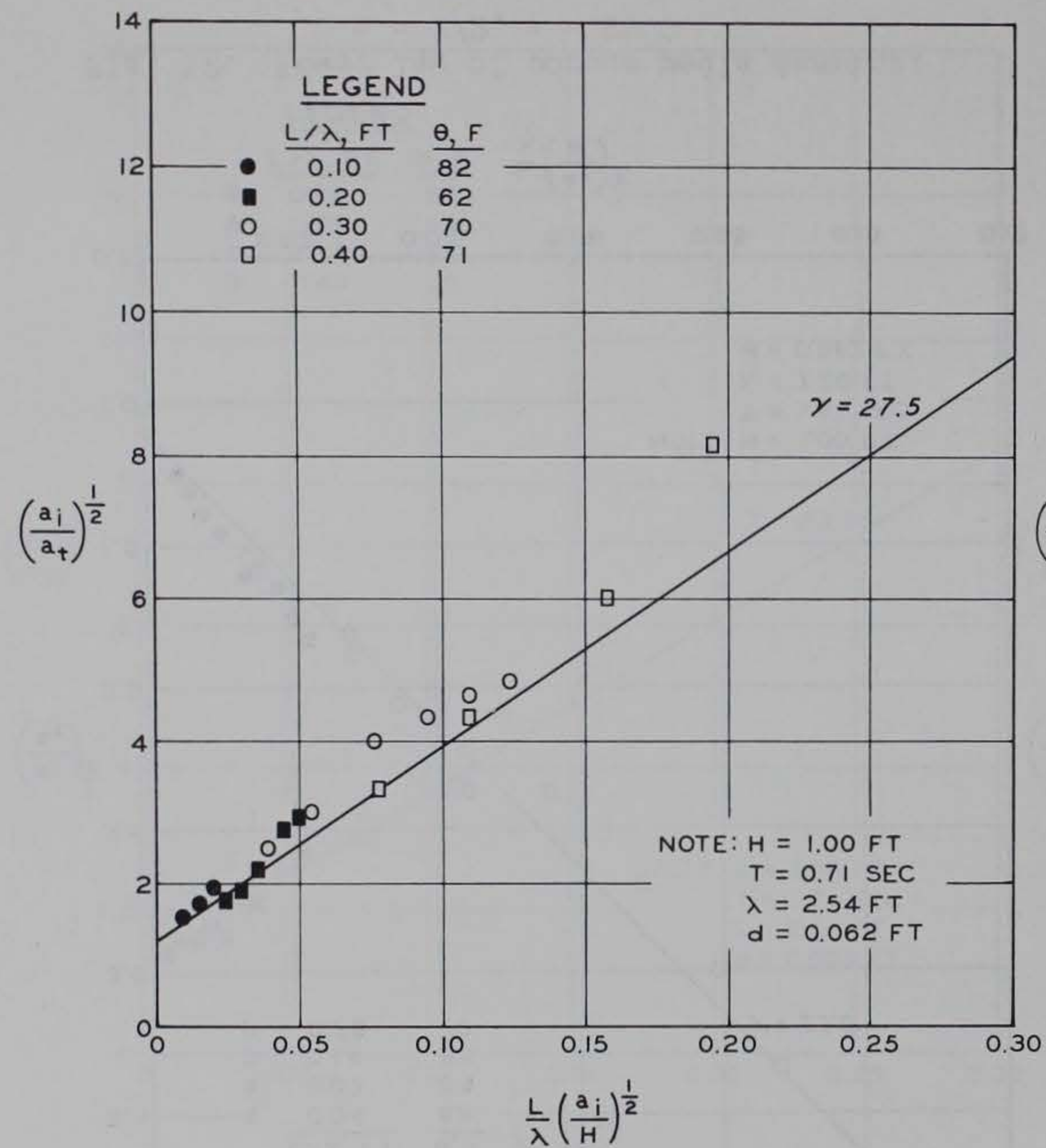


Fig. 10. Power law of porous media damping;
 $n = 1/2, \gamma = 27.5$

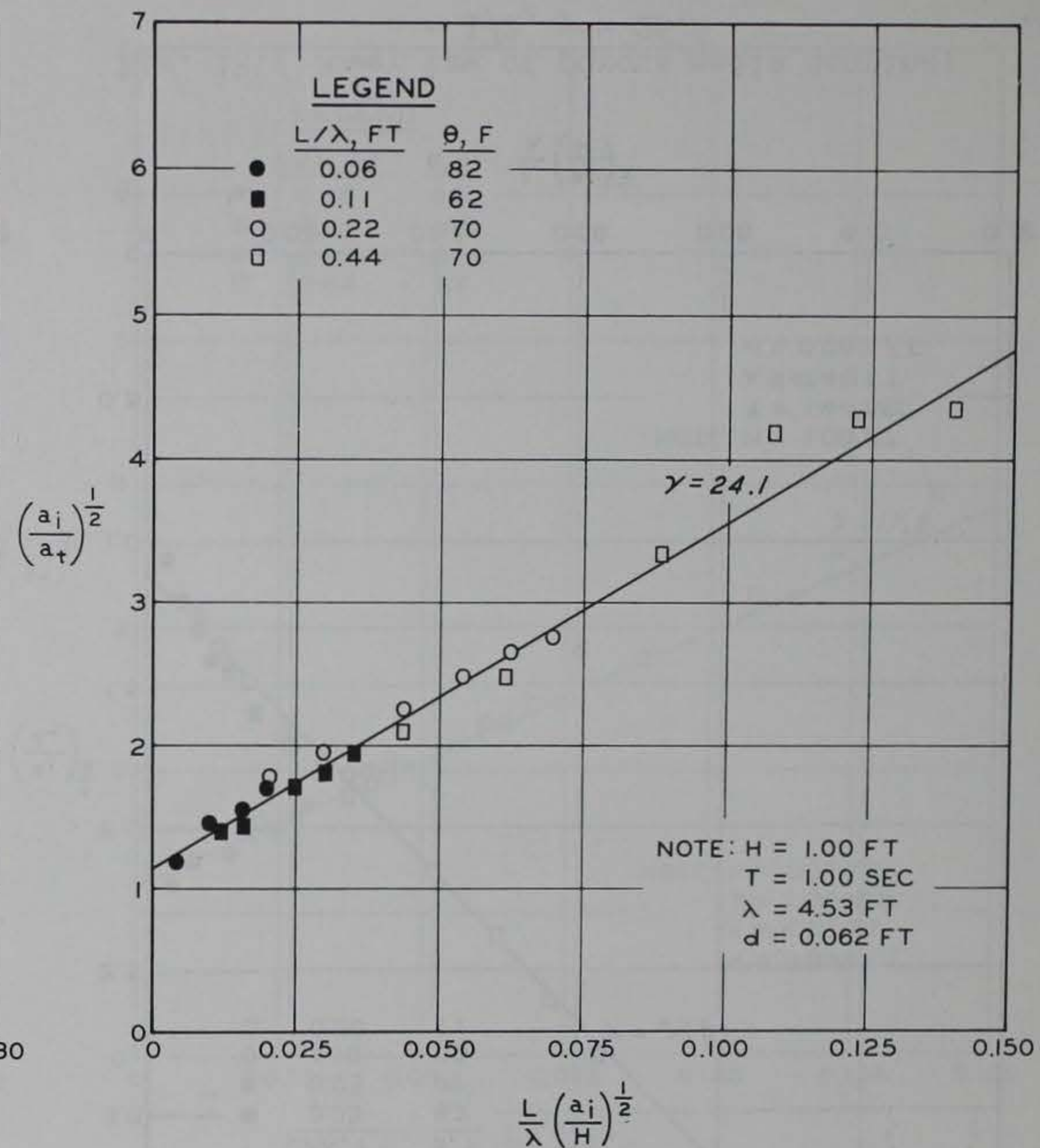


Fig. 11. Power law of porous media damping;
 $n = 1/2, \gamma = 24.1$

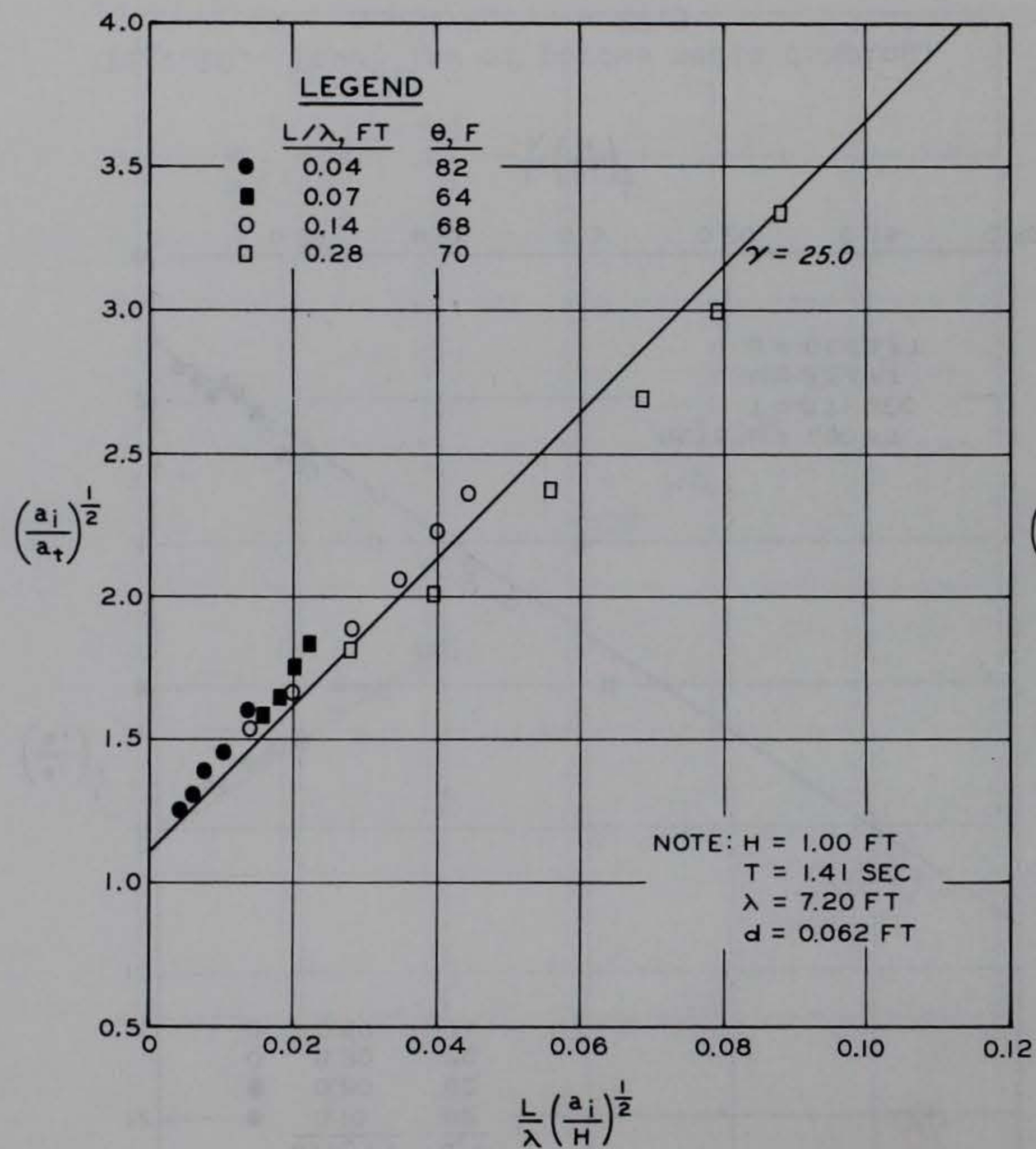


Fig. 12. Power law of porous media damping;
 $n = 1/2, \gamma = 25.0$

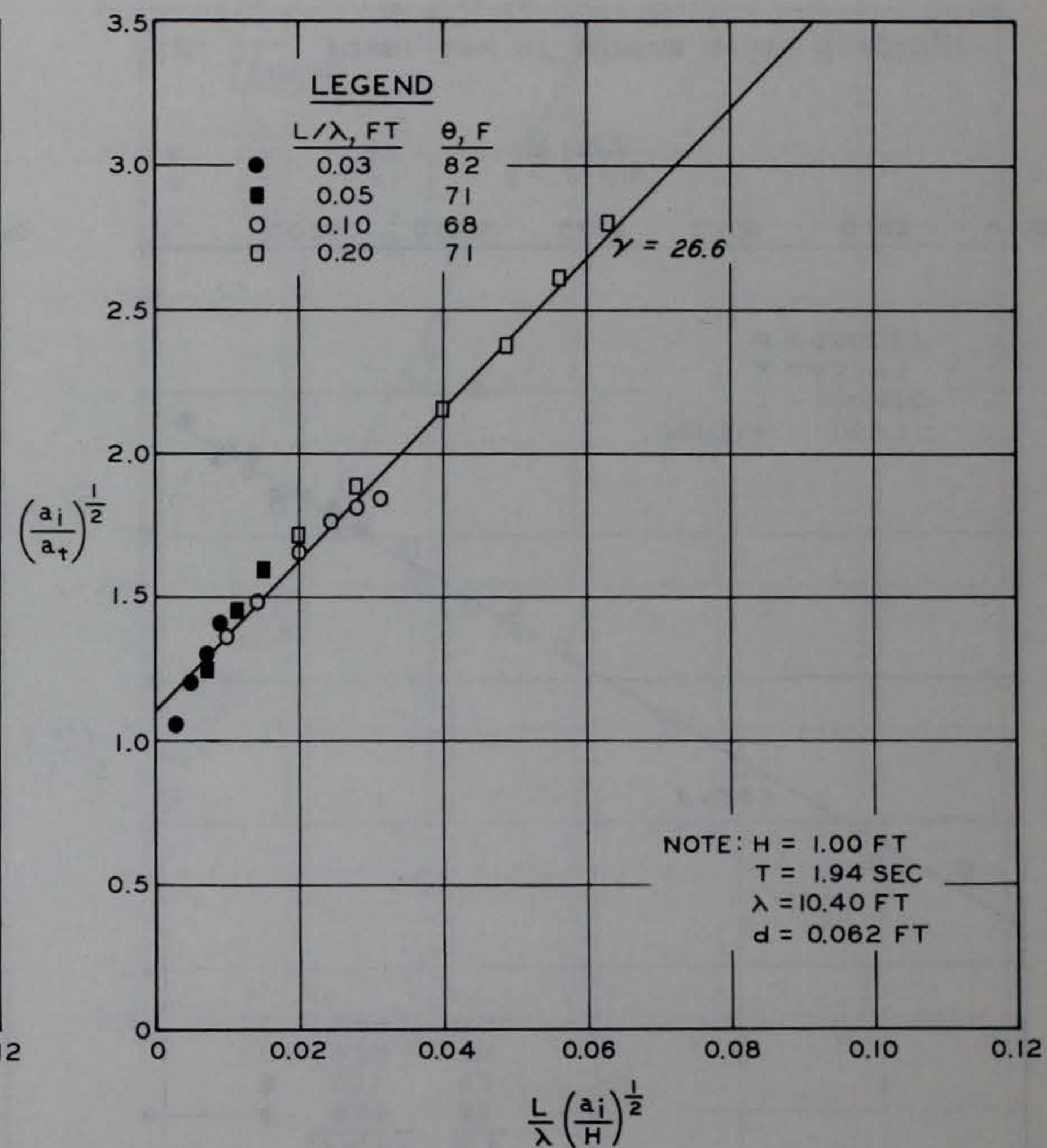


Fig. 13. Power law of porous media damping;
 $n = 1/2, \gamma = 26.6$

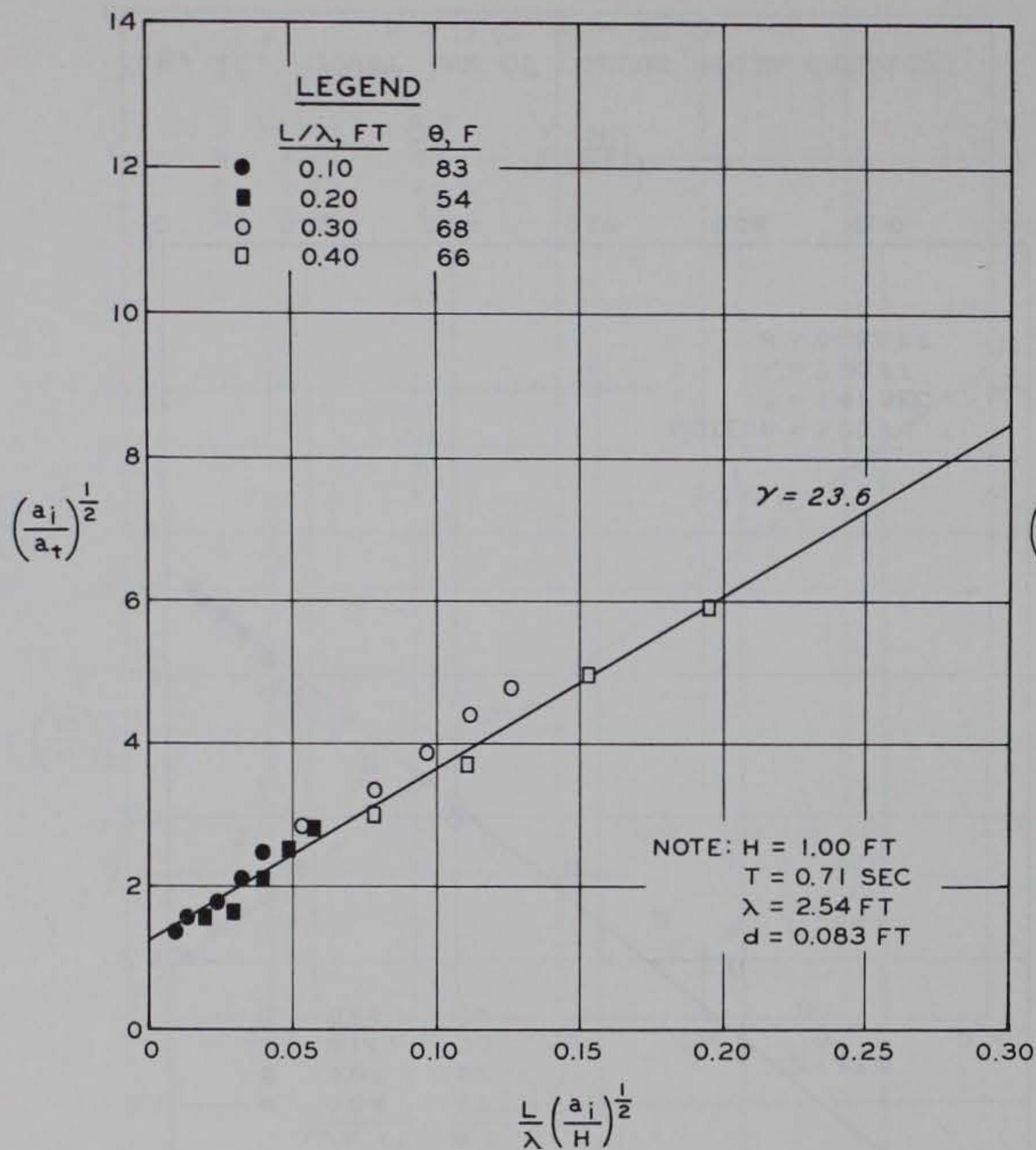


Fig. 14. Power law of porous media damping;
 $n = 1/2, \gamma = 23.6$

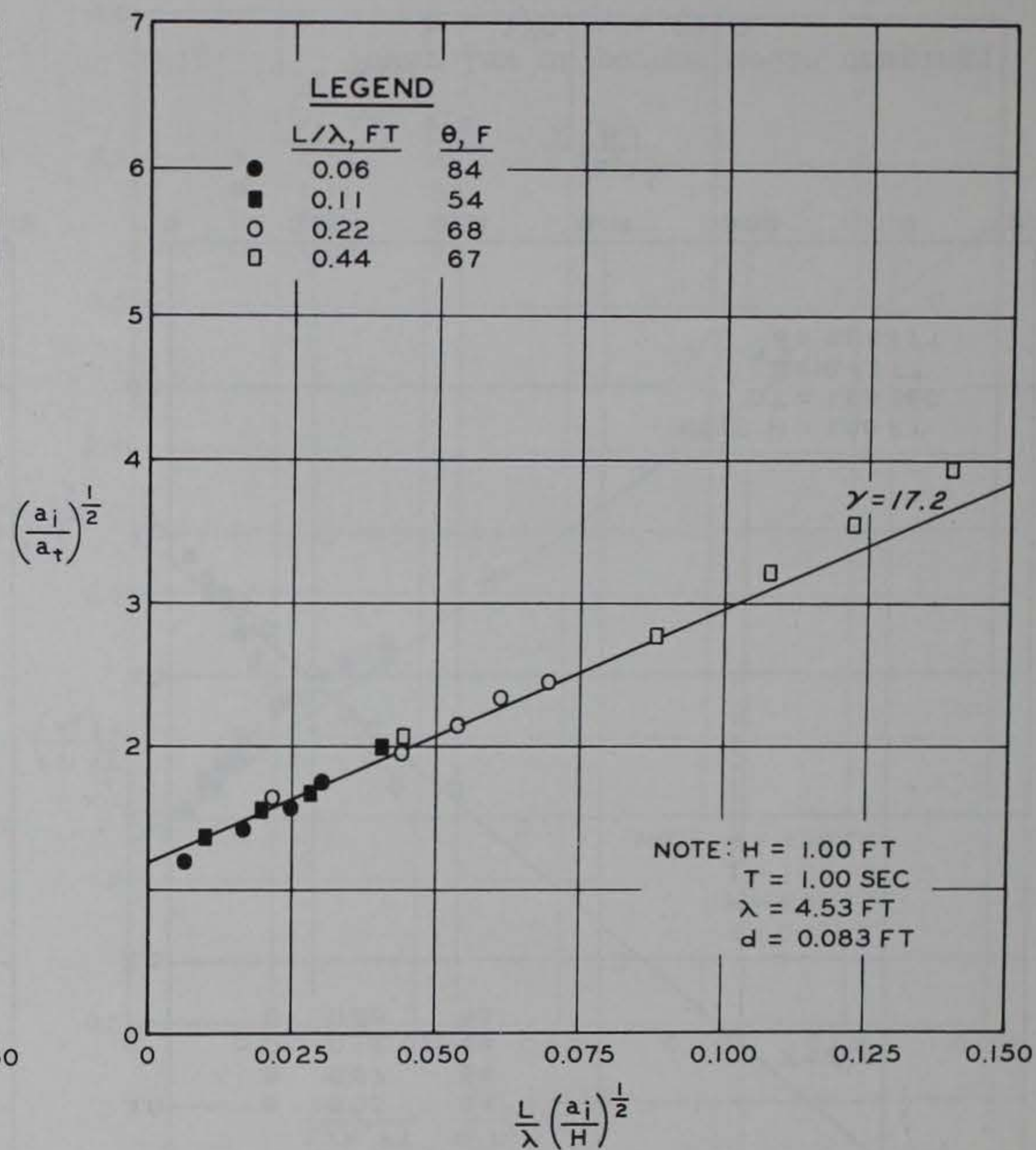


Fig. 15. Power law of porous media damping;
 $n = 1/2, \gamma = 17.2$

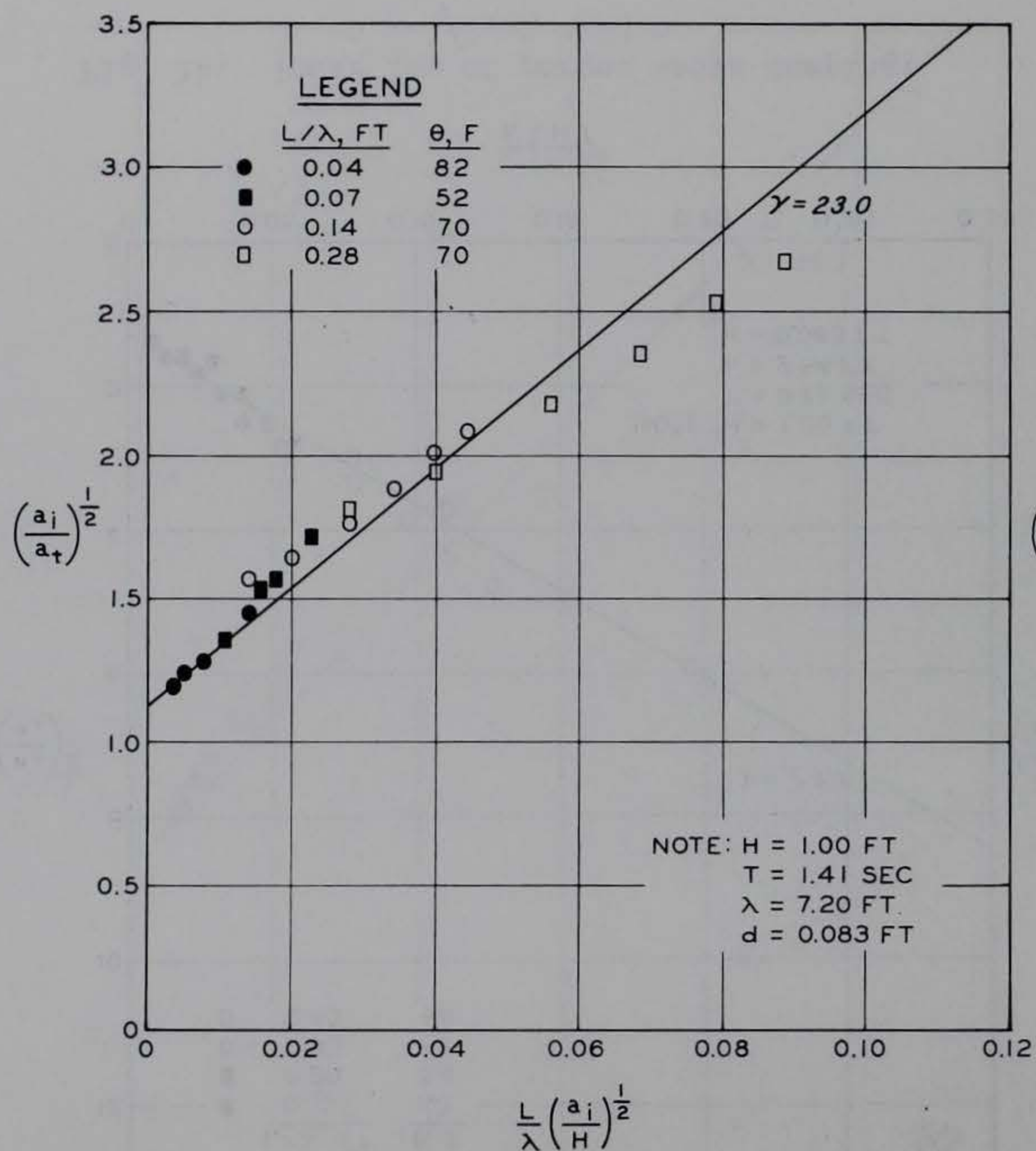


Fig. 16. Power law of porous media damping;
 $n = 1/2, \gamma = 23.0$

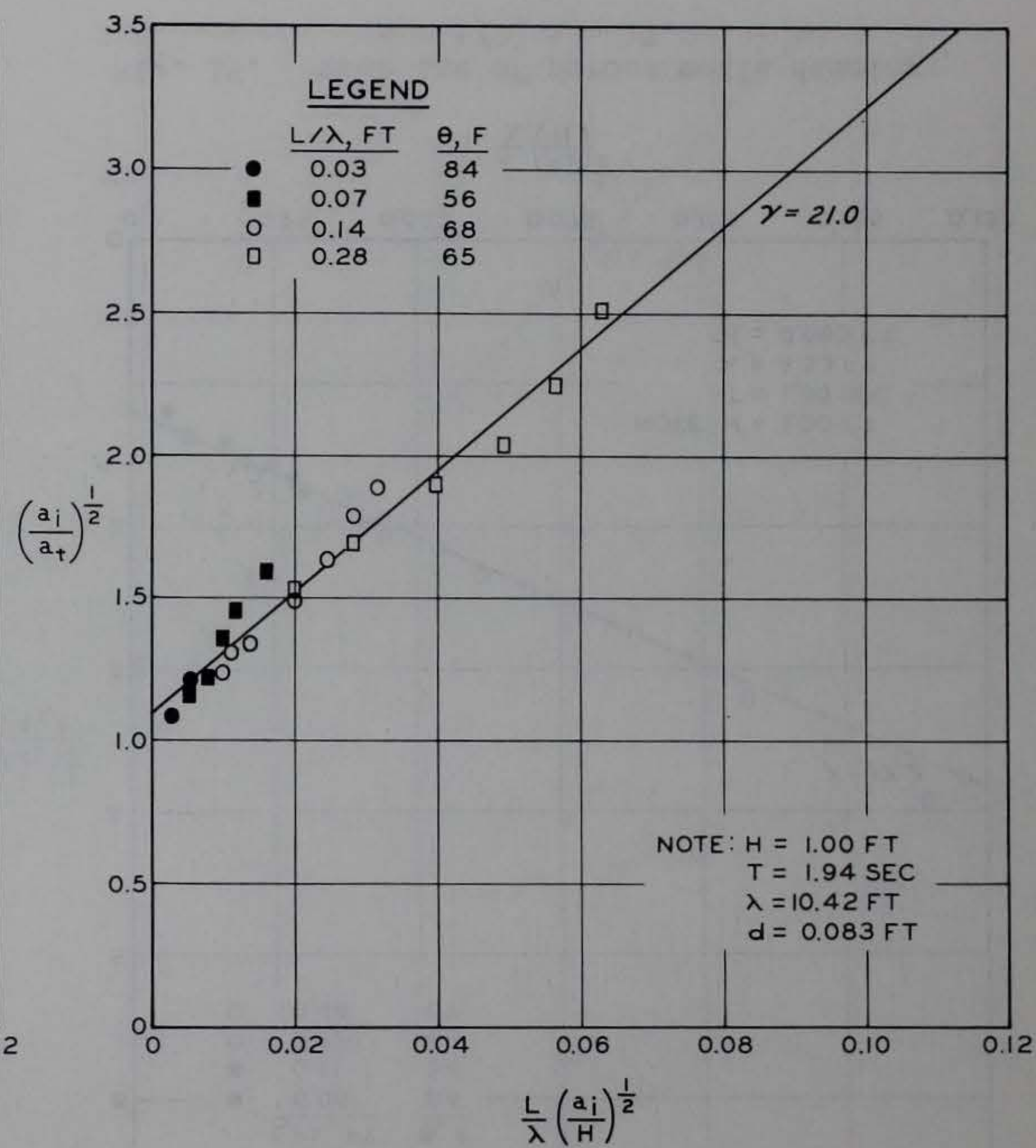


Fig. 17. Power law of porous media damping;
 $n = 1/2, \gamma = 21.0$

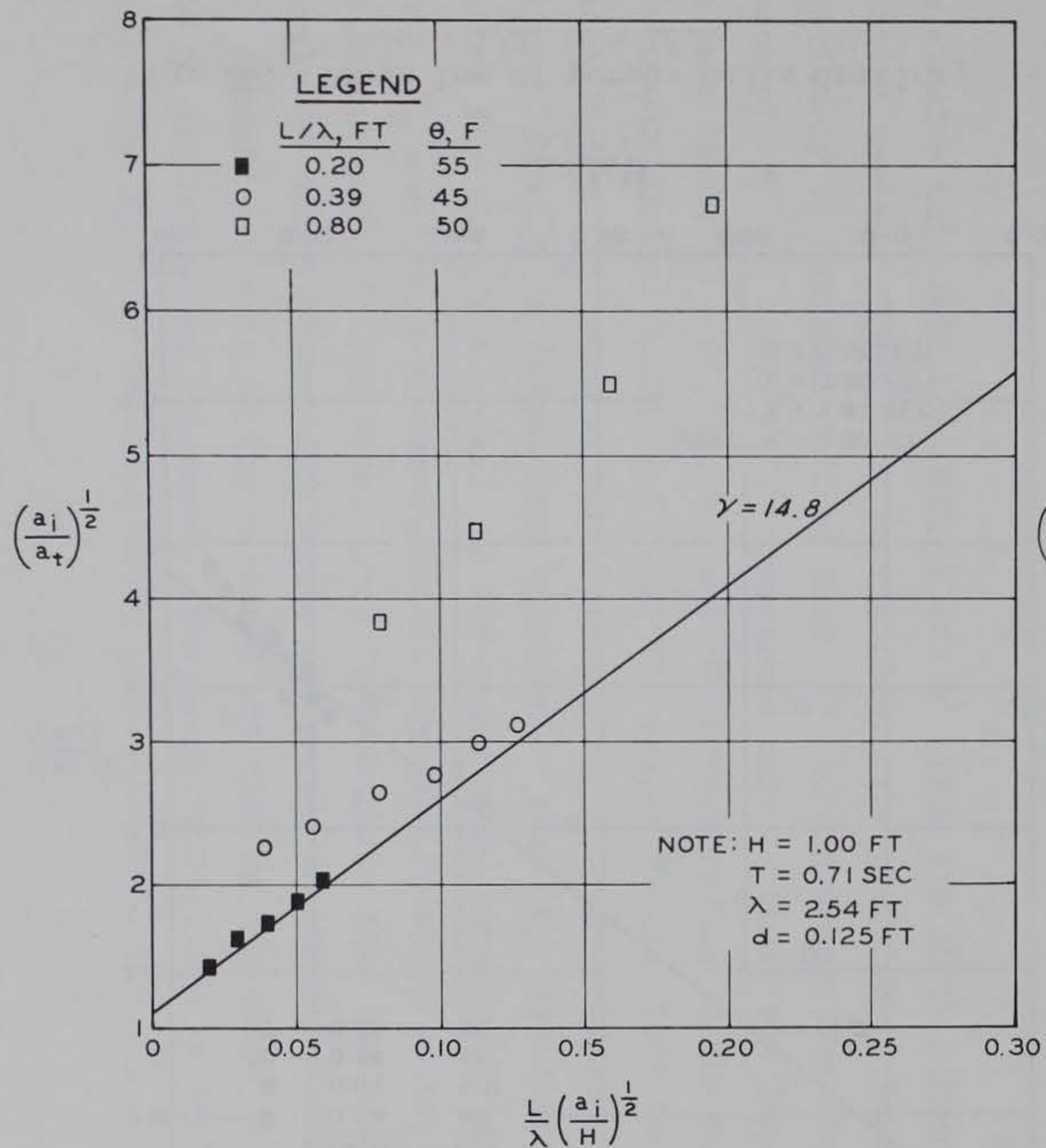


Fig. 18. Power law of porous media damping;
 $n = 1/2, \gamma = 14.8$

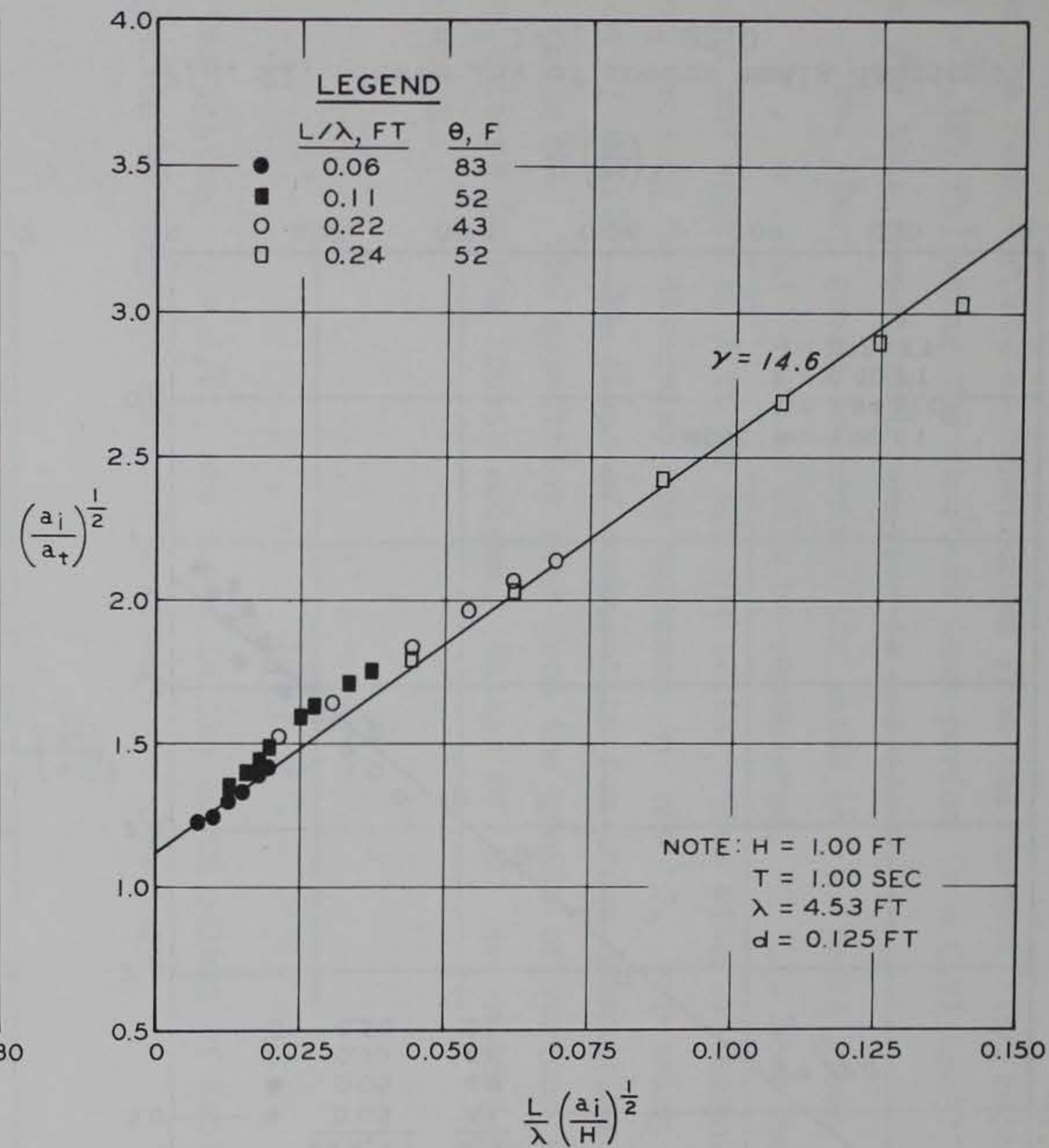


Fig. 19. Power law of porous media damping;
 $n = 1/2, \gamma = 14.6$

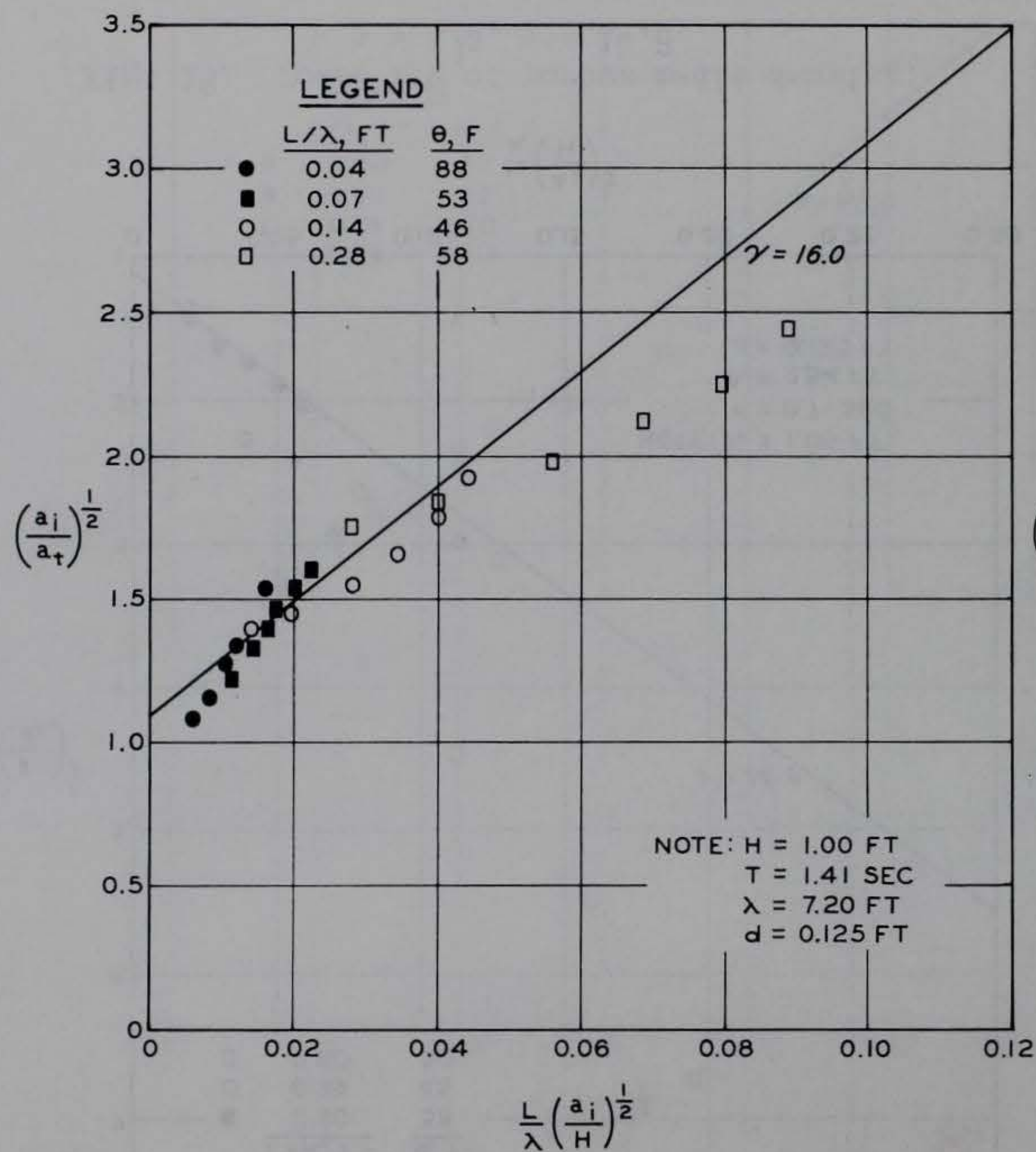


Fig. 20. Power law of porous media damping;
 $n = 1/2, \gamma = 16.0$

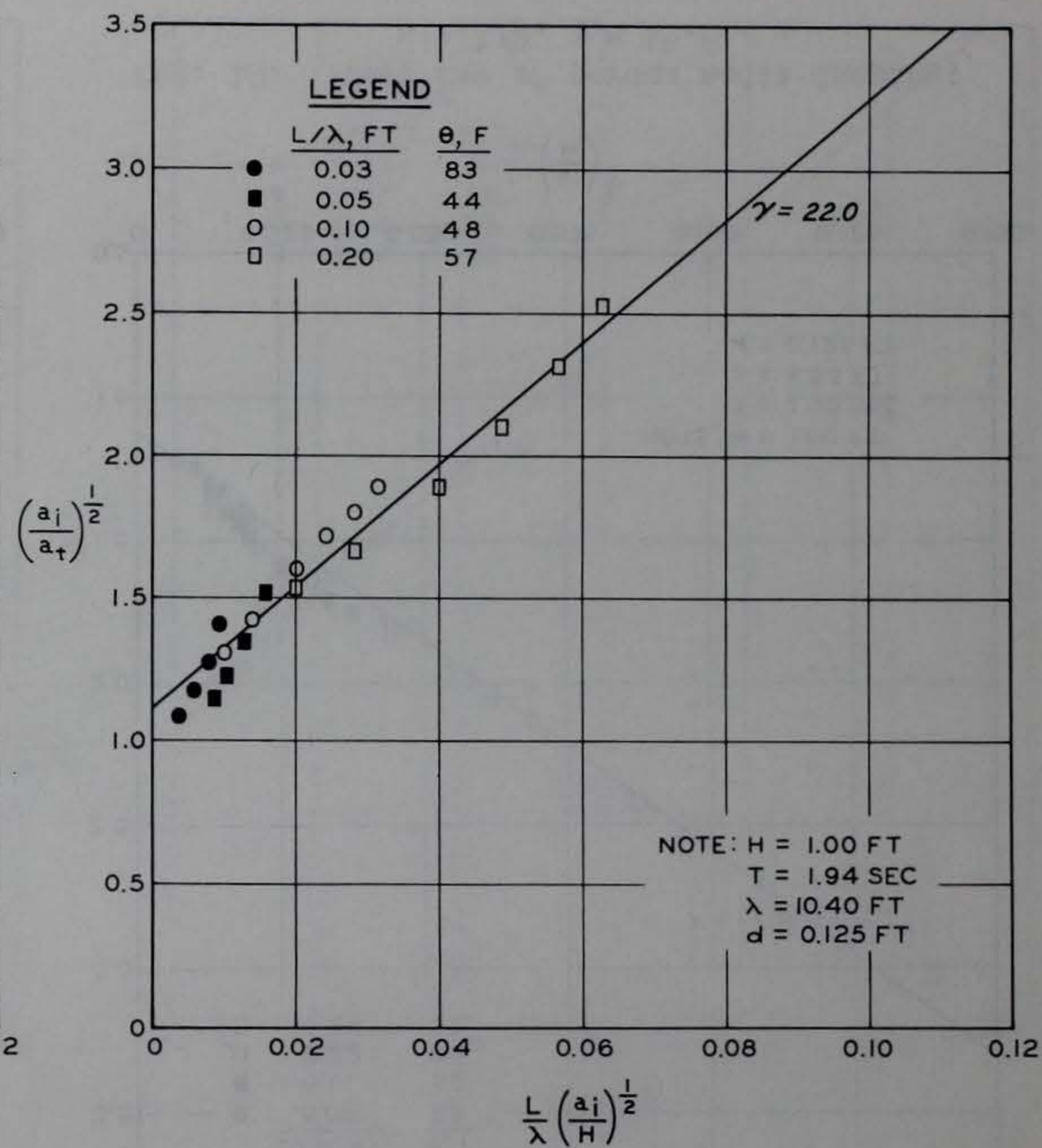


Fig. 21. Power law of porous media damping;
 $n = 1/2, \gamma = 22.0$

for the rock structures constructed from the largest size rock ($d = 0.125$ ft) using the shortest wave period ($T = 0.71$ sec). The reason for this exceptional behavior is not clear.

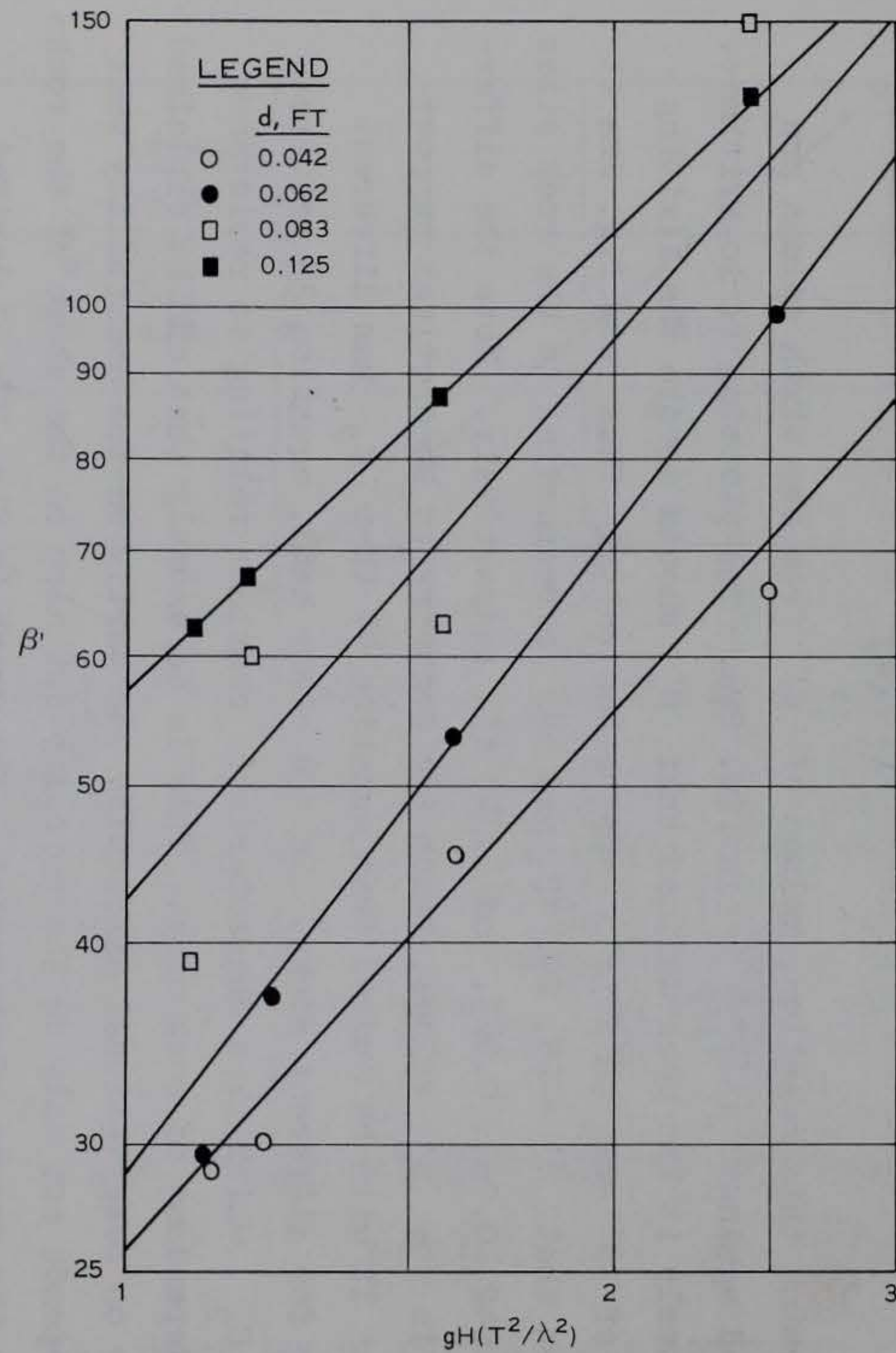
24. Each graph determines the value of γ for a given set of T , λ , v , and d . As will be remembered, H has been kept constant and equal to 1 ft in all the tests. Thus with γ known, we may next proceed to determine the exponent m and C_2 appearing in equation 53 (bis). The steps of computation for these quantities are shown in table 17. The quantities read from the graphs of various figures are entered in the fourth column. Values of β' determined from the relation

$$\beta' = \frac{\gamma}{\left(\frac{vT}{d\lambda}\right)^n \frac{\lambda}{d}}$$

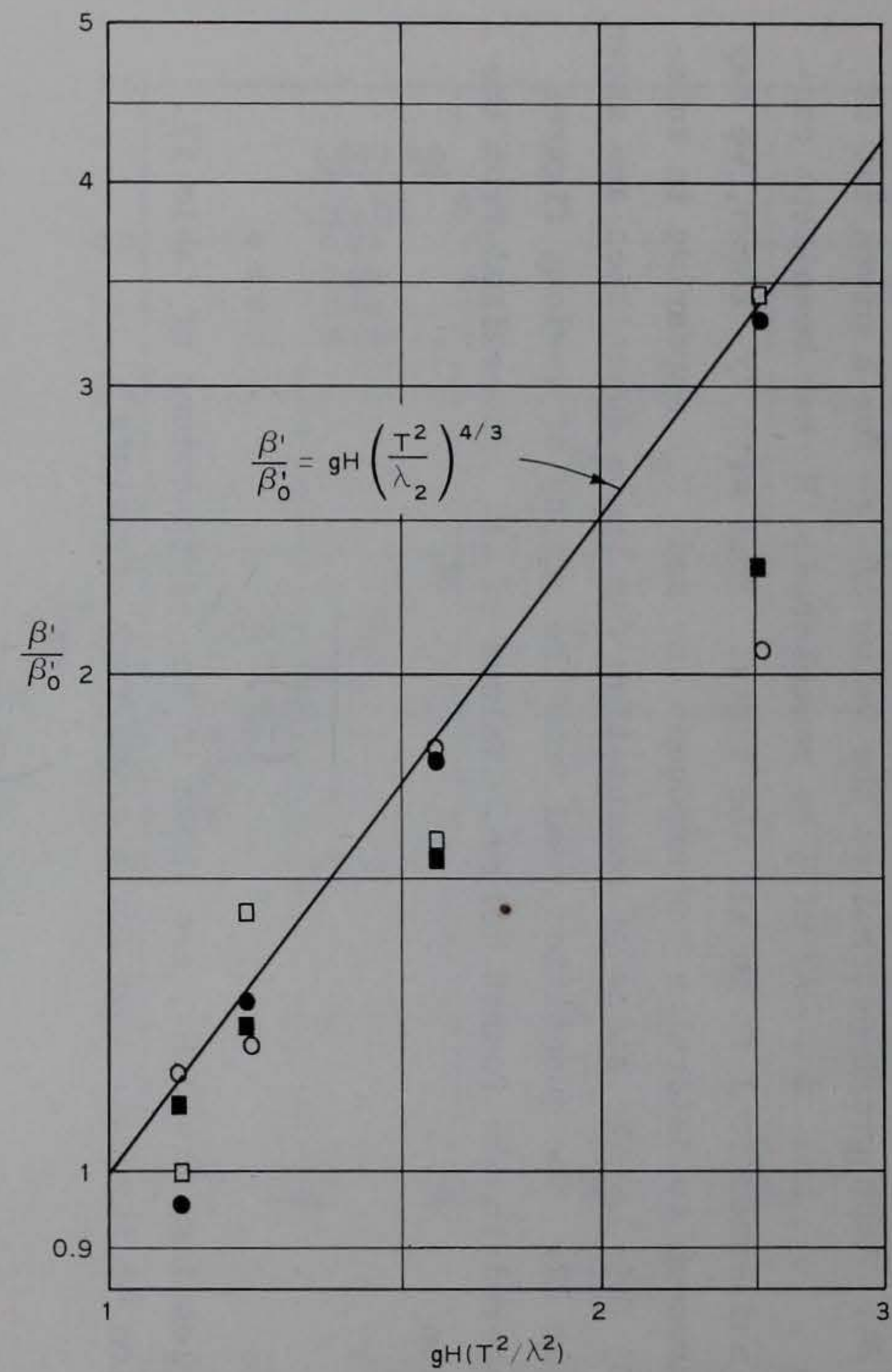
after letting $n = 1/2$, are shown in the sixth column of table 17. Equation 46 suggests that for a given rock structure

$$\beta' \sim \left(gH \frac{T^2}{\lambda^2}\right)^m$$

To examine this relation, values of β' from the sixth column are plotted against $gH(T^2/\lambda^2)$ in fig. 22a. The plotting is logarithmic. Previously it has been defined that β'_0 should denote the limiting value of β' as $gH(T^2/\lambda^2)$ approaches unity. Thus from fig. 22a it is seen that $\beta'_0 = 25, 29, 42$, and 58 corresponding to the rock sizes $d = 0.042, 0.062, 0.083$, and 0.125 ft, respectively. Note the differences in the β'_0 values, which are contrary to theoretical suppositions. It will be judged from equation 47 that C_2 has different values for different rocks. On the other hand, equation 35 requires that C_2 , which is a characteristic constant relating to resistance, be independent of rock size. This is an anomaly that can be explained partly on observational difficulty and partly on the eventuality that C_2 depends not only on the porosity but also on the shape of the rocks and on the manner of placement. The graph in fig. 22b is derived next. The slope of the line drawn indicates that $m = 4/3$. Using



a



b

Fig. 22. Effect of wave period on damping in porous media; $n = 1/2$

this, the entries of the seventh column of table 17 are computed from

$$\beta'_0 = \beta' \left(gH \frac{T^2}{\lambda^2} \right)^{-4/3} \quad (48 \text{ bis})$$

It has been indicated by equation 47 that

$$C_2 = \frac{2\beta'_0}{(1-n)N\epsilon} \quad (47 \text{ bis})$$

Letting $n = 1/2$, $N = 1$, and $\epsilon = 0.46$,

$$C_2 = 8.7\beta'_0$$

The values of C_2 computed by this expression are shown in the eighth column of table 17. Accordingly, for the rocks with sizes $d = 0.042$, 0.062 , 0.083 , and 0.125 ft, $C_2 = 198$, 242 , 343 , and 414 , respectively. Taking the mean, the resistance law on the basis of the present analysis would be

$$C_2 = 299 \left(\frac{ud}{v} \right)^{-1/2} \quad (54)$$

Thus, the power law of the transmission coefficients relating to rocks of porosity $\epsilon = 0.46$ becomes

$$\left(\frac{a_i}{a_t} \right)^{1/2} = 1 + \gamma \left(\frac{a_i}{H} \right)^{1/2} \frac{L}{\lambda} \quad (55)$$

with

$$\gamma = 34.3 \left(\frac{vT}{d\lambda} \right)^{1/2} \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2} \right)^{4/3} \quad (56)$$

The coefficient 34.3 of the latter expression is the mean value of all β'_0 values shown in table 17.

25. The power law expression just given is not a unique determination. In the above, n was taken equal to $1/2$, but almost an equally serviceable representation can be established by taking n equal to $1/3$

or $1/4$. The reason for this is that the fractions a_i/H and a_t/a_i fall within limited ranges, and experimental data are not always free of observational uncertainties and errors. Suppose that n is taken equal to $1/3$ and the data establish the relation

$$\left(\frac{a_i}{a_t}\right)^{2/3} = 1 + \gamma_o \left(\frac{a_i}{H}\right)^{2/3} \frac{L}{\lambda}$$

Placing $(a_i/a_t)^{1/6} = 1 + \Delta_1$ and $(a_i/H)^{1/6} = 1/2(1 + \Delta_2)$, the last relation becomes

$$(1 + \Delta_1) \left(\frac{a_i}{a_t}\right)^{1/2} = 1 + \frac{1}{2} \gamma_o (1 + \Delta_2) \left(\frac{a_i}{H}\right)^{1/2} \frac{L}{\lambda}$$

or

$$\left(\frac{a_i}{a_t}\right)^{1/2} = (1 - \Delta_1) + \frac{1}{2} \gamma_o (1 + \Delta_2 - \Delta_1) \left(\frac{a_i}{H}\right)^{1/2} \frac{L}{\lambda}$$

If a_t/a_i falls in the range from 0.05 to 1.00 and a_i/H , from 0.01 to 0.10, then Δ_1 and Δ_2 are fractions less than unity. In these ranges the average values of Δ_1 and Δ_2 are about 0.30 and 0.25, respectively. To the degree that these quantities may be neglected with respect to unity

$$\left(\frac{a_i}{a_t}\right)^{1/2} = 1 + \frac{1}{2} \gamma_o \left(\frac{a_i}{H}\right)^{1/2} \frac{L}{\lambda} \quad (57)$$

which suggests that by using a given set of observational data we may obtain two power law expressions for the transmission coefficient with $n = 1/2$ or $n = 1/3$.

26. This fact is illustrated in the graphs of figs. 23-26, where the data relate to the transmissions observed with the structures constructed of rocks with $d = 0.062$ ft. In these figures, the quantity $(a_i/a_t)^{2/3}$ is plotted against $(a_i/H)^{2/3} L/\lambda$. Note that in equation 52 (bis), n is now taken to equal $1/3$. The alignments of the points in these four figures are practically linear and the overall distributions are similar to those shown in figs. 10-14. The γ values

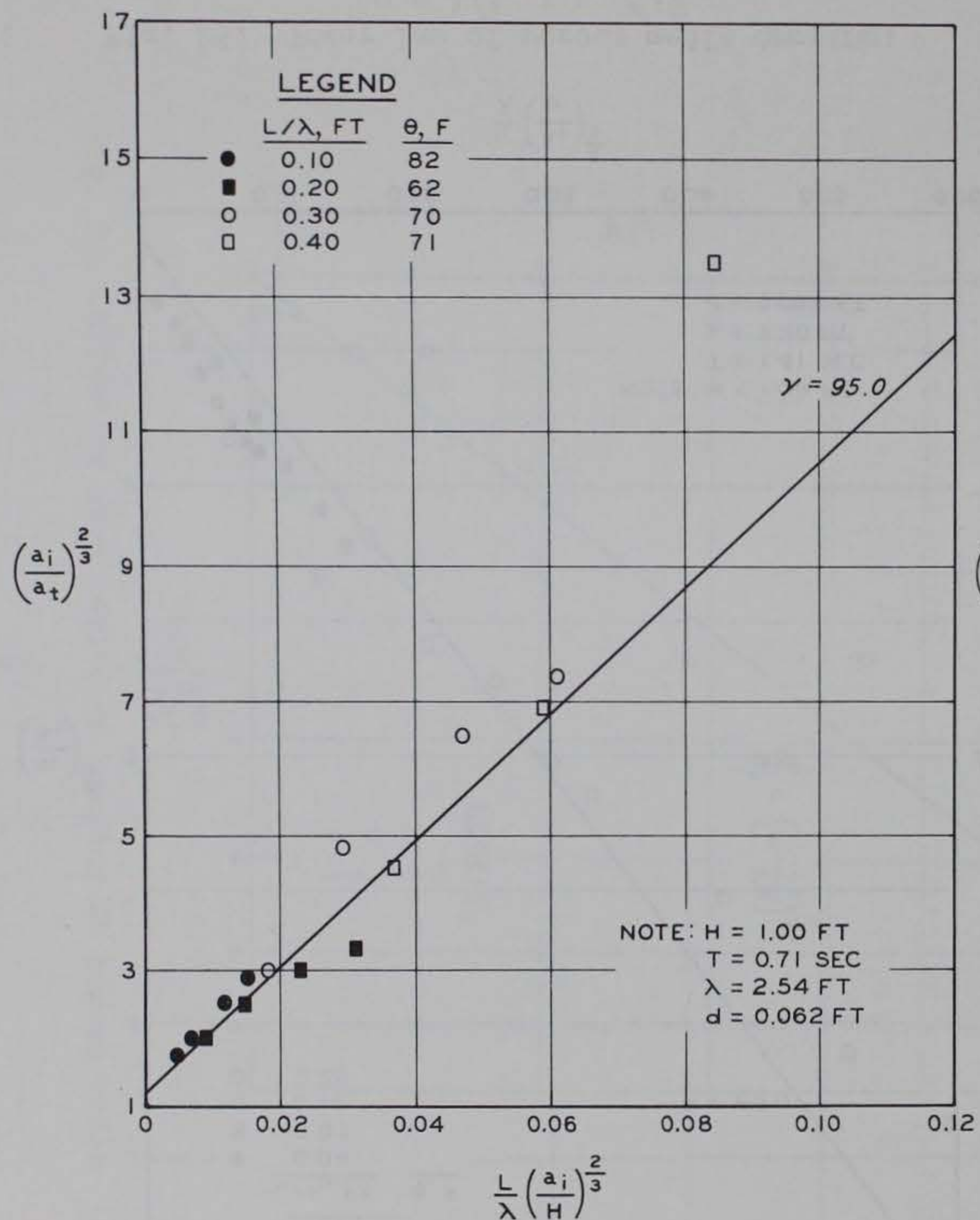


Fig. 23. Power law of porous media damping;
 $n = 1/3, \gamma = 95.0$

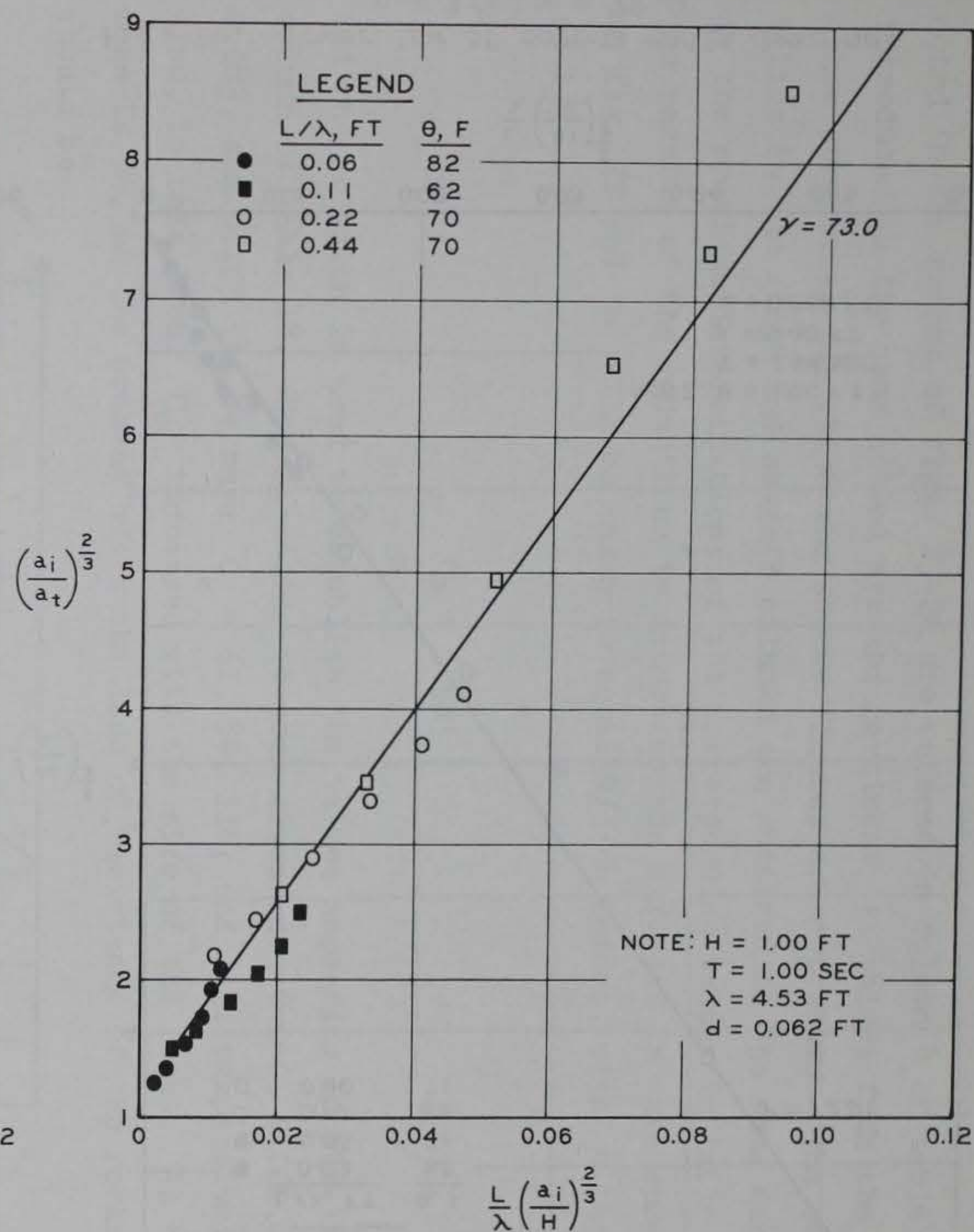


Fig. 24. Power law of porous media damping;
 $n = 1/3, \gamma = 73.0$

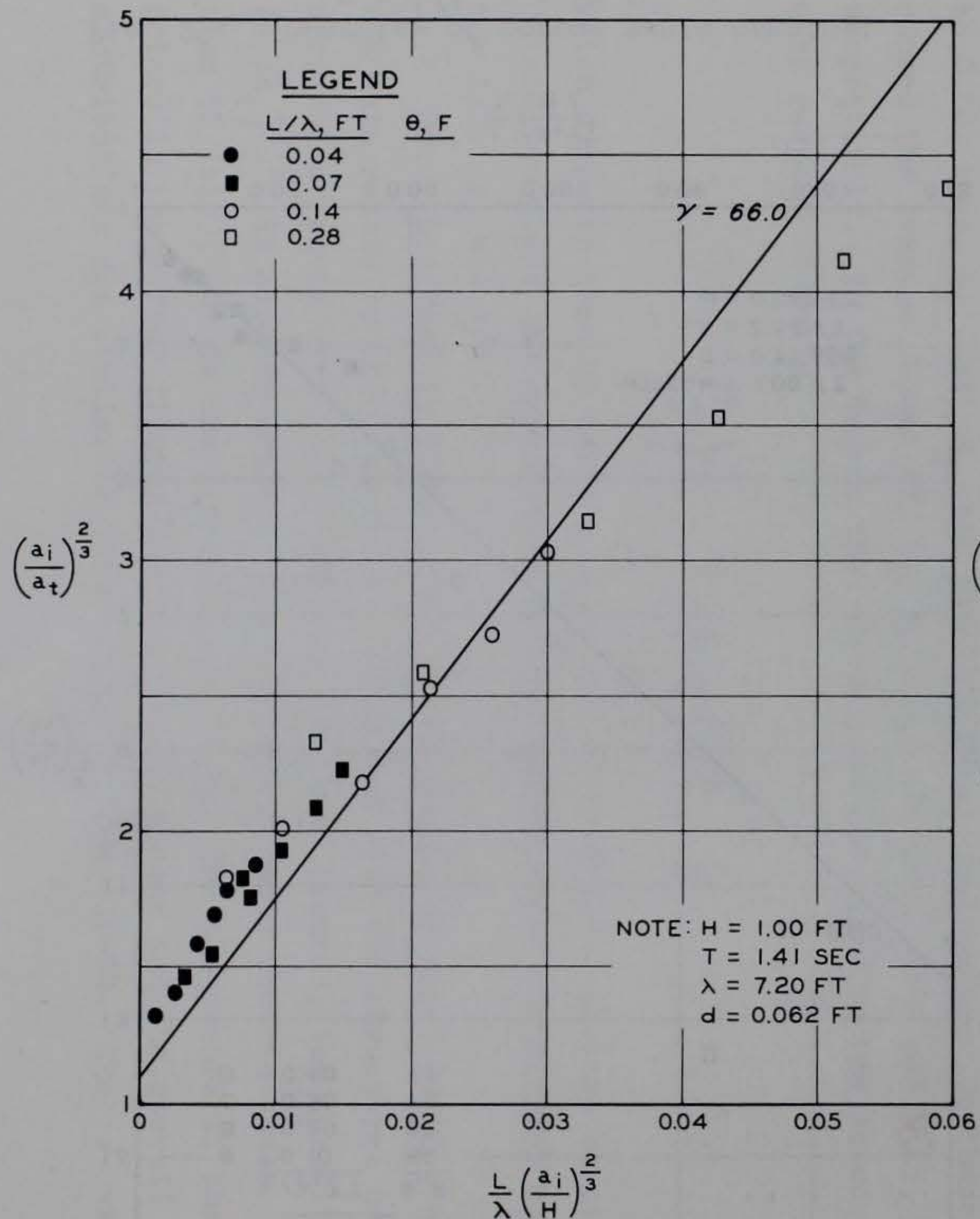


Fig. 25. Power law of porous media damping;
 $n = 1/3, \gamma = 66.0$

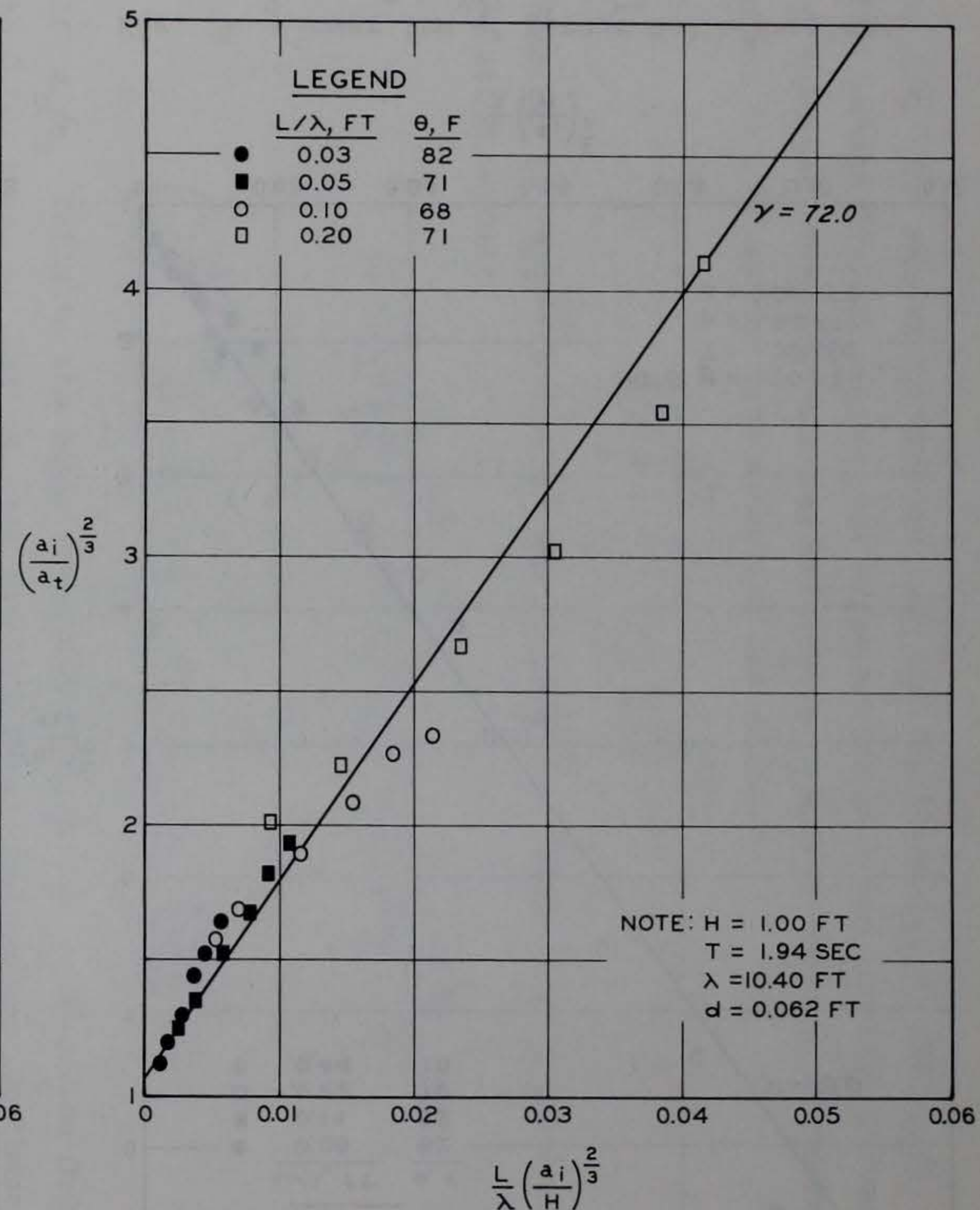


Fig. 26. Power law of porous media damping;
 $n = 1/3, \gamma = 72.0$

noted in the graphs of figs. 23-26 are entered in column 4 of table 18. Introduced in the same column are the pertinent γ values from the other structures for which the source figures are not shown. The quantities in the sixth and seventh columns are obtained in the same manner as the previous determinations of the corresponding values in table 17. The results of the reductions to determine the exponent m are shown in fig. 27 and m will be taken to equal $4/3$ as before, with n equal to $1/3$

$$C_2 = 6.5\beta'_0$$

and the relation is used to establish the values appearing in the last column of table 18. Corresponding to rock sizes $d = 0.042, 0.062, 0.083, \text{ and } 0.125 \text{ ft}$ we have $C_2 = 61, 89, 116, \text{ and } 118$, respectively. Also, in this case, C_2 increased with the size of the rocks. As the mean is 96, on the present basis the loss of resistance of the rocks should be

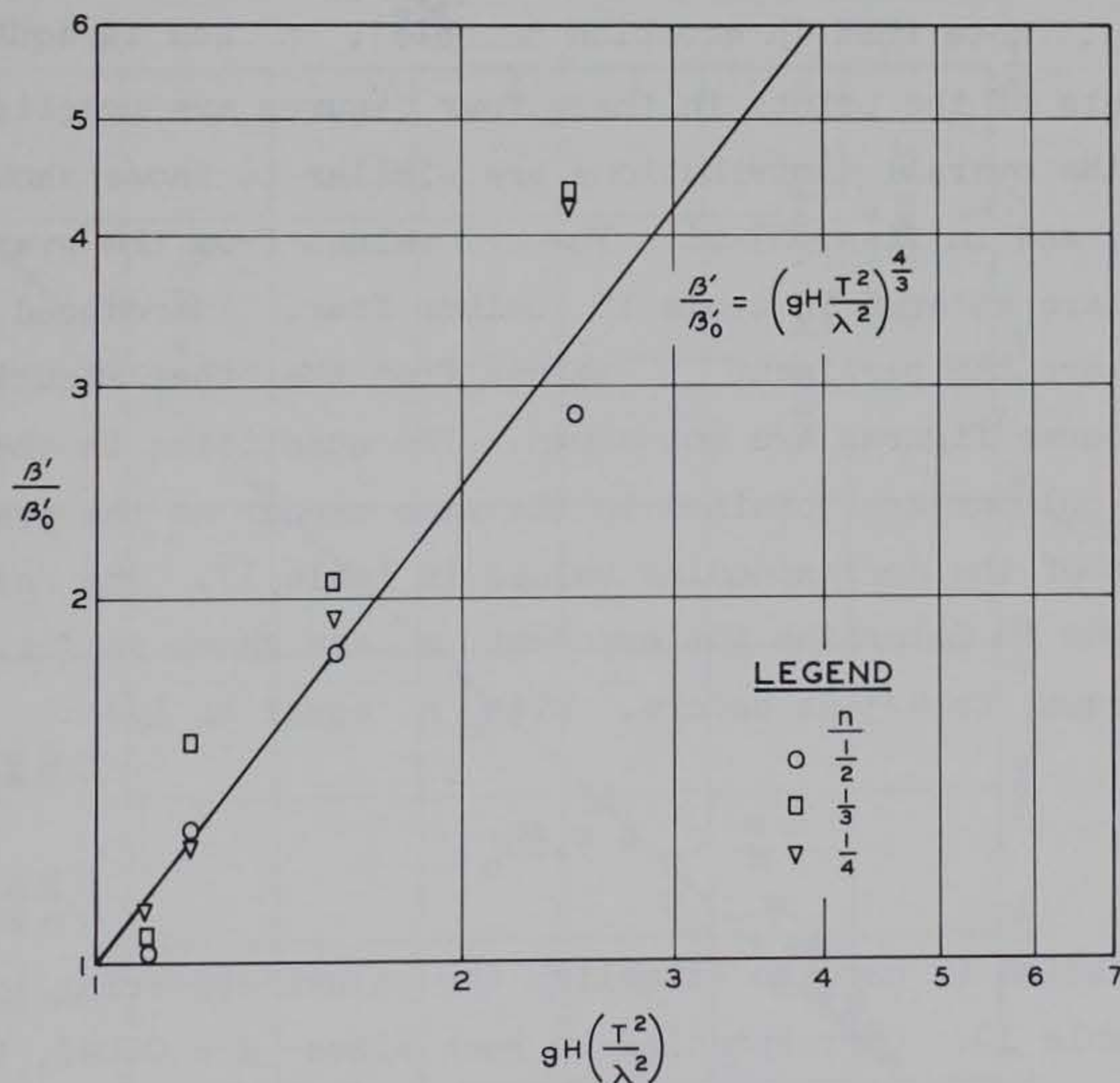


Fig. 27. Effect of wave period on damping in porous media

$$c_f = 96 \left(\frac{ud}{v} \right)^{-1/3} \quad (58)$$

Thus, the power law of the transmission coefficients relating to rocks with porosity $\epsilon = 0.46$ becomes

$$\left(\frac{a_i}{a_t} \right)^{2/3} = 1 + \gamma \left(\frac{a_i}{H} \right)^{2/3} \frac{L}{\lambda} \quad (59)$$

with

$$\gamma = 14.7 \left(\frac{vT}{d\lambda} \right)^{1/3} \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2} \right)^{4/3} \quad (60)$$

The coefficient 14.7 of the latter expression is the mean of β'_0 shown in table 18.

27. Consider further the illustrations given in figs. 28-31, where the data relate to the transmissions observed for rock structures with $d = 0.062$ ft. In these figures, $(a_i/a_t)^{3/4}$ is plotted against $\frac{L}{\lambda} (a_i/H)^{3/4}$. Note that in equation 52 (bis), n now is equal to $1/4$. The alignments of the points in these four figures are practically linear and the overall distributions are similar to those shown in figs. 10-14, and in figs. 23-26. The γ values from the graphs of figs. 23-26 are entered in table 19, column four. Introduced in the same column are the pertinent γ values from the other structures for which the source figures are not shown. The quantities in the sixth and seventh columns are obtained in the same manner as the previous determinations of the corresponding values in table 17. The results of the reductions to determine the exponent m are shown in fig. 27 and m will be equal to $4/3$ as before. With n equal to $1/4$

$$c_2 = 5.8\beta'_0$$

and this relation is used to establish the values appearing in the last column of table 19. Corresponding to rock sizes $d = 0.042, 0.062, 0.083$, and 0.125 ft we have $c_2 = 46, 55, 71$, and 73 , respectively.

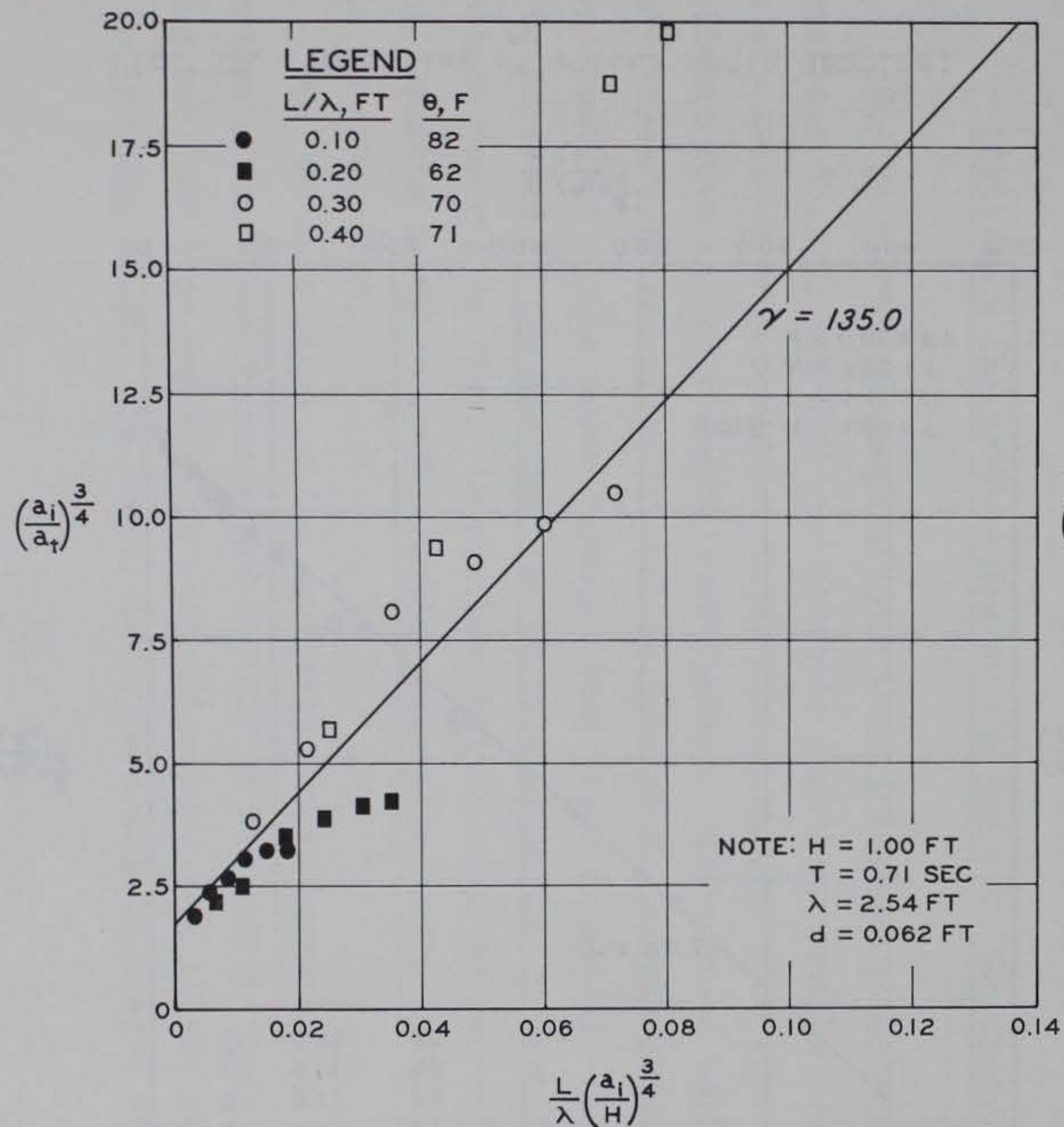


Fig. 28. Power law of porous media damping;
 $n = 1/4, \gamma = 135.0$

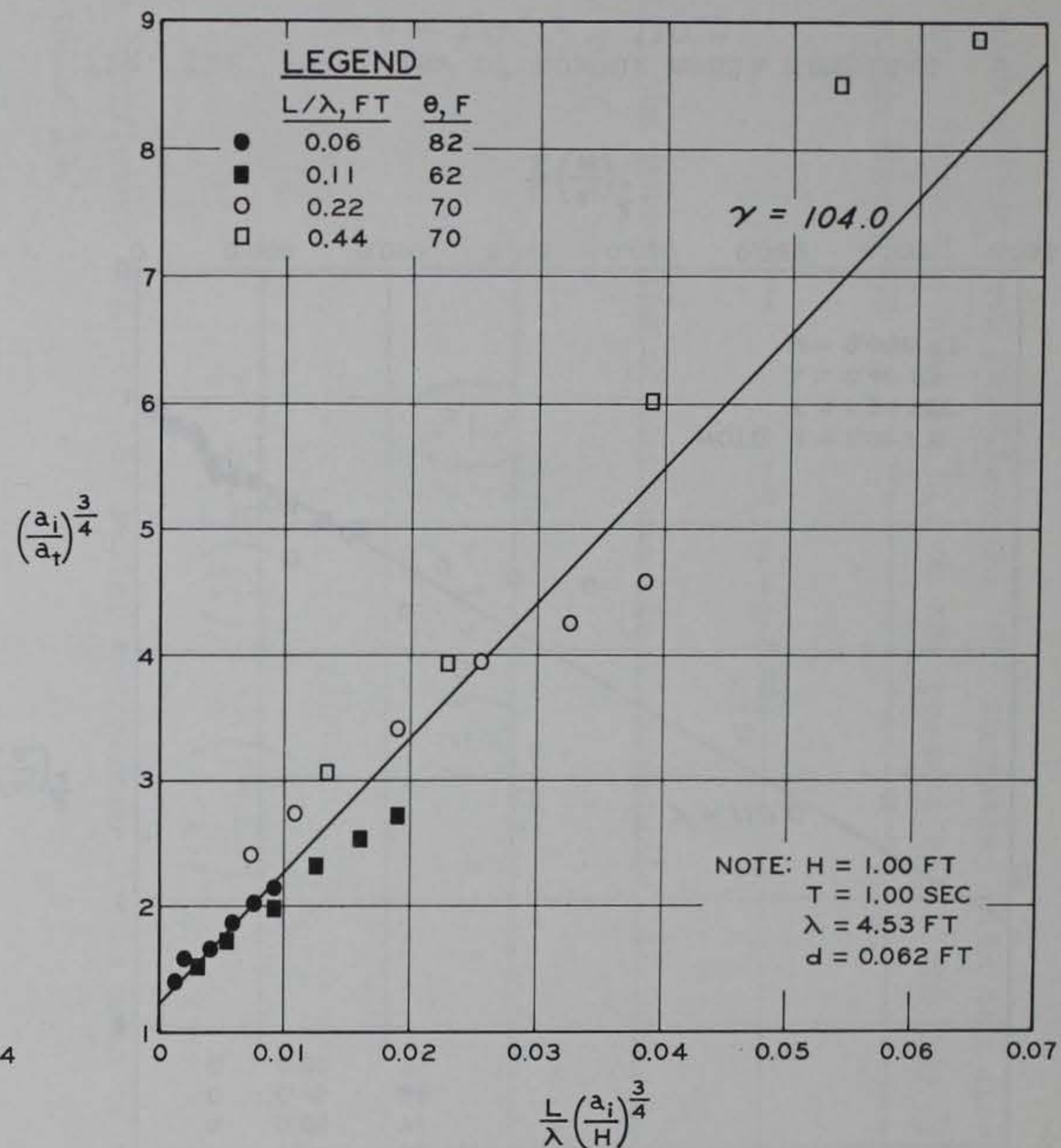


Fig. 29. Power law of porous media damping;
 $n = 1/4, \gamma = 104.0$

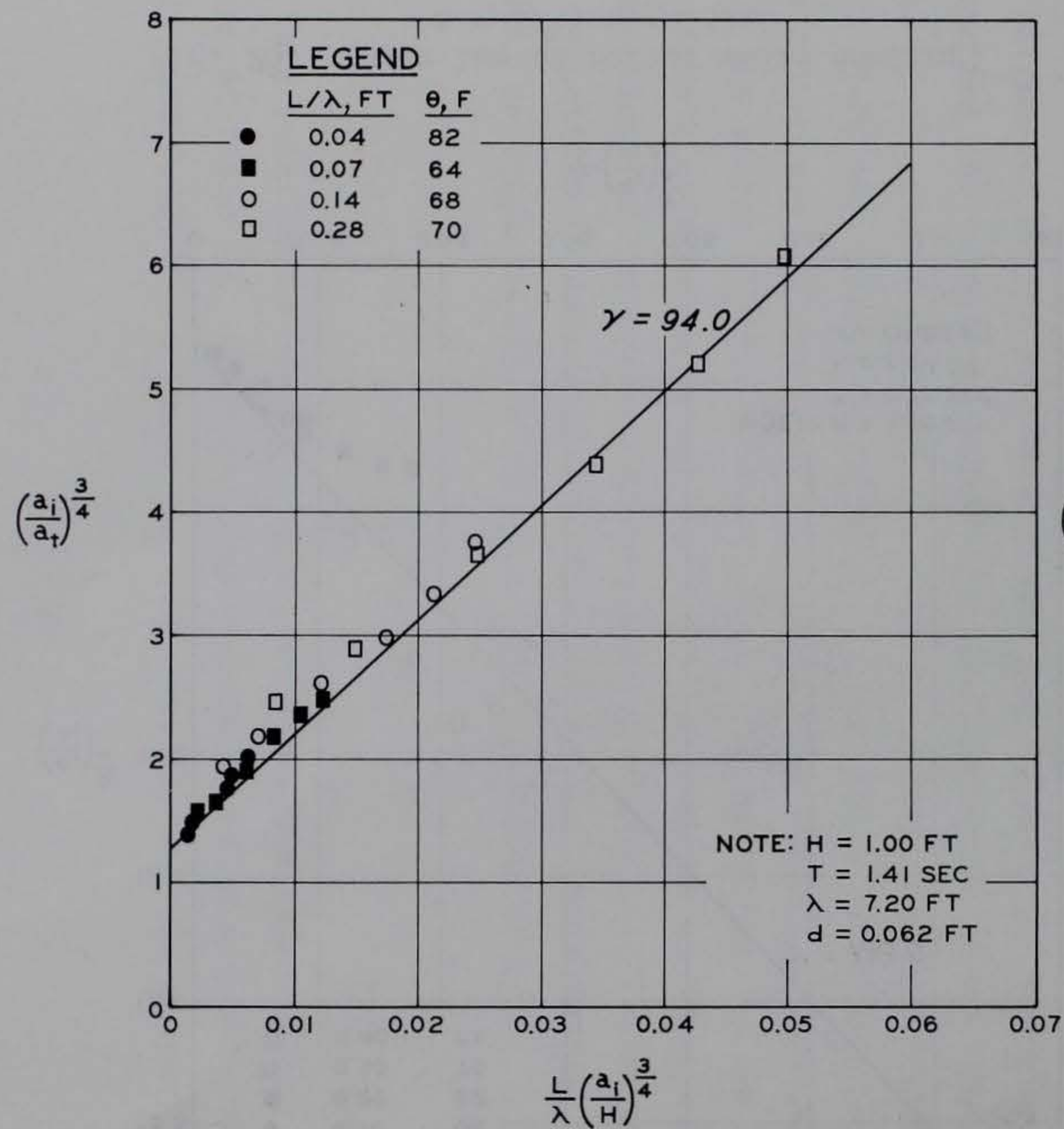


Fig. 30. Power law of porous media damping;
 $n = 1/4, \gamma = 94.0$

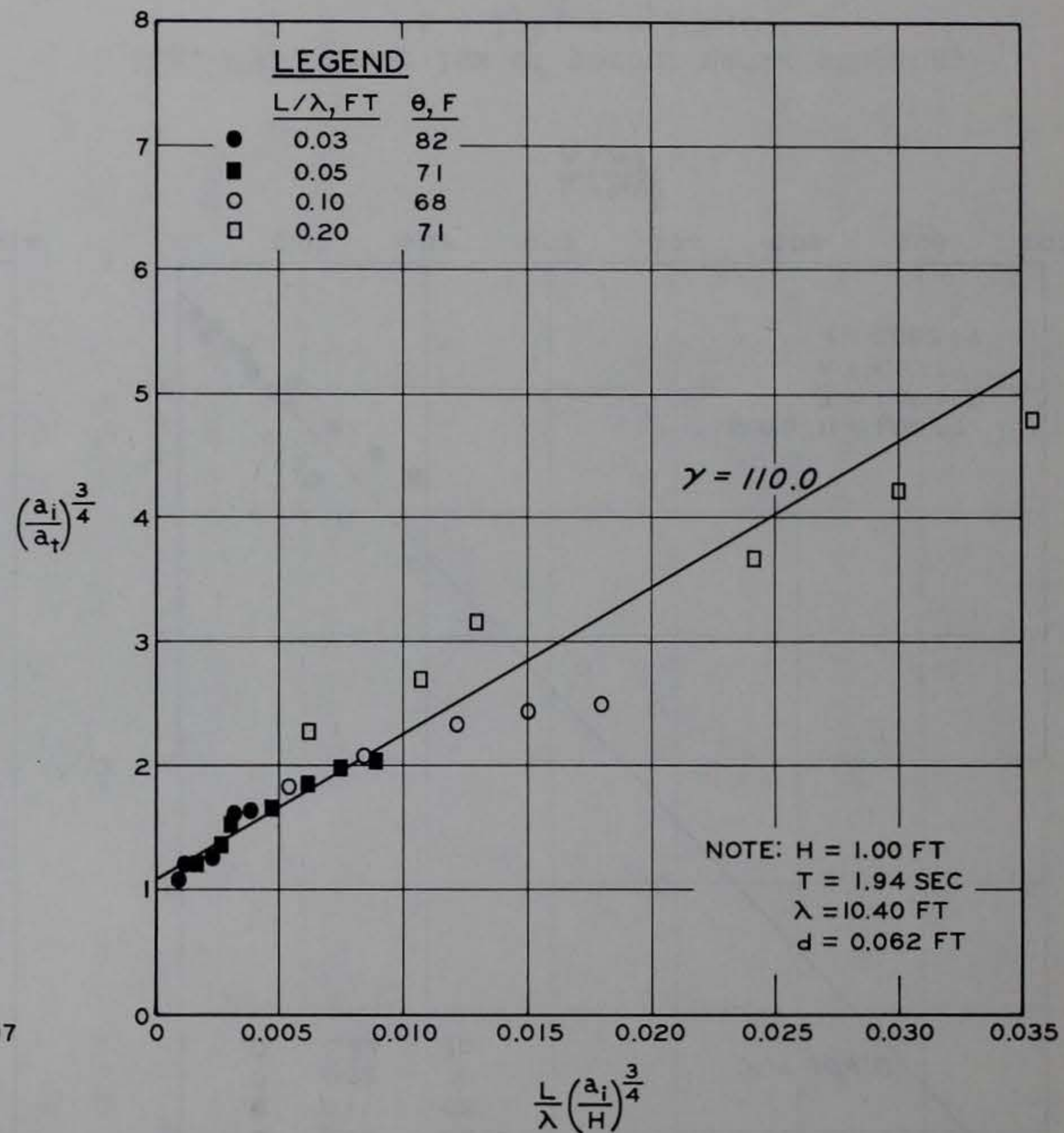


Fig. 31. Power law of porous media damping;
 $n = 1/4, \gamma = 110.0$

We also notice that in this case C_2 increases with the size of the rocks. Since the mean is 61, on the present basis the law of resistance of rocks should be

$$C_f = 61 \left(\frac{ud}{v} \right)^{-1/4} \quad (61)$$

Thus, the power law of the transmissions relating to rocks with porosity $\epsilon = 0.46$ becomes

$$\left(\frac{a_i}{a_t} \right)^{3/4} = 1 + \gamma \left(\frac{a_i}{H} \right)^{3/4} \frac{L}{\lambda} \quad (62)$$

with

$$\gamma = 10.4 \left(\frac{vT}{d\lambda} \right)^{1/4} \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2} \right)^{4/3} \quad (63)$$

The coefficient 10.4 of the latter expression is the mean of β'_0 shown in table 19.

Resistance Characteristics of the Rocks in the Study

28. In the above computations three different power formulas (equations 55, 59, and 61) are obtained, each representing adequately the attenuation of progressive waves moving through rock structures of prismatic form in the present investigation. Since these individually imply three different laws of rock resistance for steady flow, it is obvious that two of the power formulas need to be rejected. The formula to be accepted is the one that can be related to the correct law of resistance for steady flow. We now see that it would be necessary to conduct separate tests to establish the resistance of the rocks used in this investigation.

29. At this stage of the analysis, it was requested of Mr. Donald Davidson of the Wave Dynamics Branch that he reconstruct rock structures using the same type and size of rocks that were used in the structures of the present study. The actual tests were made under the direct

supervision of Mr. Davidson. All the structures were of the same size, 2 ft long, 1 ft wide, and 1 ft deep. The porosities were determined by the formula

$$G = \gamma_s (1 - \epsilon)V \quad (64)$$

where G denotes the weight of rocks, V , the volume of the structure, and γ_s , the specific weight of the rocks. The smaller rocks were of limestone, $\gamma_s = 165.5$ pcf; the larger ones of granite, $\gamma_s = 166.6$ pcf. The porosities of the rocks studied were

Sieve Size, in.	Rock Size, ft, d	Porosity, ϵ
1/8 to 1/4	0.021	0.497
3/8 to 1/2	0.042	0.491
1/2 to 3/4	0.062	0.470
7/8 to 1	0.083	0.487
1-1/4 to 1-1/2	0.125	0.489
2 to 2-1/2	0.209	0.462

The structures were fitted in the midportion of the wave flume used in this study. A known discharge Q (cfs) was introduced upstream of the structures and the same was removed from the downstream side. The mean water depth on the two sides was H , and ΔH is the difference in the elevations on the two sides. The resistance coefficient is computed as

$$C_f = \frac{\Delta H}{L} \frac{gd}{u^2}, \quad Q = uBH \quad (65)$$

The data from the measurements are shown in table 20, where C_f is given as a function of ud/ν for the different rock sizes.

30. An examination of the data reveals the fact that the coefficient of resistance at a given Reynolds number increases with the rock size. This is illustrated in the following tabulation for the Reynolds number $ud/\nu = 1000$ as inferred from the measurements.

Rock Size, ft, d	Resistance Coefficient, C_f	Proportionality Factor, N	Material
0.021	0.68	1.52	Limestone
0.042	0.88	1.17	Limestone
0.062	0.90	1.14	Limestone
0.083	0.90	1.14	Limestone
0.125	1.40	0.74	Granite
0.209	1.40	0.74	Granite

This variation of the resistance factor, an increase with increasing rock size, was also noted in the determinations of the wave transmission coefficient values. Obviously, factors other than the Reynolds number can have an influence, such as the porosity, the manner of placement, and the general shape of the rocks. Unable to pinpoint the pertinent factor, it should now suffice to normalize the observed data reducing to a common behavior. This is established by multiplying the observed data by the proportionality factor shown in the above tabulation. The normalized data are entered in table 21, which is the source of the plottings shown in fig. 32. Study of this figure reveals that for flows

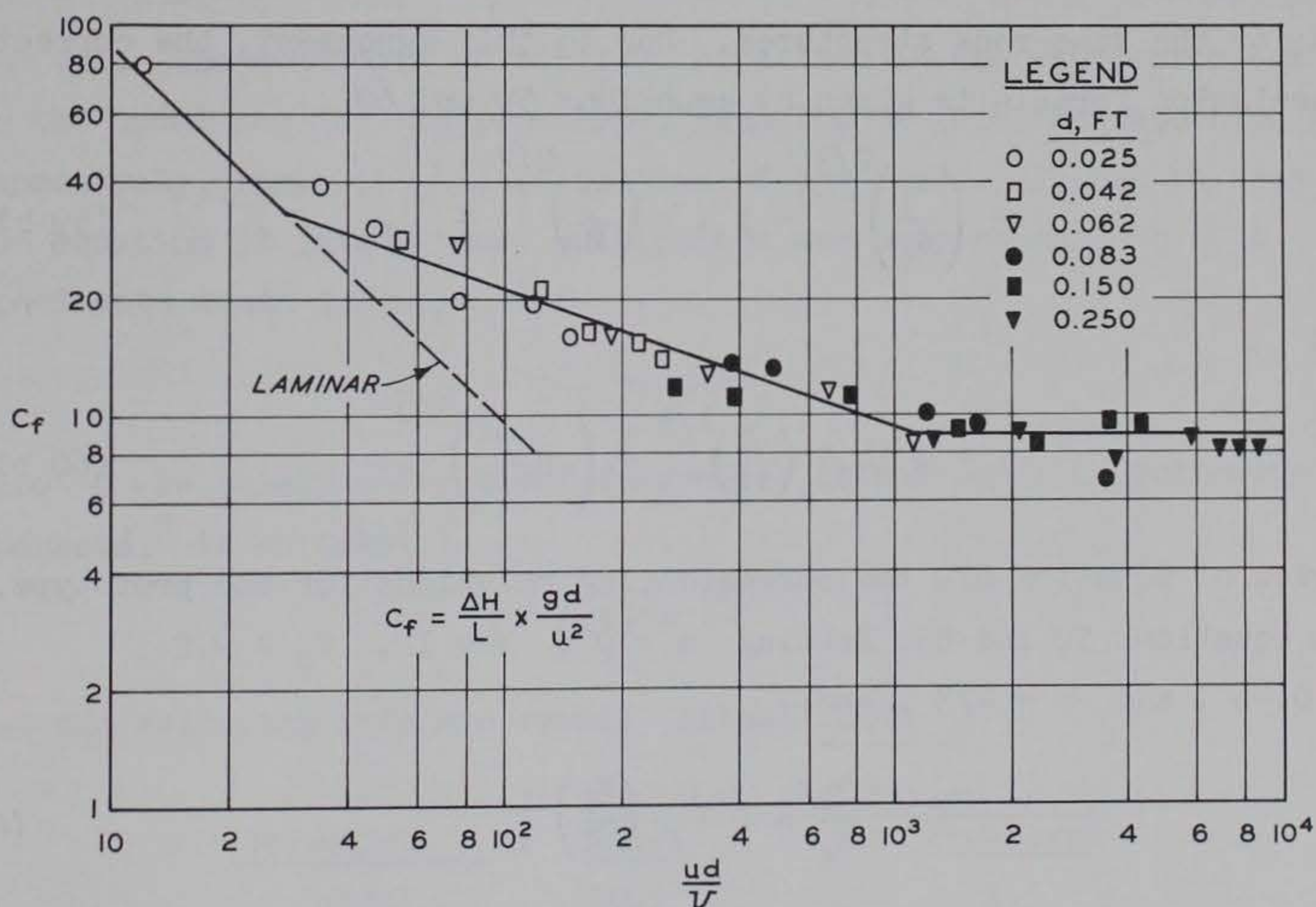


Fig. 32. Resistance coefficient of rocks, $\epsilon = 0.46$

with a Reynolds number less than 20 the flow is laminar and the coefficient of resistance is

$$C_f = 780 \left(\frac{ud}{\nu} \right)^{-1} \quad (66)$$

For the flows with a Reynolds number greater than 2000, the coefficient is independent of the Reynolds number and

$$C_f = 9.2 \quad (67)$$

For a Reynolds number lying between 20 and 2000,

$$C_f = 95 \left(\frac{ud}{\nu} \right)^{-1/3} \quad (68)$$

Formulas of Wave Transmission and Scale Effects

31. Curiously enough, the resistance law expressed by equation 68 is in agreement with equation 58 which was derived from the transmission data for the same rock structures. Due to this agreement, the correct transmission formula is given by equations 59 and 60

$$\left(\frac{a_i}{a_t} \right)^{2/3} = 1 + \gamma \left(\frac{a_i}{H} \right)^{2/3} \frac{L}{\lambda} \quad (59 \text{ bis})$$

with

$$\gamma = 14.7 \left(\frac{\nu T}{d \lambda} \right)^{1/3} \frac{\lambda}{d} \left(g H \frac{T^2}{\lambda^2} \right)^{4/3} \quad (60 \text{ bis})$$

In view of equation 67, the corresponding relations for the prototype, from equations 52 and 53, letting $n = 0$, $N = 1$, $C_2 = 9.2$, $\epsilon = 0.46$, and $m = 4/3$, are

$$\frac{a_i}{a_t} = 1 + \gamma \left(\frac{a_i}{H} \right) \frac{L}{\lambda} \quad (69)$$

and

$$\gamma = 2.11 \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2} \right)^{4/3} \quad (70)$$

These equations are fundamental for the problem of scale effects. The procedure to determine what rock size in the model will assure similarity in the wave transmission would be as follows. Using prototype values for λ , T , d , and H the ratio a_i/a_t is obtained from equation 69. Inserting this value in equation 59 (bis) the model value of d is obtained from this very equation. To make further progress in this direction, it will be necessary to firmly establish C_f for the large prototype rocks. Having learned with sufficient detail the resistance characteristics of large rocks, it is our aim to take up at some future time the problem of scales and to compare the ideas and findings with the significant deliberations of LeMéhauté⁵ regarding this matter.

Effect of Porosity of Rocks on Transmission

32. The numerical multipliers appearing in the expressions of γ for the model and for the prototype, equation 60 (bis) and equation 70, respectively, vary with the porosities of the rocks. As can be seen from equation 53 (bis), these multipliers are proportional to C_2^ϵ . Experiments would indicate that

$$C_2 = C f(\epsilon)$$

where C is solely a function of Reynolds number ud/ν . Following LeMéhauté,⁵ if we take

$$f(\epsilon) = \epsilon^{-5}$$

then the following sets are readily established:

<u>Porosity, ϵ</u>	<u>Numerical Multiplier in γ</u>	
	<u>Model</u>	<u>Prototype</u>
0.50	10.5	1.51
0.46	14.7	2.11

(Continued)

Porosity, ϵ	Numerical Multiplier in γ	
	Model	Prototype
0.45	16.1	2.31
0.40	25.7	3.70
0.35	43.8	6.29
0.30	81.2	11.65

Reexamination of Kamel's Data for Transmissions
Through Structures Composed of Cubes

33. The formula proposed by Kamel¹ for the transmission of waves is at variance with the one developed in the present study. Kamel's study relates to laboratory structures made of cubes or spheres of uniform size. Focusing the attention on the cubes, there should be an interest in comparing the transmissions through different structures composed of cubes and of rocks. This interest arises from the fact that unlike the cubes, the rocks of uniform size established by sieving differ in their individual shapes. Kamel's data on cubes are not sufficiently extensive. Cube sizes were limited in number. With a given structure length and a given period, the transmissions were noted only for three initial wave heights. In analyzing these data, using the observed initial wave heights a_i and the transmission a_t , the corresponding coefficient γ was determined using equation 69. The average values of γ for the three initial wave heights were taken and these are shown in table 22. Kamel does not specifically mention the temperatures. Assuming $\theta = 75^\circ \text{ F}$, $\nu = 1 \times 10^{-5}$. Using equation 55, the quantity β' was determined which is in the fifth column of table 22. Using equation 58 and putting $m = 4/3$ in the latter, the quantity β'_0 is determined and these are given in the sixth column of table 22. The quantity C_2 in the last column of table 22 is obtained using equation 58, putting in the latter $N = 1$, $n = 1/3$, and $\epsilon = 0.40$. The mean value of C_2 is 156 and according to transmissions observed, the law for the resistance of flow through cubes of the porosity $\epsilon = 0.40$ would be

$$C_f = 156 \left(\frac{ud}{\nu} \right)^{-1/3} \quad (71)$$

As the mean value of β'_0 is 21.0, the transmissions are given by

$$\left(\frac{a_i}{a_t}\right)^{2/3} = 1 + \gamma \left(\frac{a_i}{H}\right)^{2/3} \frac{L}{\lambda} \quad (59 \text{ ter})$$

where

$$\gamma = 21.0 \left(\frac{vT}{d\lambda}\right)^{1/3} \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2}\right)^m \quad (60 \text{ ter})$$

Comparing cubes with rocks, equations 59 (bis) and 60 (bis), it is seen that these results are compatible with each other except that the resistance for the cubes is somewhat higher. Note that for the cubes the porosity $\epsilon = 0.40$ and for the rocks used in the present study $\epsilon = 0.46$.

Reflections

34. We are not in a position at the present time to offer a theoretical determination of the reflections by a rock structure. From the data given in tables 1-16, apparently a consistent and well-discernible dependence of the reflections on the initial wave heights for a given structure and a given wave period does not exist. This was mentioned before. In the case of L/λ constant for a given T , the variation of a_r/a_i from the mean is small. Thus, it is appropriate to consider these mean values which are shown in the sixth column of tables 1-16. The ratio a_r/a_i from this source is plotted against L/λ in fig. 33. Here λ is the wavelength of the incident waves. The points in the figure are further identified regarding the rock sizes of the structures. Some scattering of the data points is noticeable, but this cannot be attributed to the rock sizes. The curve indicates that as L/λ is increased from 0 to 0.20 the reflection coefficient, starting with the least value 0.3, increases to a maximum value of 0.65. For L/λ greater than 0.20, there is a slight decrease and finally it nears 0.58 asymptotically. When the length of the rock structure is small in comparison with the wavelength of the incident wave, an approximate

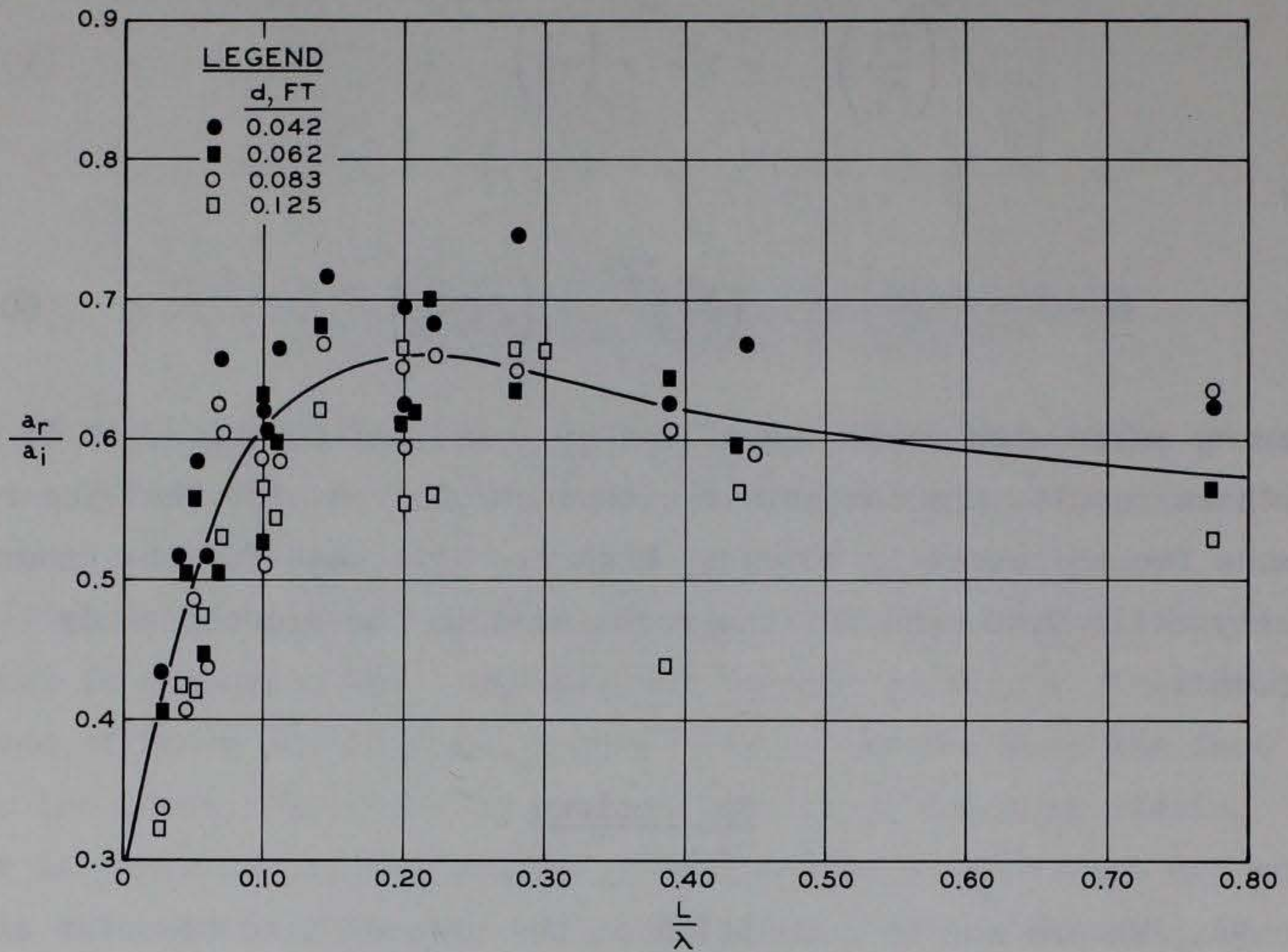


Fig. 33. Variation of reflection coefficient with structure relative length

analysis of the reflection from the structure may be carried out. It is supposed that this is analogous to the reflections noted in channels with sudden changes in the cross section. If the depth of water in the channel remains constant, the appropriate relation is

$$\frac{a_r}{a_i} = \frac{B_1 - B_2}{B_1 + B_2}$$

where B_1 is the width of channel on the incident side of the cross-section discontinuity, and B_2 is the width on the leeside. Bowers and Herbich⁶ find the approximation to be reasonably good for plate filters. With the rock structures, we note that B_1 again represents the channel width, and B_2 is to be identified as the total area of the voids in a cross section. Accordingly

$$B_2 = \epsilon B_1$$

and

$$\frac{a_r}{a_i} = \frac{1 - \epsilon}{1 + \epsilon}$$

In the structures of this study, $\epsilon = 0.46$; hence,

$$\frac{a_r}{a_i} = 0.37$$

The curve in fig. 33 indicates that the theoretical value of the coefficient of reflection just given is somewhat in agreement with the coefficient of reflection observed from structures whose length is a small fraction of the length of the incident waves. Other than this, it appears that with structures of greater length, reflection is an accumulative process. The exact mechanism for the effect cannot be definitely explained at the present.

PART V: COMPENDIUM

35. The method of energy balance is found to be quite serviceable in deriving an expression for the coefficient of transmission for very long waves traversing a prismatic structure composed of rocks of uniform size. This formula is given by equation 41 and is a power form expression. To adopt the formula for short waves, the term $(gHT^2/\lambda^2)^m$ is introduced in the manner shown in equation 42. The exponent m , with the value $4/3$, is determined experimentally from data with T/λ varying for a given structure.

36. The power formula of the present analysis

$$\left(\frac{a_i}{a_t}\right)^{2/3} = 1 + \gamma \left(\frac{a_i}{H}\right)^{2/3} \frac{L}{\lambda} \quad (59 \text{ bis})$$

where

$$\gamma = 14.7 \left(\frac{\nu T}{d\lambda}\right)^{1/3} \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2}\right)^{4/3} \quad (60 \text{ bis})$$

is adequate to estimate the transmission coefficient a_t/a_i of waves passing through prismatic rock structure composed of rocks of uniform size and of porosity $\epsilon = 0.46$. Here H is the depth of water, λ the wavelength, T the period of the incident waves, d the size of rocks, ν the kinematic viscosity, and L the structure length. The formula implies that the resistance coefficient for rock in regard to steady flow should be

$$C_f = 96 \left(\frac{ud}{\nu}\right)^{-1/3} \quad (58 \text{ bis})$$

The steady-flow experiments using the same size rocks as those of the structures tested with waves gave for the coefficient

$$C_f = 96 \left(\frac{ud}{\nu}\right)^{-1/3} \quad (35 \text{ bis})$$

for the range of the Reynolds number

$$20 < \frac{ud}{\nu} < 2000$$

Here, u is the seepage velocity. The Reynolds number corresponding to peak seepage velocity during a wave cycle through the structure may be evaluated by

$$R = \frac{\epsilon a_0 \lambda d}{HT\nu} \quad (17 \text{ bis})$$

For Reynolds number greater than 2000, the coefficient is

$$C_f = 9.2 \quad (67 \text{ bis})$$

and accordingly the transmission coefficient for the prototype structure would be

$$\frac{a_i}{a_t} = 1 + \gamma \left(\frac{a_i}{H} \right) \frac{L}{\lambda} \quad (69 \text{ bis})$$

where

$$\gamma = 2.11 \frac{\lambda}{d} \left(gH \frac{T^2}{\lambda^2} \right)^{4/3} \quad (70 \text{ bis})$$

These transition formulas for the model and for the prototype are sufficient to determine the scale effect relating to the rock size in the model. The numerals appearing in equations 60 (bis) and 70 (bis) are for a rock porosity $\epsilon = 0.46$. For other porosities, the following values apply:

Porosity, ϵ	Numerical in γ , Equation 60 (bis)	Numerical in γ , Equation 70 (bis)
0.50	10.5	1.51
0.45	16.1	2.31
0.40	25.7	3.70
0.35	43.8	6.29
0.30	81.2	11.65

37. The phenomenon of reflection from the rock structures at the

moment is not amenable to an accurate analysis. The experiments show no definite dependence of the coefficient of reflection on wave height, Reynolds number, wave period, or rock size. If such effects are present, the variations produced by them are small and probably are of the same order of magnitude as the errors from observation. The total collection of the data shows that the coefficient of reflection depends on the relative length of the structures, L/λ . In the range of L/λ from 0 to 0.2, the coefficient a_r/a_i increases from the least value of 0.3 to a maximum value 0.65. For L/λ greater than 0.20, there is a slight decrease and finally the coefficient of reflection reaches 0.58 asymptotically for the greater increases.

PART VI: ACKNOWLEDGMENT

38. The author expresses gratefully his thanks to Mr. Claude E. Chatham, Jr., Chief, Harbor Wave Action Section, Wave Dynamics Branch, for reading the report and suggesting changes to improve presentation. Thanks also are due Dr. Donald L. Durham, Research Oceanographer, Wave Dynamics Branch, for his careful and detailed examination of the mathematical developments in the theoretical section of the report. The assistance rendered by Mr. D. Donald Davidson, Chief, Wave Research Section, Wave Dynamics Branch, in furnishing the desired data on the steady-flow losses of the rocks of this study is deeply appreciated. The tests on the losses were under Mr. Davidson's direct supervision and were carried out by SP⁴ John E. Dick. The generous help of SP⁴ Tom Anderson in reducing the data of experiments was most valuable.

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Table 1
Wave Transmission and Reflection Coefficients
 $H = 1.0$ ft, $T = 0.71$ sec, $\lambda = 2.54$ ft, $d = 0.042$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.10	78	0.390	0.695	0.605	0.01	0.39	74	0.147	0.692	0.623
0.02			0.328	0.640		0.02			0.108	0.650	
0.04			0.263	0.589		0.04			0.079	0.608	
0.06			0.235	0.535		0.06			0.060	0.566	
0.08			--	--		0.07			--	--	
0.10			--	--		0.08			--	--	
0.01	0.20	74	0.248	0.692	0.623	0.01	0.78	80	0.054	0.672	0.635
0.02			0.098	0.640		0.02			0.035	0.638	
0.04			0.142	0.588		0.04			0.016	0.604	
0.06			0.122	0.536		0.06			0.012	0.570	
0.07			--	--		0.07			--	--	
0.08			--	--		0.08			--	--	

* These mean values were determined as described in paragraph 21.

Table 2
Wave Transmission and Reflection Coefficients
 $H = 1.0$ ft, $T = 1.00$ sec, $\lambda = 4.53$ ft, $d = 0.042$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.06	72	0.581	0.520	0.532	0.01	0.22	74	0.246	0.705	0.684
0.02			0.519	0.525		0.02			0.206	0.701	
0.04			0.432	0.530		0.04			0.161	0.797	
0.06			0.386	0.535		0.06			0.140	0.793	
0.08			0.354	0.540		0.08			0.120	0.789	
0.10			0.332	0.545		0.10			0.105	0.685	
0.01	0.11	74	0.401	0.632	0.668	0.01	0.44	76	0.130	0.688	0.663
0.02			0.346	0.638		0.02			0.090	0.682	
0.04			0.284	0.644		0.04			0.060	0.676	
0.06			0.247	0.651		0.06			0.050	0.670	
0.08			0.215	0.647		0.08			0.045	0.663	
0.10			0.195	0.665		0.10			0.038	0.655	

* These mean values were determined as described in paragraph 21.

Table 3

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 1.41$ sec, $\lambda = 7.20$ ft, $d = 0.042$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.04	83	0.570	0.480	0.515	0.01	0.14	76	0.327	0.739	0.711
0.02			0.513	0.487		0.02			0.281	0.732	
0.04			0.439	0.494		0.04			0.238	0.719	
0.06			0.395	0.508		0.06			0.195	0.706	
0.08			0.366	0.522		0.08			0.173	0.693	
0.10			0.340	0.536		0.10			0.154	0.680	
0.01	0.07	72	0.440	0.662	0.662	0.01	0.28	72	0.191	0.763	0.741
0.02			0.392	0.662		0.02			0.154	0.756	
0.04			0.334	0.662		0.04			0.108	0.742	
0.06			0.247	0.662		0.06			0.084	0.728	
0.08			0.269	0.662		0.08			0.073	0.714	
0.10			0.246	0.662		0.10			0.063	0.700	

* These mean values were determined as described in paragraph 21.

Table 4

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 1.94$ sec, $\lambda = 10.40$ ft, $d = 0.042$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.03	83	0.688	0.432	0.430	0.01	0.10	75	0.428	0.710	0.646
0.02			0.607	0.430		0.02			0.381	0.680	
0.04			0.490	0.427		0.04			0.292	0.650	
0.06			0.422	0.425		0.06			0.238	0.620	
0.08			0.376	0.422		0.08			0.203	0.590	
0.10			0.342	0.420		0.10			0.179	0.560	
0.01	0.05	71	0.584	0.638	0.586	0.01	0.20	76	0.251	0.780	0.635
0.02			0.506	0.608		0.02			0.170	0.760	
0.04			0.402	0.578		0.04			0.126	0.740	
0.06			0.338	0.548		0.06			0.098	0.720	
0.08			0.247	0.518		0.08			0.083	0.700	
0.10			0.269	0.488		0.10			0.076	0.680	

* These mean values were determined as described in paragraph 21.

Table 5

Wave Transmission and Reflection Coefficients

H = 1.0 ft, T = 0.71 sec, $\lambda = 2.54$ ft, d = 0.062 ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.10	82	0.437	0.535	0.526	0.01	0.39	70	0.152	0.620	0.644
0.02			0.338	0.519		0.02			0.098	0.640	
0.04			0.248	0.502		0.04			0.061	0.660	
0.06			0.212	0.486		0.06			0.052	0.680	
0.08			--	--		0.08			--	--	
0.10			--	--		0.10			--	--	
0.01	0.20	62	0.360	0.645	0.618	0.01	0.78	71	0.102	0.595	0.564
0.02			0.273	0.629		0.02			0.055	0.566	
0.04			0.198	0.613		0.04			0.020	0.537	
0.06			0.166	0.547		0.06			0.015	0.508	
0.08			--	--		0.08			--	--	
0.10			--	--		0.10			--	--	

* These mean values were determined as described in paragraph 21.

Table 6

Wave Transmission and Reflection Coefficients

H = 1.0 ft, T = 1.00 sec, $\lambda = 4.53$ ft, d = 0.062 ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.06	82	0.694	0.480	0.446	0.01	0.22	70	0.307	0.710	0.700
0.02			0.610	0.472		0.02			0.258	0.704	
0.04			0.505	0.464		0.04			0.196	0.698	
0.06			0.435	0.456		0.06			0.162	0.692	
0.08			0.394	0.448		0.08			0.140	0.686	
0.10			0.355	0.440		0.10			0.125	0.680	
0.01	0.11	62	0.550	0.625	0.596	0.01	0.44	70	0.229	0.635	0.595
0.02			0.480	0.609		0.02			0.152	0.633	
0.04			0.398	0.593		0.04			0.090	0.631	
0.06			0.333	0.577		0.06			0.058	0.629	
0.08			0.292	0.561		0.08			0.052	0.627	
0.10			0.258	0.545		0.10			0.046	0.625	

* These mean values were determined as described in paragraph 21.

Table 7

Wave Transmission and Reflection Coefficients

H = 1.0 ft, T = 1.41 sec, $\lambda = 7.20$ ft, d = 0.062 ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.04	82	0.642	0.480	0.505	0.01	0.14	68	0.415	0.705	0.684
0.02			0.586	0.486		0.02			0.355	0.692	
0.04			0.506	0.492		0.04			0.286	0.679	
0.06			0.458	0.498		0.06			0.243	0.666	
0.08			0.426	0.504		0.08			0.215	0.653	
0.10			0.405	0.510		0.10			0.181	0.640	
0.01	0.07	64	0.573	0.605	0.605	0.01	0.28	70	0.292	0.660	0.639
0.02			0.514	0.612		0.02			0.243	0.654	
0.04			0.430	0.619		0.04			0.179	0.648	
0.06			0.374	0.626		0.06			0.145	0.642	
0.08			0.331	0.633		0.08			0.116	0.636	
0.10			0.298	0.640		0.10			0.095	0.630	

* These mean values were determined as described in paragraph 21.

Table 8

Wave Transmission and Reflection Coefficients

H = 1.0 ft, T = 1.94 sec, $\lambda = 10.40$ ft, d = 0.062 ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.03	82	0.859	0.370	0.408	0.01	0.10	68	0.535	0.660	0.626
0.02			0.760	0.394		0.02			0.456	0.635	
0.04			0.632	0.418		0.04			0.371	0.610	
0.06			0.565	0.442		0.06			0.326	0.585	
0.08			0.516	0.466		0.08			0.302	0.560	
0.10			0.476	0.490		0.10			0.288	0.535	
0.01	0.05	71	0.762	0.555	0.552	0.01	0.20	71	0.341	0.665	0.620
0.02			0.645	0.554		0.02			0.291	0.630	
0.04			0.518	0.553		0.04			0.214	0.595	
0.06			0.455	0.552		0.06			0.183	0.560	
0.08			0.409	0.551		0.08			0.148	0.525	
0.10			0.381	0.550		0.10			0.123	0.490	

* These mean values were determined as described in paragraph 21.

Table 9

Wave Transmission and Reflection Coefficients

H = 1.0 ft, T = 0.71 sec, $\lambda = 2.54$ ft, d = 0.083 ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.10	83	0.530	0.580	0.511	0.01	0.39	68	0.185	0.600	0.607
0.02			0.455	0.538		0.02			0.122	0.605	
0.04			0.369	0.496		0.04			0.083	0.610	
0.06			0.320	0.454		0.06			0.062	0.615	
0.08			0.283	0.412		0.08			0.049	0.620	
0.10			--	0.370		0.10			--	0.625	
0.01	0.20	54	0.395	0.620	0.590	0.01	0.78	66	0.102	0.640	0.625
0.02			0.323	0.596		0.02			0.069	0.636	
0.04			0.243	0.572		0.04			0.038	0.632	
0.06			0.203	0.548		0.06			0.027	0.628	
0.08			0.180	0.524		0.08			0.020	0.624	
0.10			--	0.500		0.10			--	0.620	

* These mean values were determined as described in paragraph 21.

Table 10

Wave Transmission and Reflection Coefficients

H = 1.0 ft, T = 1.00 sec, $\lambda = 4.53$ ft, d = 0.083 ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.06	84	0.676	0.440	0.440	0.01	0.22	68	0.371	0.680	0.668
0.02			0.624	0.440		0.02			0.323	0.668	
0.04			0.554	0.440		0.04			0.260	0.656	
0.06			0.502	0.440		0.06			0.229	0.644	
0.08			0.470	0.440		0.08			0.186	0.632	
0.10			0.425	0.440		0.10			0.160	0.620	
0.01	0.11	54	0.527	0.600	0.600	0.01	0.44	67	0.237	0.580	0.592
0.02			0.470	0.600		0.02			0.189	0.600	
0.04			0.400	0.600		0.04			0.131	0.620	
0.06			0.349	0.600		0.06			0.105	0.640	
0.08			0.306	0.600		0.08			0.079	0.660	
0.10			0.268	0.600		0.10			0.062	0.680	

* These mean values were determined as described in paragraph 21.

Table 11

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 1.41$ sec, $\lambda = 7.20$ ft, $d = 0.083$ ft

a_i/H	L/λ	θ , F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ , F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.04	82	0.723	0.340	0.404	0.01	0.14	70	0.412	0.690	0.671
0.02			0.668	0.372		0.02			0.371	0.678	
0.04			0.591	0.404		0.04			0.315	0.666	
0.06			0.538	0.436		0.06			0.276	0.654	
0.08			0.491	0.468		0.08			0.246	0.642	
0.10			0.450	0.500		0.10			0.220	0.630	
0.01	0.07	52	0.603	0.595	0.605	0.01	0.28	70	0.325	0.660	0.650
0.02			0.541	0.601		0.02			0.268	0.656	
0.04			0.455	0.607		0.04			0.215	0.652	
0.06			0.393	0.613		0.06			0.182	0.648	
0.08			0.351	0.619		0.08			0.158	0.644	
0.10			0.315	0.625		0.10			0.140	0.640	

* These mean values were determined as described in paragraph 21.

Table 12

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 1.94$ sec, $\lambda = 10.40$ ft, $d = 0.083$ ft

a_i/H	L/λ	θ , F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ , F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.03	84	0.790	0.300	0.338	0.01	0.10	68	0.545	0.660	0.594
0.02			0.761	0.308		0.02			0.545	0.644	
0.04			0.677	0.316		0.04			0.440	0.628	
0.06			0.620	0.324		0.06			0.372	0.612	
0.08			0.575	0.332		0.08			0.313	0.596	
0.10			0.532	0.340		0.10			0.276	0.580	
0.01	0.05	56	0.710	0.490	0.505	0.01	0.20	65	0.415	0.700	0.652
0.02			0.617	0.496		0.02			0.365	0.660	
0.04			0.508	0.502		0.04			0.295	0.620	
0.06			0.453	0.508		0.06			0.242	0.580	
0.08			0.415	0.514		0.08			0.200	0.540	
0.10			0.384	0.520		0.10			0.160	0.500	

* These mean values were determined as described in paragraph 21.

Table 13

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 0.71$ sec, $\lambda = 2.54$ ft, $d = 0.125$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.10	--	--	--	--	0.01	0.39	45	0.194	0.400	0.430
0.02			--	--		0.02			0.169	0.420	
0.04			--	--		0.04			0.140	0.440	
0.06			--	--		0.06			0.125	0.460	
0.08			--	--		0.08			0.112	0.480	
0.10			--	--		0.10			--	0.500	
0.01	0.20	55	0.445	0.552	0.552	0.01	0.78	50	0.065	0.520	0.520
0.02			0.417	0.552		0.02			0.049	0.520	
0.04			0.352	0.552		0.04			0.033	0.520	
0.06			0.302	0.552		0.06			0.023	0.520	
0.08			0.265	0.552		0.08			--	0.520	
0.10			0.237	0.552		0.10			--	0.520	

* These mean values were determined as described in paragraph 21.

Table 14

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 1.00$ sec, $\lambda = 4.53$ ft, $d = 0.125$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.06	83	0.708	0.460	0.475	0.01	0.22	43	0.435	0.560	0.558
0.02			0.659	0.466		0.02			0.375	0.556	
0.04			0.598	0.472		0.04			0.303	0.553	
0.06			0.561	0.478		0.06			0.263	0.548	
0.08			0.536	0.484		0.08			0.239	0.544	
0.10			0.510	0.490		0.10			0.220	0.540	
0.01	0.11	52	0.565	0.550	0.543	0.01	0.44	52	0.310	0.567	0.567
0.02			0.492	0.542		0.02			0.244	0.567	
0.04			0.411	0.534		0.04			0.173	0.567	
0.06			0.369	0.526		0.06			0.139	0.567	
0.08			0.350	0.518		0.08			0.119	0.567	
0.10			0.340	0.510		0.10			0.108	0.567	

* These mean values were determined as described in paragraph 21.

Table 15

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 1.41$ sec, $\lambda = 7.20$ ft, $d = 0.125$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.04	83	0.862	0.370	0.405	0.01	0.14	46	0.526	0.670	0.667
0.02			0.780	0.392		0.02			0.477	0.650	
0.04			0.660	0.414		0.04			0.412	0.630	
0.06			0.579	0.436		0.06			0.358	0.610	
0.08			0.514	0.458		0.08			0.309	0.590	
0.10			0.458	0.480		0.10			0.280	0.570	
0.01	0.07	53	0.723	0.533	0.533	0.01	0.28	58	0.334	0.720	0.667
0.02			0.662	0.533		0.02			0.352	0.700	
0.04			0.561	0.533		0.04			0.257	0.610	
0.06			0.492	0.533		0.06			0.221	0.660	
0.08			0.428	0.533		0.08			0.195	0.640	
0.10			0.390	0.533		0.10			0.164	0.620	

* These mean values were determined as described in paragraph 21.

Table 16

Wave Transmission and Reflection Coefficients

$H = 1.0$ ft, $T = 1.94$ sec, $\lambda = 10.40$ ft, $d = 0.125$ ft

a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean* a_r/a_i	a_i/H	L/λ	θ, F	a_t/a_i	a_r/a_i	Mean a_r/a_i
0.01	0.03	83	0.962	0.325	0.325	0.01	0.10	48	0.575	0.568	0.568
0.02			0.853	0.325		0.02			0.494	0.568	
0.04			0.729	0.325		0.04			0.398	0.568	
0.06			0.626	0.325		0.06			0.348	0.568	
0.08			0.564	0.325		0.08			0.311	0.568	
0.10			0.516	0.325		0.10			0.287	0.568	
0.01	0.05	44	0.851	0.420	0.420	0.01	0.20	57	0.420	0.667	0.667
0.02			0.768	0.420		0.02			0.355	0.667	
0.04			0.648	0.420		0.04			0.280	0.667	
0.06			0.568	0.420		0.06			0.230	0.667	
0.08			0.505	0.420		0.08			0.191	0.667	
0.10			0.455	0.420		0.10			0.162	0.667	

* These mean values were determined as described in paragraph 21.

$$n = 1/2, \quad \epsilon = 0.46, \quad (1 - n)\epsilon/2 = 1/8.7, \quad H = 1 \text{ ft}$$

d , ft	T , sec	λ , ft	γ	v , ft ² /sec	β'	β'_o	C_2
0.042	0.71	2.54	35.5	0.99×10^{-5}	67.0	19.3	168
	1.00	4.53	30.2	1.00	45.0	24.4	213
	1.41	7.20	35.0	0.98	29.6	22.1	192
	1.94	10.40	48.0	0.98	28.6	25.1	219
0.062	0.71	2.54	27.5	1.03×10^{-5}	100.0	28.9	251
	1.00	4.53	24.1	1.04	54.5	29.6	257
	1.41	7.20	25.0	1.04	37.8	28.2	245
	1.94	10.40	26.6	1.02	28.6	25.1	218
0.083	0.71	2.54	23.6	0.85×10^{-5}	146.5	42.2	368
	1.00	4.53	17.2	0.84	66.8	36.3	316
	1.41	7.20	23.0	0.83	60.5	45.2	393
	1.94	10.40	21.0	0.82	38.8	34.0	296
0.125	0.71	2.54	14.8	1.41×10^{-5}	133.3	38.4	335
	1.00	4.53	14.6	1.23	86.5	47.0	408
	1.41	7.20	16.0	1.20	67.6	50.4	444
	1.94	10.40	22.0	1.25	61.4	53.8	468
Mean						34.3	299

$$n = 1/3, \quad \epsilon = 0.46, \quad (1 - n)\epsilon/2 = 1/6.5, \quad H = 1 \text{ ft}$$

d , ft	T , sec	λ , ft	γ	v , ft ² /sec	β'	β'_0	C_2
0.042	0.71	2.54	95	0.99×10^{-5}	38.6	11.2	73
	1.00	4.53	73	1.00	17.6	9.5	62
	1.41	7.20	83	0.98	13.4	10.0	65
	1.94	10.40	67	0.98	7.6	6.7	43
0.062	0.71	2.54	94	1.03×10^{-5}	64.8	18.8	122
	1.00	4.53	73	1.04	26.1	14.2	92
	1.41	7.20	66	1.04	17.2	12.8	85
	1.94	10.40	72	1.02	10.3	9.0	59
0.083	0.71	2.54	67	0.85	72.3	20.9	136
	1.00	4.53	50	0.84	32.8	17.7	115
	1.41	7.20	57	0.83	24.6	18.3	119
	1.94	10.40	55	0.82	16.5	14.5	95
0.125	0.71	2.54	59	1.41	52.7	15.3	100
	1.00	4.53	32	1.23	31.7	17.2	112
	1.41	7.20	41	1.20	26.9	20.0	130
	1.94	10.40	51	1.25	23.1	20.2	131
Mean						14.7	96

Table 19
Determination of Resistance Constant C_2 for Flow Through Rocks

$$n = 1/4, \quad \epsilon = 0.46, \quad (1 - n)\epsilon/2 = 3/17.4, \quad H = 1 \text{ ft}$$

d , ft	T , sec	λ , ft	γ	ν , ft^2/sec	β'	β'_0	C_2
0.042	0.71	2.54	150	0.99×10^{-5}	29.4	8.5	49
	1.00	4.53	130	1.00	15.0	8.2	53
	1.41	7.20	116	0.98	8.7	6.5	37
	1.94	10.40	157	0.98	8.5	7.5	43
0.062	0.71	2.54	135	1.03×10^{-5}	40.0	11.6	67
	1.00	4.53	104	1.04	18.2	9.9	59
	1.41	7.20	94	1.04	11.8	8.8	51
	1.94	10.40	110	1.02	8.3	7.3	42
0.083	0.71	2.54	129	0.85×10^{-5}	58.7	17.0	99
	1.00	4.53	73	0.84	19.6	10.6	62
	1.41	7.20	86	0.83	15.0	11.2	65
	1.94	10.40	93	0.82	11.5	10.1	59
0.125	0.71	2.54	81	1.41×10^{-5}	52.7	15.3	89
	1.00	4.53	58	1.23	23.4	12.7	74
	1.41	7.20	62	1.20	16.3	12.2	71
	1.94	10.40	88	1.25	16.3	9.8	57
Mean						10.4	61

Table 20
Resistance of Flow Through Rocks

ud/ν	C_f	ud/ν	C_f	ud/ν	C_f
$d = 0.02 \text{ ft}$		$d = 0.042 \text{ ft}$		$d = 0.062 \text{ ft}$	
8.7	62.2	52.2	24.8	75	22.7
13.2	50.9	120	17.2	182	14.5
32.2	25.0	160	15.6	318	11.4
44.5	19.4	213	13.4	685	9.3
73.8	12.7	247	11.9	1110	7.7
116	12.6				
149	10.0				
$d = 0.083 \text{ ft}$		$d = 0.125 \text{ ft}$		$d = 0.204 \text{ ft}$	
373	11.5	261	16.1	1170	12.0
481	11.4	370	14.8	2360	12.3
1150	8.8	705	15.3	3970	10.5
1550	8.4	1470	12.4	5870	12.1
3500	6.0	2380	11.3	6970	11.8
		3820	13.8	7850	11.7
		4350	13.2	8700	12.0

Table 21

Normalized Data of Resistance of Flow Through Rocks

ud/ν	C_f	ud/ν	C_f	ud/ν	C_f
<u>$d = 0.0204$ ft</u>		<u>$d = 0.042$ ft</u>		<u>$d = 0.062$ ft</u>	
8.7	94.2	52.2	28.0	75	25.9
13.2	79.2	120	20.2	182	16.0
32.2	38.0	160	16.2	318	13.0
44.5	29.4	213	15.7	685	11.1
73.8	19.4	247	13.9	1110	8.7
116	19.1				
149	15.2				
<u>$d = 0.083$ ft</u>		<u>$d = 0.125$ ft</u>		<u>$d = 0.204$ ft</u>	
373	13.2	261	11.9	1170	8.9
481	13.0	370	11.0	2360	9.2
1150	10.0	705	11.3	3970	7.8
1550	7.6	1470	9.4	5870	9.0
3500	6.8	2380	8.4	6970	8.8
		3820	10.2	7850	8.7
		4350	9.8	8700	9.0

Table 22

Determination of Resistance Constant C_2 for Flow Through

Cubes Using Data from Kamel

$$n = 1/3, \quad \epsilon = 0.40, \quad \nu = 1 \times 10^{-5} \text{ ft}^2/\text{sec}, \quad (1 - n)\epsilon/2 = 4/30$$

d , ft	T , sec	λ , ft	γ	β'	β'_o	C_2
0.042	0.71	2.54	149	63	18	135
	1.00	4.53	126	40	22	166
	1.41	7.20	147	24	18	136
	1.94	10.49	194	23	20	150
0.052	0.71	2.54	104	57	16	120
	1.00	4.53	107	43	23	173
	1.41	7.20	88	19	14	105
	1.94	10.49	132	20	17	127
0.109	1.00	4.53	51	55	30	220
	1.41	7.20	49	29	22	160
Mean					21	156

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13. ABSTRACT Application of the energy method leads to a theoretical expression of the coefficient of transmission through a prismatic structure, constructed from rocks of uniform size, for long waves traversing the structure. When the dissipation in the voids is due partly to turbulent forces, the transmissions are expressed by a power formula. The formula is modified by a parametric factor to respond to the conditions met with in the ordinary model studies of the shorter waves. The factor form is determined from observations made with waves of varying periods. The relation of the adapted formula to the resistance of the structures for steady flow is shown. With the reflections not being amenable to an analytical treatment, test results are consolidated in a single curve, irrespective of the rock sizes, with the porosity remaining constant.			

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