APPENDIX 10
Example of the determination of a design storm

A10.1 Statistics of individual observations

In this computational example we will look for the design storm in front of the coast of the Netherlands. We will try to find a storm with an exceedance frequency of 1/225 per year. This storm yields 20% probability of failure in a lifetime of 50 years as has been explained in Chapter 3. It is stressed again that the use of these values does not constitute a recommendation on the part of the authors.

Figure A10-1 The Noordwijk measuring station

As described in Section 5.3.3, we start from a series of wave observations with a fixed interval, made at "Meetpost Noordwijk", off the Dutch coast (Figure A10-1). Each observation (e.g. with a duration of 20 minutes) results in a given value of the H_s. When an observation programme is continued during a long time, a time series is created of H_s-values. This time series is the basis of the statistical operations explained in this section.

In the example case we use, we have 20 years of data available from observations off the Dutch coast[1]. Every half hour a wave observation is made, but in the file used in

---

[1] The dataset is provided by the Netherlands Ministry of Public Works, the location Meetpost Noordwijk is at a waterdepth of approximately 18 m. Data from this, and other Dutch sites, can be downloaded from http://www.golfklimaat.nl

this example, these data are reduced to one observation every three hours. A one-month sample is plotted in Figure A10-2.

In Table A10-1 the clustered data are presented. The number of observations of each wave height bin is given both per bin as well as cumulatively. The probability of any wave height H_s' being equal or less than a specific wave height H_s is defined as:

$$P = P(H_s' \le H_s) \tag{A10.1}$$

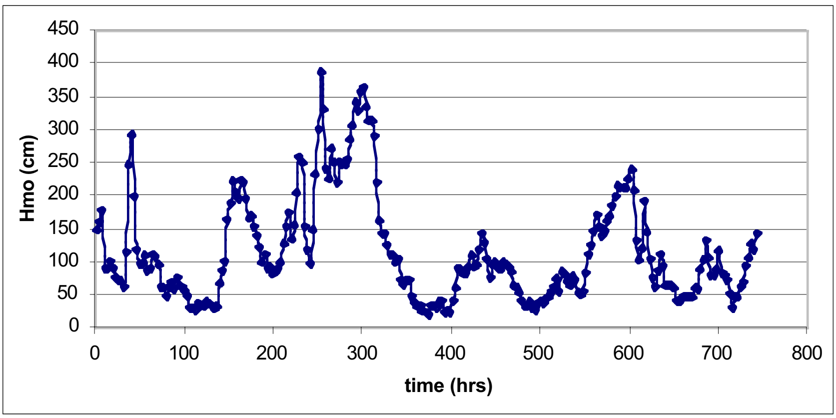

Figure A10-2 Data from Meetpost Noordwijk for one month (January 1979)

A probability of exceedance that H_s' is greater than a specific wave height H_s may also be defined as:

$$Q = Q(H_s' > H_s) = 1 - P \tag{A10.2}$$

Because a log relation is assumed, in the table also the value of –ln(Q) is given. Analysis shows that the correlation between H_s and –ln(Q) is 98.8 %. Plotting these data results in Figure A10-3. Although the correlation coefficient is quite large, it is clear that a log-distribution is not fully correct. The data are not on a straight line. For a small extrapolation this is not really a problem. The values from the regression line can be used to calculate the probability of exceedance of a given H_s. For example, a value of H_s = 6 m gives:

$$-\ln(Q) = 1.51 \cdot 6 - 0.5 = 8.5$$

$$Q = 2.035 \cdot 10^{-4} = 0.0002$$

Wave height class Hs (cm)		Number of observations		P	Q	–ln(Q)
		per bin	cumulative			
0	25	35	35	0.000599	0.999401	0.000599
25	50	8260	8295	0.141940	0.858060	0.153082
50	75	11424	19719	0.337423	0.662577	0.411618
75	100	10004	29723	0.508607	0.491393	0.710511
100	125	7649	37372	0.639493	0.360507	1.020245
125	150	5563	42935	0.734685	0.265315	1.326838
150	175	4389	47324	0.809788	0.190212	1.659615
175	200	3167	50491	0.863980	0.136020	1.994954
200	225	2360	52851	0.904363	0.095637	2.347200
225	250	1671	54522	0.932957	0.067043	2.702419
250	275	1234	55756	0.954073	0.045927	3.080692
275	300	851	56607	0.968634	0.031366	3.462047
300	325	556	57163	0.978149	0.021851	3.823487
325	350	392	57555	0.984856	0.015144	4.190168
350	375	276	57831	0.989579	0.010421	4.563938
375	400	206	58037	0.993104	0.006896	4.976819
400	425	136	58173	0.995431	0.004569	5.388507
425	450	82	58255	0.996834	0.003166	5.755400
450	475	66	58321	0.997964	0.002036	6.196632
475	500	38	58359	0.998614	0.001386	6.581307
500	525	30	58389	0.999127	0.000873	7.043930
525	550	20	58409	0.999470	0.000530	7.541769
550	575	22	58431	0.999846	0.000154	8.778531
575	600	9	58440	1.000000	0.000000	
Total		58440				

Table A10-1: Wave data from Meetpost Noordwijk

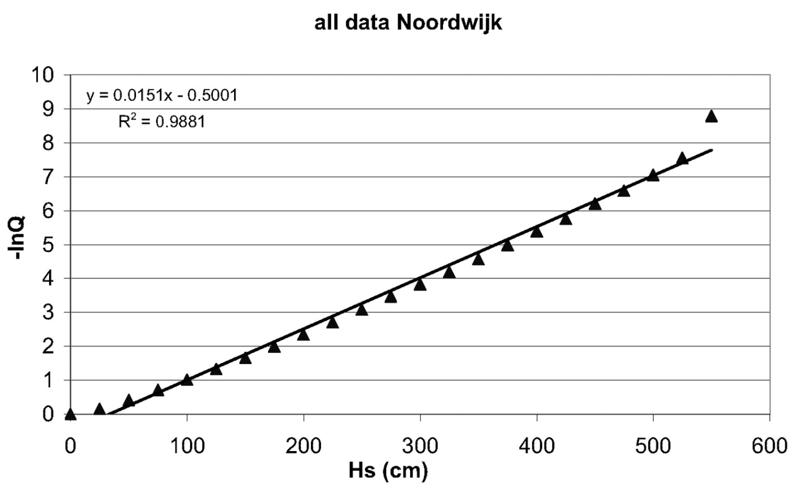

Figure A10-3 Cumulative data for Noordwijk

This means that a wave height $H_s > 6$ m will occur during 0.02 % of the time.
This result can also be obtained in a more direct manner. Plotting the values of Q on log-paper gives Figure A10-4. In this graph the exceedance of an observation larger than H_s' is given. The advantage of this plot is that the exceedance values can be read from the graph. (The statistical analysis and the plotting of data can easily be carried out using a spreadsheet software package. In this case, MS Excel has been used. It is beyond the scope of this book to outline the settings of the software). It is evident that the upper boundaries of the wave height bin should be used on the wave height axis (in this example 86% of the waves is higher than 50 cm, 99% is higher than 25 cm and 100% is higher than 0 cm).

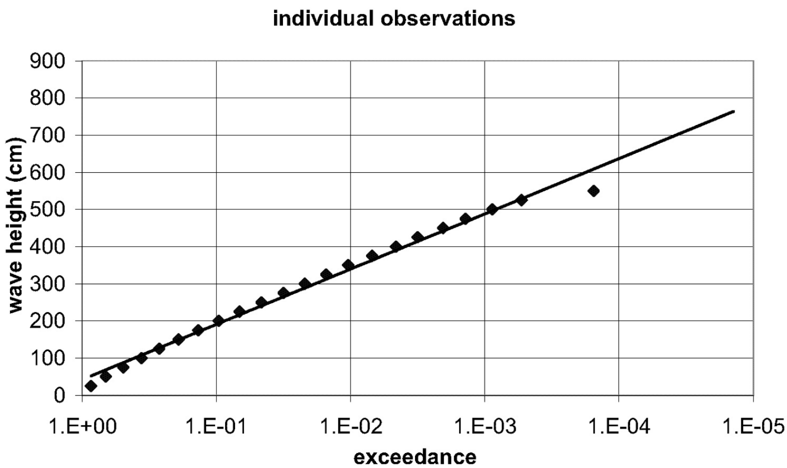

Figure A10-4 Exceedance graph for Noordwijk

However, for design purposes, these values have no real meaning. It means that, if one goes at a random moment to the sea, there is at that given moment a probability of 0.0002 (i.e. 0.02%) of observing a H_s of 6 m or more. But this has nothing to do with a design storm with a H_{ss} of 6 m.
This form of statistics can be applied for the assessment of workability. In fact the above example means that 0.02 % of the time a wave condition can be met in which H_s is larger than 6 m. Suppose that for a port in this area pilot service has to be suspended when $H_s > 3.5$ m, it can easily be read that during 1% of the time there is no pilot service.
For the design of structures, we need the exceedance of a design storm and not the exceedance of an individual wave condition.

A10.2 The Peak over Threshold method

In order to transform these individual observations into storms, we may use the fact that sequential wave height observations are not random (see also Figure A10.2).

When we measure at 12:00 hrs a H_s of 4 m, it is very unlikely that we measure at 15:00 hrs a H_s of 0.4 m. Usually the observations of 12:00 hrs and 15:00 hrs will belong to the same storm. So we will define a certain threshold, e.g. H_t = 1.5 m. And we will look in our record when the wave height exceeds 1.5m. The threshold-value of H_t = 1.5 m is arbitrarily selected. Later we will investigate the sensitivity of this choice. The month of the record in Figure A10-2 (Jan 1979) shows 9 storms, providing 9 data points.

The reason for introducing a threshold is to avoid that small variations in wave height during long, calm periods have significant influence on the final result. Basically one should assume the threshold as high as possible, as long as the base for statistics contains sufficient data for a reliable analysis. Later it will be shown that the final answer is not very sensitive to the choice of the threshold value.

When we process the whole data set of 20 years, we find 1746 storms in total with a H_{ss} higher than 1.5 m. The 1746 storms are classified in wave height bins, according to the maximum $H_{\text{ss of}}$ each storm which results in Table A10-2.

Wave height class H_{ss} (m)		Number of storms per bin	cum.	P	Q	ln(Qs)	–ln(H)	Qs	G	α=1.24 W
1.50	1.75	384	384	0.21993	0.78007	4.22098	-0.56	68.10	-0.415046	0.32522
1.75	2.00	381	765	0.43814	0.56186	3.89284	-0.69	49.05	0.192121	0.64136
2.00	2.25	266	1031	0.59049	0.40951	3.57655	-0.81	35.75	0.640938	0.91261
2.25	2.50	157	1188	0.68041	0.31959	3.32863	-0.92	27.90	0.954366	1.11202
2.50	2.75	148	1336	0.76518	0.23482	3.02042	-1.01	20.50	1.318085	1.34858
2.75	3.00	111	1447	0.82875	0.17125	2.70471	-1.1	14.95	1.672191	1.58094
3.00	3.25	81	1528	0.87514	0.12486	2.38876	-1.18	10.90	2.014645	1.80552
3.25	3.50	63	1591	0.91123	0.08877	2.04769	-1.25	7.75	2.375535	2.04065
3.50	3.75	31	1622	0.92898	0.07102	1.82455	-1.32	6.20	2.608194	2.19099
3.75	4.00	32	1654	0.94731	0.05269	1.52606	-1.39	4.60	2.916351	2.38832
4.00	4.25	23	1677	0.96048	0.03952	1.23837	-1.45	3.45	3.210883	2.57486
4.25	4.50	11	1688	0.96678	0.03322	1.06471	-1.5	2.90	3.387796	2.68590
4.50	4.75	20	1708	0.97824	0.02176	0.64185	-1.56	1.90	3.816515	2.95184
4.75	5.00	9	1717	0.98339	0.01661	0.37156	-1.61	1.45	4.089424	3.11883
5.00	5.25	7	1724	0.98740	0.01260	0.09531	-1.66	1.10	4.367707	3.28731
5.25	5.50	9	1733	0.99255	0.00745	-0.43078	-1.7	0.65	4.896399	3.60263
5.50	5.75	8	1741	0.99714	0.00286	-1.38629	-1.75	0.25	5.854211	4.15922
5.75	6.00	5	1746	1	0		-1.79	0.00		
		1746								

Table A10-2: Data from Noordwijk using PoT of 1.5 m

Using the same data, one can also determine the wave steepness. Without presenting the analysis, we found when looking at only those storms with a $H_T \geq 5$ m, a total of 29 storms in 20 years. The average duration of these heavy storms is 6.6 hrs, and the average wave steepness in these storms is 5.8 %, with a standard deviation of 0.6%.

These figures will be used in Section A10.4.3, where the probabilistic approach will be discussed.

The exponential distribution

Like was done before in equations (A10.1) and (A10.2) for H_s, the values of P and Q are computed for H_{ss}:

$$P = P(H'_{ss} \le H_{ss}) \quad \text{(A10.3)}$$

$$Q = Q\left(H'_{ss} > H_{ss}\right) = 1 - P \quad \text{(A10.4)}$$

Plotting the values of Q logarithmically results in Figure A10-5, which gives the probability of exceedance of a storm. On can read this graph as: Given there is a storm (according to our definition H_s>1.5 m), then the probability that that storm has an H_{ss} of more than 4 m is 0.05 (or 5%). [4 = 0.78 ln(x) + 1.63]

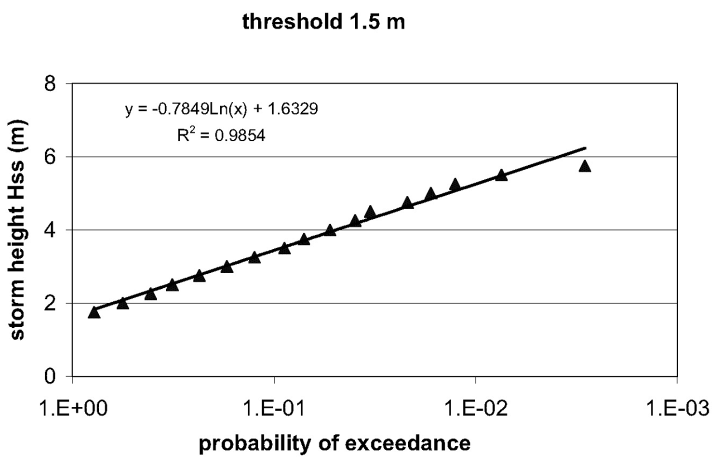

Figure A10-5 Wave exceedance using a threshold

Still we do not know the probability of a storm that occurs with a probability of for example 1/225 per year. In the analysis done so far we looked only at the probability of a single storm. We still need to transform our information to a probability per year. This can only be done when we know the number of storms per year. This number is known, since we have 1746 storms in 20 years, the average number of storms per year N_s = 1746/20 = 87.3. When we have 87.3 storms per year, a storm with a probability of exceedance of 1/87.3 has to be the "once-per-year" storm (in correct statistical terms: the storm with a probability of exceedance of once per year).

For transformation of the general probability of exceedance (Q) to the probability of exceedance of a storm in a year (Q_s), we multiply Q with the number of storms in a year.
Thus:

$$Q_s = N_s Q$$

The values of Q_s are also given in Table A10-2. Values of $Q_s > 1$ should not be called a "probability", because probabilities cannot be larger than 1. Physically however, these values do have a meaning. They represent the expected number of storms in a year.
The values of Q_s can again be plotted on log-paper. This results in Figure A10-6. In fact, Figure A10-6 is the design graph to be used. So we can calculate our 1/225 per year storm with:

$$\begin{aligned} H_{ss} &= -0.785 \ln(Q_s) \\ &= -0.785 \ln\left(\frac{1}{225}\right) + 5.141 \\ &= 9.39 \text{ m} \end{aligned}$$

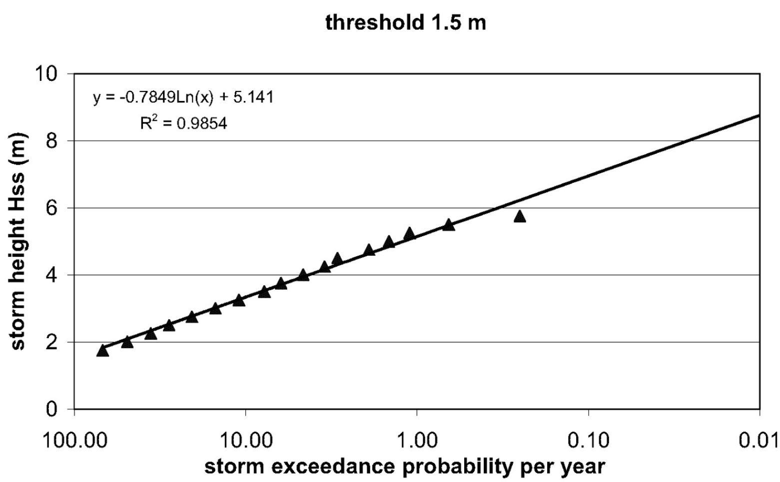

Figure A10-6 Storm exceedance using a threshold

The Gumbel distribution

A more detailed inspection of Figures A10-4 and A10-5 reveals that, like in Figures A10-2 and A10-3, the points are not exactly on a straight line. This is caused by the fact that a simple exponential relation has been used. Because we deal with extreme

values, an extreme value distribution like Gumbel or Weibull may result in predictions that are more reliable.
The Gumbel distribution is given by:

$$P=\exp\left[-\exp\left(-\frac{H_{ss}-\gamma}{\beta}\right)\right] \tag{A10.5}$$

The coefficients β and γ can be found by regression analysis on the data. In order to do so, one has to reduce the equation of the Gumbel distribution to a linear equation of the type $y = Ax + B$. After that, standard regression will provide the values A and B, and subsequently values of β and γ. Taking two times the log, the Gumbel distribution can be reduced to:

$$\begin{aligned} \ln P &= -\exp\left(-\frac{H_{ss}-\gamma}{\beta}\right) \\ \underbrace{-\ln(-\ln P)}_{G} &= \frac{H_{ss}-\gamma}{\beta} = \underbrace{\frac{1}{\beta}H_{ss}-\frac{\gamma}{\beta}}_{AH_{ss}-B} \end{aligned} \tag{A10.6}$$

The left-hand side of equation (A10.6) we call the reduced variate G:

$$G=-\ln\left(\ln\frac{1}{P}\right)$$

The values of G can be calculated for all P-values in Table A10-2. The values of G and H_{ss} are plotted in Figure A10-7. A linear regression leads to:

$$G=AH_{ss}+B$$

For the given dataset A = 1.365 and B = –2.535

So: β = 1/A = 1/1.365 = 0.733

$\gamma = -\beta B$ = –0.733 · – 2.535 = 1.877

Like with the exponential distribution, the analysis given so far results in an absolute exceedance of a storm, not in a probability per year. As given before:

$$Q_s=N_sQ$$

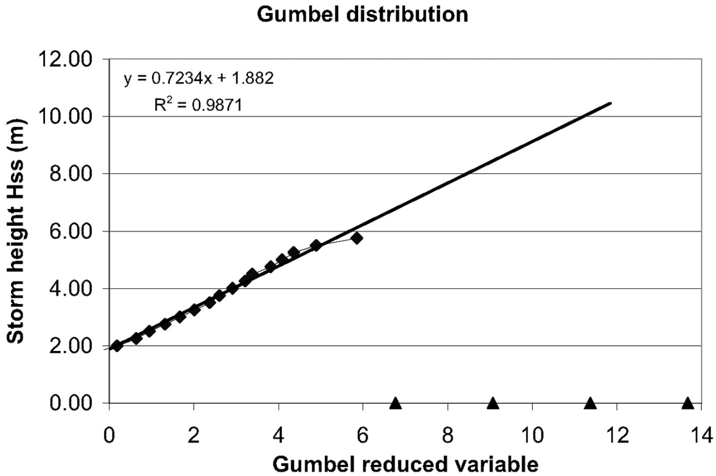

Figure A10-7 The Gumbel exceedance graph

From the Gumbel distribution it follows:

$$\begin{aligned} H_{ss} &= \gamma - \beta \ln\left(\ln \frac{1}{P}\right) \\ &= \gamma - \beta \ln\left(\ln \frac{1}{1-Q}\right) \\ &= \gamma - \beta \ln\left(\ln \frac{1}{1-Q_s/N_s}\right) \\ &= \gamma - \beta \ln\left(\ln \frac{N_s}{N_s - Q_s}\right) \end{aligned} \tag{A10.7}$$

So:

$$H_{s1/225} = 1.88 - 0.73 \ln\left(\ln \frac{87.3}{87.3 - 1/225}\right) = 9.10$$

A disadvantage of this approach is that in Figure A10-7 on the horizontal axis the reduced variable G is plotted, and not the probability of exceedance Q_s. Of course this can be transformed by:

$$G = -\ln\left(\ln \frac{N_s}{N_s - Q_s}\right) \tag{A10.8}$$

So, for this given example (with N_s = 87.3 storms/year) this results in:

Q_s	G
1/10	6.77
1/100	9.07
1/1000	11.38
1/10000	13.68

Table A10-3

These values can be inserted in Figure A10-7. One should realize that when the number of storms per year (N_s) changes, also the units on the converted horizontal axis will have to be changed.

The Weibull distribution

Instead of a Gumbel distribution, we also may try a Weibull distribution. The Weibull distribution is given by

$$Q = \exp\left[-\left\{\frac{H_{ss}-\gamma}{\beta}\right\}^{\alpha}\right] \tag{A10.9}$$

Also here, in order to find the values α, β and γ from regression analysis, we have to reduce the equation.

$$\begin{aligned} -\ln Q &= \left[\frac{H_{ss}-\gamma}{\beta}\right]^{\alpha} \\ (-\ln Q)^{1/\alpha} &= \frac{H_{ss}-\gamma}{\beta} \\ \underbrace{(-\ln Q)^{1/\alpha}}_{W} &= \underbrace{\frac{1}{\beta}H_{ss}-\frac{\gamma}{\beta}}_{AH_{ss}-B} \end{aligned} \tag{A10.10}$$

So the reduced Weibull variate is

$$W = -(\ln Q)^{1/\alpha} \tag{A10.11}$$

The Weibull distribution has three variables (α, β and γ). Linear regression will provide only two constants, A and B (and subsequently β and γ). So the determination of the third coefficient (α) will require some trial and error. Assuming different values of α will change the curvature of the points in the plot of W vs. H_{ss}. The calculation can be carried out quite easily in a spreadsheet. Changing the α

immediately provides a new graph and a new value of the correlation coefficient. The value of α that provides the straightest line and the highest correlation coefficient is the best value for α. In our example this proves to be α = 1.24, resulting in β = 1.17 and γ = 1.22. See also Table A10-2 and Figure A10-8. (Note: β and γ are not the values given in the regression line in Figure A10-8, because in this figure H_{ss} is plotted on the vertical axis; if one plots H_{ss} on the horizontal and W on the vertical, on gets the values of β and γ also directly).

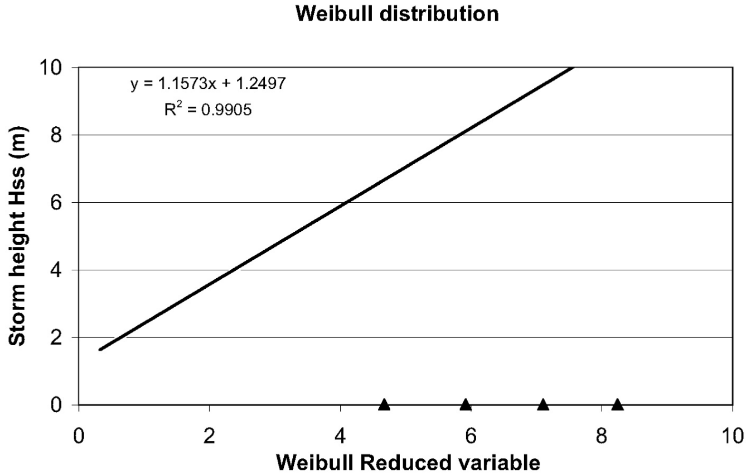

Figure A10-8 The Weibull exceedance graph

Also here the exceedance has to be transformed to a probability per year. From the Weibull distribution it follows:

$$H_{ss} = \gamma + \beta\{-\ln Q\}^{1/\alpha} \tag{A10.12}$$

Using $Q_s = H_s\, Q$ gives

$$H_{ss} = \gamma + \beta\left\{-\ln\left(\frac{Q_s}{N_s}\right)\right\}^{1/\alpha} \tag{A10.13}$$

So:

$$\begin{aligned} H_{ss\,1/225} &= 1.22 + 1.17\left\{-\ln\left(\frac{1/225}{87.3}\right)\right\}^{1/1.24} \\ &= 1.22 + 1.17 \cdot 6.34 \\ &= 8.64\, m \end{aligned}$$

Again, a manual plot has to be made of H_{ss} as a function of W (see also Figure A10-8). This can be transformed using

$$W = -\ln\left(\frac{Q_s}{N_s}\right)^{1/\alpha} \tag{A10.14}$$

So, for the given example (with N_s = 87.3 and α = 1.24), Q_s and W are:

Q_s	W
1/10	4.68
1/100	5.92
1/1000	7.11
1/10000	8.24

Table A10-4

Summary

The coefficient γ theoretically has the meaning of the threshold value H_T as used in the PoT-analysis. So as a check, we can compare these values:

H_T threshold value	1.50	2.00	2.50	3.00	3.50	4.00
γ Exponential	1.63	1.65	1.65	1.65	1.65	1.65
γ Gumbel	1.86	2.37	2.84	3.44	3.98	4.52
γ Weibull	1.23	1.77	2.29	2.93	3.51	4.10

Table A10-5

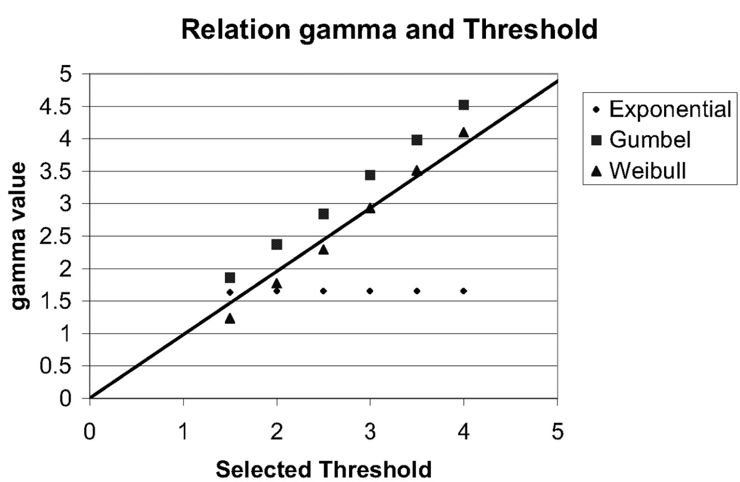

Figure A10-9: Relation between γ and the selected threshold

In this respect it is clear that the Gumbel and the Weibull distribution give a much better result. This becomes more relevant when the number of available storms in the database is lower.
Summarizing we find the following H_{ss} values for our 1/225 design storm for different threshold values:

H_T	1.50	2.00	2.50	3.00	3.50	4.00
N_s	87.3	59.6	38.9	19.4	10.5	5.3
$H_{ss\ 1/225}$ Exponential	9.39	9.30	9.00	8.85	8.58	8.31
$H_{ss\ 1/225}$ Gumbel	9.10	8.95	8.66	8.46	8.11	7.81
$H_{ss\ 1/225}$ Weibull	8.64	8.55	8.32	8.20	7.97	7.77

Table A10-6

In this comparison for the Weibull distribution a value $\alpha = 1.24$ has continuously been used. As explained before, α has to be determined by optimizing the correlation coefficient and visually on the fact that the points are on a straight line as much as possible. For easy reference we have continued to use α= 1.24 as derived for a H_T of 1.5 m. For large values of H_T the number of wave height bins available for use in the analysis becomes lower. Consequently, the calculation becomes less reliable.

A10.3 What to do if only random data are available?

The above PoT-analysis can only be carried out in case a sequential database with observations is available. In case only the grouped statistics of observations are available (like Table A10-1) a PoT-analysis is not possible. The same problem occurs when data are collected by random observations, like the visual observations from the Global Wave Statistics[2]. Often the number of observations in each wave class bin is normalized in such a way that the total is 100% or 1000 ‰.
A typical sample of such a table is given in Table A10-7 (In fact the input data are identical to Table A10-1). In such a case, one can determine the exceedance frequency of a single H_s, as indicated in the section before. Because the data are not sequential, and because the number of storms per year (N_s) is not known, the PoT-analysis is not possible. Some of the data in the table may come from the same storm.

[2] The Global Wave Statistics [HOGBEN & LUMB, British Maritime Techonology, 1986] is a book containing visual observations collected by ships at given times, but random locations (i.e. the position of the ship at that moment). This makes that all observations in a given area are completely uncorrelated.

Hs-bin		Class Obs	Cumul	*P*	*Q*	LN(Q)	12 hrs *s/y*	storm duration ln()	ln(*H*)	$\alpha = 0.8$ *W*
0	25	35	35	0.00060	0.99940	0.00	729.56	6.592	3.219	-10.564
25	50	8260	8295	0.14194	0.85806	0.15	626.38	6.440	3.912	-10.259
50	75	11424	19719	0.33742	0.66258	0.41	483.68	6.181	4.317	-9.747
75	100	10004	29723	0.50861	0.49139	0.71	358.72	5.883	4.605	-9.161
100	125	7649	37372	0.63949	0.36051	1.02	263.17	5.573	4.828	-8.562
125	150	5563	42935	0.73469	0.26531	1.33	193.68	5.266	5.011	-7.978
150	175	4389	47324	0.80979	0.19021	1.66	138.85	4.933	5.165	-7.353
175	200	3167	50491	0.86398	0.13602	1.99	99.29	4.598	5.298	-6.733
200	225	2360	52851	0.90436	0.09564	2.35	69.81	4.246	5.416	-6.095
225	250	1671	54522	0.93296	0.06704	2.70	48.94	3.891	5.521	-5.464
250	275	1234	55756	0.95407	0.04593	3.08	33.53	3.512	5.617	-4.808
275	300	851	56607	0.96863	0.03137	3.46	22.90	3.131	5.704	-4.165
300	325	556	57163	0.97815	0.02185	3.82	15.95	2.770	5.784	-3.573
325	350	392	57555	0.98486	0.01514	4.19	11.05	2.403	5.858	-2.992
350	375	276	57831	0.98958	0.01042	4.56	7.61	2.029	5.927	-2.422
375	400	206	58037	0.99310	0.00690	4.98	5.03	1.616	5.991	-1.822
400	425	136	58173	0.99543	0.00457	5.39	3.34	1.205	6.052	-1.262
425	450	82	58255	0.99683	0.00317	5.76	2.31	0.838	6.109	-0.801
450	475	66	58321	0.99796	0.00204	6.20	1.49	0.396	6.163	-0.315
475	500	38	58359	0.99861	0.00139	6.58	1.01	0.012	6.215	-0.004
500	525	30	58389	0.99913	0.00087	7.04	0.64	-0.451	6.263	0.369
525	550	20	58409	0.99947	0.00053	7.54	0.39	-0.949	6.310	0.936
550	575	22	58431	0.99985	0.00015	8.78	0.11	-2.185	6.354	2.657
575	600	9	58440	1	0	0	0	0	0	0

Table A10-7 Uncorrelated data (example from Noordwijk, but not taking into accout the fact that they are observed every three hours)

In order to solve this problem, we assume that we can divide the year in N_s periods of t_s hours, during which the wave height H_s does not vary. The basic idea behind this assumption is that, because of the persistence of winds, storm periods will have more-or-less the same duration. Therefore, the assumed time interval t_s is called "storm duration". Now, one can stipulate that each of the random observations describe an observation of one such storm.

This means that we usually have an unknown number of observations, however we know the percentage of exceedance of each observation. Because such an observation does in fact not represent the wave height of one (half-hour) sample (which we called H_s), but represents the average H_s during a t_s-hour storm, we will use the symbol H_{ss}. for this observation

Let us assume for the time being that we have a storm-duration of 12 hours. This means that we have $365 \cdot 24/12 = 730$ periods of 12 hours in a year ($N_s = 730$). This means that a "once-per-year" storm occurs in 1/730 times of the cases

($=1.37 \cdot 10^{-3}$). And a “once-in-ten-year” storm occurs in $0.137 \cdot 10^{-3}$ times of the cases.
In the example one can see that observations with $H_{ss} > 4.75$ m occur in 0.001386 of the cases. It is clear that the “once-a-year” storm is approximately 4.75 m. The value $Q_{ss} = Q_s \cdot 730$ indicates the probability of a storm in a year.
It is obvious that from a statistical point of view values of $Q_s > 1$ have no meaning. However, physically this number indicates the number of storms per year.
The values of Q_{ss} can be plotted on log paper, which results directly in a design line, showing the probability of exceedance of a given design storm.
Statistically one can process the values of Q_s in the same way as has been done before with Q. Practically this computation can be done in a spreadsheet in a simple way. In fact, Table A10-3 is a print from a spreadsheet. A multiplier is computed with the value:

$$M = \frac{\text{number of observations}}{365 \cdot 24 / \text{storm duration}} = \frac{58440}{730} = 80$$

Now we calculate the total number of storms smaller than our bin-value by taking {total-(cumulative number)} and divide that by M. The result is given in the column with the heading s/y (storms per year). And this value can be plotted, resulting in Figure A10-10.
Also in this case, we calculate the slope and intercept of the regression line. In our example the slope (A) is 1.02, and intercept (B) equals 4.16. (In order to calculate these values, one should either make an extra column with ln Q_s or make a plot where Q_s is on the y axis and H_{ss} on the x-axis). For extrapolation one can use the equation:

$$Q = \exp\left\{-\beta\left(H_{ss} - \gamma\right)\right\} \tag{A10.15}$$

or in reversed order:

$$H_{ss} = \gamma - \frac{\ln Q}{A} \tag{A10.16}$$

in which A = slope of regression line and γ = $-B/A$ (= 4.16/1.02 = 4.07).
So, the 1/225 storm-height is

$$H_{ss\,1/225} = 4.07 - \frac{\ln\left(1/225\right)}{1.02} = 9.39\,m$$

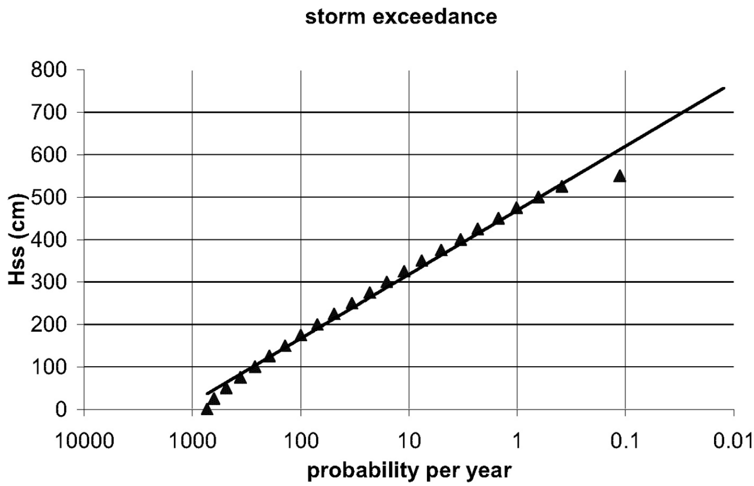

Figure A10-10 Storm exceedance graph using Random Observations

The value of 9.39 found in this way should be compared with the values between 8.31 and 9.39 found in the PoT analysis. This analysis too, can be improved by using the Weibull distribution instead of a simple log-distribution.
As was done with the PoT-analysis, the value of W_s is calculated using

$$W_s = \left(\ln \frac{1}{Q_s} \right)^{1/\alpha} \tag{A10.17}$$

See Table A10-3. At some place in the spreadsheet, a value of α is given and the data of the last column are calculated using that value of α. One can easily calculate slope, intercept and correlation. With trial and error, one can find that value of α which results in the highest correlation. In this example a value of $\alpha = 0.85$ (which is different than from the previous case) gives a correlation of 99.8 %. The parameters of β and γ follow from the slope and intercept of the regression line:

$$\beta = 1/A$$

$$\gamma = -B/A$$

The values of α, β and γ can be used directly in the equation to calculate the probability:

$$W_s = \exp\left\{ -\left(\frac{H_{ss} - \gamma}{\beta} \right)^{\alpha} \right\} \tag{A10.18}$$

or in reversed order:

$$H_{ss} = \gamma + \beta\left(-\ln W_s\right)^{1/\alpha} \tag{A10.19}$$

So, the 1/225 storm height is:

$$H_{ss\,1/225} = 4.62 + 0.43\left\{-\ln\left(1/225\right)\right\}^{1/0.85} = 8.16\,m$$

One can make a plot of H_{ss} vs. W_s,. This indicates that all points are nicely on a straight regression line. However, the value of W_s has no direct technical meaning. It becomes useful in case W_s is translated into Q_s using:

$$W_s = \left(\ln\frac{1}{Q_s}\right)^{1/\alpha} \tag{A10.20}$$

This means that one has to redefine the horizontal axis. Be aware that values of $W <$ 0 do not represent probabilities, but are only used to have a sufficient basis for extrapolation.
The resulting value of H_{ss} = 8.16 m should be compared with the values between 7.77 and 8.64 found in the PoT analysis. As stated before, the anticipated storm duration was 12 hours. Different durations result in different values for H_{ss}.

	3 hrs	6 hrs	9 hrs	12 hrs
Exponential	5.41	6.73	8.06	9.39
Weibull	8.48	8.32	8.23	8.16

Table A10-8

From this example it follows that in case a PoT-analysis is not possible, an analysis on the basis of tabulated (random) data can very well be done. The choice of the storm duration however remains a problem. Given the accuracy of the final answer, this choice is not extremely sensitive, at least when using the Weibull distribution.

A10.4 Computation of the armour units

A10.4.1 The classical computation

The classical way of computing the required block size is using the design formula, applying a design wave height with an exceedance of P_f during the lifetime of the structure. For example a lifetime of 50 years and a probability of failure of 20% during lifetime gives the following exceedance:

$$f = -\frac{1}{t_L}\ln(1-p) = -\frac{1}{50}\ln(1-0.2) = 4.5\cdot 10^{-3} = 1/225 \tag{A10.21}$$

Using the Weibull distribution from Section A10.2 for the example of Noordwijk, this results in a H_{ss} of 8.64 m. The design formula, as given by Van der Meer for cubes is:

$$\frac{H_{ss-design}}{\Delta d_n} = \left(6.7\frac{N_{od}^{0.4}}{N^{0.3}} + 1.0\right)s_m^{-0.1} \tag{A10.22}$$

For N_{od} Van der Meer recommends a value of 0.5. The wave analysis in Chapter 5 has shown a wave steepness of 5.6%. There are approximately 4000 waves in a storm. For cubes with normal concrete one may use a concrete density of 2400 kg/m^3, which results in a value of Δ = 1.75.
Substituting these values in the above design formula yields a d_n of 2.4 m, or a block weight of 91 tons. In case one applies the Hudson formula with a K_D value of 5 (head) and a slope of 1:1.5, one gets a d_n of 3.3 m, and a weight of 83 tons (realise that the use of basalt split in the concrete may increase the density to 2800 kg/m^3, which will give with Van der Meer a block weight of "only" 50 tons).

Because the depth in front of the Scheveningen breakwater is limited to 6 m below MSL (i.e. 9.5 m below design level), one may assume that waves never can become larger than 0.5 · 9.5 = 4.75 m (see Figure 5-26). Filling in these values results in a block weight of 15 tons.

However, one should realize that the number of occurrence of a storm with an H_{ss} of more than 4.75 is much more frequent. Using the Weibull distribution, one can find that a H_{ss} of 4.75 is exceeded once every 0.6 years. This means that during the design life (50 years) the breakwater will encounter 50/0.6 = 85 storms, with in total 400*85 = 33000 waves. Using this number of waves, we find a block size of 2.1 m, and a weight of 24 tons. Because the number of waves is not included in the Hudson formula, Hudson gives a block weight of only 14 tons.

During the design of the breakwater of Scheveningen, only the Hudson formula was available; the design of this breakwater however was not only based on this formula but also verified with model tests. The applied blocks in Scheveningen have a weight of 25 tons and a density of 2400 kg/m^3.

A10.4.2 The method of Partial Coefficients

The method of partial coefficients is worked out in PIANC [1992]. In this method safety coefficients are added to the design formula. There are safety coefficients for load and for strength.

The partial safety coefficients for load

The probability of exceedance during service life of the design storm with a recurrence equal to the design life

$$P_{f-\text{lifetime}} = 1 - \left(1 - \frac{1}{t_L}\right)^{t_L} \tag{A10.23}$$

in which t_L is the design life. For a life time of 50 years, $P_{f\text{-lifetime}}$ is 0.64

The design storm to be applied has of course a much smaller probability, and consequently a longer return interval. The probability that a construction may fail during life time P_f is for example set to 20 %. The return period of the design storm then becomes

$$t_{Pf} = \left(1 - (1 - P_f)^{1/t_{LL}}\right)^{-1} \tag{A10.24}$$

which gives 225 years according to our example.
To determine the partial safety coefficient one has to start with an extreme value distribution, for example the Weibull distribution. For this purpose the distribution is given as:

$$Q_{tL} = \left\{1 - \exp\left[-\left(\frac{H_{ss} - \gamma}{\beta}\right)^{\alpha}\right]\right\}^{N_s t_L} \tag{A10.25}$$

In this equation α and β are the parameters of the Weibull distribution, and γ is the threshold value. N_s is the number of observations per year.
This equation can be reworked to

$$H_{ss} = \gamma + \beta\left[-\ln\left\{1 - \exp\left(\frac{\ln Q_{tL}}{N_s T_L}\right)\right\}\right]^{1/\alpha} \tag{A10.26}$$

For Q_{tL} one should enter the non exceedance probability for $N_s t_L$ events during lifetime. This means that $Q_{tL} = P_{f\text{-lifetime}}$. For a design life of 50 years Q_{tL} is 0.364, for 100 years Q_{tL} is 0.366.
For a practical case this leads to the following values of H_{ss}:

Input from wave climate (from Meetpost Noordwijk):

α 1.24
β 1.17
γ 1.22
N_s 87.3

		lifetime 50 years	lifetime 100 years
H_{ss} for $t = t_L$ (50, 100)	H_{ss}^{tL}	7.71	8.15
H_{ss} for $t = 3\ t_L$ (150, 300)	H_{ss}^{3tL}	8.39	8.81
H_{ss} for $t = t_{20\%}$ (225, 450)	H_{ss}^{tpf}	8.64	9.06

Table A10-9

The safety coefficient is given as:

$$\gamma_{H_{ss}} = \frac{H_{ss}^{t_{pf}}}{Q_{tL}} + \sigma'_{Q_L}\left(1+\left(\frac{H_{ss}^{3tL}}{H_{ss}^{tL}}-1\right)k_\beta P_f\right) + \frac{0.05}{\sqrt{P_f N}} \quad \text{(A10.27)}$$

In this equation P_f is the allowable failure during lifetime (not to be mistaken with $P_{flifetime}$, which has been defined as the probability of exceedance of the "once in t_L-years storm" during the lifetime t_L.
The standard deviation σ'_{QtL} is given in PIANC report [PIANC 1992] (copied in Table A10-10) as a function of the type of observations available. The data from Noordwijk are based on accurate observations, so a value of $\sigma'_{QtL} = 0.05$ is realistic.
P_f was 20%, N is the number of "storms" in the PoT-analysis, for this example it is 1746. $k_\alpha = 0.027$ and $k_\beta = 38$ (see Table A10-2).

This leads to:

$$\gamma_{H_{ss}} = \frac{8.64}{7.71} + 0.2^{\left(1+\left(\frac{8.39}{7.71}-1\right)38\cdot 0.2\right)} + \frac{0.05}{\sqrt{0.2\cdot 1746}} = 1.13 \quad \text{(A10.28)}$$

The safety coefficient consists of three parts. The first term gives the correct partial safety coefficient, provided no statistical uncertainty and measurement errors related to H_{ss} are present. The second term signifies the measurement errors and the short-term variability related to the wave data. The last term signifies the statistical uncertainty of the estimated extreme distribution of H_{ss}. The statistical uncertainty treated in this way depends on the total number of wave data N, but not on the length of the observation period.

Parameters	Method of determination	Typical value for σ'
Wave height		
Significant wave height offshore	Accelerometer buoy, pressure cell, vertical radar	0.05 – 0.1
	Horizontal radar	0.15
	Hindcast, numerical model	0.1 – 0.2
	Hindcast, SMB method	0.15-0.2
	Visual observation (Global wave statistics	0.2
H_{ss} nearshore determined from offshore	Numerical models	0.1-0.2
H_{ss} taking into account typical nearshore effects (refraction, shoaling, breaking)	Manual calculation	0.15-0.35
Other wave parameters		
Mean wave period offshore on condition of fixed H_{ss}	Accelerometer buoy	0.02-0.05
	Estimates from amplitude spectra	0.15
	Hindcast, numerical model	
Duration of sea state with H_{ss} exceeding a specific level	Direct measurements	0.1-0.2
	Hindcast, numerical model	0.02
Spectral peak frequency offshore	Measurements	0.05-0.1
	Hindcast, numerical models	0.05 0.15
Spectral peakedness offshore	Measurements and hindcast with numerical models	0.1-0.2
		0.4
Mean direction of wave propagation offshore	Pitch-roll buoy	
	Measurement of horizontal velocity components and waterlevel or pressure	5°
	Hindcast, numerical model	10°
		15-30°
Water level		
Astro tides	prediction from constants	0.001-0.07
storm surges	numerical models	0.1-0.25

Table A10-10 Typical variation coefficients for sea state parameters [from PIANC 1992]

If extreme wave statistics are not based on N wave data, but for example on estimates of H_{ss} from information about water level variations in shallow water, then the last term disappears and instead the value chosen for σ' must account for the inherent uncertainty.

In Table A10-11 the values of σ' and N have been changed to the values for simple manual calculations and a shorter dataset. It is clear from this table that the effect of the length of a dataset is less important than accurate observations.

	base example	use $\sigma' = 0.35$	use $N = 10$ storms	use $\sigma' = 0.35$ and $N = 10$
basic safety coefficient	100%	87%	99%	84%
measurement and short term errors	0%	13%	0%	13%
statistical uncertainty	0%	0%	1%	3%

Table A10-11

The partial safety coefficient for strength

The safety coefficient for the strength can be calculated using

$$\gamma_z = 1 - (k_\alpha \ln P_f) \tag{A10.29}$$

in which k_α and k_β are coefficients determined by optimisation and given in the PIANC manual [PIANC 1992]. These values are copied in Table A10-12. The value of P_f is the allowable probability of failure during lifetime.

Formula, type of construction	k_α	k_β
Hudson, rock	0.036	151
Van der Meer, rock, plunging waves	0.027	38
Van der Meer, rock, surging waves	0.031	38
Van der Meer, Tetrapods	0.026	38
Van der Meer, Cubes	0.026	38
Van der Meer, Accropodes	0.015	33
Van der Meer, rock, low crested	0.035	42
Van der meer, rock, berm	0.087	100

Table A10-12 Coefficients used to determine the partial safety factor γ_z

In the Coastal Engineering Manual the same approach is followed, however in that manual the values of γ_H and γ_z are directly given as a function of P_f and σ'.
For cubes, one can apply the Van der Meer equation:

$$\frac{1}{\gamma_z}\left(6.7\frac{N_{od}^{0.4}}{N^{0.30}} + 1.0\right) s_m^{-0.1} \Delta d_n \geq \gamma_{Hss} H_{ss}^{tL} \tag{A10.30}$$

For the harbour of Scheveningen, one may use the wave data of Noordwijk. Filling in values of $N_{od} = 1$, $N = 1500$, $\Delta = 1.75$, $s_m = 2.5\%$, this gives $d_n = 2.07$, or 25 tons.
Notice that in the above equation for the wave height the H_{ss} is used which has a probability of exceedance of once in the lifetime of the structure, i.e. the “once in 50 years storm” (7.71 m). This wave height is multiplied by γ_H (1.13), resulting in a total wave height of 8.71 m.
Traditionally one should use a wave height with a probability of 20% during the lifetime of 50 years. This wave has a yearly exceedance of $1/225 = 4.4 \cdot 10^{-3}$.
See also Chapter 3. The “once in 225 years wave” is 8.64 m (in the PIANC guidelines it is called H_{ss}^{tpf}) and compares quite well to the calculated value of 8.71 m.

Note that in the latter example, the limited water depth has not been taken into account.

A10.4.3 Probabilistic approach

Instead of the method with partial safety coefficients, one may apply a full probabilistic computation, either on level 2 or level 3. For level 2 one may apply the FORM method, for level 3 one may apply the Monte-Carlo method. In the examples below the computer program VaP from ETH-Zürich has been applied.

The first step is to rewrite the design equation as a *Z*-function. For cubes the *Z*-function is:

$$Z=\left(A\frac{N_{od}^{0.4}}{N^{0.30}}+1.0\right)s_m^{-0.1}\Delta d_n-H_{ss}^{tL} \tag{A10.31}$$

In this equation seven variables are used. In a probabilistic approach one has to determine the type of distribution for each parameter.

The constant 6.7 from the Van der Meer equation is replaced here by a coefficient *A* with a normal distribution. This coefficient has a mean of 6.7 and a standard deviation describing the accuracy of the equation itself. According to Van der Meer the standard deviation of the coefficient *A* is approximately 10% of its value.

Wave steepness is assumed to have a Normal distribution. The average steepness as well as the standard deviation of the steepness can be calculated from the dataset of Meetpost Noordwijk. If one considers only the heavier storms (i.e. storms with a threshold of e.g. 4.5 m), the average steepness is 0.058, with a standard deviation of 0.0025.

We then have 56 storms in 20 years (i.e. 2.8 storms per year) with an average duration of 7.2 hours. This means that the average period is 6.9 seconds, and that there are consequently 3700 waves in a storm. However, the duration of the storms varies quite a lot (it may go up to 20 hours, which means that we have 10000 waves. This means that *N* will have a large standard deviation. One can fit the calculated waves in each storm and fit this to a distribution. However, the effect of the number of waves is not that high so one may assume a Lognormal distribution (on cannot use a Normal distribution, because the number of waves may become negative when using high standard deviations).

Because the standard deviation in the block size (concrete cubes) is so small, this parameter can be considered as a deterministic value. The acceptable damage level is a target value, therefore this should also be a deterministic value.

H_{ss} has a Weibull or Gumbel distribution. For the wave-height one may enter for example a Weibull distribution, using the values of α, β and γ as determined before.

This results in the following input table:

parameter	type	mean	standard deviation
A	Normal	6.7	0.67
N_{od}	Deterministic	1	-
N	Lognormal	3000	3000
s_m	Normal	0.058	.0025
Δ	Normal	1.75	0.05
d_n	Deterministic	2.4	-

Table A10-13

One can compute the probability of ($Z < 0$), which is the probability of failure.

The target probability is 1/225 = 0.0044 per year. However, VaP gives the probability per event. Because we have 87.3 storms per year, the target probability per event becomes 0.0044 / 87.3 = $50{*}10^{-6}$. The weight of a cube of 2.4 m is 37.5 tons.
For this example using the FORM method a probability of failure of $45{*}10^{-6}$ is found, which is quite near the target value. A Monte-Carlo computation gives a probability of failure of $70{*}10^{-6}$, so quite comparable.

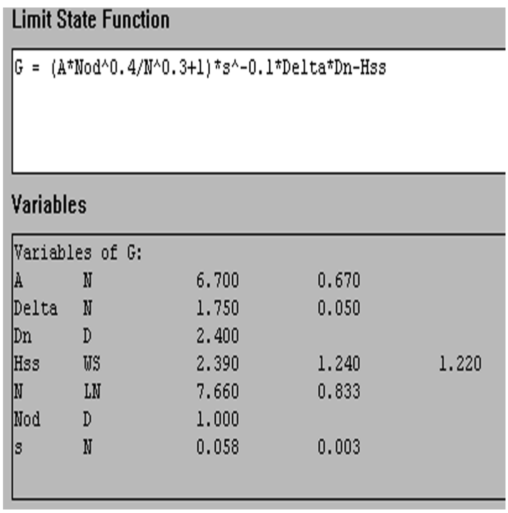

Figure A10-11

This computation is based on the fact that we have defined $N_{od} = 1$ as start of failure. However, this is quite some damage. Lowering the value of N_{od} to 0.5 means that we have to increase the block size to 2.65 m (39 tons) in order to obtain the same probability of failure. Using a N_{od} of 0 means an increase to 3.80 m (44 tons)
In the above calculations the uncertainty in the determination of the parameters of the Weibull distribution was not taken into consideration. Although this is common practice, it is not fully correct. The mathematically correct way is to consider α, β

and γ as stochastic parameters with a mean and a standard deviation. One can determine these values directly from the dataset, but it is not possible to apply these values directly in a probabilistic computation. In practice this problem is solved by introducing an extra variable M. This variable has a mean value of 1 and a standard deviation which expresses the variation in the prediction of H_{ss} using a Weibull or Gumbel distribution. The Van der Meer formula then becomes:

$$Z=\left(A\frac{N_{od}^{0.4}}{N^{0.30}}+1.0\right)s_m^{-0.1}\Delta d_n-MH_{ss}^{tL} \tag{A10.32}$$

A problem is that the standard deviation of M depends on the value of H_{ss}. On can determine this value using the design value for H_{ss}.
The standard deviation is given as:

$$\sigma'_M=\frac{\sigma_M}{H_{ss-design}} \tag{A10.33}$$

For σ_M an expression has been derived by Goda [Goda, 2000]:

$$\sigma_M=\sigma_z\sigma_x \tag{A10.34}$$

in which σ_x is the standard deviation of all H_{ss} values in the basic dataset and σ_z is defined by:

$$\begin{aligned}\sigma_z&=\frac{\left[1.0+a(y-c)^2\right]^{1/2}}{\sqrt{N}}\\ a&=a_1\exp\left[a_2N^{-1.3}\right]\end{aligned} \tag{A10.35}$$

in which the coefficients are given by:

distribution	a_1	a_2	c
Gumbel	0.64	9.0	0
Weibull, $\alpha=0.75$	1.65	11.4	0
Weibull, $\alpha=1.0$	1.92	11.4	0.3
Weibull, $\alpha=1.4$	2.05	11.4	0.4
Weibull, $\alpha=2.0$	2.24	11.4	0.5

Table A10-14

y is the reduced variate for the design value, our design value is a 1/225 wave, so y = $[\ln(87.3*22)]^{1/1.24}$ = 6.34.

In the example used, we have a Weibull with $\alpha = 1.24$; because this value is not in the table, an interpolation is needed. For the example the following values were being used: $a_a = 2.0$, $a_2 = 11.4$ and $c = 0.35$.
This results in a value of $\sigma_z = 0.024$. Given a σ_x of 6.85 (follows from dataset), this means:

$$\sigma'_M = \frac{\sigma_M}{H_{ss-design}} = \frac{\sigma_x \sigma_z}{H_{ss-design}} = \frac{6.85 \cdot 0.024}{8.64} = 0.02 \tag{A10.36}$$

A recalculation with these figures gives a probability of failure of $157*10^{-6}$. In order to bring this back to the required $50*10^{-6}$ we have to increase the cube size to 2.42 m. So, this can be neglected.
However, if our dataset would have been considerably smaller (for example only 100 storms), this would change the value of σ_z to 0.175 and consequently σ_M' to 0.14. In order to get a probability of failure in the order of the required $50*10^{-6}$ we have to increase the cube size to 2.70 m(from 37.5 to 55 tons). Again this is for deep (18 m) water conditions.
This shows that the size of the dataset has a considerable impact on the required block size.

A10.4.4 Probabilistic calculation in case of a shallow foreshore

Statistical software does discard physical limitations. E.g., applying statistics in deep water circumstances will give very high waves with very low probability. But in shallow water these high waves cannot exist at all. In general the significant wave height in shallow water is limited by the depth. So:

$$H_{ss} = \gamma_b h \tag{A10.37}$$

in which the breaker index γ_b has a value in the order of 0.6. For individual waves γ_b may have values up to 0.78, but for the significant wave height this value is much lower, sometimes even down to 0.45.
For foreshores with a gentle slope one may assume that γ_b may have an average value of 0.55, with a standard deviation of 0.05. The water depth in this equation is the total depth, i.e. the depth below mean sea level + the rise of waterlevel due to tide and storm surge.
With this information, one can rewrite the Van der Meer equation for cubes to:

$$Z = \left(A \frac{N_{\mathrm{od}}^{0.4}}{N^{0.30}} + 1.0 \right) s_m^{-0.1} \Delta d_n - \gamma_b (h_{\mathrm{surge}} + h_{\mathrm{depth}}) \tag{A10.38}$$

The water depth below mean sea level has a Normal distribution, but the standard deviation in this parameter is usually so low, that it can be considered as a deterministic value. Of course in case large bed fluctuations are to be expected, one may also enter this value as a stochastic parameter with a Normal distribution. The surge has an extreme value distribution. For this value one can apply a Gumbel distribution.

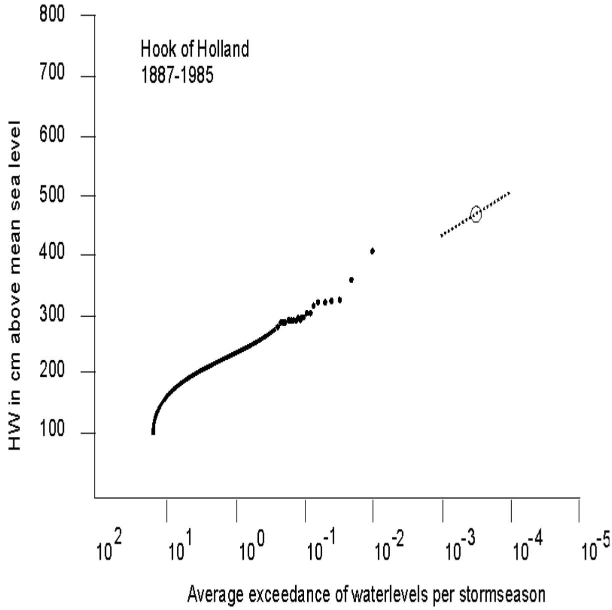

Figure A10-12 Extreme waterlevels in Hook of Holland

Again we look at the example of Scheveningen harbour. The water depth in front of the breakwater in Scheveningen is 6 m below mean sea level. Long term waterlevels are available from Hook of Holland, which are approximately identical to Scheveningen. The exceedance can be given by:

$$Q = 1 - \exp\left[-\exp\left(-\frac{h_{surge} - \gamma}{\beta}\right)\right] \tag{A10.39}$$

In this equation is γ the intercept at the 10^0-line, and β is the slope parameter. From the diagram one can derive that γ equals 2.3 and that the slope β is 0.30.

The equation is based on the maximum surges per year, so the exceedance is also per year, and not per storm. This implies that in this case the target probability of failure is 1/225 = 0.0044.

This leads to d_n = 1.85 m, or a block weight of 17.7 tons.

In principle one can in this case also add the statistical uncertainty by adding a factor M in front of the surge height in the equation. However, because of the long dataset and the limited extrapolation, this uncertainty is very small and may be neglected.

Limit State Function

```
G = (A*Nod^0.4/N^0.3+1)*s^-0.1*Delta*Dn-gamma*(h+6)
```

Variables

Variables of G:

A	N	6.700	0.670
Delta	N	1.750	0.050
Dn	D	1.850	
N	LN	7.660	0.833
Nod	D	1.000	
gamma	N	0.550	0.050
h	GL	2.300	3.289
s	N	0.025	0.002

Figure A10-13

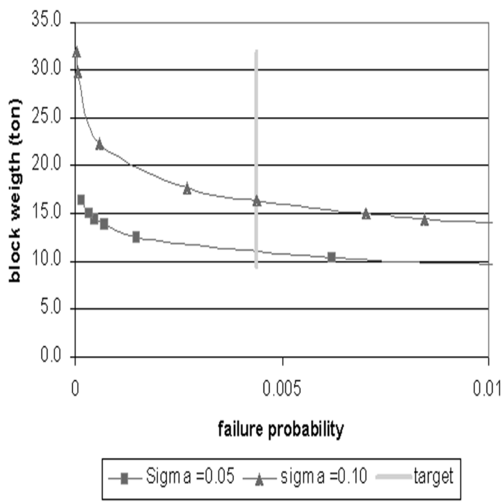

Figure A10-14 Effect of standard deviation of γ_b on block weight

Changing the standard deviation in γ from 0.05 to 0.1 has a considerable effect. To obtain the same target probability of failure, one has to increase the blocks to d_n = 2.2 m (30 tons).

A simplification made in this computation is that is assumed that in deep water, wave steepness remains the same after breaking. This is probably not the case. The higher waves will break (usually as spilling breakers) which decreases their height considerably, but usually not the period. As a consequence the wave steepness in broken waves is much less than the wave steepness at sea. Because in the Van der Meer formula for cubes a low steepness gives smaller cubes than a high steepness, neglecting the change in steepness is a conservative approach. Be aware that in case of natural rock the opposite is true.