# On Wavelet Based Spectrum Estimation for Dynamic Spectrum Access

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# ABSTRACT

One of the important functionalities of Dynamic Spectrum Access is spectrum estimation. Accuracy and speed of estimation are the key indicators to select the appropriate spectrum estimation technique. In this thesis work, the possibility of employing wavelet packet decomposition as a basis for a new spectrum estimation approach is investigated. Once the new approach is developed, four types of sources, namely partial band, single tone, multitones, and swept tone, are used to investigate the performance of the proposed wavelet based approach. Preliminary comparative analysis between the performance of wavelet based approach with conventional techniques, such as Periodogram and Welch technique has also been conducted. The studies show that the wavelet based approach offers great flexibility, reconfigurability and adaptability.

Key to the successful operation of the wavelet based spectrum estimation is the choice of the wavelet used. Commonly known wavelets are not suitable for spectrum estimation because they result in estimates with poor frequency resolution. To alleviate this problem, we design and develop a family of wavelets that are maximally frequency selective in nature as our second contribution in this thesis work. To this end, the design constraints are first enlisted. Then the problem, originally non-convex, is reformulated into a convex optimization problem and solved using Semi Definite Programming (SDP) tools. Through simulation studies the benefits of the newly designed wavelets are demonstrated.

The next contribution of this thesis work is to combine the existing wavelet packet multi-carrier modulation (WPMCM) technique with our wavelet based spectrum estimator in order to form a wavelet packet transceiver for a dynamic spectrum access environment. To enable the wavelet packet transceiver cognitive radio (CR) system to co-exist with other Licensed Users (LU), a common spectrum pool is maintained and the WPMCM transmission waveform characteristics are shaped to communicate in the idle time-frequency gaps of the licensed user. This is achieved by dynamically vacating wavelet packet carriers in and near the region of the licensed user spectrum. The spectrum estimation unit is tagged to the WPMCM transceiver structure by exploiting the filter bank infrastructure used for Discrete Wavelet Packet Transform implementation. Thus spectrum analysis is done at no additional cost. In the studies, four types of LUs are employed, namely, partial band, single tone, swept and multiple tone. The simulation results show that in the presence of an LU, the proposed spectrum adaptation method offers significant BER improvements allowing the CR to operate invisibly to the LU.

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# LIST OF ACRONYMS AND ABBREVIATIONS

AR	Auto Regressive
ARMA	Auto Regressive Moving Average
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPF	Band Pass Filter
CDMA	Code Division Multiple Access
CR	Cognitive Radio
CWT	Continuous Wavelet Transform
DFT	Discrete Fourier Transform
DSA	Dynamic Spectrum Access
DWPT	Discrete Wavelet Packet Transform
DWT	Discrete Wavelet Transform
FBSE	Filter Bank Spectrum Estimation
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
HPF	High Pass Filter
ICI	Inter Carrier Interference
IDWPT	Inverse Discrete Wavelet Packet Transform
IDWT	Inverse Discrete Wavelet Transform
ISI	Inter Symbol Interference
ITOM	Interpolated Tree Orthogonal Multiplexing
IWPT	Inverse Wavelet Packet Transform
LPF	Low Pass Filter
LU	Licensed Users
MA	Moving Average
MCM	Multi Carrier Modulation
MTSE	Multi Taper Spectrum Estimator
NTIA	National Telecommunication and Information Administration
OFDM	Orthogonal Frequency Division Multiplexing
OSI	Open System Interconnection
PSD	Power Spectral Density
QoS	Quality of Service
QMF	Quadrature Mirror Filters
QPSK	Quadrature Phase Shift Keying
ROC	Receiver Operating Characteristic
SCF	Spectral Correlation Function
SDP	Semi Definite Programming
SNR	Signal to Noise Ratio
STFT	Short Time Fourier Transform
WP	Wavelet Packet
WPMCM	Wavelet Packet Multi Carrier Modulation
WPT	Wavelet Packet Transform

# LIST OF MAJOR SYMBOLS

$S_{yy}^{p}(e^{j\omega})$	Periodogram estimate of Power Spectrum Density (PSD)
N	Number of samples
$S_{xx}^{c}(e^{j\omega})$	Correlogram estimate of Power Spectrum Density
$S_{xx}^{BT}(e^{j\omega})$	Blackman Tukey estimate of Power Spectrum Density
$S_{xx}^{ap}(e^{j\omega})$	Bartlett estimate of Power Spectrum Density
w[k]	Window function
$W(e^{j\omega})$	Window kernel of window function <i>w</i> [ <i>k</i> ]
$S_{xx}^{wp}(e^{j\omega})$	Welch estimate of Power Spectrum Density
$M_s$	Number of segments in Welch and Bartlett PSD estimate
$K_s$	Number of samples per segment
$S_{xx}^{ar}(e^{j\omega})$	Autoregressive estimate of Power Spectrum Density
$\kappa_k(f_i)$	Expansion coefficients in the calculation of Multi Taper Spectrum Estimation (MTSE) PSD
$\hat{S}_{MTSE}(f_i)$	MTSE based PSD estimate at frequency $f_i$
$\hat{S}_{PSE}(f_i)$	Periodogram estimate at frequency $f_i$ derived from MTSE
$\hat{S}_{FBSE}(f_i)$	Filter Bank Spectrum Estimation (FBSE) of PSD
$CWT_f(a,b)$	Continuous Wavelet Transform of a continuous signal $f(t)$ consisting of several wavelet coefficients, which are a function of scale $a$ and translation $b$
$\varphi$	Scaling function
Ψ	Wavelet function
$STFT{x(t)}$	Short time Fourier Transform of $x(t)$
<i>h</i> [ <i>n</i> ]	Filter coefficients of wavelet low pass analysis filter
g[n]	Filter coefficients of wavelet high pass analysis filter
$L^{2}(\mathbb{R})$	Set of square integrable function in Hilbert space
$\chi_{\mathrm{a,b}}$	Sub space spanned by wavelet transform coefficients at wavelet packet node
L	<i>b</i> and level <i>a</i> . Length of filter
L K	Regularity index
Н	Half band low pass analysis filter
G	Half band high pass analysis filter
H'	Half band low pass synthesis filter
G'	Half band high pass synthesis filter
GC(n)	Gray Code permuted integer Energy collected at <i>m</i> th wavelet packet node
$\hat{PSD}$	PSD estimate at frequency hand spanned by <i>m</i> -th wavelet packet node
$f_{WD}$	The width of frequency band spanned by single wavelet packet node
$\rho_{X,Y}$	Correlation coefficient between two random variables X and Y
Cov[X,Y]	Covariance between random variables x and Y
Var[X]	Variance of X
$\omega_{p}$	Normalized pass band frequency
$\omega_{s}$	Normalized stop band frequency
$\Delta$	The maximum value of the tolerance or ripple in filter design
$r_h$	Autocorrelation sequence of filter <i>H</i>
$H(\omega)$	Frequency response of half band low pass analysis filter
$Q(\omega)$	Factor of $H(\omega)$ that does not have zeros at $\omega = \pi$

S(n)	Wavelet packet multi-carrier	modulated signal
	1	U

- Wavelet Packet Multi-carrier modulation (WPMCM) subcarrier Number of WPMCM subcarriers
- S( ξ C

# CHAPTER 1 INTRODUCTION

### **1.1 Motivation of the Research**

In the last few years, the demand for digital wireless communication has increased dramatically. Aided by the interoperability of wireless communication network, mainly due to the flexible protocols and standards, new and valuable applications such as mobile internet access, electronic healthcare monitoring service, sensor networking and many others have emerged. With the elegant implementation of layering approach based on Open System Interconnection (OSI) model, which supports the convergence between wired and wireless network, newer and newer applications and services have emerged and are flourishing like never before. This trend is placing great demands on premium radio resources especially the radio spectrum.



Figure 1.1 The NTIA's frequency allocation chart [1]

Figure 1.1 shows the National Telecommunication and Information Administration's (NTIA) chart of spectrum frequency allocations illustrating how the frequency bands are allocated to various services. Figure 1.1 reinforces the fact that most of the available spectrum is licensed leaving very little room for newer services. Even when the clamor for free spectrum has grown into a shrill note, an interesting study conducted in Berkeley [2] showed that much of the licensed frequencies, especially in 3-6MHz band, are rarely used. The study also showed that the usage of allocated spectrum varies from 15% to 85% depending on time and geographical location. Such a static spectrum allocation policy is clearly wasteful. This paradox of non-availability of spectrum even when large swathes of licensed spectrum are under utilized most of the time has prompted a rethinking in existing spectrum regulatory policies, leading to the idea of Cognitive Radio and a Dynamic Spectrum Access regime.

Cognitive Radio [2][3] is an intelligent wireless communication system that is cognizant (hence the name) of its environment, learns from it and adapts its transmission features according to statistical variations in the environment to maximize utilization of premium resources such as spectrum while ensuring good Quality of Service (QoS). This dynamic spectrum access paradigm

introduced two main entities, namely, primary users or licensed users and secondary users or unlicensed users. The primary users are basically the owner of the licensed spectrum while the secondary users, also called the cognitive radio, are allowed to transmit and receive signals over the licensed spectra or portion of it when the primary user(s) is (are) currently not active. The secondary users should have the ability to gauge the radio environment, intelligently exploit the unused licensed spectrum and relinquish it when primary users are active, and adapt their transmission parameters (including frequency, power, and modulation scheme) in accordance with the changing environment and requirements has been promoted. This mechanism should be performed in such a way that the secondary users are invisible to primary user. The primary users usually do not possess the intelligence on the opportunistic spectrum sharing mechanism.

In dynamic spectrum access environment, spectrum estimation plays important key to gauge the wireless environments over wide frequency bands and identify spectrum holes and occupied bands. The challenge is in the identification and detection of primary user signals amidst harsh and noisy environments. The result of spectrum estimation is used to provide guidance for the secondary users to decide when they may exploit the unused licensed spectrum and when they have to relinquish it. In general, speed and accuracy of measurement are the main metrics to determine the suitable spectrum analysis tool. These two metrics are important to answer the questions of which band is occupied and at what instance. Accuracy of the estimation depends on frequency resolution, bias or leakage and variance of the estimated power. The better the frequency resolution, the better the accuracy of the estimated power in each frequency point. The bias or leakage is related to the side lobe level. High side lobe level reduces the accuracy of power estimate at neighboring frequency. Meanwhile variance of the estimate is also important to ensure that the power estimate at particular frequency band is always accurate at any time of measurement. There are other important metrics too. One is to strike the right trade-off between the time and frequency resolution achievable. Due to uncertainty principle, it is not possible to have the best frequency and time resolution at the same instance.

Apart from time and frequency resolution, complexity is very important to assess any candidate of spectrum estimation modules for cognitive radio. Since Cognitive Radio systems are envisioned to operate on wireless nodes with small size and power, the spectrum estimation implementation should be kept as simple as possible. One way to evaluate the complexity is to investigate whether the addition of the proposed spectrum estimation approach into the receiver would require significant modification and costs.

Conventional spectrum estimation techniques, like periodogram, are based on Fourier expansion which offers excellent frequency resolution but are poor in time detail. Though this can be improved through windowing, as in Short Time Fourier Transform (STFT), the results have been found unsatisfactory. It is in this context that the possibility of using wavelets and wavelet packet transforms, which offer a time-frequency resolution trade-off that can be tuned, has emerged as an enticing option.

# 1.2 Theme of the Research

This thesis investigates the possibility of exploiting Discrete Wavelet Packet Transform to build new spectrum estimators for Cognitive Radio. The wavelet based spectrum estimator is implemented using filter banks by taking advantage of the fact that compactly supported wavelets can be derived from perfect reconstruction filter banks [4]. In reality, the implementation of filter banks simulates the wavelet packet decomposition, which basically split the given signal into the coarse version (low frequency component) and detail version (high frequency component) in each decomposition stage. The number of stages is usually limited by the desired level of frequency resolution and available computational power.

While it is possible to implement two-band filter banks based on commonly available wavelets such as Symlets, Daubechies and Coiflet, the frequency response of the wavelet decomposition filter based on these standard wavelet families is simply not frequency selective by nature. Since frequency resolution is absolutely important in spectrum estimation, a new wavelet with excellent frequency selectivity is necessary to be used as a basis function for decomposition filter. In this thesis, we design a new wavelet using semi definite programming by expressing the

design as an optimization problem. The idea is to obtain wavelet decomposition filter having optimum frequency selectivity under a set of wavelet constraints.

Multi-carrier modulation, which divides the incoming high data rate among multiple carriers modulated at lower rate, has been mooted as a strong physical layer candidate for Cognitive Radio (CR) system design [5]. By merely vacating a set of subcarriers, the spectrum of a Multi Carrier Modulation based Cognitive Radio can be easily and flexibly shaped to occupy spectral holes without interfering with the licensed users. It has been shown that adaptive Multi Carrier Modulation based Cognitive Radio is a robust method to achieve good quality of communication and efficient use of the spectrum [5]. Orthogonal Frequency Division Multiplexing (OFDM) is an elegant and popular multi-carrier modulation scheme in which the generation and modulation of the sub-channels is accomplished using Fourier exponential basis function. However, another multi carrier modulation technique based on wavelet called Wavelet Packet Multi Carrier Modulation (WPMCM) has emerged as a new candidate. As it is found in [6], it appears that the performance of WPMCM is adequate to compete with OFDM when the multi carrier modulation technique is expected to provide not only modulation functionality but also spectrum adaptability. By considering the possibility of future employment of WPMCM in Cognitive Radio transceiver, we propose a method to combine the wavelet based spectrum estimator with WPMCM modules for Cognitive Radio System in this thesis. By taking advantage of this single wavelet technology, the cohabitation of WPMCM CR system and licensed user (LU) is possible by dynamically activating or de-activating CR subcarriers based on the spectrum estimation information provided by wavelet based spectrum estimation. Furthermore, the WPMCM receiver structure, which is used for demodulation of data, is also used for analysis of the radio environment to identify active/idle bands at no additional cost.

# **1.3 Objectives and Major Contribution of the Thesis Work**

The primary objectives of this thesis work are:

- To investigate the possibility of implementing spectrum estimation technique based on wavelets and wavelet packet transform.
- To establish a simulation setup in MATLAB for wavelet based spectrum estimation by exploiting filter bank implementation of discrete wavelet transform
- To evaluate the performance of established wavelet packet based spectrum estimator and provide preliminary comparative analysis between it and traditional approaches like Periodogram, Welch, Windowed Periodogram and Multitaper Spectrum estimator (MTSE).
- To design and develop new wavelet decomposition filter that are maximally frequency selective and hence best suited to applicability to wavelet based spectrum estimation
- To evaluate the performance of spectrum estimation module based on the optimal wavelet designed using semi definite programming
- To realize wavelet packet transceiver for spectrum sensing and dynamic spectrum access by merging the spectrum estimation module with a Wavelet Packet based Multi Carrier Modulation (WPMCM) module.

### **1.4 Organization of thesis works**

This thesis report is organized as follows. Chapter 2 provides the theoretical foundation for wavelet based spectrum estimation. There are two main parts in this chapter. The first part gives a broad overview on the theoretical aspect of spectrum estimation. This includes a survey on conventional techniques such as periodogram, Multi Taper Spectrum Estimation as well as new techniques proposed for Cognitive Radio. The second part of the chapter gives the details of wavelet theory, wavelet transform and its filter bank implementation. It also gives a prelude to spectrum estimation using wavelet.

Chapter 3 discusses about the first contribution of the thesis works, namely, the construction of wavelet based spectrum estimation using filter bank approach. The discussion also includes some important issues such as the frequency ordering of wavelet packet coefficients, the relationship between wavelet packet coefficients and power spectrum density as well as Parseval relationship and energy conservation. In the final part of this chapter, we elaborate the experiments

conducted to investigate the performance of the wavelet based spectrum estimation. We also try to find a preliminary overview on the position of our technique along with conventional periodogram approach.



Figure 1.2 The organization of the thesis chapters.

In chapter 3 the spectrum estimator uses standard wavelets (which were not originally intended for spectrum estimation) that are available in Matlab Toolbox. Hence, in chapter 4, we focus on the design and development of a wavelet family that best suits the applicability for spectrum estimation. The process basically involves the formulation of design problem into optimization problem, transformation of non-convex to convex optimization problem as well as deriving the solution by taking advantage of spectral factorization algorithm. In the final part of chapter 4, we examine the performance of spectrum estimation based on the designed wavelets and their standing vis-à-vis the standard wavelets. A comparative analysis between this spectrum estimator and periodogram based approach is also provided here.

In chapter 5, we demonstrate the combination of the wavelet based spectrum estimator and a wavelet packet multi-carrier modulation (WPMCM) technique. The combination of these two modules forms wavelet packet transceiver for dynamic spectrum access environment.

Chapter 6 is an extremely important one since it gives elaborate explanation about some challenges found in this research. The most important challenge is the presence of an infarction called spectrum carving which limits the performance of the wavelet based approach. Other important challenges and suggested improvements for the future are also documented in this chapter.

Finally chapter 7 concludes this thesis work and gives overview about possible future researches. Figure 1.2 illustrates the organization of the thesis chapters.

# CHAPTER 2 SPECTRUM ESTIMATION AND WAVELET THEORY

The theory of wavelet and spectrum estimation plays an important basis of this thesis work and thus this chapter gives a comprehensive elaboration about these topics. Section 2.1 discusses conventional spectrum estimation technique that is popular in telecommunication world. This section is started by elaborate discussion about periodogram and its variants. Parametric spectrum estimation is also reviewed to give complementary discussion. Even though, spectrum sensing is not exactly the same as spectrum estimation, they are built on the same basis and both of them are equally important when we talk about dynamic spectrum access. Hence, section 2.2 talks about some spectrum sensing and estimation technique that is proposed for dynamic spectrum access. Since the wavelet based spectrum estimation discussed in this thesis is built based on filter bank architecture, section 2.3 elaborates about how to represent periodogram spectrum estimates from filter bank point of view. This section also provides some information about two important techniques that were developed mainly based on filter bank paradigm. This includes Filter Bank Spectrum Estimator proposed by Farhang-Boroujeny for cognitive radio. We leave the discussion about spectrum estimation theory and explore the wavelet theory in section 2.4 that plays an important role in this thesis. Finally, section 2.5 gives an example of previous work on spectrum estimation based on wavelet.

### 2.1 Common Spectrum Estimation Techniques

In general, spectrum estimation can be categorized into direct and indirect methods. In direct method (usually recognized as frequency domain approach), the power spectrum is estimated directly from signal being estimated x[n]. On the other hand, in indirect method, also known as time domain approach, the autocorrelation function of the signal being estimated  $R_{xx}[k]$  is calculated. From this autocorrelation value, the power spectrum density can be found by applying the Discrete Fourier Transform on  $R_{xx}[k]$ .

Another way to categorize spectrum estimation methods is by classifying them into parametric or non-parametric methods. Parametric method is basically model based approach [7]. In this method, a signal is modeled by Auto Regressive (AR), Moving Average (MA) or Auto Regressive Moving Average (ARMA) process. Once the signal is modeled, all parameters of the underlying model can be estimated from the observed signal. Estimator based on parametric method provides higher degree of detail. The disadvantage of parametric method is that if the signal is not sufficiently and accurately described by the model, the result is less meaningful. Non Parametric methods, on the other hand, do not have any assumption about the shape of the power spectrum and try to find acceptable estimate of the power spectrum without prior knowledge about the underlying stochastic approach. The following sub-sections give review on some of the spectrum estimation methods.

### 2.1.1 Periodogram

The most commonly known spectrum estimation technique is periodogram, which is classified as a non parametric estimator. The procedure starts by calculating the Discrete Fourier Transform (DFT) of the random signal being estimated, followed by taking the square of it and then dividing the result with the number of samples N. The basic idea of periodogram can be illustrated as:

$$S_{xx}^{p}(e^{j\omega}) = \frac{1}{N} \left| X(e^{j\omega}) \right|^{2} = \frac{1}{N} \left| \sum_{n=0}^{N-1} x[n] \exp(-j\omega n) \right|^{2}$$
(2.1)

The  $S_{xx}^{p}(e^{j\omega})$  in (2.1) is the periodogram estimate of the power spectra while x[n] and  $X(e^{j\omega})$  are the sequence whose spectrum is to be estimated and the corresponding transform in frequency domain, respectively.

The main issue in periodogram is the use of rectangular windowing of waveform to obtain finite length samples. This windowing process introduces a discontinuity (illustrated in figure 2.1)

between the original signal and the aliased version produced by a DFT transformation. In the frequency domain, the rectangular window results in a Dirichlet Kernel described by the width of the main lobe and the level of side lobe [9]. The width of the main lobe is related to the frequency resolution of the power spectra, and the level of side lobe is related to the ratio between maximum and minimum spectral power that is distinguishable by the estimator. The rectangular window compromises the frequency resolution, producing leakage and a biased estimate. In order to mitigate the impact of rectangular window, various window functions can be applied on the data before the computation of periodogram. This is equivalent to replacing each periodogram coefficient by weighted coefficients. This is what happens in the Blackman-Tukey method illustrated in sub-section 2.1.3.



Figure 2.1 The effect applying DFT on the truncated version of signal  $y_0(k)$  has resulted in periodic signal y(k) which contains the windowed version of  $y_0(k)$  and its aliases [8]

Another problem with the periodogram is that the estimates of the power spectral density (PSD) are coarse with low precision and large variance which does not improve with more data. The only way to improve the variance of the periodogram is to average the PSD coefficients. This can be done by computing several (shorter) periodograms and use these to compute averages of each PSD coefficient. This method is known as Bartlett method described in sub-section 2.1.4. Bartlett method and Blackman-Tukey method can also be combined, so that one computes an average of several windowed periodograms. This is the Welch method, which is also explained in sub-section 2.1.4.

#### 2.1.2 Correlogram

While periodogram is categorized as direct method since it calculates the power spectral density directly from input signal x[n], correlogram is classified as indirect method. In correlogram, the autocorrelation function of the input signal  $R_{xx}[k]$  is computed. The power spectral density (PSD) is then obtained from the Fourier Transform of  $R_{xx}[k]$  illustrated as:

$$S_{xx}^{c}(e^{j\omega}) = \sum_{k=-\infty}^{\infty} R_{xx}[k] \exp(-j\omega k)$$
(2.2)

It is clear from (2.2) that the true autocorrelation value is required for the PSD calculation. However, the computation of true autocorrelation value requires infinite length of data and thus only approximation is possible. In general, there are two possible ways to compute the approximation of autocorrelation value, namely standard biased and standard unbiased estimates [8]. The standard unbiased estimate is illustrated as:

$$\widehat{R}_{xx}[k] = \frac{1}{N-k} \sum_{n=k}^{N-1} x^*[n-k]x[n], \ 0 \le k \le N-1$$
(2.3)

The standard biased estimate is illustrated as:

$$\widehat{R}_{xx}[k] = \frac{1}{N} \sum_{n=k}^{N-1} x^*[n-k]x[n], \ 0 \le k \le N-1$$
(2.4)

In (2.3) and (2.4),  $\hat{R}_{xx}[k]$  is the approximation of autocorrelation value. Correspondingly, the PSD

in (2.2) can be calculated by using  $\hat{R}_{xx}[k]$  as a replacement for  $R_{xx}[k]$ . The difference between standard unbiased and standard biased in autocorrelation value calculation can be explained as follows. Both in (2.3) and (2.4), the shift is over k data points meaning that there are k terms missing in the series. Hence, the average should be taken only over N-k terms, which has been properly done in (2.3). However, this fact has been ignored in (2.4) and the average is carried out over N samples no matter the value of k is. This is why (2.4) is called a biased estimate.

Another problem within (2.2) is the assumption that the autocorrelation value is of infinite length. This is addressed by applying a rectangular window over approximation of autocorrelation value and accordingly, the correlogram estimate is expressed as:

$$S_{xx}^{c}(e^{j\omega}) = \sum_{k=-L_{w}}^{L_{w}} \widehat{R}_{xx}[k] \exp(-j\omega k)$$
(2.5)

In (2.5), the length of rectangular window is  $2L_w+1$  and  $L_w$  is usually less than the total number of samples of available data.

### 2.1.3 Blackman-Tukey method (Windowed Correlogram)

Blackman-Tukey method is a variant of correlogram that computes the approximated autocorrelation  $\hat{R}_{xx}[k]$  according to either (2.3) or (2.4) and later applies a suitable window function w[k]. The power spectra density is then obtained by computing the Fourier Transform of  $\hat{R}_{xx}[k]$ . Blackman-Tukey method can generally be described as [7]:

$$S_{xx}^{BT}(e^{j\omega}) = \sum_{k=-L_{W}}^{L_{W}} \widehat{R}_{xx}[k]w[k]\exp(-j\omega k)$$
(2.6)

In (2.6),  $S_{xx}^{BT}(e^{j\omega})$  is the power spectra density according to Blackman-Tukey method and w[k] is the selected window. It is trivial to find that Correlogram can actually be thought as Blackman-Tukey method with rectangular window. Since (2.6) is the Fourier Transform of the product of finite length approximated autocorrelation value and the selected window, (2.6) can be represented in fully frequency domain representation, which is actually nothing but the convolution between the window kernel and the correlogram found in (2.5). Since the length of the window in (2.6) is finite, namely  $2L_w+1$ , the frequency domain representation for (2.6) is:

$$S_{xx}^{BT}(e^{j\omega}) = \frac{1}{2\pi} W(e^{j\omega}) * S_{xx}^{c}(e^{j\omega})$$
(2.7)

 $S_{xx}^{c}(e^{j\omega})$  in (2.7) is the basic correlogram of signal x[n] calculated through (2.5) while  $W(e^{j\omega})$  is the window kernel of w[k]. Equation (2.7) clearly illustrates that Blackman-Tukey method can actually be viewed as a process of smoothing the correlogram by convolving the correlogram with the kernel of selected window. This smoothing process plays an important role to reduce the bias of estimated PSD but this convolution process would reduce the frequency resolution. The amount of frequency resolution reduction is strongly related to the size of the main lobe of the window kernel.

#### 2.1.4 Averaging of periodogram and Welch approach

Averaging of Periodogram, which is also recognized as Bartlett Method can be employed to reduce the PSD variance in the periodogram estimates. The samples are divided into several segments and the periodograms of each segment is averaged [10]. Figure 2.2 illustrates the basic procedure to implement Bartlett Method. The important thing is to identify a trade off between number of samples per segment and number of segments. In theory, the number of segments should be maximized in order to minimize the variance of estimated power. However, this also means lowering the number of samples in each sub-sequence resulting in larger bias and poorer frequency resolution. By this well known trade-off, Bartlett method is famously recognized as a method that compromises the resolution to get smaller variance as a return. Finally, the Bartlett method can be illustrated by the following equation.

$$S_{xx}^{ap}(e^{j\omega}) = \frac{1}{M_s} \sum_{m=0}^{M_s-1} \left\{ \frac{1}{K_s} \left| \sum_{n=0}^{K_s-1} x[n+mK_s] \exp(-j\omega n) \right|^2 \right\}$$
(2.8)

In (2.8)  $S_{xx}^{ap}(e^{j\omega})$  is Bartlett power spectrum estimate,  $M_s$  is the number of segments and  $K_s$  is the number of samples per segment.



Figure 2.2 Basic procedures for Averaging on Periodogram

In [11], Welch modified the Bartlett method by letting the segments overlap and introduced arbitrary windows on data segments before the calculation of periodogram. This method can be easily explained by firstly expressing the data segments as:

 $x_m[l] = x[l + mD_s], \ l = 0, 1, ..., K_s - 1 \text{ and } m = 0, 1, ..., M_s - 1$  (2.9)

In (2.9),  $K_s$  represents the number of samples per segment and  $M_s$  stands for the total number of segments. It is interesting to notice that when  $D_s = K_s$ , the segments are not overlapped each other. When this is the case and the rectangular window is employed, the Welch Method is identical to Bartlett Method. Therefore,  $iD_s$  can be regarded as the starting point of the *i*-th segment. In general, the Welch procedure can be illustrated as [11]:

$$S_{xx}^{wp}(e^{j\omega}) = \frac{1}{M_s} \sum_{m=0}^{M_s - 1} \left\{ \frac{1}{K_s} \left| \sum_{l=0}^{K_s - 1} w[l] x[l + mD_s] \exp(-j\omega l) \right|^2 \right\}$$
(2.10)

In (2.10),  $S_{xx}^{wp}(e^{j\omega})$  is the power spectrum estimate calculated based on Welch approach. While in [11] Welch suggests two types of window (Triangular and Hann window), the type of window that can be used in this approach is actually arbitrary (such as Blackman, Hamming or Kaiser window). Almost all types of time window applied on each segment give smaller weight on samples located around the edges of the segment. Therefore, in the final computation of the PSD, different data samples are not equally represented. In order to mitigate this issue, segment overlap between two segments is introduced. In [7], Porat introduces 50 percent segment overlap. In this case, all data samples have equal representation on the average since samples located nearby the edges of a particular segment will be placed around the centre of the adjacent segments.

As in Bartlett method, the choice of segment size and the number of segments determine the frequency resolution and the variance that Welch estimators can offer. Apart from these two parameters, the choice of window will play important role as well. As it is mentioned in [7], different windows introduce different window Kernels in frequency domain (for example: Dirichlet Kernel for rectangular window case) with different levels of side lobe and hence the "leakage". As a result, the choice of window will also determine the dynamic spectrum range of the estimator. Different window Kernel also introduces different width of main lobe, which is strongly related to frequency resolution. In conclusion, the use of windows can be used as a new lever to tune the resolution and the range of the estimator.

#### 2.1.5 Parametric spectrum estimation

As mentioned earlier, parametric method is a model based approach in which a signal is modeled by Auto Regressive (AR), Moving Average (MA) or Auto Regressive Moving Average (ARMA) process. In this method, the data sequence is modeled as an output of difference equation excited by discrete time white noise [9]. The difference equation can be represented as follows:

$$x[n] = -\sum_{m=1}^{p} a_m x[n-m] + \sum_{k=0}^{q} b_k w[n-k]$$
(2.11)

Equation (2.11) illustrates ARMA model or general pole-zero model. In this case, w[n], which is commonly known as innovation process [8], is a zero-mean white process. When the value of p is equal to zero, (2.11) becomes:

$$x[n] = \sum_{k=0}^{q} b_k w[n-k]$$
(2.12)

The model illustrated by (2.12) is called all-zero model or Moving Average model. If, instead of p, q in (2.11) is equal to zero, the model in (2.11) becomes Auto Regressive model, which is represented as:

$$x[n] = -\sum_{m=1}^{p} a_m x[n-m] + b_0 w[n]$$
(2.13)

In this discussion of parametric power spectrum estimator, only Auto Regressive model is considered as an example.

For the sake of simplicity, it is firstly assumed that  $b_0$  in (2.13) is equal to 1. In order to derive the model for power spectrum estimator, we can apply z-transform on both side of equation (2.13) resulting in:

$$X(z) = H(z)W(z), \text{ where } H(z) = \frac{1}{1 + \sum_{m=1}^{p} a_m z^{-m}}$$
(2.14)

By taking into account (2.14), the relationship between power spectrum density of x[n] and w[n] can be represented as follows:

$$S_{xx}^{ar}(e^{j\omega}) = \left| H(e^{j\omega}) \right|^2 S_{ww}(e^{jw}) = \frac{\sigma_w^2}{\left| 1 + \sum_{m=1}^p a_m \exp(-j\omega m) \right|^2}$$
(2.15)

In (2.15),  $S_{xx}^{ar}(e^{j\omega})$  is Auto Regressive estimate of power spectrum density. In this case, it is obvious that the next step to complete the model is to find the value of white noise variance  $\sigma_w^2$  as well as the coefficients  $a_m$ . Since w[n] is white noise, w[n] is uncorrelated with past value of x[n]. Therefore, it is possible to find  $\sigma_w^2$  and  $a_m$  by multiplying both side of (2.13) with x[n-k] and take the expectation of both sides of the equation. Assuming  $b_0=1$ , the result would be:

$$R_{xx}[k] = -\sum_{m=1}^{p} a_m R_{xx}[k-m] + E[w[n]x[n-k]]$$
(2.16)

For positive value of k, (2.16) is represented as:

$$R_{xx}[k] = -\sum_{m=1}^{p} a_m R_{xx}[k-m] \qquad k > 0$$
(2.17)

For alternative representation, a collection of *p* equations can be obtained from (2.17) with  $1 \le k \le p$ . These *p* equations can be expressed altogether as a matrix-vector relation as follows:

$$\begin{bmatrix} R_{xx}[0] & R_{xx}[-1] & \dots & R_{xx}[1-p] \\ R_{xx}[1] & R_{xx}[0] & \dots & R_{xx}[2-p] \\ \vdots & \vdots & \vdots & \vdots \\ R_{xx}[p-1] R_{xx}[p-2] & \dots & R_{xx}[0] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = -\begin{bmatrix} R_{xx}[1] \\ R_{xx}[2] \\ \vdots \\ R_{xx}[p] \end{bmatrix}$$
(2.18)

For zero value of k, (2.16) can be represented as:

$$R_{xx}[0] = -\sum_{m=1}^{p} a_m R_{xx}[-m] + E[w[n]x[n]]$$
(2.19)

If we substitute x[n] in (2.19) with (2.13), (2.19) can be represented as:

$$\sigma_w^2 = R_{xx}[0] + \sum_{m=1}^p a_m R_{xx}[-m]$$
(2.20)

Due to the symmetricity of autocorrelation sequence, both (2.18) and (2.20) can then be represented as:

$$\begin{bmatrix} R_{xx}[0] & R_{xx}[1] & \dots & R_{xx}[p-1] \\ R_{xx}[1] & R_{xx}[0] & \dots & R_{xx}[p-2] \\ \vdots & \vdots & \vdots & \vdots \\ R_{xx}[p-1] & R_{xx}[p-2] & \dots & R_{xx}[0] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = -\begin{bmatrix} R_{xx}[1] \\ R_{xx}[2] \\ \vdots \\ R_{xx}[p] \end{bmatrix}$$
(2.21)  
$$\sigma_w^2 = R_{xx}[0] + \sum_{m=1}^p a_m R_{xx}[m]$$
(2.22)

These two equations, (2.21) and (2.22) are famously called Yule Walker equation [7]. From the illustrated derivation, it is obvious that once the Auto Regressive model for power spectrum density has been established, Yule Walker equations should be used to determine the parameter in the Auto Regressive model.

As it is stated in [7], parametric model tries to overcome the limitation inherent in nonparametric spectrum estimation. As already mentioned, both correlogram and Blackman-Tukey approaches apply window on autocorrelation value. In the other words, both assume that the value of correlation function  $\hat{R}_{xx}[k]$  in (2.3) and (2.4) is zero for k > N-1. While this windowing process results in reduced frequency resolution, parametric estimation offers better frequency resolution and avoids bias in the estimate by performing extrapolation based on a-priori knowledge. In this case, the a-priori knowledge is the samples that have already been received. Based on the received samples, the model is established and the parameters underlying the model are calculated (the noise variance and the coefficient  $a_ms$ ). The complete model is later used to estimate the power spectrum density through (2.15).

# 2.2 A Review of Existing Spectrum Sensing and Estimation Techniques available for Dynamic Spectrum Access

With respect to dynamic spectrum access paradigm, the secondary user (cognitive radio (CR) system) should gauge the wireless environment over particular frequency bands in order to identify the spectrum holes and occupied band. This functionality is performed by spectrum sensing or spectrum estimation module within the CR system. Based on the information provided by spectrum estimation module, the CR system can shape its transmitted signal in order to eliminate or at least minimize the distortion to other licensed users (LUs). As it is mentioned in [42], the challenge of spectrum sensing module is in identification and detection of primary user signals amidst harsh and noisy environs. The accuracy and the speed of sensing or estimation would play significant roles and thus, in term of spectrum estimation, the time resolution and

frequency resolution are the main issue. It is also worth to emphasize that the CR system should give up the spectrum when an LU begins transmission. This means that the spectrum sensing or estimation process should be conducted continuously. The time difference between two consecutive sensing or estimation processes would determine how up-to-date the spectrum information is. This is very important especially if we consider the requirement for minimizing the interference between the LU signals and CR signals.

We would like to underline the slight difference between spectrum sensing and spectrum estimation here. The interest of spectrum sensing is to identify the presence of user over particular frequency band without the need for finding the exact value of power in that band. On the other hand, the goal of spectrum estimation technique is to obtain the exact power spectrum density over the bands of interest. The problem in spectrum sensing is more detection problem and thus it is not as thorough as spectrum estimation approach. In CR system, spectrum sensing module is more desirable due to the fact that the interest is to identify only the presence of primary users in a particular band. However, it is also interesting to find that most of spectrum sensing techniques are built on the existing spectrum estimation approach. Some spectrum sensing approaches, however, are not developed from spectrum estimation technique.

### 2.2.1 Pilot detection via match filtering

Cabric et al in [14] suggest the use of conventional match filter for pilot detection. As already well known, the important advantage offered by match filter is the maximization of signal to noise ratio. This method assumes that the primary user sends pilot signal with data. The pilot signal should be known by secondary users to allow them to perform timing and carrier synchronization to achieve coherency [13]. Secondary users should have full prior knowledge of modulation type, pulse shaping and packet format. In this scenario, secondary users should provide separate dedicated receiver for each primary user class, which is impractical from complexity point of view. Other drawbacks of this approach are susceptibility to frequency offsets and the resultant loss of synchronization [13]. On the positive side, pilot detection requires minimum sensing time because it exploits available a priori knowledge especially the carrier frequency of the primary users. Figure 2.3 illustrates Pilot Detection via matched filtering.



Figure 2.3 Pilot detection through match filtering [13]



Figure 2.4 Implementation of Energy Detection (a) with analog pre-filter and square-law device (b) with Periodogram: FFT magnitude squared and averaging [13]

#### 2.2.2 Energy detection

Another approach is Energy Detection, a non-coherent detection technique, where prior knowledge of pilot data is not required. Figure 2.4 illustrates the implementation of Energy Detection. The first implementation of Energy Detection (figure 2.4a) consists of a low pass filter to remove out of band noise and adjacent interference, an analog to digital converter as well as square law device to compute the energy. However, this implementation is not flexible for narrowband signals and sine waves [13]. Therefore, in [13], a periodogram solution (figure 2.4b) is proposed through square magnitude of FFT (Fast Fourier Transform). The result is then averaged. Some disadvantages of non-coherent detection are the susceptibility of the detection threshold to noise, in-band interference and fading [14].

#### 2.2.3 Cyclostationary feature detection

This method takes advantage of the cyclostationarity of the modulated signal [14]. Generally, the transmitted data is taken to be a stationary random process. However, when it is modulated with sinusoid carriers, cyclic prefixes (as in OFDM) and code or hoping sequences (as in CDMA(Code Division Multiple Access)), a *cyclostationarity* is induced i.e. the mean, autocorrelation and statistics show periodic behavior. This feature is exploited in a detector (depicted in figure 2.5) that measures a signal property called Spectral Correlation Function (SCF). When parameter  $\alpha_{cf}$  in figure 2.5 (called cycle frequency) is 0, the SCF yields the power spectral density.



Figure 2.5 Cyclostationarity feature detector [14]

#### 2.3 Spectrum Estimation as a Filter Bank Analysis Problem

From the perspective of spectrum estimation, a filter bank can be considered as an array of band pass filters that separates the input signal into several frequency components, each one carrying a single frequency sub-band [15]. The filter banks are usually implemented based on single prototype filter, which is a low pass filter. This low pass filter is normally used to realize the zero-th band of the filter bank while filters in the other bands are formed through the modulation of the prototype filter [16]. Figure 2.6 illustrates the main idea of filter bank concept.

This section basically tries to explore the filter bank paradigm in spectrum estimation. In the beginning of this section, periodogram spectral estimator previously illustrated is represented based on filter bank point of view. The two elegant approaches, Multi Taper Spectrum Estimation (MTSE) and Filter Bank Spectrum Estimation (FBSE), which are purely based on filter bank architecture, are also discussed. Since our proposed wavelet based spectrum estimation is entirely based on filter bank theory, the discussion about this filter bank paradigm is advantageous for a comparative analysis between the proposed wavelet based technique and the existing spectrum estimation approach.



Figure 2.6 Graphical representation of filter bank concept [16]

### 2.3.1 Periodogram spectral estimator realization through filter banks

Spectrum estimation is about finding the power spectrum density (PSD) of a finite sample set  $\{x[n], n = 1, 2, ..., N\}$  for frequency  $|\omega| \le \pi$ . The classical approach to spectrum estimation is to use Fourier transforms to obtain a Periodogram, given as [17]:

$$S_{xx}^{p}(e^{j2\pi f}) = \frac{1}{N} \left| \sum_{n=1}^{N} x[n] e^{(-j2\pi fn)} \right|^{2}$$
(2.23a)

For any given frequency  $f_i$ , (2.23a) can be written as:

$$S_{xx}^{p}(e^{j2\pi f_{i}}) = \frac{1}{N} \left| \sum_{n=1}^{N} x[n] e^{(-j2\pi f_{i}n)} \right|^{2} = \frac{1}{N} \left| \sum_{n=1}^{N} x[n] e^{(j2\pi f_{i}(N-n))} \right|^{2}$$
(2.23b)

It should be noted that (2.23b) is possible since  $|e^{(j2\pi f_i N)}| = 1$ . By introducing new variable k = N - n, we can rewrite (2.23b) as:

$$S_{xx}^{p}(e^{j2\pi f_{i}}) = \frac{1}{N} \left| \sum_{k=0}^{N-1} x[N-k] e^{(j2\pi f_{i}k)} \right|^{2} = \left| \sum_{k=0}^{N-1} h_{i}[k] x[N-k] \right|^{2}$$
(2.23c)

where

$$h_i[k] = \frac{1}{\sqrt{N}} e^{(j2\pi f_i k)} \text{ for } k = 0, 1, 2, \dots, N-1$$
(2.23d)

We can now concentrate on the summation within the magnitude operation in (2.23c) and express this summation as:

$$y[N] = \sum_{k=0}^{N-1} h_i[k]x[N-k]$$
(2.23e)

By considering the fact that (2.23e) is actually nothing but the truncated convolution sum at particular point N, we can rewrite (2.23e) as general convolution sum at the same point associated with a linear causal system by padding  $h_i[k]$  with zeros [43]. When this is the case, (2.23e) can be represented as:

$$y_a[N] = \sum_{k=0}^{\infty} h_i[k]x[N-k]$$
(2.24a)

with

$$h_{i}[k] = \begin{cases} w[k]e^{(j2\pi f_{i}k)} & \text{for } k = 0, 1, 2, \dots, N-1 \\ 0 & \text{otherwise} \end{cases}$$
(2.24b)

and window function  $w[k] = 1/\sqrt{N}$ . It is clear that (2.24a) can be perceived as passing N samples through a filter having impulse response  $h_i[k]$  and then taking only single sample of the filtered signal at point N. Based on this perspective, it is worth finding the frequency response of the linear filter having impulse response  $h_i[k]$  through the following evaluation:

$$H_{i}(\omega) = \sum_{k=0}^{\infty} h_{i}[k]e^{-j\omega k} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{j(\omega_{i}-\omega)k}$$
$$= \frac{1}{\sqrt{N}} \frac{e^{j(\omega_{i}-\omega)N} - 1}{e^{j(\omega_{i}-\omega)} - 1}$$
(2.25a)

(2.25a) finally gives:

$$H_{i}(\omega) = \frac{\sin[N(\omega_{i} - \omega)/2]}{\sqrt{N} \sin[(\omega_{i} - \omega)/2]} \exp\left[j\left(\frac{N-1}{2}\right)(\omega_{i} - \omega)\right]$$
(2.25b)

If w[k] in (2.24b) is taken to be a prototype FIR (Finite Impulse Response) low pass filter, then  $h_i[k]$  s will constitute a bank of band pass filters centered at frequencies  $f_i$ s. This filter bank is constructed by modulating the prototype filter. By considering (2.23)-(2.25), we can easily find that the periodogram estimate at particular frequency point  $f_i$  can actually be obtained by passing the received samples through the band pass filter centered at  $f_i$ . The power calculation of this estimate

is performed *based only on a single sample* of the output of the filter (see (2.23c), (2.23e) and (2.24a)) [43].

The entire periodogram estimates can then be related to the output of several filters in the filter bank constructed by modulating a single prototype filter w[k]. For the case of simple periodogram, the window function w[k] is rectangular with  $w[k] = 1/\sqrt{N}$ . As it is clear from (2.25), the frequency response of filter based on prototype filter having rectangular window as its impulse response would have significant level of side lobes. This is actually the main reason why the periodogram estimates have high side lobe or large leakages. This problem can be alleviated by replacing the rectangular window with a window function with a taper that smoothly decays on both sides to obtain a prototype filter with much smaller side lobes. A few popular windows are Hanning, Kaiser and Blackman [8].

### 2.3.2 Multi Taper Spectral Estimator

The Multi Taper Spectrum Estimator (MTSE), proposed by Thomson [18], uses multiple orthogonal prototype filters to improve the variance and reduce the sidelobe and leakage. The process is initiated by collecting the last *N* received samples in a vector  $\mathbf{x}[n] = [x[n] \ x[n-1] \ \dots \ x[n-N+1]]^T$  and representing it as an incomplete expansion of set of orthogonal slepian base vectors [16] [18]:

$$\mathbf{x}[n] \approx \sum_{k=0}^{K_{pf}-1} \kappa_k(f_i) \mathbf{D} \mathbf{q}_k$$
(2.26)

In (2.26),  $\kappa_k(f_i)$  is the expansion coefficients,  $K_{pf}$  is the total number of orthogonal prototype filters,  $\mathbf{q}_k$  is the set of orthogonal slepian basis vectors (prolate spherical sequences) derived using a minimax algorithm and **D** is a diagonal matrix with the diagonal elements of 1,  $e^{j2\pi f_i}$ , ...,  $e^{j2\pi(N-1)f_i}$ . As in other orthogonal expansion, the expansion coefficients  $\kappa_k(f_i)$  can be computed from the inner product between expanded signals and the basis as follows [16][18]:

$$\boldsymbol{\kappa}_{k}(f_{i}) = \left(\mathbf{D} \ \mathbf{q}_{k}\right)^{H} \mathbf{x}[n]$$
(2.27)

Based on (2.27), the MTSE is formulated as:

$$\hat{S}_{MTSE}(f_i) = \frac{1}{K_{pf}} \sum_{k=0}^{K_{pf}-1} |\kappa_k(f_i)|^2$$
(2.28)

In order to investigate the relationship between MTSE and Periodogram estimator, we can try to derive periodogram estimator from MTSE illustrated in (2.28). If  $\hat{S}_{PSE}(f_i)$  denotes the periodogram spectral estimates at frequency  $f_i$  derived from MTSE in (2.28),  $\hat{S}_{PSE}(f_i)$  can be obtained by assuming  $K_{pf}=1$  and thus, there is only one basis vector  $\mathbf{q}_0$ . Indeed if there is only a vector  $\mathbf{q}_0$  in (2.26) containing  $1/\sqrt{N}$  s as its elements, (2.28) becomes periodogram with rectangular window. The derivation can be illustrated as follows:

$$\hat{S}_{PSE}(f_i) = \left| \kappa_0(f_i) \right|^2$$

$$= \left| (\mathbf{D} \mathbf{q}_0)^H \mathbf{x}[n] \right|^2$$

$$= \left| \sum_{k=0}^{N-1} \frac{1}{\sqrt{N}} \exp(j2\pi f_i k) x[n-k] \right|^2$$

$$= \frac{1}{N} \left| \sum_{k=0}^{N-1} x[n-k] \exp(j2\pi f_i k) \right|^2$$
(2.29)

It is obvious that (2.29) is nothing but periodogram estimates defined in (2.23)

If we manipulate the elements in  $\mathbf{q}_0$  such that the elements in  $\mathbf{q}_0$  exhibit a window function, (2.29) becomes the windowed periodogram with window type determined by  $\mathbf{q}_0$ . Hence, (2.28) can

actually be interpreted as average of several periodograms with different windows. The averaging process in (2.28) is conducted on the data set in its entirety and in this sense MTSE is different from the Welch approach [11] where the data samples are segmented and averaged. Moreover, in Welch approach the same window is applied on different segments.

Equation (2.27) can be regarded as Fourier Transform of convolution between the received samples x[n] and a filter having  $\mathbf{q}_k$  as its impulse response. Since there are  $K_{pf}$  orthogonal prototype filters, the MTSE at frequency point  $f_i$  is related to multiple outputs obtained from  $K_{pf}$  band pass filters. The impulse responses of these  $K_{pf}$  band pass filters are the modulated version of the responses of corresponding  $K_{pf}$  prototype filters. Therefore,  $\kappa_k(f_i)$  can be viewed as the output of  $k^{\text{th}}$  filter of a group of  $K_{pf}$  band pass filters. Since the PSD estimate at every point  $f_i$  is related to outputs of  $K_{pf}$  band pass filters with centre frequency  $f_i$ , the filters' pass band  $\Delta f$  gives its frequency resolution. Hence, (2.28) is the estimate of PSD over the frequency band  $[f_i - \Delta f/2, f_i + \Delta f/2]$ .

For a given resolution  $\Delta f$ , the prototype low pass filters should have pass band between [- $\Delta f/2$ , + $\Delta f/2$ ] and minimum energy at stop band to minimize leakage. The variance of the estimate is reduced by taking advantage of the presence of multiple prototype (prolate) filters having impulse responses derived from the vectors  $\mathbf{q}_k$ . For given frequency band, the output of each band pass filter corresponding to different prototype filter is collected and averaged. The output of each band pass filter should be independent from each other to effectively reduce the variance of estimated power. This is achieved from the orthogonality of the Slepian sequences. In summary, we have two constrains in defining the prototype filters based on given frequency resolution  $\Delta f$ :

- How to minimize the energy at stop band
- How to obtain  $K_{pf}$  prototype filters having coefficients orthogonal to each other

As mentioned in [16], the minimax theorem is used to derive the Slepian sequences. Firstly, the autocorrelation matrix **R** of the observation vector **x**[*n*] is computed. The set of eigenvalues  $\lambda_0 > \lambda_1 > ... > \lambda_{N-1}$  of correlation matrix **R** and the corresponding eigenvectors  $\mathbf{q}_0, \mathbf{q}_1, ..., \mathbf{q}_{N-1}$  are obtained through the following optimizations [16]:

$$\lambda_{\max} = \lambda_0 = \max_{\|\mathbf{q}_0\|=1} E\left[\left|\mathbf{q}_0^T \mathbf{x}[n]\right|^2\right]$$
(2.30a)

$$\lambda_{i} = \max_{\|\mathbf{q}_{i}\|=1} E\left[\left|\mathbf{q}_{i}^{T}\mathbf{x}[n]\right|^{2}\right] \text{ for } i = 1, 2, 3, \dots, N-1$$
subject to  $\mathbf{q}_{i}^{T}\mathbf{q}_{i} = 0$ , for  $0 \le j < i$ 

$$(2.30b)$$

In (2.30a) and (2.30b),  $\|\mathbf{q}_i\|$  denotes the Euclidean norm of vector  $\mathbf{q}_i$ . The last step is basically to choose the  $K_{pf}$  eigenvectors out of N eigenvectors of the correlation matrix  $\mathbf{R}$ . These  $K_{pf}$  eigenvectors correspond to the largest  $K_{pf}$  eigenvalues and play roles as the Slepian Sequences. These would finally become the orthogonal bases vectors  $\mathbf{q}_k$  in (2.26) as well as the prototype filters coefficients.

While  $K_{pf}$  prototype filters having minimum energy in stop band are expected, not all of the prototype filters fulfill the expectation. The filter having  $\mathbf{q}_0$  as its impulse response tends to have minimum energy in stop band [16]. However, the filter having  $\mathbf{q}_1$  as its impulse response does not have stop band attenuation as good as that of  $\mathbf{q}_0$ . The reason for this can be explained as follows. Filter having  $\mathbf{q}_0$  as its impulse response has the best stop band attenuation since it is chosen to maximize the corresponding eigenvalue in (2.30a) without any constraint. On the other hand, filter having  $\mathbf{q}_1$  is chosen to maximize the corresponding eigenvalue in (2.30b) but with additional constraint  $\mathbf{q}_1^T \mathbf{q}_0 = 0$  mentioned also in (2.30b). The performance of the next derived filter (filters having  $\mathbf{q}_2, \mathbf{q}_3, \dots, \mathbf{q}_{K_{p_F}-1}$  as their impulse response) has more deterioration.

With regard to the need for having minimum leakage, a careful treatment is needed when the outputs of each filter corresponding to different prototype filters are averaged. Obviously, they should not have the same weight. The output of filter having better stop band attenuation should be given more weight. Thomson offers an iterative algorithm to compute the estimate of power spectrum which is illustrated in [18]. Figure 2.7 illustrates the magnitude response of the first seven MTSE prolate filters of length 128. In this figure, only the even numbered filters are shown for the sake of clarity. These first seven prolate (prototype) filters have  $\mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_6$  as their impulse response.



Figure 2.7 Magnitude responses of the first seven prolate filters of length 128. For clarity, only the even numbered filters are shown. The odd numbered filters have responses that fall in between the presented ones [16].

### 2.3.3 Filter Bank Spectral Estimator (FBSE)

One example of spectrum estimation technique based on filter bank paradigm, which is proposed for cognitive radio, is filter bank spectrum estimation. FBSE is proposed by Farhang-Boroujeny in [16] by employing a pair of matched root Nyquist-filter. The proposal is based on the assumption that multicarrier modulation is used as the underlying communication technique. Similar to filter bank paradigm employed by periodogram estimator and MTSE, the entire frequency spectrum is considered as the output of multiple filters (called filter banks) covering the entire frequency bands. While in Thomson's MTSE, the estimate at frequency point  $f_i$  is obtained by averaging the output of multiple filters constructed based on different prototype filters, FBSE is intended to simplify the complexity of MTSE by introducing only one prototype filter in the zero-th band shown in figure 2.6.



Figure 2.8 The demodulation of received signal with respect to i<sup>th</sup> subcarrier before it is processed through root-Nyquist filter [16].

The root-Nyquist filter can be explained as follows [19]. Given that H(z) is the transfer function of a filter and P(z) the product filter  $P(z) = H(z)H(z^{-1})$ , H(z) is said to satisfy the Nyquist criterion if :

$$\sum_{k=0}^{O_S - 1} P(ze^{-j2\pi fk}) = O_S$$
(2.31)

In (2.31),  $O_s$  is an integer called the over sampling factor [19]. In multi-carrier communication, such filters are useful to design a pair of matched transmit and received filters whose cascade is a Nyquist pulse shape. When |z|=1, then  $P(z) = H(z)H(z^{-1})=|H(z)|^2$ . P(z) is called Nyquist filter and  $|H(z)|=P(z)^{1/2}$  is called a <u>root Nyquist filter</u>.
Based on the previous sub-sections, the implementation of a spectrum estimator using filter bank for signal analysis is clear, namely by passing an input signal through a bank of filters. The output power of each filter is a measure of the estimated power over the corresponding sub-band. Hence the power spectral density (PSD) estimate of *i*-th sub band of the filter bank is represented as [16]:

$$\hat{S}\left(\frac{i}{N}\right) = avg\left[\left|y_i[n]\right|^2\right]$$
(2.32)

In (2.32), avg[...] describes time average operator while  $y_i[n]$  is the output signal of *i*-th sub band filter. The basic idea of FBSE is to assume that filter bank-based multicarrier communication technique is used as underlying communication system. The same filter bank can then be used for spectrum estimation. In this filter bank architecture, it is presumed that the filters at the receiver and transmitter side are a pair of matched root-Nyquist filters H(z) shown in figure 2.8 [19]. In the receiver side, the received multicarrier signal is demodulated. For each subcarrier, the corresponding portion of the received signal is down-converted to baseband, low pass filtered, and decimated [16] before finally forwarded to the root-Nyquist filter as illustrated in figure 2.8. In the same time, the receiving module also performs spectrum estimation. Figure 2.9 illustrates the simplicity of FBSE. As already clear, H(f) is the prototype filter, which is root-Nyquist filter while the rest of the filters are modulated copies of H(f). The frequency response of the prototype filter H(f) and its modulated version is described in figure 2.10.



Figure 2.9 Simple illustration of Filter Bank Spectral Estimator (FBSE) proposed by Farhang-Boroujeny [16].  $S_{FBSE}(f_i)$  is FBSE estimate at *i*-th frequency sub-band



Figure 2.10 Optimally designed Root Nyquist Filter by Farhang-Boroujeny in [19] as prototype filter for FBSE

According to [16], the correlation properties of the decimated signal samples of each subcarrier band are related to the variance of the estimates. In order to investigate the correlation properties of the demodulated signal, we start by considering figure 2.8, which shows that:

$$S_{y_i y_i}(f) = S_{xx}(f+f_i) \left| H(e^{j2\pi f}) \right|^2$$
(2.33)

In (2.33),  $S_{y_i y_i}(f)$  is power spectrum density of  $y_i[n]$ , which is the output signal. Assuming that H(z) is narrowband,  $S_{xx}(f+f_i)$  can be approximated by  $S_{xx}(f_i)$ . Based on this approximation, it is possible to write (2.33) in z domain as:

$$\Phi_{y_i y_i}(z) = S_{xx}(f_i) H(z) H(z^{-1})$$
(2.34)

 $S_{xx}(f_i)$  in (2.34) is a constant. It can be noted that the correlation coefficients of  $y_i[n], R_{y_i y_i}[k]$ , can be obtained from the inverse Z-transform of  $\Phi_{y_i y_i}(z)$ .

Since H(z) is designed as root-Nyquist (*N*) filters (*N* gives the zero-crossings interval),  $G(z) = H(z)H(z^{-1})$  is Nyquist (*N*) filter. It is required that the time domain function of G(z) satisfies [16]:

$$g[n] = \begin{cases} 1, \text{ if } n = 0\\ 0, \text{ if } n = mN \ m \neq 0 \end{cases}$$
(2.35)

As a result, the correlation  $R_{y_i y_i}[k]$  bears a resemblance to Nyquist (*N*) sequence  $g_N[n]$ , where the subscript *N* indicates the zero-crossing spacing of the autocorrelation coefficient. For an observation vector  $\mathbf{y_i}[n] = [y_i[n], y_i[n-L_{spc}], \dots, y_i[n-(K_{obs}-1)L_{spc}]]$  with  $L_{spc}$  is the sample spacing and  $K_{obs}$  is the size of observation vector  $\mathbf{y_i}[n]$ , the correlation matrix is given as [16]:

$$\mathbf{R}_{\mathbf{y}_i \mathbf{y}_i} = S_{xx}(f_i) \mathbf{A} \tag{2.36}$$

with A given as:

$$\mathbf{A} = \begin{bmatrix} g_{N}[0] & g_{N}[L_{spc}] & \dots & g_{N}[K_{obs} - 1]L_{spc} \\ g_{N}[-L_{spc}] & g_{N}[0] & \dots & g_{N}[K_{obs} - 2]L_{spc} \\ \vdots & \vdots & \vdots & \vdots \\ g_{N}[-(K_{obs} - 1)L_{spc}] & g_{N}[-(K_{obs} - 2)L_{spc}] & \dots & g_{N}[0] \end{bmatrix}$$
(2.37)

An eigenvalue decomposition is then performed on matrix **A**. These resultant eigenvalues are used to measure the degree of freedom which can later be used to adjust the variance of the estimates.

Based on (2.32), we can formulate FBSE estimate in term of elements of observation vector  $\mathbf{y}_{i}[n]$  as follows:

$$\hat{S}_{FBSE}(f_i) = \frac{1}{K_{obs}} \sum_{k=0}^{K_{obs}-1} y_i [n - kL_{spc}]^2$$
(2.38)

It is very interesting to compare (2.38) and (2.28) showing how FBSE and MTSE perform averaging process in order to reduce the variance of the estimated PSD. From (2.38) as well, the importance of matrix **A** in (2.37) becomes more obvious. Since the independency between elements of  $y_i[n]$  influences the variance of the estimated PSD and the correlation matrix  $\mathbf{R}_{\mathbf{y},\mathbf{y}_i}$  depends on matrix **A**, the problem definition can be stated as how to adjust the eigenvalues of

A in order to approach the desired variance of estimated PSD. More detail on this issue can be found in [16].

A comparative analysis of MTSE and FBSE has been performed by Farhang Boroujeny in [16]. It is generally concluded that FBSE is usually better when PSD estimation is performed based on larger number of samples. On the other hand, when the available samples of received signal are not that much, MTSE is preferable. Another important advantage of FBSE underlined in [16] is the possibility of no additional cost to implement FBSE when filter-bank multicarrier modulation is employed in the transceiver. This is due to the fact that the same filter bank in receiver module can be used for double functionality namely: spectrum estimation and signal demodulation. Finally, it is also worthy to note the higher complexity introduced by MTSE.

# **2.4 Wavelet Theory**

The emergence of interest in using wavelets in various applications is mainly motivated by the need for frequency content analysis of a signal locally in time. An interesting analogy for the frequency content analysis within a given time window is provided by Daubechies [20] who uses music notation as an example. Music notation informs the musician about which notes (frequency information) to play at particular time. While Fourier transform gives accurate information on the frequency content of a signal, the information about time localization is not readily available. Fourier Transform is nothing but orthogonal expansion of time-domain signals in terms of sine and cosine basis function. Wavelet Transform is also orthogonal expansion. However, instead of decomposing the signals in terms of sine and cosine, a new waveform called wavelet is used as a basis function. Similar to other orthogonal expansions, a wavelet expansion of a square integrable function  $f \in L^2$  generally expresses the expanded function as a complete, orthonormal set of basis functions for the Hilbert Space of square integrable functions. As in the Fourier domain, which is more familiarly recognized as frequency domain, the analysis of signals in wavelet domain is conducted by analyzing the transform coefficients.

This section elaborates some important information about wavelet theory which is used as the basis of this thesis. Firstly, the Wavelet Transform is discussed. Two types of wavelet transform, namely Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT) are explained. The discussion about DWT is preceded by review about time-frequency tiling and multi resolution, which play a role as important basis for DWT. This is then followed by filter bank implementation of DWT and Wavelet Packet Transform (WPT) which is a slight modification of DWT. Finally, a review on the most popular wavelet employed in this thesis is given.

#### 2.4.1 Continuous Wavelet Transform

Continuous wavelet transform (CWT) can be defined as an expansion of continuous-time functions in terms of two variables, shift and scale. The scale parameter in CWT basically has apparent similarity with frequency in Fourier Transform. It describes how a wavelet basis function is stretched or contracted. Meanwhile, the shift variable, also known as translation parameter, represents the speeding up or the delay of the wavelet. It basically tells the location of the wavelet in time. The CWT can be expressed as the inner product between the signal being analyzed and the set of wavelet basis functions[4]:

$$CWT_f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi^* \left(\frac{t-b}{a}\right) f(t) dt$$
(2.39)

In (2.39),  $CWT_f(a,b)$  is the CWT of a continuous signal f(t) consisting of several wavelet coefficients, which are a function of scale a and translation b. Equation (2.39) clearly illustrates that the CWT of a continuous signal f(t) is transform coefficients, which can be viewed as the sum of entire time components of the signal multiplied by scaled, shifted versions of the wavelet  $\psi(t)$ . In the other words, there are multiple wavelet basis functions and all of them are generated from only single prototype (mother) wavelet. This relation is commonly expressed as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$
(2.40)

The relation shows in (2.40) underlines an enormous advantage offered by wavelet transform by allowing it to provide dynamic resolution capability through the use of short basis function (contracted version) to obtain good time domain analysis and long basis function to get fine frequency domain analysis. The original time domain signal can later be constructed by inverse Continuous Wavelet Transform illustrated as [4]:

$$f(t) = \frac{1}{C_{\psi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} CWT_f(a,b) \ \psi\left(\frac{t-b}{a}\right) \frac{da \ db}{a^2}$$
(2.41)

where

$$C_{\psi} = \int_{-\infty}^{\infty} \frac{\left|\Psi(\omega)\right|^2}{\left|\omega\right|} \, d\omega \tag{2.42}$$

In (2.42),  $\Psi(\omega)$  is the Fourier transform of  $\psi(t)$ .



<sup>(</sup>b)







# 2.4.2 Discussion about Time-Frequency Tiling

One simple technique, which provides both time and frequency information, is Short Time Fourier Transform (STFT) or Gabor Transform illustrated as [12]:

$$STFT\{x(t)\} \equiv X(\tau, f) = \int_{t} \left[x(t)w(t-\tau)\right] \exp(-j2\pi ft) dt$$
(2.43)

From (2.43), a trade off between time and frequency resolution can be achieved in STFT by altering the dimensions of the window function. Smaller windows mean better time resolution but poorer frequency resolution. If the size of the window is extended, time resolution is compromised. It should be noted that STFT suggests the use of same window for each collection samples, so the frequency resolution for the whole frequency range is uniform. Figure 2.11 illustrates this problem. This technique is less appropriate since the signal is always dynamic (even for stationary signal). Sometimes, there is discontinuity in the signal and some times there is a signal composed of long period sinusoid. Small window is required to locate discontinuity in the time domain though the frequency is traded as compensation. On the other hand, large window is enough for long period signal and thus best frequency resolution is obtained in this case. This means that dynamic resolution in time and frequency is preferred even for single signal realization.



Figure 2.13 Time-frequency tiling in wavelet transform [22]

Figure 2.12 clearly shows the advantage of the wavelet transform over Short Time Fourier Transform illustrated in figure 2.11. By taking advantage of scaling coefficients *a* in (2.39), we can define contracted basis function, which is sharp in time but low in frequency resolution as well as stretched basis function having low time resolution but it is sharp in frequency. Short Time Fourier Transform (STFT) is based on conventional Fourier Transform having sines and cosines as its basis function. This limits its ability to localize the signal property, such as transients and edges in time domain. On the other hand, due to the use of irregular wave shape as basis functions, wavelet transform can be exploited to analyze sharp variation and time local features of the signal [22].

At low frequencies, wavelet transform provides good frequency resolution but it is poor in time information. In contrast, it gives good time resolution and poor frequency resolution at high frequency. Thus, the wavelet transform approach can be customized according to the inherent properties of the signal. The encountered signals usually have low frequency components for long period of time but they have high frequency components for short durations. Figure 2.13 illustrates the time-frequency tiling concept that is employed by wavelet transform. Though it is true for most of types of wavelet transforms, such as Discrete wavelet transform, this is not the case for wavelet packet transform as it would be clear in later discussion.

In figure 2.13, each block covers one coefficient of the wavelet transform in the timefrequency plane. It is clear from this figure that for higher frequencies, the blocks have narrow width but long height illustrating good time resolution but poor frequency resolution. On the other hand, the blocks at low frequencies have broad width but short height illustrating poor time resolution but good frequency resolution.

#### 2.4.3 Multi-resolution analysis

The orthogonal wavelet expansion can also be seen as a multi-resolution formulation. According to Burrus, et al in [23], there are two main components in the multi-resolution formulation of wavelet analysis, namely scaling and wavelet functions. The scaling function can be defined as:

$$\varphi_k(t) = \varphi(t-k), \quad k \in \mathbb{Z} \quad \varphi \in L^2 \tag{2.44}$$

In (2.44), the subspace  $L^2(\mathbb{R})$  represents square integrable functions in Hilbert space while *k* implies discrete step translation. The subspace that is spanned by linear combination of the scaling function in (2.44) and its translated version is imaginable. However, it can be easily found that the size of the subspace can be increased by manipulating the scale of the scaling function. This manipulation results in two dimensional functions that are generated by basic scaling function through both scaling *j* and translation *k* as follows:

$$\varphi_{j,k}(t) = 2^{j/2} \varphi(2^j t - k), \text{ with } j, k \in \mathbb{Z} \text{ and } \varphi \in L^2$$

$$(2.45)$$

From (2.45) it is clear that the scaling function can be expressed as a linear combination of the half scale scaling function and its shifted versions which are orthogonal to each other. In this case, the space spanned by the scaling function with larger scale is included in the space spanned by the scaling function with larger scale is a subspace of the space spanned by the scaling function with larger scale is a subspace of the space spanned by the scaling function with smaller scale. In other word, the space spanned by the scaling function with larger scale is a subspace of the space spanned by the scaling function with smaller scale. In order to clarify this idea we can define  $V_0$  as a subspace spanned by the set of basis functions in (2.44). Then we can also construct  $\varphi_k(t)$  for k = 0 via the following way:

$$\varphi(t) = \sum_{n} h[n]\varphi(2t-n)\sqrt{2}, \ n \in \mathbb{Z}$$
(2.46)

Since  $\varphi(t)$  in (2.46) is expressed as linear combination of its shifted half scale versions, h[n] defines the weight of each half scale component. If  $V_1$  is a subspace spanned by set of scaling functions  $\varphi(2t-n)$ , it is clear that  $V_0 \subset V_1$ . In general, we can define the subspace  $V_i$  as

$$V_{j} = \overline{Span}_{k} \{ \varphi_{k}(2^{j}t) \} = \overline{Span}_{k} \{ \varphi_{j,k}(t) \}$$
(2.47)

Therefore, the multiresolution analysis can be defined as a nesting of closed subspaces as follows [24]:

$$\{0\} \subset \dots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \dots \subset L^2$$

$$(2.48)$$

It is obvious that as j goes to infinity  $V_j$  enlarges to cover all energy signals. On the other hand, as j goes to minus infinity  $V_j$  shrinks down to cover only the zero signal.

The difference between space spanned by scaling function and its half scale version is expressed as the orthogonal complement. This orthogonal complement is spanned by the corresponding wavelet function. This means, if we have a certain scaling function with particular scale, the space spanned by that scaling function can be decomposed into a subspace and its orthogonal complement. The subspace is spanned by the scaling function with double scale of the previous scaling function while the orthogonal complement is spanned by the corresponding wavelet function. Therefore we can define the space spanned by wavelet  $W_j$  as:

$$V_{j+1} = V_j \oplus W_j \tag{2.49}$$

By performing repetition on (2.49), we may formulate:

$$V_{j+1} = (V_{j-1} \oplus W_{j-1}) \oplus W_j$$
  

$$= (V_{j-2} \oplus W_{j-2}) \oplus W_{j-1} \oplus W_j$$
  

$$V_{j+1} = (V_{j-3} \oplus W_{j-3}) \oplus W_{j-2} \oplus W_{j-1} \oplus W_j$$
  

$$= (V_0 \oplus W_0) \oplus W_1 \oplus \dots \oplus W_{j-2} \oplus W_{j-1} \oplus W_j$$
  
with:  $V_0 \perp W_0 \perp W_1 \perp \dots \perp W_{j-2} \perp W_{j-1} \perp W_j$   
(2.50)

As a consequence of (2.50), the entire square integrable functions in Hilbert space can later be represented as:





Figure 2.14 Illustration of multi-resolution analysis concepts and nested subspaces



**Figure 2.15** 1-level wavelet decomposition applied on noisy signal *s* having time-varying frequency. The approximation component is denoted by a<sub>1</sub> while d<sub>1</sub> illustrates the detail component

The next step would be to express the wavelet function  $\psi(t)$  in multi-resolution concept. Since it is clear from (2.49) that  $W_0 \subset V_1$  and by taking (2.46) into consideration, we can express the wavelet function  $\psi(t)$  as:

$$\psi(t) = \sum_{n} g[n]\varphi(2t-n)\sqrt{2} \quad n \in \mathbb{Z}$$
(2.52)

g[n] in (2.52) defines the weight of each half scale component. Since  $V_0$  and  $W_0$  are orthogonal to each other,  $\psi(t)$  is orthogonal to  $\varphi(t)$ . This means there should be special relationship between weight coefficients h[n] in (2.46) and g[n] in (2.52) to ensure the orthogonality. This relationship is given by [4]:

$$g[n] = (-1)^n h[L-1-n], \text{ for } h[n] \text{ with length of } L$$
(2.53)

As for the case of scaling function in (2.45), the wavelet functions can be manipulated through scaling and translation as follows:

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k), \text{ with } j, k \in \mathbb{Z} \text{ and } \psi \in L^2$$
 (2.54)

Finally, based on the philosophy illustrated above, we can give visual illustration of signal decomposition. Given signal  $f(t) \in V_0$ , we can apply signal decomposition on f(t) as follows [24]:

$$\begin{split} f(t) &= D_1(t) + A_1(t) \\ &= D_1(t) + D_2(t) + A_2(t) \\ &= D_1(t) + D_2(t) + D_3(t) + A_3(t) \\ &= D_1(t) + D_2(t) + D_3(t) + D_4(t) + A_4(t) \end{split} \tag{2.55}$$

$$\begin{aligned} \text{where } D_j(t) \in W_{-j} \text{ and } A_j(t) \in V_{-j} \end{aligned}$$

In (2.55),  $D_j(t)$  is the detail at level *j* while  $A_j(t)$  is recognized as the approximation at level *j*. Hence, the scaling function corresponds to the approximation of a signal while the wavelet function describes the detail version of the signal at particular level of decomposition. Figures 2.15-2.17 depict the decomposition of noisy signals having time-varying frequency into 1-level, 2-level and 3-level decomposition, respectively. We can see how the behavior of detail and approximation components at different level and how the time-varying frequency property is described by wavelet and scaling function as a function of scale and translation index.



Figure 2.16 2-level wavelet decomposition applied on noisy signal *s* having time-varying frequency. The approximation component at level-2 is denoted by  $a_2$  while  $d_i$  illustrates the detail component at level *j* 

#### 2.4.4 Discrete Wavelet Transform

Due to practical limitation of CWT, Discrete Wavelet Transform (DWT) is usually more preferable to solve practical problems. DWT is developed based on multi-resolution analysis and it basically can be used to decompose any function  $f(t) \in L^2(\mathbb{R})$  into scaling and wavelet basis function spanning the entire  $L^2(\mathbb{R})$ . By considering (2.51), the reconstruction formula for DWT using finite resolution of wavelet function can be illustrated as follows [23]:

$$f(t) = \sum_{k=-\infty}^{\infty} c(j_0, k) \varphi_{j_0, k}(t) + \sum_{j=j_0}^{\infty} \sum_{k=-\infty}^{\infty} d(j, k) \psi_{j, k}(t)$$

$$f(t) = \sum_{k=-\infty}^{\infty} c(j_0, k) 2^{j_0/2} \varphi(2^{j_0} t - k) + \sum_{j=j_0}^{\infty} \sum_{k=-\infty}^{\infty} d(j, k) 2^{j/2} \psi(2^j t - k)$$
(2.56)

In (2.56), the DWT coefficients  $c(j_0,k)$  and d(j,k) denote the weight of scaling function  $\varphi_{j_0,k}(t)$  and wavelet function  $\psi_{j,k}(t)$ , respectively, while  $j_0$  defines coarsest scale spanned by the scaling function [23]. Correspondingly, the DWT coefficients c(j,k) and d(j,k) can now be defined respectively as (2.57) and (2.58):

$$c(j,k) = \left\langle f(t), \varphi_{j,k}(t) \right\rangle$$

$$= \int f(t) \varphi_{j,k}(t) dt \qquad (2.57)$$

$$= \int f(t) 2^{j/2} \varphi(2^{j}t - k) dt$$

$$d(j,k) = \left\langle f(t), \psi_{j,k}(t) \right\rangle$$

$$= \int f(t) \psi_{j,k}(t) dt \qquad (2.58)$$

$$= \int f(t) 2^{j/2} \psi(2^{j}t - k) dt$$

In (2.57) and (2.58),  $\langle a(t), b(t) \rangle$  denotes the inner product operation between a(t) and b(t).



Figure 2.17 3-level wavelet decomposition applied on noisy signal *s* having time-varying frequency. The approximation component at level-3 is denoted by  $a_3$  while  $d_i$  illustrates the detail component at level *j* 

# 2.4.5 Filter bank representation of Discrete Wavelet Transform

Based on the previous discussion about how scaling and wavelet basis function together with corresponding translation and dilatation can be used to construct any function  $f(t) \in L^2(\mathbb{R})$ , this sub-section intends to elaborate the relationship between DWT, multi-resolution analysis, lowpass and high pass filtering as well as to discuss how DWT can be practically implemented using multi-rate filter bank system. We start the discussion by considering a scaling function  $\varphi_k(t)$ . Since  $\varphi_k(t)$  basically can be expressed as a linear combination of  $\varphi_k(2t)$ ,  $\varphi_k(t)$  is the coarse version of  $\varphi_k(2t)$  and thus  $\varphi_k(t)$  occupies the lower half part of the frequency band occupied by  $\varphi_k(2t)$ . On the other hand, the wavelet function  $\psi_k(t)$ , as the orthogonal complement of  $\varphi_k(t)$ , resides in the upper half band. Figure 2.18 illustrates the relationship between space  $V_1$  (spanned by  $\varphi_k(2t)$ ) and its corresponding subspaces,  $V_0$  (spanned by  $\varphi_k(t)$ ) and  $W_0$  (spanned by  $\psi_k(t)$ ).



Figure 2.18 Frequency domain illustration of the relationship between  $V_1$ , and its two sub-spaces  $V_0$  and  $W_0$ 

From the description provided in the previous paragraph and figure 2.18, it is logical to associate coarse version of a signal with low frequency component and the detail version with high frequency component. Projection of signal with respect to scaling and wavelet basis function can logically be actualized through low pass and high pass filtering. Since multiple-level signal analysis based on DWT is nothing but signal decomposition into different frequency bands, successive high pass and low pass filtering of the time domain signal can be employed. These successive filtering should be implemented based on (2.46) and (2.52) famously recognized as *two-scale equation*. In order to find the exact relationship between DWT and the filtering process, we modify two-equation (2.46) and (2.52) by replacing t with  $2^{j}t - k$  in order to obtain more general form of two-scale equation. The general form of two-scale equation for scaling function with scale j and translation k is represented as:

$$\varphi(2^{j}t-k) = \sum_{n} h[n]\varphi(2(2^{j}t-k)-n)\sqrt{2}$$
  
=  $\sum_{n} h[n]\varphi(2^{j+1}t-2k-n)\sqrt{2}$   
=  $\sum_{m} h[m-2k]\varphi(2^{j+1}t-m)\sqrt{2}, m, n \in \mathbb{Z} \text{ and } m = 2k+n$  (2.59)

Likewise, (2.60) illustrates the general form of two-scale equation for wavelet function with scale j and translation k.

$$\psi(2^{j}t-k) = \sum_{n} g[n]\varphi(2(2^{j}t-k)-n)\sqrt{2} = \sum_{n} g[n]\varphi(2^{j+1}t-2k-n)\sqrt{2}$$
$$= \sum_{m} g[m-2k]\varphi(2^{j+1}t-m)\sqrt{2}, \quad n,m \in \mathbb{Z} \text{ and } m = 2k+n$$
(2.60)

The general form of two-scale equations (2.59) and (2.60) have exactly the same meaning as the original ones ((2.46) and (2.52)), namely the scaling and wavelet function at scale j are weighted sum of the multiple translated versions of scaling function with half-scale (at scale j+1).

Based on (2.59) and (2.60), the computation of DWT coefficients c(j,k) at (2.57) and d(j,k) at (2.58) can also be redefined as (2.61) and (2.62), respectively.

$$c(j,k) = \int f(t) \ 2^{j/2} \varphi(2^{j}t-k) \ dt$$
  

$$= \int f(t) \ 2^{(j+1)/2} \sum_{m} h[m-2k] \varphi(2^{j+1}t-m) \ dt$$
  

$$= \sum_{m} h[m-2k] \int f(t) \ 2^{(j+1)/2} \varphi(2^{j+1}t-m) \ dt$$
  

$$= \int h[m-2k] c(j+1,m), \ m \in \mathbb{Z}$$
  

$$d(j,k) = \int f(t) \ 2^{j/2} \psi(2^{j}t-k) \ dt$$
  

$$= \int f(t) \ 2^{(j+1)/2} \sum_{m} g[m-2k] \varphi(2^{j+1}t-m) \ dt$$
  

$$= \sum_{m} g[m-2k] \int f(t) \ 2^{(j+1)/2} \varphi(2^{j+1}t-m) \ dt$$
  

$$= \sum_{m} g[m-2k] \ c(j+1,m), \ m \in \mathbb{Z}$$
  
(2.62)

Equation (2.61) and (2.62) express how the DWT coefficients for wavelet and scaling function at particular scale *j* can be obtained from linear combination of DWT coefficients from smaller scale scaling function (at scale *j*+1). These two equations also inform us that a convolution between DWT coefficients at scale *j*+1 with filter having impulse response h[n] and g[n] followed by down sampling each output with factor 2 will produce new scaling and wavelet DWT coefficients at scale *j*. As a result, the filtering representation of DWT is realized by developing half-band low pass filter *H* and high pass filter *G*. The low pass filter *H* and high pass filter *G* have weight values h[n] in (2.46) and g[n] in (2.52), respectively, as their impulse responses. The term half-band is used here since g[n] and h[n] are related according to (2.53) which ensures the orthogonality between scaling and wavelet function illustrated in (2.46) and (2.52). As a result of having relation described by (2.53), the frequency response of *G* appears as the mirror image at normalized frequency of  $0.5\pi$  of low pass filter *H*. The filters satisfying (2.53) are also commonly known as the Quadrature Mirror Filters (QMF).

Two scale equations (2.46) and (2.52) can now be seen as discrete time filtering with filters H and G, respectively [25]. The sampled version of original signal x[n] is passed through filter G and H. The half-band low pass filter H removes all frequencies component higher than half of the highest frequency. On the contrary, the high pass filter G eliminates all frequency components below half of the highest frequency. Consequently, the number of samples at the output of the filter is redundant if Nyquist criterion is considered. In order to eliminate this redundancy, down sampling by factor 2 is applied on the output of the filter. The down sampling (decimation) by factor 2 is expressed as:

$$y_0[n] = x_0[2n] \tag{2.63}$$

In (2.63),  $x_0[n]$  and  $y_0[n]$  are the sequence of samples before and after the decimation process, respectively. In order to prevent the aliasing caused by the lost of information due to the down sampling process, the signal is often passed through anti-aliasing filter prior to the decimation. This filter is called the decimation filter. In our case, however, this is not necessary since exactly half frequency component is removed prior to down sampling by-2 operation.

Figure 2.19 illustrates the successive filtering implementation of DWT known as tree structured filter bank. As mentioned before, the impulse responses of half band low pass filter H and high pass filter G are nothing but the weight values h[n] and g[n] obtained from two-scale equations (2.46) and (2.52), respectively. The action of low pass and high pass filtering followed by decimation process make up a single-stage decomposition process famously known as *two-channel filter bank*. The most important feature in filter bank implementation of DWT is the fact that the iteration of the two-channel filter bank is only performed on low pass branch of the tree.

For example, it is assumed that the discrete-time input signal is  $x[n] \in V_4$  spanning the normalized frequency band of  $[0, \pi]$ . Since  $V_3 \oplus W_3 = V_4$ , the signal can be projected into subspace  $V_3$  and  $W_3$  by performing the inner product between x[n] and the scaling function as well as wavelet function that scale subspace  $V_3$  and  $W_3$ , respectively. In filter bank domain, these operations are equivalent to the convolution between x[n] and h[n] as well as x[n] and g[n] followed by two-rate down sampling. The results are level-1 DWT coefficients illustrating the projection of x[n] on the two subspaces. The normalized frequency band spanned the projection of x[n] on  $V_3$  is  $[0, 0.5\pi]$  while the projection of x[n] on  $W_3$  spans the normalized frequency band  $[0.5\pi, \pi]$ . The same process can be conducted iteratively on the output of the first-stage low pass filter H in order to obtain the level-2 DWT coefficients illustrating the projection of x[n] on  $V_2$  and  $W_2$ .



Figure 2.19 The tree structured filter bank implementation of Discrete Wavelet Transform. The down arrow following number 2 illustrates decimation or down sampling by factor 2 [12].

In general, at every level, the filtering and down sampling will result in half the number of samples (and thus half the time resolution) and half the frequency bands being spanned (and hence doubles the frequency resolution). Figure 2.20 gives rough illustration of the frequency bands spanned by each subspaces of the signal x[n] at each output branch of DWT tree. This concept can be related to time-frequency tiling shown in figure 2.13. In DWT, the iterative two-channel filter bank is not performed on high frequency component. As a result, the higher frequency component

has wider bandwidth resulting in poor frequency resolution. However, the higher frequency component has larger number of samples resulting in good time resolution. On the other hand, the decomposition process is always applied on low-frequency component. Therefore, the lower frequency component has lower bandwidth but smaller number of samples resulting in good frequency resolution but poor time resolution.



Figure 2.20 The frequency bands spanned by subspaces of signal x[n] at each output branch of DWT tree shown in figure 2.19.



Figure 2.21 The tree structured filter bank implementation of Wavelet Packet Transform. The down arrow followed by number 2 illustrates decimation or down sampling by factor 2 [12].  $\chi_{a,b}$  illustrates sub space spanned by transform coefficients at wavelet packet node b at level a.

#### 2.4.6 Wavelet Packet Transform

The filter bank implementation of Discrete Wavelet Transform (DWT) performs iterative decomposition only on the low pass filter output. While this approach is good to obtain the multi resolution version of the signal being decomposed, recursive decomposition on both low and high frequency component is also possible. This is what happens on Wavelet Packet Transform (WPT). As it can be seen in figure 2.21, WPT performs the iteration of the 2-channel filter bank on both

low pass and high pass branch. The signal to be decomposed is split into the detail (high frequency component) and the approximation (low frequency components parts). Instead of just further decomposing the approximation part, wavelet packet decomposition also splits the detail parts further. For *n* level of decomposition, the WPT produces  $2^n$  different sets of coefficients. In order to simplify the discussion, every output point of each filter is named *wavelet packet node*. For example, at decomposition level 5 there would be 32 wavelet packet nodes. The output of each wavelet packet node corresponds to particular frequency band. In the example shown in figure 2.21, the notation  $\chi_{a,b}$  is used to address the sub space spanned by transform coefficients at node *b* and level *a*.

While figure 2.20 clearly shows the non-uniformity of the frequency resolution of DWT, each final outputs of the WPT branch has uniform frequency resolution as it is shown in figure 2.22. This uniformity is due to the same manner of decomposition in both low and high frequency components. Hence, the outputs at wavelet packet nodes in the same level have evenly spaced frequency bands. As a result, the time-frequency tiling illustrated in figure 2.13 is not necessarily true for WPT. As it is clear from figure 2.21 and 2.22, the coefficients of the wavelet packet transform are not naturally ordered by increasing order of frequency. This issue would be clearer in chapter 3.

The mathematical basis of WPT is similar to DWT. However, it is always interesting to investigate the relationship between filter bank implementation of WPT and its theoretical counterpart. The interested reader is referred to [26] to find out more about the topic.



Figure 2.22 The frequency bands spanned by subspaces of signal x[n] at each wavelet packet node of WPT tree shown in figure 2.21.

#### 2.4.7 Inverse Wavelet Transform and Synthesis Filter Bank

While we have discussed about how signal decomposition using DWT and WPT are implemented using tree structured filter bank in the previous two sub-sections, this sub-section discusses about signal reconstruction using inverse of DWT or WPT using filter bank as well. It is very common to address the tree structured filter banks illustrated in figure 2.19 and 2.21 as *analysis filter-banks* since they are basically employed to analyze a complex signal into either its DWT coefficients or wavelet packets coefficients. In order to find the filter bank implementation of signal reconstruction, one may recall the DWT reconstruction formula illustrated by (2.56) but this time we consider different signal  $f_1(t)$  in the j+1 scaling function space  $(f_1(t) \in V_{j+1})$  [23]. Hence,  $f_1(t)$  complex signal interval.

 $f_1(t)$  can be simply expressed in term of scaling function at scale j+1 as:

$$f_1(t) = \sum_{k=-\infty}^{\infty} c(j+1,k) 2^{(j+1)/2} \varphi(2^{(j+1)}t - k)$$
(2.64)

It is also trivial to express this function in terms of scaling and wavelet function at scale *j* as:

$$f_1(t) = \sum_{k=-\infty}^{\infty} c(j,k) 2^{j/2} \varphi(2^j t - k) + \sum_{k=-\infty}^{\infty} d(j,k) 2^{j/2} \psi(2^j t - k)$$
(2.65)

Next, we can modify (2.65) by substituting the general form of two-scale equations given by (2.59) and (2.60) into (2.65). This results in (2.66).

$$f_1(t) = \sum_{k=-\infty}^{\infty} c(j,k) \sum_n h[n] 2^{(j+1)/2} \varphi(2^{j+1}t - 2k - n) + \sum_{k=-\infty}^{\infty} d(j,k) \sum_n g[n] 2^{(j+1)/2} \varphi(2^{j+1}t - 2k - n)$$
(2.66)

Finally, we can multiply both sides of (2.66) by  $\varphi(2^{(j+1)}t-k')$  and also take (2.64) into account. After integrating the results of this multiplication, we can express scaling coefficients at scale j+1c(j+1, k) in terms of scaling and wavelet coefficients at scale j [23]. This is given by (2.67).



Figure 2.24 The tree structured filter bank implementation of Inverse Discrete Wavelet Transform. The up arrow following number 2 illustrates up-sampling by factor 2.

$$c(j+1,k) = \sum_{m} c(j,m)h[k-2m] + \sum_{m} d(j,m)g[k-2m]$$
(2.67)

Equation (2.67) actually does the opposite of what is performed by (2.61) and (2.62). It basically states that the DWT coefficients at particular level j+1 can be obtained from linear combination of both weighted scaling and wavelet coefficients at double scale j. The filter bank implementation of (2.67), which is famously known as *synthesis filter-bank* can be illustrated by figure 2.23.

As it is obvious from figure 2.23, in two-band synthesis filter bank, the scaling and wavelet coefficients are first up-sampled by factor 2. The up-sampling process by factor 2 can be generally expressed as:

$$y_0[n] = \begin{cases} x_0[n/2], \text{ if } \frac{n}{2} \text{ is an integer} \\ 0 & \text{otherwise} \end{cases}$$
(2.68)

In (2.68),  $x_0[n]$  and  $y_0[n]$  are the sequence of samples before and after the up-sampling process, respectively. This up-sampling process basically doubles the number of samples in the input signal by inserting a zero between each pair of samples. The scaling coefficients are later filtered by half band low pass filter H'. Correspondingly, the wavelet coefficients are filtered by half band high pass filter G'. The outputs of the two filters are summed in order to construct the scaling coefficients at the next scale (which is half-scale). By considering figure 2.23 as well as equations (2.67), (2.59) and (2.60), it is obvious that the impulse responses of H' and G' are time reversed of the impulse responses of H and G, respectively. For simplicity, we might address H' and G' as synthesis filters while H and G can be addressed by analysis filters.



Figure 2.25 The tree structured filter bank implementation of Inverse Wavelet Packet Transform. The up arrow following number 2 illustrates up-sampling by factor 2.

When the signal is decomposed using DWT, it is always possible to reconstruct the original signal by employing the inverse of DWT called IDWT (Inverse Discrete Wavelet Transform). IDWT does the opposite of what is performed by DWT. Figure 2.24 depicts the synthesis filter bank realization of IDWT counterpart of figure 2.19. Likewise, Inverse Wavelet Packet Transform (IWPT) is used to reconstruct the original signal that is previously decomposed by WPT. The filter bank implementation of IWPT is illustrated by figure 2.25.

# **2.4.8 Popular Wavelet Families**

In this thesis, several popular wavelet families are used as a basis of wavelet based spectrum estimation. This sub-section is dedicated to give brief introduction about these wavelet families.



Figure 2.26 Illustration of some wavelet family function (a) Haar (Daubechies-1) (b) Daubechies-2 (c) Daubechies-4 (d) Coiflet-2 (e) Coiflet-3 (f) Coiflet-4.

One of the most popular wavelet families is **Daubechies**, named after Ingrid Daubechies, the leading researcher in the world of wavelet. Daubechies wavelets are compactly supported orthonormal wavelets, which allow the discrete wavelet analysis becomes practicable. In practical world, Daubechies family is commonly written as dbN, where N is the order. As discrete wavelets, the length of Daubechies is 2N. One of the members of Daubechies family is db1, which is

famously known as Haar wavelet. Haar wavelet is the simplest wavelet. It is discontinuous and not differentiable. The shape of the Haar wavelet is similar to the unit step function.

**Coiflets** is another wavelet family that is used in this thesis. Similar to Daubechies, Coiflets family is also normally written as coifN with N as the order and 6N as the length of the wavelet. This discrete wavelet is designed by Ingrid Daubechies to be more symmetrical than the Daubechies wavelets. Another wavelet family that is nearly symmetrical is **Symlets**, which is also the variant of Daubechies wavelets. Similar to Daubechies, Symlets are also written as symN with N as the order and 2N as the length of the wavelet. Figure 2.26 gives the illustrations of some wavelet family functions. Other wavelet families, such as Discrete Meyer, Biorthogonal wavelets and Reverse Biorthogonal wavelets are also used in our research. The Biorthogonal wavelets are the wavelets where the corresponding wavelet transform can be inverted but they are not necessarily orthogonal [4]. The design of biorthogonal wavelets provides more degrees of freedoms than orthogonal wavelet. The intention of using biorthogonal wavelets in spectrum estimation experiments here is to find the importance of wavelets orthogonality in the performance of the spectrum estimation.

#### 2.5 Existing literature on wavelet based spectrum estimation

In the context of dynamic spectrum access and cognitive radio, the use of multiple narrowband Band Pass Filter (BPF) might be required by some spectrum sensing technique such as match filtering illustrated in sub-section 2.2.1. In [27], Tian and Giannakis propose a wavelet based edge detection for spectrum sensing in cognitive radio. In general, this technique eliminates the need of multiple narrowband BPF. Moreover, the number of spectrum bands lies within the band of interest can be assumed to be unknown. Under this assumption, the use of multiple BPF-s is not only challenging but also useless because we even do not know the number of required BPF.

In Tian & Giannakis proposal, the wide band of interest should be known and it can be defined as a band in the frequency range  $[f_0, f_{N_B}]$  with bandwidth of  $B = f_{N_B} - f_0$ . In this band of interest, there could be  $N_B$  spectrum bands and some of them could be occupied. The task is how to find  $N_B$ , the occupied and unoccupied bands and the frequency boundaries of each band. By defining the *n*-th band as  $B_n$ :  $\{f \in B_n : f_{n-1} \le f < f_n\}$ ,  $n = 1, 2, ..., N_B$ , we need to find  $f_0 < f_1 < f_2 < .... < f_{N_B-1} < f_{N_B}$ . Once the number of spectrum bands  $N_B$  and the boundaries of every band are found, the spectral density estimation for every band is conducted by assuming the smooth and flat power spectrum density (PSD) within each band and the presence of discontinuities and irregularities around the boundary between two adjacent bands. In addition, additive zero mean white is assumed [27]. Figure 2.27 illustrates this idea.



Figure 2.27 Illustration of the assumption used by wavelet based edge detection proposed by Tian and Giannakis in [27]

Given the wide band of interest  $[f_0, f_{N_B}]$ , Tian and Giannakis method employs wavelets to locate the discontinuities and irregularities within the wide band. In other words, the wavelets play important roles in order to find the boundaries between bands, and thus the number of narrow

bands within the wide band of interest as well. In general, Continuous Wavelet Transform (CWT) is used for this purpose. However, instead of applying the CWT on the time domain version of the signal, Tian and Giannakis apply the CWT on the PSD. Given a wavelet function  $\psi(f)$ , the dilated version by *s* of  $\psi(f)$  can be defined as:

$$\psi_s(f) = \frac{1}{s} \psi(\frac{f}{s}) \tag{2.69}$$

The CWT of the PSD can then be defined as [27]:

$$CWT\{S_x(f)\} = CWT_{S_x(f)}(s) = S_x(f) * \psi_s(f)$$
(2.70)

The next step is by taking into account the fact that the PSD  $S_x(f)$  is the Fourier Transform of the autocorrelation function  $R_x(\tau)$  and defined the inverse Fourier Transform of the wavelet function  $\psi_s(f)$  as:

$$\Psi_s(\tau) = IFT\{\psi_s(f)\} = \Psi(s\tau) \tag{2.71}$$

In (2.71), *IFT*  $\{ \}$  denotes Inverse Fourier Transform. By taking (2.71) into account, (2.70) can be represented as:

$$CWT\{S_x(f)\} = FT\{R_x(\tau)\Psi(s\tau)\}$$
(2.72)

*FT* {} in (2.72) denotes Fourier Transform. Given (2.72), Tian and Giannakis investigate the shape of the first and second order derivative of  $CWT\{S_x(f)\}$  in order to locate the irregularities and discontinuities in the wide band of interest. In general, the first and second order derivative of (2.72) can be described by (2.73) and (2.74), respectively [27].

$$CWT'\{S_x(f)\} = s \frac{d}{df} (S_x * \psi_s)(f) = -s FT\{\tau R_x(\tau) \Psi_s(s\tau)\}$$
(2.73)

$$CWT"\{S_x(f)\} = s^2 \frac{d^2}{df^2} (S_x * \psi_s)(f) = s^2 FT\{\tau^2 R_x(\tau) \Psi_s(s\tau)\}$$
(2.74)

In (2.73) and (2.74), *CWT* '{ $S_x(f)$ } and *CWT* "{ $S_x(f)$ } are the first and second order derivative of CWT of the PSD  $S_x(f)$ . These two derivatives actually describe the first and second order derivatives of  $S_x(f)$  smoothed by the wavelet  $\psi_x(f)$ .

According to [27], the local maxima of the first order derivative  $CWT'\{S_r(f)\}$  can be used to indicate the irregularities of the PSD. Therefore, with regard to the assumption that the PSD is smooth within each band, the boundaries of each band can be located by the location of the local maxima of  $CWT'\{S_r(f)\}$ . The same goal can also be achieved by tracking the location of zero crossing of the second order derivative  $CWT''\{S_r(f)\}$ . Both of these two procedures give the location of  $f_0, f_1, f_2, \dots, f_{N_B-1}, f_{N_B}$ . The problem that might emerge in this approach is the possibility of noise that could induce local maxima in the shape of first order derivative  $CWT'\{S_r(f)\}$ . However, this problem can be avoided by varying the value of scale variable s. By assuming that the noise is random, the local maxima induced by the noise in  $CWT'\{S_r(f)\}$  for given scale s is less likely to reappear again for different value of s. Hence, the actual boundaries of each band are described by the local maxima that always presents in  $CWT'\{S_r(f)\}$  for any scale s.

After the boundaries of each band are identified, the level of PSD of the received signal is measured with respect to noise PSD. After calculating the average of total PSD, the PSD of the received signal is obtained by subtracting noise PSD from the average of total PSD. Tian and Giannakis experiment in [27] has shown that the wavelet based edge detection approach has successfully identified the number of occupied band within wide band of interest. This method has also offered what is called good dynamic spectrum range.

Another use of wavelet approach for spectrum sensing is offered by Hur, et al in [28]. The idea is to provide combination of coarse and fine sensing resulting in Multi Resolution Spectrum Sensing. The received signal is correlated with the modulated wavelet and the result of the correlation process represents the spectral contents of the input signal at the band around the carrier

frequency that is modulated by the wavelet. The resolution is adjusted by either using wavelet with large resolution bandwidth (sparse resolution) or small resolution bandwidth (precise resolution). With regard to cognitive radio, the coarse sensing is basically used to examine a wideband spectrum in fast manner and to produce information about candidate spectrum segments that are unoccupied. If it is needed, the fine sensing can be used to further investigate the candidate spectrum segments [28]. However, the speed issues related to the use of double sensing needs to be investigated further.

# CHAPTER 3 DEVELOPMENT OF WAVELET BASED SPECTRUM ESTIMATOR

In this chapter, we elaborate on the construction of the novel wavelet based spectrum estimation technique developed in our research. Our novel wavelet based spectrum estimation method is based on Wavelet Packet Transform (WPT) introduced in the previous chapter. The filter bank architecture is employed in order to realize the WPT. This chapter starts with section 3.1 which describes how the spectrum estimator is built based on wavelet packet representation. This is then followed by a discussion on spectral foot print of the wavelet packet coefficients and their frequency ordering. This section also presents the wavelet packet based spectrum estimator as a filter bank problem. This representation is useful to simplify the comparative analysis between the wavelet based technique with existing spectrum estimation methods such as Periodogram and Welch approaches. Parseval relationship and energy conservation is crucial in Fourier Transform theory and this energy conservation concept is the main reason why Fourier Transform can be used for spectrum estimation. Section 3.2 shows how the wavelet transform is a lossless unitary transform like Fourier Transform, and thus can be employed for spectrum estimation. Section 3.3 talks about the relationship between wavelet packet coefficient and power spectrum density (PSD). Section 3.4 explains the simulation setup, experiments, scenarios and results. The results of wavelet based estimation is provided together with the estimates of Periodogram and Welch method. Section 3.5 gives remarks on the comparative analysis performed in section 3.4 while section 3.6 concludes this chapter.

# 3.1 Spectrum Estimation through Wavelet Packet (WP) Tree Construction

# 3.1.1 Wavelet Packet Representation

In this section, we will describe the proposed spectrum estimation approach based on discrete wavelet packet transform using filter banks. It is well known from the theory of wavelets that compactly supported wavelet can be derived from perfect reconstruction filter banks [4]. Two channel filter banks split the given signal into the coarse version (low frequency component) and detail version (high frequency component). The use of high pass and low pass filter removes the lower half and the upper half frequency components, respectively. As a result, the output signal only spans the half of the frequency band spanned by the input signal. However, the time scale of the signal remains unchanged. To retain the same number of samples, the filter outputs are down sampled by factor 2. Therefore, one step decomposition process consisting of half band filtering and down sampling basically reduces the time resolution by a half and reduces the frequency band spanned by the signal by half as well. The scheme is then iterated successively on both the coarse and detailed versions until the desired degree of resolution to form a cascaded tree structure.

Through this hierarchical coding scheme, the signal to be estimated is successively split into high and low frequency components. The cascaded two channel filter banks structure recursively decomposes the signal being estimated and maps the signal components into the frequency domain. This process may be likened to passing the received signal into a sieve of filters (filter banks) where the output point of each filter is a wavelet packet node. The output of each wavelet packet node corresponds to a particular frequency band. The decomposition of the signal into different frequency bands with different resolutions is possible. The resolution of the estimate can be adjusted by increasing or decreasing the levels of iteration. The greater the degree of decomposition, the better the frequency resolution is. The number of successions is usually limited by the desired level of frequency resolution and available computational power. Such successive high and low pass filtering results in the decomposition of the signal into wavelet packet coefficients at different frequency bands.

We may recall from chapter 2 that the impulse response of analysis low pass filter h[n] and high pass filter g[n] in figure 3.1 should satisfy the two-scale equations illustrated in (2.46) and (2.52). Moreover, g[n] and h[n] must be tightly coupled by the Quadrature Mirror Filters (QMF) relation, as in (2.53) to ensure the orthogonality between scaling and wavelet functions. In figure 3.1, a level-4 decomposition procedure generating 16 wavelet packet coefficients are illustrated. These 16 wavelet packet coefficients are produced at 16 wavelet packet nodes corresponding to 16 outputs of the 16 filters at level 4. It is also possible to utilize the outputs at level 2 or level 3 of the tree. This is the advantage of using wavelet packet - the output of every node at every level can be chosen according to the desired frequency resolution. The wavelet packet coefficients at the filters output actually describe the projection of the signal on the corresponding wavelet and scaling basis functions.



# Frequency Ordering

Figure 3.1 Wavelet packet tree for four levels wavelet packet decomposition. Here H(z) and G(z) denote the low and high pass decomposition filters, respectively. The down arrows represent decimation by 2. It should be noted that the coefficients of the wavelet packet transform are not ordered by increasing order of frequency. Grey code permutation is required to obtain the correct frequency order.

# **3.1.2 Frequency Ordering of Wavelet Packet Coefficients**

It is of great importance to understand the spectral foot print of the wavelet packet coefficients and their frequency ordering to be able to identify and isolate coefficient that lie near the interference spectrum. The coefficients of the wavelet packet transform are not naturally ordered by increasing order of frequency. Instead, they are numbered on the basis of a sequential binary grey code value. For example if each coefficient in the level basis is numbered with a sequential decimal order (0000, 0001, 0010, 0011,...) the frequency ordering of the coefficients can be ordered by frequency by sorting them into Gray code value (0000, 0001, 0010, 0110,...) [29].

To understand the working of the wavelet packet transform, consider the example shown in Figures 3.2 and 3.3 where the decomposition of a signal spanning 0-8 Hz is considered for up to two levels. The output of a decomposition process is the result of the scaling function (the low pass filter) and the result of the wavelet function (the high pass filter) followed by down sampling. Down sampling generates two new filter results with half the number of elements in the time domain. In addition to this, it also results in mirroring of the high pass components in the frequency

domain. This switches the low and high pass components in a subsequent decomposition as exemplified in the figures. When the wavelet packet algorithm is recursively applied the resultant wavelet packet coefficients obtained follow the Gray code sequence.



Figure 3.2 Level 1 Decomposition: Mirroring of high pass components due to down sampling. In the figure, 2↓ denotes down sampling by 2



**Figure 3.3** Level 2 Decomposition (continued from Figure 3.2): The 2-levels wavelet packet decomposition is applied. Due to down sampling all the high frequency parts are mirrored. The low and high pass part is swapped in a subsequent transform. In the figure,  $2\downarrow$  denotes down sampling by 2. Note that the output of the 1<sup>st</sup> wavelet packet node correspond to 0-2Hz, 2<sup>nd</sup> wavelet packet node correspond to 2-4Hz, 3<sup>rd</sup> and 4<sup>th</sup> node correspond to 6-8Hz and 4-6Hz respectively

Because of the Gray code ordering there is a need to formulate the frequency ordering of the output of wavelet packet node given the order of the node. Jensen and la Cour-Harbo in [29] has found that this relationship is expressible as Gray Code permutation. For example, given the decimal number n = 5 having binary representation (in the form of  $b_4 b_3 b_2 b_1$ )  $b_1 = 1$ ,  $b_2 = 0$ ,  $b_3 = 1$ ,  $b_4 = 0$ , the Gray Code permuted integer GC(n) is defined via the following formula [29]:

$$GC(b_i) = (b_i + b_{i+1}) \mod 2$$
Hence, given the order of wavelet packet node as follows:  

$$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$
(3.1)

, having binary representation: 0000 0001 0010 0011 0100 0101 0110 0111 The Gray code permutation of the binary representation above would be: 0000 0001 0011 0010 0110 0111 0101 0100 Therefore, the wavelet packet node number in increasing frequency would be: 3 2 7 5 0 1 6 4

Instead of the usual Gray Code Permutation, we here present an alternative algorithm to convert the sequence from Gray to binary. This method is easier since it does not involve any binary to decimal conversion and vice versa. If the wavelet packet nodes are in sequence (from the smallest number to largest number), the algorithm for obtaining the frequency band order is as follows:

- Initialize a vector **alpha** with elements 0 and 1 (**alpha = [0 1]**)
- Define the required level of wavelet packet decomposition L
- For j = 2 to L-1 do
   beta = alpha + 2<sup>j</sup>;
   Flip the element of beta
   Append beta into the end of alpha

 Table 3.1 Relationship between wavelet packet node number, its frequency ordering and the spanned frequency band at 4<sup>th</sup> decomposition level

Wavelet	Frequency	Spanned frequency band	Spanned frequency band (Hz)
packet node	order number	(Case 1: in terms of normalized	(Case 2: the entire spanned
number		frequency)	frequency is [0, 500 Hz]
0	0	$0 - 0.0625\pi$	0 - 31.25
1	1	$0.0625\pi - 0.125\pi$	31.25 - 62.5
2	3	0.1875π - 0. 25π	93.75-125
3	2	0.125π - 0.1875π	62.5 - 93.75
4	7	0.4375π - 0.5π	218.75-250
5	6	0.375π - 0.4375π	187.5-218.75
6	4	$0.25\pi - 0.3125\pi$	125-156.25
7	5	0.3125π - 0.375π	156.25-187.5
8	15	0.9375π - π	468.75-500
9	14	0.875π - 0.9375π	437.5-468.75
10	12	0.75π - 0.8125π	375-406.25
11	13	0.8125π - 0.875π	406.25-437.5
12	8	0.5π - 0.5625π	250-281.25
13	9	0.5625π - 0.625π	218.25-312.5
14	11	0.6875π - 0.75π	343.75-375
15	10	0.625π - 0.6875π	312.5-343.75

The bottom part of figure 3.1 shows the relationship between the order of wavelet packet node number and its frequency ordering for 4-level decomposition. There are 16 nodes in the lowest level shown in figure 3.1 corresponding to 16 frequency bands. These 16 frequency bands span the normalized frequency range  $[0,\pi]$  or given sampling frequency  $f_s$  Hz, they span the frequency range of  $[0 \text{ Hz}, 0.5f_s \text{ Hz}]$ . If the spanned frequency range is [0 Hz, 500 Hz], the width of frequency range spanned by single wavelet packet node at the 4<sup>th</sup> level is 500 Hz/16 = 31.25 Hz. Table 3.1 gives the relationship between wavelet packet node number, its frequency ordering and the spanned frequency band. For clarity, two cases are provided here. The first case is in terms of normalized frequency and the second case is for the spanned frequency range of [0 Hz, 500 Hz]. Finally, figure 3.4 illustrates the modified structure of wavelet packet tree with 3-level of decomposition in order to match the frequency ordering. We can note the difference of this structure with the first 3 level of the tree shown in figure 3.1, especially the order of analysis low pass filter *H* and high pass filter *G* in each level.



**Figure 3.4** Wavelet packet decomposition of a signal. Here *H* and *G* denote the frequency responses of the low and high pass decomposition filters, respectively. The down arrows represent decimation by 2. The  $x_i$ 's denote the wavelet packet coefficients. Besides the decomposition, the Power Spectral Density (PSD) of the decomposed signal components in successive octave bands normalized to the Nyquist frequency is shown. The order of filter in each level is modified in order to match frequency ordering from 0 to  $\pi$ .



Figure 3.5 Wavelet packet based spectrum estimation concept from the point of view of filter bank paradigm. Here 3level decomposition is employed resulting in 8 virtual filters splitting the normalized frequency band  $[0, \pi]$  into eight sub-bands corresponding to eight estimate points.

#### 3.1.3 Wavelet Packet based spectrum estimation as a Filter Bank analysis problem

From filter bank paradigm point of view, the wavelet packet approach is a natural extension to the Multi Taper Spectrum Estimation (MTSE) in the sense that this method also uses different orthogonal filters as prototype filters but instead of Slepian sequences the filters are derived from tree structures constructed by cascading wavelet packet decomposition filters. Akin to MTSE and periodogram, every single point of the wavelet packet spectrum estimates can be viewed as an output of a virtual filter having pass band around that point. However, in contrast to MTSE and periodogram, these filters are realized by cascading several analysis low pass and/or high pass filters, which are derived from single prototype according to two scale equations and quadrature mirroring relationship illustrated by (2.46), (2.52), and (2.53), respectively. The impulse

response of these cascaded filters called wavelet packet duals  $\tilde{\psi}_i[k]$  [6], which can be represented as:

$$\psi_{i}[k] = f(k) * f(2k) * \dots * f(2^{J-2}k) * f(2^{J-1}k);$$
  
where,  $0 \le i \le 2^{J} - 1$  (3.2)  
and,  $f(k) = \begin{cases} h[k], \text{ for lowpass branches} \\ g[k], \text{ for highpass branches} \end{cases}$ 

Figure 3.5 illustrates the filter bank paradigm of the proposed wavelet packet based spectrum estimator. As it is obvious from the figure, there are eight virtual filters dividing normalized frequency range of  $[0,\pi]$  into 8 sub-bands. In this figure, the decomposition level is 3 which results in 8 estimate points. The impulse response of each virtual filter in this figure can be derived from (3.2). Clearly, higher level of decomposition would increase the number of sub-bands (or estimate points) and thus it would increase the frequency resolution.

# 3.2 Wavelet Packet Transform and Energy Conservation

As in the case of Fourier Transform, the relationship between the amplitude of the signal and wavelet coefficients needs to be defined in order to develop valid wavelet based spectrum estimation. As already known, the Parseval relation proves that the Fourier transform is a lossless unitary transform. Likewise, we need to assert if the wavelet packet transforms preserves energy too. In order to verify this relation, we can start by representing a function f(x) in Hilbert Space as linear combination of the basis function  $\varphi_i(x)$ :

$$f(x) = \sum_{i} \alpha_{i} \varphi_{i}(x)$$
(3.3)

It is clear from (3.3) that  $\alpha_i$  can be obtained from inner product between basis function  $\varphi_i(x)$  and function f(x):

$$\alpha_i = \left\langle \varphi_i(x), f(x) \right\rangle \tag{3.4}$$

The norm of the function can be computed from the transform coefficients:

$$\left\|f(x)\right\| = \sum_{i} \left|\alpha_{i}\right|^{2} = \sum_{i} \left|\left\langle\varphi_{i}(x), f(x)\right\rangle\right|^{2}$$
(3.5)

By assuming that a function g(x) has transform coefficients  $\beta_i$ , we can derive the generalized Parseval equation by taking the inner product between two functions f(x) and g(x) in Hilbert Space:

$$\left\langle f(x), g(x) \right\rangle = \sum_{i} \overline{\alpha}_{i} \beta_{i} = \sum_{i} \left\langle f(x), \varphi_{i}(x) \right\rangle \left\langle \varphi_{i}(x), g(x) \right\rangle$$
(3.6)

In (3.6),  $\overline{\alpha_i}$  indicates the complex conjugate version of  $\alpha_i$ . According to Todorovska and Hao in [30], the Parseval relation for Discrete Orthogonal Wavelet Transform and its inverse is obtained by substitution on generalized Parseval Equation in (3.6). While the general equations for Discrete Wavelet Transform (DWT) have already been given by (2.56)-(2.58), we try to rewrite the DWT equations specifically for discrete signal x[n] with respect to our filter bank implementation shown in figure 3.4. Discrete wavelet transforms pairs for discrete signal x[n] can be represented as follows [30]:

$$x[n] = \sum_{k=1}^{N/2^{J}} c(J,k)\varphi_{J,k}[n] + \left(\sum_{j=1}^{J} \sum_{k=1}^{N/2^{J}} d(j,k)\psi_{j,k}[n]\right)$$
(3.7)

$$c(J,k) = \left\langle \varphi_{J,k}[n], x[n] \right\rangle \text{ and } d(j,k) = \left\langle \psi_{j,k}[n], x[n] \right\rangle$$
(3.8)

In (3.7) and (3.8), *J* is the decomposition level and *N* is total number of samples. The two equations, (3.7) and (3.8) are nothing but the synthesis and analysis equations, respectively. The first component in the right side of (3.7) is the coarse part of signal x[n], which is represented as linear combination of the scaling function  $\varphi_{J,k}[n]$ . On the other hand, the second part in the right side of (3.7) is the detail version of x[n], which is represented as linear combination of wavelet function  $\psi_{j,k}[n]$ . If we have another signal, y[n] with  $d^{(i)}(j,k)$  and  $c^{(i)}(J,k)$  as its wavelet packet coefficients, the Parseval relation for y[n] and x[n] can be described using (3.6) as:

$$\left\langle x[n], y[n] \right\rangle = \sum_{n=-\infty}^{\infty} x[n] y[n] = \left( \sum_{j=1}^{J} \sum_{k=1}^{N/2^{j}} d^{(i)}(j,k) d(j,k) \right) + \left( \sum_{k=1}^{N/2^{j}} c^{(i)}(J,k) c(J,k) \right)$$
(3.9)

Using (3.9) and by taking x[n] = y[n], the Parseval relation describing the norm of y[n] can be given as [30]:

$$\left\|y[n]\right\|^{2} = \sum_{n=-\infty}^{\infty} \left|y[n]\right|^{2} = \left(\sum_{j=1}^{J} \sum_{k=1}^{N/2^{j}} \left|d(j,k)\right|^{2}\right) + \sum_{k=1}^{N/2^{j}} \left|c(J,k)\right|^{2}$$
(3.10)

Equation (3.10) clearly illustrates the lossless nature of wavelet transform. Hence, the discrete wavelet transform preserves the time domain energy in wavelet domain. This lossless feature is really important and a fundamental reason why the spectrum estimation technique based on wavelet can be built.

Parseval relation holds well for both conventional discrete wavelet transform and wavelet packet decomposition. The Parseval relation for wavelet packet can be practically proved as follows:

- Perform wavelet packet decomposition
- Extract the wavelet coefficients of each node. For three level decomposition, the result would be eight vectors of wavelet coefficient namely:  $cf_{node0}, cf_{node1}, ..., cf_{node7}$ .
- Calculate the total energy in wavelet packet domain  $(E_{WP})$  from the wavelet coefficients by performing an inner product (dot product) of the vector with itself as follows:

$$E_{WP} = cf_{node0} * cf_{node0} + cf_{node1} * cf_{node1} + \dots + cf_{node7} * cf_{node7}$$
(3.11)

Based on Parseval relationship with respect to discrete wavelet transform and wavelet packet decomposition illustrated above, we can start to define the wavelet based spectrum estimates. Given a certain level of wavelet packet decomposition, the wavelet packet nodes span the frequency band from 0 to  $0.5f_s$  with  $f_s$  as the sampling frequency. This fact is the result of filter banks theory proposed in [4]. If we have *n*-level wavelet packet decomposition, there will be  $2^n$ packet nodes as tree's leaves. This decomposition will divide  $[0 \ 0.5f_s]$  into  $2^n$  equal band and the output of each node in the leaves correspond to each band. The energy contained in particular band can be found from inner product of the wavelet packet coefficients vector of the corresponding node with itself. It can be noted that we can find the energy for different frequency resolution simply by investigating the wavelet coefficients in different levels. The advantage of the wavelet packet decomposition is that we do not only have wavelet coefficients of the nodes in the tree's leaves but also those of nodes in different level of the tree.

# 3.3 Calculating Power Spectrum Density from Wavelet Packet Coefficients

As a consequence of the conformance of WPT to Parseval relationship, the power spectrum density in  $m^{\text{th}}$  frequency band ( $P\hat{S}D_{WPm}$ ) corresponding to  $m^{\text{th}}$  wavelet packet node can be computed from the energy ( $E_{WPm}$ ) found in the  $m^{\text{th}}$  node, the frequency band that is spanned by single wavelet packet node and total number of samples N as follows:

$$P\hat{S}D_{WPm} = \frac{E_{WPm}}{N f_{WP}} (watt / (radian / sample))$$
(3.12)

# **3.4 Experiments and Results**

#### **3.4.1 Experiment scenarios, sources and their characteristics**

In order to investigate the performance of the wavelet packet based spectrum estimation technique, four different types of sources are considered, namely, partial-band, single tone, multitones and swept tone. The partial-band source has its energy spread over a continuous range of frequencies and it occupies the normalized frequency band from  $0.25\pi$  to  $0.75\pi$ . The single tone source has all of its energy at one frequency and it sits right in the middle of the range spanned by wavelet based spectrum estimation, namely at  $0.5\pi$ . The multi-tones source consists of seven single tone sources located at normalized frequency from  $0.125\pi$  to  $0.875\pi$  and they are equally spaced. Finally, a swept tone source is introduced to test how well the estimation schemes perform when there are temporal variations in the frequency occupied. A swept tone source is just like a partial band source except that it occupies different bands at different instances. The vastly different nature of the test sources will give interesting insights into the operation of the spectrum estimation tool. Table 3.2 summarizes the description of sources used in the experiment

	Type of sources	Description
1	Partial band	• Frequency occupied: $[0.25\pi, 0.75\pi]$
2	Single tone	<ul> <li>Frequency occupied: 0.5π</li> </ul>
3	Multi-tones	Consist of 7 single tones occupying normalized
		frequency: 0.125π, 0.25π, 0.375π, 0.5π, 0.625π,
		$0.75\pi$ and $0.875\pi$
4	Swept tone	<ul> <li>Different bands are occupied at different</li> </ul>
		instances
		• Sweeping the frequency band of $[0.2\pi 0.8\pi]$
		• 20 sweeps (each of 640 samples)
		• One sweep can be divided into 5 sub sweeps
		• The estimate of each sub sweep is displayed

Table 3.2 Description of four types of sources used in the experiments

To gauge the swept tone source, 20 sweeps (each of 640 unit samples) are considered. The sweep spans the normalized frequency band  $0.2\pi$  to  $0.8\pi$ . In order to present the effect of highly time-varying frequency on spectrum estimation, the estimate for five portions of a single sweep is displayed. The estimation technique depicts the first 128 unit samples of a single sweep followed by the next 128 unit samples of the same sweep and so on until the fifth 128 unit samples of the same sweep. For this experiment, several wavelet families are investigated namely Daubechies families, Coiflet, Symlet, Discrete Meyer, Biorthogonal and Reverse Biorthogonal.

# **3.4.2 Result and Analysis**

# A. Partial Band Source

# A.1 Analysis on various wavelet families

In this type of source, we try to provide some preliminary assessment on our wavelet based estimation approach. A comprehensive analysis is provided in chapter 4. For partial band source, the performance of the estimation techniques is mainly evaluated with respect to three different metrics:

- Leakage suppression (rejection at unoccupied band) or sometimes known as side lobe suppression
- Variance of the estimated power spectrum density (PSD)
- Transition band (transition between active band and unoccupied band).



**Figure 3.6** Periodogram and wavelet based estimates (Daubechies 15, Symlet 15, and Discrete Meyer) for partial band source. The number of samples in this experiment is 12800. 7-level decomposition is used in wavelet based estimation.

Figures 3.6 and 3.7 depict Periodogram and Welch approach as well as various wavelet families based estimates for the case of partial band source. In these figures, the number of samples used in the experiment is 12800. For the purpose of this experiment, the Welch approach divides the received samples into 399 segments of 64 samples. Two consecutive segments overlap to one another by 50%. Before performing the averaging process, Hamming window is applied on each segment.

For the case of wavelet based estimates, 7-level decomposition is used in the experiment shown in figures 3.6-3.7. Daubechies-15, Coiflet-5, Symlet-15, Biorthogonal-3.9 and Reverse Biorthogonal 3.9 are chosen because they have roughly comparable wavelet filters length. Daubechies-15, Coiflet-5, and Symlet-15 have filter length of 30 while both Biorthogonal 3.9 and Reverse Biorthogonal 3.9 have filter length of 20. On the other hand, Discrete Meyer having filter length of 102 is included here in order to give a rough idea on how the length of the decomposition filter impacts the quality of the estimate.

From figures 3.6 and 3.7, it can be seen that Discrete Meyer wavelet is better than other wavelets families in terms of transition band (transition between active band and unoccupied band) as well as the variance of the estimated power spectrum density (PSD). However, it should be noted that the length of decomposition filter for Discrete Mayer is 102 and thus it introduces more complexity in its actual implementation. Among other wavelet families, the PSD estimates based on Daubechies-15, Symlet-15 and Coiflet-5 are quite acceptable. All of these three wavelet families have filter length of 30. We can notice the poor transition band on the estimates based on these three wavelets, which are very likely caused by poor frequency selectivity of their corresponding wavelet decomposition filter. On the other hand, the performance of non orthogonal wavelet families (Biorthogonal 3.9 and its reverse counter part) is extremely poor and thus their usage is not recommended.

When the performance of the wavelet based estimate is compared to Fourier based periodogram, it appears that the transition band of periodogram is moderately superior to wavelet based estimate. However, on account of the variance of the estimated PSD, the wavelet approach performs significantly better than the periodogram. In the context of dynamic spectrum access as in cognitive radio applications, a large variance in the estimate could lead to erroneous judgements in

the presence/absence of a source. Hence, in the metric of variance of estimated PSD, it can be said that orthogonal wavelet based estimate is preferable in comparison to the periodogram for partial band sources with number of samples of 12800.

The Welch approach shows a slightly better performance than orthogonal wavelet based estimate. The averaging of estimates in the Welch approach plays an important role in ensuring that the PSD has a small variance while maintaining sharp transition band. However, the transition band found in Welch approach is only marginally better than that is found in Discrete Meyer wavelet based estimate. This means that there is a great scope for improvement in the wavelet based approach especially when the length of the decomposition filter is increased.



Figure 3.7 Welch approach and wavelet based estimates (Coiflet-5, biorthogonal 3.9, and reverse biorthogonal 3.9) for partial band source. The number of samples in this experiment is 12800. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 7-level decomposition is used in wavelet based estimation.

Figure 3.6 and 3.7 also show that the level of estimated power in unoccupied band for Welch approach is higher than for simple periodogram meaning that the Welch approach offers poorer rejection in the unoccupied band. This is understandable since Welch approach divides the received samples into several segments with lower number of samples before estimating each segment. In the other words, the size of window in Welch approach for each segment is significantly smaller than in simple periodogram and thus Welch approach introduced wider main lobe in its window kernel. As a result, Welch method introduces more leakage than the periodogram. The introduction of Hamming window in Welch approach, however, helps to improve the rejection in unoccupied band to the level as shown in figure 3.7. The wavelet based approach offers slightly better rejection than both Welch and periodogram in the normalized unoccupied frequency band of  $[0, 0.15\pi]$  and  $[0.85\pi, \pi]$ . This means that once the transition from active band to unoccupied band is completed, the leakage rejection in unoccupied band offered by wavelet based estimates is not that poor.

Figure 3.8 illustrates the effect of filter length on the performance of the wavelet based estimates. In this case, the Daubechies family is selected for the experiment. It should be noted that the length of the filter is twice the index of the wavelet. For example, Daubechies-4 has filter length of 8. It can also be noted that Haar is actually Daubechies-1 and thus it has a filter length of

2. From figure 3.8, it is obvious that the longer the filter length, the smaller the transition band. Longer filter length appears to correspond to a better suppression of power in the unoccupied band as well. However, a longer filter length also means a higher complexity in the implementation.



Figure 3.8 Wavelet based estimates for partial band source using Daubechies family with different filter length. 7-level decomposition is used here. The number of samples in this experiment is 12800



Figure 3.9 PSD estimates of partial band source according to various decomposition level of Daubechies-20. The number of samples in this experiment is 12800.

# A.2 Analysis on decomposition level

Figures 3.9-3.12 demonstrate the effect of wavelet decomposition levels on the PSD estimates. The results are provided in four different figures for ease of depiction. In figure 3.9, wavelet based PSD estimates are displayed at four different decomposition levels, namely 5-level, 7-level, 9-level and 11-level. Of particular interest with the wavelet packet approach is the variance of estimated PSD with increasing / decreasing of decomposition levels. With a decrease in depth of signal decomposition, the variance of the estimated PSD achievable is reduced. This is reasonable since the lower the decomposition level, the larger the band spanned by single wavelet packet node. Therefore, the total energy obtained from single wavelet packet node would be averaged over larger frequency band resulting in smaller variance.



Figure 3.10 PSD estimates of partial band source according to different decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. This figure is included to describe how the effect of decomposition level on wavelet estimates with respect to Periodogram and Welch estimates. The number of samples in this experiment is 12800. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach.

# a) Comparison with Welch method and Periodogram

Even though 11-level wavelet packet decomposition introduces a large variance, it is still much smaller than in periodogram (see figure 3.10). The variance is considerably reduced as the decomposition level is lowered to 6 though is still larger than in Welch estimates. It can thus be inferred from figure 3.9 and 3.10 that the decomposition level of wavelet based spectrum estimation can be adjusted to get a variance somewhere in between the variance found in Welch and periodogram. As it is clear from section 3.1, the wavelet packet based spectrum estimator allows us to exploit not only the wavelet coefficients produced by the node in the leaves of the tree but also the coefficients produced by all nodes in all levels of the tree. For example, if the decomposition level is set to 11, we do not only get the coefficients of the nodes at the 11<sup>th</sup> level but also at the 10<sup>th</sup> level, the 9<sup>th</sup> level and so on. Therefore, we can actually obtain multiple estimates from different levels of the tree and hence multiple estimates with different degree of variance, in one snapshot and one operation. Unfortunately, for partial band source, varying the decomposition level does not significantly improve the transition band and the side lobe level (the rejection in the unoccupied band). This is clear from figures 3.9 and 3.10.



Figure 3.11 PSD estimates of partial band source according to different decomposition level of Daubechies-20 together with Thomson's MTSE and Periodogram using Hann window. The number of samples in this experiment is 12800.



Figure 3.12 PSD estimates of partial band source according to different decomposition level of Daubechies-20 together with Periodogram using Hamming window and Blackman window. The number of samples in this experiment is 12800.

#### b) Comparison with windowed Periodogram and MTSE

Figures 3.11 and 3.12 show that applying the window to the periodogram reduces the side lobes in the estimates. However, the windowing technique does not reduce the large variance of the periodogram estimates. In fact, the variance of the estimated PSD in periodogram with various windows introduced here are much larger than in 11-level wavelet packet based estimation scheme. Lastly, as already discussed in chapter 2, Multi Taper Spectrum Estimation (MTSE) tries to minimize the variance of estimated PSD by employing multiple orthogonal prototype filters. As a result, it offers much smaller variance compared to periodogram. The derivation of MTSE prototype filters based on Slepian sequence has resulted in excellent leakage suppression and thus the MTSE easily outperforms the wavelet based approach with respect to rejection at unoccupied band. However, the variance of MTSE is still significantly larger than the one found in wavelet based estimates presented here. Moreover, the MTSE is complex and difficult to implement.

#### A.3 Analysis for the case of small number of samples

Figures 3.13 and 3.14 depict the performance of wavelet based estimates along with periodogram and Welch approach for small number of samples. The number of samples used in the experiments is 384. The setting of Welch approach used here is exactly the same as the one used in figure 3.7. The purpose of this experiment is to learn about the behavior of the estimators when higher speed of estimation is demanded as in the case of cognitive radio. Assuming the sampling rate is constant, higher speed of estimation would correspond to smaller number of samples.



Figure 3.13 Periodogram and wavelet based estimates (Symlet 15, and Discrete Meyer) for partial band source. The number of samples in this experiment is 384. 7-level decomposition is used in wavelet based estimation.

In general, there is no significant different between the Welch estimates found in figure 3.7 and in figure 3.14 in terms of rejection level in the unoccupied band. This is logical since the size of each segment for both cases is the same, namely 64. However, the variance of the Welch estimates is increased when the number of samples is reduced because the estimates are now averaged over 11 segments instead of 399 (refer figure 3.7). On the other hand, the periodogram estimates for 384 samples introduce more leakage than the estimates for 12800 samples shown in figure 3.6. This is also reasonable since the size of the window in the case of 384 samples is much smaller. Therefore, when this time domain window is transformed into sinc function at frequency domain, the width of the main lobe for 384 samples case is larger followed by larger distance

between lobes. This results in more leakage as it is clearly shown in figure 3.13. The performance of wavelet based estimates also deteriorates when the number of samples is lowered from 12800 to 384. Apart from the frequency selectivity issue in the wavelet decomposition filters, which is independent from number of samples, it seems that the rectangular windowing effect when we take finite number of samples from the received signals also happens in wavelet based estimates. Another issue which would be discussed in chapter 6 is the impact of spectrum carving in the wavelet decomposition filter. With respect to rectangular windowing issue, further analytical and mathematical study is needed.



Figure 3.14 Welch approach and wavelet based estimates (Coiflet-5, Daubechies-15) for partial band source. The number of samples in this experiment is 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 7-level decomposition is used in wavelet based estimation.

# A.4 Summary of Inferences

Based on the investigation on the estimates of partial band sources conducted in subsection A.1 to A.3, we can summarize our important findings as follows:

- Wavelet based estimates has significant transition band due to poor frequency selectivity of their corresponding wavelet decomposition filter. The reason for this is due to the fact that wavelet families used in this experiment are standard wavelets which are not designed specifically for spectrum estimation.
- The decomposition level of wavelet based spectrum estimation can be adjusted to get a variance somewhere in between the variance of Welch and periodogram.
- Varying the decomposition level of wavelet based estimates does not improve the transition band and the rejection in the unoccupied band.
- Even though MTSE and windowed periodogram has successfully minimized the leakage, the variance of estimates based on MTSE and windowed periodogram is still poorer than the variance of wavelet based estimates with presented decomposition level.
- As the number of samples is decreased, the leakage introduced by periodogram and wavelet based estimates in the unoccupied band become more significant.
# B. Single Tone Source

# B.1 Analysis on various wavelet families

In single tone source, some preliminary evaluation on our wavelet based estimation approach is provided for three different metrics:

- Variance of the estimated power spectrum density (PSD)
- Frequency resolution
- Leakage suppression or power rejection at unoccupied band.

Figures 3.15 and 3.16 describe Periodogram, Welch as well as wavelet based estimates for the case of single tone source. The number of samples in these experiments is 12800 while the configuration of Welch method used here is the same as in the case of partial band source. The same wavelet families as in figure 3.6 and 3.7 are employed here with decomposition level of 7. The reasons for selecting these families are the same as in Partial band case. As expected, Discrete Meyer wavelet, having much longer filter length, has a slightly better performance than other wavelet families in terms of low variance of the estimated PSD in the unoccupied band. However, in terms of frequency resolution, all orthogonal wavelet based estimates perform equally well. Again, the performance of biorthogonal wavelets is very poor making them unsuitable candidates.



**Figure 3.15** Periodogram and wavelet based estimates (Daubechies 15, Symlet 15, and Discrete Meyer) for single tone source. The number of samples in this experiment is 12800. 7-level decomposition is used in wavelet based estimation.

From the two figures, it is quite interesting to note that the variance of orthogonal wavelet based estimate in the case of single tone source is better than that of Periodogram. In terms of frequency resolution, the wavelet based estimates also outperforms Welch estimates. While the averaging of estimates employed in the Welch approach is the key reason to suppress the variance of estimated PSD in partial band sources, the same feature seems to be the key for its poorer frequency resolution compared to orthogonal wavelet estimate of the single tone source. It is also interesting to consider the well known trade off between the variance of the estimated PSD, size of the side lobes and the frequency resolution. So far, Welch approach is considered as an approach that best trades off the frequency resolution for the variance. However, more prices are also paid here because Welch approach split the signals into smaller segment resulting in more leakage in the unoccupied band. Even though the Hamming window is applied on each segment to mitigate this leakage, the overall side lobe suppression in Welch method is worse than in orthogonal wavelet based estimates. This makes the orthogonal wavelet based estimate is more preferable than Welch approach for the case of single tone source with number of samples of 12800.

Meanwhile, the periodogram shown in figure 3.15 seems to have a very good frequency resolution and side lobe suppression comparable to wavelet based estimates. However, the large variance found in periodogram estimate should be considered. Since the frequency resolution of orthogonal wavelet based estimate is just slightly poorer than periodogram estimates, orthogonal wavelet approach can still be considered as good alternative for spectrum estimation especially if the variance issue is important.



Figure 3.16 Welch approach and wavelet based estimates (Coiflet-5, biorthogonal 3.9, and reverse biorthogonal 3.9) for single tone source. The number of samples in this experiment is 12800. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 7-level decomposition is used in wavelet based estimation.

Figure 3.17 illustrates the effect of filter length on the performance of the Daubechies wavelet based estimates for the case of single tone source. There is no clear relationship between the length of filter and the frequency resolution of the Daubechies wavelet based estimate. However, on account of variance of estimate at the unoccupied band, a clearer pattern emerges with increase in filter lengths. It is obvious from figure 3.17 that the longer the decomposition filters the smaller the variance of estimated PSD in the unoccupied band.

### B.2 Analysis on decomposition level

### a) Comparison with Welch method and Periodogram

Similar to the case of partial band source, the effect of wavelet decomposition levels on the PSD estimates is illustrated in figure 3.18-3.21. In terms of variance of the estimated PSD, figures 3.18 and 3.19 exemplifies what has been shown in the partial band case, namely the fact that the wavelet based estimates with various level of decomposition have characteristics in between of that of periodogram and Welch approach. However, some other aspects are found in these two figures. As shown in figure 3.19, the frequency resolution of simple periodogram is excellent but it has moderate side lobe suppression with large variance of PSD in the unoccupied band. On the other hand, the averaging of periodogram employed by Welch approach is the key reason for its poor

frequency resolution in the estimation of single tone source. Welch approach also has slightly worse side lobe suppression (due to smaller window size per segment) and much better variance of estimated PSD than periodogram.



Figure 3.17 Wavelet based estimates for single tone source using Daubechies family with different filter length. 7-level decomposition is used here. The number of samples in this experiment is 12800



Figure 3.18 PSD estimates of single tone source according to various decomposition level of Daubechies-20. The number of samples in this experiment is 12800

Meanwhile, it is very interesting to note from figures 3.18 and 3.19 that the frequency resolution of wavelet packet decomposition tends to approach that of periodogram as the decomposition level is raised from 5 to 11. On the other hand, the frequency resolution of wavelet based estimates has a tendency to approach Welch estimates when the decomposition level is reduced. This is logical since the higher the decomposition level, the smaller the frequency band that is spanned by single wavelet packet node resulting in better frequency resolution. In this regard one can say that the performances of the WP approach can be made to operate between the strengths and weaknesses of Welch approach (minimum variance of the estimated PSD but poor frequency resolution) and periodogram (excellent frequency resolution but poor variance of the estimated PSD) without compromising too much on either of these metrics by merely increasing or decreasing the levels of decomposition.



**Figure 3.19** PSD estimates of single tone source according to different decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. This figure is included to describe how the effect of decomposition level on wavelet estimates with respect to Periodogram and Welch estimates. The number of samples in this experiment is 12800. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach.

### b) Comparison with windowed Periodogram and MTSE

Figure 3.20 and 3.21 show the impact of windowing on the reduction of side lobe level of the periodogram estimates. As it is clear from theoretical aspect, the introduction of window maintain the excellent frequency resolution of the simple periodogram and hence it is still better than the frequency resolution of wavelet based estimate with presented decomposition levels. However, as in figures 3.18 and 3.19, the same trend also appears here since the frequency resolution of wavelet based estimate tends to approach that of the windowed periodogram estimates as the wavelet packet decomposition level is increased. This means that there is a promise offered by wavelet based estimates as long as the required decomposition level can be fulfilled. The MTSE also offer better frequency resolution. In terms of side lobe suppression (rejection of power in the unoccupied band), windowed periodogram and MTSE clearly outperforms the wavelet based estimate in the mediocre transition band for the case of partial band course, the fact that the number of samples is finite (or rectangular windowing effect) as well as spectrum carving issue (discussed in chapter 6) are mainly suspected for the significant level of PSD at unoccupied band. With respect to the excellent performance of MTSE, which is caused by the employment of Slepian

sequence as the basis derivation of prototype filters and averaging process, the complexity issue in MTSE implied by Farhang-Boroujeny [16] should still be considered.



Figure 3.20 PSD estimates of single tone source according to different decomposition level of Daubechies-20 together with Thomson's MTSE and Periodogram using Hann window. The number of samples in this experiment is 12800.



**Figure 3.21** PSD estimates of single tone source according to different decomposition level of Daubechies-20 together with Periodogram using Hamming window and Blackman window. The number of samples in this experiment is 12800.



Figure 3.22 Periodogram and wavelet based estimates (Symlet 15, and Discrete Meyer) for single tone source. The number of samples in this experiment is 384. 7-level decomposition is used in wavelet based estimation.



**Figure 3.23** Welch approach and wavelet based estimates (Coiflet-5, Daubechies-15) for single tone source. The number of samples in this experiment is 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 7-level decomposition is used in wavelet based estimation.



Figure 3.24 Periodogram and wavelet based estimates (Daubechies 15, Symlet 15, and Discrete Meyer) for multi-tones source. The number of samples in this experiment is 12800. 7-level decomposition is used in wavelet based estimation.



Figure 3.25 Welch approach and wavelet based estimates (Coiflet-5, biorthogonal 3.9, and reverse biorthogonal 3.9) for multi-tones source. The number of samples in this experiment is 12800. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 7-level decomposition is used in wavelet based estimation

### B.3 Analysis for the case of small number of samples

As in the case of partial band source, we also tries to investigate the performance of wavelet based estimates along with periodogram and Welch approach for much smaller number of samples, which is also set to 384 here. This is described by figures 3.22 and 3.23. Since the reduction of number of samples from 12800 to 384 does not alter the size of each segment in the Welch approach, this reduction has no impact on the side lobe level. On the other hand both periodogram and wavelet based estimates introduce more leakage in 384 samples case. For the case of periodogram, the main reason for this is the smaller window size employed in order to take 384 samples from the received signal. Meanwhile, the deterioration of the wavelet based performance is likely to be caused by the rectangular windowing effect which becomes more obvious for smaller number of samples.

### B.4 Summary of Inferences

Based on the investigation on the estimates of single tone sources conducted in subsection B.1 to B.3, we can summarize our finding as follows:

- The variance of the wavelet based estimates with various decomposition level is between the variance of estimates based on periodogram and Welch approach.
- The frequency resolution of wavelet packet based estimates approach that of periodogram as the decomposition level is raised. On the other hand, it approaches the variance of Welch estimates as the decomposition level is reduced.
- More leakage in the estimates is introduced by both periodogram and wavelet based approach when the number of samples is significantly reduced.
- Varying the decomposition level of wavelet based estimates does not improve the rejection in the unoccupied band.

### C. Multi-Tones Source

### C.1 Analysis on various wavelet families

Figures 3.24 and 3.25 illustrate Periodogram, Welch as well as wavelet based estimates for the case of multi-tones source. The number of samples in these experiments is 12800 while the configuration of Welch method used here is the same as in the case of partial band and single tone source. Same wavelet families as in figure 3.6 and 3.7 are employed here with decomposition level of 7. Again, the performance of biorthogonal wavelets is much poorer than the orthogonal wavelet based estimates making them unsuitable candidates. The trends in multi-tones estimation are similar to those of single-tone estimation. There are no palpable differences in the performance of the orthogonal wavelet based estimates in terms of frequency resolution. In terms of variance of the estimated PSD, Discrete Meyer is slightly better than other orthogonal wavelet families.

The performance comparison of Fourier based periodogram and wavelet based estimates follow similar trends as in the case of single tone source. The performance of orthogonal wavelet based estimates is better than Welch approach estimates in terms of frequency resolution but they have comparable power suppression in unoccupied band. The reason for this is the same as in other sources namely the averaging of periodogram feature employed by Welch approach. Similar to the results found for the case of single tone source, simple periodogram has slightly better frequency resolution compared to orthogonal wavelet based approach. The large variance of estimated PSD issue inherent in periodogram estimate also appears here. Other similarities with the case of single tone source are also found when we compare the relationship of filter length with the performance of orthogonal wavelet based approach (not shown here).

### C.2 Analysis on decomposition level

Figures 3.26-3.29 illustrate the effect of the decomposition level on the wavelet based PSD estimates. In general, the results are similar to single tone source, namely the higher the wavelet packet decomposition level, the more similar the estimates to the simple periodogram estimates. On the other hand, lowering decomposition level would make the wavelet based estimates approach the estimates given by Welch method.



Figure 3.26 PSD estimates of multi-tones source according to various decomposition level of Daubechies-20. The number of samples in this experiment is 12800.



Figure 3.27 PSD estimates of multi-tones source according to different decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. This figure is included to describe how the effect of decomposition level on wavelet estimates with respect to Periodogram and Welch estimates. The number of samples in this experiment is 12800. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach.



Figure 3.28 PSD estimates of multi-tones source according to different decomposition level of Daubechies-20 together with Thomson's MTSE and Periodogram using Hann window. The number of samples in this experiment is 12800.



Figure 3.29 PSD estimates of multi-tones source according to different decomposition level of Daubechies-20 together with Periodogram using Hamming window and Blackman window. The number of samples in this experiment is 12800.

Similar inferences to single tone case are also obtained when the wavelet based estimates are compared to windowed periodogram and MTSE. While increasing the level of decomposition would make the frequency resolution of wavelet based estimates approach the frequency resolution

of windowed periodogram and MTSE, both MTSE and windowed periodogram outperforms wavelet based estimates in terms of power rejection in unoccupied bands.

### C.3 Analysis for the case of small number of samples

Figures 3.30 and 3.31 compare the wavelet packet estimates with Welch and Periodogram when the number of samples is low. In general, no significant influence is found on Welch based estimates after the number of samples is reduced from 12800 to 384. This is due to no alteration of the size of each segment. Both periodogram and wavelet based approach suffers more side lobes when the number of samples is reduced.

### C.4 Summary of Inferences

Based on the investigation on the estimates of multi-tones sources conducted in subsection C.1 to C.3, we find that most of findings found here are similar to the findings that are found in the case of single tone estimation, namely:

- The variance of the wavelet based estimates with various decomposition level is between the variance of estimates based on periodogram and Welch approach.
- The frequency resolution of wavelet packet based estimates approach that of periodogram as the decomposition level is raised. On the other hand, it approaches the variance of Welch estimates as the decomposition level is reduced.
- More leakage in the estimates is introduced by both periodogram and wavelet based approach when the number of samples is significantly reduced.
- Varying the decomposition level of wavelet based estimates does not improve the rejection in the unoccupied band.



Figure 3.30 Periodogram and wavelet based estimates (Symlet 15, and Discrete Meyer) for multi-tones source. The number of samples in this experiment is 384. 7-level decomposition is used in wavelet based estimation.

### D. Swept Tone Source

### D.1 Analysis on various wavelet families

As mentioned in section 3.4.1, 20 sweeps have been conducted to estimate the swept tone source. Here, we do not investigate the spectral estimate of the whole 20 sweeps in one snapshot

since it would not give different information from partial band source case. Instead, we investigate the snapshot of a portion of single sweep. The total number of samples is 12800 and this leads to 640 samples per sweep. The goal of this kind of observation is related to the possibility of using the wavelet based spectrum estimation for dynamic spectrum access, in which, the occupancy of particular frequency band is time varying.



Figure 3.31 Welch approach and wavelet based estimates (Coiflet-5, Daubechies-15) for multi-tones source. The number of samples in this experiment is 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 7-level decomposition is used in wavelet based estimation.

Figures 3.32-3.34 illustrate Periodogram and Welch approach as well as various wavelet families based estimate for five portions (snapshots) of a single sweep. For convenience, we address a single snapshot here as a *sub-sweep*. In these three figures, there are five sub-sweeps observed in a single sweep and each sub-sweep corresponds to 20% of the period of a single sweep and this leads to 128 samples per sub-sweep. As it is clear from the figures, the first sub-sweep corresponds to the left most lobe in those figures due to the fact that the sweep goes from the normalized low frequency  $(0.2\pi)$  to high frequency  $(0.8\pi)$ . The fifth snapshot, on the other hand corresponds to the right most lobe.

The configuration of Welch approach in figure 3.33 is exactly the same as in single tone and partial band source, namely 64 samples per segment with 50% overlapping between segments. Hamming window is applied on each segment before averaging.

As it can be seen from figures 3.32-3.34, the performance of biorthogonal wavelet is worse than its orthogonal counterparts and it does not even give the clear location of the five portions of the sweep. Among orthogonal wavelets, Discrete Meyer shows slightly better performance than others in terms of variance of the estimated PSD in the pass band (the band that is swept). However, in terms of transition band and unoccupied band power suppression, there is no clear performance difference among the five families of orthogonal wavelets.



**Figure 3.32** Periodogram and Discrete Meyer wavelet based estimates for a single sweep of swept tone source. Five portions of single sweep is captured (the most left lobe is the first 20% of the sweep; the most right lobe is the fifth 20% of the sweep). The number of samples in single sweep is 640 samples. 7-level decomposition is used in wavelet based estimation



Figure 3.33 Welch approach and wavelet based estimates (Symlet 15 and Daubechies-15) for a single sweep of swept tone source. Five portions of single sweep are captured (the most left lobe is the first 20% of the sweep; the most right lobe is the fifth 20% of the sweep). The number of samples in single sweep is 640 samples. 7-level decomposition is used in wavelet based estimation. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach.



**Figure 3.34** Coiflet-5 and Biorthogonal 3.9 wavelet based estimates for a single sweep of swept tone source. Five portions of single sweep is captured (the most left lobe is the first 20% of the sweep; the most right lobe is the fifth 20% of the sweep). The number of samples in single sweep is 640 samples. 7-level wavelet decomposition is used here.



**Figure 3.35** Three dimensional plot of energy of spectrum estimates using Daubechies-20 wavelet (2 sweeps, each of 640 samples)

When those five orthogonal wavelet based estimates are compared to periodogram and Welch estimates, some interesting results are found. In general, none of wavelet based estimates can surpass the performance of Welch approach. The use of Hamming window at each segment averaged by Welch approach clearly results in excellent side lobe suppression and extremely small variance of the estimated PSD. Moreover, the Welch approach still has comparable frequency resolution with that of orthogonal wavelet based estimates. On the other hand, the performance of orthogonal wavelet based estimates is quite comparable with the periodogram in terms of side lobe or unoccupied band power suppression with one advantage found in the use of wavelet based estimates due to their moderately smaller variance of the estimated PSD. It should be noted that even though the Welch approach seems much stronger in this kind of swept tone source cases, the number of samples that is needed to do some averaging might not be sufficient especially when the time duration of a sub-sweep or snapshot is extremely small. Figure 3.35 shows a 3-dimensional plot of Daubechies-20 wavelet estimate for 2 sweeps of swept tone source.

Figure 3.36 illustrates the effect of filter length on the performance of the wavelet based estimates for the five portions of a single sweep in the case of swept tone source. For clarity of expression, only two Daubechies wavelets are depicted in the figure. It is simple to conclude from the graph that the estimate based on wavelet with longer filter (Daubechies 20) has a much better performance both in terms of variance of the estimated PSD as well as side lobes or power suppression at unoccupied band. However, the frequency resolution of the estimate based on both Daubechies 20 is roughly the same.



Legend: Red => Daubechies 4 Black Circle => Daubechies 20

Figure 3.36 Wavelet based estimate for a single sweep of swept tone source (using Daubechies family with different filter length). Five portions of single sweep are captured (the most left lobe is the first 20% of the sweep, the most right lobe is the fifth 20% of the sweep). The number of samples in single sweep is 640 samples. 7-level wavelet decomposition is used here

### D.2 Analysis on decomposition level

In this part, a more elaborate investigation is provided on swept-tone source. Apart from the inquiry on the impact of decomposition level of the wavelet, we try to enrich the experiments by varying the size of a single sub sweep. Three different sizes of sub sweep are examined here. In the first case (shown in figure 3.37-3.40), the size of a single sub sweep is exactly the same as in part D.1, namely 128 samples. In the second case (shown in figures 3.41-3.46), a single sweep is divided into 10 portions (sub-sweeps) and each sub-sweep consists of 64 samples. For the sake of visibility, only first four or five sub-sweeps are displayed in these figures. Finally, figures 3.47-

3.52 illustrates the third case in which a single sweep is divided into 20 sub-sweeps. Each subsweep contains 32 samples. Again, only the first four or five sub-sweeps are shown in these figures. The investigation on different size of sub-sweeps is actually intended to illustrate how fast a snapshot is taken by the spectrum estimator. The smaller the size of sub-sweep, the faster the snapshot is taken.



Figure 3.37 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 wavelet based approach. In this result, the size of a single sub-sweep is 128 samples.

Based on figures 3.37 and 3.38 we can observe how the performance of wavelet based estimates with various decomposition levels is contrasted to simple periodogram and Welch approach for sub-sweep size of 128. In terms of the variance of the estimated power spectrum density (PSD), similar behavior found in single tone and partial band cases is observed here. For higher level of decomposition (11 in this case), the variance of the estimate is larger and the wavelet based estimate tends to be comparable to simple periodogram estimate. As the decomposition level is decreased, the variance of the wavelet based PSD estimates is decreased as well and this variance tends to approach the variance of Welch periodogram estimates. Apart from the variance of the estimated PSD, there is no clear difference found when the decomposition level of wavelet based estimates is varied. The side lobe suppression and transition band of wavelet based estimates for different decomposition level look similar. Meanwhile, figures 3.38 shows that the Welch estimates clearly have good side lobe suppression, extremely small variance and comparable resolution with the frequency resolution of orthogonal wavelet based estimates. The reason for its low side lobe and variance of the PSD is clearly the combination of Hamming windowing and averaging process. On the other hand, the performance of orthogonal wavelet based estimates is quite comparable with simple periodogram in terms of side lobe or power suppression in the uninhabited band. Figure 3.39 and 3.40 basically shows how windowing technique has successfully improved side lobe suppression in periodogram estimate and thus it outperforms the wavelet based approach with various decomposition levels especially in terms of side lobe suppression. The MTSE approach has significantly lowered the estimate variance and improved side lobes suppression thanks to the employment of Slepian Sequence as the basis for deriving the orthogonal prototype filters. However, it is very interesting to notice that the frequency resolution of MTSE is still comparable to wavelet based estimates for the case of sub-sweep size of 128 samples.



Legend: Black Star => Periodogram, Pink Dash => db20 (11-level decomposition) Green Circle => db20 (6-level decomposition), Blue Rectangle => Welch

Figure 3.38 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. In this result, the size of a single sub-sweep is 128 samples. This figure is included to describe how the effect of decomposition level on wavelet estimates with respect to Periodogram and Welch estimates. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach.



Legend: Red => Hann windowed periodogram

Figure 3.39 PSD estimates of swept tone source according to Thomson's MTSE and periodogram with Hann window. In this result, the size of a single sub-sweep is 128 samples.



Figure 3.40 PSD estimates of swept tone source according to periodogram with Hamming and Blackman window. In this result, the size of a single sub-sweep is 128 samples.



Figure 3.41 PSD estimates of swept tone source according to Thomson's MTSE and periodogram with Hann window. In this result, the size of a single sub-sweep is 64 samples.



Figure 3.42 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 wavelet based approach. In this result, the size of a single sub-sweep is 64 samples. First and second sub-sweeps are displayed here



Figure 3.43 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 wavelet based approach. In this result, the size of a single sub-sweep is 64 samples. Third and fourth sub-sweeps are displayed here



Figure 3.44 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. In this result, the size of a single sub-sweep is 64 samples. The overlap percentage and the length of each segment employed in Welch approach is 50% and 32 samples, respectively. Hamming window is used in this Welch approach. First and second sub-sweeps are displayed here.



Figure 3.45 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. In this result, the size of a single sub-sweep is 64 samples. The overlap percentage and the length of each segment employed in Welch approach is 50% and 32 samples, respectively. Hamming window is used in this Welch approach. Third and fourth sub-sweeps are displayed here.

The next step would be to observe how all of these estimates behave when the size of a subsweep is reduced to 64. Firstly, it can be observed in figure 3.41 how the frequency resolution of Thomson's MTSE approach has been very bad for the case of sub-sweep size of 64 samples and it is not comparable anymore to wavelet based estimates with various decomposition levels shown in figure 3.42 and 3.43. The wavelet based estimates still clearly illustrate distinguishable sub-sweeps as shown in those two figures. By considering the complexity of MTSE solution as mentiond by Farhang-Boroujeny in [16], the use of MTSE for investigating this kind of highly time varying source is questionable. Meanwhile, the frequency resolution of wavelet based estimates tends to be worse when the decomposition level is reduced. This is clear from figures 3.42 and 3.43. On the other hand, when the decomposition level is increased, the wavelet based estimates have frequency resolution similar to that of simple periodogram. This is shown in figures 3.44 and 3.45.



Figure 3.46 PSD estimates of swept tone source according to periodogram with Hamming and Blackman window. In this result, the size of a single sub-sweep is 64 samples.

Since the size of sub-sweep is reduced to 64, we also modify the segment size in the Welch approach from 64 to 32 in order to keep the presence of averaging process in this approach. It can be found from figure 3.44 and 3.45 that the use of smaller segment in Welch approach has resulted in worse frequency resolution. However, the use of Hamming windowing and averaging still keep the variance of estimated PSD and side lobe level remain low. Figure 3.41 and 3.46 generally shows the effect of windowing technique on simple periodogram. In terms of frequency resolution, the performance of periodogram with Hann, Blackmann and Hamming Windows is still comparable to that of wavelet based estimates with high level of decomposition (8-level or 11-level). However, periodogram with windowing offer better side lobe or suppression at the unoccupied band compared to wavelet based estimates with various decomposition levels.

The last case would be to observe the behavior of the estimates when the size of the subsweep is further reduced to to 32 samples. In this experiment, we modify the configuration of the Welch approach. Since, it is only 32 samples available as a snapshot, the segment size in Welch approach is reduced from 32 to 16 in order to keep the averaging process in the Welch approach. However, this segment size reduction has completely jeopardized the frequency resolution as it is shown in figures 3.47 and 3.48. This makes Welch unsuitable for estimating the swept tone source with 32 samples per sub-sweep. In general, the important lesson that can be acquired from the use of Welch periodogram is the fact that the number of available samples that is needed to do some averaging might be just simply not sufficient enough to get acceptable result especially when the time duration of a snapshot is extremely small.



**Figure 3.47** PSD estimates of swept tone source according to various decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. In this result, the size of a single sub-sweep is 32 samples. The overlap percentage and the length of each segment employed in Welch approach is 50% and 16 samples, respectively. Hamming window is used in this Welch approach. First and second sub-sweeps are displayed here.



Figure 3.48 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 together with Periodogram and Welch Estimate. In this result, the size of a single sub-sweep is 32 samples. The overlap percentage and the length of each segment employed in Welch approach is 50% and 16 samples, respectively. Hamming window is used in this Welch approach. Third and fourth sub-sweeps are displayed here.

Figures 3.47–3.52 generally show that the performance of all spectrum estimation techniques is worse when the size of a sub sweep is further reduced to 32 samples. For the case of wavelet based estimates, higher decomposition levels have resulted in better frequency resolution as it shown in figures 3.47-3.50. Similar to the case of sub-sweep size of 128 and 64 samples, higher decomposition level leads to frequency resolution similar to simple periodogram. The phenomena showing how all spectrum estimation techniques get worse when the snapshot size is extremely reduced is quite understandable for the case of swept tone source. Swept tone source basically simulates how a particular user occupies a certain frequency point at certain time point. This means that both high time and frequency resolution are required to get accurate power spectrum density snapshot of this kind of source. In fact, the concept of uncertainty principle expressed by Vitterli and Herley in [21] has clearly excluded the possibility to have both good time and frequency resolution in the same occasion. However, this kind of interference is quite good to be used as indicator to assess the performance of spectrum estimation techniques with respect to dynamic spectrum access.



Figure 3.49 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 wavelet based approach. In this result, the size of a single sub-sweep is 32 samples. First and second sub-sweeps are displayed here

### D.3 Summary of Inferences

Based on the investigation on the estimates of swept tone sources conducted in subsection D.1 to D.2, we can summarize our finding as follows:

- For different sub-sweep size, the wavelet based estimates with various decomposition level has comparable variance, frequency resolution as well as side lobe level with periodogram estimates.
- Periodogram has windowing techniques as additional weapons to suppress the side lobe level and this has resulted in more favorable estimates especially for larger size of subsweep.
- Welch based estimates get worse when the size of one sub-sweep is reduced. The reason for this issue is the fact that the size of each segment to be averaged is just simply too small especially for sub-sweep size of 32 samples.

• MTSE also performs badly for sweep tone sources when the size of sub-sweep is still moderate (64 samples) and when wavelet based estimates still have fair performance.



Figure 3.50 PSD estimates of swept tone source according to various decomposition level of Daubechies-20 wavelet based approach. In this result, the size of a single sub-sweep is 32 samples. Third and fourth sub-sweeps are displayed here



Figure 3.51 PSD estimates of swept tone source according to Thomson's MTSE and periodogram with Hann window. In this result, the size of a single sub-sweep is 32 samples.



Legend : Red Circle => Hamming windowed Periodogram, Black => Blackman windowed Periodogram

Figure 3.52 PSD estimates of swept tone source according to periodogram with Hamming and Blackman window. In this result, the size of a single sub-sweep is 32 samples

# **3.5 Remarks on the Comparative Analysis with Existing Techniques**

We would like to emphasize that we have no intention to say that one approach is better than the other when we compare our wavelet based approach to periodogram and Welch. It should be remember that we only employ one setting of Welch approach namely with segement length of 64 and overlap percentage of 50% (though we also employ segment length of 32 and 16 in swept tone case). There is still a great deal of possible parameter combinations of Welch approach that can end up with different estimation performance. As an example, when we say that we can vary the variance of wavelet based estimates by tuning the decomposition level, the similar way can also be done in Welch by varying the number of segments and the segment length. However, they cannot be simply compared since both of them are completely different mechanism. Our intention in the comparative analysis provided in section 3.4 is to give some tastes and flavors about how the wavelet based estimates behave and where its position with regard to conventional approach.

# **3.6 Summary of the Chapter**

In this chapter, the application of wavelet packet transform for spectrum estimation technique was proposed and investigated. Four classes of sources with different features and characteristics are used to gauge the operation of the developmental system and the results were compared with that of well-known periodogram and Welch estimates. The performance metrics used were variance and frequency resolution of the estimated PSD as well as side-lobe level or suppression at the unoccupied band. We also investigated the impact of decomposition level on the wavelet based estimates.

In general, it is easily found that orthogonal wavelet based spectrum estimates with various decomposition level tend to behave in between the performances of periodogram and Welch estimates especially in terms of variance of the estimated PSD and frequency resolution. When the decomposition level is increased, the variance of the orthogonal wavelet based estimate is higher but its frequency resolution gets better and the performance approaches that of periodogram. Meanwhile, the variance of the wavelet based estimated PSD and its frequency resolution approach those of Welch approach once the decomposition level is reduced. The Welch approach has

problems with tracking irregularities and locating sharp band sources due to its limited frequency resolution while the periodogram has problem with its large variance when it is used to estimate wide band sources. Thus, the pliability of wavelet packet based method to tune the variance and its frequency resolution by adjusting the decomposition level is advantageous especially if we consider the possibility to obtain multiple wavelet based estimates from different level of wavelet packet tree.

MTSE proposed by Thomson in [18], is quite promising for suppressing the variance of the estimates in wide band cases and it also has extremely good frequency resolution in narrow band cases. However, the variance of the estimate for the wide band cases is still worse than wavelet based estimates. Moreover, the complexity issue introduced in MTSE should be considered.

Of particular interest is the estimation of a swept tone source that varies with time. In this type of source, wavelet based estimates with various decomposition level has comparable variance, frequency resolution as well as side lobe level with periodogram. However, again, the periodogram has windowing techniques as additional weapons to suppress the side lobe level making it more favorable than wavelet based approach. It is also very interesting to note how the Welch based estimates gets worse when the size of one sub-sweep is reduced due to its too little segment size in order to do averaging process. This makes wavelet based estimates more competitive than Welch estimates when the window of opportunity available to collect sufficient samples for averaging is limited. Apart from Welch approach, MTSE also performs badly for sweep tone sources when the size of sub-sweep is still moderate (64 samples) and when wavelet based estimates still have fair performance.

The wavelet packet based approach gives all wavelet coefficients at all decomposition levels. The presence of all of these coefficients allows the possibility to obtain multiple estimates from different level of the tree with different degree of variance and frequency resolution, in one snapshot and one operation. This feature can be exploited to construct an adaptable and reconfigurable spectrum estimation mechanism. Since the wavelet packet based approach always operates between the strengths and the weaknesses of the periodogram and Welch approaches in terms of variance and the frequency resolution as well as given that it is impractical to employ both periodogram (for narrowband sources) and Welch (for wideband sources) estimation apparatus in one receiver. And also that it is meaningless to switch between these two techniques during the run time as there would be no apriori knowledge on whether the incoming signal is narrow band or wide band signal. The possibility of using a single wavelet based spectrum estimator block to deal with both narrowband and wideband sources looks desirable and attractive. Clearly this kind of flexibility offered by wavelet based spectrum estimation technique is of enormous advantage in a dynamic and time variant environment. This is the first point that can be inferred from this chapter.

We should not ignore, however, the possibility of Welch approach to tune the variance of the estimated PSD and the frequency resolution by employing different segment size and consequently, different number of segments. However, given the received samples, this theoretically requires different operation. On the other hand, in wavelet based approach, we may obtain multiple estimates from different level of wavelet packet tree in single run time (since wavelet packet coefficients at certain level are basically obtained from the wavelet coefficients at the previous level). Further research in complexity issues, however, still needs to be conducted and so far we cannot say that one technique is better than the other.

The wavelets used in this chapter were standard wavelets available in the Matlab toolbox. These wavelets were originally developed for applications such as image processing or encryption, and hence may not be suitable for spectrum estimation. Therefore, it is important to derive new and frequency selective wavelets that best suit the applicability of wavelet theory for spectrum estimation. This is the second point deduced from this chapter and we shall delve on this in the next chapter.

# CHAPTER 4 OPTIMAL DESIGN OF WAVELET FOR WAVELET BASED SPECTRUM ESTIMATION

The attributes of the wavelet packet based spectrum estimation greatly depend on the set of filter banks it uses. Choosing the right filter is a delicate task. The filters cannot be arbitrarily chosen and instead have to satisfy a number of constraints. These constrains are fundamental constraints for valid wavelet construction. They include orthogonality constraint, compact support, K-regularity and vanishing moments. With respect to spectrum estimation, additional constraints might be required. In the previous chapter, we found that commonly available wavelets are not frequency selective in nature. Therefore, the spectrum estimator based on these wavelets results in estimates with poor transition between the occupied band and unoccupied band. It would therefore be interesting to design new wavelets that best suit applicability for spectrum estimation.

The wavelet tool is actually a double edged sword. On one hand there is an enormous opportunity and scope for customization and adaptation. On the other hand there are no clear guidelines to choose the best wavelet for a given application. In this chapter, we focus on the design and development of a wavelet family that is maximally frequency selective in nature. We start the discussion by investigation of relationship between the properties of wavelet decomposition filter and the performance of the estimation (section 4.1). In this section, we generate different types of sources and develop important metrics to justify the quality of estimation on those sources. We later try to figure out the relationship between these metrics and the properties of wavelet decomposition filter via the computation of correlation coefficient. Based on the inferences drawn from the studies, in section 4.2, we try to formulate our design as an optimization problem. This optimization problem basically contains objective function and some budgets. Apart from constraints that are related to desired frequency response of the decomposition filter, there are other constraints on wavelet bases that should be considered in order to guarantee the designed wavelet is valid. Turning out that the optimization problem is non-convex susceptible to instabilities and inaccuracies, we reformulate the non-convex optimization problem into linear optimization problem, which is definitely convex. This is discussed in section 4.3. Hence, in order to do so, the optimization problem that is originally defined in terms of filter coefficients as variable constraint is defined in term of autocorrelation sequence of the filter coefficients. Hence, the solution would be in term of autocorrelation sequence as well. We need a special technique called spectral factorization discussed in section 4.4 in order to obtain the solution in term of filter coefficients. Section 4.5 delves on a method to simplify the problem and make it computationally more palatable. The convex optimization problem is solved by using semi definite programming tool called Sedumi. The performance of wavelet based spectrum estimator based on our optimal wavelet solution is discussed in section 4.6. The performance of our optimal wavelet based spectrum estimator is compared with that of standard wavelet based estimator as well as with that of conventional techniques such as Periodogram, in the same section. Section 4.7 summarizes the chapter.

# **4.1 Relationship between the properties of wavelet decomposition filter and the performance of the estimation**

In this section, the relationship between the properties of decomposition filter and the performance of the corresponding wavelet based estimates is investigated by running simulations that involve nine different types of sources occupying different band in the range of normalized frequency from 0 to  $\pi$ . Table 4.1 lists these nine types of sources. The wavelet packet decomposition level used in this investigation is 7.

To measure the performance of the wavelet based estimates, we develop some important metrics, namely:

- The average of power spectrum density (PSD) at the unoccupied band
- The maximum side lobe
- The PSD variance at the unoccupied band

• Frequency resolution of the estimate

Those four indicators are common for all sources involved in the investigation. Meanwhile, there are additional indicators that are available for all sources except for single tone source, namely:

- Width of the transition band
- The variance of the PSD at the occupied (active) band

The width of the transition band in the estimate basically illustrates the transition from the occupied band to the unoccupied band. Ideally, this width should be zero meaning there should be immediate decrease or discontinuity of the PSD curve in the border of occupied and unoccupied band.

 Table 4.1 Nine types of source that is used for the purpose of investigation on the correlation between the properties of wavelet decomposition filter and the performance indicator of the estimation

Name	Type of Source	Normalized Active Frequency
Type-A	Partial Band Source	[0.25π, 0.75π]
Type-B	Single Tone Source	0.5π
Type-C	Partial Band Source	[0.375π, 0.625π]
Type-D	Partial Band Source	[0.4375π, 0.5625π]
Type-E	Band Stop Source	$[0, 0.25\pi]$ and $[0.75\pi, \pi]$
Type-F	Band Stop Source	$[0, 0.375\pi]$ and $[0.625\pi, \pi]$
Type-G	Band Stop Source	$[0, 0.4375\pi]$ and $[0.5625\pi, \pi]$
Туре-Н	Multiple Partial Band Source	$[0, 0.2\pi], [0.4\pi, 0.61\pi]$ and $[0.82\pi, \pi]$
Type-I	Multiple Partial Band Source	$[0, 0.14\pi], [0.29\pi, 0.42\pi], [0.57\pi, 0.7\pi]$ and $[0.86\pi, \pi]$

Based on these metrics, we try to investigate which properties of the wavelet decomposition filter that has strong correlation with those metrics. The properties of the frequency response of decomposition filters to be investigated are as follows:

- The width of transition band
- The variance of the pass-band
- The variance of the stop-band

represented as follows:

• The average power at the rejection band (stop band) relative to the pass band

It can be easily found that those four indicators are commonly used to judge the quality of the filter. For each indicator of the wavelet decomposition filter and each indicator of the estimates, correlation coefficient is calculated in order to investigate the relationship between each pair of indicators. The formula for correlation coefficient of two random variables, *X* and *Y* can be

$$\rho_{X,Y} = \frac{Cov[X,Y]}{\sqrt{Var[X]Var[Y]}}$$
(4.1)

In (4.1), Cov[X,Y] denotes the covariance between random variables X and Y while Var[X] represents the variance of X. The correlation coefficient between each pair of indicators mentioned above is computed by collecting data from several orthogonal wavelet families namely Daubechies (db1 to db 20), Symlet (sym1 to sym20), Coiflet (coif1 to coif 5) and Discrete Meyer. For each wavelet, all indicators with respect to the decomposition filter and the estimates are collected. The correlation coefficient for each pair of indicators is calculated 100 times for each source. Later, the average and the standard deviation of the correlation coefficients,  $E[\rho]$  and  $\sigma_{\rho}$  respectively, are calculated. The results are all tabulated in Appendix A.1 (tables A.1.1 to A.1.9) which illustrate the average and standard deviation of 100 correlation coefficients computed for each pair of indicators in each source. For the case of single tone source, the correlation coefficients obtained from Haar (Daubechies-1) and Symlet-1 wavelets are excluded because the performance of the wavelet based estimates for single tone source using those two wavelets are exceptional and thus, they can be considered as outliers in the statistic world. The criteria that we use to describe the correlation between the two indicators are indicated in table 4.2.

Based on the analysis conducted (which are tabulated in A.1.1 to A.1.9), it can be inferred that:

- All four indicators chosen from the frequency response of the wavelet decomposition filters have strong influence on two indicators found in the estimates, namely **average of PSD at the unoccupied band** and **maximum side lobes**.
- In general, moderate correlation is also found between the four filter's indicators and the width of the transition band and PSD variance at the unoccupied band.
- The frequency resolution seems do not have correlation with the four filter's indicators. However, the frequency resolution of the estimation can be enhanced by increasing the decomposition level. Moreover, from the results found from the previous chapter, it can be learnt that the orthogonality of the wavelets used in the spectrum estimation approach seems to play important role on the frequency resolution of the estimate for the case of single tone sources.
- Unclear correlation is also found between the four filter's indicators and the PSD variance of the occupied band even though a quite significant relation is found between width of the transition band and PSD variance in the occupied band.

**Table 4.2** Criteria used to describe the correlation between the properties of wavelet decomposition filter and the performance indicator of the estimation. In this table,  $E[\rho]$  and  $\sigma_{\rho}$  represents the mean and the standard deviation of correlation coefficients between the two indicators, respectively

Criterion	Inference
$E[\rho] > 0.8$	Strongly correlated
$(0.8 > E[\rho] > 0.5)$ and $(\sigma_{\rho} < 0.1)$	Correlated
$(0.8 > E[\rho] > 0.5)$ and $(\sigma_{\rho} > 0.1)$	Correlated (with some note that the variance of the
	correlation is significant)
$(0.5 > E[\rho] > 0.3)$ and $(\sigma_{\rho} < 0.1)$	Weakly correlated
$(0.3 > E[\rho] > 0)$ and $(\sigma_{\rho} < 0.1)$	Uncorrelated
$(0.5 > E[\rho] > 0)$ and $(\sigma_{\rho} > 0.1)$	Not clear

In conclusion, the four indicators of the frequency response of the wavelet decomposition filters, namely the width of transition band, the variance of the pass-band, the variance of the stopband, and the average power at the rejection band (stop band) can be used as guides in the design of the wavelet for spectrum estimation. Orthogonality is another important factor in this wavelet design.



# 4.2 Stating Wavelet Design as an Optimization Problem

With respect to the design of wavelet for wavelet packet based spectrum estimation, the actual design problem is to design the wavelet filters that are used to compose the wavelet packet tree structure shown in figure 3.4, which is used as the basis of the spectrum estimator. The fundamental idea is about finding the filter coefficients of the low pass decomposition filter. Once the filter coefficient of the half band low pass filter is obtained, the filter coefficients of high pass decomposition filter can later be acquired through the use of Quadrature Mirror Relationship between the half band low pass filter can be expressed as optimization problem based on two sets of constraints namely the wavelet constraints and desired design constraints. The optimization problem can later be expressed as convex optimization problem allowing the use of convex optimization tools to obtain the solutions.

From common knowledge of the filter design, it is generally not possible to minimize the width of the transition band, the variance of the pass and stop band as well as the mean of the stop band simultaneously. As a result, we try to adopt Parks and McClellan equi-ripple design of FIR filter [31] as our basis. Figure 4.1 illustrates the magnitude response of the low pass filter being optimized. In the figure,  $\omega_p$  and  $\omega_s$ , denote normalized pass and stop band frequencies, respectively,  $[0, \omega_p]$  is called the pass-band,  $[\omega_s, \pi]$  is called the stop-band and  $[\omega_p, \omega_s]$  is the transition band B.  $\Delta$  is the maximum value of the tolerance or ripple, In general, Parks and McClellan equi-ripple design tries to minimize the ripple  $\Delta$  in figure 4.1 given the transition band  $B = \omega_s - \omega_p$  by using the so-called Remez Exchange algorithm [31].

However, in addition to Parks and McClellan proposal, we should also incorporate additional wavelet constraints into our design. As it is clear from [23], those additional contraints are regularity condition, double shift orthogonality, and the fact that the wavelet should be compactly supported allowing decomposition filter to have finite impulse response. These three constraints are mandatory for the design of valid wavelets. These constraints are presented in sub section 4.2.1-4.2.3 in term of low pass filter coefficient as the variable constraint.

# 4.2.1 Compact support or admissibility constraint

This constraint is necessary to ensure that the wavelet has finite non-zero coefficient and thus the impulse response of the wavelet decomposition filter is finite as well. According to [24], this property can be derived by simply integrating both sides of the two-scale equation in (2.46) as follows

$$\int_{-\infty}^{\infty} \varphi(t)dt = \int_{-\infty}^{\infty} \sum_{n} h[n]\varphi(2t-n)\sqrt{2}dt$$
$$\int_{-\infty}^{\infty} \varphi(t)dt = \sqrt{2}\sum_{n} h[n]\int_{-\infty}^{\infty} \varphi(2t-n)dt$$
$$\int_{-\infty}^{\infty} \varphi(t)dt = \sqrt{2}\sum_{n} h[n]\int_{-\infty}^{\infty} 0.5\varphi(2t-n)d(2t-n)$$
(4.2a)

By assuming u = 2t-*n*, equation (4.2a) can be further processed as follows:

$$\int_{-\infty}^{\infty} \varphi(t)dt = 0.5\sqrt{2}\sum_{n} h[n] \int_{-\infty}^{\infty} \varphi(u)d(u)$$
$$\int_{-\infty}^{\infty} \varphi(t)dt$$
$$= 0.5\sqrt{2}\sum_{n} h[n]$$
$$\int_{-\infty}^{\infty} \varphi(u)du$$

Finally we obtain the compactly supported wavelet constraint as:

$$\sum_{n} h[n] = \sqrt{2} \tag{4.2b}$$

It should be noted that the derivation that is given above is only possible if the scaling function is absolutely integrable and the integration of the scaling function is non-zero. Due to this fact, (4.2b) is also recognized as the wavelet existence constraint.

### 4.2.2 Paraunitary or double shift orthogonality constraint

The paraunitary or the orthogonality condition, which is one of the pivotal wavelet properties, actually emerges from the concept of multi-resolution concept defining the so-called two scale equation [23]. This constraint becomes a reason why it is possible to generate orthonormal wavelets and why it is possible to ensure perfect reconstruction of the decomposed signals.

This constraint is derived from the orthonormality between scaling function and its shifted version as follows:

$$\int_{-\infty}^{\infty} \varphi(t)\varphi(t-k)dt = \delta(k)$$
(4.3a)

Taking into account two-scale equation (2.46) in (4.3a) we can obtain:

$$\int_{-\infty}^{\infty} \sum_{n}^{n} h[n] \varphi(2t-n) \sqrt{2} \sum_{m}^{\infty} h[m] \varphi(2(t-k)-m) \sqrt{2} dt = \delta(k)$$

$$2 \sum_{n}^{\infty} h[n] \sum_{m}^{\infty} h[m] \int_{-\infty}^{\infty} \varphi(2t-n) \varphi(2(t-k)-m) dt = \delta(k)$$

$$2 \sum_{n}^{\infty} h[n] \sum_{m}^{\infty} h[m] \int_{-\infty}^{\infty} 0.5 \varphi(2t-n) \varphi(2(t-k)-m) d(2t) = \delta(k)$$

$$\sum_{n}^{\infty} h[n] h[n-2k] = \delta(k) \text{ for } k = 0, 1, \dots, (L/2) - 1$$
(4.3b)

Equation (4.3b) is called double shift orthogonality relation of the wavelet low pass filters impulse responses. In (4.3b), L illustrates the length of the low pass wavelet filter impulse response.

### 4.2.3 K-Regularity constraint

*K*-regularity is also an important measure for it determines the smoothness of the wavelets. It is defined in terms of the regularity index *K* which gives the number of times the wavelet is continuously differentiable [4]. Regularity index (*K*) also gives the number of zeros that the filter has at normalized frequency  $\omega = \pi$ . Therefore, the frequency response of the low pass filter should have the following structure [23]:

$$H(\omega) = \left(\frac{1 + e^{-j\omega}}{2}\right)^{K} Q(\omega) \text{ with } Q(\pi) \neq 0$$
(4.4a)

In (4.4a),  $Q(\omega)$  is a factor of  $H(\omega)$  that does not have any single zero at  $\omega = \pi$ . Having K number of zeros at  $\omega = \pi$  also mean that  $H(\omega)$  is K-times differentiable and its derivatives are zero when they are evaluated at  $\omega = \pi$ . By considering that:

$$H(\omega) = \sum_{n} h[n] \exp(-j\omega n)$$

The *k*-th order derivative of  $H(\omega)$  would be:

$$H^{(k)}(\omega) = \sum_{n} h[n](-jn)^{k} \exp(-j\omega n)$$
(4.4b)

The evaluation on (4.4b) at  $\omega = \pi$  would result in:

$$H^{(k)}(\pi) = \sum_{n} h[n](-jn)^k \exp(-j\pi n)$$

$$0 = \sum_{n} h[n](-j)^{k} (n)^{k} (e^{-j\pi})^{n}$$
$$0 = \sum_{n} h[n](n)^{k} (-1)^{n}$$

Therefore, we can formulate our K-regularity constraint in term of low pass filter coefficients as:

$$\sum_{n} h[n](n)^{k}(-1)^{n} = 0 \text{ for } k = 0, 1, 2, \dots K-1$$
(4.5)

### 4.2.4 Optimization Problem

The design goal is to generate filters with the desired transition band  $B = \omega_s \cdot \omega_p$  illustrated in figure 4.1 and minimum error  $\Delta$  even while satisfying the wavelet constraints. For a given the transition band *B*, the optimization problem can be formally stated as:

### $MINIMIZE \ \Delta$

under the constraints of:

$$\sum_{n} h[n] = \sqrt{2} \tag{4.2b}$$

$$\sum_{n} h[n]h[n-2k] = \delta(k) \text{ for } k = 0, 1, \dots, (L/2) - 1$$
(4.3b)

$$\sum_{n} h[n](n)^{k}(-1)^{n} = 0 \text{ for } k = 0, 1, 2, \dots, K-1$$
(4.5)

$$0 \le |H(\omega)|^2 \le \Delta \text{ for } \omega \in [\omega_s, \pi]$$
(4.6)

with fixed value of *L* and *K*.

From the optimization problem listed above, we can say that the design procedure actually comprises of defining a low pass FIR filter, satisfying the regularity, paraunitary, compact support and frequency selectivity conditions, expressed in the form of an impulse response h[n] or a transfer function H(z). In this case, regularity, paraunitary and compact supports are mandatory for the design of valid wavelets. Meanwhile, frequency selectivity represented by stop band constraint in (4.6) is an additional condition to the three mandatory ones. This actually shows the flexibility and adaptation properties of wavelet based approach. For our wavelet packet based spectrum estimation, stop band constraint represents the engineering requirement for frequency selectivity. Based on the specifications, other constraints may be incorporated.

We can also see that, for a filter of length *L*, the design problem is essentially solving *L* unknown filter coefficients from *L* linear equations. Of these *L* linear equations, L/2 equations come from the paraunitary and admissibility constraints, *K* equations come from the regularity or flatness constraint and the remaining L/2 - K conditions offer the room for maneuverability to establish the desired wavelet property such as frequency selectivity in our case. The larger the value of L/2-*K* is, the greater the degree of freedom for frequency selectivity and the greater the loss in regularity. There is therefore a trade-off between frequency selectivity and regularity. Wavelets such as the Daubechies family are maximally flat with regularity order *K*=L/2 and hence they are not frequency selective

It should be noted that we define the stop band constraint only within the range of  $\omega \in [\omega_s, \pi]$  due to the built in anti-symmetry of  $(/H(\omega))^2 - 1)$  about  $\omega = \pi/2$  in figure 4.1 [32]. The stop band constraint can be further expanded by performing some manipulation on  $|H(\omega)|^2$  term as follows:

$$|H(\omega)|^{2} = H(e^{j\omega})H(e^{-j\omega})$$
$$= \left(\sum_{n} h[n]e^{-j\omega n}\right) \left(\sum_{m} h[m]e^{j\omega m}\right)$$
$$|H(\omega)|^{2} = \sum_{n} \sum_{m} h[n]h[m]e^{-j\omega(n-m)}$$
(4.7)

Hence, the stop band constraint can be written as:

$$0 \leq \sum_{n} \sum_{m} h[n]h[m]e^{-j\omega(n-m)} \leq \Delta$$
  
for all  $\omega \in \left\lceil \omega_{s}, \pi \right\rceil$  (4.8)

From (4.3b) and (4.8), it is obvious that both double shift orthogonality and stop band constraints are non linear and non convex. Therefore, the whole optimization problem becomes non-linear as well as non-convex. It is possible to solve this non convex optimization problem but there is susceptibility that the solution found is only locally optimal instead of globally optimal [33]. In the other words, trying to solve non-convex optimization problem may not yield the optimal solution. Furthermore, they may warrant the usage of sophisticated solvers that can blow up the complexity.

# 4.3 Transformation of non-convex problem into linear optimization problem

Fortunately, it is possible to transform our non-convex optimization problem into linear optimization problem by transforming the variable constraint h[n] in the four constraints into new variable constraints, namely autocorrelation sequence  $r_h[l]$ , which is defined as:

$$r_{h}[k] = \sum_{m \in \mathbb{Z}} h[m]h[m+k]$$
(4.9)

By taking into account inherent symmetry property of the autocorrelation sequence at k=0, namely:  $r_h[l] = r_h[-l]$ , the four constraints in (4.2b),(4.3b),(4.5) and (4.6) can be rewritten.

# 4.3.1 Compact support or admissibility constraint

It is possible to represents the compact support constraint in (4.2b) in terms of autocorrelation sequence  $r_h[l]$  by first defining the autocorrelation sequence in a more precise way:

$$r_h[l] = \sum_{n=0}^{L-l-1} h[n]h[n+l] \text{ for } l \ge 0$$
(4.10)

In (4.10), *L* is the length of the FIR filter to be designed. It should be noted that due to symmetry property at l=0, we have:

$$r_h[-l] = r_h[l]$$
 for  $l < 0$  (4.11)  
The compact support constraint in (4.2b) can then be modified through the following way:

$$\sum_{n=0}^{L-1} h[n] = \sqrt{2}$$
$$\sum_{n=0}^{L-1} h[n] \sum_{m=0}^{L-1} h[m] = 2$$

By taking m = n + l, we have:

$$\sum_{n=0}^{L-1} \sum_{l=-n}^{L-n-1} h[n]h[n+l] = 2$$

Then, by reverse the order of the summation operation and by taking the fact that the impulse response of filter h[n] only has non-zero value at  $0 \le n \le L$ -1, we obtain:

$$\sum_{l=-(L-1)}^{L-1} \sum_{n=0}^{L-l-1} h[n]h[n+l] = 2$$
(4.12a)

The compact support constraint in (4.2b) can then be formulated as:

$$\sum_{l=-(L-1)}^{L-1} r_h \left[ l \right] = 2 \tag{4.12b}$$

Due to the double shift orthonormality constraints presented by (4.3b) and the fact that the symmetry property holds for autocorrelation sequence, (4.12b) can further presented as:

$$r_h[0] + 2\sum_{l=1}^{L-1} r_h[l] = 2$$

$$\sum_{l=1}^{L-1} r_h \left[ l \right] = \frac{1}{2} \tag{4.13}$$

Equation (4.13) becomes the compactly supported wavelet constraint in term of autocorrelation sequence  $r_h[l]$ .

### 4.3.2 Double shift orthogonality constraint

The double shift orthogonality constraint presented in (4.3b), can be reformulated in terms of autocorrelation sequence  $r_h[l]$  through the following way:

$$\sum_{m} h[m]h[m+2k] = r_h[2k] = \delta(k)$$
(4.14)

It should be noted that (4.14) is obtained by applying n - 2k = m on (4.3b). Hence the final double shift orthogonality constraint in term of autocorrelation sequence  $r_h[l]$  is:

$$r_{h}[2k] = \delta(k) = \begin{cases} 1, \text{ for } k = 0\\ 0, \text{ otherwise} \end{cases}$$
with  $k = 0, 1, \dots, \left\lfloor \frac{L-1}{2} \right\rfloor$ 

$$(4.15)$$

Again we make use of the symmetry property to limit the number of constraints. In contrast to (4.3b) which was non-convex, (4.15) consists of linear equalities and is also convex.

### 4.3.3 K-Regularity constrain

The regularity constraint can be reformulated in terms of autocorrelation sequence  $r_h[l]$  by taking the square of the absolute value of equation (4.4a) as follows:

$$\left|H(\omega)\right|^{2} = \left(\frac{1+e^{-j\omega}}{2}\right)^{K} \left(\frac{1+e^{j\omega}}{2}\right)^{K} \left|Q(\omega)\right|^{2}$$

$$(4.16)$$

By comparing (4.4a) to (4.16), we can find that requiring the transfer function  $H(\omega)$  to have *K* zeros at Nyquist frequency ( $\omega = \pi$ ) is equivalent to requiring  $|H(\omega)|^2$  to have 2*K* zeros at  $\omega = \pi$ . By considering the fact that  $|H(\omega)|^2$  is the Fourier transform of autocorrelation sequence of  $r_h[I]$ , we can represent the 2*k*-th order derivative of  $|H(\omega)|^2$  as follows:

$$\left(\left|H(\omega)\right|^{2}\right)^{(2k)} = \sum_{l} r_{h} \left[l\right] (-jl)^{2k} \exp(-j\omega l)$$
(4.17)

The evaluation on (4.17) at  $\omega = \pi$  would result in:

$$\left(\left|H(\pi)\right|^{2}\right)^{(2k)} = \sum_{l} r_{h}[l](-jl)^{2k} \exp(-j\pi l)$$

$$0 = \sum_{l} r_{h}[l](-j)^{2k} (l)^{2k} \left(e^{-j\pi}\right)^{l}$$

$$\sum_{l} r_{h}[l](l)^{2k} (-1)^{l} = 0$$
(4.18)

By considering the fact that the value of *l* for the case of FIR filter having length of *L* is  $-(L-1) \le l \le (L-1)$ , (4.18) can be represented as:

$$\sum_{l=-L+1}^{L-1} (-1)^l (l)^{2k} r_h [l] = 0 \text{ for } k = 0, 1, ..., K-1$$
(4.19)

In (4.19), *K* represents the regularity index of the wavelet that is required to be fulfilled. Making use of symmetry property of the autocorrelation sequence  $r_h[l]$  and the fact that the term with l=0 has zero value, (4.19) can be further simplified as:

$$\sum_{l=1}^{L-1} (-1)^l (l)^{2k} r_h[l] = 0 \text{ for } k = 0, 1, \dots, K-1$$
(4.20)

Equation (4.20) imposes the regularity constraint in term of autocorrelation sequence  $r_h[l]$ .

### 4.3.4 Stop band constraint

The stop band constraint presented in (4.8) can be represented in terms of autocorrelation sequence by defining n=m+k. Hence, (4.7) can be represented as:

$$\left|H\left(\omega\right)\right|^{2} = \sum_{m} \sum_{k} h[m]h[m+k]e^{-j\omega(k)} = \sum_{k} r_{h}\left[k\right]e^{-j\omega k}$$
(4.21)

Therefore, the stop band constraint in (4.8) can be written as:

$$0 \le \sum_{k} r_h \left[ k \right] e^{-j\omega k} \le \Delta \quad \text{for all } \omega \in \left[ \omega_s, \pi \right]$$
(4.22)

The autocorrelation sequence  $r_h[k]$  is symmetric at k=0, (i.e,  $r_h[l] = r_h[-l]$ ) [34]. Hence, (4.21) can be modified as:

$$|H(\omega)|^{2} = r_{h} [0] + \sum_{l} r_{h} [l] \left( e^{-j\omega l} + e^{j\omega l} \right)$$
  

$$|H(\omega)|^{2} = r_{h} [0] + 2\sum_{l} r_{h} [l] \cos(\omega l)$$
  
for  $l = 1, 2, ..., L-1$  and  $\omega \in [\omega_{s}, \pi]$   
(4.23)

Consequently, the stop band constraint in (4.22) is written as:

$$0 \le r_h \left[ 0 \right] + 2\sum_l r_h \left[ l \right] \cos(\omega l) \le \Delta$$
  
for  $l = 1, 2, ..., L - 1$  and  $\omega \in \left[ \omega_s, \pi \right]$  (4.24)

### 4.4 Spectral factorization and discretization on stop band constraint

The reformulated optimization problem consists of the objective function and constraints expressed in terms of autocorrelation sequence  $r_h[l]$  and therefore the optimal solution will also be in the autocorrelation domain. Since our interest is the filter coefficients h[n], we need to be able to obtain h[n] from  $r_h[l]$ . In general, there are infinite sequences of filter coefficients that can be obtained from given  $r_h[l]$ . However, by using spectral factorization algorithm proposed in [35], it is possible to obtain unique sequence of filter coefficients having minimum-phase property [36]. The spectral factorization of an autocorrelation sequence  $r_h[l]$  can be performed as long as the logarithmic function of its Fourier transform  $R_h(\omega)$ , which is nothing but  $|H(\omega)|^2$ , remains in  $\Re$ . To ensure this an additional constraint is enforced:

$$|H(\omega)|^2 \ge 0 \quad \text{for } \omega \in [0,\pi]$$

$$(4.25)$$

Based on (4.23), time domain representation of (4.25) can be written as:

$$r_{h}\left[0\right] + 2\sum_{l} r_{h}\left[l\right]\cos(\omega l) \ge 0$$
for  $l = 1, 2, \dots, L-1$  and  $\omega \in [0, \pi]$ 

$$(4.26)$$

Since we have infinite number of inequalities in the constraints defined in (4.26), a discretization process needs to be performed in the interval  $\omega \in [0,\pi]$ . This is necessary in order to make the optimization problem practically solvable using available optimization programs. One way to do discretization proposed in [36] is by replacing continuous variable  $\omega$  with a discrete variable  $\omega_i = i\pi/d$ , which is defined on a finite set i = [0, ..., d]. A typical value of *d* according to [36] is 15*n*. As a result, the constraint required for successful spectral factorization after applying the discretization process can be represented as:

$$r_{h}[0] + 2\sum_{l=1}^{L-1} r_{h}[l] \cos(i\pi l / d) \ge 0$$
  
for  $i = 0, 1, ..., d$  (4.27)

For clarity, we refer (4.27) as spectral factorization constraint.

Similar to the spectral factorization constraints, the number of stop band constraints defined in (4.24) is also infinite. Hence, discretization process is needed on stop band constraints in order to make the optimization problem becomes practically solvable. After implementing discretization, the stop band constraints in (4.24) can be rewritten as:

$$0 \le r_h \left[0\right] + 2\sum_{l=1}^{L-1} r_h \left[l\right] \cos(i\pi l / d) \le \Delta$$
  
for  $i = \left[\frac{\omega_s}{\pi}\right]^* d, \dots, d$  (4.28)

If we compare (4.27) with (4.28) and take into account the fact that we exploit the built in anti-symmetry of  $(/H(\omega))^2$ -1) about  $\omega = \pi/2$  in figure 4.1 [32], it can be easily found that spectral factorization constraint (4.27) will be automatically satisfied if the stop band constraint (4.28) is satisfied. In other word, the stop band constraint (4.28) is more stringent than the spectral factorization constraint (4.27).

In summary, our optimization problem in term of autocorrelation sequence  $r_h[l]$  can be defined as:

### $MINIMIZE \Delta$

under the constraints of:

1) 
$$\sum_{l=1}^{L-1} r_h[l] = \frac{1}{2}$$
 (4.13)

$$r_{h}[2k] = \delta(k) = \begin{cases} 1, \text{ for } k = 0 \\ 0, \text{ otherwise} \end{cases}$$
with  $k = 0, 1, \dots, \left| \frac{L-1}{2} \right|$ 
(4.15)

3) 
$$\sum_{l=1}^{L-1} (-1)^l (l)^{2k} r_h[l] = 0$$
 for  $k = 0, 1, \dots, K-1$  (4.20)

$$0 \le r_h \left[ 0 \right] + 2 \sum_{l=1}^{L-1} r_h \left[ l \right] \cos(i\pi l / d) \le \Delta$$
(4.28)  
for  $i = \left[ \frac{\omega_s}{\pi} \right] * d, \dots, d$ 

This optimization problem is clearly linear and convex.

### 4.5 Reformulating the optimization problem in $Q(\omega)$ function domain

As it can be noticed from section 4.4, the optimization problem is entirely linear. Hence, any linear programming technique can theoretically be used to solve this optimization problem. Since, in general, linear optimization problem is a subset of convex optimization problem, any tools or algorithm that commonly used to solve convex optimization problem should be capable to solve this optimization problem as well. However, a numerical problem may arise when the optimization problem formulated in section 4.4 to be practically solved by using the available convex or linear optimization program. This numerical problem is caused by the fact that matrix of the linear system composed by regularity constraint in (4.20) becomes ill-conditioned when the value of L and K increases [32][34]. In order to alleviate this problem, the optimization problem should be further reformulated in term of autocorrelation sequence  $r_q[l]$ . In the other word, the analysis of the optimization problem is shifted from the domain of  $H(\omega)$  into the domain of  $Q(\omega)$  based on (4.4a). In order to simplify the reformulation process, (4.16) is represented as:

$$|H(\omega)|^{2} = \left(\frac{1+e^{-j\omega}}{2}\right)^{K} \left(\frac{1+e^{j\omega}}{2}\right)^{K} |Q(\omega)|^{2}$$
$$|H(\omega)|^{2} = \left(\frac{\left(1+e^{j\omega}\right)\left(1+e^{-j\omega}\right)}{4}\right)^{K} |Q(\omega)|^{2}$$
$$|H(\omega)|^{2} = \left(\frac{\left(1+\cos(\omega)\right)}{2}\right)^{K} |Q(\omega)|^{2}$$
(4.29)

Hence, the time domain representation of (4.29) can be denoted as [34]:
$$r_{h}[l] = 2^{-2K} \sum_{n=-K}^{K} {\binom{2K}{n+K}} r_{q}[l-n] \qquad l=0, 1, \dots, L-1$$
(4.30)

As for  $r_h[l]$ , symmetry property also holds for autocorrelation sequence  $r_q[l]$ . The constraints now can be redefined in term of autocorrelation sequence  $r_q[l]$ .

1) Compactly Supported Wavelet constraint: The reformulation of compactly supported wavelet constraint in term of autocorrelation sequence  $r_q[l]$  is obtained by combining (4.2b) and (4.29) as well as setting  $\omega = 0$ . It can be noticed that:

$$\sum_{n} h[n] = H(\omega) \Big|_{\omega=0} = \sum_{n} h[n] \exp(-j\omega n) \Big|_{\omega=0} = \sqrt{2}$$

$$\left| H(\omega) \right|^{2} \Big|_{\omega=0} = 2$$
(4.31a)

By substituting (4.29) into (4.31a) we obtain:

$$\begin{split} &\left\{ \left( \frac{1+e^{-j\omega}}{2} \right)^{K} \left( \frac{1+e^{j\omega}}{2} \right)^{K} \left| Q(\omega) \right|^{2} \right\} \bigg|_{\omega=0} = 2 \\ &\left| Q(\omega) \right|^{2} \bigg|_{\omega=0} = 2 \\ &\left\{ r_{q} \left[ 0 \right] + 2 \sum_{l=1}^{L_{q}-1} r_{q} \left[ l \right] \cos(\omega l) \right\} \bigg|_{\omega=0} = 2 \end{split}$$

Hence, we finally come up with the compactly supported wavelet constraint in term of autocorrelation sequence  $r_q[l]$  as follows:

$$r_{q}\left[0\right] + 2\sum_{l=1}^{L_{q}-1} r_{q}\left[l\right] = 2$$
(4.31b)

2) Double Shift Orthogonality constraint: Based on (4.15) and (4.30), the double shift orthogonality constraint in term of autocorrelation sequence  $r_q[l]$  can be represented as:

$$r_{h}[2l] = 2^{-2K} \sum_{n=-K}^{K} {\binom{2K}{n+K}} r_{q}[2l-n] = \delta[l], \quad l = 0, 1, \dots, \left\lfloor \frac{L-1}{2} \right\rfloor$$

$$\sum_{n=-K}^{K} {\binom{2K}{n+K}} r_{q}[2l-n] = 2^{2K} \delta[l], \quad l = 0, 1, \dots, \left\lfloor \frac{L-1}{2} \right\rfloor$$
(4.32)

Equation (4.32) defines double shift orthogonality constraints in term of autocorrelation sequence  $r_q[l]$ .

3) Spectral Factorization constraint: The easiest way to reformulate the spectral factorization constraint in term of autocorrelation sequence  $r_q[l]$  is by combining (4.25) and (4.29) as follows:

$$|H(\omega)|^{2} \ge 0 \quad \text{for } \omega \in [0,\pi]$$

$$\left(\frac{1+\cos(\omega)}{2}\right)^{K} |Q(\omega)|^{2} \ge 0 \quad \text{for } \omega \in [0,\pi] \quad (4.33)$$

Since the term  $1 + \cos(\omega)$  in (4.33) is always positive, the constraint in (4.33) can be simplified as:

$$|Q(\omega)|^2 \ge 0 \quad \text{for } \omega \in [0,\pi]$$

$$(4.34)$$

By taking into account discretization in the interval  $\omega \in [0, \pi]$  as already mentioned in section 4.4, the spectral factorization constraint in term of autocorrelation sequence  $r_q[l]$  can be defined as:

$$r_{q}[0] + 2\sum_{l=1}^{L_{q}-1} r_{q}[l] \cos(i\pi l / d) \ge 0 \text{ for } i = 0, 1, ..., d$$
(4.35)

It is clear from (4.4) that since  $Q(\omega)$  has K zeros less than  $H(\omega)$ , the length of the filter q[n] would be  $L_q = L - K$ .

4) Stop band constraint: Similar to the case of spectral factorization constraints, the stop band constraints in term of autocorrelation sequence  $r_q[l]$  is obtained by combining (4.7), (4.8) and (4.29) as follows:

$$0 \le |H(\omega)|^2 \le \Delta \quad \text{for } \omega \in [\omega_s, \pi]$$
$$0 \le \left(\frac{1 + \cos(\omega)}{2}\right)^K |Q(\omega)|^2 \le \Delta \quad \text{for } \omega \in [\omega_s, \pi]$$

By taking into account discretization in the interval  $\omega \in [\omega_s, \pi]$  as already mentioned in section 4.4, the stop band constraint in term of autocorrelation sequence  $r_q[l]$  can be defined as:

$$0 \le \left(\frac{1 + \cos(i\pi / d)}{2}\right)^{K} \left(r_{q}\left[0\right] + 2\sum_{l=1}^{L_{q}-1} r_{q}\left[l\right]\cos(i\pi l / d)\right) \le \Delta$$
  
for  $i = \left\lceil \frac{\omega_{s}}{\pi} \right\rceil^{*} d, \dots, d$  and  $L_{q} = L - K.$  (4.36)

It is clear from equation (4.29) that when the optimization problem is expressed in term of autocorrelation sequence  $r_q[l]$ , the necessity for  $|H(\omega)|^2$  to have 2K zeros at  $\omega = \pi$  has been imposed implicitly. Therefore, the regularity constraints are not explicitly expressed when the optimization problem is conducted in  $Q(\omega)$  domain.

Similar to the case of  $H(\omega)$  domain in section 4.4, we find that the spectral factorization constraint in (4.33) and (4.35) will be automatically satisfied if the stop band constraint (4.36) is satisfied by considering the fact that we take into account the built in anti-symmetry of  $(/H(\omega))^2$ -1) about  $\omega = \pi/2$  in figure 4.1 [32]. In other word, the stop band constraint (4.36) is more stringent than the spectral factorization constraint (4.35).

In summary, our optimization problem in term of autocorrelation sequence  $r_q[l]$  can be defined as:

#### $MINIMIZE \ \Delta$

under the constraints of: 1)  $r_q [0] + 2 \sum_{l=1}^{L_q - 1} r_q [l] = 2$  (4.31b) 2)  $\sum_{n=-K}^{K} {2K \choose n+K} r_q [2l-n] = 2^{2K} \delta[l]$  (4.32) for  $l = 0, 1, \dots, \lfloor \frac{L-1}{2} \rfloor$  (4.32) 3)  $0 \le \left(\frac{1+\cos(i\pi/d)}{2}\right)^K \left(r_q [0] + 2 \sum_{l=1}^{L_q - 1} r_q [l] \cos(i\pi l/d)\right) \le \Delta$ for  $i = \left\lceil \frac{\omega_s}{\pi} \right\rceil^* d, \dots, d$  and  $L_q = L - K$ . (4.36)

Once we find the optimal autocorrelation sequence  $r_q[l]$ , the spectral factorization is employed in order to derive the optimal sequence q[l] from  $r_q[l]$ . Finally, the optimal wavelet low pass filter coefficients can be computed using the time domain equivalent of (4.4a) [34]:

$$h[l] = 2^{-K} \sum_{k=0}^{K} {K \choose k} q[l-k]$$
(4.37)

## 4.6 Results and Analysis

## 4.6.1 Solving the Convex Optimization Problem

Since the optimization problem illustrated in section 4.5 is linear optimization problem and thus it is also convex optimization problem, any linear programming tools as well as convex optimization solver can be used to solve this problem. In this case, we choose SeDuMi [37] as generic Semi Definite Programming (SDP) solvers to solve the convex optimization problem listed in section 4.5. In addition to SeDuMi, we also incorporate Yalmip toolbox [38] in order to allow the optimization problems to be expressed in higher level language. We have used Yalmip to describe our optimization problem and SeDuMi solver to obtain the optimal solution for given parameters. At the end of the filter design process, the coefficients of the analysis low pass filter will be generated. From the analysis low pass filter (LPF) h[n], the high pass filter (HPF) g[n] can be obtained through the Quadrature Mirror Filter Banks (QMF) equations illustrated by (2.53). And from these set of filters, the wavelet packet based spectrum estimator structure can be realized. Figure 4.2 illustrates the flow chart describing the design process. From this figure, it is clear that the design process can be divided into two main parts namely analytical part and numerical part. In analytical part, we try to modify our non-convex problem into convex problem followed by the conversion of the expression from autocorrelation  $r_h[l]$  domain into autocorrelation  $r_a[l]$  domain. The numerical part basically tries to solve the convex problem in term of variable constraints  $r_o[l]$ . After that, another analytical process is performed in order to derive optimum low pass filter coefficients h[n] from sequences q[n], which is obtained by applying spectral factorization on  $r_a[l]$ .



Figure 4.2 Flow chart of the optimum wavelet design process for wavelet packet based spectrum estimation

We use the spectral factorization algorithm that is proposed by Stephen Boyd from Stanford University [35]. From given autocorrelation sequence, this spectral factorization algorithm tries to derive filter coefficients with length L having minimum phase property. We decide to use this spectral factorization algorithm since a filter having minimum phase property is definitely

stable. We would like to emphasize that as long as the spectral factorization constraint (4.33) is satisfied, it is guaranteed that we can obtain at least one solution of filter coefficients q[n] [35].



Figure 4.3 Frequency response of Daubechies-15, Coiflet-5 and the designed wavelet low pass (LPF) and high pass (HPF) filter with L=30, K=7,  $B=0.2\pi$ 

## 4.6.2 Comparison between the designed wavelet and standard wavelet family

#### A. Frequency and Impulse Response of Designed Filter

In this part, we tries to present illustration about the frequency response of our optimally designed wavelet low pass and high pass filter. We consider examples with filter lengths L=30 and L=40. Indeed it is possible to consider filters with other lengths too. In the first example shown in figure 4.3, the frequency response of the designed wavelet filters is compared with Daubechies and Coiflet wavelet filters. All of these wavelet filters have filter length of 30. In this figure, *K*-regularity index of 7 and transition band (*B*) of  $0.2\pi$  is applied on the designed wavelet filters. From figure 4.3, it can be found that our proposed wavelet filters have better frequency selectivity than its Daubechies and Coiflet counterparts. A small price however is paid in terms of the small ripples introduced in the side lobe. However, the frequency response of our proposed wavelet filter is generally still more preferable. Figure 4.4 presents similar comparison for L=40. In this figure, only the frequency response of our optimal wavelet and Daubechies-20 filters are displayed.

Figure 4.5 and 4.6 describes the impulse response of the high and low pass filters of the optimally designed wavelets for L=30 K=7  $B=0.2\pi$  and L=40 K=8  $B=0.2\pi$ , respectively. The coefficients of the designed wavelet filter for L=30 K=7  $B=0.2\pi$  and L=40 K=8  $B=0.2\pi$  are presented in appendix A.2 (tables A.2.1 and A.2.2, respectively).



Figure 4.4 Frequency response of Daubechies-20 and the designed wavelet low pass (LPF) and high pass (HPF) filter with L=40, K=8, B=0.2 $\pi$ 



Figure 4.5 Impulse response of the designed optimal wavelet filter with length =30, K-regularity = 7, overall transition band =  $0.2\pi$ .



Figure 4.6 Impulse response of the designed optimal wavelet filter with length = 40, K-regularity = 8, overall transition band =  $0.2\pi$ .



**Figure 4.7** Estimates of Partial Band Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 12800

## **B.** Evaluation of Spectrum Estimator performance

Next, we try to examine the performance of spectrum estimation based on our designed wavelet. For this purpose, three types of sources are considered. Two of them have been already introduced in chapter 3, namely partial band source and single tone source. A partial band source is constructed to occupy the normalized frequency band of  $0.25\pi$ - $0.75\pi$ . The single tone source has all of its energy at one frequency and it sits right in the middle of the range spanned by the wavelet based spectrum estimation, namely at  $0.5\pi$ . The third source is the new source introduced here as multi-band source. In this kind of source, three active bands are constructed to occupy the normalized frequency bands of  $0.08\pi$ - $0.19\pi$ ,  $0.47\pi$ - $0.58\pi$ , and  $0.86\pi$ - $0.97\pi$ , respectively.



**Figure 4.8** Estimates of Single Tone Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 12800

## B.1 Partial Band source

Figure 4.7 illustrates how spectrum estimation with the newly designed wavelet compares with estimates based on standard wavelet family for partial band case. Here, the number of samples is set to 12800. The specifications for the optimal wavelet are L (length) = 30, K (regularity index) = 7 and B (transition bandwidth) =  $0.2\pi$ . It is clear from the figure that the optimal wavelet outperforms Daubechies, Coiflet and Symlet wavelets of the same length. The improvements are with regard to frequency selectivity and the sharp transition between occupied band and unoccupied band. This is logical since our wavelet filter is specifically designed to have optimum frequency selectivity.

## B.2 Single Tone source

On the other hand figure 4.8 illustrates that, in single tone source, no significant difference is found between the performance of the estimator based on our designed wavelet and that based on standard wavelets. For extremely narrow band source like single tone source, the frequency resolution issue is more related to decomposition level rather than the frequency selectivity of the response of decomposition filter. This is why there is no perceivable difference between the results when different wavelets are used.

## B.3 Multiple-Bands source

The newly developed wavelets perform better than existing standard wavelets of comparable lengths in estimating multiple narrower band source shown in figure 4.9. Due to better frequency selectivity in decomposition filter, our designed wavelet offers greater side lobe suppression compared to standard wavelet families. This side lobe appears due to spectrum carving effect produced by iterative decomposition process when the frequency response of decomposition filter is not frequency selective enough. Since the spectrum carving issue is extremely important in our wavelet based spectrum estimation proposal, chapter 6 would discuss this issue. Other noticeable advantages of designed wavelet compared to standard wavelet families are in terms of frequency selectivity of the estimates and sharper transition between occupied and unoccupied bands.



**Figure 4.9** Estimates of Multiple Band Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 12800

## B.4 Estimation for small number of samples

The next investigation is to find out how the wavelet based spectrum estimator performs for much smaller number of samples. The goal of this investigation is to examine our proposed estimators in dynamic spectrum environment where the speed of estimation is an issue and thus the number of samples might be limited. For this purpose, we consider the three types of source mentioned before for 1152 samples and 384 samples. These are depicted in figures 4.10-4.12 for the case of 1152 samples and in figures 4.13-4.15 in the case of 384 samples. In general, it can be found that the reduction of samples space has resulted in poorer side lobe level and higher variance of estimated power spectral density (PSD) both in the occupied and non unoccupied band. However, it is also found from those six figures that the spectrum estimation based on our designed wavelet performs better than that based on standard wavelets in term of transition from occupied to unoccupied band (see figures 4.10 and 4.13) and side lobe level (see figures 4.12 and 4.15). The key reason for the optimal designed wavelet based approach to have lower side lobe level in the

estimation of the multi band source is better resistance to the spectrum carving effect (to be discussed in chapter 6).



Normalized Frequency (x  $\pi$  rad/sample)

**Figure 4.10** Estimates of Partial Band Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 1152



**Figure 4.11** Estimates of Single Tone Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 1152



**Figure 4.12** Estimates of Multiple Band Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 1152



**Figure 4.13** Estimates of Partial Band Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 384



**Figure 4.14** Estimates of Single Tone Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 384



Figure 4.15 Estimates of Multiple Band Source based on Coiflet-5, Daubechies-15, Symlet-15 and the designed optimal wavelet filter with length=30, K-regularity = 7, overall transition band =  $0.2\pi$ . The wavelet decomposition level used here is 7. The number of samples in this experiment is 384

Apart from higher variance produced by all wavelet based estimator, the results for single tone source shown in figures 4.11 and 4.14 tell the same story as in the case of 12800 samples. No significant difference is found between the performance of spectrum estimation based on designed wavelet and that based on standard wavelet. This, once again, underlines the lack of impact of the frequency selectivity of decomposition filters on the estimation of extremely narrow band source.



**Figure 4.16** Detection and false alarm probability of spectrum estimation based on various wavelet families. In this scenario, the length of wavelet decomposition filter is 30, the wavelet decomposition level is 7 and the sample space is of size 12800. The K-Regularity of the designed wavelets with SDP is 7 with a normalized transition band of  $0.2\pi$ .

## C. Evaluation of Receiver Operating Characteristic

To better gauge the system performance of the spectrum estimator based on optimally designed wavelet in comparison with standard wavelet, the receiver operating characteristic (ROC) is used as the second figure of merit. To obtain the probability of detection  $(P_d)$  and false alarm  $(P_{fa})$ , we divide the normalized frequency range  $[0,\pi]$  into 128 equal bands (or frequency bins). Each bin is occupied by 1 source meaning that we have overall 128 sources. These 128 sources are randomly activated / deactivated and the  $P_d$  and  $P_{fa}$  are calculated for each given threshold out of total 100 experiments. The way the source activation and deactivation is conducted is similar to the activation and deactivation of sub carriers in OFDM transmission. The threshold is varied manually from -3dB to -15dB. The number of samples here is 12800. An active source would have around -2.1 dB power. Figure 4.16 depicts the  $P_d$  and  $P_{fa}$  as a function of threshold level which clearly underlines the superiority of the newly designed wavelet in relation to other wavelet families of the same filter length. The frequency selectivity inherent in the proposed wavelet has allowed spectrum estimator built on it to have much better  $P_d$  and  $P_{fa}$  for all thresholds in comparison to Daubechies, Symlet and Coiflet based estimators. Similar result is also found when the size of sample space is reduced to 384 as shown in figure 4.17. Here the spectrum estimator based on newly designed wavelet moderately outperforms that based on standard wavelets. Finally, the receiver operating characteristic (ROC) shown in figure 4.18 gives more justification on the superiority of the estimator based on the designed wavelet.



**Figure 4.17** Detection and false alarm probability of spectrum estimation based on various wavelet families. In this scenario, the length of wavelet decomposition filter is 30, the wavelet decomposition level is 7 and the sample space is of size 384. The K-Regularity of the designed wavelets with SDP is 7 with a normalized transition band of  $0.2\pi$ .



**Figure 4.18** Receiver operating characteristic of spectrum estimation based on various wavelet families. In this scenario, the length of wavelet decomposition filter is 30, the wavelet decomposition level is 7 and the sample space is of size 384. The K-Regularity of the designed wavelets with SDP is 7 with a normalized transition band of  $0.2\pi$ .



**Figure 4.19** Detection and false alarm probability of spectrum estimation based on newly designed wavelet with variations on filter lengths. In this scenario, the wavelet decomposition level is 7 and the sample space is of size 12800. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 7 with a transition band of  $0.2\pi$ .



**Figure 4.20** Detection and false alarm probability of spectrum estimation based on newly designed wavelet with variations on filter lengths. In this scenario, the wavelet decomposition level is 7 and the sample space is of size 384. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 7 with a transition band of  $0.2\pi$ .

The key point that makes it possible is to play with *K*-Regularity. In Daubechies and Symlet wavelet families, all of *L* equations provided by *L* impulse response in filter with length of *L* has been exploited to provide double shift orthogonality (requiring L/2 equations) and L/2 regularity index (also requiring L/2 equations). Hence, there is no degree of freedom that remains in order to be exploited for providing frequency selectivity feature. On the other hand, it is possible to have regularity index of K < L/2 in our designed wavelet and this has provide L/2-K equations as degree of freedom which is basically used by the optimization tool to minimize the ripple in pass band and stop band given certain transition band as it is shown in figure 4.1.



**Figure 4.21** Receiver operating characteristic of spectrum estimation based on newly designed wavelet with variations on filter lengths. In this scenario, the wavelet decomposition level is 7 and the sample space is of size 384. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 7 with a transition band of  $0.2\pi$ .

## **4.6.3 Other Evaluation and Studies**

In addition to the results presented in the previous section, the impact of some parameters on receiver operating characteristic is studied based on the same experiment setup employed in part C of sub-section 4.6.2. In figure 4.19, the results shows that for a given regularity order, the longer the decomposition filters, the better the  $P_d$  and  $P_{fa}$  of the estimates. This is reasonable since we have more degree of freedom to minimize the pass band and stop band ripple. Likewise, for a given filter length, lower K regularity index results in greater degree of freedom available to minimize the pass/stop band ripple yielding better performance results. Figure 4.20 generally shows that the impact of different decomposition filter length on the  $P_d$  and  $P_{fa}$  of the estimates is less significant for much smaller number of samples. However, it is still clearly illustrated that longer decomposition results in better performance. This fact is also justified by receiver operating characteristic (ROC) shown in figure 4.21.

Figure 4.22 describes the influence of transition band variation on the detection and false alarm probability. The result basically exemplifies the importance of frequency selectivity on the quality of the estimates. Here, narrower transition band produces lower false alarm and higher detection probability which is valid since narrower transition band means better frequency selectivity and the use of wavelet filter with better frequency selectivity would theoretically leads to spectrum estimator with better performance. Figure 4.23 shows the same phenomenon for smaller number of samples (384 samples). The impact of varying transition band on the  $P_d$  and  $P_{fa}$  of the estimates is less significant here. However, it is still obvious that narrow transition band of

the decomposition filter corresponds to better performance of the spectrum estimator. This is justified by the receiver operating characteristic (ROC) shown in figure 4.24.



Figure 4.22 Detection and false alarm probability of spectrum estimation based on newly designed wavelet with variations on transition band. In this scenario, the length of wavelet decomposition filter is 40, the wavelet decomposition level is 7 and the sample space is of size 12800. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 6.



Figure 4.23 Detection and false alarm probability of spectrum estimation based on newly designed wavelet with variations on transition band. In this scenario, the length of wavelet decomposition filter is 40, the wavelet decomposition level is 7 and the sample space is of size 384. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 6.



**Figure 4.24** Receiver operating characteristic of spectrum estimation based on newly designed wavelet with variations on transition band. In this scenario, the length of wavelet decomposition filter is 40, the wavelet decomposition level is 7 and the sample space is of size 384. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 6.



Figure 4.25 The impact of varying number of samples on detection and false alarm probability. In this scenario, the length of wavelet decomposition filter is 40 and the wavelet decomposition level is 7. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 8 with a normalized transition band of  $0.3\pi$ .



**Figure 4.26** The impact of varying number of samples on receiver operating characteristic. In this scenario, the length of wavelet decomposition filter is 40, the wavelet decomposition level is 7. The K-Regularity of the designed wavelets with Semi Definite Programming (SDP) is 8 with a normalized transition band of  $0.3\pi$ .



**Figure 4.27** Detection and false alarm probability of spectrum estimation based on periodogram, Welch approach and newly designed wavelet with filter lengths of 40, *K*-regularity index of 8 and transition band  $0.3\pi$ . In this scenario, the wavelet decomposition level is 9 and the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 128 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated.

Figure 4.25 describes the impact of varying number of samples on detection and false alarm probability. As it is clear from the figure, we should aware that the quality of the performance drops for smaller number of samples. This obviously shows that the size of snapshot windows does have an impact on the performance of wavelet based spectrum estimation. The figure also shows that the smaller the number of sample, the larger the drop in performance quality. The receiver operating characteristic in figure 4.26 justifies the inference.

# 4.6.4 Comparison between the designed wavelet based spectrum estimators, Periodogram and Welch approach

#### A. Investigation on Detection and False Alarm Probability with varying thresholds level

In this sub-section, we try to compare the performance of the spectrum estimator based on the designed wavelet with that of periodogram and Welch approach. With regard to the designed wavelet used here, the length of the decomposition filter is 40 with K-regularity index of 8 and transition band of  $0.3\pi$ . The wavelet decomposition level is 9. The sample space in this experiment is of size 384 because we would like to assess the performance for few numbers of samples. Though it is certainly possible to obtain the variation of the performance by using different parameters of the designed wavelet, we just fix the parameters here in order to obtain rough view about the position of our wavelet based spectrum estimators along with periodogram and Welch. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. The setup of this experiment used here is similar to the setup in part C of sub-section 4.6.2. We basically divide the normalized frequency range  $[0,\pi]$  into 128 frequency bins.



Figure 4.28 Detection and false alarm probability of spectrum estimation based on periodogram, Welch approach and newly designed wavelet with lengths of 40, *K*-regularity index of 8 and transition band  $0.3\pi$ . In this scenario, the wavelet decomposition level is 9 and the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 64 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated. Each source occupies 2 frequency bins

In figure 4.27, 128 sources occupy 128 frequency bins in the normalized frequency range of  $[0,\pi]$ . These 128 sources are randomly activated / deactivated and the  $P_d$  and  $P_{fa}$  are calculated for each given threshold out of total 100 experiments. An active source would have around -2.1 dB power. The result shows in figure 4.27 generally exemplifies what has been found in chapter 3. The performance of our wavelet based approach is somewhere in between that of Welch and Periodogram. In most of threshold values, Welch approach offers better probability of detection but very poor probability of false alarm due to its poorer frequency resolution. Since the length of each segment employed in Welch approach is 64 samples which is only 1/6 of the number of samples, the averaging process introduced in Welch does not offer much significant improvement of the variance of estimated PSD when it is compared to periodogram. On the other hand, periodogram has better probability of false alarm but poor probability of detection due to higher variance of estimated PSD. This underlines some advantages offered by wavelet approach. In wavelet based estimates the performance can be adjusted by varying the parameters of the designed wavelet such as length of filter, transition band and especially decomposition level in the wavelet packet tree. Even though, wavelet based approach seems to be not competitive enough compared to periodogram in term of false alarm probability, the re-configurability and adaptability feature of wavelet based approach might be promising. The reason of poorer false alarm probability in wavelet based spectrum estimation is clearly the spectrum carving issue that might introduce side lobes as high as -9dB as it would be explained in chapter 6.



Figure 4.29 Detection and false alarm probability of spectrum estimation based on periodogram, Welch approach and newly designed wavelet with lengths of 40, *K*-regularity index of 8 and transition band  $0.3\pi$ . In this scenario, the wavelet decomposition level is 9 and the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 32 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated. Each source occupies 4 frequency bins

While in figure 4.27, 128 narrow band sources are randomly activated and deactivated along 128 frequency bins in the normalized frequency band of  $[0,\pi]$ , we try to investigate the performance of the three spectrum estimators for wider-band sources. In figure 4.28, 64 sources are randomly activated and deactivated over 128 frequency bins in the range of  $[0,\pi]$ . Each source occupies 2 frequency bins. As it can be seen from this figure, the performance of the periodogram, Welch approach and the spectrum estimation based on the design wavelet is better than that is

shown in figure 4.27. This means, the spectrum estimators generally perform better for wider band sources. This fact is also exemplified when the width of the band spanned by each source is increased to 4 and 8 frequency bins in figure 4.29 and 4.30, respectively. From figures 4.28-4.30, we also observe similar phenomenon to that is found in figures 4.27. The performance of wavelet based approach is basically in between that of periodogram and Welch.



**Figure 4.30** Detection and false alarm probability of spectrum estimation based on periodogram, Welch approach and newly designed wavelet with lengths of 40, *K*-regularity index of 8 and transition band  $0.3\pi$ . In this scenario, the wavelet decomposition level is 9 and the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 16 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated. Each source occupies 8 frequency bins

## B. Investigation on Receiver Operating Characteristic

In this part, we try to compare the receiver operating characteristic of the estimator based on the designed wavelet with that of periodogram and Welch approach. The sample space in this experiment is of size 384 and the setup of the experiment is similar to part A of sub-section 4.6.4. The setting of Welch approach is exactly the same as in the previous part. In figure 4.31, 128 sources occupying 128 frequency bins in the normalized frequency range of  $[0,\pi]$  are randomly activated /deactivated. From this figure, it is found that the performance of Welch approach is the poorest. We also found that wavelet based spectrum estimator with decomposition filter length of 40, K-regularity of 8, transition band of  $0.3\pi$  and decomposition level of 9 has poorer performance than periodogram for false alarm probability less than 0.25. When the length of decomposition filter is increased to 50 and the K-regularity is decreased to 7, the performance of wavelet based estimator is slightly improved. This is valid since increasing the length of the filter and decreasing the K-regularity would provide more degree of freedom for improving the frequency selectivity of the wavelet filter. From this stage, we might exploit the flexibility of wavelet based approach. Since it is possible to obtain the wavelet coefficients from all wavelet packet nodes at all levels within one snapshot and one operation, it is also reasonable to display the estimates from decomposition level less than 9 in the plots. In this case, we display the estimates based on wavelet with decomposition levels of 7 (the filter length, K-regularity and transition band are fixed to 50, 7 and  $0.3\pi$ , respectively) and we can see that the wavelet based approach with this setting outperforms periodogram for almost all value of false alarm probability. Once again, we would like to emphasize that the estimates based on wavelet approach with decomposition levels of 7 and 9 are obtained from the same snapshot and from single operation.



**Figure 4.31** Receiver operating characteristic of spectrum estimation based on periodogram, Welch approach and newly designed wavelet. In this scenario, the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 128 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated.

We now try to investigate the receiver operating characteristic of the estimators for wider band sources. In figure 4.32-4.34, the number of frequency bins occupied by each source is 2, 4, and 8 bins, respectively. From these three figures, it is clear that the wider the bandwidth of the source, the better the performance of the estimation. The most significant improvement, however, happens on Welch based estimation. While in figure 4.32, the receiver operating characteristic (ROC) of Welch approach is still clearly poorer than periodogram and wavelet based approach, figure 4.34 shows that the performance of Welch approach surpasses that of periodogram and wavelet based approach with decomposition level of 9. Again, in these three figures, we can simply reduce the decomposition level from 9 to 7 in order to improve the receiver operating characteristic of wavelet based approach.

The results shown in figures 4.31-4.34 emphasize the inference that is drawn from chapter 3. Welch approach performs worse for narrower band source due its poor frequency resolution. Once the bandwidth of the source to be estimated is increased, the drawback caused by limited frequency resolution is less significant and this results in dramatic improvement of the Welch ROC. Periodogram, on the other hand, still has to deal with the larger variance of the estimated PSD. This variance problem still has significant impact on the performance of periodogram estimates for wider band making the improvement is less significant compared to the one observed from the case of Welch.

It is important to learn that we should not make careless conclusions on the ROC of the spectrum estimations. The performance of every method clearly depends on the setting of the estimators. For example, the size of the segment employed in Welch method is clearly vital and different size would end up with completely different performance. However, at least we can figure out that increasing the size of each segment would make the performance of Welch method approaches that of periodogram assuming the use of the same window. Similar case also holds for

wavelet based spectrum estimation. The use of different decomposition level while keeping the same wavelet specification (filters length, regularity index and transition band) may results in different ROC. Varying the decomposition filter length, regularity index and transition band would results in even more variations in performance. Even though, it is irrelevant to make simple justification among the three techniques, we might at least highlight the advantage of wavelet based spectrum estimation here. The possibility to obtain wavelet coefficients from all wavelet packet nodes at all levels provides multiple estimates with different trade off. This is done based only on one snapshot and through one-time operation.



Figure 4.32 Receiver operating characteristic of spectrum estimation based on periodogram, Welch approach and newly designed wavelet. In this scenario, the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 64 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated.

## 4.7 Summary and Conclusion of the Chapter

In this chapter, we presented the design of optimum wavelets which have greater frequency selectivity than common wavelet families such as Symlet, Coiflet and Daubechies. The need for frequency selectivity feature was deduced after an exhaustive analysis on the relationship between the property of wavelet decomposition filter and the performance indicators. In our design, we exploited Semi Definite Programming (SDP) after stating the entire constraints and objective functions as a non-convex optimization problem. We later manipulated and modified the problems into linear optimization problem and used SDP tools to solve it. In the modification process, we had expressed the problem in terms of autocorrelation sequence of filter coefficients as variable constraints instead of merely the filter coefficients. The key idea is to compromise the regularity constraints in order to get more degree of freedom allowing the optimization tool to obtain more suppressed pass and stop band ripple in the frequency response of decomposition filters given the transition band. The simulation results revealed that the spectrum estimator based on designed wavelet has a better performance compared to estimator based on well known wavelets, such as Daubechies, Symlets, and Coiflet. Therefore, it is reasonable to conclude that the Semi Definite Programming and optimization approach has provide efficient tools to design new wavelet with desired frequency selectivity feature, which suits spectrum estimation application.



Figure 4.33 Receiver operating characteristic of spectrum estimation based on periodogram, Welch approach and newly designed wavelet. In this scenario, the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 32 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated.



Figure 4.34 Receiver operating characteristic of spectrum estimation based on periodogram, Welch approach and newly designed wavelet. In this scenario, the sample space is of size 384. The overlap percentage and the length of each segment employed in Welch approach is 50% and 64 samples, respectively. Hamming window is used in this Welch approach. 16 sources occupying 128 frequency bins in the normalized frequency range  $[0,\pi]$  are randomly activated and deactivated.

From the investigation of receiver operating characteristic, we also found that the performance of our wavelet based spectrum estimation strongly depends on the decomposition level as well as the wavelet settings such as decomposition filter length, regularity index, and transition band. By keeping the same decomposition filter length, regularity index and transition band, it is possible to exploit the wavelet coefficients produced by wavelet packet nodes in the different levels to provide multiple estimates with different performance. Since wavelet based estimates, Welch and periodogram estimates perform differently with different settings, it is therefore irrelevant to simply say that one approach is better than the others. However, it is reasonable to conclude that the flexible alteration of the parameters such as wavelets filter length, configured transition band in the frequency response of the decomposition filter, and especially decomposition levels has converted wavelet packet based spectrum estimation into a reconfigurable and adaptable system giving it a competitive edge. The main inference from the use of wavelet based estimate is the possibility to obtain wavelet coefficients from all wavelet packet nodes at all levels, which provides multiple estimates with different trade off. This is done based only on one snapshot and through one-time operation.

## CHAPTER 5 A WAVELET PACKET TRANSCEIVER FOR SPECTRUM ESTIMATION AND DYNAMIC SPECTRUM ACCESS

The paradox of non-availability of spectrum even when large swathes of licensed spectrum is underutilized most of the time has prompted a rethinking in existing spectrum regulatory policies. While traditional spectrum allocation schemes follow a static approach where established frequency bands are allocated and assigned to fixed licensees, the new approach envisioned is a Dynamic Spectrum Access (DSA) model where unlicensed users may *rent* unused spectrum from licensed users on a need-to-need basis. To actualize this vision, the development of Cognitive Radios [39] or wireless systems that intelligently adapt their transmission parameters (including frequency, power, and modulation scheme) in accordance with the changing environment and requirements has been promoted.

Multi-carrier modulation (MCM) has been mooted as a strong physical layer candidate for Cognitive Radio system design [5]. By merely vacating a set of subcarriers, the spectrum of a MCM based Cognitive Radio can be easily and flexibly shaped to occupy spectral gaps without interfering with the Licensed Users. It has been shown that adaptive MCM based Cognitive Radio is a robust method to achieve good quality of communication and efficient use of the spectrum [5].

In traditional implementations of MCM, as in Orthogonal Frequency Division Multiplexing (OFDM), the generation and modulation of the sub-channels is accomplished digitally using Fourier bases. In [6], the replacement of the conventional Fourier-based complex exponential carriers of OFDM with orthonormal wavelet packet bases is proposed. The wavelet packet bases are derived from perfect reconstruction two-band FIR filter banks. Cohabitation of the Wavelet Packet Multi Carrier Modulation (WPMCM) based Cognitive Radio systems with existing licensed users is actualized by shaping its transmission waveform by adaptively activating or vacating sub-carriers in a way that it utilizes the unoccupied time-frequency gaps of the Licensed Users. The idea is to dynamically sculpt the Cognitive Radio transmission signal so that it has no or very little time-frequency components competing with the Licensed Users. This way the Cognitive Radio can seamlessly blend with the Licensed Users operation. Furthermore, the WPMCM receiver structure, which is used for demodulation of data, could also be used for analysis of the radio environment to identify active/idle bands - *at no additional cost*!

In this chapter, we demonstrate the wavelet based spectrum adaptation for Dynamic Spectrum Access by combining the wavelet based spectrum estimator proposed in the previous chapters with the WPMCM setup. The wavelet based spectrum estimation simply exploits the existing filter bank infrastructure used to conduct the Discrete Wavelet Packet Transform (DWPT) and its inverse in WPMCM. Based on the wavelet based estimates, the WPMCM Cognitive Radio (CR) system dynamically activates and deactivates the carriers in order to allow the cohabitation between CR and Licensed User (LU) system. The spectrum estimator and the spectrum shaping module are connected by an intermediate module that processes the radio spectrum information and converts it into a data vector that can readily be used by the spectrum shaping module to activate and deactivate subcarriers. In the simulation study, we employ four types of LU, namely partial band, single tone, multi-tones and swept tone LU. The organization of this chapter is as follows. In section 5.1, the blocks of the proposed system are elucidated. Section 5.2 gives elaborate discussion about the experiment scenario considered in the simulation studies as well as the results of the experiments. Finally section 5.3 summarizes this chapter.

## **5.1 System Description**

Figure 5.1 depicts the proposed WPMCM based CR transceiver setup. The major blocks of the system are a) WPMCM transceiver, b) Spectrum estimator, c) Spectral Vector generator and d) spectrum adaptor.

## 5.1.1 Wavelet Packet Multicarrier Modulation

The wavelet packet theory can be viewed as an extension of Fourier analysis. The basic

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idea of both transformations is the same: projecting an unknown signal on a set of known basis functions to obtain insights on the nature of the signal. Any function S(n) in  $L^2(\mathcal{R})$  can be expressed as the sum of weighted wavelet packets. In communication systems, this means that a signal can be seen as the sum of modulated wavelet packets leading to the idea of WPMCM. The WPMCM signal is composed of multicarrier symbols obtained from a sum of modulated and weighted wavelet packet waveforms  $\xi$ . In the discrete time domain this signal S(n) can be expressed as:



**Figure 5.1** Block diagram of the proposed WPMCM Transceiver. The receiver contains discrete wavelet packet transformer (DWPT) used to estimate the spectrum and extract the data transmitted. The transmitter contains inverse discrete wavelet packet transformer (IDWPT) used to construct multi-carrier modulated signals. IDWPT and DWPT are implemented using filter bank analysis. H' and G' are the low and high pass reconstruction filters while H and G are the low and high pass decomposition filters. Down and up arrows refer to down and up-sampling, respectively

In (5.1) *C* is the number of subcarriers while *u* and *k* are the symbol and subcarrier indices, respectively. The constellation symbol modulating  $k^{\text{th}}$  subcarrier in  $u^{\text{th}}$  symbol is represented as  $a_{u,k}$ . Time and frequency limited wavelet packet bases  $\xi(t)$  can be derived by iterating discrete half-band high g[n] and low-pass h[n] filters, recursively defined by [4]:

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$$\xi_{l+1}^{2p}(t) = \sqrt{2} \sum_{n} h[n] \xi_{l}^{p}(t-2^{l}n)$$

$$\xi_{l+1}^{2p+1}(t) = \sqrt{2} \sum_{n} g[n] \xi_{l}^{p}(t-2^{l}n)$$
(5.2)

In (5.2) subscript l denotes the level in the tree structure and superscript p indicates the sub-carrier index at given tree depth. Equation (5.2) is nothing but two-scale equation which has already been expressed in more general form by (2.46). In the case of (5.2), the low pass filter h[n] and high pass filter g[n] play a role as analysis filters. As already explained in chapter 2, the analysis filters have *mirrors* called synthesis filters which are also a pair of low-high pass filter combo  $\{h'[n], g'[n]\}$ . The two synthesis and analysis filters share a tight Quadrature Mirror Filters (QMF) relationship. For paraunitary QMF filter pairs of length L, the relationship is given as:

$$g[L-1-n] = (-1)^n h[n]$$
(5.3)

It should be noted that (5.3) is exactly the QMF relationship expressed by (2.53). We rewrite it here for the sake of convenience. The analysis and synthesis filters are basically complex conjugate time reversed versions of one another i.e.

$$h'[n] = h^*[-n] \text{ and } g'[n] = g^*[-n]$$
 (5.4)

The linear combination relationship presented by (5.1) is realized by implementing wavelet packet reconstruction in the transmitter by using Inverse Discrete Wavelet Packet Transform (IDWPT). As it is clear from figure 5.1, wavelet packet (WP) reconstruction employs multichannel filter bank consisting of cascaded two-channel synthesis filters (H' and G').

An incoming high-rate serial data stream is divided into several lower-rate parallel data streams. The data in each parallel branch is used as an input of the corresponding branch (WP node) in WP reconstruction tree. In every stage of reconstruction, the data in each branch is up-sampled by 2 before they are passed through the corresponding low or high pass half-band filter. These half-band filtering and up-sampling processes form one stage reconstruction process. The entire cascaded reconstruction process is actually analogous to sub-carriers modulation process in OFDM and it simulates the IDWPT as found in figure 5.1. The wavelet packet sub-carries with index *i* at tree level  $J \xi_j^i[k]$  used at the transmitter end are derived from the synthesis filters through a simple convolution rule leading to:

$$\xi_{J}^{i}[k] = f(k) * f(k/2) * \dots * f(k/2^{J-2}) * f(k/2^{J-1});$$
  
where,  $0 \le i \le 2^{J} - 1$  (5.5)  
and,  $f(k) = \begin{cases} h'[k], \text{ for lowpass branches} \\ g'[k], \text{ for highpass branches} \end{cases}$ 

In (5.5), h'[k] and g'[k] stand for the impulse responses of low pass and high pass synthesis filters, respectively.

Later in the receiver, the Discrete Wavelet Packet Transform (DWPT) block (see figure 5.1), which is also used for spectrum estimation, is employed to perform WP data demodulation. The wavelet packet duals  $\tilde{\xi}_{J}^{s}[k]$ , for sub-carriers with index *s* at tree level *J*, are used to extract the estimate of the transmitted data symbols. This is performed by taking advantage of the orthogonality between the wavelet packet  $\xi_{J}^{i}[k]$  and wavelet packet dual  $\tilde{\xi}_{J}^{s}[k]$  expressed as [6]:

$$\langle \xi_J^i[k], \tilde{\xi}_J^s[k] \rangle = \delta[i-s]$$
(5.6)

In (5.6) < .,. > represents the scalar product operator and  $\delta$  is the kronecker delta with  $\delta[i] = 1$  if i = 0, and it is 0 otherwise. The corresponding wavelet packet duals  $\tilde{\xi}_{j}^{i}[k]$  used at the receiver are obtained from the analysis filters through the discrete wavelet packet transform (DWPT):

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$$\widetilde{\xi}_{J}^{i}[k] = f(k) * f(2k) * .... * f(2^{J-2}k) * f(2^{J-1}k);$$
where,  $0 \le i \le 2^{J} - 1$ 
and,  $f(k) = \begin{cases} h[k]$ , for lowpass branches
 $g[k]$ , for highpass branches
$$\end{cases}$$
(5.7)

In (5.7), i, J, h[k] and g[k] are the sub-carrier index, tree level, the impulse responses of low and high pass analysis filter, respectively. We can note the similarity between (5.7) and (3.2) leading us to the fact that the demodulation module in WPMCM can be exploited for spectrum estimation.

In the receiver, the estimate of each sub channel  $\hat{a}_{u,k}$  is given as [6]:

$$\widehat{a}_{u,k} = (R[n] * \widehat{\xi}_{\log_2(C)}^k) \downarrow_C$$
(5.8)

In (5.8), R[n] is the received signal, \* denotes the convolution operator and the down arrow represents decimation by C with  $C = 2^J$  where J is the decomposition or reconstruction level.  $\hat{a}_{u,k}$  in (5.8) is the estimation of constellation encoded  $u^{\text{th}}$  data symbol modulating the  $k^{\text{th}}$  waveform. As shown in figure 5.1, the estimates go through constellation de-mapping process before it is passed to parallel to serial converter in order to obtain the actual data.

#### 5.1.2 Wavelet based spectrum estimation module in WPMCM receiver

The operation detail of wavelet based spectrum estimation and how it is built has already been discussed in chapter 3. Here, we just intend to emphasize that the spectrum estimation functionality can be provided by the same wavelet packet decomposition block used for multicarrier de-multiplexing of the received signal in the WPMCM receiver block. It should also be noted here that the frequency band that is spanned by spectrum estimator is exactly the same as the frequency band spanned by WPMCM subcarriers generated by the IDWPT block. Therefore, when cognitive radio systems use WPMCM as their multi-carrier modulation technique, the spectrum estimation module can be included with virtually no additional cost.

## 5.1.3 Thresholding and Spectral Notching

The information about the radio environment given by spectrum estimation module is passed to the spectrum vector generator (see figure 5.1). Here, the spectral information is mapped into a spectrum vector containing ones and zeros. The zeros correspond to bands which are occupied and the ones represent bands that are free (spectrum holes). The pattern of ones and zeros effectively characterizes the desired magnitude of the spectral estimate. The threshold is performed on a sub-band-by-sub-band basis whereby the power contained in each sub-band is independently compared to a predetermined threshold. The threshold value is defined in terms of the noise power. When sub-band power exceeds the threshold, interference is declared present and all of the sub-band coefficients are set to a value of zero. If sub-band power does not exceed the threshold, all of the sub-band coefficients are retained (set to one).

It is quite possible that the frequency bands adjacent to the occupied bands (the band that has power above threshold) have power just below the threshold. Therefore, sometimes it is required to deactivate the carriers surrounding the occupied band. For this purpose, spectrum vector manipulation block is added to do some manipulation on spectrum vector in order to obtain the desired activated and de-activated frequency bands. Furthermore, the decomposition levels of the tree structure used for spectrum estimator and WPMCM demodulation can be different. Hence, the spectrum vector should be modified accordingly into a format that is matched to the reconstruction level in the transmitter. This spectrum-vector matching process is also performed by spectrum vector manipulation block.

Finally, the spectrum manipulation block produces final spectrum vector that is used to inform the Inverse Discrete Wavelet Packet Transform (IDWPT) block in the transmitter about deactivated carriers.

## 5.1.4 Transmission Waveform Shaping

Based on the spectrum vector, sub-channels of the WP MCM system that lie in and around the spectrum of the LU are vacated to facilitate coexistence. This way the CR transmission signal is dynamically sculpted such that it has no or very little time-frequency components competing with the LU and the CR operation is made invisible to the LU. In order to keep all of this mechanism to work properly, a pilot channel is required for communication between transmitter and receiver regarding to the state of the occupied and unoccupied frequency bands.

## 5.2 Experiments, Results and Analysis

## **5.2.1 Experiment Scenarios**

In order to investigate the performance of the wavelet packet transceivers for dynamic spectrum access, we consider a WPMCM system operating with 128 equally spaced carriers derived from a level-7 cascaded tree. The same tree structure is used at the receiver to gauge the radio environment. To simplify the evaluation of the system performance, no distortions such as Inter Symbol Interference (ISI) or Inter Carrier Interference (ICI) are considered. The modulation scheme used is Quadrature Phase Shift Keying (QPSK). We use the optimal wavelet family designed in chapter 4 using Semi Definite Programming (SDP) tool. The optimal wavelet has decomposition filter length of 50 *K*-regularity of 7 and transition band of  $0.2\pi$ . This was chosen based on empirical data which gave the best Probability of detection ( $P_d$ ) and Probability of false alarm ( $P_{fa}$ ) combination. A recap of the analysis conducted in sub-sections 4.6.2 and 4.6.3 would also be useful to understand these choices. Of course, more thorough experiments can be conducted in order to arrive at optimal specifications that give the best performance while maintaining acceptable level of complexity.

**Table 5.1** Probability of detection and false alarm of optimal wavelet based spectrum estimates for various threshold levels. The length of wavelet decomposition filter used here is 50 with K-regularity index of 7 and transition band of  $0.2\pi$ 

Threshold Level (dB)	<b>Probability of Detection (P<sub>d</sub>)</b>	Probability of False Alarm (P <sub>fa</sub> )
-3	0.8825	0
-5	0.9997	4.6627e-004
-7	1	0.0512
-9	1	0.1285
-11	1	0.2097
-13	1	0.3496
-15	1	0.4923
-17	1	0.5910
-19	1	0.7295
-21	1	0.7883
-23	1	0.7903
-25	1	0.7959

Table 5.1 illustrates the value of  $P_d$  and  $P_{fa}$  of the specified optimal wavelet based estimates for different threshold level. By considering the performance of the spectrum estimator based on the selected wavelet shown in table 5.1, we select the threshold level of -7dB to evaluate the presence/absence of a LU since this threshold value gives  $P_d = 1$  with minimum probability of false alarm. It should be noted that the power level of the active band that is used in this experiment would be exactly the same as the one that is used in sub-sections 4.6.2 and 4.6.3, and thus the choice of threshold level is definitely valid. Lastly, the cognitive modules at the transmitter and receiver are taken to be always aware of the transmission characteristics, including details of the active and deactivated carriers. To gauge the performance of the proposed WPMCM system, four types of LU are considered namely: partial band, single tone, multi-tones and swept tone sources. The detail descriptions about these four sources can be found in sub-sections 3.4.1 and 3.4.2. For partial band and swept tone source, we decide to employ 7-level wavelet decomposition while 11level wavelet decomposition is used for estimating single tone and multi-tone source. The fundamental reason for this is found in chapter 3 showing the fact that higher level of decomposition gives better frequency resolution, which is appropriate to locate extremely narrow

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band sources such as single tone and multi-tone. On the other hand, increasing the decomposition level tends to increase the variance of estimated PSD. For partial band source, high variance of estimated PSD in the pass band is undesirable since it might reduce the probability of detection for a given threshold.



**Figure 5.2** Result of spectrum adaptation (partial band LU case) based on Wavelet approach. Only carriers correspond to frequency bands with LU energy above threshold are deactivated (66 carriers). The wavelet decomposition filters used here have length of 50, K-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition level is used in spectrum estimation module

## 5.2.2 Results and Analysis

## A. Partial Band Source

The blue curve in figure 5.2 illustrates the wavelet packet based Power Spectral Density (PSD) estimates of the partial band LU. Based on this estimate, the shape of the CR spectrum is adapted so the interference between existing LU signal and CR signal is minimized. The red curve in figure 5.2 depicts the PSD of the CR signal (with carriers coinciding with LU deactivated). Meanwhile, figure 5.3 basically shows the effect of deactivation of four additional carriers in the neighboring of carriers coinciding with LU.

Figure 5.4 illustrates the Bit Error Rate (BER) performance of WPMCM CR in the presence of a partial band LU. It is clear from the BER curves that spectral adaptation of the CR source greatly reduces the interference energy between CR and LU and improves its operation. However, the interference energy is not completely suppressed by merely deactivating the carriers that coincide with LU since a transition band between the occupied and unoccupied band exist for both LU and CR. Hence, deactivation of neighboring carriers is required. When additional carriers buffering the sides of the LU are removed the interference energy is reduced further and for the case of removal of six additional neighboring carriers, the performance reaches the theoretical limit. In general, the reduction in Bit Error Rate (BER) for different number of vacated carriers is much clearer for higher number of signal to noise ratio (SNR)



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**Figure 5.3** Result of spectrum adaptation (partial band LU case) based on Wavelet approach. Two carriers in the left and two carriers in the right side of bands having energy above threshold are also deactivated (In total, 70 carriers are vacated). The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition level is used in spectrum estimation module



Figure 5.4 Performance of wavelet based spectrum estimation and adaptation in WPMCM CR system for partial band LU case. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition level is used in spectrum estimation module





**Figure 5.5** Result of spectrum adaptation (multi-tone LU case) based on wavelet based approach. Only carriers correspond to the frequency bands with energy above threshold are deactivated (14 carriers). The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 11-level wavelet decomposition level is used in spectrum estimation module



Figure 5.6 Result of spectrum adaptation (multi-tone LU case) based on wavelet based approach. Two frequency bands in the left and two bands in the right side of bands having energy above threshold are also deactivated (total 42 carriers). The wavelet decomposition filters used here have length of 50, K-regularity index of 7 and transition band of  $0.2\pi$ . The 11-level wavelet decomposition level is used in spectrum estimation module

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**Figure 5.7** Performance of wavelet based spectrum estimation and spectrum adaptation in WPMCM CR system for multi tone licensed user case. The wavelet decomposition filters used here have length of 50, K-regularity index of 7 and transition band of 0.2π. The 11-level wavelet decomposition level is used in spectrum estimation module

#### **B.** Multi-tones Source

Figure 5.5 shows the wavelet packet based estimated PSD of the multi-tone LU as well as the spectrum shaped CR. Similar to the case for partial band LU, the spectrum estimate provided by wavelet based approach (blue color) is used as an input for spectrum vector generation block (see figure 5.1) to decide which carriers to be turn off. Then, the spectrum adaptation process would be performed correspondingly. In figure 5.5, only the CR carriers that coincide with LU are deactivated. The figure shows how the adapted CR's signal perfectly fit with the LU signal even though no additional carriers apart from the ones that coincide with LU bands are deactivated. Meanwhile, figure 5.6 basically shows the effect of deactivation of two additional carriers in the neighboring of each carrier coinciding with LU resulting in total of 42 vacated carriers. The corresponding BER performance curves are plotted in Figure 5.7. The PSD and BER curves clearly show the ameliorative impact of spectrum shaping on the performance of the CR system. Interestingly, unlike the case of the partial band source, it is enough to vacate only those CR carriers (totaling 14) that co-exist with the LU to obtain excellent performances. This means that no additional carriers neighboring the LU band have to be deactivated.

#### C. Single Tone Source

Figure 5.8 illustrates the wavelet packet based estimates on the single tone LU as well as the spectrum adapted CR. In general, the result observed in single tone LU case is actually similar to the result observed in multi-tones LU case and this validates the fact that wavelet based spectrum estimate actually perform very well for narrow band source case. From figure 5.8, it can be found that no significant leakage is introduced by the adapted CR signal into the band occupied by LU even though there is no extra carriers deactivation. Figure 5.9, once again, justifies the excellent performance of wavelet based spectrum estimation. The very accurate estimation provided by wavelet based approach allows the carrier deactivation only at the band coinciding with LU to be good enough. Additional carriers deactivation makes almost no improvement on the performance of the WPMCM CR system.





Normalized Frequency (x  $\pi$  rad/sample)

**Figure 5.8** Result of spectrum adaptation (single tone LU case) based on wavelet based approach. Only carriers correspond to the frequency bands with energy above threshold are deactivated (2 carriers). The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 11-level wavelet decomposition is used in spectrum estimation module



**Figure 5.9** Performance of wavelet based spectrum estimation and spectrum adaptation in WPMCM CR system for single tone licensed user case. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of 0.2*π*. The 11-level wavelet decomposition is used in spectrum estimation module

## D. Swept tone Source

Similar to the experiments conducted in chapter 3, to gauge the swept tone LU, 20 sweeps (each of 640 unit samples) in the normalized frequency band  $0.2\pi$ -  $0.8\pi$  are considered. In order to track the temporal variations in the frequency, three different scenarios are provided.

In the first scenario, the spectrum estimation module catches a snapshot (or sub-sweep) containing 128 samples corresponding to 20% of a single sweep. Hence, five snapshots of a single sweep are available. Based on each 128-samples snapshot, spectrum vector generator has to determine the carriers to be turned off and the WPMCM transmitter will adapt the spectrum of the transmitted signal correspondingly. Figure 5.10 depicts the LU and CR PSD curve for this first scenario. In this figure, only the PSD of the fourth and the fifth sub-sweeps of LU signal are displayed together with the corresponding adapted CR PSD. It should also be noted that only the carriers that coincide with LU are deactivated in this figure. If the interference between the LU and CR signals needs to be reduced, it is possible to additionally deactivate the carriers adjacent to the band occupied by LU. Figure 5.11 illustrates the effect of deactivation of four additional carriers that are adjacent to the LU band.



**Figure 5.10** Result of spectrum adaptation based on wavelet approach for the case of swept tone LU with sub-sweep size of 128 samples. In this case, every single sweep contains 5 sub-sweeps and only the 4<sup>th</sup> and 5<sup>th</sup> sub-sweeps are displayed. Only carriers correspond to the bands with energy above threshold are deactivated. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module

In the second scenario, the size of the snapshot is reduced to 64 samples corresponding to 10% of a single sweep. The process of carrier deactivation and spectrum adaptation are now based on smaller number of samples. Figure 5.12 illustrates the combination between adapted WPMCM CR PSD and LU PSD viewed by wavelet based spectrum estimator. In this figure, only the PSD of the ninth and the tenth sub-sweeps of LU signal are displayed together with the corresponding adapted CR PSD. In addition, only the carriers corresponding to the band occupied by LU are deactivated in this figure. The effect of deactivation of four additional carriers adjacent to the LU band for the purpose of interference reduction is illustrated in figure 5.13.
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**Figure 5.11** Result of spectrum adaptation based on wavelet approach for the case of swept tone LU with sub-sweep size of 128 samples. Every single sweep contains 5 sub-sweeps and only the 4<sup>th</sup> and 5<sup>th</sup> sub-sweeps are displayed. In this case, two bands in the left and two bands in the right side of bands having energy above threshold are also deactivated. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module



**Figure 5.12** Result of spectrum adaptation based on wavelet approach for the case of swept tone LU with sub-sweep size of 64 samples. In this case, every single sweep contains 10 sub-sweeps and only the 9<sup>th</sup> and 10<sup>th</sup> sub-sweeps are displayed. Only carriers correspond to the bands with energy above threshold are deactivated. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module

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**Figure 5.13** Result of spectrum adaptation based on wavelet approach for the case of swept tone LU with sub-sweep size of 64 samples. Every single sweep contains 10 sub-sweeps and only the 9<sup>th</sup> and 10<sup>th</sup> sub-sweeps are displayed. In this case, two bands in the left and two bands in the right side of bands having energy above threshold are also deactivated. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module



**Figure 5.14** Result of spectrum adaptation based on wavelet approach for the case of swept tone LU with sub-sweep size of 32 samples. In this case, every single sweep contains 20 sub-sweeps and only the  $19^{th}$  and  $20^{th}$  sub-sweeps are displayed. Only carriers correspond to the bands with energy above threshold are deactivated. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module

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**Figure 5.15** Result of spectrum adaptation based on wavelet approach for the case of swept tone LU with sub-sweep size of 32 samples. Every single sweep contains 20 sub-sweeps and only the  $19^{th}$  and  $20^{th}$  sub-sweeps are displayed. In this case, two bands in the left and two bands in the right side of bands having energy above threshold are also deactivated. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module

Finally, in the third scenario, the size of snapshot is further reduced to 32 samples corresponding to 5% of a single sweep. Figure 5.14 and 5.15 illustrates the PSD of the last two subsweeps of LU signal together with the associate adapted CR PSD for no additional carrier deactivation and the deactivation of two additional carriers in each side of LU band, respectively.

Figures 5.16-5.18 show the BER performance of WPMCM based CR system for sweepsizes of 128, 64 and 32, respectively. In general, the greater the number of sub-sweep samples, the greater the information available on the LU features and hence the greater the scope for adapting the CR characteristics to evade the LU. Thus significant improvements in the CR performance can be achieved when the size of the radio *snapshot* is 128 samples or more (see Figure 5.16). And when the number of samples available to judge the spectrum is low (say 64 or 32 samples) the accuracy of the judgment on occupied and idle bands falters and the CR performance suffers (Figures 5.17 and 5.18). This issue can also be related to the frequency resolution aspect. If we consider some results related to swept tone source shown in figures 3.32-3.52, we can find that the frequency resolution and thus the accuracy of the estimates tend to deteriorate when the size of the snapshot is reduced. This phenomenon is found not only for wavelet based estimates but also for periodogram and Welch estimates. This fact explains the inaccuracy of the judgment on occupied and idle bands.

On the other hand, a smaller sample space also means a faster spectrum analysis and greater opportunity to track temporal variations. For the case of swept tone case, these temporal variations are quite significant since different frequency is occupied at different instant. Due to uncertainty principle [21], it is not possible to have the best frequency and time resolution at the same time. Hence it is important to strike the right balance between the time and frequency resolutions achievable. Herein lies the advantage of employing the wavelet packet transforms for spectrum analysis – one can tailor the level of decomposition to obtain the required frequency resolution.

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**Figure 5.16** Performance of wavelet based spectrum estimation and spectrum adaptation in WPMCM CR system for swept tone LU case with sub-sweep size of 128 samples. The wavelet decomposition filters used here have length of 50, K-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module



**Figure 5.17** Performance of wavelet based spectrum estimation and spectrum adaptation in WPMCM CR system for swept tone LU case with sub-sweep size of 64 samples. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module

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**Figure 5.18** Performance of wavelet based spectrum estimation and spectrum adaptation in WPMCM CR system for swept tone LU case with sub-sweep size of 32 samples. The wavelet decomposition filters used here have length of 50, *K*-regularity index of 7 and transition band of  $0.2\pi$ . The 7-level wavelet decomposition is used in spectrum estimation module

#### **5.3 Summary of the Chapter**

In this chapter, the wavelet based spectrum adaptation for Dynamic Spectrum Access was investigated by combining the novel wavelet based spectrum estimation and the proposed WPMCM in [6]. The wavelet packet transform is used for spectrum estimation, spectrum shaping as well as multi carrier modulation technique, thus paving way for an efficient and low cost Cognitive Radio (CR) system. The spectrum estimation unit is tagged to the WPMCM transceiver structure by exploiting the filter bank infrastructure used for Discrete Wavelet Packet Transform implementation and hence the spectrum analysis is done at virtually no additional cost. Based on the result of wavelet based estimates, the cohabitation of the CR system with Licensed User (LU) is actualized by dynamically activating and deactivating the CR carriers in a way that the CR and LU systems do not have any competing time-frequency components. Threshold operation and some manipulation on the spectrum estimates results are required before the final decision about the carriers to be deactivated is made.

Through simulation studies, the usefulness and potential of the WPMCM based for developing CR systems was demonstrated. These studies exemplify how well the proposed wavelet based spectrum adaptation performs in the presence of LU and Additive White Gaussian Noise (AWGN). In general, the performance of wavelet based estimates in the case of narrow band LU is excellent since virtually no additional carrier deactivation is required apart from the ones coinciding with the LU. The additional carrier deactivation is still required for partial band case. Of particular interest is the estimation of a swept tone LU that varies with time. The wavelet based spectrum adaptation performs well for the snapshot size of 128 samples. However, poor performance is shown for much smaller snapshot size (32 samples) due to poor estimates provided by wavelet based spectrum estimation block. This phenomenon can be understood by considering uncertainty principle which states that it is not simultaneously possible to have very good time and frequency resolution.

The analysis conducted in this chapter are preliminary and needs to be carried further particularly with respect to the demonstration of the flexibility, adaptability and reconfigure ability

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offered by wavelet packet modulation based transceiver. We still have not considered more complex scenario including how to distinguish the signal transmitted by LU from the signal produced by other CR users. Moreover, we also has not taken into account the need for echo cancellation to make sure that the spectrum estimator of a CR system is not burdened by the signal that is produced by its own transmitter. It should also be noted that the mechanism that is introduced for WPMCM transceiver here requires a pilot channel. The pilot channel is needed by a CR system to inform its communication pair (another CR system) about the subcarriers that are currently used. Overall, the performance results of the simulation studies make us to conclude that this wavelet based system indeed can be a useful tool in the design of adaptive systems for dynamic spectrum access.

### CHAPTER 6 CHALLENGES AND BOTTLENECKS

We have thus far investigated the spectrum estimation tool based on wavelet and also developed a family of new optimal wavelets which are maximally frequency selective and are best suited for wavelet based spectrum estimator. In the fifth chapter, we combined the wavelet based spectrum estimator with a Multi-carrier Modulation scheme based on Wavelet packets and demonstrated its operation in the context of dynamic spectrum access.

In this chapter we enlist and document the challenges and problems encountered in the realization of the systems.

### 6.1 Introduction to spectrum carving issue

Spectrum carving is a major issue in the wavelet packet based spectrum estimation. Spectrum carving occurs because the wavelet filters used for decomposing the given signal are non-ideal filters having a non-zero transition band. The wavelet packet estimator uses multiple orthogonal filters derived from tree structures constructed by cascading wavelet packet decomposition filter. Hence, the spectrum estimates can be considered as outputs of series of virtual filters spanning the normalized frequency range of  $[0,\pi]$ . For example, if we have 6-level decomposition, the normalized frequency range of  $[0,\pi]$  is split into 64 equal bands and the estimate on each band correspond to the output of single virtual filter. Hence, there would be 64 virtual filters in total. The impulse response of each virtual filter is cumulative convolution of the impulse responses of the cascaded analysis low pass and/or high pass filter from the wavelet packet tree root to the leaf (while taking into account down sampling by factor 2 at each filtering stage). As already mentioned in chapter 3, the impulse response of this cascaded filters structure is called

wavelet packet duals  $\psi_i[k]$  [6], which can be represented as:

$$\psi_{i}[k] = f(k) * f(2k) * ... * f(2^{J-2}k) * f(2^{J-1}k);$$
  
where,  $0 \le i \le 2^{J} - 1$  (6.1)  
and,  $f(k) = \begin{cases} h[k], \text{for lowpass branches} \\ g[k], \text{for highpass branches} \end{cases}$ 

Equation (6.1) is exactly the same as (3.2). We have reproduced it here for ready reference. When the wavelet packet tree structure is constructed by iterating the wavelet filters, it results in residual artifacts in the stop band.

We will illustrate this phenomenon by considering level-5 wavelet packet decomposition and explain how spectrum carving progresses at each level. In this example, we use the maximally frequency selective wavelets developed in chapter 4 using SDP. The length of the wavelet decomposition filter here is 40 with *K*-regularity index of 6 and transition band of  $0.2\pi$ . While level-5 decomposition results in 32 virtual filters, we will only focus on one branch for ease and clarity of depiction. In this example, the virtual filter considered would ideally span the normalized frequency range of  $[0.53125\pi, 0.5625\pi]$ .

The first two stages of decomposition are shown in figure 6.1. Since we would like to obtain the response of virtual filter spanning the frequency range of  $[0.53125\pi, 0.5625\pi]$  which is in the upper half band of  $[0, \pi]$  band, the first half band filtering should be high pass filtering. This is described by the blue curve in figure 6.1. The output of this first high pass filtering should logically span the frequency band of  $[0.5\pi,\pi]$ . Hence the next filtering should be the one that removes the upper half band of this frequency range, which is  $[0.75\pi, \pi]$  and keep the lower half frequency component, which is in the range of  $[0.5\pi, 0.75\pi]$ . However, we need to remember that between these two filtering stages, down sampling by factor-2 is performed. As a result, instead of doing low pass half band filtering in the  $2^{nd}$  stage, we perform high pass filtering again. The high pass filter would cover the frequency range of  $[0.25\pi, 0.5\pi]$  but its alias version would span the range of  $[0.5\pi, 0.75\pi]$ . Therefore, we actually take the advantage of the alias version of high pass filter covering  $[0.25\pi, 0.5\pi]$  in order to maintain the lower half of  $[0.5\pi,\pi]$ . This aliasing or

mirroring caused by down sampling is the reason why the ordering of wavelet packet nodes follow the Gray code sequence (refer chapter 3, especially figure 3.2).

The frequency response of second stage filtering including its mirror version is illustrated by the red curve in figure 6.1. The resultant of these two filtering stages and down sampling process between stages is illustrated by the black curve in figure 6.1. An interesting phenomenon can be found from this figure. Apart from the desired response at normalized frequency range of  $[0.5\pi, 0.75\pi]$ , there are some residual components between  $0.2\pi$  and  $0.5\pi$  as well as between  $0.75\pi$ and  $0.9\pi$ . The reason for the emergence of these residues is the fact that the frequency response of the half band filter is not perfectly flat and square. Instead, it has non-zero transition band as well as a few ripples in the stop band. Even the maximally frequency selective filter banks designed in Chapter 4 are not ideal filters and hence they too are marginally affected by these infarctions. Figure 6.2 shows the precise location of the cumulative convolution between the first and second stage of half band filtering with down-sampling by-2 between stages described by figure 6.1 in the wavelet packet tree.



**Figure 6.1** The first two half-band filtering stages (with down sampling by factor 2 in between) in order to derive the cumulative frequency response of the virtual filter at level-5 of the wavelet packet tree. In this example, we try to derive the virtual filter spanning the normalized frequency range of  $[0.53125\pi, 0.5625\pi]$ . We use our designed wavelet with decomposition filter length of 40, K-regularity of 6 and transition band of  $0.2\pi$ 

As the decomposition process is carried on into the higher level, some of the existing residual components grow and other new residual components would emerge to form side lobes with significant level. This formation of side lobes in the frequency response of cumulative filters is known as spectrum carving effect.

Figure 6.3 illustrates the progression for the level third and fourth decomposition and Figure 6.4 gives the progression for level fifth decomposition. It is clear from these figures that spectrum carving becomes more and more significant as the decomposition level is increased. Figure 6.5 shows the precise location of the cumulative convolution between the first, second, third, fourth and fifth stage of half band filtering with down-sampling by-2 between stages described by figures 6.3 and 6.4 in the wavelet packet tree.



**Figure 6.2** Wavelet packet decomposition of a signal. Here *H* and *G* denote the frequency responses of the low and high pass decomposition filters, respectively. The down arrows followed by '2' represent decimation by 2. The order of filter in each level has already matched the frequency ordering from 0 to  $\pi$ . The cascaded two high pass filter with down sampling process in between within the red dashed box illustrates the first two half band filtering stages described in



**Figure 6.3** The third and fourth stages of half-band filtering process (with down sampling by factor 2 in between) in order to derive the cumulative frequency response of the virtual filter at level-5 of the wavelet packet tree. In this example, we try to derive the virtual filter spanning the normalized frequency range of  $[0.53125\pi, 0.5625\pi]$ . We use our designed wavelet with decomposition filter length of 40, K-regularity of 6 and transition band of  $0.2\pi$ 



**Figure 6.4** The fifth stage of half-band filtering process (with down sampling by factor 2 in between) in order to derive the cumulative frequency response of the virtual filter at level-5 of the wavelet packet tree. In this example, we try to derive the virtual filter spanning the normalized frequency range of  $[0.53125\pi, 0.5625\pi]$ . We use our designed wavelet with decomposition filter length of 40, K-regularity of 6 and transition band of  $0.2\pi$ 



**Figure 6.5** Wavelet packet decomposition of a signal. Here *H* and *G* denote the low and high pass decomposition filters, respectively. The down arrows followed by '2' represent decimation by 2. The order of filter in each level has already matched the frequency ordering from 0 to  $\pi$ . The cascaded high pass and low pass filters with down sampling process in between surrounded by the blue dashed lines illustrates the first four half band filtering stages described in figure 6.3. Meanwhile, the first five half band filtering stages with down sampling process in between described in figure 6.4 is illustrated by cascaded structure surrounded by pink dash lines



Figure 6.6 Spectrum carving effect on 3-level wavelet packet decomposition based on Daubechies-20. From this figure, it is clear that two -25 dB side lobes emerge due to the spectrum carving effect



Figure 6.7 Spectrum carving effect on 3-level wavelet packet decomposition based on our designed wavelet. The designed wavelet has decomposition filter length of 40, K-regularity of 6 and transition band of  $0.2\pi$ . No significant side lobe appears in this figure

# **6.2 Maximally frequency selective wavelet versus standard wavelets in tackling Spectrum Carving**

Figure 6.6 and 6.7 illustrates the impact of spectrum carving on 3-level wavelet packet decomposition based on Daubechies-20 and our designed wavelet, respectively. As in the previous five figures, the length of the decomposition filter of the designed wavelet is 40 with *K*-regularity index of 6 and transition band of  $0.2\pi$ . From figure 6.6 and 6.7, it is obvious that the wavelet packet decomposition based on the newly designed wavelet is more resistant to spectrum carving effect. For example, the 3-level Daubechies-20 wavelet packet decomposition has side lobes at -25 dB while no significant side lobes appear for the case of the designed wavelet.



Figure 6.8 Spectrum carving effect on 5-level wavelet packet decomposition based on Daubechies-20. From this figure, it is clear that around -7 dB side lobe emerges due to the spectrum carving effect

Figure 6.8 and 6.9 illustrate how the spectrum carving causes greater harm for higher levels of decomposition. For level-5 decomposition case, the wavelet packet decomposition based on our designed wavelet still outperforms that based on Daubechies-20 in term of side lobe level.

We can now summarize our finding about spectrum carving effect as the main issue in the performance of wavelet based spectrum estimation. It can be concluded that the reason for the emergence of spectrum carving is due to the combination of two factors. The first aspect is due to the fact that all the wavelet decomposition filters have a transition band which tapers between the pass and stop bands. The poorer the frequency selectivity, the greater the impact of spectrum carving is. The second aspect is the mirroring or aliasing effect on the frequency response of decomposition filters due to the presence of down sampling by 2 factors following the half band filtering in every decomposition stage. The imperfection of frequency selectivity features in decomposition filters at different stages (the down sampling process is also taken into account here). As the decomposition process is carried on into the higher level, some of the existing residual components would grow and other new residual components would emerge to form side lobes with significant level. This spectrum carving effect becomes more significant at higher level of decomposition.



Figure 6.9 Spectrum carving effect on 5-level wavelet packet decomposition based on our designed wavelet. The designed wavelet has decomposition filter length of 40, K-regularity of 6 and transition band of  $0.2\pi$ . Side lobe level of around -9 dB appears due to the spectrum carving effect.

The analysis on spectrum carving effect can be used to provide thorough evaluation on some results found in chapter 3. As it is found in chapter 3, the performance of wavelet based spectrum estimation for extremely narrow band sources is much better than that of the estimation for partial band (very wide band sources). In partial band (wide band) signal, the received energy is spread over frequency band spanned by a large number of virtual filters. Each virtual filter introduced different amount of residual response at the different location in the frequency domain. Hence, for partial band case, more residual energy is produced in the unoccupied band especially the bands adjacent to the frequency bands occupied by the source resulting in poor transition band. On the other hand, in single tone and multi-tone cases, the received energy is focused on particular frequency that is spanned by one or two virtual filters. Since the number of virtual filters that pass the receive signal is minimum, the number of residual energy in the stop band is also minimized resulting in much better transition band than in the case of partial band.

### 6.3 The challenges encountered in wavelet design

There are issues in the design of the maximally frequency selective wavelets, that best suit applicability to spectrum estimation, too. The wavelet design problem is expressed as an optimization problem by incorporating the wavelet constraints as well as frequency selectivity constraints. This problem in its original form is non-convex. Though non convex problem can be solved, they are susceptible to be caught in locally optimal solution instead of being globally optimal. We solve this challenge by representing our design problem in terms of autocorrelation sequence of the low pass filter coefficients instead of the filter coefficients itself. This transformation successfully converted the design problem from non-convex to convex. At this juncture, however, a new problem emerged. This challenge is related to the fact that the regularity constraints are vulnerable to numerical problem. This was addressed by reformulating the optimization problem into  $Q(\omega)$  domain instead of  $H(\omega)$  (refer section 4.5). The last issue in design process is to get back stable low pass filter coefficients from the autocorrelation sequence.

Fortunately, this was easily addressed by employing spectral factorization algorithm proposed in [35]

### 6.4 Time resolution issue in spectrum estimation for Dynamic Spectrum Access

When we talk about spectrum estimation for Dynamic Spectrum Access, it is natural to discuss its employability for cognitive radio (CR) system. In the context of CR system, speed and accuracy of measurements are very important to determine the suitable spectrum estimation technique for CR. Speed and accuracy are important to answer the questions of which band is occupied and at what instance. While accuracy (which is strongly related to frequency resolution, bias or leakages and variance of the estimates) was investigated in this thesis, we have not touched the time resolution aspect. Time resolution can be related to how fast estimation is performed for a given sampling rate. When the time period between successive estimation is shorter, we obtain better information about the instance at which the frequency bands are occupied. In addition, faster estimation also means smaller number of samples as well as smaller snapshot window.

Wavelet transform has been widely known as transformation that provides both time and frequency information. This fact has naturally encourages a hypothesis promoting the possibility of employing wavelet transform for CR since both time and frequency information about occupied spectrum are urgently needed. It is thus interesting to relate this time resolution issue mentioned in the previous paragraph with time frequency tiling in wavelet transform shown in figure 2.13. This figure clearly shows the trade off between time resolution and frequency resolution in continuous wavelet transform. In discrete implementation, the trade off between time and frequency resolution can be clearly seen from one stage of wavelet decomposition. In wavelet decomposition, every stage consists of half band filtering followed by down sampling by factor 2. Therefore, after single stage decomposition, the frequency resolution is increased by 2 since the output signal only spans the half of the frequency band covered by the input signal. On the other hand, the time resolution is reduced by half due to the down sampling process. It should be noted here that the number of samples are reduced by half but the snapshot window remains the same. This is slightly different from the time resolution issue mentioned in the previous paragraph stating that reduction of time resolution is identical to reduction of snapshot window. Due to this issue, it is quite challenging to expect a new wavelet based spectrum estimation technique that can also provide tunable time resolution for cognitive radio.

### 6.5 The effect of fading channel on the wavelet based spectrum estimation

When fading channels separate the receivers from the licensed users in the dynamic spectrum access environment, new challenges emerge. Due to the multi-path propagation in fading channel, different multi-path components may reach the receiver at different instances. Due to phase difference, difference components can combine constructively/destructively resulting in a resultant component having much larger/smaller amplitude than the original signal.

There are two types of fading, narrow band fading (or flat fading) and wide band fading (or frequency selective fading). These two types of fading are easily explained in terms of digital communication. In narrow band fading, the root mean square of the delay spread is smaller than the duration of a transmitted pulse in digital communication. Delay spread describes the variation of time difference among multi-paths components. When the channel delay spread is transformed into frequency domain, the result is called channel coherence bandwidth. In flat fading, the bandwidth of the signal is smaller than channel coherence bandwidth meaning that all frequency components experience the same magnitude of fading. On the other hand, in frequency selective fading, the channel delay spread is larger than the duration of the transmitted pulse while the bandwidth of the transmitted signal is wider than the channel coherence bandwidth. As a result, different frequency components of the signal experience different fading.

In a fading channel, the wavelet based spectrum estimates may become inaccurate since it may not describe the actual power spectrum density (PSD) of the transmitted signals. Instead, it actually gives the multiplication of the actual PSD and the channel frequency response. In the context of cognitive radio (CR), some frequency components of the signal transmitted by licensed users (LU) that experiences fading might end up at level below the threshold set by the CR

receivers. The spectrum estimation module in the CR receiver would decide that the corresponding bands are currently not occupied and thus the CR transmitter might exploit that band. As a result, the LU and CR transmission may hinder one-another. This challenge, however, is common to all spectrum estimators (and not necessarily with wavelet based estimators alone) and is usually classified as channel estimation problem rather than spectrum estimation problem.

### CHAPTER 7 CONCLUSIONS AND FUTURE RESEARCH TOPICS

### 7.1 Conclusions

In this thesis work, we developed a wavelet packet based spectrum estimator and conducted investigations on its performance. Since the theory of wavelets and wavelet packets are tightly coupled with filter bank analysis, wavelet based spectrum estimation is formulated as a filter bank analysis problem. To gauge the performance of the estimator, four different sources with different characteristics were utilized. The sources were single tone, multi-tones, partial band and swept tone (a source which occupied different frequencies at different instances). The performances were compared and contrasted with that of traditional approaches like Periodogram, Welch, Windowed periodogram and MTSE.

We also designed new wavelets with excellent frequency selectivity features to optimize the performance of the estimator. A comparative analysis between spectrum estimation based on the newly designed wavelets and standard wavelets were also carried out.

Finally, a wavelet packet transceiver that combined the wavelet packet based spectrum estimator with a Multi-carrier modulator was established.

The core conclusions of all these efforts can be summarized as follows:

- The wavelet transform is a unitary transform and conserves energy. It is also possible to seamlessly move from the wavelet domain to the frequency domain without any loss.
- A valid spectrum estimation based on wavelet can be built by exploiting filter banks structure of wavelet packet decomposition.
- The decomposition level of the wavelet packet tree can be tuned to adjust the performance of the wavelet based estimates with respect to variance of the estimated PSD and frequency resolution.
- The wavelet based estimates at various decomposition levels tend to behave between the performance of Welch approach (with the particular setting mentioned in chapter 3 and 4) and periodogram in term of variance of the estimates and frequency resolution offered. Welch estimates have low variances but poor frequency resolution while Periodogram estimates have large variation but guarantees. This inference is supported by the investigations on receiver operating characteristic carried out in chapter 4.
- The wavelet packet based approach gives all wavelet coefficients at all decomposition levels. The presence of all of these coefficients allows obtaining multiple estimates from different level of the tree with different degree of variance and frequency resolution, in one snapshot and one operation. This feature can be exploited to construct an adaptable and reconfigurable spectrum estimation mechanism. Clearly this kind of flexibility offered by wavelet based spectrum estimator (and not available in periodogram and Welch estimates) is of enormous advantage in a dynamic and time variant environment.
- Maximally frequency selective wavelets that suppressed the pass and stop band ripples were developed by compromising the regularity constraints. The solver Semi Definite Programming used for solving the optimization problem was found to be quite efficient.
- The estimators based on the newly designed optimum wavelet decomposition filter yielded better performance than ones based on standard wavelets such as Coiflets, Symlets and Daubechies.
- We also successfully tagged the wavelet based spectrum estimation unit to a WPMCM transceiver structure by exploiting the filter bank infrastructure used for Discrete Wavelet Packet Transform implementation. Hence the spectrum analysis is done at virtually no additional cost. From simulation studies, assuming the presence of Licensed Users (LU), it was found that the performance of complete WPMCM transceiver is excellent for extremely narrow band LU and good for wide band LU.
- Spectrum carving was identified as the main issue that limits the performance of the wavelet based spectrum estimator. Spectrum carving emerges because the wavelet filters have a non-zero transition band. When the wavelet packet tree structure is constructed by

iterating the wavelet filters, it results in residual artifacts which sometimes occur in the stop band

• The impact of spectrum carving is not significant for extremely narrow band cases but can be deleterious for wideband sources. This also explains why WPMCM transceivers are excellent for extremely narrow band LU

### 7.2 Future Research Topics

### 7.2.1 Addressing the spectrum carving issue

As already discussed before, in section 6.1.1, spectrum carving emerges because the cascaded filters that should theoretically be confined to a particular frequency band have residual component in other bands. With respect to WPMCM transmitter, a particular multi-carrier component that should theoretically only have frequency components at particular band may also have residual components at other band. In the context of cognitive radio applications this may complicate the shaping of the CR characteristics in order to enable it to cohabitate with LU.

Fred Harris in [40] has proposed the employment of Interpolated Tree Orthogonal Multiplexing (ITOM) as an antidote for spectrum carving. While, in WPMCM (as well as in the wavelet based spectrum estimation), only two half band filters are employed in the filter bank structure, ITOM uses four half band interpolating filters in its filter bank structure. These four half band filters are low pass filter, high pass filter and two Hilbert Transform filters [41]. The mitigation of spectrum carving is done by careful selection of half band filters in each branch of ITOM. Unfortunately, both [40] and [41] do not provide the specifics on the actual rules that govern the placement of particular half band filters in different branches of the ITOM tree. The results provided in [40]-[41] appear promising and hence it will be interesting to investigate the possibility of exploiting the tree structured used by ITOM for spectrum estimation purposes.

### 7.2.2 Investigation on time resolution issue

Time information is very important when the spectrum estimation is employed as spectrum sensing tool for cognitive radio system to detect the licensed user. However, it is also clear that there is ambiguity about the definition of time resolution from the perspective of spectrum estimation and the time resolution from the perspective of wavelet packet decomposition. Before we perform a research on the time resolution aspect of the wavelet based spectrum estimation, this ambiguity should be resolved.

If we assume that the sampling rate is fixed, the time resolution would be related to the size of the snapshot window and hence, the number of samples. From chapter 3, it can be found that difference number of samples (and snapshot window) seems to impact the performance of the wavelet based estimates. While in periodogram this can be easily explained analytically by using Fourier Transform and the concept of rectangular windowing, analytical explanation for the case of wavelet based estimates still needs to be investigated.

### 7.2.3 Investigation on the impact of fading channel on WPMCM Transceiver

In chapter 5, we combined the wavelet based spectrum estimator and WPMCM in one transceiver and tested its operation. However the channel considered was a benign AWGN channel. Hence it would be important that the work is extended to study the operation of the WPMCM transceiver under realistic wireless communication channel with multi-paths and in the presence of shadowing.

# **7.2.4** Comparative analysis with the combination of traditional OFDM and FFT based spectrum estimators

While we have already combined the WPMCM and wavelet based spectrum estimators to form spectrum shaping technique based on wavelet, it is always possible to develop a similar technique by combining OFDM as a mature multi-carrier modulation technique with FFT based spectrum estimator. Even though it is not quite appropriate to compare these two spectrum shaping techniques due to the existence of broad range of performance metrics, it is always interesting to observe the BER performance of the both system in order to find out the position of one technique with respect to the other. The impact of both AWGN channel and fading channel should also be investigated. The result of this study can play a role as a basis for further comprehensive comparative analysis between two systems.

#### 7.2.5 Analysis of WPMCM based CR impact on primary user

In Chapter 5 we studied as to how various sources (single tone, multi tone, partial band and swept tone) influenced the performance of a WPMCM transceiver. A similar study on how these sources were affected by the WPMCM based CR operation is also due and necessary.

### 7.2.6 Pilot Channel and Echo Cancellation issue in WPMCM transceiver

As already mentioned in chapter 4, we have not considered the mechanism to distinguish the signal transmitted by LU from the signal produced by other CR users. Furthermore, we also has not taken into account the need for echo cancellation to make sure that the spectrum estimator of a CR system is not burdened by the signal that is produced by its own transmitter. It has also been mentioned that the mechanism that is introduced for WPMCM transceiver here requires a pilot channel. The pilot channel is needed by a CR system to inform its communication pair (another CR system) about the subcarriers that are currently used. We propose all this issues in WPMCM transceiver as another important research topic.

### 7.2.7 Investigation on the complexity issues of wavelet based spectrum estimation

Last but not the least, a thorough complexity analysis of the Wavelet packet based spectrum estimator as well as the WPMCM transceiver is important. Theoretically speaking, the wavelet based spectrum estimation is quite simple since it is just implemented based on iterative half band filtering and down sampling by factor-2. Nevertheless, it is important to find out the position of this technique in relation to existing spectrum estimation approaches from the stand point of system complexity. Information about complexity aspect would be extremely important for comparative analysis between different spectrum estimation techniques in order to assess whether the advantages offered by each technique is worthy or not after the complexity aspect is considered.

Some aspects that might be included with respect to investigation on complexity are the optimal decomposition levels, how the complexity grows with increasing decomposition level and/or increasing wavelet decomposition filter length, what kind of optimizations can be employed in the computer simulation programs to reduce the complexity and in the hardware realization. From the design perspective, it would also be useful to try other semi definite programming tools apart from SeDuMi especially to solve wavelet design optimization problems with considerably large filter length and large regularity index. Further study on convex optimization problems might also be required.

### 7.2.8 Treatment of Finite Length Data Samples

The wavelet based spectrum estimator that is developed in this thesis work is based on Wavelet Packet Transform. As any spectral estimation method, the wavelet transform suffers from discontinuities (abrupt transition) in the edge of the data blocks which may lead to the emergence of additional high frequency components in the estimates. This undesirable effect is known as the edge effect [44]. The current wavelet based spectrum estimator has not included additional technique to handle this edge effect. With respect to this issue, we propose further study on existing techniques such as circular convolution and symmetric extension and the possibility of employing them in our wavelet based spectrum estimator. In addition, the windowing and segment overlap approach introduced by Welch on Periodogram may also be employed in wavelet based approach. The window may be applied on the received samples before passing the samples through the wavelet packet tree. Apart from the investigation on the employment of these possible techniques, further analysis with respect to Parseval theory as well as some trade offs related to the possibility of information loss needs to be conducted. We consider all of these issues as possible further research.

## **7.2.9** Position of wavelet based spectrum estimation with respect to existing cognitive radio spectrum sensing techniques

While it is our desire to use wavelet based spectrum estimation technique for dynamic spectrum access, we confined ourselves in this work to address the problems of spectrum estimation and hardly touched the real issues of spectrum sensing like signal detection in the presence of noise, exploitation of signal features such as cyclo-stationarity for detection. Furthermore, throughout this thesis work, we always compared the wavelet based approach with Periodogram and Welch methods. This is because these are also estimation techniques as against Energy Detection and Cyclostationary Feature Detection, which are spectrum sensing techniques and hence deal with detection problems. Hence, in the future we would like to alter the framework of the research topic to include spectrum sensing problems, particularly, in the context of Cognitive Radio and Dynamic Spectrum Access applications.

### 7.3 Concluding Remarks

At this juncture it is worth underlining the fact that the work carried out in this thesis, on applying wavelets and wavelet packet transform for spectrum estimation, is pioneering (albeit preliminary) in many ways. Much work remains to be done before the proposed methodology can be considered mature and viable for real-time systems.

In this work we focused on realizing the system and on evaluating the frequency resolution and accuracy offered by the wavelet based solution. The challenges contending the implementation and possible improvements, as suggested in Chapter 6 and Section 7.2, have to be addressed in the future.

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### APPENDIX A: ADDITIONAL TABLES FOR CHAPTER 4

# A.1 Investigation on the relationship between the properties of decomposition filter and the performance of wavelet based estimates

As already mentioned in section 4.1, the relationship between the properties of decomposition filter and the performance of the corresponding wavelet based estimates is investigated by running simulations that involve nine different types of sources occupying different band in the range of normalized frequency from 0 to  $\pi$ . These nine types of sources (from type A to type I) have been listed in table 4.1. The correlation coefficient for each pair of indicators is calculated 100 times for each source. Later, the average and the standard deviation of the correlation coefficients,  $E[\rho]$  and  $\sigma_{\rho}$  respectively, are calculated. The results are all tabulated in tables A.1.1 to A.1.9 of this appendix. These tables illustrate the average and standard deviation of 100 correlation coefficients computed for each pair of indicators in each source. The criteria that we use to describe the correlation between the two indicators are indicated in table 4.2.

<b>Table A.1.1</b> Correlation coefficient between the indicators found in the frequency response of wavelet decomposition					
filter and wavelet based estimates for the case of Type-A source					
Wanalat	Average of Standard				
vv avelet	Indicators of the Wavelet based	Wavelet based the Deviation of the			

Wavelet Decomposition Filter Indicators	Indicators of the Wavelet based Estimates	Average of the Correlation Coefficients	Standard Deviation of the Correlation Coefficients	Inferences
Width of transition band	Average power at the unoccupied band	0.99931	0.00013	Strongly correlated
Variance of the pass-band	Average power at the unoccupied band	0.80659	0.00263	Strongly correlated
Variance of the stop-band	Average power at the unoccupied band	0.83217	0.00251	Strongly correlated
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.89532	0.00207	Strongly correlated
Width of transition band	Width of the transition band	0.94444	0.02343	Strongly correlated
Variance of the pass-band	Width of the transition band	0.63351	0.06876	Correlated
Variance of the stop-band	Width of the transition band	0.66249	0.06622	Correlated
Average power at the rejection band (stop band)	Width of the transition band	0.74236	0.05849	Correlated
Width of transition band	Maximum side lobe	0.98150	0.00630	Strongly correlated
Variance of the pass-band	Maximum side lobe	0.80968	0.03611	Strongly correlated
Variance of the stop-band	Maximum side lobe	0.83265	0.03341	Strongly correlated
Average power at the rejection band (stop band)	Maximum side lobe	0.88981	0.02615	Strongly correlated
Width of transition band	Power variance in the unoccupied band	0.81792	0.01783	Strongly correlated
Variance of the pass-band	Power variance in the unoccupied band	0.31594	0.02827	Weakly correlated
Variance of the stop-band	Power variance in the unoccupied band	0.35339	0.02806	Weakly correlated
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.46181	0.02694	Weakly correlated
Width of transition band	Frequency resolution of the estimate	0.59106	0.19066	Correlated (Note: the variance of the correlation is significant)

Variance of the pass-band	Frequency resolution of the estimate	0.25523	0.24093	Not clear
Variance of the stop-band	Frequency resolution of the estimate	0.28390	0.23724	Not clear
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.36014	0.22949	Not clear
Width of transition band	Power variance in the occupied band	0.62644	0.14075	Correlated (Note: the variance of the correlation is significant)
Variance of the pass-band	Power variance in the occupied band	0.13265	0.15825	Not clear
Variance of the stop-band	Power variance in the occupied band	0.16664	0.15909	Not clear
Average power at the rejection band (stop band)	Power variance in the occupied band	0.26754	0.15992	Not clear

 Table A.1.2 Correlation coefficient between the indicators found in the frequency response of wavelet decomposition filter and wavelet based estimates for the case of Type-B source

Wavelet Decomposition Filter Indicators	Indicators of the Wavelet based Estimates	Average of the Correlation Coefficients	Standard Deviation of the Correlation Coefficients	Inferences
Width of transition band	Average power at the unoccupied band	0.55062	0.00000	Correlated
Variance of the pass- band	Average power at the unoccupied 0.47323		0.00000	Weakly Correlated
Variance of the stop- band	Average power at the unoccupied band	0.46972	0.00000	Weakly Correlated
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.51847	0.00000	Correlated
Width of transition band	Maximum side lobe	0.65689	0.00000	Correlated
Variance of the pass- band	Maximum side lobe	0.76122	0.76122 0.00000	
Variance of the stop- band	Maximum side lobe	0.75728	0.00000	Correlated
Average power at the rejection band (stop band)	Maximum side lobe	0.73158	0.00000	Correlated
Width of transition band	Power variance in the unoccupied band	0.46729	0.00000	Weakly Correlated
Variance of the pass- band	Power variance in the unoccupied band	0.30188	0.00000	Weakly Correlated
Variance of the stop- band	Power variance in the unoccupied band	0.30023	0.00000	Weakly Correlated
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.37162	0.00000	Weakly Correlated
Width of transition band	Frequency resolution of the estimate	0.00000	0.00000	Uncorrelated
Variance of the pass- band	Frequency resolution of the estimate	0.00000	0.00000	Uncorrelated
Variance of the stop- band	Frequency resolution of the estimate	0.00000 0.00000 Uncor		Uncorrelated
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.00000	0.00000	Uncorrelated

Wavelet Decomposition Filter Indicators	Indicators of the Wavelet based Estimates	Average of the Correlation Coefficients	Standard Deviation of the Correlation Coefficients	Inferences	
Width of transition band	Average power at the unoccupied band	0.99256	0.00069	Strongly correlated	
Variance of the pass-band	Average power at the unoccupied band	0.85694	0.00279	Strongly correlated	
Variance of the stop-band	Average power at the unoccupied band	0.87916	0.00262	Strongly correlated	
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.93213	0.00205	Strongly correlated	
Width of transition band	Width of the transition band	0.97187	0.01488	Strongly correlated	
Variance of the pass-band	Width of the transition band	0.74314	0.04199	Correlated	
Variance of the stop-band	Width of the transition band	0.77079	0.04012	Correlated	
Average power at the rejection band (stop band)	Width of the transition band	0.83909	0.03498	Strongly Correlated	
Width of transition band	Maximum side lobe	0.94954	0.00940	Strongly correlated	
Variance of the pass-band	Maximum side lobe	0.85827	0.02128	Strongly correlated	
Variance of the stop-band	Maximum side lobe	0.87855	0.01893	Strongly correlated	
Average power at the rejection band (stop band)	Maximum side lobe	0.92382	0.01379	Strongly correlated	
Width of transition band	Power variance in the unoccupied band	0.92633	0.01257	Strongly correlated	
Variance of the pass-band	Power variance in the unoccupied band	0.51394	0.02696	Correlated	
Variance of the stop-band	Power variance in the unoccupied band	0.54962	0.02663	Correlated	
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.64760	0.02484	Correlated	
Width of transition band	Frequency resolution of the estimate	0.69129	0.17072	Correlated (Note: the variance of the correlation is significant)	
Variance of the pass-band	Frequency resolution of the estimate	0.53018	0.24751	Correlated (Note: the variance of the correlation is significant)	
Variance of the stop-band	Frequency resolution of the estimate	0.54951	0.24426	Correlated (Note: the variance of the correlation is significant)	
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.59824	0.23185	Correlated (Note: the variance of the correlation is significant)	
Width of transition band	Power variance in the occupied band	0.77501	0.13590	Correlated (Note: the variance of the correlation is significant)	
Variance of the pass-band	Power variance in the occupied band	0.41006	0.18044	Not clear	
Variance of the stop-band	Power variance in the occupied band	0.44137	0.17820	Not clear	
Average power at the rejection band (stop band)	Power variance in the occupied band	0.52661	0.17213	Correlated (Note: the variance of the correlation is significant)	

Table A.1.3 Correlation coefficient between the indicators found in the frequency response of wavelet
decomposition filter and wavelet based estimates for the case of Type-C source

Wavelet Decomposition Filter Indicators	Indicators of the Wavelet based Estimates	Average of the Correlation Coefficients	Standard Deviation of the Correlation Coefficients	Inferences
Width of transition band	Average power at the unoccupied band	0.98966	0.00114	Strongly correlated
Variance of the pass- band	Average power at the unoccupied band	0.86821	0.00375	Strongly correlated
Variance of the stop- band	Average power at the unoccupied band	0.88944	0.00352	Strongly correlated
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.93971	0.00271	Strongly correlated
Width of transition band	Width of the transition band	0.94940	0.02322	Strongly correlated
Variance of the pass- band	Width of the transition band	0.69501	0.05658	Correlated
Variance of the stop- band	Width of the transition band	0.72547	0.05448	Correlated
Average power at the rejection band (stop band)	Width of the transition band	0.79983	0.04819	Correlated
Width of transition band	Maximum side lobe	0.89691	0.00749	Strongly correlated
Variance of the pass- band	Maximum side lobe	0.95643	0.00591	Strongly correlated
Variance of the stop- band	Maximum side lobe	0.96711	0.00483	Strongly correlated
Average power at the rejection band (stop band)	Maximum side lobe	0.98431	0.00237	Strongly correlated
Width of transition band	Power variance in the unoccupied band	0.96424	0.00994	Strongly correlated
Variance of the pass- band	Power variance in the unoccupied band	0.61181	0.02753	Correlated
Variance of the stop- band	Power variance in the unoccupied band	0.64527	0.02714	Correlated
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.73444	0.02488	Correlated
Width of transition band	Frequency resolution of the estimate	0.69845	0.16642	Correlated (Note: the variance of the correlation is significant)
Variance of the pass- band	Frequency resolution of the estimate	0.58134	0.26444	Correlated (Note: the variance of the correlation is significant)
Variance of the stop- band	Frequency resolution of the estimate	0.59857	0.25845	Correlated (Note: the variance of the correlation is significant)
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.63984	0.24118	Correlated (Note: the variance of the correlation is significant)
Width of transition band	Power variance in the occupied band	0.74891	0.14368	Correlated (Note: the variance of the correlation is significant)
Variance of the pass- band	Power variance in the occupied band	0.44006	0.18129	Not clear
Variance of the stop- band	Power variance in the occupied band	0.46776	0.17894	Not clear
Average power at the rejection band (stop band)	Power variance in the occupied band	0.54249	0.17316	Correlated (Note: the variance of the correlation is significant)

Table A.1.4 Correlation coefficient between the indicators found in the frequency response of wavelet
decomposition filter and wavelet based estimates for the case of Type-D source

Wavelet Decomposition Filter Indicators	Indicators of the Wavelet based Estimates	Average of the CorrelationStandard Deviation of the CorrelationCoefficientsCorrelation Coefficients		Inferences	
Width of transition band	Average power at the unoccupied band	0.99935	0.00012	Strongly correlated	
Variance of the pass- band	Average power at the unoccupied band	0.80592	0.00257	Strongly correlated	
Variance of the stop- band	Average power at the unoccupied band	0.83152	0.00246	Strongly correlated	
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.89477	0.00203	Strongly correlated	
Width of transition band	Width of the transition band	0.93717	0.02959	Strongly correlated	
Variance of the pass- band	Width of the transition band	0.61136	0.07941	Correlated	
Variance of the stop- band	Width of the transition band	0.64109	0.07615	Correlated	
Average power at the rejection band (stop band)	Width of the transition band	0.72361	0.06758	Correlated	
Width of transition band	Maximum side lobe	0.98103	0.00678	Strongly correlated	
Variance of the pass- band	Maximum side lobe	0.81110	0.03869	Strongly correlated	
Variance of the stop- band	Maximum side lobe	0.83408	0.03573	Strongly correlated	
Average power at the rejection band (stop band)	Maximum side lobe	0.89111	0.02813	Strongly correlated	
Width of transition band	Power variance in the unoccupied band	0.81715	0.01806	Strongly correlated	
Variance of the pass- band	Power variance in the unoccupied band	0.31510	0.02885	Weakly correlated	
Variance of the stop- band	Power variance in the unoccupied band	0.35248	0.02865	Weakly correlated	
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.46081	0.02747	Weakly correlated	
Width of transition band	Frequency resolution of the estimate	0.54509	0.24180	Correlated (Note: the variance of the correlation is significant)	
Variance of the pass- band	Frequency resolution of the estimate	0.21580	0.29142	Not clear	
Variance of the stop- band	Frequency resolution of the estimate	0.24219	0.29038	Not clear	
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.31514	0.28600	Not clear	
Width of transition band	Power variance in the occupied band	0.63421	0.13500	Correlated (Note: the variance of the correlation is significant)	
Variance of the pass- band	Power variance in the occupied band	0.13317	0.16224	Not clear	
Variance of the stop- band	Power variance in the occupied band	0.16777	0.16188	Not clear	
Average power at the rejection band (stop band)	Power variance in the occupied band	0.27015	0.16058	Not clear	

Table A.1.5 Correlation coefficient between the indicators found in the frequency response of wavelet
decomposition filter and wavelet based estimates for the case of Type-E source

Wavelet Decomposition Filter Indicators	Indicators of the Wavelet based Estimates Coefficients Coefficients Coefficients		Inferences	
Width of transition band	Average power at the unoccupied band	0.99245	0.00182	Strongly correlated
Variance of the pass- band	Average power at the unoccupied band	0.85634	0.00822	Strongly correlated
Variance of the stop- band	Average power at the unoccupied band	0.87859	0.00744	Strongly correlated
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.93166	0.00548	Strongly correlated
Width of transition band	Width of the transition band	0.93781	0.03238	Strongly correlated
Variance of the pass- band	Width of the transition band	0.66750	0.07803	Correlated
Variance of the stop- band	Width of the transition band	0.69543	0.07506	Correlated
Average power at the rejection band (stop band)	Width of the transition band	0.76897	0.06693	Correlated
Width of transition band	Maximum side lobe	0.96260	0.00856	Strongly correlated
Variance of the pass- band	Maximum side lobe	0.84985	0.03230	Strongly correlated
Variance of the stop- band	Maximum side lobe	0.86957	0.02951	Strongly correlated
Average power at the rejection band (stop band)	Maximum side lobe	0.91646	0.02263	Strongly correlated
Width of transition band	Power variance in the unoccupied band	0.93981	0.02252	Strongly correlated
Variance of the pass- band	Power variance in the unoccupied band	0.55519	0.05918	Correlated
Variance of the stop- band	Power variance in the unoccupied band	0.58990	0.05655	Correlated
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.68346	0.05020	Correlated
Width of transition band	Frequency resolution of the estimate	0.45059	0.25808	Not clear
Variance of the pass- band	Frequency resolution of the estimate	0.12011	0.31270	Not clear
Variance of the stop- band	Frequency resolution of the estimate	0.14440	0.31262	Not clear
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.21458	0.30757	Not clear
Width of transition band	Power variance in the occupied band	0.75371	0.12573	Correlated (Note: the variance of the correlation is significant)
Variance of the pass- band	Power variance in the occupied band	0.40546	0.17126	Not clear
Variance of the stop- band	Power variance in the occupied band	0.43507	0.16959	Not clear
Average power at the rejection band (stop band)	Power variance in the occupied band	0.51677	0.16264	Correlated (Note: the variance of the correlation is significant)

Table A.1.6 Correlation coefficient between the indicators found in the frequency response of wavelet
decomposition filter and wavelet based estimates for the case of Type-F source

Wavelet Decomposition Filter Indicators	Indicators of the Wavelet based Estimates	Average of the Correlation Coefficients	Standard Deviation of the Correlation Coefficients	Inferences	
Width of transition band	Average power at the unoccupied band	0.98907	0.00249	Strongly correlated	
Variance of the pass- band	Average power at the unoccupied band	0.86925	0.00845	Strongly correlated	
Variance of the stop- band	Average power at the unoccupied band	0.89038	0.00766	Strongly correlated	
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.94031	0.00561	Strongly correlated	
Width of transition band	Width of the transition band	0.85296	0.09626	Strongly correlated	
Variance of the pass- band	Width of the transition band	0.65903	0.11969	Correlated (Note: the variance of the correlation is significant)	
Variance of the stop- band	Width of the transition band	0.68287	0.11776	Correlated (Note: the variance of the correlation is significant)	
Average power at the rejection band (stop band)	Width of the transition band	0.74254	0.11269	Correlated (Note: the variance of the correlation is significant)	
Width of transition band	Maximum side lobe	0.91682	0.02458	Strongly correlated	
Variance of the pass- band	Maximum side lobe	0.84327	0.03704	Strongly correlated	
Variance of the stop- band	Maximum side lobe	0.85720	0.03430	Strongly correlated	
Average power at the rejection band (stop band)	Maximum side lobe	0.89388	0.02660	Strongly correlated	
Width of transition band	Power variance in the unoccupied band	0.95505	0.02323	Strongly correlated	
Variance of the pass- band	Power variance in the unoccupied band	0.60875	0.06844	Correlated	
Variance of the stop- band	Power variance in the unoccupied band	0.64082	0.06596	Correlated	
Average power at the rejection band (stop band)	verage power at the ejection band (stop band) Power variance in the unoccupied band		0.05864	Correlated	
Width of transition band	th of transition Frequency resolution of the band estimate		0.22749	Not clear	
Variance of the pass- band	Variance of the pass- band Frequency resolution of the estimate		0.28952	Not clear	
Variance of the stop- band	Variance of the stop- band Erequency resolution of the estimate		0.28833	Not clear	
Average power at the rejection band (stop band)	Average power at the rejection band (stop band) Frequency resolution of the estimate		0.28157	Not clear	
Width of transition band	Width of transition Power variance in the occupied band band		0.22860	Correlated (Note: the variance of the correlation is significant)	
Variance of the pass- band	Power variance in the occupied band	0.36587	0.24378	Not clear	
Variance of the stop- band	Power variance in the occupied band	0.38770	0.24323	Not clear	
Average power at the rejection band (stop band)	verage power at the 'ejection band (stop band) Power variance in the occupied band		0.24219	Not clear	

Table A.1.7 Correlation coefficient between the indicators found in the frequency response of wavelet
decomposition filter and wavelet based estimates for the case of Type-G source

Wavelet Decomposition Filter Indicators	Wavelet ecomposition Filter Indicators Indicators		Standard Deviation of the Correlation Coefficients	Inferences
Width of transition band	Width of transition Average power at the unoccupied band band		0.00110	Strongly correlated
Variance of the pass- band	Average power at the unoccupied band	0.85102	0.00605	Strongly correlated
Variance of the stop- band	Average power at the unoccupied band	0.87497	0.00549	Strongly correlated
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.93039	0.00409	Strongly correlated
Width of transition band	Width of the transition band	0.89661	0.05495	Strongly correlated
Variance of the pass- band	Width of the transition band	0.67414	0.09552	Correlated
Variance of the stop- band	Width of the transition band	0.69983	0.09226	Correlated
Average power at the rejection band (stop band)	Width of the transition band	0.76538	0.08321	Correlated
Width of transition band	Maximum side lobe	0.99379	0.00298	Strongly correlated
Variance of the pass- band	Maximum side lobe	0.78070	0.03222	Correlated
Variance of the stop- band	Maximum side lobe	0.80746	0.02982	Strongly correlated
Average power at the rejection band (stop band)	Maximum side lobe	0.87345	0.02405	Strongly correlated
Width of transition band	Power variance in the unoccupied band	0.99212	0.00413	Strongly correlated
Variance of the pass- band	Power variance in the unoccupied band	0.75848	0.03053	Correlated
Variance of the stop- band	Power variance in the unoccupied band	0.78872	0.02853	Correlated
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.86121	0.02347	Strongly correlated
Width of transition band	7idth of transition Frequency resolution of the band estimate		0.08288	Strongly correlated
Variance of the pass- band	Frequency resolution of the estimate	0.56308	0.19682	Correlated (Note: the variance of the correlation is significant)
Variance of the stop- band	Frequency resolution of the estimate	0.59285	0.18915	Correlated (Note: the variance of the correlation is significant)
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.67023	0.16926	Correlated (Note: the variance of the correlation is significant)
Width of transition band	Power variance in the occupied band	0.89810	0.04922	Strongly Correlated
Variance of the pass- band	Power variance in the occupied band	0.54856	0.10572	Correlated (Note: the variance of the correlation is significant)
Variance of the stop- band	iance of the stop- band Power variance in the occupied band		0.10223	Correlated (Note: the variance of the correlation is significant)
Average power at the rejection band (stop band)	Power variance in the occupied band	0.67196	0.09312	Correlated

Table A.1.8 Correlation coefficient between the indicators found in the frequency response of wa	welet
decomposition filter and wavelet based estimates for the case of Type-H Source	

Wavelet Decomposition Filter Indicators	Wavelet mposition Filter Indicators Indicators of the Wavelet based Estimates		Standard Deviation of the Correlation Coefficients	Inferences	
Width of transition band	Average power at the unoccupied band	0.99779	0.00037	Strongly correlated	
Variance of the pass- band	Average power at the unoccupied band	0.81262	0.00575	Strongly correlated	
Variance of the stop- band	Average power at the unoccupied band	0.83862	0.00531	Strongly correlated	
Average power at the rejection band (stop band)	Average power at the unoccupied band	0.90150	0.00414	Strongly correlated	
Width of transition band	Width of the transition band	0.89467	0.02671	Strongly correlated	
Variance of the pass- band	Width of the transition band	0.54922	0.05721	Correlated	
Variance of the stop- band	Width of the transition band	0.58022	0.05555	Correlated	
Average power at the rejection band (stop band)	Width of the transition band	0.66722	0.05068	Correlated	
Width of transition band	Maximum side lobe	0.66816	0.06678	Correlated	
Variance of the pass- band	Maximum side lobe	0.66139	0.06954	Correlated	
Variance of the stop- band	Maximum side lobe	0.66702	0.07076	Correlated	
Average power at the rejection band (stop band)	Maximum side lobe	0.67699	0.07224	Correlated	
Width of transition band	Power variance in the unoccupied band	0.99240	0.00342	Strongly Correlated	
Variance of the pass- band	Power variance in the unoccupied band	0.77416	0.02759	Correlated	
Variance of the stop- band	Power variance in the unoccupied band	0.80385	0.02570	Strongly Correlated	
Average power at the rejection band (stop band)	Power variance in the unoccupied band	0.87411	0.02090	Strongly Correlated	
Width of transition band	Frequency resolution of the estimate	0.86444	0.10055	Strongly Correlated	
Variance of the pass- band	Frequency resolution of the estimate	0.60784	0.22027	Correlated (Note: the variance of the correlation is significant)	
Variance of the stop- band	Frequency resolution of the estimate	0.63514	0.21212	Correlated (Note: the variance of the correlation is significant)	
Average power at the rejection band (stop band)	Frequency resolution of the estimate	0.70569	0.19026	Correlated (Note: the variance of the correlation is significant)	
Width of transition band	ransition Power variance in the occupied band		0.02973	Strongly Correlated	
Variance of the pass- band	Power variance in the occupied band	0.63746	0.06851	Correlated	
Variance of the stop- band	Power variance in the occupied band	0.66863	0.06639	Correlated	
Average power at the rejection band (stop band)	rage power at the ection band (stop band) Power variance in the occupied band		0.05949	Correlated	

Table A.1.9 Correlation coefficient between the indicators found in the frequency response of wavelet
decomposition filter and wavelet based estimates for the case of Type-I source

# A.2 Example of impulse responses of the high and low pass filters of the optimally designed wavelets using SDP Approach

Tables A.2.1 and A.2.2 give the coefficients of the designed wavelet filter illustrated in figure 4.5 and 4.6, respectively

Indox	Low Poss Filtor	High Pass Filtor	Indox	Low Poss Filtor	High Pass Filtor
muex	Low 1 ass Filter	ingi i ass ritter	muex	Low 1 ass Filter	ingn i ass ritter
1	0.0000	-0.0201	16	-0.0611	-0.0074
2	-0.0000	0.1437	17	0.0206	-0.0395
3	0.0001	-0.4279	18	0.0892	-0.0017
4	0.0002	0.6521	19	-0.0357	0.0223
5	-0.0006	-0.4454	20	-0.1296	0.0055
6	0.0001	-0.0789	21	0.0469	-0.0097
7	0.0024	0.3037	22	0.1943	-0.0049
8	-0.0026	-0.0350	23	-0.0350	0.0026
9	-0.0049	-0.1943	24	-0.3037	0.0024
10	0.0097	0.0469	25	-0.0789	-0.0001
11	0.0055	0.1296	26	0.4454	-0.0006
12	-0.0223	-0.0357	27	0.6521	-0.0002
13	-0.0017	-0.0892	28	0.4279	0.0001
14	0.0395	0.0206	29	0.1437	0.0000
15	-0.0074	0.0611	30	0.0201	0.0000

**Table A.2.1** Optimal filter coefficients for filter length L=30, K-regularity=7 Transition Band= $0.2\pi$ 

<b>Table A.2.2</b> Optimal filter coefficients for filter length $L$ =40, $K$ -regularity=8 Transition Band=0.2 $\pi$						
Index	Low Pass Filter	High Pass Filter	Index	Low Pass Filter	High Pass Filter	
1	0.0000	-0.0071	21	-0.0404	-0.0018	
2	0.0000	0.0630	22	0.0035	0.0261	
3	0.0000	-0.2416	23	0.0586	0.0047	
4	0.0000	0.5128	24	-0.0110	-0.0154	
5	0.0001	-0.6110	25	-0.0818	-0.0053	
6	-0.0000	0.2958	26	0.0194	0.0077	
7	-0.0001	0.1849	27	0.1122	0.0044	
8	0.0004	-0.2882	28	-0.0249	-0.0030	
9	0.0001	-0.0275	29	-0.1538	-0.0028	
10	-0.0013	0.2128	30	0.0179	0.0006	
11	0.0006	-0.0179	31	0.2128	0.0013	
12	0.0028	-0.1538	32	0.0275	0.0001	
13	-0.0030	0.0249	33	-0.2882	-0.0004	
14	-0.0044	0.1122	34	-0.1849	-0.0001	
15	0.0077	-0.0194	35	0.2958	0.0000	
16	0.0053	-0.0818	36	0.6110	0.0001	
17	-0.0154	0.0110	37	0.5128	-0.0000	
18	-0.0047	0.0586	38	0.2416	0.0000	
19	0.0261	-0.0035	39	0.0630	-0.0000	
20	0.0018	-0.0404	40	0.0071	0.0000	

Figure 4.5 and 4.6 describes the impulse response of the high and low pass filters of the optimally designed wavelets for L=30 K=7  $B=0.2\pi$  and L=40 K=8  $B=0.2\pi$ , respectively. The coefficients of the designed wavelet filter for L=30 K=7  $B=0.2\pi$  and L=40 K=8  $B=0.2\pi$  are presented in appendix A.2 (tables A.2.1 and A.2.2, respectively).

### APPENDIX B: LIST OF PUBLICATION

### Accepted Paper:

- [1] D. D. Ariananda, M.K. Lakshmanan, and H. Nikookar," A Survey on Spectrum Sensing Techniques for Cognitive Radio", IEEE Proceeding of 2nd International Workshop on Cognitive Radio and Advanced Spectrum Management (CogART) 2009 pages 74-79, May 2009
- [2] D. D. Ariananda, M.K. Lakshmanan, and H. Nikookar, "A Study on Application of Wavelets and Filter Banks for Cognitive Radio Spectrum Estimation", to appear in Proc. European Wireless Technology Conference (EuWiT2009), October 2009.
- [3] D. D. Ariananda, M.K. Lakshmanan, and H. Nikookar, "A Study on the Application of Wavelet Packet Transforms to Cognitive Radio Spectrum Estimation", IEEE Proceeding of 4th International Conference on Cognitive Radio Oriented Wireless Networks and Communications 2009 (CrownCom '09), June 2009.
- [4] D. D. Ariananda, M.K. Lakshmanan, and H. Nikookar, "An Investigation of Wavelet Packet Transform for Spectrum Estimation", to appear in Proc. The 12<sup>th</sup> International Symposium on Wireless Personal Multimedia Communications (WPMC '09), September 2009
- [5] M.K. Lakshmanan, D.D. Ariananda, and H. Nikookar,"Cognitive Radio Transmission and Spectrum Sensing using a Wavelet Packet Transceiver", to appear in Proc. The 20<sup>th</sup> Personal, Indoor and Mobile Radio Communications Symposium 2009 (PIMRC'09), September 2009
- [6] D. D. Ariananda, M.K. Lakshmanan, and H. Nikookar, "Design of Best Wavelet Packet Bases for Spetrum Estimation", to appear in Proc. The 20<sup>th</sup> Personal, Indoor and Mobile Radio Communications Symposium 2009 (PIMRC'09), September 2009