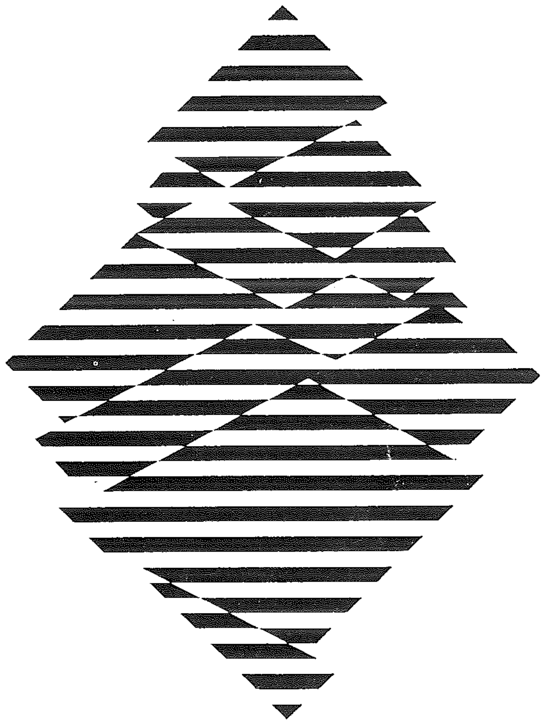
CTwa3320

Syllabus Grondwatermechanica CTwa3320

Maart 2000

Prof.dr.ir. F.B.J. Barends



TU Delft
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Syllabus Grondwatermechanica CTwa3320

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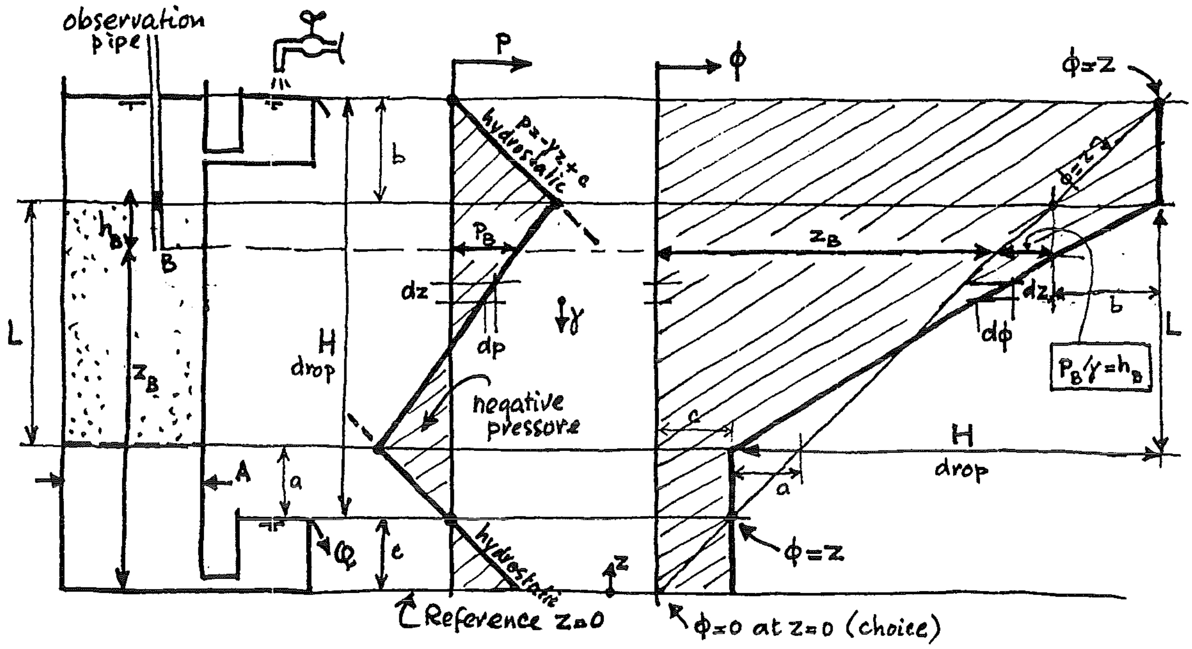
Darcy's law



Observation pipe measures the pressure at opening (point B) by watercollumn inside h_B (pressurehead): $h_B = p_B/\gamma$. Position of pipe opening B at elevationhead z_B

Sum is the piezometric head: ϕ

Referenc (z=0) is arbitrary.

In hydrostatic state ϕ is constant, because then $p=-\gamma z$ + constant.

$$\boxed{\phi = \frac{p}{\gamma} + z}$$ PIEZOMETRIC HEAD OR POTENTIAL HEAD

Darcy found $\boxed{Q = KA\,H/L}$; H/L : gradient piezometric head

Discharge [m³/s] ; constant (depending on type of material) [m/s]

Generalized $\boxed{q = \frac{Q}{A} = K\frac{H}{L} = -K\frac{d\phi}{dz}}$ (flow in vertical direction)

specific discharge [m/s]; $\boxed{q = n.v}$ (n: porosity ; v: absolute velocity real velocity)

Because q depends on the difference of ϕ, the reference (z=0) does not matter.

In more dimensions $\boxed{q_x = -K\frac{d\phi}{dx}, \; q_y = -K\frac{d\phi}{dy}, \; q_z = -K\frac{d\phi}{dz}}$ q, q_z, q_x, q_y

minus sign because q flows towards lower potential or: q is positive when $d\phi$ is negative.

intrinsic permeability [m²]

$\boxed{K = \frac{k\gamma}{\mu}}$ dynamic viscosity $\mu = \rho\nu$; ν: kinematic viscosity (water 20° $\nu \approx 10^{-6}$ m²/s)

hydraulic permeability [m/s] $\boxed{\gamma = \rho g} \approx 10^4$ N/m³ specific weight of water

Darcy's law in terms of pressure p:

$$\boxed{q_x = -\frac{k}{\mu}\frac{dp}{dx}, \quad q_y = -\frac{k}{\mu}\frac{dp}{dy}, \quad q_z = -\frac{k}{\mu}\left(\frac{dp}{dz}+\gamma\right)}$$

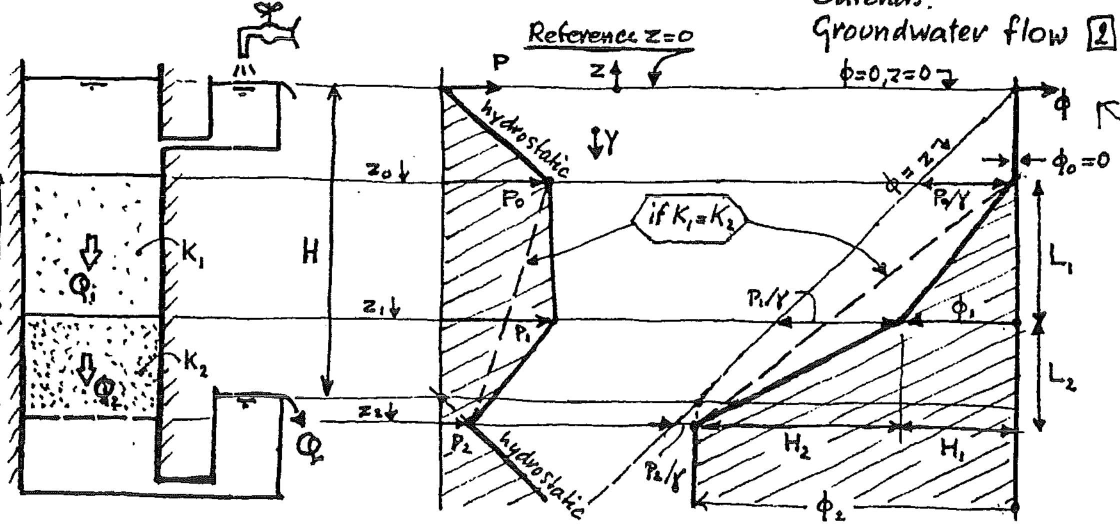
$k \approx c\tilde{T}D^2 \frac{n^3}{(1-n)^2}$; n: porosity

grainsize D_{20} (heterogeneity)

tortuosity (anisotropy)

constant

Darcy's law
Heterogeneity



drop: $H=H_1+H_2$

Continuity: $Q_1 = Q_2 = Q$

$K_1 A H_1/L_1 = K_2 A H_2/L_2$ $\quad H_1 = \phi_0-\phi_1 \quad H_2=\phi_1-\phi_2 \quad \phi_1$ is unknown

$$\phi_1 = \frac{(K_1/L_1)\phi_0 + (K_2/L_2)\phi_2}{(K_1/L_1) + (K_2/L_2)} = \frac{4\phi_0+3\phi_2}{7} = \frac{3}{7}\phi_2 = -\frac{3}{7}H$$

$K_1 = 2K_2$, $L_1=\frac{3}{2}L_2$ $\qquad \phi_0 = 0$ choice of reference $\qquad \phi_2$ is negative !

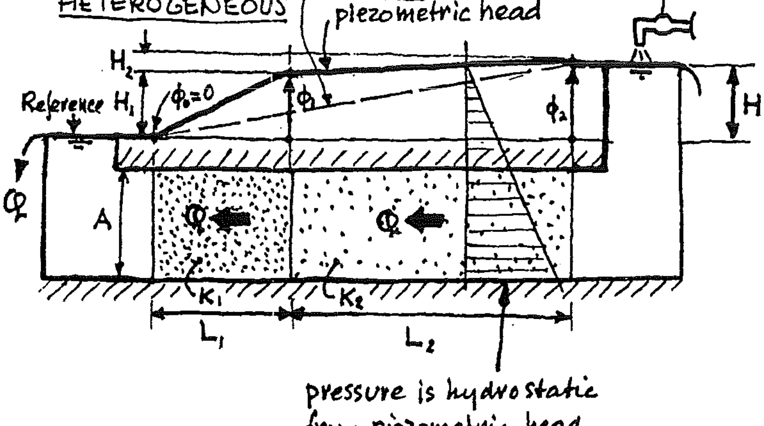
Assume: $L_1 = 30$ cm ; $L_2 = 20$ cm ; $H = 65$ cm ; $\gamma = 10^4$ N/m^3 ; $z_0 = -20$; $z_1=-50$; $z_2=-70$ cm

position z_0 $\qquad \phi_0 = 0$ (choice of reference) $\qquad p_0 = \gamma(\phi_0 - z_0) = 0.2\ 10^4$ N/m^2

position z_1 $\qquad \phi_1 = -\frac{3}{7}H = -\frac{3}{7}65 = -27.8$ cm $\qquad p_1 = \gamma(\phi_1 - z_1) = 0.22\ 10^4$ N/m^2

position z_2 $\qquad \phi_2 = -H = -65$ cm $\qquad p_2 = \gamma(\phi_2 - z_2) = 0.05\ 10^4$ N/m^2

HORIZONTAL HETEROGENEOUS



$K_1 = 5$ m/day; $K_2 = 20$ m/day

$L_1 = 10$ cm ; $L_2 = 20$ cm

$H = 6$ cm ; $A = 40$ cm^2

Calculate ϕ_1 and Q

answer:

$\phi_1 = 4$ cm ; $Q = 0.008$ m^3/day

Conservation of mass

$$\boxed{M = \rho V}$$

ρ: (specific) density; mass per unit volume [kg/m³]

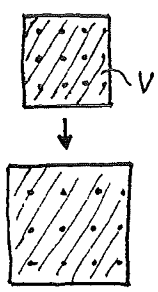
V: Volume [m³] volume occupied by number of molecules

M: total mass: number of molecules * molecular mass in volume V

Variation of M is zero (no molecules are produced or taken out of volume V)

$$d(M)=0 \rightarrow d(\rho V) = 0 \rightarrow \rho dV + V d\rho = 0$$

$$\rightarrow \boxed{\frac{d\rho}{\rho} = -\frac{dV}{V}}$$



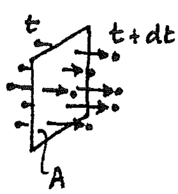
V larger
ρ smaller

relative variation of density is equal to the negative relative variation of volume; if volume V containing fixed number of molecules changes (becomes larger), the density changes (becomes smaller). If volume does not change, density does not change.
For an <u>incompressible fluid</u> (no density change) the volume occupied by a fixed number of molecules is constant. The volume may deform (change its shape). Incompressible: $d\rho = 0$, $dV = 0$

Mass flux

$$\rho q = \rho \frac{Q}{A} = \rho \frac{V}{At} = \frac{M}{At}$$

number of molecules passing through surface A per time t

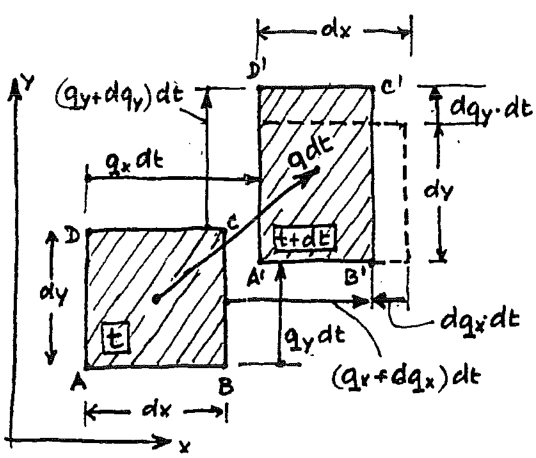


variation of mass flux: q is a vector!

$$d(\rho q) = d\left(\frac{M}{At}\right) = \frac{dM}{At} = 0 \rightarrow \rho dq + q d\rho = 0 \rightarrow \boxed{dq = 0}$$

for any surface, and any time

incompressible fluid $d\rho = 0$



Moving volume $V = dxdy$ with fixed number of molecules at $t=t$ and $t=t+dt$

For incompressible flow Volume ABCD remains equal to Volume A'B'C'D'.
The shape changes, such that the total mass flux through the surface equals zero

$(Q_{out} - Q_{in})\,dt = 0$ (<u>out</u> is positive, <u>in</u> is negative)

$$\rightarrow dq_y . dt . dx + dq_x . dt . dy = 0$$

devide by $dt \cdot dxdy$ $(dxdy = V)$

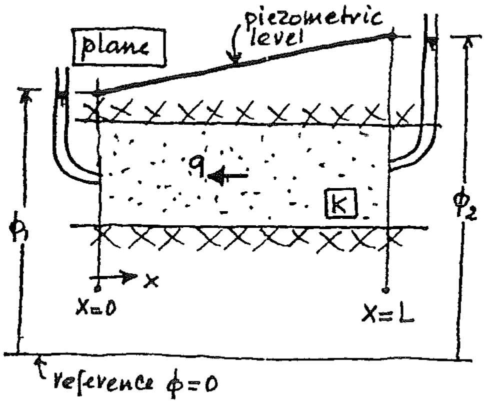
unit volume arbitrary choice

$$\boxed{\frac{dq_y}{dy} + \frac{dq_x}{dx} = 0}$$

Mathematical description of mass conservation for incompressible fluid

CONFINED AQUIFER

1-DIMENSIONAL



$$\boxed{\frac{dq_x}{dx} = 0}$$

conservation of mass

$$\boxed{q_x = -K\frac{d\phi}{dx}}$$

Darcy's law

$$\boxed{\frac{d}{dx}\left(-K\frac{d\phi}{dx}\right) = 0}$$

homogeneous field: K is constant (not dependent on x)

$$\boxed{\frac{d^2\phi}{dx^2} = 0}$$

flow equation

Integration:

$$\underbrace{\frac{d\phi}{dx} = a}_{\text{constant}} \rightarrow \boxed{\phi = ax + b}$$

General Solution; linear function of x; constant

The constants are determined by the boundary conditions

$x=0 \quad \phi_1 = a(0) + b = b \rightarrow b = \phi_1$

$x=L \quad \phi_2 = a(L) + b = aL + \phi_1 \rightarrow a = \dfrac{\phi_2-\phi_1}{L}$

Final Solution

$$\boxed{\phi = \frac{\phi_2-\phi_1}{L}x + \phi_1}$$

At any position x the piezometric level in an observation pipe will rise to the calculated pressure head (piezometric head):

$$\phi = \phi_1 + \frac{\phi_2-\phi_1}{L}x$$

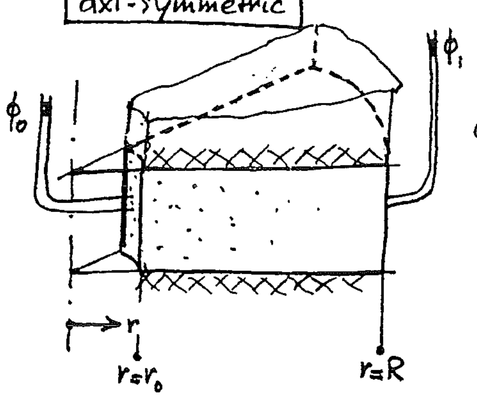
Flow in the aquifer:

$$q = -K\frac{d\phi}{dx} = -K\frac{\phi_2-\phi_1}{L}$$

is constant, and negative (opposite to positive x-direction)

The slope of the piezometric line determines the direction ~~and size~~ of the flow.

axi-symmetric



$$\boxed{\frac{1}{r}\frac{d}{dr}(rq_r) = 0}$$

Conserv. of mass

$$\boxed{q_r = -K\frac{d\phi}{dr}}$$

Darcy's law

$$\boxed{\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = 0}$$

flow equation

Integration: $r\dfrac{d\phi}{dr} = a \rightarrow \dfrac{d\phi}{dr} = \dfrac{a}{r}$

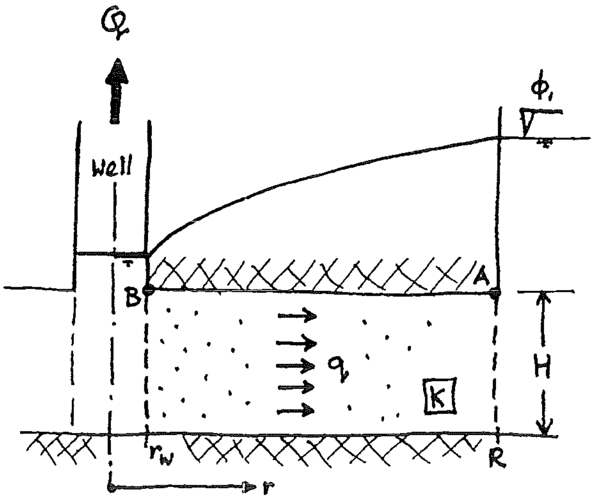
$$\rightarrow \boxed{\phi = a\ln(r) + b}$$

logarithmic function of r

General solution

Constants a and b determined by boundary conditions.
Result:

$$\boxed{\phi = \phi_1 + \frac{\phi_1-\phi_0}{\ln(R/r_0)}\ln\left(\frac{r}{R}\right)}$$



Axi-symmetric confined aquifer

Boundary condition at $r=r_w$

$$\boxed{Q = -q\,H\,2\pi r}$$

choice: production positive

height

perimeter well-screen

$q=-K\frac{d\phi}{dr}$ Darcy's law

$\rightarrow\ Q = K\frac{d\phi}{dr}H2\pi r$

$\rightarrow\ \boxed{\frac{d\phi}{dr} = \frac{Q}{2\pi r KH}}$ condition at $r=r_w$

(for confined aquifer valid $r_w \le r \le R$)

General solution

$$\boxed{\phi = a\ln(r) + b}$$

at $r=r_w$ $\quad \frac{d\phi}{dr} = \frac{a}{r} = \frac{Q}{2\pi r KH} \rightarrow a = \frac{Q}{2\pi KH}$

at $r=R$ $\quad \phi_1 = a\ln(R) + b \rightarrow b = \phi_1 - a\ln(R)$

$$\boxed{\phi = \phi_1 + \frac{Q}{2\pi KH}\ln\left(\frac{r}{R}\right)}$$

Specific solution

$$\boxed{q = -K\frac{d\phi}{dr} = -\frac{Q}{2\pi r H}}$$

Traveltime *How long it takes for a particle to travel from A to B?*

Specific velocity q

real velocity $v = q/n$

$v = \frac{dr}{dt} \rightarrow dt = dr/v = \frac{dr}{q}\cdot n$

Integration $\int_{t_A}^{t_B} dt = \int_{r_A}^{r_B} n\frac{dr}{q}$

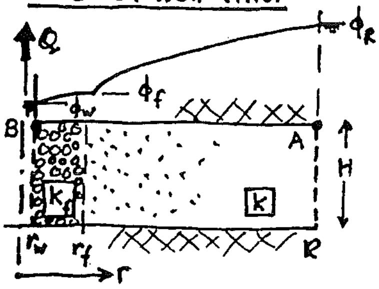
at A: $t_A = 0$; $r=R$ at B: $t_B = T$; $r=r_w$

$$\int_0^T dt = \int_R^{r_w} n\frac{dr}{q} = n\int_R^{r_w} -\frac{2\pi r H dr}{Q} = -\frac{2\pi n H}{Q}\left(\tfrac{1}{2}r^2\right)\Big|_R^{r_w}$$

$$\rightarrow\ \boxed{T_{A-B} = \frac{\pi n H}{Q}(R^2 - r_w^2)}$$

if $r_w \ll R$ $\quad T = \frac{\pi n H R^2}{Q}$

Effect of well-filter



How much is the drawdown $(\phi_R - \phi_w)$? Traveltime A→B?

$K_f = 0.01$ m/s ; $K = 0.0001$ m/s

$Q = 10^{-3}$ m³/s ; $H = 10$ m ; $n = 0.4$

$r_w = 0.2$ m ; $r_f = 1.0$ m ; $R = 2000$ m

suggestion: choose ϕ_R as reference
calculate first ϕ_f, then ϕ_w.

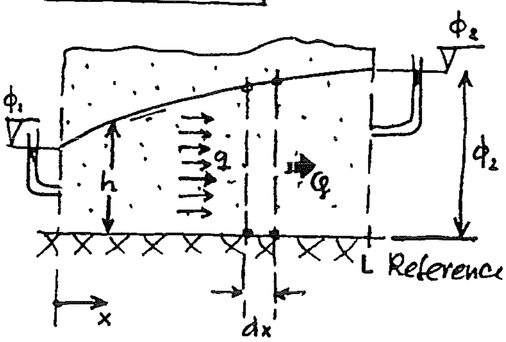
answer

$(\phi_R - \phi_w) = 1.212$ m

$T_{A-B} = 1589$ year !

PHREATIC AQUIFER

1-DIMENSIONAL



At any position x the piezometric level will rise to the water table which is at position h above the base. The watertable is the piezometric head (if flow is steady).

→ choose reference at the base

At any position x : $\boxed{\phi = h}$

Flow in the aquifer:

$$q = -k\frac{d\phi}{dx} = -k\frac{dh}{dx}$$

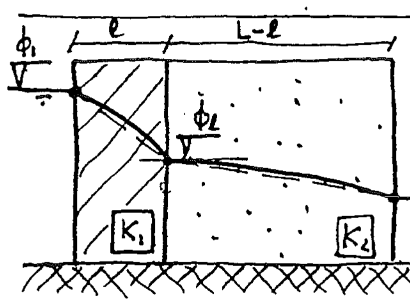
is assumed approximately horizontal. Usually this causes a small error (negligible).

$$q = -k\frac{dh}{dx} \rightarrow hq = -kh\frac{dh}{dx} \quad (Q/B)$$

$$\rightarrow Q = -\frac{1}{2}\frac{dh^2}{dx} = -\frac{\phi_2^2 - \phi_1^2}{2L}\cdot kB$$

$$\rightarrow \boxed{Q = -kB\frac{\phi_2^2 - \phi_1^2}{2L}}$$ Formula of Dupuit

(Also valid for exact 2-D flow)



Conservation of mass $Q = hqB$ is constant (B: meter width)

$$\boxed{\frac{d}{dx}(hq) = 0} \qquad \boxed{q = -k\frac{dh}{dx}} \text{ Darcy's law}$$

$$\boxed{\frac{d}{dx}\left(-hk\frac{dh}{dx}\right) = 0}$$

homogeneous field : k is constant

$$\frac{d}{dx}\left(h\frac{dh}{dx}\right) = 0 \quad \text{or} \quad \boxed{\frac{d^2(h^2)}{dx^2} = 0}$$

flow equation

Equation is linear for h^2 !

Integration: $\frac{d}{dx}(h^2) = a$ ← constant

$$\rightarrow \boxed{h^2 = ax + b} \rightarrow \boxed{h = \sqrt{ax+b}}$$

General Solution — h is parabolic in x

Boundary condition:

$x = 0 \quad \phi = \phi_1 \rightarrow h = \phi_1 \rightarrow h^2 = \phi_1^2$

$x = L \quad \phi = \phi_2 \rightarrow h = \phi_2 \rightarrow h^2 = \phi_2^2$

Final solution: (Specific solution)

$$h^2 = \frac{\phi_2^2 - \phi_1^2}{L}x + \phi_1^2 \rightarrow \boxed{h = \sqrt{\frac{\phi_2^2 - \phi_1^2}{L}x + \phi_1^2}}$$

Calculate discharge Q through two-layer dam

$$Q = Q_1 = Q_2 \rightarrow k_1\frac{\phi_1^2 - \phi_\ell^2}{2\ell} = k_2\frac{\phi_\ell^2 - \phi_2^2}{2(L-\ell)} \rightarrow \phi_\ell = \cdots$$

$$\rightarrow Q_1 = \frac{\phi_1^2 - \phi_\ell^2}{2\ell}k_1B = \cdots$$

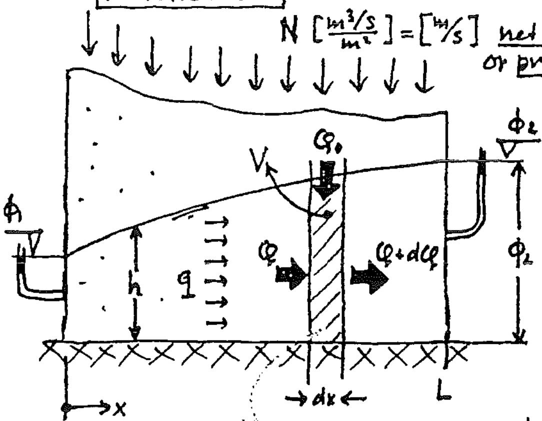
Prove that: $Q = k_2B\frac{\phi_1^2 - \phi_2^2}{2L}\cdot\beta$ with $\beta = \frac{L}{L + (\frac{k_2}{k_1} - 1)\ell}$

$$\boxed{\begin{array}{l} k_1 = 0.2k_2 = 0.1 \text{ m/day} \\ \ell = 0.2L = 2\text{m} \\ \phi_1 = 10\text{m}; \ \phi_2 = 2\text{m} \end{array}} \rightarrow Q = 1.33 \text{ m}^3\text{/day per meter width}$$

$\phi_\ell = 6.83$ m

INFILTRATION

$N\ [\frac{m^3/s}{m^2}] = [m/s]$ net infiltration or precipitation is a "velocity".



Conservation of mass

at any position x : $Q = qhB$ (width system)

consider elementary volume $V = hB\,dx$

conservation of mass for that volume (incompressible water): $Q_{in} = Q_{out}$

$\rightarrow Q + Q_0 = Q + dQ \rightarrow \boxed{dQ = Q_0}$

$Q_0 = N\,dx\,B$

$\rightarrow d(qhB) = N\,dx\,B$

Devide by elementary volume to obtain general formula (per unit volume):

$$\frac{d(qhB)}{hB\,dx} = \frac{N\,dx\,B}{hB\,dx} \rightarrow$$

$$\boxed{\frac{d(qh)}{dx} = N}$$ Conservation of mass

Darcy's law $q = -k\frac{dh}{dx}$

$$\rightarrow \frac{d}{dx}\left(-kh\frac{dh}{dx}\right) = N \rightarrow \boxed{\frac{d^2}{dx^2}(h^2) + \frac{2N}{k} = 0}$$

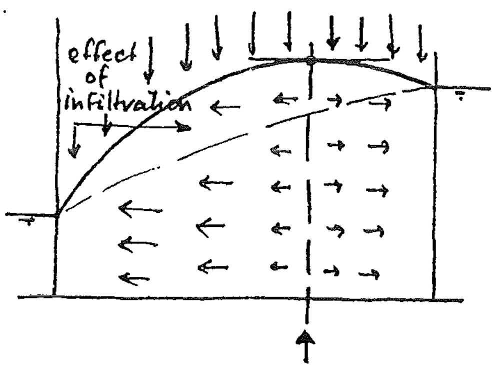
General flow equation

General Solution:

$$h^2 = -\frac{N}{k}x^2 + ax + b \rightarrow \boxed{h = \sqrt{-\frac{N}{k}x^2 + ax + b}}$$

a and b determined by boundary conditions:

$b = \phi_1^2$ $\quad a = \frac{\phi_2^2 - \phi_1^2}{L} + \frac{NL}{k}$



Point of horizontal water table
No gradient: $dh/dx = 0$
No flow
Stagnation point. It acts as a point of waterdevide

Important for water quality management.

For which infiltration N the stagnation point is in $x = L$:

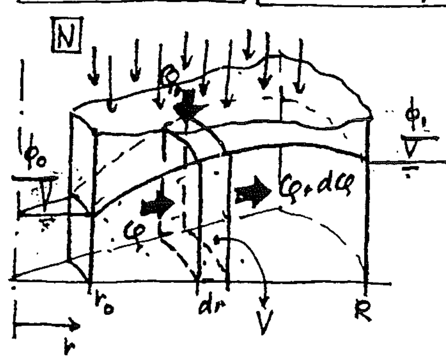
$\rightarrow \frac{dh}{dx} = 0$ in $x = L$

$\frac{dh}{dx} = 0 \rightarrow \frac{d(h^2)}{dx} = 0 \rightarrow -\frac{2N}{k}x + a = 0$

for $x = L \rightarrow -\frac{2NL}{k} + a = 0$

$\rightarrow N = \frac{k}{L^2}(\phi_2^2 - \phi_1^2)$

Axi-symmetric | phreatic aquifer



$Q = q2\pi rh$; $Q_0 = N\,dr\,2\pi r$; $V = 2\pi rh\,dr$

$Q + Q_0 = Q + dQ \rightarrow dQ = Q_0$

$\frac{dQ}{V} = \frac{Q_0}{V} \rightarrow \boxed{\frac{1}{r}\frac{d}{dr}(rqh) = N}$ mass conservation

Darcy's law $\rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}h^2\right) + \frac{2N}{k} = 0$

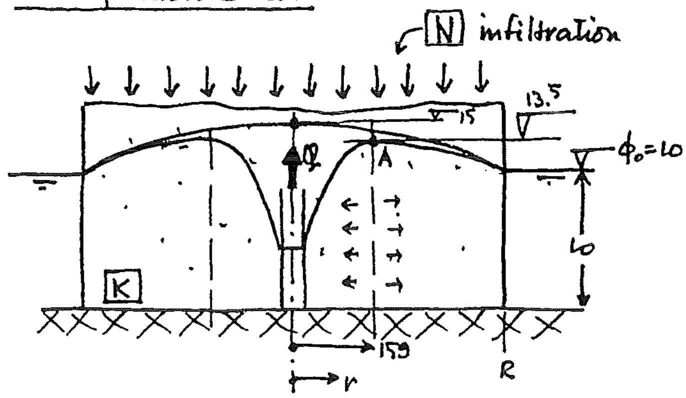
General Solution $h^2 = -\frac{Nr^2}{2k} + a\ln(r) + b$

Specific Solution $\boxed{h^2 = \phi_1^2 + \frac{N}{2k}(R^2 - r^2) + \frac{Q}{\pi k}\ln\left(\frac{r}{R}\right)}$

Barends
Groundwater flow $\boxed{8}$

Phreatic aquifer

Axi-symmetric flow



$N = 0.01$ m/day ; $K = 10$ m/day
$\phi_o = 10$ m ; $R = 500$ m
$r_w = 0.1$ m

A: point of the waterdevide

Questions

a) What is the watertable height at $r=0$ (centre of the island)?

b) A well is placed at $r=0$. What is the maximum discharge if no intrusion at the side takes place?

c) What is the drawdown at the well?

d) If maximum height is 3m, what is than the discharge?

e) Where is than the waterdevide and what is the waterlevel there?

Answers

a) $h^2(r=0) = \phi_o^2 + \frac{N}{2K}R^2 = 2.25 = (15)^2 \rightarrow h = 15$ at $r=0$ (raise of 5m).

b) waterdevide at $r=R$ (no intrusion) $\rightarrow \frac{dh}{dr} = 0$ at $r=R$. $\rightarrow \frac{d(h^2)}{dr} = 0$ at $r=R$

$$h^2 = \phi_o^2 + \frac{N}{2K}(R^2-r^2) + \frac{Q}{\pi K}\ln\left(\frac{r}{R}\right) \rightarrow \frac{dh^2}{dr} = -\frac{Nr}{K} + \frac{Q}{\pi K r} \rightarrow \frac{dh^2}{dr} = 0 \text{ if } Q = \pi N r^2$$

$\rightarrow Q = \pi N R^2 = 7854$ m³/day (infiltration over total area: πR^2 into discharge)

c) $$\phi_w^2 = \phi_o^2 + \frac{N}{2K}(R^2 - r_w^2) + \frac{Q}{\pi K}\ln\frac{r_w}{R} = 10^2 + \frac{0.01}{2\cdot 10}(500^2 - 0.1^2) + \frac{7854}{\pi\cdot 10}\ln\left(\frac{0.1}{500}\right) = -1904\text{m}$$

not possible; toe of well at level 0. Discharge is not realistic.

d) $$\phi_w^2 = 3^2 = 10^2 + \frac{0.01}{2\cdot 10}(500^2 - 0.1^2) + \frac{Q}{\pi\cdot 10}\ln\left(\frac{0.1}{500}\right) \rightarrow Q = 796 \text{ m}^3/\text{day}$$

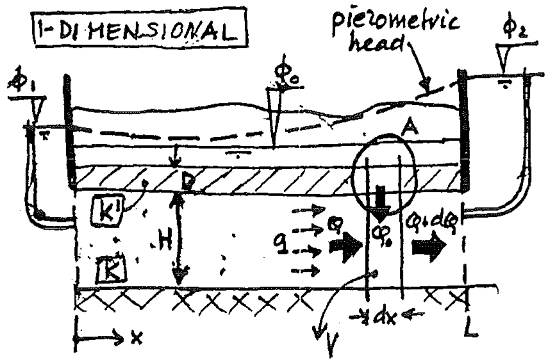
e) waterdevide at $\frac{dh}{dr} = 0 \rightarrow \frac{dh^2}{dr} = 0 \rightarrow Q = \pi N r^2 \rightarrow r = \sqrt{Q/\pi N} = \sqrt{\frac{796}{\pi\cdot 0.01}} = 159$m

$$h^2(r=159) = 10^2 + \frac{0.01}{2\cdot 10}(500^2 - 159^2) + \frac{796}{\pi\cdot 10}\ln\left(\frac{159}{500}\right) = 183.4 = (13.5)^2 \rightarrow h = 13.5\text{m}$$
at $r = 159$m

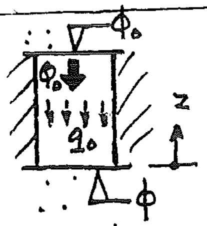
In reality the filter at the well-screen and the capillary zone will play a role.

Semi-confined aquifer

1-DIMENSIONAL



Leakage through semi-permeable top layer



DETAIL A

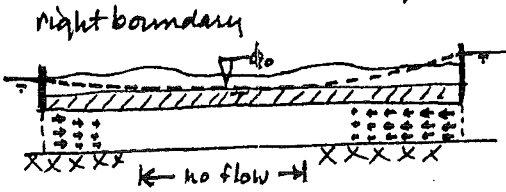
Darcy's law $q_0 = -k' \frac{d\phi}{dz}$

$$\rightarrow q_0 = -k' \frac{\phi - \phi_0}{D} = -\frac{\phi - \phi_0}{c}$$

$\boxed{c = D/k' : \text{hydraulic resistance [s]}}$

$Q_0 = q_0 B dx$

Special case when $L >>> \lambda$
Then no connection between left and right boundary



Conservation of mass

$Q + Q_0 = Q + dQ$; $Q = qBH$; $V = HBdx$

$$\rightarrow \frac{dQ}{V} = \frac{Q_0}{V} \rightarrow \frac{d(qBH)}{HBdx} = \frac{q_0 Bdx}{HBdx}$$

$$\rightarrow \boxed{\frac{dq}{dx} = \frac{q_0}{H}}$$

Darcy's law: $q = -K\frac{d\phi}{dx}$; $q_0 = -\frac{\phi - \phi_0}{c}$

$$\rightarrow \boxed{\frac{d^2\phi}{dx^2} = \frac{\phi - \phi_0}{\lambda^2}} \; ; \; \boxed{\lambda = \sqrt{KHc}}$$

equation of flow — Leakage factor [m]

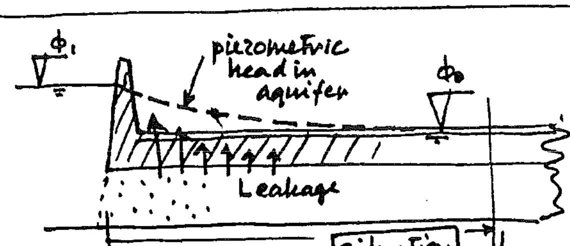
General Solution

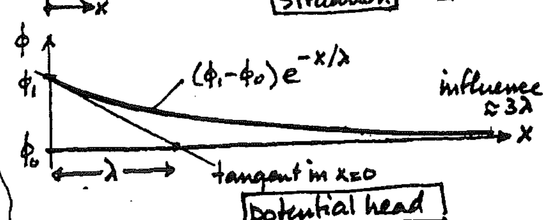
$$\phi = \phi_0 + a e^{x/\lambda} + b e^{-x/\lambda}$$

a, b depend on boundary condition

Specific Solution

$$a = [\phi_2 - \phi_0 - (\phi_1 - \phi_0)e^{-L/\lambda}]/N$$
$$b = [(\phi_1 - \phi_0)e^{L/\lambda} - (\phi_2 - \phi_0)]/N$$
$$N = e^{L/\lambda} - e^{-L/\lambda}$$





Example leakage in a polder or lowland

Solution is zero for $x \rightarrow \infty$: $a = 0$
at $x = 0$: $\phi_1 = \phi_0 + b e^{-0/\lambda} \rightarrow b = (\phi_1 - \phi_0)$
$\rightarrow$ solution: $\phi = \phi_0 + (\phi_1 - \phi_0)\exp[-x/\lambda]$

Calculate total leakage Q_0^T in the polder

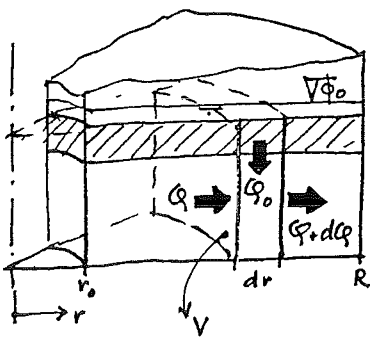
$$Q_0^T = \int_0^L q_0 B dx = -\int_0^L \frac{B}{c}(\phi - \phi_0)dx = -\frac{B(\phi_1 - \phi_0)}{c}\int_0^L e^{-x/\lambda}dx$$
$$= +\lambda\frac{B}{c}(\phi_1 - \phi_0)\left(e^{-x/\lambda}\right|_0^L = \frac{\lambda B}{c}(\phi_1 - \phi_0)(e^{-L/\lambda} - 1)$$

For $L \rightarrow \infty$ $Q_0^T = \frac{\lambda B}{c}(\phi_1 - \phi_0) = 2 \cdot 10^{-5} B$ m^3/s
(or 1728 liter per day per m width)

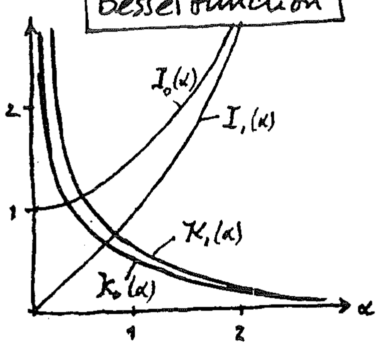
$KH = T = 10^{-3}$ m^2/s (Transmissivity)
$c = 4 \cdot 10^7$ s (Hydraulic Resistance)
$\phi_1 - \phi_0 = 4$ m
$\rightarrow \lambda = \sqrt{Tc} = 200$ m

Semi-confined aquifer

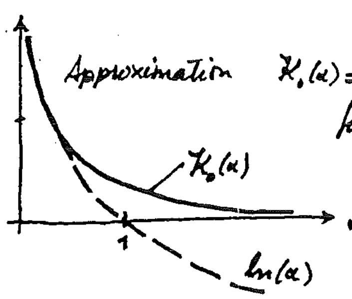
AXI-SYMMETRY



Bessel function



Bessel functions



Approximation $K_0(\alpha) = -\ln\left(\frac{\alpha}{1.123}\right)$ for $\alpha \ll 1$

$$\frac{d}{d\alpha} I_0(\alpha) = I_1(\alpha)$$

$$\frac{d}{d\alpha} K_0(\alpha) = -K_1(\alpha)$$

$$\int_0^\alpha \alpha I_0(\alpha)\, d\alpha = \alpha I_1(\alpha)$$

$$\int_0^\alpha \alpha K_0(\alpha)\, d\alpha = -\alpha K_1(\alpha) + 1$$

Conservation of mass

$Q = q\,2\pi r H$; $Q_0 = q_0\, 2\pi r\, dr$; $V = 2\pi r\, dr\, H$

$Q + Q_0 = Q + dQ \rightarrow \quad \frac{Q_0}{V} = \frac{dQ}{V}$

$$\rightarrow \boxed{\frac{1}{r}\frac{d}{dr}(rq) = \frac{q_0}{H}}$$

Darcy's law $\quad q = -K\frac{d\phi}{dr}$; $q_0 = -\frac{\phi - \phi_0}{c}$

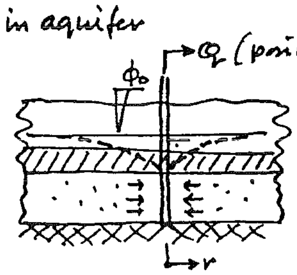
$$\rightarrow \boxed{\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = \frac{\phi - \phi_0}{\lambda^2}} \quad ; \quad \lambda = \sqrt{KHC}$$

flow equation

General Solution

$$\phi = \phi_0 + a\, I_0\left(\frac{r}{\lambda}\right) + b\, K_0\left(\frac{r}{\lambda}\right)$$

Special case Well in aquifer



$r \to \infty \quad \phi \to \phi_0 \to a = 0$

$r \to r_0 \quad q = \frac{-Q}{2\pi r H}$

↑ Darcy's law

$\frac{d\phi}{dr} = +\frac{Q}{2\pi r K H}$

$$\frac{d\phi}{dr} = b\frac{d}{dr}K_0\left(\frac{r}{\lambda}\right) = \frac{b}{\lambda}\left(-K_1\left(\frac{r}{\lambda}\right)\right)$$

at $r = r_0 \quad \frac{d\phi}{dr} = +\frac{Q}{2\pi r_0 KH} = -\frac{b}{\lambda}K_1\left(\frac{r_0}{\lambda}\right)$

$$\rightarrow b = \frac{-Q\lambda}{2\pi r_0 KH\, K_1\left(\frac{r_0}{\lambda}\right)} \cong \frac{-Q}{2\pi KH}$$

$\boxed{\text{for } \frac{r_0}{\lambda} \ll 1 \text{, then } \frac{r_0}{\lambda}K_1\left(\frac{r_0}{\lambda}\right) \to 1 \quad \text{(property Bessel function)}}$

Specific Solution:

$$\phi = \phi_0 + \frac{-Q}{2\pi KH}K_0\left(\frac{r}{\lambda}\right)$$

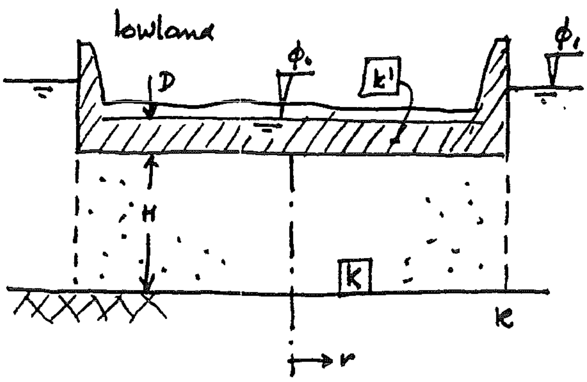
For small values of $\frac{r}{\lambda}$ $\quad \left(\frac{r}{\lambda} \ll 1\right) \rightarrow K_0\left(\frac{r}{\lambda}\right) \cong -\ln\left(\frac{r}{1.123\lambda}\right)$

$$\phi = \phi_0 + \frac{Q}{2\pi KH}\ln\left(\frac{r}{1.123\lambda}\right)$$

Drawdown $\boxed{s = \phi - \phi_0 = \frac{Q}{2\pi KH}\ln\left(\frac{r}{R_\lambda}\right)} \quad R_\lambda = 1.123\lambda$

Barends,
Groundwaterflow [11]

Axi-symmetric semi-confined aquifer



$k' = 10^{-6}$ m/s ; $k = 10^{-4}$ m/s

$D = 5$ m ; $H = 10$ m

$R = 200$

$\phi_0 = 5$ m ; $\phi_1 = 10$ m

$\rightarrow \lambda = \sqrt{kHD/k'} \cong 70$ m

$\rightarrow c = D/k' = 5.10^6$ s

1) Potential head at $r=0$

General Solution $\boxed{\phi = \phi_0 + a\, I_0(\tfrac{r}{\lambda}) + b\, K_0(\tfrac{r}{\lambda})}$

at $r=0$: $K_0(0)$ tends to infinity, cannot be used $\rightarrow \boxed{b=0}$

at $r=R$ $\quad \phi = \phi_1 = \phi_0 + a\, I_0(\tfrac{R}{\lambda}) \longrightarrow \boxed{a = \dfrac{\phi_1-\phi_0}{I_0(R/\lambda)}}$

Specific solution $\boxed{\phi = \phi_0 + \dfrac{\phi_1-\phi_0}{I_0(\frac{R}{\lambda})} I_0(\tfrac{r}{\lambda})}$

at $r=0 \longrightarrow \quad \phi = \phi_0 + \dfrac{\phi_1-\phi_0}{I_0(\frac{R}{\lambda})} I_0(\tfrac{r}{\lambda})\Big|_{r=0} = 5 + \dfrac{10-5}{I_0(\frac{200}{70})} I_0(0) = 5 + \dfrac{5}{4.3}\cdot 1 = \underline{\underline{6.16\text{ m}}}$

2) What is the total leakage?

$$Q_0 = \int_0^R \Big(2\pi r\, dr \underbrace{\frac{\phi-\phi_0}{c}}_{q_0}\Big) = \frac{2\pi}{c}\int_0^R r\, \frac{\phi_1-\phi_0}{I_0(\frac{R}{\lambda})} I_0(\tfrac{r}{\lambda})\, dr$$

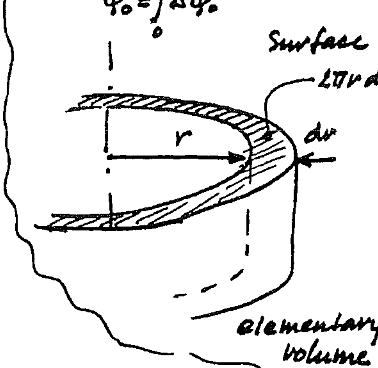
$$= \frac{2\pi(\phi_1-\phi_0)}{c\, I_0(\frac{R}{\lambda})}\, \lambda \Big(r\, I_1(\tfrac{r}{\lambda})\Big|_0^R$$

$$= \frac{2\pi(\phi_1-\phi_0)}{c\, I_0(\frac{R}{\lambda})}\, R\lambda\, I_1(\tfrac{R}{\lambda})$$

$$= \frac{2\pi(10-5)}{5.10^6\ \ 4.3}\, 200\cdot 70\ (3.4) = \underline{\underline{0.024}}\ \text{m}^3/\text{s}\ \Big(= 2\,10^6\ \frac{\text{liter}}{\text{day}}\Big)$$

$\Delta Q_0 = 2\pi r\, dr\, q_0$

$Q_0 = \int_0^R \Delta Q_0$



$\int r\, I_0(\tfrac{r}{\lambda})\, dr = r\lambda\, I_1(\tfrac{r}{\lambda})$

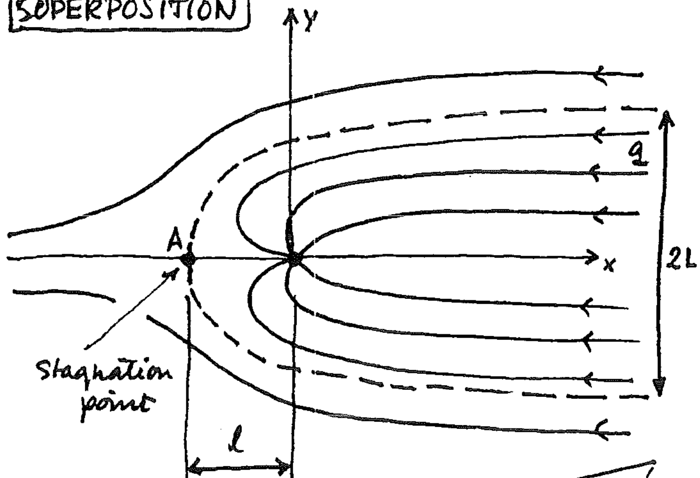
$I_0(\tfrac{R}{\lambda}) = I_0(\tfrac{200}{70}) = 4.3$

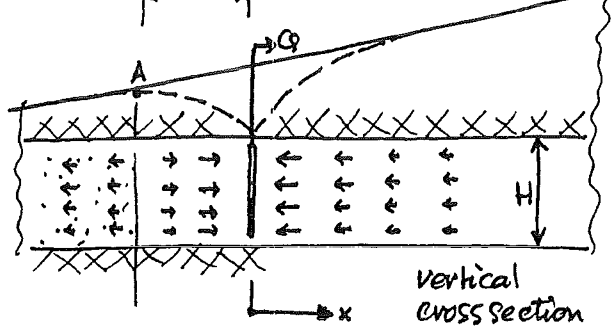
$I_1(\tfrac{R}{\lambda}) = I_1(\tfrac{200}{70}) = 3.4$

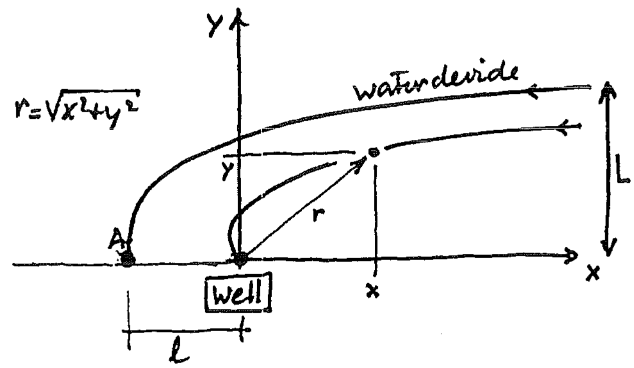
$\uparrow$

From Mathematical Handbook

SUPERPOSITION







WELL IN UNIFORM FLOW

confined aquifer

q is constant in uniform flow (far from the well)

$\boxed{\phi = ax + b}$ choice: reference $\phi = 0$ at $x = 0$ $\rightarrow b = 0$

$q = -ka$

$a > 0 \rightarrow q < 0$ negative vector

Well:

$$\boxed{\phi = \frac{Q}{2\pi kH}\ln\left(\frac{r}{R}\right)}$$

or $\phi = \phi_0 + \frac{Q}{2\pi kH}\ln(r/R)$ at $r=R$ $\phi=\phi_0=0$

$r = \sqrt{x^2+y^2}$ general coordinates

SUPERPOSITION

$$\boxed{\phi = ax + \frac{Q}{2\pi kH}\ln\left(\frac{\sqrt{x^2+y^2}}{R}\right)}$$

Constant defined by reference (far boundary)

Stagnation point A: no flow $q_x = 0$ ($q_y = 0$ is trivial because of symmetry)

$$\frac{d\phi}{dx} = \frac{d}{dx}\left\{ax + \frac{Q}{2\pi kH}\ln\frac{\sqrt{x^2+y^2}}{R}\right\} = a + \frac{Q}{2\pi kH}\frac{d}{dx}\left(\ln\sqrt{x^2+y^2}\right)$$

$$\frac{d}{dx}\left(\ln\sqrt{x^2+y^2}\right) = \frac{1}{\sqrt{x^2+y^2}}\cdot\frac{d}{dx}\left(\sqrt{x^2+y^2}\right) = \frac{1}{\sqrt{x^2+y^2}}\,\frac{\frac{1}{2}}{\sqrt{x^2+y^2}}\,2x = \frac{x}{x^2+y^2}$$

$$\rightarrow \frac{d\phi}{dx} = a + \frac{Q}{2\pi kH}\,\frac{x}{x^2+y^2}$$

At point A: $y = 0$, $\frac{d\phi}{dx} = 0$

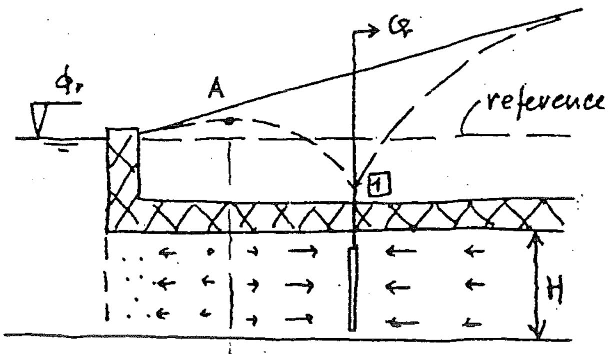
$$\rightarrow a + \frac{Q}{2\pi kH}\,\frac{1}{x} = 0 \rightarrow \boxed{x = -\frac{Q}{2\pi kHa}}$$

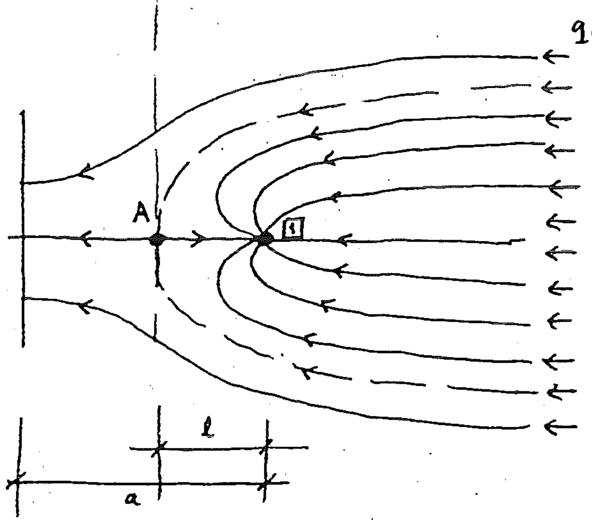
L to be determined from discharge Q

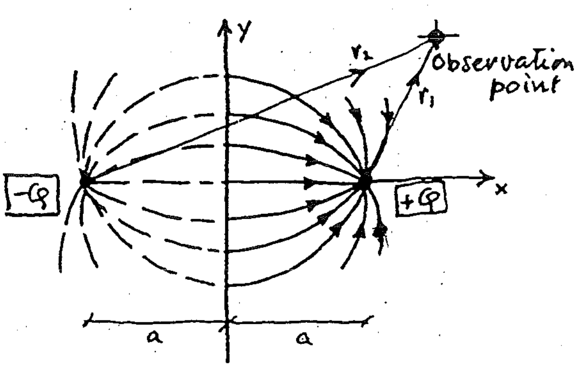
$Q = H2Lq = -2HLka$

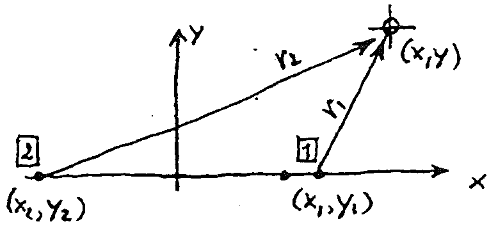
Distance stagnation point from the well $\rightarrow \boxed{l = \frac{L}{\pi}}$

WELL IN UNIFORM FLOW NEAR RIVER









$$r_1 = \sqrt{(x-x_1)^2+(y-y_1)^2}$$

$$r_2 = \sqrt{(x-x_2)^2+(y-y_2)^2}$$

confined aquifer

uniform flow

$$\boxed{\phi = \phi_r + \frac{q_0}{k}x}$$

drawdown $\boxed{s_0 = +\frac{q_0}{k}x}$

real well

$$\boxed{\phi = \phi_r + \frac{Q_1}{2\pi kH}\ln\left(\frac{r_1}{R}\right)}$$

drawdown $\boxed{s_1 = \frac{Q_1}{2\pi kH}\ln\left(\frac{r_1}{R}\right)}$

image well $\boxed{Q_1 = -Q_2}$

$$\boxed{\phi = \phi_r + \frac{Q_2}{2\pi kH}\ln\left(\frac{r_2}{R}\right)}$$

drawdown $\boxed{s_2 = \frac{Q_2}{2\pi kH}\ln\left(\frac{r_2}{R}\right)}$

Superposition

$$s = s_0 + s_1 + s_2 = \frac{q_0 x}{k} + \frac{Q}{2\pi kH}\ln\left[\frac{\sqrt{(x-a)^2+y^2}}{\sqrt{(x+a)^2+y^2}}\right]$$

Stagnation point A

along y, where $\frac{ds}{dx} = 0$ for $0 < x < a$

$$\frac{q_0}{k} + \frac{Q}{2\pi kH}\left(\frac{1}{x-a} - \frac{1}{x+a}\right) = 0$$

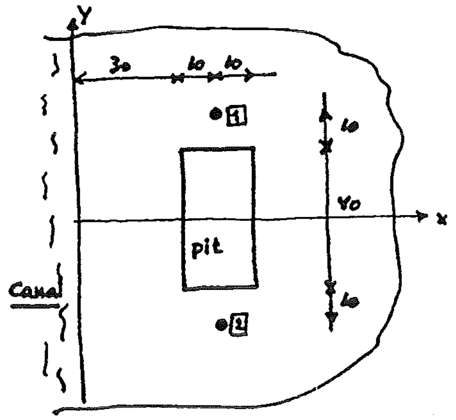
$\rightarrow \eta x^2 - \eta a^2 + a = 0$ with $\eta = \frac{q_0 \pi H}{Q}$

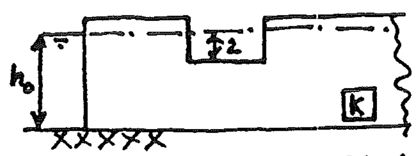
$\rightarrow x = \pm a\sqrt{1-(1/a\eta)}$ (positive root)

$\rightarrow a - \ell = a\sqrt{1-(1/a\eta)}$

A op rivier $\rightarrow \ell = a \rightarrow Q = \pi a H q_0$

Exercise Superposition





cross section (x-axis)

Building pit to be drained by two wells
Discharge equal: Q_0.
Phreatic aquifer system: $h_0 = 8$ m ; $K = 25$ m/day
Minimal drawdown 2m required.

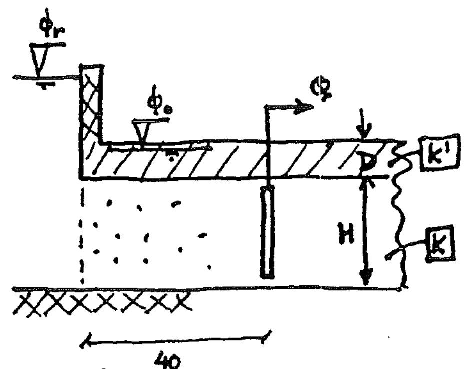
Questions

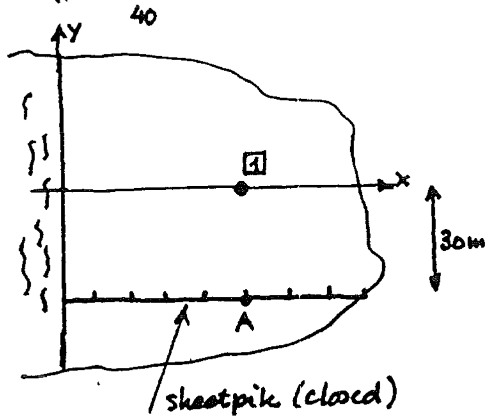
1) Where is in the pit the point of smallest drawdown?

2) Calculate required Q_0 (to achieve required drawdown) using principle of superposition and virtual wells.
Apply linearized formula's

3) Same as 2) but use the correct formula's

4) Is there a stagnation point? Sketch the flow field.





Well in semiconfined aquifer: $K' = 10^{-6}$ m/s ; $K = 10^{-4}$ m/s
$D = 5$ m ; $H = 10$ m

Near a canal: $\phi_r = 14$ m ; $\phi_0 = 12$ m

Near a sheetpile, which closes the layer of (extends far in x-direction)

Questions

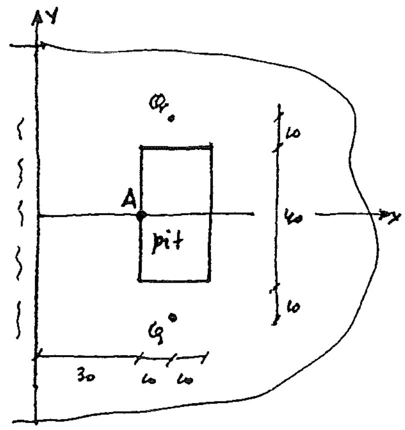
1) Make the scheme of calculation (virtual well system)

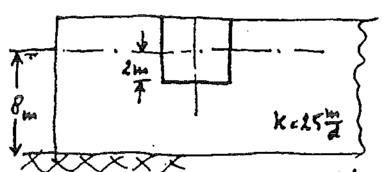
2) Calculate the discharge Q if in point A the potential head is equal to 12 m.

3) Is there a stagnation point? So, yes, where it is?
Sketch the flow field.

$$\left(\begin{array}{l} K_0(0.424) = 1.02 \\ K_0(1.208) = 0.32 \end{array} \right)$$

$\boxed{\text{Excercise Superpositie}}$



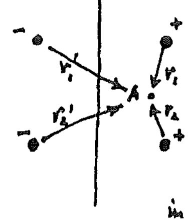


phreatic aquifer

Building pit to be draine by two wells. Bottom pit 2m below normal groundwater level.

1) <u>Where is point in the pit with smallest drawdown</u>
→ far from the well, near canal → point A

2) <u>Discharge Q?</u> Use point A as observation point. Superposition. Two virtual wells to simulate canal. Linearised formula (phreatic).



$r_1 = \sqrt{10^2 + 30^2} = 31.62 = r_2$

$r_1' = \sqrt{70^2 + 30^2} = 76.16 = r_2'$

in A $\phi = \dfrac{Q}{2\pi kH} \ln\left[\dfrac{r_1 r_2}{r_1' r_2'}\right]$

$$\rightarrow Q = \frac{2\pi kH\phi}{\ln\left[\frac{r_1 r_2}{r_1' r_2'}\right]} = \frac{2\pi(25)(8)(-2)}{\ln\left[\frac{(31.62)^2}{(76.16)^2}\right]} = 1430\ \frac{m^3}{d} = 60\ \frac{m^3}{hr}$$

minimal discharge required

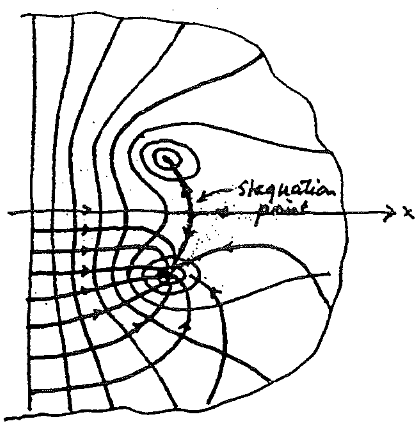
3) <u>Correct formula (quadratic)</u>

$h^2 - H^2 = \dfrac{Q}{\pi k} \ln\left(\dfrac{r}{R}\right)$; $(h^2 - H^2)$ linear with Q.

in A: $h^2 - H^2 = \dfrac{Q}{\pi k} \ln\left[\dfrac{r_1 r_2}{r_1' r_2'}\right] \rightarrow$

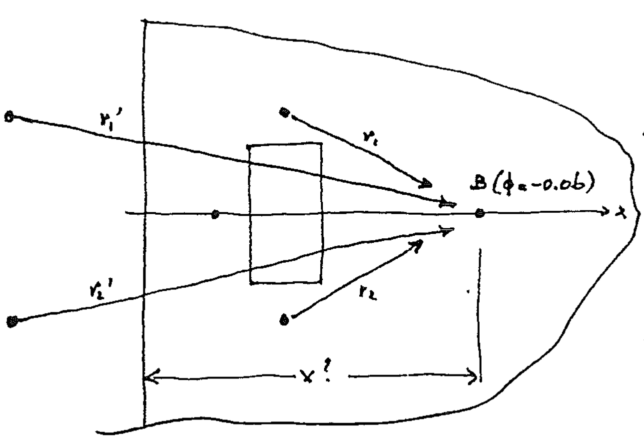
$$Q = \frac{\pi k\left[h^2 - H^2\right]}{\ln\left[\frac{r_1 r_2}{r_1' r_2'}\right]} = \frac{\pi(25)(6^2 - 8^2)}{\ln\left[\frac{(31.62)^2}{(76.16)^2}\right]} = 1250\ \frac{m^3}{d} = 52\ \frac{m^3}{hr}$$

→ Linear system is rather accurate (error 15%) and on the safe side, in this case: $Q_{linear} > Q_{quadratic}$



4 There is a <u>stagnation point</u>, between the wells some little to the right due to the canal.

Sketch of the flow pattern is given.



5) How far to the right is (on the x-axis) the point B <u>where the influence is only 3%</u> (=−0.06m). (linear formula)

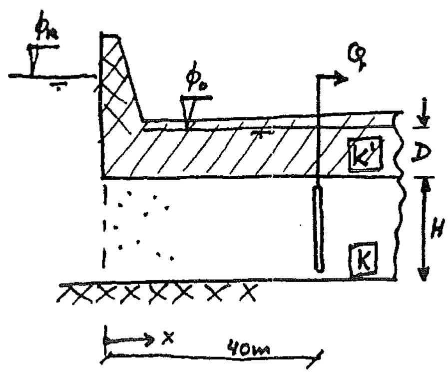
in B; $\phi = -0.06 = \dfrac{Q}{2\pi kH} \ln\left[\dfrac{r_1 r_2}{r_1' r_2'}\right] = \dfrac{1430}{2\pi(25)(8)} \ln\left[\dfrac{(x-40)^2 + 30^2}{(x+40)^2 + 30^2}\right]$

$\rightarrow \dfrac{(x-40)^2 + 30^2}{(x+40)^2 + 30^2} = e^{-0.06\ 2\pi(25)(8)/1430} = 0.949$

→ $3057x + 2500 = 0 \rightarrow$ $x = 0.82$ or $x = 3050$m !
(two points are found on the x-axis)

Note:
x-axis is the water divide line (if there is a sheetpiling, the same solution applies)

Exercise Superposition



$K' = 10^{-6}$ m/s
$K = 10^{-4}$ m/s
$D = 5$ m
$H = 10$ m
$\phi_K = 14$ m
$\phi_0 = 12$ m

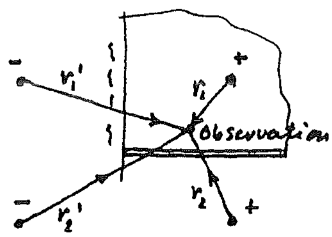
$\lambda = \sqrt{KDH/K'}$
$= 70.7$ m

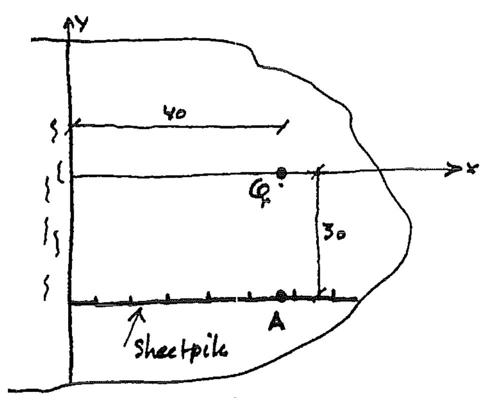
1) Scheme of calculation

(reference)
Consider ϕ_0 everywhere as initial situation

ϕ_K higher than ϕ_0 is effect 1 (uniform in y direction)

effect 2 is due to the well; the canal and Sheetpiling are simulated by virtual wells





2) Calculate Q if in point A $\phi = 12$m.
$A = (x,y)_A = (40, -30)$

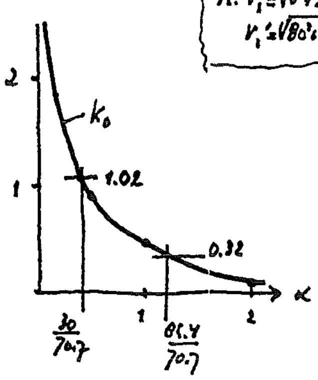
effect 1 drawdown $s_1 = (\phi_K - \phi_0)\, e^{-x/\lambda}$

effect 2 $s_2 = \dfrac{-Q}{2\pi kH}\left\{K_0\left(\frac{r_1}{\lambda}\right) + K_0\left(\frac{r_2}{\lambda}\right) - K_0\left(\frac{r_3}{\lambda}\right) - K_0\left(\frac{r_4}{\lambda}\right)\right\}$

A: $r_1 = \sqrt{0^2+30^2} = 30 = r_2$
$r_1' = \sqrt{80^2+30^2} = 85.4 = r_2'$

total $s = s_1 + s_2 =$

$$(\phi_K - \phi_0)\, e^{-x/\lambda} + \frac{-Q}{2\pi kH}\left\{K_0\left(\frac{r_1}{\lambda}\right) + K_0\left(\frac{r_2}{\lambda}\right) - K_0\left(\frac{r_3}{\lambda}\right) - K_0\left(\frac{r_4}{\lambda}\right)\right\}$$

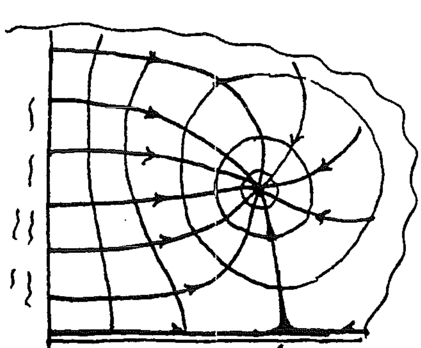


For point A: $s = 0$ (drawdown just zero)

$$0 = (14-12)\, e^{-40/70.7} + \frac{-Q}{2\pi 10^{-4}\, 10}\left\{K_0\left(\frac{30}{70.7}\right) + K_0\left(\frac{30}{70.7}\right) - K_0\left(\frac{85.4}{70.7}\right) - K_0\left(\frac{85.4}{70.7}\right)\right\}$$

$$0 = 1.136 + \frac{-Q}{2\pi 10^{-3}}\left\{1.02 + 1.02 - 0.32 - 0.32\right\}$$

$$\rightarrow \quad Q = \frac{(1.136)\, 2\pi\, 10^{-3}}{\{1.4\}} = 0.6\ 10^{-3}\ \frac{m^3}{s} = 2.2\ \frac{m^3}{hr}$$



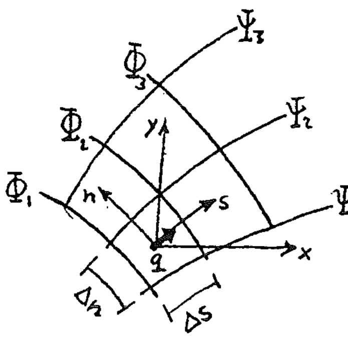
Stagnation point

3) Stagnation point

If canal level ϕ_K would be equal to ϕ_0 the stagnation point will be in point A (because of symmetry).
Due to the canal (higher level in the canal) it will be to the right of point A.

Note: if the well is situated more than 3λ from the canal, the effect of the canal is negligible; only effect 2 is important

Method of Squares



Darcy's law $q_x = -k\dfrac{\partial\phi}{\partial x}$ $\quad q_y = -k\dfrac{\partial\phi}{\partial y}$

FLOWPOTENTIAL $\boxed{\Phi = k\phi}$ $[m^2/sec]$ $\quad \boxed{\phi = z + p/\gamma}$

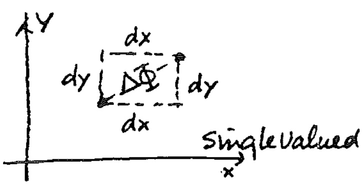
$$\boxed{q_x = -\frac{\partial\Phi}{\partial x} \quad q_y = -\frac{\partial\Phi}{\partial y}}$$

Valid for homogeneous fields

Continuity $\dfrac{\partial}{\partial x}q_x + \dfrac{\partial}{\partial y}q_y = 0 \rightarrow \boxed{\dfrac{\partial^2}{\partial x^2}\Phi + \dfrac{\partial^2}{\partial y^2}\Phi = \nabla^2\Phi = 0}$

Φ is harmonic

Φ is single-valued $\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}\Phi\right) = \dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial x}\Phi\right)$



STREAM FUNCTION Ψ $\quad \boxed{q_x = -\dfrac{\partial\Psi}{\partial y} \quad q_y = +\dfrac{\partial\Psi}{\partial x}}$

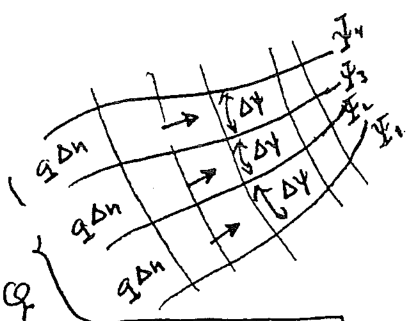
$$\boxed{\frac{\partial\Phi}{\partial x} = \frac{\partial\Psi}{\partial y}, \quad \frac{\partial\Phi}{\partial y} = -\frac{\partial\Psi}{\partial x}}$$

Cauchy-Riemann conditions

Continuity $\dfrac{\partial}{\partial x}q_x + \dfrac{\partial}{\partial y}q_y = 0 \rightarrow \dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}\Psi\right) = \dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial x}\Psi\right)$

Ψ is single-valued $\qquad \psi$ is harmonic

$$\boxed{\frac{\partial^2}{\partial x^2}\Psi + \frac{\partial^2}{\partial y^2}\Psi = \nabla^2\Psi = 0}$$



$$\boxed{q = \frac{\Delta\Psi}{\Delta n} = \frac{kH}{N\Delta n}}$$

rotation to curved coordinates $(x,y) \rightarrow (s,n)$

$$\boxed{q_s = -\frac{\partial\Phi}{\partial s} = -\frac{\partial\Psi}{\partial n}} \qquad \boxed{q_n = -\frac{\partial\Phi}{\partial n} = +\frac{\partial\Psi}{\partial s} = 0}$$

s along flow direction $\rightarrow q_n = 0$

$\dfrac{\Delta\Phi}{\Delta s} = \dfrac{\Delta\Psi}{\Delta n}$

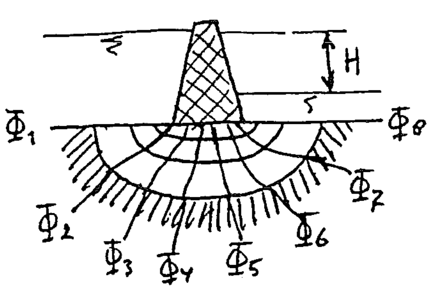
$\dfrac{\partial\Phi}{\partial n} = 0 \rightarrow \Phi$ is constant along n

$\dfrac{\partial\Psi}{\partial s} = 0 \rightarrow \Psi$ is constant along s

$\Delta s = \Delta n$ (local)

Square net

$\rightarrow \boxed{\Delta\Phi = \Delta\Psi \text{ (uniform)}}$



H: drop

$q\Delta n = \Delta\Psi = \Delta\Phi = \dfrac{kH}{N}$ $\qquad$ N: number potential intervals

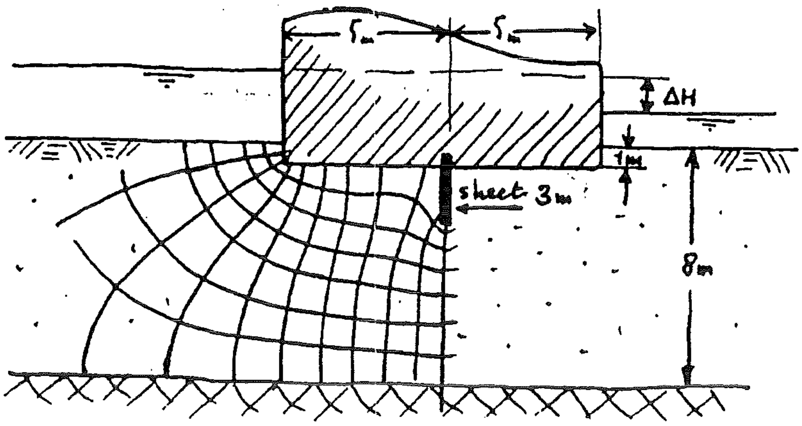
$Q = Mq\Delta n = M\Delta\Psi = \dfrac{M}{N}kH$ $\qquad$ M: number flow tubes

$M = 3,\ N = 7$

$Q = \dfrac{3}{7}kH$

$$\boxed{Q = \frac{M}{N}kH} \qquad \boxed{q = \frac{\Delta\Psi}{\Delta n} = \frac{kH}{N\Delta n}} \qquad \boxed{p = \gamma\left(\frac{\Phi}{k} - z\right)}$$

Two examples are shown:



Flow net sketched for half the area (Symmetry)

Through every flow tube similar discharge: $q\Delta n = -\Delta\Psi$

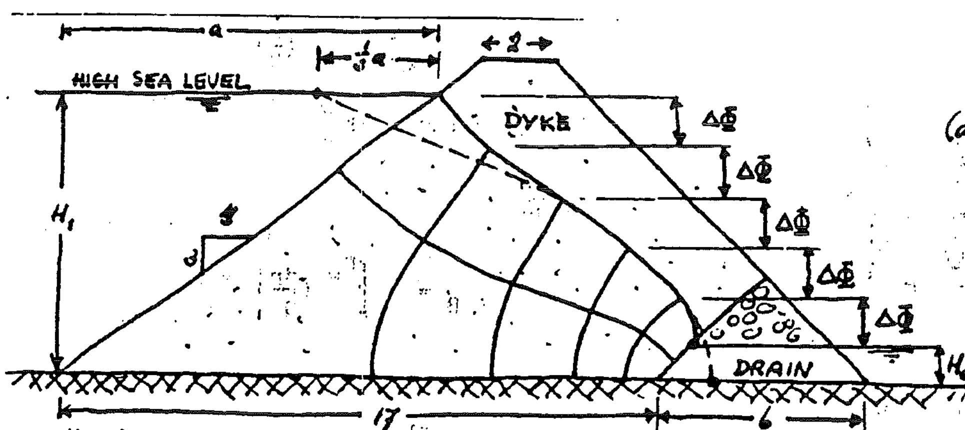
Total discharge $Q = 7\Delta\Psi$

Flow net theory $\Delta\Psi = \Delta\Phi$ because $\Delta n = \Delta s$ (squares)

Total drop ΔH over 20 equipotential intervals $\rightarrow \Delta\Phi = k\Delta H/20$

or: $F = N/M = 20/7 = 2.86$

Thus: $Q = 7\Delta\Psi = 7\Delta\Phi = 7\, k\Delta H/20 = 0.35\, k\Delta H$ $\qquad Q = k\Delta H/F = 0.35 k\Delta H$



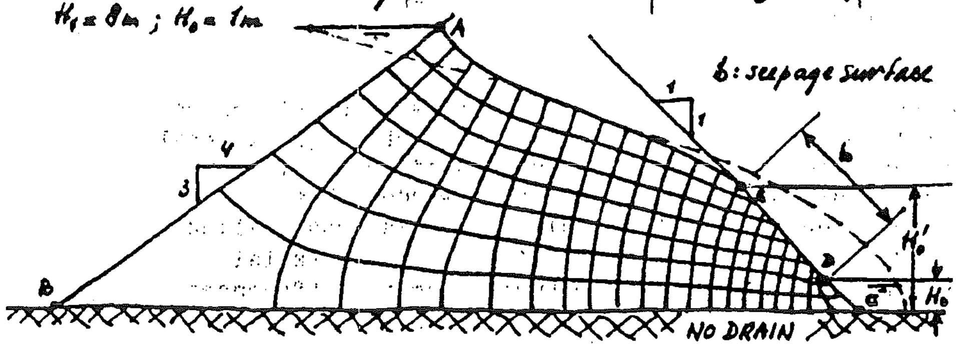
Flow net sketch (dashed: Dupuis parabola) 2 Flow tubes

$Q = 2q\Delta n = 2\Delta\Psi$

$= 2\Delta\Phi = 2\frac{k(H_1-H_0)}{5}$

$= 2.8\, k\, [m^3/ms]$

$H_1 = 8m$; $H_0 = 1m$



Without drain 7 Flow tubes

$Q = 7\Delta\Psi = 7\Delta\Phi$

$= 7k\frac{H_1-H_0}{21} = \frac{7}{3}k$

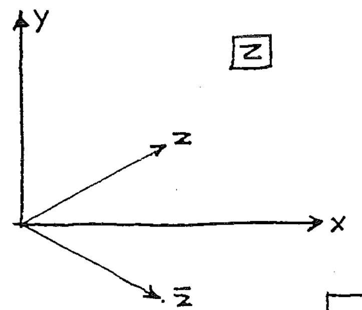
Seepage surface:

$Q = 7\Delta\Phi = 7k\frac{H_1-H_0'}{13}$

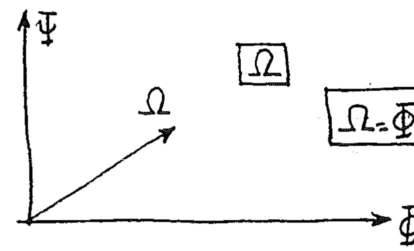
$\rightarrow H_0' = 8 - \frac{13}{3} = 3.6m$.

Complex Potential

$\boxed{z}$ $\boxed{z = x + iy}$



$\boxed{\Omega}$ $\boxed{\Omega = \Phi + i\Psi}$



$$\boxed{z = z(\Omega)} \Longleftrightarrow \boxed{\Omega = \Omega(z)}$$

Mapping

$$\boxed{\Omega = \Omega(z)} \qquad \frac{d\Omega}{dz} \equiv \lim_{\Delta z \to 0} \frac{\Omega(z+\Delta z) - \Omega(z)}{\Delta z}$$

independent on the path

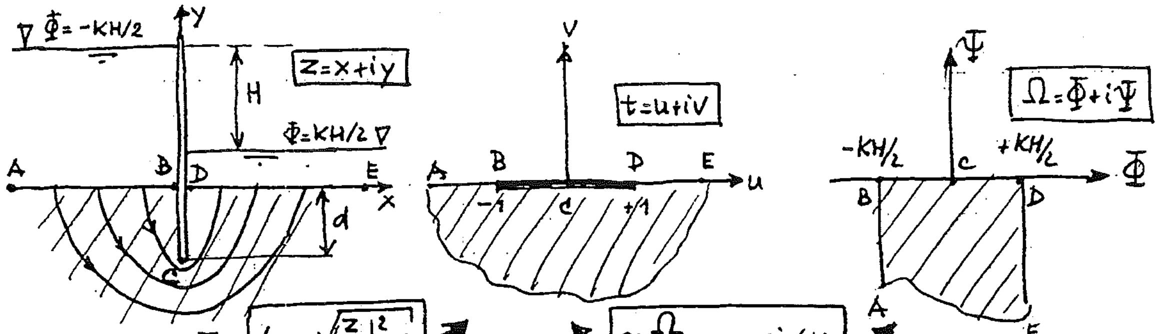
$$\boxed{\Omega \text{ is differentiable (or analytic)} \longleftrightarrow \nabla^2\Phi = 0 \ \wedge \ \nabla^2\Psi = 0}$$

$$\frac{\partial\Omega}{\partial x} = \frac{d\Omega}{dz}\frac{\partial z}{\partial x} \qquad \frac{\partial\Omega}{\partial y} = \frac{d\Omega}{dz}\frac{\partial z}{\partial y} \qquad z = x + iy \rightarrow \frac{\partial z}{\partial x} = 1, \ \frac{\partial z}{\partial y} = i$$

$$\frac{d\Omega}{dz} = \frac{\partial\Omega}{\partial x} = \frac{\partial\Omega}{i\,\partial y} = \frac{i}{i^2}\frac{\partial\Omega}{\partial y} = -i\frac{\partial\Omega}{\partial y}$$

$$\frac{\partial[\Phi + i\Psi]}{\partial x} = -i\frac{\partial[\Phi + i\Psi]}{\partial y} \rightarrow \frac{\partial\Phi}{\partial x} + i\frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{\partial y} - i\frac{\partial\Phi}{\partial y}$$

Cauchy Riemann conditions $\rightarrow$ $\boxed{\frac{\partial\Phi}{\partial x} = \frac{\partial\Psi}{\partial y}}$ $\boxed{\frac{\partial\Psi}{\partial x} = -\frac{\partial\Phi}{\partial y}}$

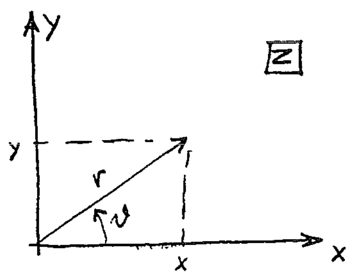


$$\boxed{t = -\sqrt{\left(\frac{z}{d}\right)^2 + 1}} \qquad \boxed{\pi\frac{\Omega}{KH} = \arcsin(t)}$$

Conformal mapping

$$\boxed{\Omega = \frac{KH}{\pi}\arcsin\left[-\sqrt{\left(\frac{z}{d}\right)^2 + 1}\right]}$$

Complex Algebra



$z = x + iy$

$z = r\cos\vartheta + i r \sin\vartheta$

$z = r e^{i\vartheta}$

$\boxed{e^{i\vartheta} = \cos\vartheta + i\sin\vartheta}$ Euler's rule

---

$$\left.\begin{aligned} e^{i\vartheta} &= \cos\vartheta + i\sin\vartheta \\ e^{-i\vartheta} &= \cos\vartheta - i\sin\vartheta \end{aligned}\right\} \rightarrow$$

$\cos(\vartheta) = \frac{1}{2}(e^{i\vartheta} + e^{-i\vartheta})$

$i\sin(\vartheta) = \frac{1}{2}(e^{i\vartheta} - e^{-i\vartheta})$

$\cos(i\vartheta) = \frac{1}{2}(e^{-\vartheta} + e^{+\vartheta}) = \cosh(\vartheta)$

$i\sin(i\vartheta) = \frac{1}{2}(e^{-\vartheta} + e^{+\vartheta}) = -\sinh(\vartheta)$

$\sin(i\vartheta) = i\sinh(\vartheta)$

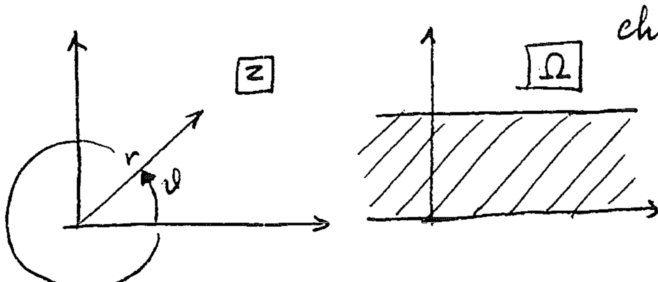
$$\frac{a+ib}{c+id} = \frac{a+ib}{c+id}\cdot\frac{c-id}{c-id} = \frac{(a+ib)(c-id)}{c^2 - i^2 d^2} = \frac{(ac+bd) + i(bc-ad)}{c^2+d^2}$$

$\Omega = \ln(z) \rightarrow \Phi + i\Psi = \ln(re^{i\vartheta}) = \ln r + i\vartheta$

$\rightarrow \Phi = \ln r \wedge \Psi = \vartheta$

ϑ ~~is meerwaardig~~ is manivalued.

choice $0 < \vartheta < 360$ (for example)



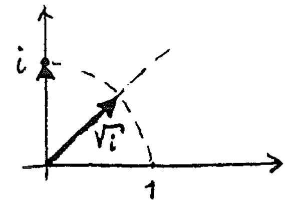
single valuedness

$$\sqrt{i} = \sqrt{r e^{i\pi/2}} = \sqrt{r}\, e^{i\pi/4}$$

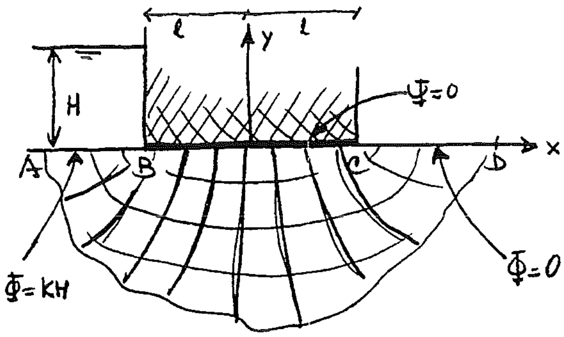
$r = 1$

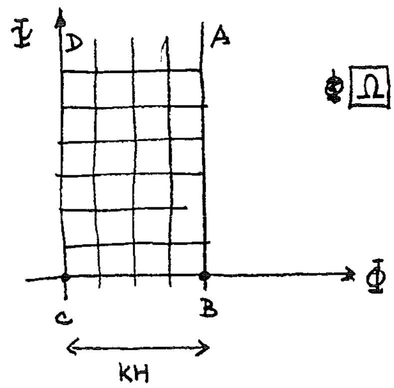
$= \cos \pi/4 + i \sin \pi/4$

$= \frac{1+i}{\sqrt{2}}$



Conformal Mapping





Φ $\boxed{\Omega}$

$$\boxed{\Omega = \frac{kH}{\pi}\arccos\left(\frac{z}{l}\right)}$$

$$\boxed{\frac{z}{l} = \cos\left(\frac{\pi\Omega}{kH}\right).} \quad\longrightarrow\quad \frac{x+iy}{l} = \cos\frac{\pi(\Phi+i\Psi)}{kH} = \cos\left(\frac{\pi\Phi}{kH}\right)\cos\left(\frac{\pi i\Psi}{kH}\right) - \sin\left(\frac{\pi\Phi}{kH}\right)\sin\left(\frac{\pi i\Psi}{kH}\right)$$

$$\longrightarrow \quad \boxed{\frac{x}{l} = \cos\frac{\pi\Phi}{kH}\cosh\frac{\pi\Psi}{kH}} + \boxed{\frac{y}{l} = -\sin\frac{\pi\Phi}{kH}\sinh\frac{\pi\Psi}{kH}}$$

$\boxed{\text{Flow lines}}$: eliminate Φ $\rightarrow$ x,y as function of Ψ
for Ψ is constant a flowline is formed.

$$\left(\frac{x}{l}\right)^2 = \cos^2\frac{\pi\Phi}{kH}\cosh^2\frac{\pi\Psi}{kH} = \left(1-\sin^2\frac{\pi\Phi}{kH}\right)\cosh^2\frac{\pi\Psi}{kH} = \left(1-\left(\frac{y}{l}\right)^2\frac{1}{\sinh^2\frac{\pi\Psi}{kH}}\right)\cosh^2\frac{\pi\Psi}{kH}$$

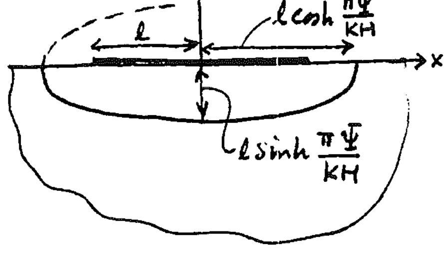
$$\left(\frac{y}{l}\right)^2 = \sin^2\frac{\pi\Phi}{kH}\sinh^2\frac{\pi\Psi}{kH}$$

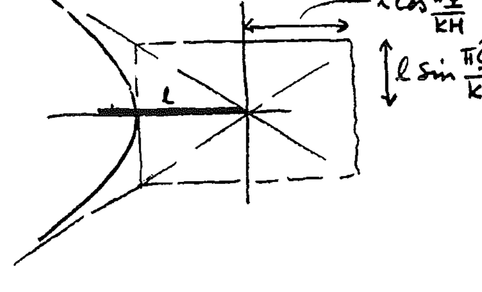
ellips for Ψ is constant

$$\boxed{\frac{1}{\cosh^2\left(\frac{\pi\Psi}{kH}\right)}\left(\frac{x}{l}\right)^2 + \frac{1}{\sinh^2\left(\frac{\pi\Psi}{kH}\right)}\left(\frac{y}{l}\right)^2 = 1}$$

$\boxed{\text{Potential lines}}$: eliminate Ψ $\rightarrow$ x,y as function of Φ
for Φ is constant a potential line is formed

hyperbola for Φ is constant
$$\frac{1}{\cos^2\left(\frac{\pi\Phi}{kH}\right)}\left(\frac{x}{l}\right)^2 - \frac{1}{\sin^2\left(\frac{\pi\Phi}{kH}\right)}\left(\frac{y}{l}\right)^2 = 1$$



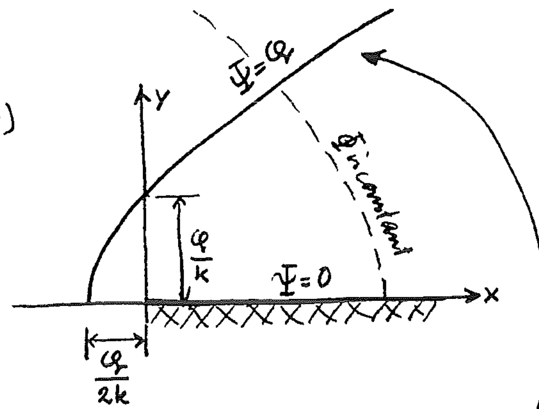


Flow in dams

$\boxed{\Omega = \sqrt{(2kQz)}}$ (Vreedenburgh)

$$\Omega^2 = \Phi^2 + 2i\Phi\Psi - \Psi^2 = 2kQz = 2kQ\,(x+iy)$$

$$\rightarrow \boxed{x = \frac{\Phi^2 - \Psi^2}{2kQ}} \quad + \quad \boxed{y = \frac{\Phi\Psi}{kQ}}$$



Flow line: eliminate Φ

$$\Phi = \frac{kQy}{\Psi} \rightarrow x = \frac{\left(\frac{kQy}{\Psi}\right)^2 - \Psi^2}{2kQ} \rightarrow 2kQx = \left(\frac{kQy}{\Psi}\right)^2 - \Psi^2$$

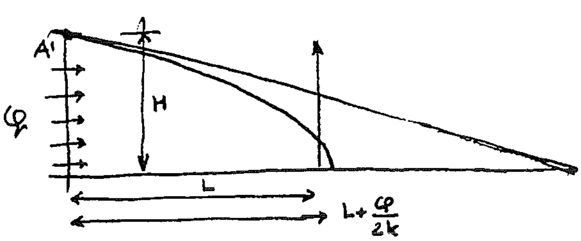
$$\text{for } \Psi = Q \rightarrow \frac{2kx}{Q} = \left(\frac{ky}{Q}\right)^2 - 1 \rightarrow \boxed{\frac{2k}{Q}\left(x + \frac{Q}{2k}\right) = \left(\frac{ky}{Q}\right)^2} \quad \text{parabola}$$

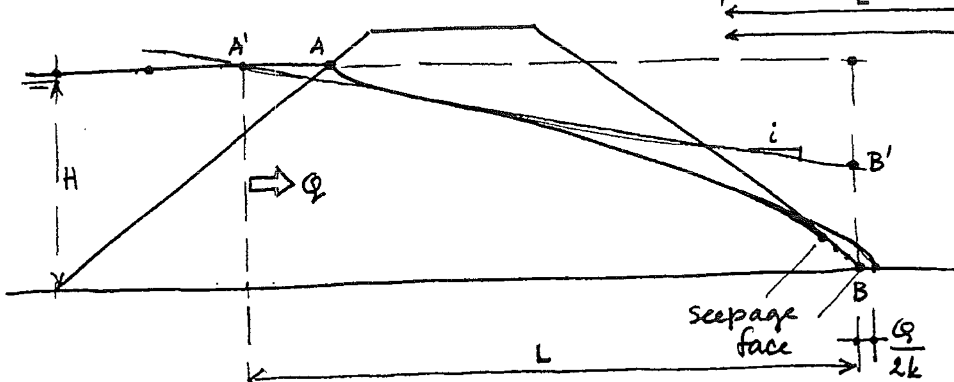
$$\Psi = 0 \rightarrow x = \frac{\Phi^2}{2kQ},\ y = 0 \rightarrow (x,y) = [0<x<\infty,\ y=0]$$

Potential line: eliminate Ψ $\rightarrow$ $2kQx = \Phi^2 - \left(\frac{kQy}{\Phi}\right)^2$ parabola

Free surface: $p=0$, $\Psi = Q$, $\quad y = \frac{\Phi\Psi}{kQ} = \frac{\Phi}{k} = \phi = y + \frac{p}{\gamma} = y$ (correct)

Graphical method free surface





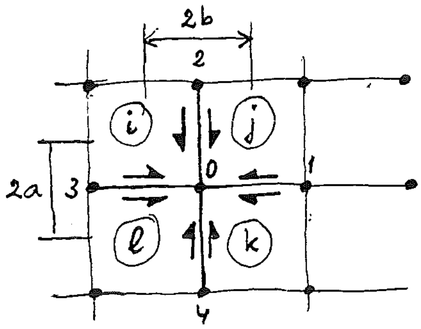
$K = 0.0001$ m/s
$H = 5$ m
$L = 25$ m

$\rightarrow \frac{Q}{2k} = 0.5 \ll L$

$$Q = Hq = Hki = Hk\frac{H/2}{L} = k\frac{H^2}{2L} \quad \text{(Dupuit)}$$

Precise: $\quad Q = Hk\frac{H}{2L + Q/k} \rightarrow Q = Lk\left(\sqrt{\left(1 + \left(\frac{H}{L}\right)^2\right)} - 1\right) \rightarrow \text{for } \frac{H}{L} < 1:\ Q = k\frac{H^2}{2L}$

FINITE DIFFERENCE METHOD



Thickness H

Conservation of mass

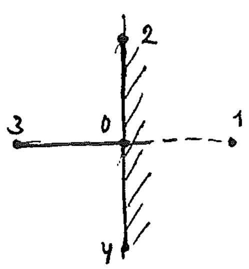
$$Q^{i}_{20} + Q^{j}_{20} + Q^{j}_{10} + Q^{k}_{10} +$$

$$Q^{k}_{40} + Q^{\ell}_{40} + Q^{\ell}_{30} + Q^{i}_{30} = Q_0$$

↑ nodal discharge

$$Q^{i}_{20} = K^{i} b H \frac{\phi_2 - \phi_0}{2a}$$

Homogeneous case, square net $(a=b)$

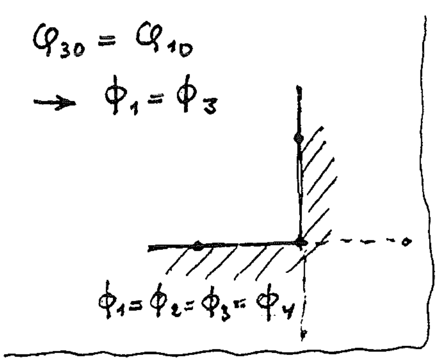


$$2KbH \frac{\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_0}{2a} = Q_0$$

$$\boxed{\phi_0 = \frac{1}{4}(\phi_1 + \phi_2 + \phi_3 + \phi_4) - \frac{Q_0}{4KH}}$$

$Q_{30} = Q_{10}$

$\rightarrow \phi_1 = \phi_3$



Solution procedure

$$\phi_0^{n} = \frac{1}{4}(\phi_1^{n-1} + \phi_2^{n-1} + \phi_3^{n-1} + \phi_4^{n-1})$$

RELAXATION

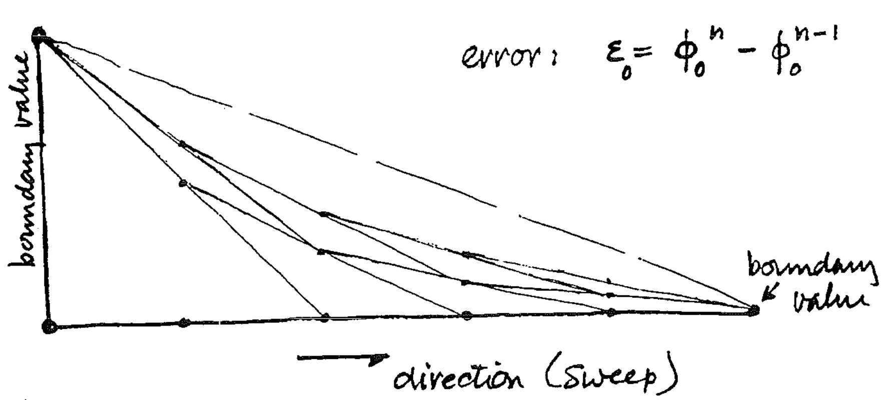
Adjust the node with largest error

OVERRELAXATION

Exaggerate too some extent (20%)

Use newest value

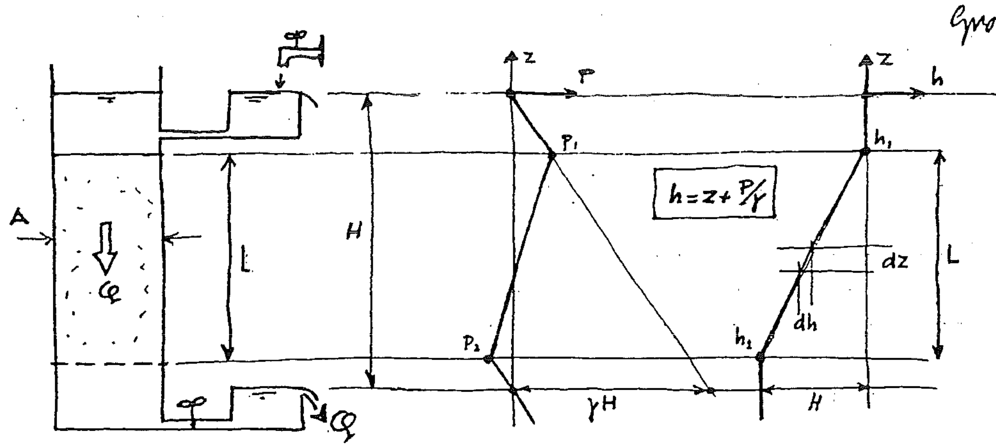
error: $\varepsilon_0 = \phi_0^{n} - \phi_0^{n-1}$



TEST

constant head

DARCY 1856

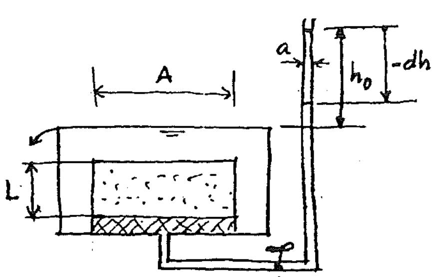


$$q_z = \frac{Q}{A} = -k\frac{\partial h}{\partial z} = -k\,\frac{h_1 - h_2}{L} = +k\frac{H}{L} \;\rightarrow\; \boxed{Q = +kA\frac{H}{L}}$$

↑ constant for sample

$$\boxed{k = \frac{QL}{AH}}$$

Falling head



observe lowering of h in time after opening the tap

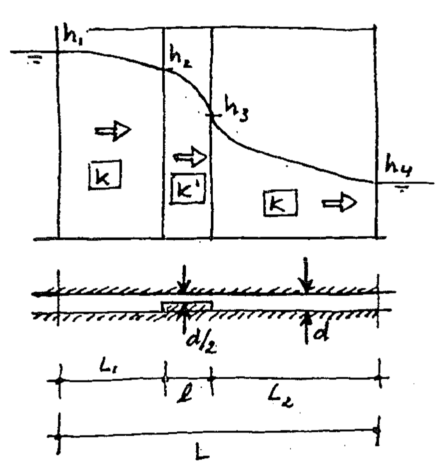
$Q = kAh/L$ (no acceleration effects)

$Q = -a\dfrac{\partial h}{\partial t}$ (storage in the pipe)

$$\frac{dh}{dt} = -\frac{kA}{aL}\,h \;\rightarrow\; h = h_0\,e^{-kAt/aL}$$

$$\rightarrow \boxed{k = \frac{aL}{At}\,\ln\left(\frac{h_0}{h}\right)}$$

Platemodel



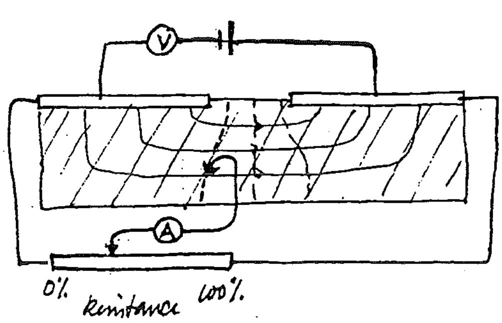
$$\frac{Q}{k} = \frac{h_1^2 - h_2^2}{2L_1} = \frac{k'}{k}\,\frac{h_2^2 - h_3^2}{2\ell} = \frac{h_3^2 - h_4^2}{2L_2}$$

$$\rightsquigarrow \boxed{\frac{Q}{k} = \frac{h_1^2 - h_4^2}{2L}\cdot f} \qquad f = \frac{\alpha L}{(1+\alpha)L_1 + \alpha L_2}$$

$$\alpha = \frac{k'}{k}\,\frac{L_1}{\ell}$$

Observe 2-dimensional flow lines

Resistance model



Ohm's law $\quad i = -\dfrac{1}{\rho}\,\dfrac{dV}{ds}$

i: current $[A/m^2]$; ρ: resistance [Ohm·m]; V: volt [V]; s: flow path

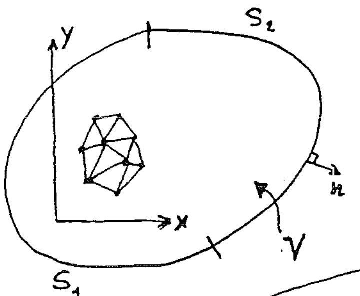
conservation electric energy $\quad \nabla\cdot i = 0$

$\rightarrow \quad \nabla^2 V = 0$

FINITE ELEMENT METHOD

Plane groundwaterflow, steady without infiltration or leakage

FUNCTIONAL $\quad U = \frac{1}{2}\int_V \left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right] dxdy \quad$ *U depends on ϕ; ϕ is a function of (x,y)*



> **Lemma:** If $\phi=$const along S_1 and U attains a minimum value then $\underbrace{\nabla^2\phi = 0 \text{ and } \partial\phi/\partial n = 0 \text{ along } S_2}_{\text{this is a groundwaterflow problem}}$

If this is true then: try approximate ϕ and condition $\delta U = 0$, then ϕ is the solution of a groundwaterflow problem

> FINITE ELEMENT METHOD: approximate ϕ by simple (linear) functions in simple segments of the flow field (triangles). Use $\delta U = 0$ to find equations for ϕ. The resulting solution is the problem solution

Prove Chose $\eta(x,y)$, with $\eta=0$ on S_1. Consider the function $\phi+\alpha\eta$, α is a scalar constant. The variation of U is then defined by: $\quad \delta U = \frac{d}{d\alpha}\left[U(\phi+\alpha\eta)\right]\Big|_{\alpha\downarrow 0}$

δU is independent of η.

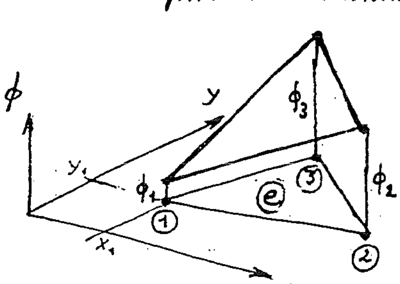
$$\delta U = \frac{1}{2}\int_V \left[\frac{d}{d\alpha}\left[\left(\frac{\partial(\phi+\eta\alpha)}{\partial x}\right)^2 + \left(\frac{\partial(\phi+\alpha\eta)}{\partial y}\right)^2\right]\right]\Bigg|_{\alpha\downarrow 0} dxdy = \iint_V \left[\frac{\partial(\phi+\alpha\eta)}{\partial x}\cdot\frac{\partial\eta}{\partial x} + \frac{\partial(\phi+\alpha\eta)}{\partial y}\frac{\partial\eta}{\partial y}\right]\Bigg|_{\alpha\downarrow 0} dxdy$$

$$= \iint_V \left[\frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial y}\right] dxdy = \iint_V \left[\frac{\partial}{\partial x}\left(\eta\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\eta\frac{\partial\phi}{\partial y}\right) - \eta\frac{\partial^2\phi}{\partial x^2} - \eta\frac{\partial^2\phi}{\partial y^2}\right] dxdy$$

$$= \underbrace{\int_S \left[\eta\frac{\partial\phi}{\partial x}dy + \eta\frac{\partial\phi}{\partial y}dx\right]}_{\eta\frac{\partial\phi}{\partial n}ds} - \int_V \eta\nabla^2\phi\, dxdy = \int_{S_1} \eta\frac{\partial\phi}{\partial n}ds + \int_{S_2}\eta\frac{\partial\phi}{\partial n}ds - \int_V \eta\nabla^2\phi\, dxdy$$

Thus $\delta U = 0$ for any η, if $\eta=0$ on S_1, $\frac{\partial\phi}{\partial n}=0$ on S_2, and $\nabla^2\phi = 0$ in V. This proves the Lemma

The functional U can be extended to include infiltration, leakage, consolidation, etcetera (See Num. Methods literature)



element e

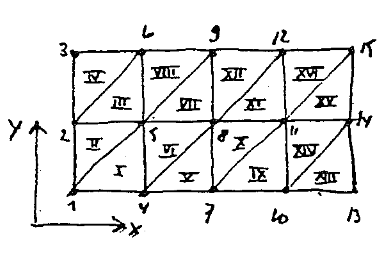
Approximate ϕ in triangular segments by linear interpolation from nodal values ϕ_1, ϕ_2, ϕ_3:

$$\boxed{\phi^e = a_1^e x + a_2^e y + a_3^e} \longrightarrow \boxed{\phi^e = b_1^e\phi_1 + b_2^e\phi_2 + b_3^e\phi_3}$$

$$b_i^e = \left\{\frac{(y_j - y_k)x + (x_k - x_j)y + x_j y_k - x_k y_j}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\right\}, \quad i,j,k \text{ cyclic node numbers } 1,2,3$$

Substitution in U leads to: $\quad U = \sum U^e = \frac{1}{2}\sum_{k=1}^{N}\sum_{l=1}^{N} P_{kl}\,\phi_k\phi_l$, $\quad$ N total nodes, k,l node numbers

Matrix P contains the interpolation functions (geometry of nodal distribution and element structure)



EXAMPLE
ELEMENT/NODE
DISTRIBUTION

$\delta U = 0$, if for any variation in ϕ_i (a nodal value) no change in U occurs

$$\frac{\partial U}{\partial \phi_i} = \boxed{\sum_{k=1}^{N} P_{ik}\phi_k = 0}$$

This provides a set of N equations to solve N unknown nodal values ϕ_i

The solution of the set equations (linear) can be by direct method (Gauss elimination), by semi-direct method (conjugate gradient) or by indirect method (Gauss-Seidel iteration)

Gauss-Seidel

$$\sum_{k=1}^{N} P_{ik}\phi_k = 0 \rightarrow P_{ii}\phi_i + \sum_{\substack{k=1\\k\neq i}}^{N} P_{ik}\phi_k = 0 \rightarrow \boxed{\phi_i = \frac{-1}{P_{ii}}\sum_{\substack{k=1\\k\neq i}}^{N} P_{ik}\phi_k}$$

Successive Substitution

Chose ϕ_k, calculate ϕ_i, and do so for all nodes 1,..,N until no change occurs

Every iteration a new estimate is formed $\boxed{\phi_i^{n+1} = \frac{-1}{P_{ii}}\sum_{\substack{k=1\\k\neq i}}^{N} P_{ik}\phi_k^{n}}$

If during the iteration the latest value is used

$$\boxed{\phi_i^{n+1} = \frac{-1}{P_{ii}}\left\{\sum_{k=1}^{i-1} P_{ik}\phi_k^{n+1} + \sum_{k=i+1}^{N} P_{ik}\phi_k^{n}\right\}}$$

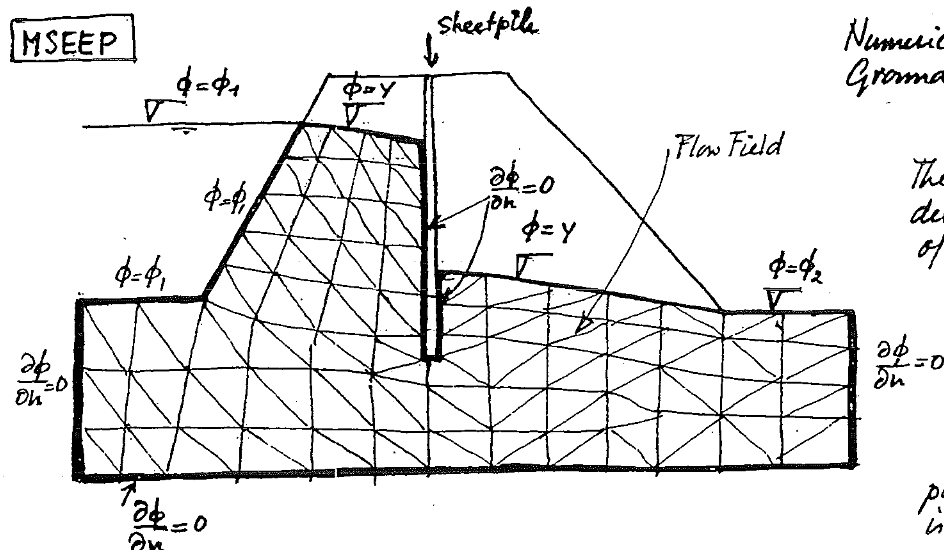
$0<r<2$

Overrelaxation: exagerate the subsequente changes: $\phi_i^{n+1} = \phi_i^{n} + r(\phi_i^{n+1} - \phi_i^{n})$

$$\boxed{\phi_i^{n+1} = (1-r)\phi_i^{n} + \frac{-r}{P_{ii}}\left\{\sum_{k=1}^{i-1} P_{ik}\phi_k^{n+1} + \sum_{k=i+1}^{N} P_{ik}\phi_k^{n}\right\}}$$

best choice for $r = 1.4$

MSEEP

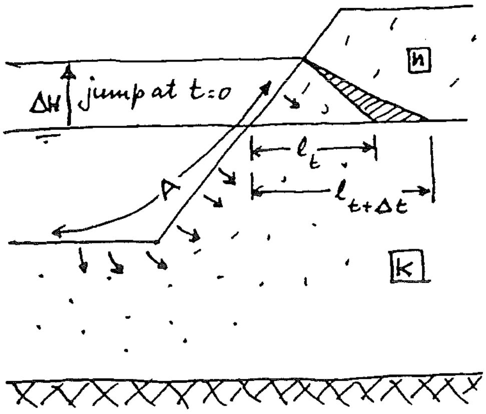


Numerical Program FEMethod Groundwaterflow MSEEP

The freatic surface is determined by a first guess of the position, calculating the flow problem assuming along the surface $\frac{\partial \phi}{\partial n} = 0$ (flow line) and checking afterwards if $\phi = Y$. If not the position of the freatic line is adjusted accordingly, and the procedure is repeated until acceptable accuracy.

TIME-VARIANT POROUS FLOW

Phreatic Storage



Volume water penetration

$$Q\Delta t = k\Delta t\, A\left(\frac{\Delta H}{\ell}\right)$$

Volume stored (dashed area)

$$n\,\Delta H\,(\ell_{t+\Delta t} - \ell_t)/2$$

Inflow = storage :

$$\Delta H \frac{\ell_{t+\Delta t} - \ell_t}{\Delta t} \cdot \frac{n}{2} = k\Delta H A/\ell$$

$$\rightarrow \ell\frac{d\ell}{dt} = 2\frac{kA}{n}$$

$$\rightarrow \frac{d(\ell^2)}{dt} = 4kA/n$$

$k = 0.0001\,\frac{m}{s}$; $A = 10\,m$; $n = 0.2$

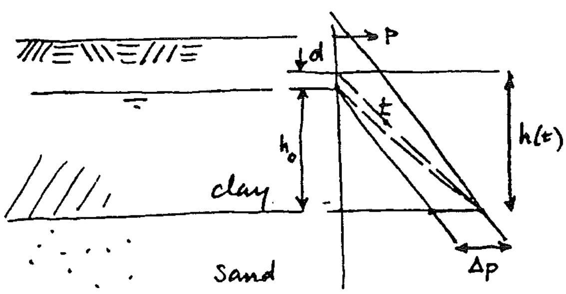
Waves $t \approx 10\,s \rightarrow \ell \approx 0.20\,m$

tydes $t = 4\,hr \rightarrow \ell \approx 8.50\,m$

rivers $t = 3\,day \rightarrow \ell = 36.0\,m$

ℓ penetration length

$$\rightarrow \ell^2 = 4kAt/n \rightarrow \boxed{\ell = 2\sqrt{kAt/n}}$$



How fast rises the watertable?

$$\boxed{n\frac{dh}{dt} = k\frac{\Delta p/\gamma + h_0 - h}{h}}$$ Storage equation

Storage = volume of water

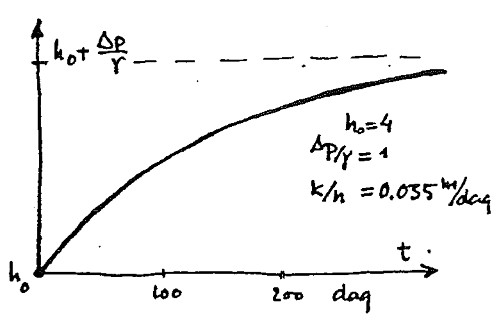
Solution Separation of variables →

$$\frac{k\,dt}{n} = \frac{h}{\frac{\Delta p}{\gamma}+h_0-h}\,dh = \frac{h-\left(\frac{\Delta p}{\gamma}+h_0\right)+\left(\frac{\Delta p}{\gamma}+h_0\right)}{\frac{\Delta p}{\gamma}+h_0-h}\,dh$$

$$= \left[-1 + \left(\frac{\frac{\Delta p}{\gamma}+h_0}{\frac{\Delta p}{\gamma}+h_0-h}\right)\right]dh$$

$$= -dh - \left(\frac{\Delta p}{\gamma}+h_0\right)d\left[\ln\left(\frac{\Delta p}{\gamma}+h_0-h\right)\right]$$

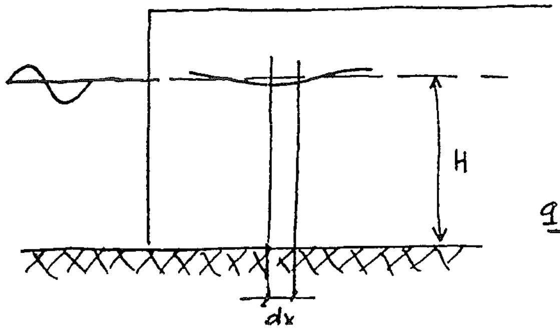
$$\frac{k}{n}\int_0^t dt = \int_{h_0}^{h} \cdots\cdots$$

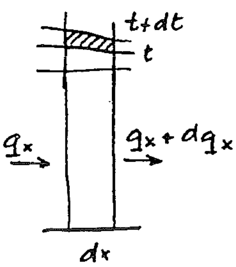


$$\frac{kt}{n} = -(h-h_0) - \left(\frac{\Delta p}{\gamma}+h_0\right)\ln\left(\frac{\frac{\Delta p}{\gamma}+h_0-h}{\frac{\Delta p}{\gamma}+h_0-h_0}\right)$$

$$\boxed{\frac{kt}{n} = -(h-h_0) + \left(\frac{\Delta p}{\gamma}+h_0\right)\ln\left[1/\left(1-\frac{h-h_0}{\Delta p/\gamma}\right)\right]}$$

TIME-VARIANT POROUS FLOW





Storage : $n\frac{\partial h}{\partial t}\Delta t \Delta x$

net outflow : $\frac{\partial h q_x}{\partial x}\Delta x \Delta t$

Phreatic storage

Storage equation

$$\boxed{n\frac{\partial h}{\partial t} + \frac{\partial h q_x}{\partial x} = 0}$$

Darcy's law $q_x = -k\frac{\partial h}{\partial x}$

$$\boxed{n\frac{\partial h}{\partial t} = \frac{\partial}{\partial x} k h \frac{\partial h}{\partial x}}$$

nonlinear partial differ. equation.

Linearisation

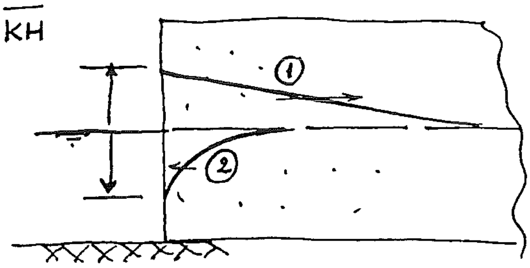
① $$\boxed{n\frac{\partial h}{\partial t} = kH\frac{\partial^2 h}{\partial x^2}}$$

$$\frac{nL^2}{kHT}\frac{\partial(h/H)}{\partial(t/T)} = \frac{\partial^2(h/H)}{\partial(x/L)^2}$$

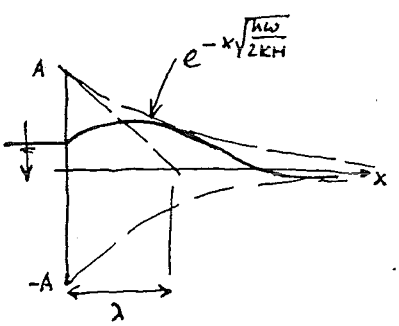
$\rightarrow$ $L = \sqrt{kHT/n}$; $T = n\frac{L^2}{kH}$

② $$n\frac{h}{H}\frac{\partial h}{\partial t} = k\frac{\partial}{\partial x}h\frac{\partial h}{\partial x}$$

$$\boxed{n\frac{\partial(h^2)}{\partial t} = kH\frac{\partial^2(h^2)}{\partial x^2}}$$



Solution for cyclic loading (method of complex harmonics)



$\lambda = \sqrt{\frac{2kH}{n\omega}}$; $\omega = \frac{2\pi}{T}$

$\lambda = \sqrt{\frac{kHT}{n\pi}} = 0.564\sqrt{\frac{kHT}{n}}$

$\beta\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2}$; $\beta = \frac{n}{kH}$; $x=0$: $h = Ae^{i\omega t}$

Take: $h = \bar{h}e^{i\omega t}$; $\bar{h} = \bar{h}(x)$

$\beta i\omega\bar{h}\cancel{e^{i\omega t}} = \frac{d^2\bar{h}}{dx^2}\cancel{e^{i\omega t}} \rightarrow \beta i\omega\bar{h} = \frac{d^2\bar{h}}{dx^2}$

$\bar{h} = Be^{\alpha x} \rightarrow \beta i\omega Be^{\alpha x} = B\alpha^2e^{\alpha x} \rightarrow \alpha^2 = \beta i\omega$

$\alpha = \pm\sqrt{\beta i\omega}$

$\bar{h} = B_1e^{+\sqrt{\alpha}x} + B_2e^{-\sqrt{\alpha}x}$

$x\to\infty$ $\bar{h} = 0 \rightarrow B_1 = 0$

$x = 0$ $\bar{h} = A \rightarrow B_2 = A$

$\bar{h} = Ae^{-\sqrt{\alpha}x}$

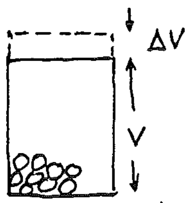
$\rightarrow h = Ae^{i\omega t - \sqrt{\alpha}x}$

$\sqrt{i} = \frac{1+i}{\sqrt{2}} \rightarrow h = Ae^{i(\omega t - x\sqrt{\beta\omega/2}) - x\sqrt{\beta\omega/2}}$

$$h = Ae^{-x\sqrt{\frac{n\omega}{2kH}}}\left\{\cos\left[\omega t - x\sqrt{\tfrac{n\omega}{2kH}}\right] + i\sin\left[\omega t - x\sqrt{\tfrac{n\omega}{2kH}}\right]\right\}$$

TIME VARIANT POROUS FLOW

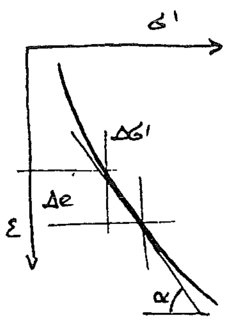
Elastic Storage



$$\frac{\Delta V}{V} \equiv \boxed{\Delta e = \frac{\Delta n V}{V} - \nabla \cdot q \; \Delta t}$$

volume strain — swell pore water — net inflow volume (outflow is positive)

Water compressibility $\quad -\beta = \dfrac{\Delta V}{V \Delta p} \rightarrow \boxed{-n\beta = \dfrac{\Delta n V}{V \Delta p}}$



Soil (Skeleton) compressibility $\quad -\alpha = \dfrac{\Delta V}{V \Delta \sigma'} \rightarrow \boxed{\Delta e = -\alpha \Delta \sigma'}$

σ' effective stress

Terzaghi $\quad \boxed{\sigma = \sigma' + p} \longrightarrow \boxed{\Delta e = \alpha(\Delta p - \Delta \sigma)}$

Suppose no change in total stress (overburden) $\rightarrow \Delta\sigma = 0$
This is the case for pumping wells.

$$\alpha(\Delta p - \cancel{\Delta \sigma}) = -n\beta \Delta p - \nabla \cdot q \; \Delta t$$

STORAGE EQUATION $\quad \boxed{(\alpha + n\beta)\Delta p = -\nabla \cdot q \; \Delta t}$

Darcy's law $\quad q = -k \nabla \phi$

$\Delta \phi = \Delta p / \gamma + \Delta z$

$\rightarrow \dfrac{\partial \phi}{\partial t} = \dfrac{\partial p}{\gamma \partial t} \quad \left(\dfrac{\partial z}{\partial t} = 0\right)$

$$\boxed{\frac{\partial \phi}{\partial t} = \frac{k}{\gamma(\alpha + n\beta)} \nabla^2 \phi}$$

$\boxed{\dfrac{k}{\gamma(\alpha + n\beta)} = c}$ coefficient of consolidation $[m^2/s]$

Comparison with phreatic storage $\quad \boxed{c = \dfrac{kH}{n}}$

DIMENSION ANALYSIS

$$\frac{\partial \phi / \phi_0}{\partial t / T} = \frac{Tc}{L^2} \left\{ \frac{\partial^2 \phi / \phi_0}{\partial (x/L)^2} + \frac{\partial^2 \phi / \phi_0}{\partial (y/L)^2} \right\}$$

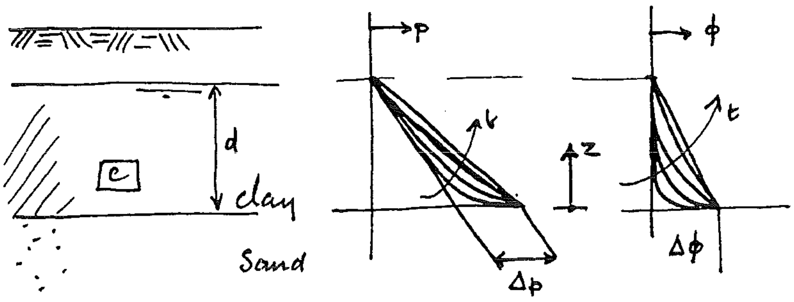
$\hookrightarrow \quad L = \sqrt{Tc} = \sqrt{\dfrac{kHT}{n}} \quad ; \quad \boxed{T = \dfrac{L^2}{c}}$

hydrodynamic period

Storage coefficient $\quad S = \dfrac{kH}{c}$

$S \approx 0.001$ for sand

TIME VARIANT POROUS FLOW



$$\boxed{\frac{\partial \phi}{\partial t} = c\,\frac{\partial^2 \phi}{\partial z^2}}$$

Laplace transform

$$\boxed{s\bar{\phi} = c\,\frac{d^2\bar{\phi}}{dz^2}}$$

$$\boxed{\bar{\phi} = \frac{\Delta\phi}{s}\,\frac{\sinh\left[(1-\frac{z}{d})d\sqrt{\frac{s}{c}}\right]}{\sinh\left[d\sqrt{\frac{s}{c}}\right]}}$$

$z=0 \quad \phi = \begin{matrix} 0 & t<0 \\ \Delta\phi & t>0 \end{matrix}$

$z=d \quad \phi = 0$

$z=0 \quad \bar{\phi} = \frac{\Delta\phi}{s}$

$z=d \quad \bar{\phi} = 0$

inverse (approx.)

$$\boxed{\phi = \Delta\phi\,\frac{\sinh\left[(1-\frac{z}{d})\delta\right]}{\sinh[\delta]}}$$

$\delta = d/\sqrt{2ct}$

<u>limit</u> $\quad t \downarrow 0 \quad \phi = \Delta\phi\, e^{-z/\sqrt{2ct}}$

$t \to \infty \quad \phi = \Delta\phi\,(1 - z/d)$

in between $\quad t = T \quad T = \frac{d^2}{2c} \quad (\delta = 1)$ hydrodynamic period

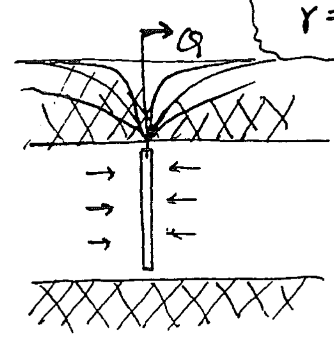
<u>application</u>

$\left.\begin{matrix} \alpha + n\beta = 10^{-7}\ \mathrm{m^2/N} \\ k = 10^{-4}\ \mathrm{m/s} \\ d = 2\mathrm{m} \\ \gamma = 10^4\ \mathrm{N/m^3} \end{matrix}\right\}$ $\quad c = 0.25\ \mathrm{m^2/s}$, $\quad T = 8\mathrm{s}$

$k = 10^{-8}$

$c = 2.5\ 10^{-5}\ \mathrm{m^2/s}$

$T = 22$ min



$$\boxed{\frac{\partial \phi}{\partial t} = \frac{c}{r}\frac{\partial}{\partial r}\, r\,\frac{\partial \phi}{\partial r}}$$

$r_0 < r \leq \infty \quad t > 0$

$\phi = 0 \quad t = 0 \quad r_0 < r < \infty$

$\phi = 0 \quad r \to \infty \quad t \geq 0$

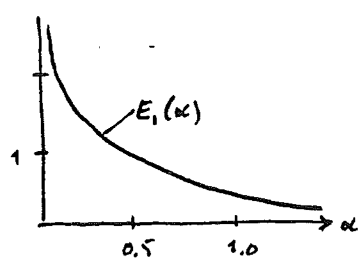
$2\pi r_0 kH \left.\frac{\partial \phi}{\partial r}\right|_{r_0} = -Q \quad t > 0 \quad r = r_0$

$\quad '' \quad = 0 \quad t = 0 \quad r = r_0$

<u>Formula of Theis</u>

Solution $\quad \boxed{\phi = \frac{Q}{4\pi kH}\,E_1\left(\frac{r^2}{4ct}\right)}$

$$E_1(\alpha) = \int_\alpha^\infty \frac{e^{-\beta}}{\beta}\,d\beta$$



Well function

Barends
Grondwaterflow

TRANSPORT achtergrond

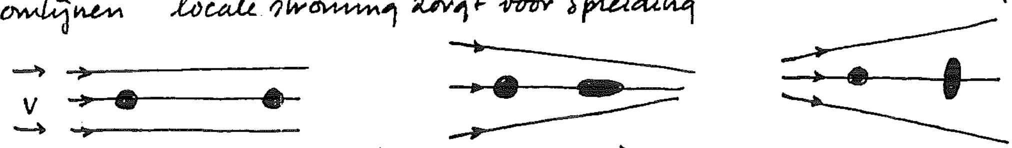
verblijftijd in grond vele jaren.

- advectie/convectie — reversibel
- dispersie — irreversibel
- numerieke dispersie — niet realistisch

Schaal
- moleculair
- microscopisch — korrels, porieruimte
- macroscopisch — permeability
- megascopisch — watervoerende lagen

advectie
$W = c \cdot v$

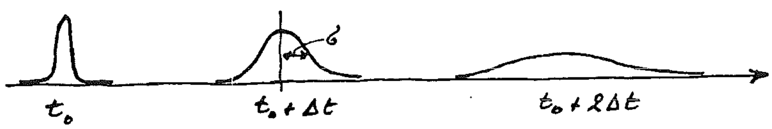
stroomlijnen — locale stroming zorgt voor spreiding



snelheid en weg
verschillende stroomlijnen
verschillend

tortuositeit

Dispersie



Gauss-curve

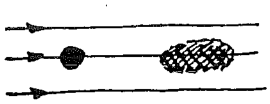
$\sigma^2 = 2 D_d t$ spreiding

Diffusie $D_d = 0.7 \frac{cm^2}{dag}$ zout in water

$\boxed{\text{Wet van Fick} \quad = - D_d \nabla c \quad \text{diffusie}}$

concentratie gradient

microdispersie = diffusie + microscopische stroming (tortuositeit)
(snelheidsafhankelijk)



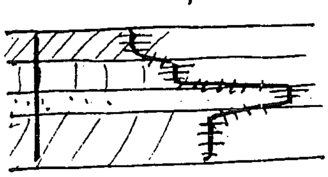
$\sigma_L = \sqrt{2 D_L t}$ $D_L = \alpha_L |v|$ $v = q/n$

$\sigma_T = \sqrt{2 D_T t}$ $D_T = \alpha_T |v|$ $\alpha_L / \alpha_T \sim 5$ a 10

dispersiviteit [cm]

macrodispersie = gelaagdheid
→ macroscopische snelheidsvariatie en transversaal mengen

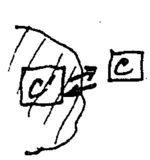
macroscopisch is advectie dominant
microscopisch is diffusie dominant



$\boxed{\sigma^2 = 2 D^m t}$?

op welke schaal gaat dit op

adsorptie massa transport van korrels naar vloeistof en andersom afhankelijk van de concentraties en de transportcapaciteit



niet evenwicht
wel evenwicht $c' = kc$ k evenwichts isotherm (lineair)

c: concentratie in vloeistof

c': concentratie in de deeltjes

TRANSPORT

vergelijkingen

transport — transfer adsorptie — verloop (afbraak)

massa balans

pollutants in de vloeistof $$n\frac{\partial c}{\partial t} = -\nabla\cdot(nw) - q - nG_f$$

pollutants in de deeltjes $$(1-n)\frac{\partial c'}{\partial t} = q - (1-n)G_s$$

adsorptie (evenwicht, lineair) $$c' = Kc$$

advectie en dispersie $$w = cv - D\nabla c$$

dispersie/diffusie $$D_{ij} = (D_d + \alpha_T v)\delta_{ij} + (\alpha_L - \alpha_T)v_i v_j / v$$

afbraak $$c = c_0 e^{-\lambda t}$$

$$\rightarrow \quad R\frac{\partial c}{\partial t} = -\nabla\cdot(cv) + \nabla\cdot D\nabla c - \lambda c$$

transportvergelijking

$R = 1 + \frac{1-n}{n}K$ retardatiecoefficient

$$\nabla\cdot(cv) = c\nabla\cdot v + v\cdot\nabla c = v\cdot\nabla c$$

in constante stroming: $\nabla\cdot v = 0$

uniforme stroming $v = v_x$, $v_y = 0$, $v_z = 0$

$$R\frac{\partial c}{\partial t} + \lambda c = -v\frac{\partial c}{\partial x} + \alpha_L v\frac{\partial^2 c}{\partial x^2} + \alpha_T v\frac{\partial^2 c}{\partial y^2} + \alpha_T v\frac{\partial^2 c}{\partial z^2}$$

adsorptie — afbraak — advectie — spreiding (longitudinaal, transversaal)

$\beta^2 = \alpha_L/\alpha_T$; $\bar{t} = t/R$; $\bar{x} = x - v\bar{t}$; $\bar{y} = \beta y$; $\bar{c} = c\,e^{\lambda t}$; $\bar{z} = \beta z$

$$\frac{\partial \bar{c}}{\partial \bar{t}} = D_L\left(\frac{\partial^2 \bar{c}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{c}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{c}}{\partial \bar{z}^2}\right)$$

intensiteit — ellipsoide — tijd

oplossing puntinjectie $$\bar{c} = \frac{A}{4\pi\bar{t}}\exp\left[-\frac{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}{4D_L\bar{t}}\right]$$

oplossing continue injectie $A = q\,d\tau$

$$\bar{c} = \frac{q}{4\pi\bar{t}}\int_0^t \frac{1}{\bar{t}}\exp\left[-\frac{\bar{x}^2+\bar{y}^2+\bar{z}^2}{4D_L\bar{t}}\right]d\tau$$

(Hantush-well functions)

books: W. Kinzelbach Numerische Methoden zur Modellierung des Transport von Schadstoffen im Grundwasser, Oldenbourg Verlag, 1987
A. Verruijt Numerical Geomechanics, TUD, 1994

$\boxed{\text{TRANSPORT}}$ numeriek

numerieke dispersie door de convectieve term

$$\boxed{D \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial t} = \frac{\partial c}{\partial t}}$$

$$\frac{\partial c}{\partial x} = \frac{c(x+\Delta x) - c(x)}{\Delta x} - \frac{\Delta x}{2} \frac{\partial^2 c}{\partial x^2} + \cdots$$

$$\left(D + \frac{\Delta x}{2} v\right) \frac{\partial^2 c}{\partial x^2} - \frac{c(x+\Delta x) - c(x)}{\Delta x} = \frac{\partial c}{\partial t} \qquad (D = \alpha v)$$

$v\left(\alpha + \boxed{\frac{\Delta x}{2}}\right)$ als $\Delta x < \alpha$ resultaat zinvol — vele elementen!

methode der karakteristieken : volg een deeltje langs een stroomlijn

$\bar{x} = x - v_x t$; $\bar{y} = y - v_y t$; $z = \bar{z} - v_z t$

De advectieterm verdwijnt, geen numerieke dispersie

deeltjes volgen via integratie (Runge-Kutta) en dispersie superponeren

random walk proces — relatie tussen α_L, α_T en de sprong in richtingen longitudinaal en transversaal

algemeen geldt $D = u^2/6\Delta t$ — u: sprong, Δt: tijdstap

$\rightarrow \boxed{\alpha = u^2/6\Delta x}$

analytisch — Straming in een uniformveld (confined) met een aantal bronnen

$$\boxed{\begin{aligned} v_x &= V_x + \frac{1}{2\pi H n} \sum_{i=1}^{N} Q_i \frac{x - x_i}{(x-x_i)^2 + (y-y_i)^2} \\ v_y &= V_y + \frac{1}{2\pi H n} \sum_{i=1}^{N} Q_i \frac{y - y_i}{(x-x_i)^2 + (y-y_i)^2} \end{aligned}}$$

uniforme straming

Programma POLLUT (gemaakt door Uffink) gebruikt bovenstaande analytische methode met random walk

SINGULIER PUNT

ANALYTISCH

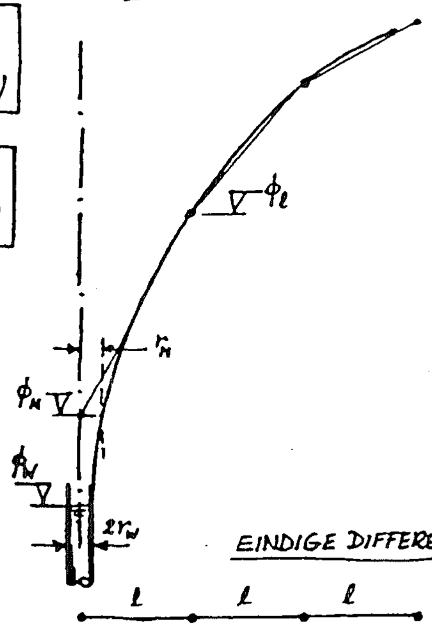
$$\boxed{\phi-\phi_\ell=-(\phi_\ell-\phi_W)\,\frac{\ln(r/\ell)}{\ln(r_W/\ell)}}$$

$$\boxed{q=-k\nabla\phi=k\,\frac{\phi_\ell-\phi_N}{r\,\ln(\frac{r_N}{\ell})}}$$

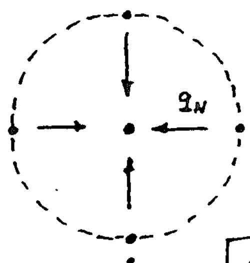
$$Q=2\pi r q D$$

$$\boxed{Q=2\pi kD\,\frac{\phi_\ell-\phi_W}{\ln(r_W/\ell)}}$$

$$\boxed{Q=2\pi kD\,\frac{\phi_\ell-\phi_N}{\ln(r_N/\ell)}}$$



EINDIGE DIFFERENTIE



$$\boxed{Q=4\ell\,q_N\cdot D}$$

$$\boxed{q_N=-k\,\frac{\phi_\ell-\phi_N}{\ell}}$$

$$\boxed{Q=-4kD\,(\phi_\ell-\phi_N)}$$

$-\frac{\pi}{2}=\ln(\frac{r_N}{\ell})$
$r_N=\ell\,e^{-\pi/2}$
$r_N=0.208\,\ell$

SCHIJNBARE STRAAL r_N

AFHANKELIJK VAN ELEMENTGROOTTE

UITWERKING

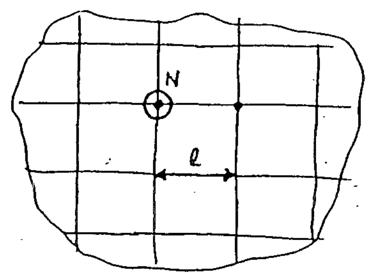
$$Q=2\pi kD\,\frac{\phi_\ell-\phi_W}{\ln r_W/\ell}=2\pi kD\,\frac{\phi_\ell-\phi_N}{\ln r_N/\ell}=2\pi kD\,\frac{\phi_\ell-\phi_N}{(-\pi/2)}$$

$$\rightarrow\quad\boxed{\phi_W=\phi_\ell-\frac{2}{\pi}(\phi_\ell-\phi_N)\,\ln(\frac{\ell}{r_W})}$$

VOORBEELD

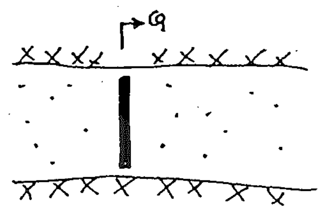
$\boxed{\phi_N=7}$; $\phi_\ell=10$

$r_W=0.25$; $\ell=25$



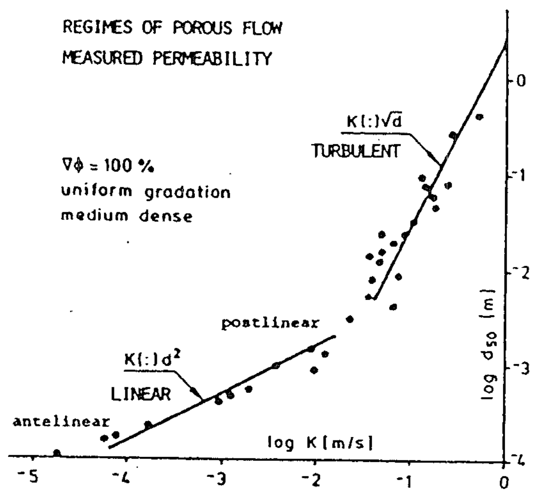
$$\phi_W=10-\frac{2}{\pi}\ln(\frac{25}{0.25})\,(10-7)$$

$$\rightarrow\quad\boxed{\phi_W=1.5}$$



RESULTAAT

Lokale snelheid groter transport sneller



toets 4

gegeven D..

berekenen gelineariseerde $q = k' i$

DETERMINATION OF PERMEABILITY

FORCHEIMER $$I = 160 \frac{\nu}{g} \frac{(1-n)^2}{n^3} \frac{1}{D_{15}^2} q + 2.2 \frac{1}{g n^2 D_{15}} q^2$$

$\nu = 10^{-6}\,\mathrm{m^2/s}$; $g = 10\,\mathrm{m/s^2}$; $n = 0.4$

$$\rightarrow I = 10^{-4}\, q/D_{15}^2 + 1.38\, q^2/D_{15}$$

SAND (linear) $\boxed{q = kI}$ DARCY $\quad D_{15} = 0.2\,10^{-3}\,\mathrm{m} \rightarrow I = 2500\, q$

$\rightarrow k = 4\,10^{-4}\,\mathrm{m/s}$

GRAVEL 20–100 mm $\quad D_{15} = 0.02\,\mathrm{m} \rightarrow I = 0.25\, q + 69\, q^2$

(linearized) $\boxed{q = k'I}$

I	q	k'
1.0	0.12	0.12 m/s
0.1	0.036	0.36
0.01	0.010	1.0

CORE 10–500 kg (≈ 30 kg) $\quad D_{15} = \sqrt[3]{\frac{30}{2650}} = 0.22\,\mathrm{m}$

$$I = 2\,10^{-3}\, q + 6.2\, q^2$$

I	q	k'
1.0	0.40	0.40
0.3	0.22	0.73
0.1	0.13	1.3

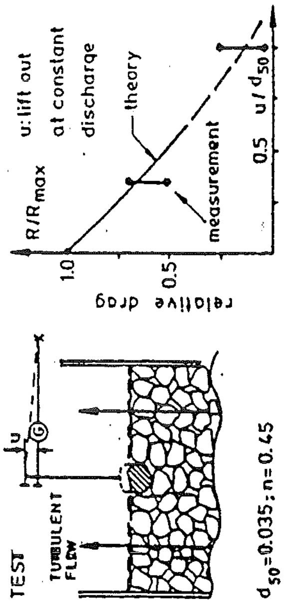
ARMOUR 1–3 ton $\quad D_{15} = \sqrt[3]{\frac{1300}{2650}} = 0.73\,\mathrm{m}$

$$I = 2\,10^{-4}\, q + 1.9\, q^2$$

I	q	k'
1.0	0.73	0.73
0.3	0.40	1.33
0.1	0.23	2.3

Barends, Groundwater Flow 36

DYNAMIC STABILITY OF BOUNDARY PARTICLES



LOCAL STABILITY OF A BOUNDARY PARTICLE UNDER TURBULENT POROUS OUTFLOW CONDITIONS TAKING INTO ACCOUNT THE EFFECT CAUSED BY THE MOTION OF THE PARTICLE INDUCED BY THE OUTFLOW

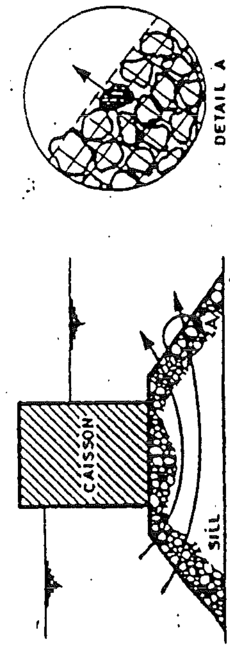
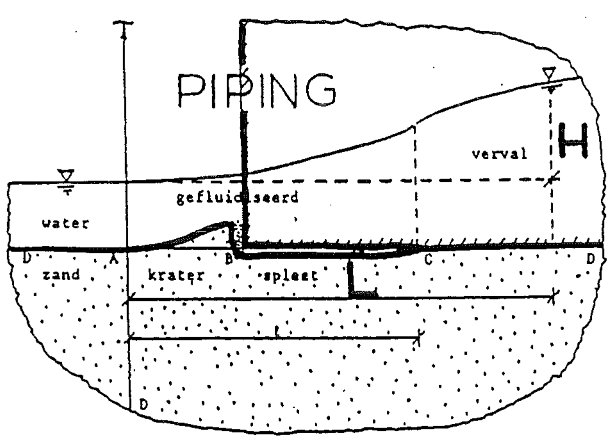
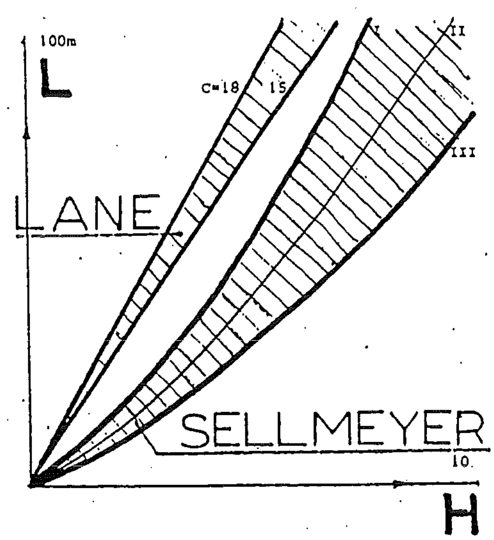


Figure 4. Selfhealing, lifting surface unit





SELLMEIJER

PIPING UNDER EMBANKMENTS

$H/L = \Gamma\delta(1-0.65(\delta/c)^{0.42})$

$\Gamma = \gamma'/\gamma \ \tan\xi/c$

$\delta = D(2/\kappa L)^{1/3}$

c effective roughness

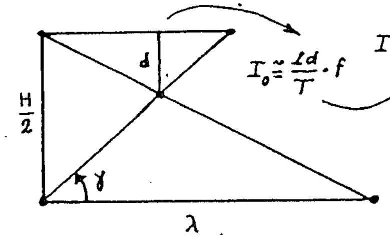
ξ bedding angle

κ intrinsic permeability

D grain size

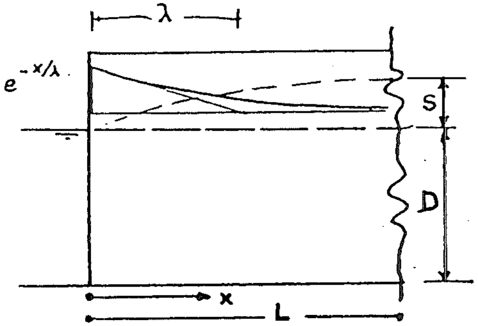
$K = \frac{k\gamma}{\mu} = \frac{kg}{\nu}$

$\mu = \rho\nu$ $\qquad \nu = 10^{-6}$



$I_0 \simeq \frac{2d}{T} \cdot f$

$I = I_0 e^{-x/\lambda}$



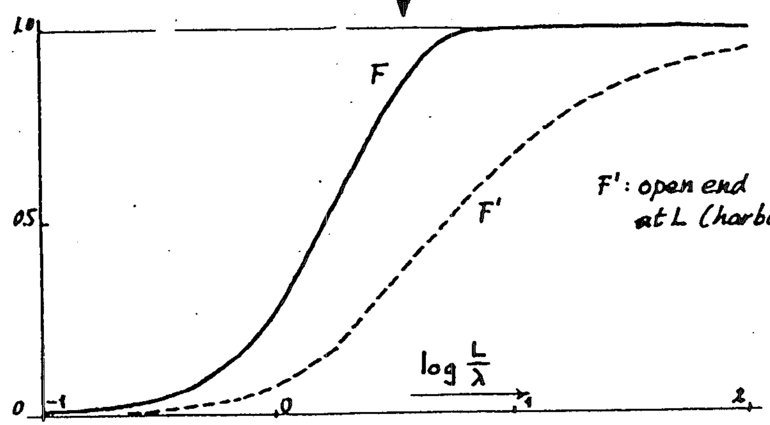
INFILTRATION

$$d = \frac{H/2}{1 + (2\lambda/H)\tan\gamma}$$

$$F = \frac{H/2\tan\gamma + \lambda}{\lambda}$$

$$I_0 \approx c f \frac{2d}{T} = c \frac{H^2}{2\lambda T \tan\gamma}$$

SOLUTION: $S = D\left(\sqrt{1 + \xi F} - 1\right)$ $\quad \xi = \frac{\pi c H^2}{4 n \lambda D \tan\gamma}$



F' : open end at L (harbour)

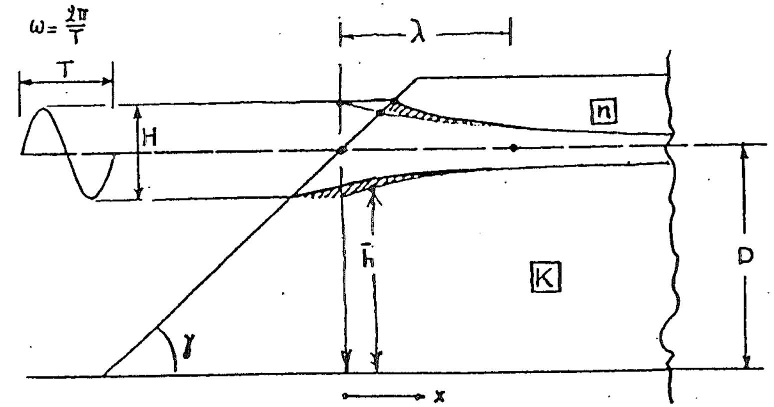
ORDER OF MAGNITUDE OF INTERNAL SETUP

$\frac{H}{D} \lesssim 0.5$, $\frac{H}{\lambda} \approx 2$, $c \approx 1.2$, $\tan\gamma \approx 0.5$, $n \approx 0.4$ $\rightarrow \xi \lesssim 1.5$

L/λ →	0.5	1.0	1.5	2.0	..... 10.0
$\frac{S}{D}$ →	0.06	0.18	0.29	0.48	..... 0.58

INTERNAL SETUP

UNSTEADY FLOW



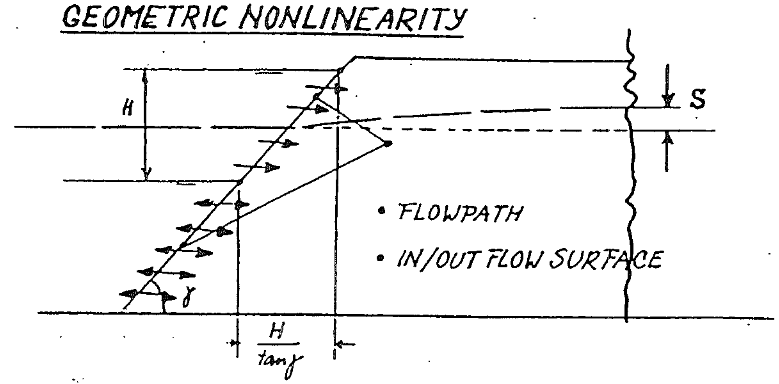
PENETRATION λ

$$h = H e^{-x/\sqrt{2KD/\omega n}} \cos\left(\omega t - x/\sqrt{2KD/\omega n}\right)$$

$$\bar{h} = H e^{-x/\sqrt{2KD/\omega n}} = H e^{-x/\lambda}$$

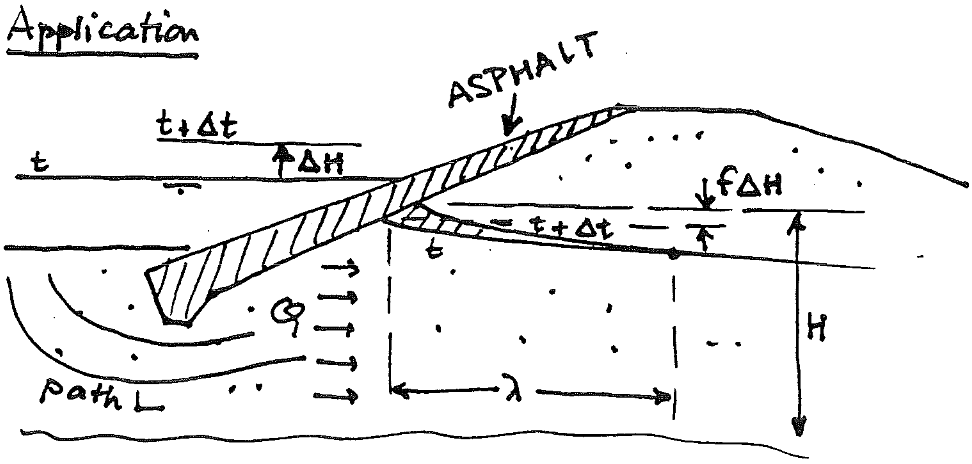
$$\lambda = \sqrt{2KD/\omega n} = \sqrt{KDT/\pi n} \cong \frac{1}{2}\sqrt{KDT/n}$$

GEOMETRIC NONLINEARITY



Barends, Groundwater flow

Application



INFLOW VOLUME $\quad Q\Delta t = KHi\Delta t = KH\left(\frac{f\Delta H}{\lambda}\right)\Delta t$

equal $\quad$ effective height $\quad$ appr. gradient

STORAGE VOLUME $\quad Q\Delta t = nf\Delta H\lambda/2 \qquad \Delta H$ is arbitrary

$KH\Delta t = n\lambda^2/2 \quad \rightarrow \quad \lambda = \sqrt{2KH\Delta t/n}$

$\boxed{\text{Waves}}$ $\quad \Delta t = \frac{2\pi}{\omega}$, $\omega \approx 0.5$ Herz $\qquad \Delta t = 10$ s

$H = 5$ m

$K_{zand} \approx 10^{-4}$ m/s

$n = 0.3$

$\rightarrow \lambda = 0.18$ m

Pressure drop over total path: $L+\lambda \quad (\lambda << L)$

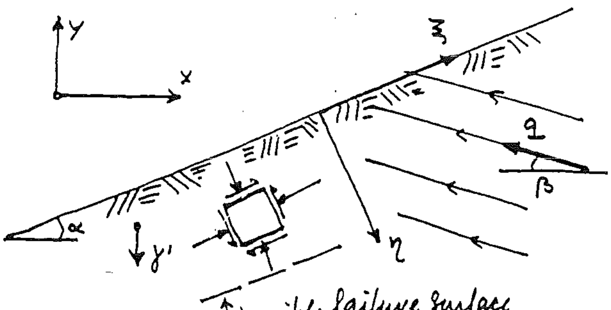
$L \approx 10$ à 15 m

$f \approx \lambda/L = 0.18/10 = 0.018 \rightarrow \underline{f \approx 2\%}$

$\boxed{\text{TYDES}}$ $\quad \Delta t \cong 5$ hrs $\cong 2\, 10^4$ s $\quad \therefore \quad \lambda = 8.17$ m

$f = \frac{\lambda}{L+\lambda} = \frac{8}{10+8} = 0.44 \rightarrow \underline{f = 45\%}$

SLOPE STABILITY AND POROUS FLOW



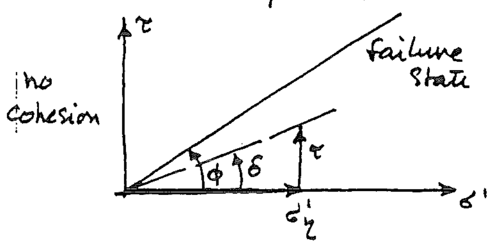
porous flow field uniform

$$\boxed{u = \gamma_w \left[\sin(\alpha)\tan(\alpha+\beta)+\cos\alpha\right]\eta}$$

equilibrium

$$\frac{\partial(\sigma'_\xi+u)}{\partial\xi}+\frac{\partial\tau}{\partial\eta}+\gamma'\sin\alpha=0$$

$$\frac{\partial\tau}{\partial\xi}+\frac{\partial(\sigma'_\eta+u)}{\partial\eta}-\gamma'\cos\alpha=0$$



Situation for infinite slope independent on ξ (symmetry condition)

$$\tau = -\gamma'\sin\alpha\,\eta$$

$$\sigma'_\eta = \gamma'\cos\alpha\,\eta\left[1-\frac{\gamma_w}{\gamma'}(\tan\alpha\tan(\alpha+\beta)+1)\right]$$

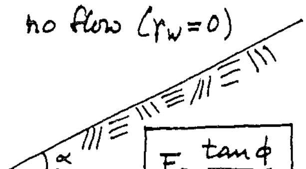
effect of porous flow

Stability factor $\boxed{F=\dfrac{\tan\phi}{\tan\delta}}$

$F>1$ stable, $F<1$ unstable

$$\rightarrow \boxed{F=\frac{\tan\phi}{\tan\alpha}\left[1-\frac{\gamma_w}{\gamma'}(\tan\alpha)\tan(\alpha+\beta)+1)\right]}$$

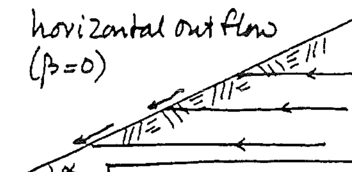
no flow ($\gamma_w=0$)



$$\boxed{F=\frac{\tan\phi}{\tan\alpha}}$$

independent on γ

horizontal out flow ($\beta=0$)

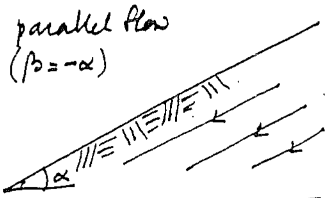


$$\boxed{F=\frac{\tan\phi}{\tan\alpha}\left(1-\frac{\gamma_w}{\gamma'\cos^2\alpha}\right)}$$

$\rightarrow F<0.5\,\dfrac{\tan\phi}{\tan\alpha}$

$$\left(\tan\alpha=\frac{-F+\sqrt{F^2+4\tan^2\phi\frac{\gamma_w}{\gamma'}\left(1-\frac{\gamma_w}{\gamma'}\right)}}{2\tan\phi\,\frac{\gamma_w}{\gamma'}}\right)$$

parallel flow ($\beta=-\alpha$)

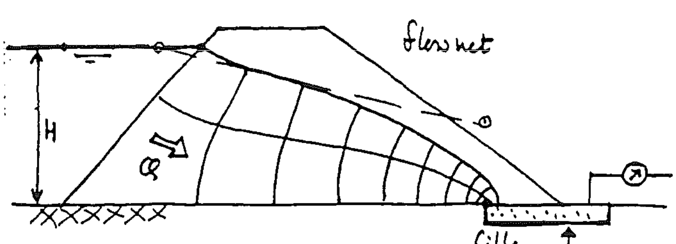


$$\boxed{F=\frac{\tan\phi}{\tan\alpha}\left(1-\frac{\gamma_w}{\gamma'}\right)}$$

$\rightarrow F\simeq 0.5\,\dfrac{\tan\phi}{\tan\alpha}$

Porous flow has a clear effect on the slope stability
(Slope equal to ϕ: natural slope)

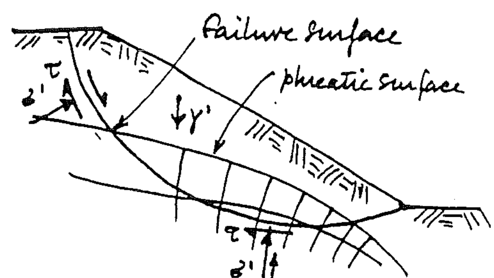
For finite slopes the stability factor is higher (toe effect)
The porous flow can be determined by the method of squares (graphical) or by computer models



square net

$$\boxed{Q = KH\frac{M}{N}}$$

total leakage, permeability, total drop, flow tubes, equipotential intervals

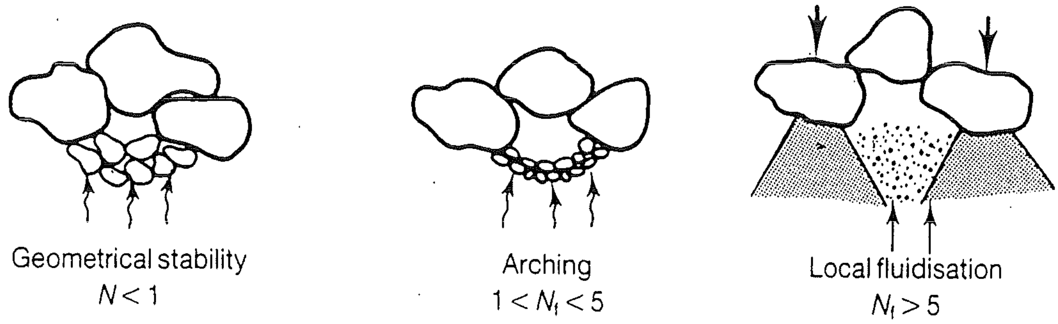


pore pressure

$$\boxed{u=\gamma_w(h-z)}$$

piezometric height, location

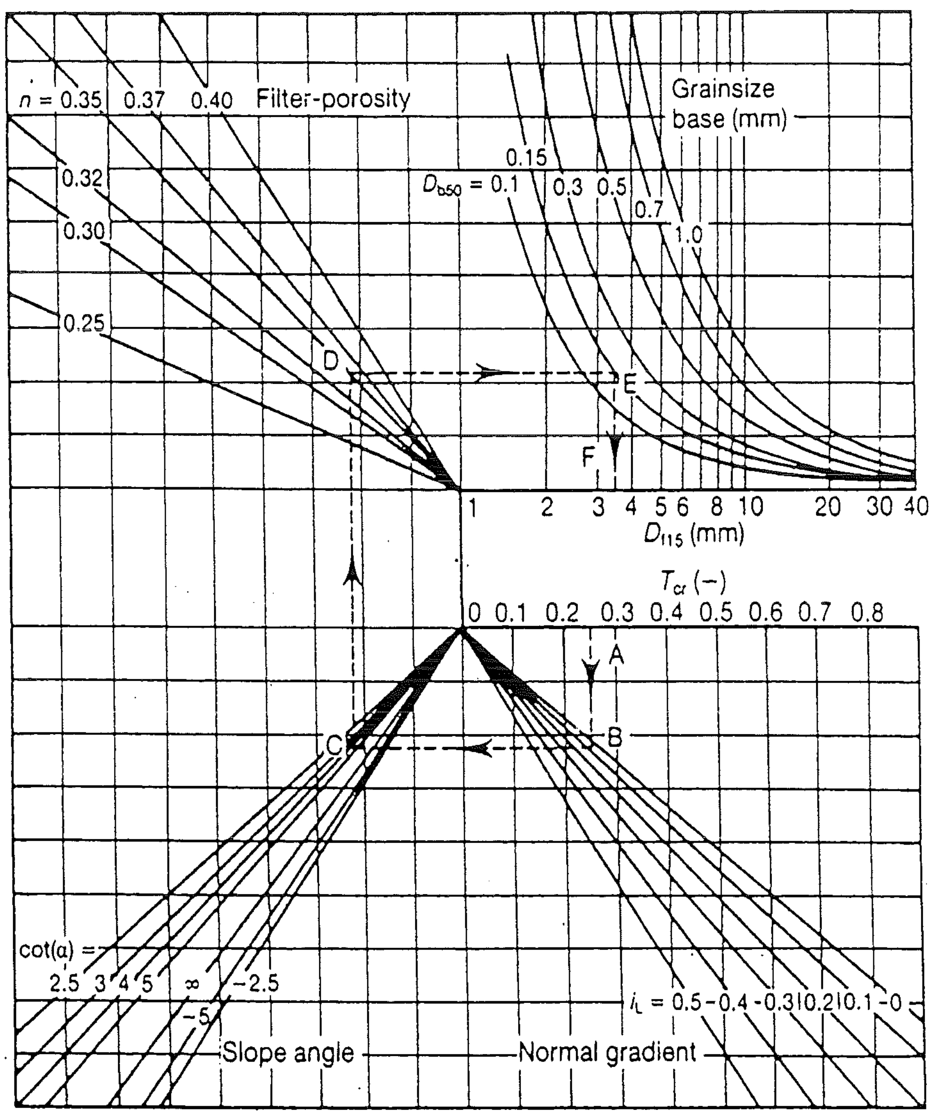
Barends
Groundw. flow 40



Mechanisms related to N_f

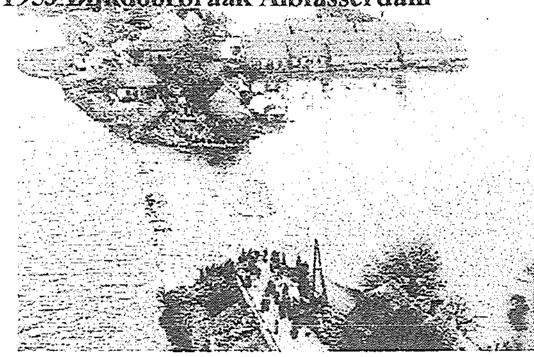
example : D_{f15} = 3.5 mm
D_{b50} = 0.15 mm
n = 0.35
cot(α) = 4
i_L = 0.1

} i_{cr} = 0.25

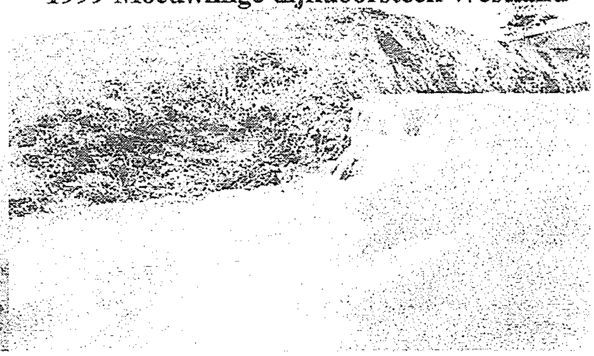


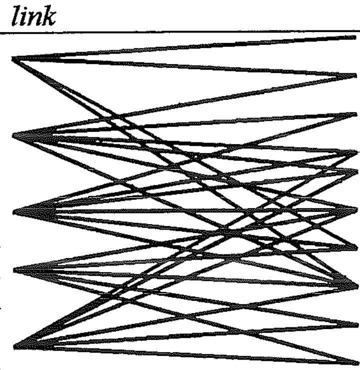
cot (α) < 0: uprush flow

1953 Dijkdoorbraak Alblasserdam



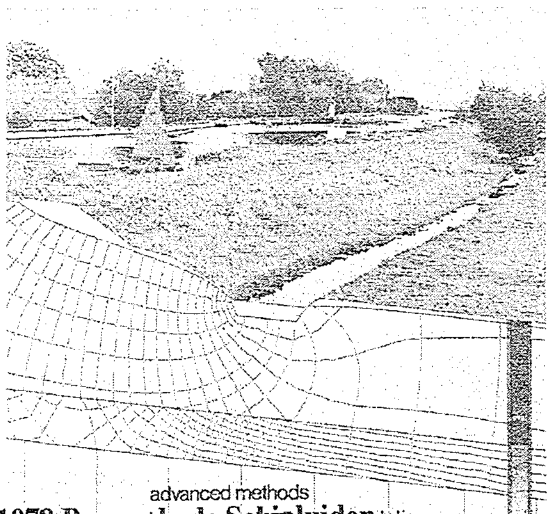
1999 Moedwillige dijkdoorsteek Westland



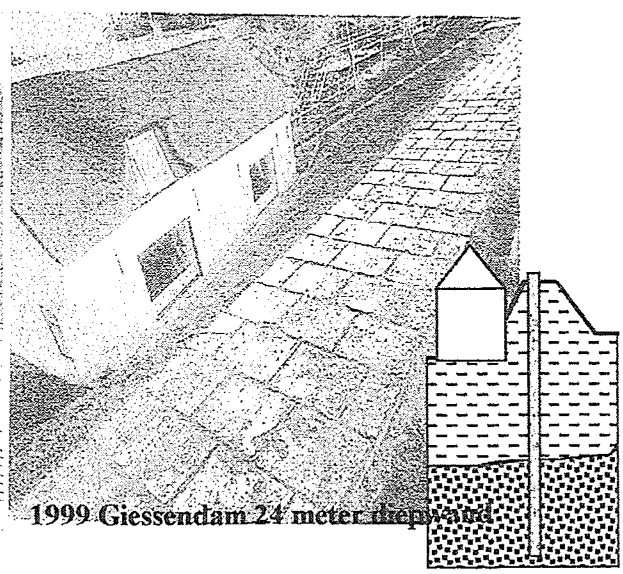
principal element	*link*	*principle limit states*
height		overtopping
		wave overtopping
outer slope		erosion outer slope
		instability outer slope
core		leakage
		settlement
inner slope		erosion inner slope
		instability inner slope
subsoil		uplift
		piping

Dike Technology

1991 Massale dijkverzakking Streefkerk



1978 Boezemkade Schipluiden

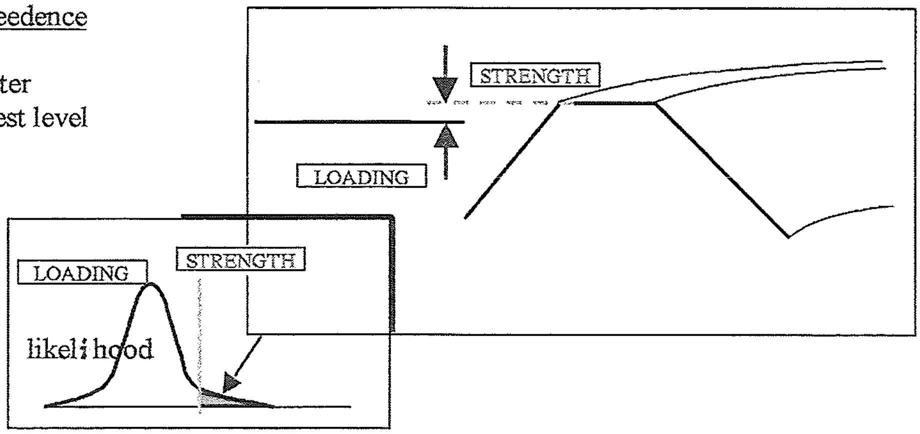


1999 Giessendam 24 meter diepwand

Safety against innundation

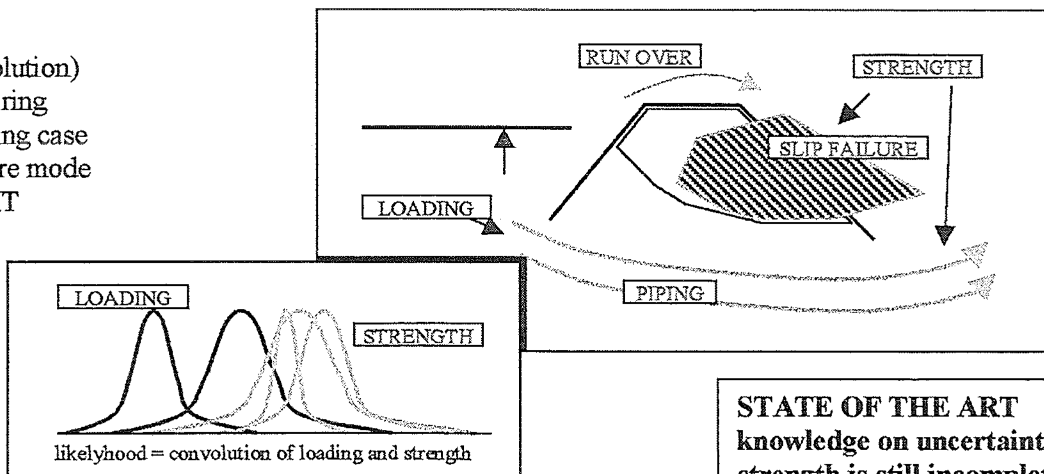
1. water level exceedence

loading = high water
strength = dike crest level
historic measure
INTUITION



2. inundation likelihood = water level exceedence (loading) ❋ failure likelihood (strength)

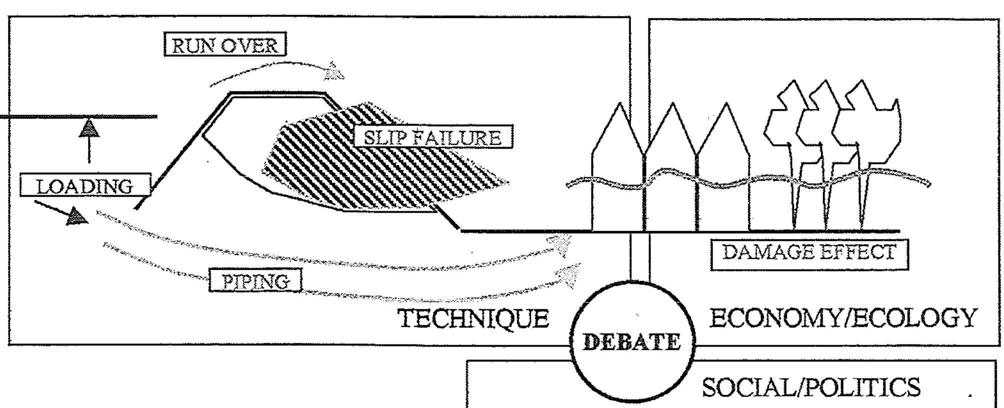
❋ (convolution)
each dike ring
each loading case
each failure mode
CONTEXT



STATE OF THE ART
knowledge on uncertainties in strength is still incomplete

3. inundation risk = inundation likelihood ❋ consequences (damage per dike ring)

consequences
integral evaluation
MEANING



4. inundation safety = (actual inundation risk < accepted inundation risk) *(bolean)*