FFSHORE TECHNOLOGY CONFERENCE
200 North Central Expressway
allas, Texas 75206

PAPER NUMBER OTC 1159, 1970

THIS IS A PREPRINT --- SUBJECT TO CORRECTION

Analysis of Peak Mooring Force Caused by Slow Vessel Drift Oscillation in Random Seas

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INTRODUCTION

Traditional naval architecture technology has not devoted extensive efforts to understanding of the requirements for mooring a vessel in the open ocean under storm conditions. Perhaps it can be said that conventional maritime practice would consider mooring under such circumstances an act of foolishness and therefore not deserving of serious technological effort. The demand on the offshore petroleum industry for mooring under trying conditions has, however, created the need for a clearer understanding of the physical phenomena involved. The offshore industry has experienced major difficulties in mooring under storm conditions and has suffered extensive financial loss. Over the years, attempts have been made to solve offshore mooring problems, utilizing a variety of vessels and mooring techniques. Results of experience and practice offer conflicting indications of the relative merits of various mooring systems. Various engineering and scientific studies have contributed toward an understanding of many factors influencing forces; however, it appears that previous studies have, for the most part, ignored a governing phenomenon. Specifically, there has been little attention devoted to the effects of slow vessel drift oscillation in random or irregular seas. It is this phenomenon which is the prime subject of the present paper.

References and illustrations at end of paper.

Fig. 1 illustrates results obtained from model tests of a moored vessel in irregular waves. Shown in the figure, as a function of time, are the variations of wave height and period, the surge or drift position of the vessel and the tension in the primary mooring line. It will be noted that the surge motion of the vessel involves both a direct wave induced surge and a gradual slow drift taking place over a period of 1 minute or more in prototype time. The drift behavior shown in Fig. 1 is the phenomenon of critical importance. This type of drift motion is found in the motion records of moored ships in an actual ocean storm environment. Moreover, the basic behavior of slow oscillations is not unique to moored vessels. For instance, such behavior has been observed in tests involving vessels towed through irregular waves with a constant towing force. In such case, it has been observed that the vessel velocity exhibits slow oscillations with periods in the range of 1 to 2 minutes.

When an ocean wave is propagated toward a moored vessel, part of the wave is reflected, the remainder being transmitted on beyond the vessel. The conservation of wave momentum results in a net force applied to the vessel for each wave. For regular waves the consequence is a steady drift force resulting in a static shift of the position of the moored vessel. For irregular waves, on the other hand, a varying sequence of drift forces arises in correspondence to changes in wave height and period. Investigations leading to this paper show that the ensuing long period drift of the vessel can for many cases be the completely dominating influence in determining maximum mooring line tension.

CONCEPTS OF THE MATHEMATICAL MODEL

In the development of concepts it is first convenient to focus attention on phenomena associated with regular waves. The hydrodynamic theory of the average steady force imposed by wave action on a stationary body or on a floating ship was first presented by Havelock in the early 1940's.[1,2] Since then, several other investigators[3,4] have examined similar problems on floating ships, primarily vessels underway, and have made important contributions. Past work on the subject has been derived rigorously from solution of boundary value problems in linear potential theory. Results of this type are exceedingly important in providing basic insight into the problem and in affording a foundation for further development. These theories, however, are not convenient for direct solution of practical problems because such solutions are difficult to obtain except for grossly simplified bodies and the theories are applicable only to small amplitude regular waves.

The basic method for calculation of wave drift force developed in this paper is based upon the same concept as the "radiation stress" introduced by Longuet-Higgins and Stewart in 1960.[5] "It is well known that surface waves possess momentum which is directed parallel to the direction of propagation and is proportional to the squares of wave amplitude. Now, if a wave train is reflected from an obstacle, its momentum must be reversed. Conservation of momentum then requires that there be a force exerted on the obstacle equal to the rate of change of a wave momentum." We prefer however to describe the radiation stress as "wave drift force" and to refer to "rate-of-change of wave momentum" as simply "momentum flux".

Consider the two-dimensional case of a regular wave train propagating from left to right in Fig. 2. TA and TB are two vertical planes fixed in space, beyond the limits of vessel excursion. If the vessel were absent, the wave field would be spatially homogeneous and the fluid properties [namely, velocity and pressure] at TA would be different from those at TB at most by a constant phase angle. Consequently, the time average properties at TA would be identical to those at TB. Let M_A and M_B indicate the average momentum flux crossing TA and TB, respectively; then, by conservation of momentum of the fluid contained between TA and TB, we would have

$$\vec{n}_A \cdot M_A + \vec{n}_B \cdot M_B = 0 \quad \text{[1]}$$

where $\vec{n}_A$ and $\vec{n}_B$ are the outward normal vectors at TA and TB, respectively. When the vessel is present, part of the wave passing through Plane TA is blocked and reflected by the vessel, the remaining part being transmitted on to Plane TB. At the same time, movements of the vessel also generate waves which propagate away from the vessel. Therefore, the average fluid properties at TA and TB are no longer the same. Let ΔM_A and ΔM_B represent changes of average momentum flux at TA and TB caused by the presence of the vessel; then the wave drift force exerted on the vessel is equal and opposite to the direction of the total change of average momentum flux crossing the two planes and can be expressed as

$$\vec{F} = -(\vec{n}_A \cdot \Delta M_A + \vec{n}_B \cdot \Delta M_B) \quad \text{[2]}$$

Hence, if the fluid properties at far-field were known, the wave drift force could be formally determined accordingly. However, such a formal calculation depends on the exact knowledge of the wave potentials for reflected and generated waves.

In this paper, a model for reflection and generation of waves is constructed utilizing simplified boundary conditions at the vessel's vertical walls as indicated by Planes T1 and T2 in Fig. 2. Since any variation of fluid parameters between T1 and TA and, likewise, between T2 and TB must satisfy the requirement of conservation of momentum,[6] the drift force can be expressed as

$$\vec{F} = -(\vec{n}_1 \cdot \Delta M_1 + \vec{n}_2 \cdot \Delta M_2)$$

$$= \vec{n}_2 (\Delta M_1 - \Delta M_2) \quad \text{[3]}$$

where $\vec{n}_1$ and $\vec{n}_2$ are outward normal vectors at T1 and T2, ΔM_1 and ΔM_2 are change of momentum flux caused at T1 and T2, respectively.

Great simplification in the problem of calculating fluid parameters at Planes T1 and T2 is achieved by assuming that all wave momentum between the free surface and vessel keel is blocked and reflected and that the wave field below the vessel keel is not disturbed. The waves generated by the movement of the vessel are defined as a function of vessel draft, and its horizontal motion relative to the incident waves. In the present model for calculation of wave drift force, the influence of vessel vertical motion, namely, heave, is incorporated in the sense that heave motion changes the effective draft of the vessel. Detail description of the model and mathematical treatments are given in Appendix A.

Reduced to the restricted case of two-dimensional flow [e.g., beam seas] Eq. [A-16] of Appendix A gives average drift force as follows, correct to second order in wave amplitude.

$$\Delta M_1 - \Delta M_2 = \frac{\rho g a^2}{2} \cdot L \cdot \left\{ 2 \cdot \left[1 + \frac{S_o}{a} \cdot \text{Cos}\ (KX_1 - \bar{\theta}) \right] \cdot \left[\frac{2\ \text{Cosh}\ K\ (h-D)}{\text{Cosh}\ Kh} - 1 \right] - \left[\frac{W_o}{a} \cdot \frac{2\ \text{Cosh}\ K(h-D)}{\text{Cosh}\ Kh} \right] \cdot \left[\frac{S_o}{a} \cdot \text{Cos}\ (\bar{\theta} - \beta) + \text{Cos}\ (KX_1 - \beta) \right] \right\}, \quad \ldots\ldots\ldots [4]$$

where parameters are as defined in list of symbols. Eq. 4 gives the magnitude of the average drift force per unit length exerted on a vessel by a regular wave train traveling in the direction normal to the surface of the vessel. Therefore, the total wave drift force on a ship caused by waves coming from beam direction should be

$$L \cdot (\Delta M_1 - \Delta M_2)$$

where L is the length of the ship. The calculation procedure developed here is best suited for evaluation of wave drift force exerted on a vessel in beam seas. However, Eq. 4 can be adopted for head seas by replacing the amplitude and phase angle of the sway motion, namely S_o and $\bar{\theta}$, with those of the vessel surge motion. For a vessel with square bow and stern, the longitudinal wave drift force is given by

$$B \cdot [\Delta M_1 - \Delta M_2]$$

where B is the beam of the ship. For waves coming at an angle with respect to the ship, Eq. 4 should be modified. The expression for the modified beam drift force is given in Appendix A. In general, in quartering seas, there is a longitudinal drift force in addition to a beam drift force.

In head seas, a vessel with a ship bow will have less longitudinal drift force than a barge with a square bow. This is because the vertical wall of a ship bow is at an angle with the incoming wave and only partially reflects wave momentum in the direction of propagation, whereas a square bow fully reflects the wave momentum. Limited by space, a detailed discussion on the influence of bow shape on vessel longitudinal drift force will not be made here. However, by following the development presented in Appendix A, one can arrive at a correction factor for vessel longitudinal drift force for a ship bow.

The method of calculation for wave drift force presented here is based on a model of wave generation and reflection by the vessel; consequently, the authors hesitate to suggest the use of the method for semisubmersibles. This is because semisubmersibles are in general less obstructive to wave propagation, thus wave reflection and generation are no longer the dominating factors in wave drift force. For semisubmersibles with structure member dimensions at the water line in the same order as the water particle motions, drag force acting on the members due to relative water particle velocity may contribute significantly to the total wave drift force.

The average wave drift force exerted on a vessel by a regular wave train has the same effect as an added average steady force. This steady force will displace the vessel to a new average position about which the vessel oscillates. Fig. 3 shows the model test results of oscillatory surge motion of a spread moored ship in regular head seas. Since no wind or current was simulated during the tank test, the shift of the average position of the vessel as a function of wave period is a clear indication of wave drift force at work.

VESSEL MOTION ESTIMATE

Eq. 4 clearly indicates that both the amplitude and phase of vessel motion have a direct influence on the wave drift force exerted on the vessel. In this paper only surge, sway and heave motions are considered. One may conceivably attempt to include the angular motions into the model, but then the model will become much more complex and perhaps lose much of its applicability. Moreover, it has been the authors' observation that the rectilinear motions, namely heave, surge and sway, seem to be of overriding importance.

One can do well in calculation of wave drift force if model tank test data on vessel motions in regular waves are available. In such a case, he can insert the measured amplitude and phase relation directly into Eq. 4. If no test data are available, one may either use sophisticated computer programs making a complete evaluation of the motions, or use simplified analysis making a reasonable estimate. As one gains experience and insight into vessel motion analysis, one can even arrive at a fairly good estimate by making an educated guess.

Simplified models for calculation of the horizontal and vertical vessel motions are given in Appendix B. These predict accurately the amplitudes and phase angles of the rectilinear vessel motions to be substituted into Eq. 4. In order to use these models, one must specify vessel displacement, hydrodynamic mass, damping, hydrostatic restoration, and mooring system spring stiffness. In general, the mooring system basically governs only the slow oscillation of the vessel, and it does not have much influence on the direct wave-induced vessel motions. An example of calculated amplitude and phase relation of vessel motion, according to the simplified model described in Appendix B, is given in Fig. 4. The amplitude of vessel horizontal motion increases steadily with the increase of wave period in the range of practical interest, whereas the phase angle stays approximately the same, lagging behind the wave profile by an angle of about 90 degrees. This means that in practical cases the vessel's horizontal velocity is about in phase with the horizontal velocity of the water particles. The maximum heave motion occurs at about the heave natural period of the vessel, and the phase angle has a sharp change at the heave natural period. For wave periods longer than the heave natural period, the vessel essentially heaves with the wave profile. Therefore, for long period waves the vessel tends to move with the water particle in both horizontal and vertical directions, and the wave drift force should be small, because the more the vessel moves with wave particles, the less disturbance it creates in the wave field.

APPLICATIONS TO IRREGULAR SEAS

For applications of the mathematical model of drift force to the case of irregular seas, two basic simplifying assumptions are made: [1] that the irregular sea can be characterized by a sequence of waves, each being assigned a defined height and period, and [2] each of the waves of an irregular sea will impart to the moored vessel the same impulse which it would were it merely one of a sequence of regular waves. The lack of demonstrable rigor in these two assumptions is readily conceded. However, our experience is that the results determined on the basis of these assumptions are meaningful and useful in describing the phenomenon of interest.

A quantitative evaluation of the slow drift oscillation can be made by applying the time dependent wave drift force on a mass and nonlinear spring system. The mass is made to simulate the total mass of the vessel and its hydrodynamic mass in a given mode of horizontal motion, and the nonlinear spring is made to simulate the total resistance of the mooring system against drift motion. The equation for this drift oscillation can be written as

$$M \cdot X'' + C \cdot X' + f(X) = g(t) \, , \quad \ldots \ldots \quad [5]$$

where M and C are the virtual mass and damping, f[X] is the nonlinear spring function, and g[t] is the wave drift force function which gives the average wave force over each wave cycle. In regular waves, this average force is the same for every wave, hence g[t] is a constant. For irregular seas, in view of the two basic simplifying assumptions, g[t] becomes a step function changing abruptly from wave to wave. Since g[t] is not a periodic function, the most expeditious way to solve Eq. 5 is by using the finite difference method on a computer, propagating a solution forward in time.

The basic calculating procedure can be described as follows.

Step 1 - For a wave condition of interest, establish either a measured wave history, or one which is mathematically reconstructed as described further in this paper.

Step 2 - For each wave of the wave history, calculate the wave drift force utilizing Eq. 4 and the methods described in the previous two sections.

Step 3 - Determine the mooring system stiffness based upon mooring line composition, weight, length and water depth.

Step 4 - Calculate the slow drift oscillation of the vessel, utilizing the force input determined in Step 2 for Eq. 5, previously described.

Step 5 - Estimate extreme vessel excursion by superimposing direct wave-induced horizontal motion on the maximum amplitude of slow vessel drift.

Step 6 - Calculate peak mooring force from the spring characteristics of the mooring system and the extreme vessel excursion determined in Step 5.

If the measured regular wave motion response of the barge and the measured spring force of a model test are used in determining the wave drift force, the results can be very good, as shown in Figs. 1 and 5. Fig. 1 shows the model tank record of the surge motion of a spread moored 150,000-ton barge in random waves of significant height 20.5 ft and average period 13 seconds. Fig. 5 shows the sway motion of the same vessel in identical wave conditions. It will be noted that in both cases the calculated slow drift motion of the vessel matches very well with the measured value determined in the model test. The high quality of correlation between calculated and model test results provides a high degree of confidence in the relevancy of the concepts introduced in the paper, and the potential usefulness of the calculating approach.

Wave condition is generally specified statistically in terms of energy spectrum, or significant wave height and average period. There are at least two ways in practice to reconstruct an irregular wave history from its statistical specifications: [1] summing a finite number of sine or cosine waves of different amplitudes and frequencies and [2] recording the output of a specially designed filter network which has been subjected to "white noise" input. Discussion of details on the selection of the amplitudes and frequencies in the first method and the construction of the filter network in the second method are beyond the scope of this paper. For a discussion on irregular wave generation, refer to Ref. 7.

In view of the nonlinear spring characteristics of conventional mooring systems and the large amplitude of the slow drift oscillation, it is doubtful whether the statistics of mooring line force can be meaningfully derived from wave height statistics using directly linear spectral analysis. On the other hand, the fact that wave drift force in irregular seas varies basically with the variation of wave heights suggests that the slow drift oscillation may be directly related to characteristics of wave packets or groups in such seas. In practical mooring system analysis and design, some wave measurements taken at the operation site will be invaluable in helping to reconstruct a more realistic wave history and to obtain more meaningful results. We suspect that characteristics of irregular seas and wave packets depend to a large extent on local geographical conditions.

Influence of Mooring Line Pre-Tension

A typical force vs horizontal displacement relation for conventional mooring line catenary is indicated by a curve for Line A in Fig. 6. The force exerted by a mooring system on a vessel against a specified displacement of the vessel is the sum of change of force in all lines caused by the displacement. For a two-dimensional case with identical mooring lines at both sides of the vessel, the force in the lines on the one side of the vessel increases while the lines on the other side of the vessel decrease. The net mooring force is the sum as indicated in Fig. 6.

Fig. 7 shows the dependency of slow drift oscillation of a given vessel on its mooring line pre-tensions. One can see that slackening the mooring lines by reducing pre-tension from F2 to F1 substantially increases the amplitude of drift oscillation from A2 to A1, and at the same time increases the mooring force from B2 to B1 and load in the stormward mooring line from C2 to C1. This is because a slackened mooring system offers very little resistance to vessel drift motion until the drift displacement is large. At that time, the vessel, having been pushed by the wave drift force without the counteraction of mooring system spring, has gathered appreciable momentum which must be absorbed by the mooring system. Consequently, to slacken mooring lines before a storm does not always lead to reduction of force in the lines. Table 1 indicates the mooring forces of two model tank tests on the 150,000-ton barge. One can see that, by lowering the mooring line pre-tension from 300 kips to 150 kips, the peak mooring force in a single line is significantly increased. For every vessel and its mooring system there is an optimum range of pre-tension. Tightening mooring lines beyond this range will result in increasing instead of decreasing of mooring line force. As shown in Fig. 7, increasing pre-tension from F2 to F3 results in increasing mooring force from B2 to B3 and force in mooring line from C2 to C3. For a given vessel and a mooring system, one can utilize the method suggested here to calculate the vessel slow drift oscillation and to determine the range of optimum mooring line pre-tension for the vessel to ride out a design storm.

An important element in the design of mooring systems is the stiffness characteristics of the mooring lines. As shown in Fig. 8, the calculated amplitude of vessel slow drift oscillation of two different mooring systems for the same vessel are about equal. However, due to difference in mooring line spring characteristics, the estimated peak mooring forces are drastically different. In general, it seems desirable to design mooring lines in such a way that their spring characteristics are as linear as possible.

Influence of Wind and Current

Storm wind and current may induce some additional vessel drift oscillation due to the fact that neither wind velocity nor current velocity is truly constant in nature. However, in this discussion, their direct contributions to the slow drift oscillation are ignored, and both wind and current are considered as constant velocities exerting a steady force on the moored vessel.

One can see from Fig. 9 that, by applying a steady force on the vessel, the vessel is displaced to a new equilibrium position, indicated by Point P. The new mooring system stiffness is determined by translating the origin of the coordinates from Point O to Point P, as indicated in Fig. 9. The stiffness of the mooring system with respect to Point P is no longer the same as with respect to Point O. It is to be pointed out that the force-displacement curve, measured from the origin of the F'X' coordinates, is appreciably stiffer than that from the FX coordinates. In other words, the steady force caused by winds and currents will in effect stiffen the mooring system.

As discussed in previous sections, stiffening a mooring system may either increase or decrease mooring force. As a matter of fact, under certain situations [namely, soft mooring system and high waves] wind and current may actually help to reduce the peak mooring force by stiffening the mooring system. This surprising result was verified by model tank tests. Consequently, contrary to intuition, superimposing wind and current on storm waves does not always increase the peak mooring force. From test results as listed in Table 2, for mooring System M1, the mooring force is recuced as a result of superimposing a 2-knot current, whereas for mooring System M2, the mooring force is increased.

Results and Conclusions

The analytic procedure of calculating the slow drift oscillation and peak mooring force has been applied to analyze the mooring system for a 150,000-ton oil storage barge. Using the recorded tank test data for direct wave-induced vessel motions, and using the measured mooring-system spring characteristics and measured waves, the calculated slow drift oscillation agreed amazingly well with the measured values as shown in Fig. 1.

The procedure was then used to forecast peak mooring force for several different mooring systems in order to optimize the design. Subsequently, additional model tank tests were conducted, and the test results confirmed the basic validity of the analytical procedure both in predicting the general behavior of the barge mooring system, including the influence of pre-tension, winds and current, and in quantitative evaluation of peak mooring force. A comparison of calculated and measured peak mooring forces is given in Table 3. It is important to point out that in the prediction of peak mooring force in a single line it has been assumed that peak mooring forces are equally distributed among parallel lines extending in the same direction. The model test results showed that this assumption is wrong. Therefore, distinction must be made between a total force in all parallel lines extending in one direction and peak force in individual lines. Additional research is needed to clarify this point. From a practical engineering point of view, mooring system design is more of a technical and economical factor in the case of large vessels than for small vessels. Therefore, the calculation procedure suggested here is of particular importance to the design of mooring systems for large hull-type vessels. The following listed general conclusions have been reached.

1. The slow drift oscillation of a vessel, particularly hull-type vessels or a large floater, caused by random waves is a governing factor contributing to the high load in mooring lines.

2. A method has been developed to calculate the slow drift oscillation and peak mooring force based on a simplified model for evaluation of wave drift force. The general validity of this method has been supported by model tank results.

3. Influence of winds, current and mooring line pre-tension on the peak mooring force can be evaluated on the basis of their influence on the slow drift oscillation. Contrary to intuition, winds and current do not always increase the peak mooring force, and likewise, slackening of mooring lines does not always reduce the peak mooring force.

4. Using the procedure suggested in this paper, one can more accurately assess the capability of different mooring systems and arrive at an optimum mooring system design.

While in this paper, the wave drift force, the slow drift oscillation and peak mooring forces are discussed in the light of a spread mooring system, the concept and the method suggested here are by no means limited to analyzing one type of mooring system. As a matter of fact, they can be directly used in the evaluation of mono-mooring systems. Furthermore, wave drift force and resultant vessel drift motion are likely to be the two most important factors in dynamic positioning. With some refinement and modification, the method for evaluation of wave drift force suggested here may even be adapted to analyze the problem of vessel speed reduction in random seaways.

NOMENCLATURE

- S,S',S" = displacement, velocity and acceleration of vessel sway motion
- W,W',W" = displacement, velocity and acceleration of vessel heave motion
- U,U' = horizontal velocity and acceleration of water particle motion in waves
- V,V' = vertical velocity and acceleration of water particle motion in waves

$\bar{Z}$ = vertical position of vessel center of buoyancy

L,B = length and beam of the vessel

K = wave number

$\omega n, \omega nl$ = natural frequency of vessel sway and heave motion, respectively

ϕ,a,p = velocity potential, surface profile, wave amplitude and pressure of the waves

$\bar{\Phi}, \bar{\bar{\Phi}}, \eta_1, \eta_2, P_1, P_2$ = fictitious velocity potentials, surface profiles and pressures at the windward and leeward sides of the vessel, respectively

X,Y,Z = spatial coordinate system with origin resting on the still water level and Z measuring positive vertically upward

$\theta_1, \alpha_1, \theta_2, \alpha_2$ = phase angles of the fictitious velocity potentials

θ = angle between the direction of propagation of incident wave train and the normal of the vessel vertical wall

A, γ = amplitude and direction angle of vessel motion in the horizontal plan, respectively

ρ = density of water

g = gravity constant

$\underline{a}, C, \omega$ = amplitude, celerity and frequency of waves

U_{n1}, U_{n2}, V_n = velocity of vessel horizontal motion in beam direction

$[\Delta M_1]n$, $[\Delta M_2]n$ = change of wave momentum flux in beam direction at the windward side and the leeward side of the vessel, respectively

$S_o, \bar{\theta}$ = amplitude and phase of vessel sway

W_o, β = amplitude and phase of vessel heave

D = vessel draft

$\vec{n}_1, \vec{n}_2, \vec{n}_A, \vec{n}_B$ = normal vectors

h = water depth

$C, C_1, \underline{C_n}, C_{n1}, \bar{C}, \bar{C}_1$ = damping coefficients

ℓ = variable of integration denoting length along ship

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APPENDIX A

Momentum Flux Calculation Model

Consider the cases of a regular wave train propagating from $-\infty$ in the direction of positive X axis as indicated in Fig. 10. This wave train, upon impinging on the floating vessel, is partially reflected back to $-\infty$ and partially transmitted to $+\infty$. Within the realm of potential flow theory, this interaction between the vessel and the incident wave train can be formally treated as a boundary value problem with the relative water particle velocity normal to the hull of the vessel equal to zero. Theoretical and numerical solutions to this boundary problem for ship hulls are generally difficult to obtain. In this appendix a simplified model for wave reflection and transmission, and average drift force exerted on the vessel in the beam direction is proposed. This model incorporates the following basic assumptions.

1. The boundary conditions posed by the vertical walls of the vessel are satisfied at Planes T_1 and T_2 which are the average positions of the walls and fixed in space.

2. The boundary conditions at the bottom of the vessel can be ignored.

3. Partial reflection and transmission are approximately modeled by superimposing fictitious velocity potentials $\bar{\Phi}$ and $\bar{\bar{\Phi}}$ at Planes T_1 and T_2, respectively.

4. The fluid field below the keel is assumed undisturbed.

For waves coming at an angle with respect to the vessel, the boundary condition to be satisfied is that the velocity of water particles in the direction normal to the beam of the vessel is the same as the vessel velocity in that direction. Consequently, if the vessel motion in the horizontal plane has an amplitude A and a direction angle γ with respect to the

$$M_b \cdot S'' - M_f \cdot U' + M_h \cdot (S''-U') + C \cdot (S'-U) + \overline{K} \cdot S = 0, \quad [B-1]$$

where M_f = displaced fluid mass
$\overline{K}$ = spring constant of the mooring system
C = damping coefficient
M_b = basic mass of the vessel
M_h = hydrodynamic mass due to relative acceleration between fluid particle and the vessel

For a floating body $M_f = M_b$, hence replacing $[M_b + M_h]$ by M, we have

$$M \cdot S'' + C \cdot S' + \overline{K} \cdot S = M \cdot U' + C \cdot U \quad [B-2]$$

If the surface profile of the wave is

$$\eta = a \cos \omega t, \quad [B-3]$$

then water particle velocity in the horizontal direction is

$$U = U_o \cos \omega t, \text{ where } U_o = a\omega \frac{\cosh K(\overline{Z}+h)}{\sinh Kh}.$$

Water particle acceleration in the horizontal direction is

$$U' = \omega U_o \sin \omega t .$$

The equation of sway motion becomes

$$M \cdot S'' + C \cdot S' + \overline{K} \cdot S = M \omega U_o \sin \omega t + C U_o \cos \omega t \quad . \quad [B-4]$$

The resulting sway motion is

$$S = S_o \cos [\omega t + \overline{\theta}],$$

where

$$S_o = a\, Q \sqrt{\frac{\omega^2 + c^2/M^2}{\omega^2 \cdot (1 - \frac{\omega_n^2}{\omega^2})^2 + c^2/M^2}},$$

$$\omega_n = \sqrt{\frac{\overline{K}}{M}}, \qquad Q = \frac{\cosh K(\overline{Z}+h)}{\sinh Kh}$$

$$\overline{\theta} = \tan^{-1}\left[\frac{\omega M^2(\omega_n^2 - \omega^2 - c^2)}{C\overline{K}}\right]. \quad [B-5]$$

$\overline{Z}$ = vertical position of vessel center of buoyancy.

To evaluate vessel surge motion, in addition to using the proper mass, spring and damping coefficients, a correction factor in the form of

$$\sin\left(\frac{KL}{2}\right) \Big/ \left(\frac{KL}{2}\right)$$

should be applied to the wave amplitude to take into account the influence of the length of the vessel. It is to be noted that by applying this factor to wave amplitude, the amplitude of motion, S_o, is reduced, but the phase relation remains the same.

Heave Motion

Let W, W' and W" denote the displacement, velocity and acceleration of heave motion. The equation of motion can be written as

$$(M_{h1} + M_{b1})(W'' - V') + C_1 \cdot (W' - V) + \overline{K}_1 (W - \eta) = 0 \quad [B-6]$$

where η = wave profile = $a \cos \omega t$
V = vertical water velocity = $V_o \sin \omega t$
$V_o = a\omega \dfrac{\sinh K[\overline{Z}+h]}{\sinh Kh}$
V' = vertical water acceleration = $V_o \omega \cos \omega t$
$\overline{K}_1$ = hydrostatic restoration in heave motion
C_1 = heave damping coefficient
M_{b1} = heave basic mass
M_{h1} = heave hydrodynamic mass

Let $M_1 = M_{h1} + M_{b1}$, the equation of motion becomes

$$M_1 \cdot W'' + C_1 \cdot W' + \overline{K}_1 \cdot W = (M_1 \omega V_o + \overline{K}_1 a) \cdot \cos \omega t + C_1 V_o \sin \omega t. \quad [B-7]$$

The resulting heave motion is

$$W = W_o \cos(\omega t + \beta)$$

where

$$W_o = \frac{a}{\omega} \sqrt{\frac{(\frac{C_1}{M_1})^2 R^2 \omega + (R\omega^2 + \omega_{n1}^2)^2}{\omega^2 \cdot (1 - \frac{\omega_{n1}^2}{\omega^2})^2 + (\frac{C_1}{M_1})^2}},$$

$$\omega_{n1} = \sqrt{\frac{\overline{K}_1}{M_1}},$$

$$R = \frac{\sinh K(\overline{Z}+h)}{\sinh Kh}, \quad [B-8]$$

and

$$\beta = \tan^{-1}\left[\frac{(1-R)\ C_1 \overline{K}_1 \omega}{M_1^2(\omega_{n1}^2 - \omega^2)(\omega_{n1}^2 + R\omega^2) - C_1^2 \omega^2 R}\right]$$

$\overline{Z}$ = vertical position of vessel center of buoyancy.

It is to be noted that C and C_1 are the damping coefficients for the linear damping term in the equations of motion, while in reality damping is generally nonlinear in nature. For a closer approximation, C and C_1 should be replaced by the equivalent linear damping coefficients so that the amount of energy dissipated will be approximately the same as the nonlinear damping. A comprehensive discussion on equivalent viscous damping is given in Ref. 8.

If damping is proportional to the second power of velocity, the equivalent linear coefficient for horizontal motion is

$$\overline{C} = \frac{8}{3\pi} C_n S_o \omega Q$$

for heave,

$$\overline{C}_1 = \frac{8}{3\pi} C_{n1} W_o \omega R$$

where C_n, C_{n1} are nonlinear damping coefficients. In such cases, the amplitude of motions have to be modified accordingly.

Table 1 - Influence of pretension on mooring line.

	PRE-TENSION 300 KIPS	PRE-TENSION 150 KIPS
TOTAL MOORING FORCE IN ALL 4 LINES	3950	4600
PEAK FORCE IN A SINGLE LINE	1400	2000

Table 2 - Influence of current on mooring force.

MOORING SYSTEM DESIGNATION	CURRENT VELOCITY KNOTS	TOTAL MOORING FORCE, KIPS
M1	0	2400
	2	1800
M2	0	1000
	2	1300

Table 3 - Mooring force - comparison of measured and predicted force.

	TOTAL MOORING FORCE (KIPS)		NO. OF MOORING LINES	PEAK FORCE IN A SINGLE LINE (KIPS)	
	MEASURED	PREDICTED		MEASURED	PREDICTED
STERN SEAS	1250	1800	2	766	900
BEAM SEAS	2730	2500	4	996	625

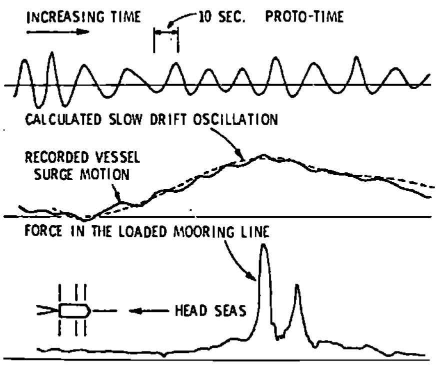

Fig. 1 - Vessel Slow drift surge oscillation.

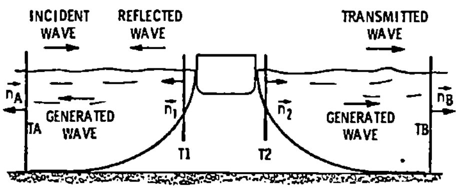

Fig. 2 - Model for calculation of wave momentum flux.

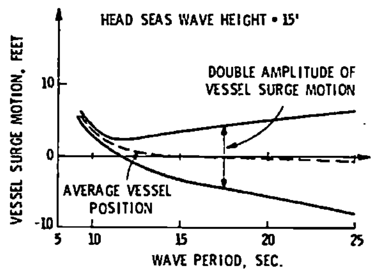

Fig. 3 - Model test results of vessel surge motion in regular waves - head seas.

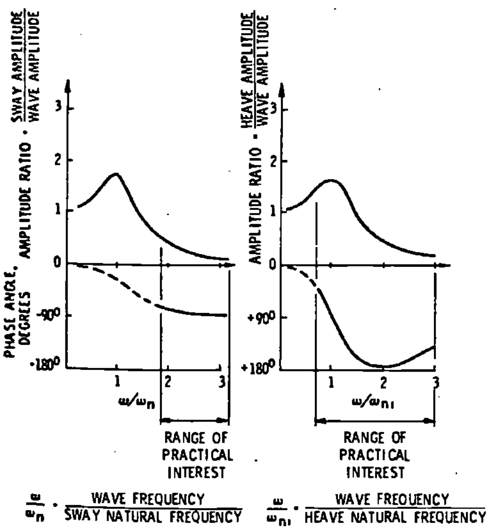

Fig. 4 - An example of vessel sway and heave response.

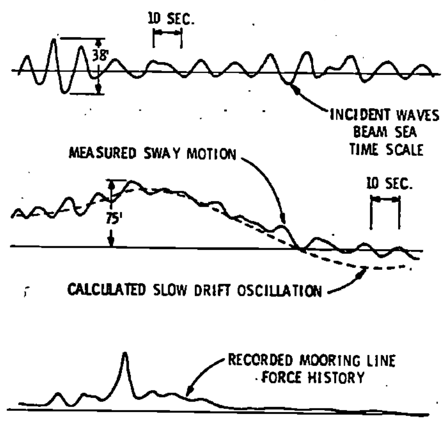

Fig. 5 - Vessel slow drift sway oscillation.

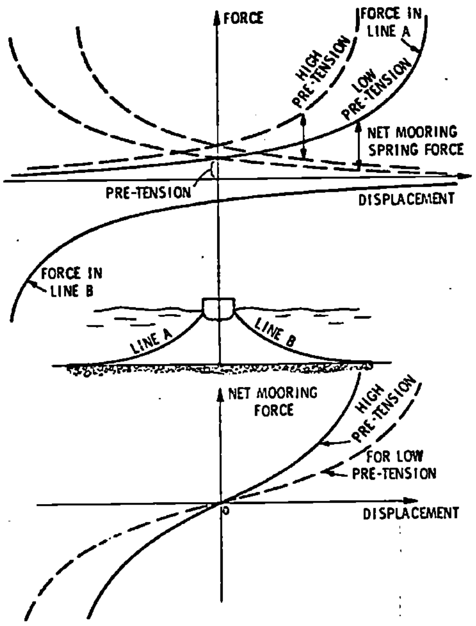

Fig. 6 - Conventional mooring line catenary springs.

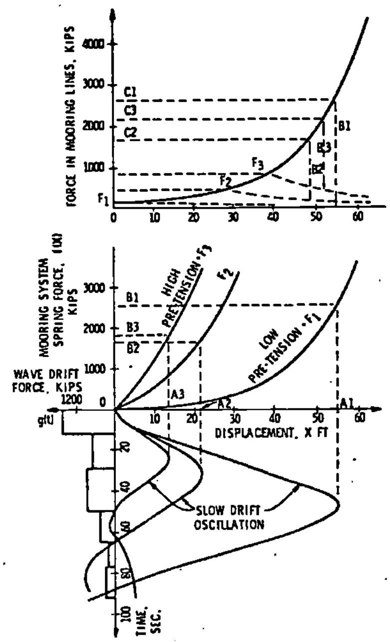

Fig. 7 - Influence of mooring line pretension on vessel slow drift oscillation.

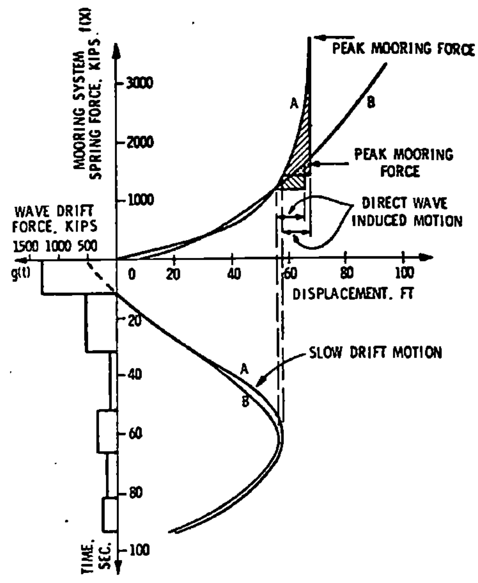

Fig. 8 - Influence of mooring line stiffness characteristics on peak mooring force.

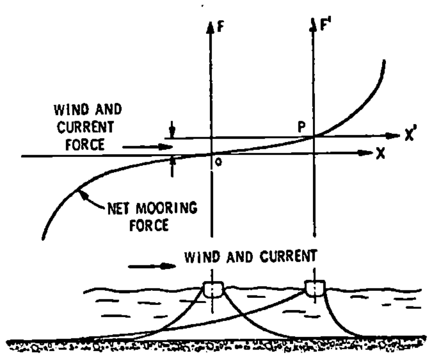

Fig. 9 - Influence of wind and current on mooring system stiffness.

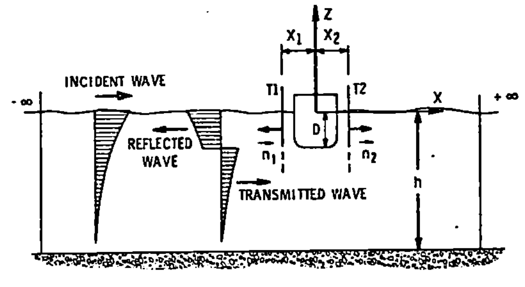

Fig. 10 - Simplified model for wave reflection.

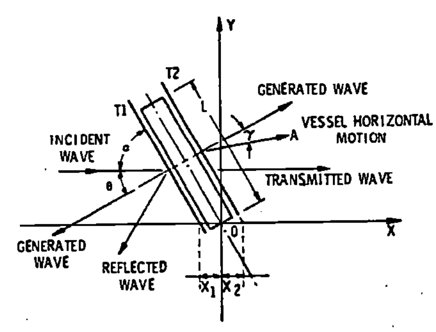

Fig. 11 - Waves coming at an angle with respect to a vessel.