

Multi-Stage Formation Flight Planning

A Fuel Saving Strategy For Civil Aviation

M.Sc.Thesis Report

M. Doole BSc. Aerospace Engineering

Technische Universiteit Delft



Multi-Stage Formation Flight Planning

A Fuel Saving Strategy for Civil Aviation

MASTER OF SCIENCE THESIS

For obtaining the degree of Master of Science in Aerospace Engineering at Delft University of Technology

M.M. Doole, BSc.

24th August 2016

Faculty of Aerospace Engineering

Delft University of Technology



Copyright © M.M. Doole, BSc

All rights reserved.

Delft University of Technology

Department of

Air Transport & Operations

The undersigned hereby certify that they have read and recommended to the Faculty of Aerospace Engineering for acceptance a thesis entitled "**Multi-Stage Formation Flight Planning**" by **M.M. Doole, BSc.** in partial fulfilment of the requirements for the degree of Master of Science.

Dated: 24th August 2016

Chairman/ Supervisor

.....
Prof. Dr. Ir. H.G. Visser

Internal Examiner

.....
Ir. P. Roling

External Examiner

.....
Dr. Ir. M. Voskuijl

Summary

This thesis investigates the concept of fuel-reduction via formation flight of long-haul commercial aircraft on a network-wide scale, particularly addressing the issue of combinatorial complexity in global formation routing and allocation. A multi-stage formation flight planning (MS-FFLIP) tool is proposed as a computationally tractable approach to formation allocation. Assuming all trailing aircraft receive a 10% reduction in the fuel-burn by flying in formation, the optimal routes including the joining and splitting points are computed in four specific stages along the flight depending on the desired formation size. Within a stage, the interconnected allocation problem takes a set of flights, their possible formation combinations and associated costs and optimally assigns them in a fuel-cost minimising formation fleet. The formation flight routing and the associated fuel burn are enumerated rapidly. This fast computation allows the large-scale allocation problem to be solved via a Mixed Integer Linear Program and optimised through an external optimiser solver. Within the context of this thesis, a stage is defined as a collection of events where all flights merge into formation at optimal joining points, driven by an event-based simulation. The MS-FFLIP tool sequentially creates stages until the largest allowable formation size is created. Additionally, the model is capable of handling large sets of flights and in creating large formations of up to sixteen-aircraft allocated to a formation within a real-time operational time limit. Similar models exist in the current literature, however they are highly prone to combinatorial complexity and as a result, their computation times diverge dramatically with increase in aircraft number and formation size. For this reason, the models were not able to capture the majority of the potential fuel-savings. The MS-FFLIP model was compared against the conventional single-stage centralised formation flight planning approach for which 50 eastbound transatlantic flights were evaluated for optimal routes and assigned to formation fleet of up to four-aircraft per formation. The experiment for the centralised formation flight planning approach showed a 5.72% reduction in fuel-burn against its corresponding solo flights, while requiring 58 CPU minutes of computation time. On the contrary, the MS-FFLIP model performed the same experiment within a time frame of 14 seconds while demonstrating potential fuel savings of 5.66%. Moreover, a case study for 267 transatlantic flights was simulated by the MS-FFLIP tool. The results revealed a 6.88% in potential formation fuel savings against solo flights, with up to sixteen-aircraft allocated to a formation, while performing the computation within 9 CPU minutes. The developed model also showed significant fuel saving potential compared to local optimisation-based cooperative planning frameworks.

Acknowledgements

I would like to begin by Praising God Almighty for blessing me with the gift of becoming an aerospace engineer.

I would like to take this opportunity to show my profound gratitude and appreciation to my MSc supervisor, Prof. Dr. Dries Visser. Without his support, guidance, motivation and kind understanding this thesis would not have been achievable. No matter how busy he was, he always made time for me, a true gentleman. It was a great honour of working with such a distinguished Scientist. I would like to thank my co-supervisor, Dr. Bruno Santos for his support and also Dr. Thomas Kent from Bristol University, for his valuable advice and support.

I wish to thank my wonderful Mother and Father for all their unconditional love, care and support. Without their support none of this would have been possible. Furthermore, I want to thank my sister, Quraisha, my niece and nephew, Halina and Duane, for all their encouragement.

Moreover, I want to extend my appreciation towards my grandfather, my aunties; Chasnin, Jessica, Veronica and aunty Threase and my uncles Jerome and Russell for all love and support. And not forgetting my cousins Allen and Eileen for all their support.

Last but not least I would thank my friends, Shashi Kellapathe, Pulkit Goyal, Noor Mujahid, Prasobh Narayanan, Imrul Asharf, Li Meng, and Yasser Muhammed for all their help and support.

Contents

Summary	v
List of Figures	xv
List of Tables	xvii
1 Introduction	1
1.1 Research Objective and Question	2
1.2 Thesis Outline	3
2 Literature Study	5
2.1 History of Formation Flight	6
2.2 Aerodynamics of Formation Flight	6
2.2.1 Ideal Separation of Formation Flight	8
2.2.2 Induced-Drag Reduction	9
2.2.3 Formation Flight Topologies	10
2.3 Fuel Consumption of Formation Flight	11
2.4 Cost Performance of Formation Flight	13
2.5 Operations Optimisation of Formation Flight	13
2.5.1 Route Optimisation	13
2.5.2 Decentralised Formation Flight Planning	14
2.5.3 Centralised Formation Flight Planning	15
2.6 Concluding Remarks	17
3 Conceptual Model	19
3.1 Model Framework	20
3.2 Combinatorial Explosion	21
3.2.1 Combinatorial Problem	22
3.3 General Overview of The Models	22
3.3.1 Validation Model: Single-Stage Centralised Formation Flight Planning . .	23

3.3.2	Decentralised Model: Local Optimisation-Based Model	23
3.3.3	Multi-Stage Formation Flight Planning (MS-FFLIP) Model	24
3.4	Governing Assumptions	25
3.5	Reference Aircraft	26
3.5.1	Aircraft Data: B777	26
3.6	Simulation Set-Up	27
3.6.1	Program Set-Up	27
3.6.2	Coordinate Reference System: Cartesian Coordinates	27
3.6.3	Simulated Scenario: Eastbound Transatlantic Flights	28
4	Routing Model	29
4.1	The Geometric Routing Approach	29
4.1.1	The Notion of Arc Weights	30
4.1.2	The Fermat Point Problem	30
4.1.3	Extending The Fermat Point Problem	31
4.1.4	The Influence of Arc Weights on Optimal Formation Flight Assembly . .	34
4.1.5	Sub-Optimal Joining and Splitting Point Scenario	34
4.2	Routing of Large Flight Formations: Single-Stage Centralised Approach	35
4.2.1	Route Optimisation For Three-Aircraft Formation Size	36
4.2.2	Route Optimisation For Four-Aircraft Formation Size	38
4.3	Fuel Consumption Modelling	40
4.3.1	High-Fidelity Breguet Range Fuel Consumption Model	41
4.3.2	Assumptions	41
4.3.3	Implemented Breguet range equation	42
4.3.4	Alternative Fuel Consumption Model	44
4.3.5	Comparison of Fuel Consumption Models	45
4.4	Formation Flight Efficiency	45
4.4.1	Optimal Airspeed For Re-introduced Breguet Range Equation	45
4.4.2	Speed Limitations of Formation Flight	46
4.5	Synchronisation of Formation Flights	47
4.5.1	Linear-Hold	48
4.5.2	Extended Joining Point	50
4.5.3	Flight Path Detour	52
5	Linear Program Formulation of Assignment Problem	55
5.1	Mixed-Integer Linear Programming	55

5.2	Notation	57
5.3	Objective Function	57
5.4	Constraints	57
5.5	Optimiser	58
5.5.1	Branch-and-Bound Algorithm Steps	58
6	Formation Flight Assignment Model	61
6.1	Concept Description	61
6.1.1	Objective	61
6.2	Model Structure	62
6.2.1	Input	63
6.2.2	Pairing Matrix $[P]$: Demonstration of A Small-Scale Modelling Example .	63
6.2.3	Optimiser	66
6.2.4	Post-Processor	66
7	Multi-Stage Process	69
7.1	Staged Formation Flight Process	69
7.2	Staging Procedure Example	70
7.2.1	Stage 1	70
7.2.2	Stage 2	71
7.2.3	Stage 3	72
7.2.4	Stage 4	75
7.3	Schematic Description of MS-FFLIP Model	75
8	Management of Formation Flight	77
8.1	Organisational Structure	78
8.2	Proposal For The Management of Formation Flight	78
9	Case Study	81
9.1	Formation Flight Routing Capability	81
9.1.1	Three-Aircraft Formation Flight Routing	81
9.1.2	Four-Aircraft Formation Routing	83
9.1.3	Eight-Aircraft Formation Routing	86
9.2	Case Study: 267 Eastbound Transatlantic Flights	89
9.2.1	MS-FFLIP Results	89
9.2.2	Decentralised Optimisation Planning Method: Greedy Model	95
9.3	Comparison: MS-FFLIP Model Against Greedy Model	100

9.4	Comparison: Validation Model Against MS-FFLIP Model	101
9.5	Interpretation of Results	102
10	Conclusion	105
10.1	Summary of Research Contribution	105
10.2	Research Conclusions	105
10.3	Recommendation for Future Work	107
	Bibliography	111

List of Figures

2.1	Preliminary research framework	5
2.2	Upwash and downwash generated in bird formation [9]	6
2.3	Vertical component of velocity induced on trailing aircraft by the wake of leading aircraft [10]	7
2.4	Induced upwash on a wing cross-section due to the trailing wing vortex field [11]	7
2.5	Trailing upwash velocity distribution in span-wise direction due to vortex generation by leading wing [11]	8
2.6	Drag reduction benefits against number of aircraft in formation [13]	9
2.7	Drag reduction vs leader-follower weight (M)TOW ratio [20]	10
2.8	The three types of formation flight architectures [10]	11
3.1	General framework of the MS-FFLIP model and the validation model	20
3.2	Illustration of maximum attainable formation size against the number of stages in the MS-FFLIP model	25
3.3	World aviation traffic flow [37]	28
4.1	The geometric construction to determine the optimal point P [25]	31
4.2	Formation angle $\angle BPC$ between AP and BP [25]	32
4.3	Two solo routes from A to C and B to D	32
4.4	Flight trajectory between back vertexes X_1 and Y_2 for the origin and destination location [5]	33
4.5	Optimal trajectory with intersection points at J and S [5]	33
4.6	Typical optimal formation flight route with 4 solo segments and 1 formation segment	33
4.7	Geometry of formation flight route for; $w_A = w_B$ for the blue route and $w_A > w_B$ for the green route	34
4.8	Typical sub-optimal formation route for which the joining point is located behind point B	35
4.9	Grey: sub-optimal route, black: optimal route, old joining point, J is shifted to point B	35

4.10	Solo flights 1, 2, 3, 4, 5	37
4.11	Flight 12 and its optimal route	37
4.12	Flight 3 added to pseudo flight 12	37
4.13	Optimal formation route for flight 1, 2 and 3	38
4.14	Pseudo flight 12 and solo flights 3,4	39
4.15	Pseudo flight 12 and pseudo flight 34	39
4.16	Flight 12 added to flight 34	39
4.17	Optimal formation flight route for the four-aircraft formation size aircraft 1,2,3,4	40
4.18	Synchronisation process	47
4.19	Fuel-burn rate vs segment travel time for two aircraft flying on AJ and BJ: B777, $W = 2.35 \times 10^6$ [N], $h = 9km$, Unshaded region indicates the allowable synchronisation range of speeds	49
4.20	Fuel-burn rate vs time for two aircraft flying on AJ and BJ: B777, $W = 2.35 \times 10^6$ [N], $h = 9km$, Unshaded region indicates the allowable synchronisation airspeed .	50
4.21	Re-location of the joining point to accommodate synchronisation of aircraft . . .	50
4.22	Flight path detour illustration for synchronisation	53
5.1	Linear programming framework for the flight formation assignment problem . . .	56
6.1	Formation flight assignment model architecture	62
7.1	Two groups of two-aircraft formation ($\{1,2\}$, $\{3,4\}$) routes from a set of 190 combinations	71
7.2	Two entities of two-aircraft formations and their optimal route	72
7.3	Optimal four-aircraft formation route for flights 1,2,3 and 4	72
7.4	Optimal four-aircraft formation route for flights 1,2,3 and 4	73
7.5	Optimal eight-aircraft formation route for flights 1,2,3,4,5,6,7,8	74
7.6	Flow chart for the developed MS-FFLIP model	75
8.1	Space-based ADS-B communication for formation flight management	79
9.1	Solo routes for flights 1, 2, and 3 between their respective origin and destinations locations	82
9.2	Illustration of formation routing of three flights $\{1,2,3\}$	83
9.3	Illustration of solo routes flown by 4 flights between their respective origin and destination points	83
9.4	Illustration of 4 optimal formation routes for flights $\{1,2,3,4\}$	84
9.5	Illustration of optimal dis-assembly for four-aircraft formation scenario	85
9.6	Illustration of four solo flights from widely spread origin and destination locations	85

9.7	Four flights in formations with large origin and destination spread	86
9.8	Illustration of 8 aircraft with 2 equal solo routes	87
9.9	Illustration of 8 aircraft in formation flight	87
9.10	Illustration of an eight-aircraft formation for randomly spaced origins and destination points	88
9.11	Projection of 267 great circle routes [47]	89
9.12	Fraction of total time in formation size for stage 2	92
9.13	Fraction of total time in formation sizes for stage 3	93
9.14	Fraction of total time in formation sizes for stage 4	95
9.15	Simulated screen shot of 267 flights at 40 th time step for the Greedy model . . .	97
9.16	Simulated screen shot of 267 flights at 60 th time step for the Greedy model . . .	97
9.17	Simulated screen shot of 267 flights at 80 th time step for the Greedy model . . .	98
9.18	Simulated screen shot of 267 flights at 125 th time step for the Greedy model . . .	98
9.19	Frequency encounter for the respective flight formation sizes	99
9.20	Formation size usage rate for 267 flights	100

List of Tables

3.1	Number of possible combinations, N_p , with increase in number of aircraft, N_a and, formation size, n	22
3.2	B777 aircraft characteristics, Boeing B777 performance manual [35]	27
4.1	Fuel burn for different Breguet range fuel model versions for two flights from JFK-ROM	45
9.1	Fuel saving against corresponding solo flights for stage 1 of MS-FFLIP model . .	90
9.2	Stage 1 average fraction of time spent in solo and in a two-aircraft formation size relative to the total flight time	90
9.3	Fuel saving against corresponding solo flights for stage 2 of MS-FFLIP model . .	91
9.4	Number of optimally assigned formation pairs in stage 2	91
9.5	Fuel saving against corresponding solo flights for stage 3 of MS-FFLIP process .	93
9.6	Number of optimally assigned formation pairs in stage 3	93
9.7	Fuel saving against corresponding solo flights for stage 4 of MS-FFLIP model . .	94
9.8	Number of optimally assigned formation pairs in stage 4	95
9.9	Greedy model configuration parameters	96
9.10	Legend descriptions for plots 9.15 to 9.18	96
9.11	Results for Greedy Simulation Model	99
9.12	Fuel saving and computational time difference between centralised validation model and MS-FFLIP model	102

Nomenclature

ρ	Free stream density	$[kg/m^3]$
θ_f	Formation angle	$[rad]$
A	Constraint matrix	$[-]$
AR	Aspect ratio	$[-]$
b	Wing span	$[m]$
C_D	Drag coefficient	$[-]$
c_j	Fuel cost	$[N]$
C_L	Lift coefficient	$[-]$
c_t	Thrust specific fuel consumption factor	$[s^{-1}]$
d_{AJ}	AJ flight segment distance	$[M]$
d_{BJ}	BJ flight segment distance	$[M]$
e	Oswald factor	$[-]$
F	Fuel consumption	$[N]$
i	Aircraft	$[-]$
j	Formation	$[-]$
k	Induced drag factor	$[-]$
n	Formation fleet size	$[-]$
N_a	Number of aircraft	$[-]$
N_p	Number of possible formations	$[-]$
P	Pairing matrix	$[-]$
q	Dynamic pressure	$[N/m^2]$
R	Range	$[m]$
R_s	Specific range	$[m/s/N]$

S	Wing surface area	$[m^2]$
$t_{AJ'}$	Time in AJ segment	$[s]$
$t_{BJ'}$	Time in BJ segment	$[s]$
V	Airspeed	$[m/s]$
$V_{\max \text{ endurance}}$	Maximum endurance airspeed	$[m/s]$
V_{AJ}	Airspeed flown in AJ segment length	$[m]$
V_{BJ}	Airspeed flown in BJ segment length	$[m]$
V_{optimal}	Optimal airspeed	$[m/s]$
w_A, w_B, w_C	Solo arc weightings	$[-]$
W_F	Total fuel load	$[N]$
W_e	Aircraft end weight	$[N]$
W_i	Aircraft start weight	$[N]$
x	Binary decision variables of whether j is chosen	$[-]$
γ	Speed ratio	$[-]$
λ	Fuel discount factor	$[-]$
μ	Number of leaders	$[-]$
A, B, C, D	Points representing airport locations	$[-]$
b	Control column vector	$[-]$
C_{Di}	Induced-drag coefficient	$[-]$
C_{Do}	Zero lift drag coefficient	$[-]$
D	Drag force	$[N]$
J	Joining point	$[-]$
J'	New joining point	$[-]$
L	Lift force	$[N]$
M_{cruise}	Cruise Mach number	$[-]$
$M_{\max}$	Maximum Mach number	$[-]$
S	Splitting point	$[-]$

Abbreviations

ATC	Air Traffic Control
ADS-B	Airborne Dependent Surveillance - Broadcast
ATM	Air Traffic Management
BMILP	Binary Mixed-Integer Linear Programming
BT	Block Time
DOC	Direct Operating Cost
FATCo	Formation Air Traffic Controller
FL	Flight Level
ISA	International Standard Atmosphere
IATA	International Air Transport Association
LP	Linear Programming
MILP	Mixed-Integer Linear Programming
MNPS	Minimum Navigation Performance Specification
MS-FFLIP	Multi-Stage Formation Flight Plan
MTOW	Maximum Take-Off Weight
NAT	North Atlantic Tracks
NASA	National Aeronautics Space Agency
NP	Non-deterministic Polynomial
OR	Operations Research
OTS	Oceanic Track System
OACC	Oceanic Area Control Center
PRM	Preferred Route Message
PCo	Planning Controller

UAV	Unmanned Aerial Vehicle
TCAS	Traffic Collision Avoidance System
RCo	En-Route Planner
ZFTOW	Zero-Fuel Take-Off Weight

1 Introduction

With the number of passengers estimated to increase to 7.4 Billion by the year 2034 [1], the aviation industry is constantly seeking for methods to deal with increase in demand whilst simultaneously aiming to increase profitability [2] and mitigating the environmental effects of aviation. Therefore, a combination of operational, economic and technological changes are required. While new aircraft technology offers great potential, the ability for it to be integrated into commercial aviation requires significant long-term financial investments. Operational change concepts such as, formation flight requires significantly low financial investments with the advantage of great potential in fuel savings for commercial airliners. The military uses the operation of formation flight to safely move large numbers of aircraft for the purpose of communication and defence benefits [3]. Flying in close proximity leads to the added benefit of drag-reductions, hence fuel savings. The majority of current commercial airliners can fundamentally obtain reductions in drag. For this reason, it is of interest to estimate the potential fuel which could be saved for commercial aviation while utilising formation flight operations.

Nature has always inspired most of the major developments in the field of aviation. One example is the migration of a flock of birds which showed birds fly in a V-formation when travelling on long distances. Scientists acclaimed this to the fact that birds in formation are able to increase their range compared to solo flight. Research identified the increase in range was correlated by the decrease in flight energy required. As the potential for fuel savings in aviation surfaced, several studies were conducted to validate formation flight benefits. The most recent study was performed as a flight experiment by Boeing researchers, Blake et al. [4], using two C-17 military aircraft. The flight test showed a 10% reduction in fuel consumption for trailing aircraft. To further contribute to the field of formation flight, the literature study was conducted in order to explore the topic, determine potential value-addition to the current research field by primarily addressing the notion of operations optimisation of formation flight.

In recent years, studies were conducted on the global optimal routing and assignment of formation flights on a network level, thereby optimising with respect to the overall fuel consumption of the fleet. This network-wide coordination approach was classified as “centralised”, since formation flights were assigned via global optimisation techniques prior to departure. A leading centralised study was published by Kent & Richards in 2015 [5], in which the authors formulated an assignment problem using a Mixed Integer Linear Programming (MILP) in order to optimally assign flights to formations. Kent & Richards conducted a case study for 210 eastbound transatlantic flights, demonstrating potential fuel savings of 8.7% for a two-aircraft formation size and 13.1% for a three-aircraft formation size. Within the same year, Verhagen et al. [6] developed a local optimisation method using a heuristic greedy algorithm as an option to assigning aircraft to formations in-flight. This local formation flight coordination approach was labelled as a “decentralised” approach. Verhagen et al. presented a case study of 347 transatlantic flights that reported a 3.7% in overall fuel burn reduction against corresponding

solo flights while creating relatively larger formation sizes.

The decentralised approach has its disadvantages such as, local optima, thereby not recovering the maximum potential in fuel savings and, it being an in-flight option, the potential fuel savings cannot be predicted prior to flight. The global optimisation approach solves these disadvantages. However, recent work has underlined the downside of global optimisation approaches to formation flight planning being NP-hard. The term NP-hard is associated with combinatorial optimisation problems and is deemed difficult to solve in terms of complexity. The complexity of the problem arises as the size of the problem increase, hence causing combinatorial explosions. As a consequence of this factorial growth, the enumeration time increases dramatically with the number of aircraft and formation size. In comparison to the local optimisation approach, that allowed for aircraft to be assigned to large formation sizes, perform fast computations and hence cope with schedule disruptions, resulted in a severe limitation on current global optimisation method. Since global optimisation methods were limited to smaller fleets with smaller formation sizes and also highly susceptible to schedule disruptions, the majority of potential fuel savings is yet to be shown. Therefore, this research focuses on the development of an optimisation-based method for centralised formation planning as an in-flight option of moderate combinatorial complexity, rendering it amenable to in-flight implementation for commercial airliners.

1.1 Research Objective and Question

The framework of the current research is the operations optimisation of formation flight planning using Mixed Integer Linear Programming. The current research poses the following objective and research question:

Research Objective:

The objective of this research is to develop an optimisation-based cooperative planning method using a Mixed Integer Linear Programming approach with the aim to reduce the overall fuel consumption for a fleet of commercial aircraft.

Research Question:

How much potential fuel could be saved when using formation flight in civil aviation by combining the strengths of both centralised and decentralised strategies?

The above research question has been broken down into several key sub-questions in order to find a feasible answer within the designated time frame of this study.

Sub-Questions:

1. When can formation flight be implemented?
2. How can the optimal rendezvous and splitting locations for flight formation be calculated?
3. What effective strategies could be used to synchronise the formation flights at the rendezvous location?

4. How to effectively model the fuel consumption for solo and formation flight?
5. How can formation flights be optimally assigned to formations in a cooperative manner?
6. How can the combinatorial impact of the problem be avoided in order to reduce the computational time for en-route formation flight planning?
7. How to create a simulation framework for the developed model?
8. How can the model be validated?
9. What are the differences in results for the developed model against leading literature sources?
10. What are the differences in results for the developed model against the decentralised model by Verhagen et al?
11. What challenges need to be addressed for future research work related to the developed model?

This study addresses the aspect of the large combinatorial problem, hence allowing the number of aircraft and formation size to increase in the formation flight planning process. A Multi-Stage Formation Flight Planning (MS-FFLIP) model has been developed for which aircraft are allocated to optimal formations using a global optimisation method formulated with a MILP formulation. The model is not adversely affected by the computational complexity (NP-hard problem) observed in conventional centralised formation methods and hence, it is capable of solving for large number of aircraft, with large formation sizes, within a real-time operational time frame. The developed model was validated against an existing centralised model which showed promising results.

1.2 Thesis Outline

This report provides a thorough overview of the conducted research. Chapter 2 examines the fundamentals of formation flight which cover the current literature. Chapter 3 presents the conceptual model of the developed models. The geometric routing model and complimentary fuel consumption model is detailed in Chapter 4. Chapter 5 describes the linear program model followed by the MILP formulation for the formation flight allocation problem in Chapter 6. Thereafter, Chapter 7 uses the material in the prior chapters to develop the framework for the MS-FFLIP model. Next, Chapter 8 studies the aspects concerning the implementation of formation flight in civil aviation. Chapter 9 investigates the results of the developed model and concludes with a comprehensive interpretation of the observed results. Finally, Chapter 10 summarises the research contribution and provides conclusions of this study and in addition, it presents potential recommendations for future research.

2 Literature Study

The interest in utilising formation flight as a fuel savings measure has grown over the past twenty years. This interest has been generated in a range of distinct but interconnected research areas. This chapter provides a concise examination of the current research areas in formation flight. Three key topics in the literature were identified: Aerodynamics, (Fuel)Performance and Operations Optimisation. Figure 2.1, shows an overview of the significant topics covered in this chapter.

In this chapter, Section 2.1 and 2.2 examine the (aero)dynamics of formation flight. Next, Section 2.3 investigates the fuel-burn modelling techniques observed in the current literature. Then, Section 2.4 reviews the encountered literature material on performance of formation flight from a costs perspective. Thereafter, in Section 2.5 a comprehensive study is performed on the operations optimisation of formation flight. Finally, Section 2.6 briefly outlines the concluding remarks of the conducted literature review.

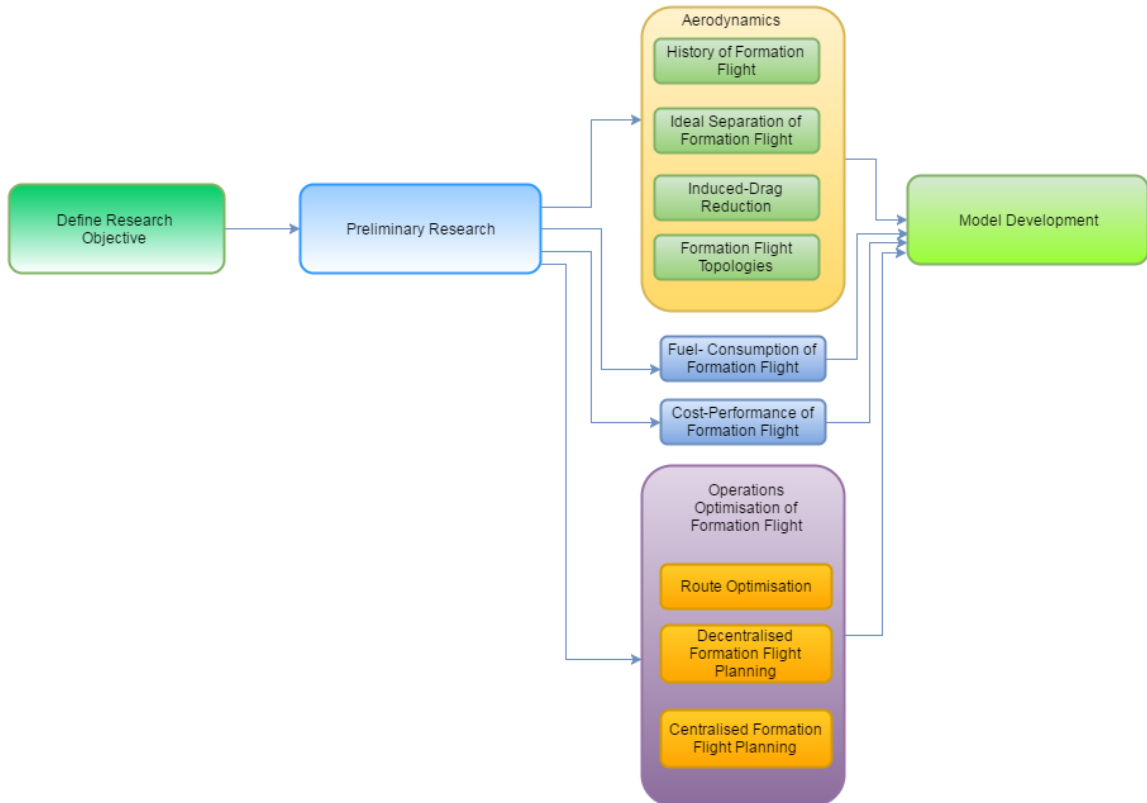


Figure 2.1: Preliminary research framework

2.1 History of Formation Flight

Before the fundamentals of Aerodynamics of formation flight is presented, it is important to first study the inspiration behind formation flight.

Formation flight for commercial use was first inspired by birds that fly in formation on long distance migration journeys. Lissaman first investigated this phenomenon back in 1970 [7]. Research conducted by Lissaman, highlighted interesting findings that explained the intuition behind birds in creating formations. The findings showed that birds in formation were able to conserve energy, allowing them to increase endurance and range [7]. The reason birds were able to save energy was caused by the upwash generated by the leading bird as shown in Figure 2.2. This upwash provided the trailing birds with an additional lift force, which allowed the birds to reduce the flapping frequency of their wings, hence saving energy. The findings also showed that the leading bird would switch roles between other trailing birds in order to minimise the energy of the entire flock in formation. These findings from Lissaman was validated by Weinerskirch et al [8]. Weinerskirch et al. conducted experiments on a flock of birds in formation. The researchers embedded sensors to a flock of birds in order to monitor the heart rate. When the birds flew in formation they were able to deduce key results. Their investigations revealed a 11 to 14% decrease in heart rate for a trailing bird relative to a leader. These results inspired researchers in the field of aviation to initiate a series of research efforts in order to reduce the fuel consumption in commercial aircraft.

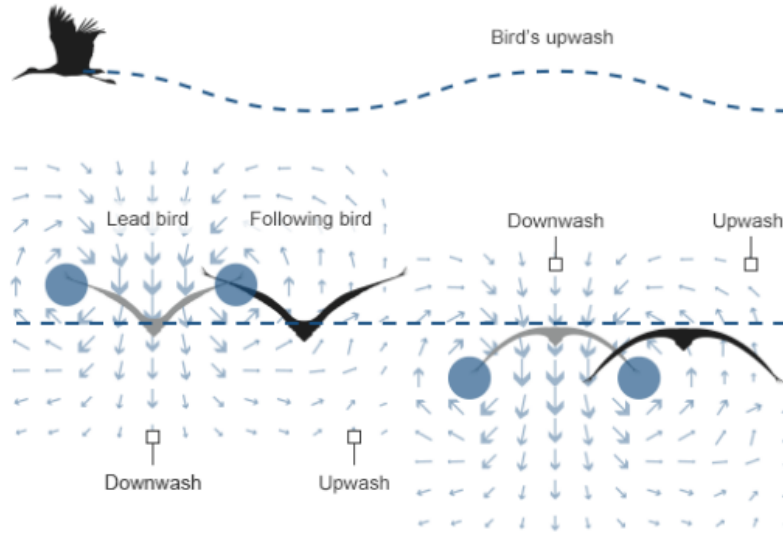


Figure 2.2: Upwash and downwash generated in bird formation [9]

2.2 Aerodynamics of Formation Flight

This thesis does not primarily focus on the aerodynamics of formation flight, however, the underlying factors are highly influential towards the practicality and overall effectiveness of formation flying. This section presents the key aspects of formation flight aerodynamics, which are: optimal positioning (formation leader-follower separation) of aircraft in formation, induced-drag savings, ideal flight speed in formation and ideal formation topologies (architectures). These key sub-areas are the main drivers of this section. Several key authors contributed to

these sub-topics which led to various different but interconnected research publications.

Before presenting the key sub-topics, the mechanisms of formation flight drag savings ability will be explained in order to understand the fuel saving potential of formation flight.

As a body passes through a fluid, it exerts a downward momentum on the fluid, which in turn causes an equal and opposite lift force on the body. The downward force, spirals around the edge of the wake creating upwash and downwash regions as shown in Figure 2.3. When a second aircraft flies through the upwash region, the upwash increases the local apparent angle of attack of the aircraft and generates an additional lift force greater than what it would generate in still air. The red arrows illustrated in Figure 2.4 clearly demonstrate the additional aerodynamic forces acting on the main wing of the trailing aircraft due to the upwash. As the definition of lift (L) and drag (D') needs to be perpendicular and parallel to the freestream, the figure shows that the lift force, L , tilted forward and the drag, D tilted upwards. Figure 2.4 also depicts the additional local freestream, V' , of the wake acting on the body. This additional V' changes the lift and drag force and results in a new L^* and D^* . Therefore, in formation flight, the lift increases and drag decreases when the wing is in the upwash region. This results in a reduction in induced drag and a increase in lift for the same angle of attack. Hence, aircraft in formation will require to decrease its angle of attack compared to out-of-formation flight in order to keep the same lift force. This is the basic premise behind formation flight drag reductions.

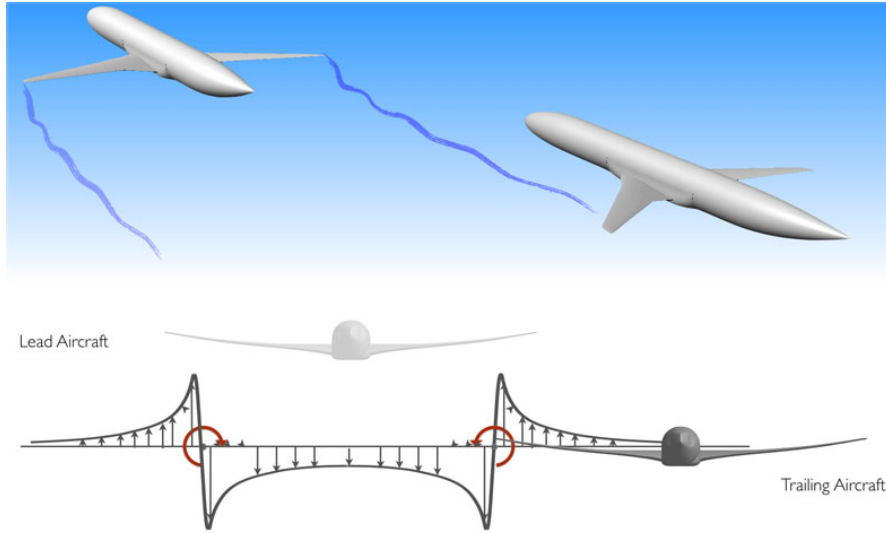


Figure 2.3: Vertical component of velocity induced on trailing aircraft by the wake of leading aircraft [10]

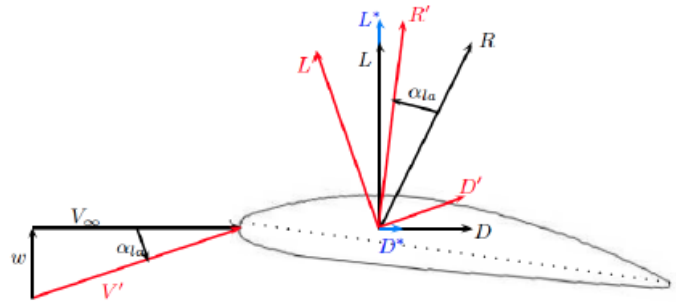


Figure 2.4: Induced upwash on a wing cross-section due to the trailing wing vortex field [11]

2.2.1 Ideal Separation of Formation Flight

The first encountered publications on aerodynamics of formation flight was from Veldhuis et al. [11]. Veldhuis et al. presented a detailed background on the aerodynamics of formation flight. The authors developed a model which allowed to investigate a crucial element of formation flight. From their model they were able to deduce ideal separation distances between the leader-follower. Moreover, for the trailing aircraft to benefit from the upwash generated by the wake of the leading aircraft, the wings of the trailing aircraft need to be positioned within the upwash as illustrated in Figure 2.5. The results of their model showed that an ideal separation of 10 to 100 wing spans between leader-follower is required. Furthermore, Veldhuis et al. claimed that optimal positioning of the wings of the aircraft was crucial to the safety aspect of the formation flight. In particular, the authors claimed that the additional lift force generated by the upwash on the wing, as seen in Figure 2.4, created a rolling and yawing moment. This adverse dynamic interaction would need to be compensated for by trimming the aircraft and as a result would diminish the effects of the induced drag.

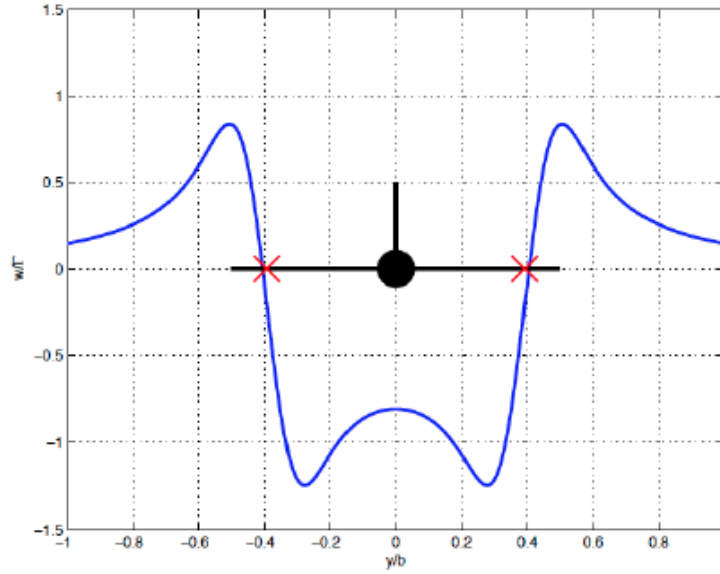


Figure 2.5: Trailing upwash velocity distribution in span-wise direction due to vortex generation by leading wing [11]

These findings were further supported by Herinckx et al. and Nangia & Palmer [12]. Herinckx et al. claimed that a separation of 10 wingspans would be ideal for trailing aircraft [13]. These ideal separations were also confirmed by the leading formation flight Aerodynamists, Ning et al. These leading authors claimed that flying such separation distances would not diminish the upwash generated. Ning et al. further stated that the vertical separation is more drag sensitive to induced-drag benefits than lateral separation, hence the authors recommended the wing tips of the trailing aircraft to overlap the wing tips of the leading aircraft. The authors also stressed the need to trim the aircraft in order to compensate for the adverse span-wise moments [14], [15], [16], [10]. Similarly, Blake et al. claimed that in order to prevent instability caused by flow separation on the trailing wing, positioning the trailing aircraft was critical towards the safety of flight [17]. Research contribution towards formation aircraft positioning was also provided by Haissig [3]. Haissig examined the tight formations of military aircraft which regularly fly in formations with lateral and longitudinal spacings of 500 *ft* and 4000 *ft*. Haissig stressed the issue

of possible activation of the aircraft Traffic Collision Avoidance System (TCAS) when flying in close proximity. The author suggested a possible use of the TCAS and Airborne Dependent Surveillance Broadcast System (ADS-B) for inter-formation navigation. However, Ning et al. dismissed the use of such systems as formation navigation aids, since it may confuse the pilots.

With respect to formation positioning, the authors Okolo et al. [18], presented unique results in which they discovered the weight of the leading aircraft highly influenced the optimal positioning of the trailing aircraft. This was due to the larger upwash generated by the leading aircraft. As a result, when the weight of the leader increased, the position in which the trailing aircraft would gain the optimal upwash benefit was moved in a lateral direction away from the wing of the leading aircraft. Furthermore, Okolo et al. [4], studied the ride quality for aircraft in formation. Their results showed that aircraft in formation experienced equivalent comfort levels as in out-of-formation flight.

2.2.2 Induced-Drag Reduction

Induced-drag reduction, which accounts for almost 50% of the total drag [19] is the critical factor that enable the decrease in trailing aircraft fuel consumption in formation flight. In the theoretical work of Veldhuis et al. identical trailing aircraft experienced a total drag reduction of 13%, which relates to approximately 26% reduction of induced drag [11]. These drag savings are highly dependent upon the modelling technique and the governing assumptions, hence different drag savings are estimated in various publications. In the research work conducted by Nangia & Palmer, a 30% induced drag reduction was reported [12]. This 30% induced drag savings was supported by Ning et al. [15]. Furthermore, Herinckx et al. reported a total drag savings of 10% for a two-aircraft formation which translates to approximately 25% of induced-drag savings. Additionally, Herinckx et al. examined the effect on potential savings in total drag with increase in number of aircraft in formations. The study concluded that the drag savings for trailing aircraft reach a plateau beyond a ten-aircraft formation size [13]. This effect can be seen in Figure 2.6.

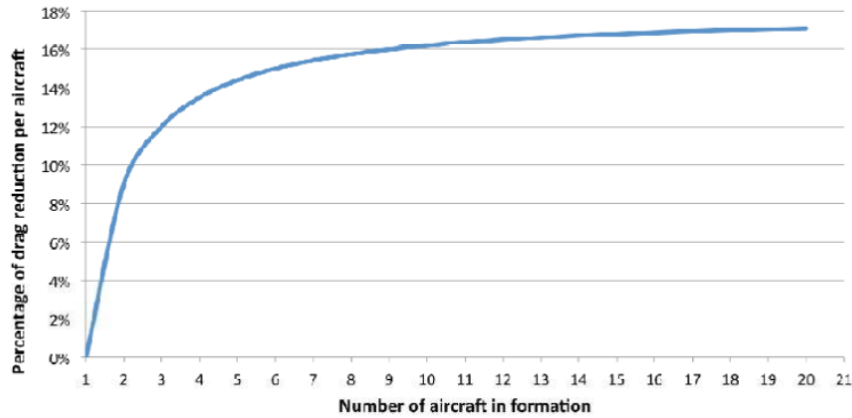


Figure 2.6: Drag reduction benefits against number of aircraft in formation [13]

Furthermore, the work of Xue & Hornby presented a compelling study on drag reduction for heterogeneous (non-identical) and homogeneous (identical) aircraft in formation flight. The study concluded that the leader-follower combination highly influenced the accumulated drag

savings [20]. This data is clearly presented in Figure 2.7, in which the induced drag and total drag are plotted against the Maximum Take-off Weight (M)TOW ratio between two respective aircraft. The work of Xue & Hornby claimed that when the leader to follower aircraft ratio is 0.8:1.0, i.e., an A320:A330 type aircraft configuration would lead to a 21% decrease in induced drag, while two equal (1.0:1.0) homogeneous aircraft achieved a 25% decrease of induced drag. However, if the leader to follower aircraft was relatively larger, i.e., on a scale of 2.5:1.0 (A380:B777), the induced drag savings were approximately 45%.

The above results were obtained through wind tunnel test and via sophisticated computational models and as a result, these results required proper validation. For this reason, the National Aeronautics and Space Agency (NASA) conducted flight tests to validate the drag savings for trailing aircraft in formations. NASA Engineers, Ray et al., used two F/A-18 aircraft for the purpose of a formation test flight. The flight test revealed a 20% reduction in the induced drag by the trailing aircraft [21]. Additionally, the researchers showed that the decrease in drag caused a 18% decrease in the trailing aircraft fuel flow rate. This confirmed the fact that trailing aircraft indeed benefit from the upwash of the leading aircraft [21].

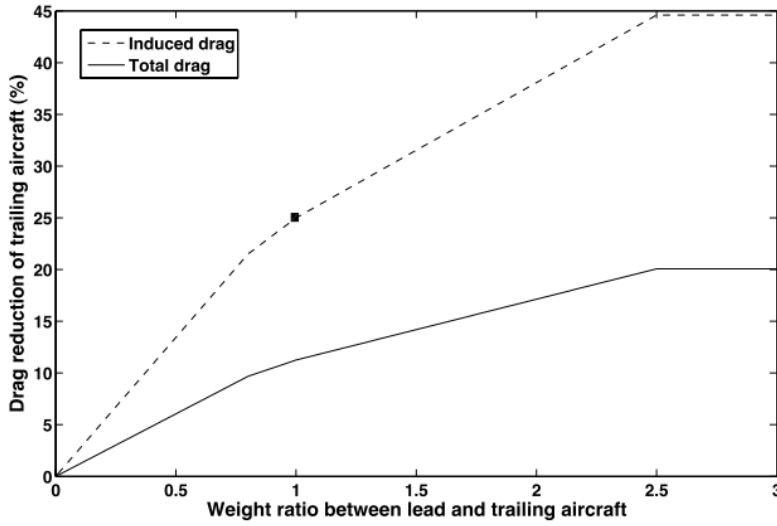


Figure 2.7: Drag reduction vs leader-follower weight (M)TOW ratio [20]

2.2.3 Formation Flight Topologies

The number of aircraft joined in formation and the type of formation architecture, govern flight safety and the potential fuel savings. The drag against formation size data shown in Figure 2.6, clearly illustrates the correlation between the number of aircraft in a formation and the experienced drag reductions. As mentioned previously, the increase in drag-saving benefits from formation flight only improves up to a ten-aircraft formation. Additionally, Herinckx et al. reported that a ten-aircraft formation will occupy an entire 500ft vertical block in the airspace, hence causing implication to procedural vertical separation in air traffic control [13].

The literature moreover, presented studies on the safety implications of formation flight. The authors Blake et al. and Haissig conducted research on the safety implications of formation flight. Their tests recommended the assembly of smaller formations comprising of two to three aircraft in order to maintain the proper safety of flight [17], [3]. This meant that a compromise between potential fuel savings and formation size has to be made since larger formation sizes

result in more followers than leaders and as a consequence, it leads to higher attainable fuel savings. This was seen in the work of Verhagen et al. in which a sixteen-aircraft formation was observed in the simulation results and, subsequently yielding higher overall fuel savings [6].

Moreover, the current literature also examined the optimal formation architectures or, topologies. Three types of formation architectures were seen in the literature, namely, V-formation, echelon formation and inverted-V formation, as shown in Figure 2.8. With respect to these formation types, Ning et al. reported that both echelon and inverted-V formation allow for favourable potential drag reduction savings [10], [15]. In addition, Bower et al. validated the benefit of flying in formations with an inverted-V formation architecture. Both Ning et al. and Bower et al. reported that fewer control deflections would be required when using an inverted-V formation. This is due to the positioning of the aircraft, since inverted-V formation experience symmetrical loading acting on the aircraft, compared to the asymmetrical load in echelon formation. The two groups of authors also reported that inverted-V formation showed higher fuel saving potential than echelon formation. However, for real-world applications, inverted formations are relatively more complicated to coordinate than echelon formation.

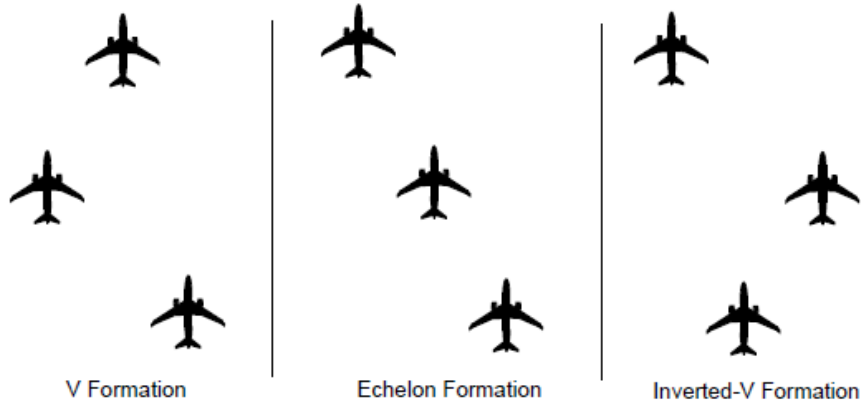


Figure 2.8: The three types of formation flight architectures [10]

2.3 Fuel Consumption of Formation Flight

The literature indicated the fuel reduction potential for trailing aircraft to be highly dependent on drag savings and also on the employed fuel consumption models. As discussed in Section 2.2, induced drag savings of 10 to 30% were experimentally determined for trailing aircraft in formation, resulting in a potential reduction to the fuel burn rate. The majority of the literature assumed a 10% reduction in fuel consumption in trailing aircraft [4]. This was later validated by Boeing researchers using two C-17 military aircraft. The test aircraft were flown from two identical airports towards two distinct destination airports. Their research devised an optimal routing strategy for the aircraft to be flown. The optimal routes were derived by minimising the distance flown while assuming a direct correlation between the minimum distance and minimum fuel consumed. Taking into account the respective deviations from solo flight, the flight test showed an overall savings of 7.2%.

The authors Bower et al. also conducted a case study experiment for a fleet of five FedEx cargo aircraft [22]. The five flights were of three aircraft types: DC-10-30, A300-600F and B727-200. Bower et al. assumed the aircraft to fly at their respective maximum payload weight at an altitude of 33,000ft and at Mach 0.82. The authors also assumed formations comprising of

two and three aircraft, respectively. An examination of the authors results revealed a 11.5% potential fuel savings for a three-aircraft formation. Bower et al. attributed the relatively high fuel savings to the three-aircraft formation flying longer routes, i.e., longer flight time spent in formation. Additionally, the authors also investigated the influence of fuel savings against the type of formation architecture. Their findings for the respective fuel-burn, were a 10% decrease for echelon formation and a 16% decrease while flying in an inverted-V formation.

Two other interesting case studies were conducted by Xu et al. for a fleet of 31 aircraft on a long-haul South African Airline (SAA) network and 150 transatlantic Star Alliance members [23], [24]. In both cases, flights were assumed to form assemblies of up to a three-aircraft formation size. The case study comprising of 31 SAA aircraft showed an overall fuel savings of 5.8% while the 150 Star Alliance fleet saved 7.7% in overall fuel burn. The difference in fuel savings was primarily due to the larger fleet number.

In 2015, Kent & Richards conducted a case study for 210 eastbound transatlantic flight routes. The authors used a version of the Breguet range equation to simulate fuel consumption for groups of two and three-aircraft formation sizes. Furthermore, Kent & Richards made an optimistic assumption on the fuel flow rate, hence assuming a 20% reduction in fuel burn for trailing aircraft on a two-aircraft formation and a 25% for a three-aircraft formation. The results reported potential fuel-burn savings against solo flight of 8.7% for a two-aircraft formation and 13.1% for a three-aircraft formation [25], [26], [5],[27]. In the same year, Verhagen et al. performed a case study for 347 eastbound transatlantic flights. The authors assumed a potential fuel burn reduction of 10% for trailing aircraft. They simulated their fuel-burn model using a Breguet range equation, similar to the one used by Kent & Richards. Uniquely, the authors created large formations up to sixteen-aircraft in formation and achieved a 3.7% fuel-burn savings against corresponding solo flights. The most recent formation flight fuel savings analysis was conducted by Marks et al. for 20 flights on the North Atlantic Tracks (NAT). The authors found 3.1% of potential fuel savings on a two-aircraft formation against solo flight [28].

The current literature study indicated that the majority of the authors used the Breguet range equation in order to model fuel-burn for aircraft in formation. Both Kent & Richards and Verhagen et al. used the same version of the Breguet range equation, i.e., a version governed by varying airspeed, constant lift coefficient and constant altitude. However, the authors assumed constant airspeed therefore, this particular version violated the definition of $W = L = \frac{1}{2}\rho V^2 SC_L$ since if velocity is varied then either C_L or altitude has to change in order to satisfy the above equation.

The authors Ning et al. and Xu et al. used sophisticated aerodynamic fuel-burn models. These high-fidelity fuel consumption models have the advantage of estimating better realistic fuel-burn values. However, when assessing the fuel-burn for scenarios of large number of aircraft, the Breguet range equation becomes more attractive than the sophisticated models. There are two key reasons for using the Breguet range equation. First, the computational time is kept low since the Breguet range equation allows the fuel-burn to be computed for an entire flight segment. Second, important factors such as fuel flow rate, altitude and airspeed can be manipulated in order to study the influence it has on the fuel consumption of formation flight. This second factor in particular, makes the Breguet range equation an attractive fuel-burn model for the conceptual studies of formation flight.

2.4 Cost Performance of Formation Flight

The performance of formation flight has a direct impact on the overall flight efficiency, primarily for commercial flights where the performance is a key aspect of profitability. The authors Ning et al. investigated the impact of formation flight to the overall performance. Their investigations showed that commercial aircraft which normally fly at transonic speeds experience compressibility effects when flying in formation, hence causing flow separation over the wing of the trailing aircraft [15]. Therefore, to mitigate compressibility effects, aircraft in formation will require to fly at a 2.5% lower Mach number compared to the nominal out-formation cruise Mach number. This means that for typical long-haul jets, such as the B777/A330 aircraft with typical cruise Mach numbers of 0.85, will be required to fly at Mach 0.82. As a result of lower airspeed in formation flight, the block time (BT) would slightly increase, hence directly affecting the direct operating cost (DOC) of commercial flight [29], [30]. For this reason, a trade-off has to be made between fuel-burn and BT/DOC minimisation. Furthermore, the increase in BT is not only adversely affected by lower flight speed but also influenced by the increase in overall flight distance caused by the detours created by aircraft that require to rendezvous into formations.

2.5 Operations Optimisation of Formation Flight

This section focuses on the operations optimisation planning of formation flight, i.e., it aims to answer the following question pertaining to formation flight, *when, where and with whom* to create formations. This section will review the current literature on route optimisation. Thereafter, the two types of formation flight planning approaches in the current literature, namely, centralised and decentralised will be studied.

2.5.1 Route Optimisation

This section of the literature review focuses on *when and where* formation flight should form, in order to join and break away from formations in an optimum way. The literature indicated two leading groups of authors namely, Bower et al., and Kent & Richards. Both groups of authors claimed that the route optimisation was the most computationally expensive part of the formation flight problem since all combinations of formation need to be evaluated. Bower et al. used MATLAB's built in (*fmincon*) gradient-based optimiser to solve the large-scale route optimisation problem [22]. The authors considered each formation segment to be a great circle track, hence making the optimisation a 4-D trajectory problem. The optimisation was constrained to range, drag, time and weight respectively [22]. Furthermore, each of the constraint variables were defined at each flight state. In addition, aircraft within a formation were assigned the same Mach number and altitudes to fly at. Subsequently, the rendezvous and splitting points were optimised for minimum fuel burnt in formation flight with design variables being the aircraft weight, altitude and Mach number. Notably, the authors accounted for 10% of the aircraft weight as contingency fuel due to required detours for rendezvous of formation flights.

The authors Kent & Richards uniquely derived an optimal routing model for a pair of aircraft in order to estimate the rendezvous and splitting points such that it minimises the fuel cost. Using a geometric approach based on the Fermat-Torricelli problem, allowed the authors to determine the minimum weighted distance for a formation flight route [5]. This routing method allowed for evaluating large number of potential formations by decomposing the problem and, hence obtaining relatively faster results than the method presented by Bower et al. Furthermore, the

authors used the notion of arc weights in order to incorporate the associated drag reduction benefits from flying in formation. This was done by using aircraft performance data to derive constant nominal fuel-burn rate for a specific aircraft. This fuel-burn constant was then used as arc weights for a particular formation segment in order to represent the fuel burn. The authors claimed that their approach can be easily scaled for large formation sizes.

The routing model proposed by Kent & Richards was later extended by Verhagen et al. The authors extended the arc weights and used the model to solve for relatively larger formation sizes. Furthermore, Verhagen et al. developed a methodology in order to synchronise the joining of the respective formation flights. The authors claimed that expediting the formation was crucial for the overall potential fuel saving benefit, since the time spent, for example, in a holding pattern in order to synchronise with a formation partner would lead to fuel burn penalties. Verhagen et al. reported that aircraft requiring to synchronise were bound by a maximum speed of 251 m/s and a minimum speed of 201 m/s for the considered aircraft type (Boeing 777) and its characteristics. The airspeed for two merging flights were then varied with respect to the distance flown in order for the flights to rendezvous at the same time. This method was dependent on the solo flight leg distances to the joining point for the two respective flights. Therefore, in order to tackle the scenario, Verhagen et al. shifted the rendezvous point towards the splitting point in order to achieve synchronisation.

2.5.2 Decentralised Formation Flight Planning

Decentralised formation planning is defined as a local optimisation-based planning method that achieves cooperation through inter-aircraft communication. The current literature on decentralised planning only allows for the formations to be created only when there is a potential to decrease fuel consumption, i.e., formation flight is not imposed as a priority, rather the formation flight is an in-flight option to a cost minimisation problem [31], [32]. The first decentralised approach was formulated by Frazzoli & Ribichini in 2003 [31], [32]. The authors used a greedy algorithm as a means to communicating and negotiating with neighbouring aircraft in order to coordinate formation flights. The communication of aircraft were limited to a specific radius and allowed only two aircraft to cooperate with one another at a given time. Since the optimisation for the formation flight planning embodied a greedy algorithm, the formations were driven purely on individual gains. As a result, the overall fuel saving potential was not considered since the decision to create a formation flight was solely based on the minimum energy consumption principle. Therefore, aircraft were ranked according to the highest cost savings and they partnered with other aircraft by only evaluating the self-benefits towards formation flight, rather on the overall cooperation of the fleet. As cooperation is not attained for the overall fleet, this optimisation of formation flight planning was a local method, hence the fuel saving potential were also local.

In 2015, Verhagen et al. extended the work of Frazzoli & Ribichini with the objective of finding the potential fuel savings of formation flight using the decentralised optimisation-based cooperative planning approach. Verhagen et al. created a simulation model which achieved fast solutions for a larger fleet with flight formation greater than two-aircraft formation sizes. Similar to the work of Frazzoli & Ribichini, Verhagen et al. defined a communication perimeter for each aircraft [6]. The perimeter was defined as a circle with a radius of 250km that extended from the centre of the aircraft. Communications between aircraft were governed by the intersection of two circumferences. As a result, only two aircraft were allowed to communicate at a time which formed a communication set determined by the greedy algorithm. These communication sets were then ranked according to their respective flight numbers. Notably, the authors modelled a

flight to be the flight number, origin, destination and departure time. The flight number is set from 1 – N where N is the total number of flights. The lowest flight number within a set was elected as the formation leader in which it does not benefit from the potential fuel savings of formation flight. Their model then loads these sets into the routing algorithm. The algorithm then compute the formation routes for the respective communication set and then evaluates the cost of formation flight, hence producing the fuel costs and time costs. The formation flight costs are then compared against solo flight and if there is potential to save fuel, a commitment is made within a communication set for a possible formation flight. This local optimisation-based algorithm is executed on pairs of aircraft formation, hence the potential fuel savings were strictly local optima. The greedy algorithm allows for fast computations with the downside of not evaluating other possible/better partners options and as a result, better potential fuel savings were unrealised. Verhagen et al. concluded the research with a transatlantic case study of 347 flights. Their model was able to cope with schedule disruptions, however not all the fuel saving potential was captured. As a result, the authors Verhagen et al. and Frazzoli & Ribichini recommend developing a centralised global optimisation-based cooperative formation flight approach with the capability of performing fast computations whilst achieving global solutions.

2.5.3 Centralised Formation Flight Planning

Centralised formation flight planning is a strategy conducted pre-flight, hence complete information is known with regard to formation flight joining and splitting locations, as well as predictable fuel savings. This optimisation-based approach stems from Operations Research (OR), allowing to solve the global assignment problem. Since this approach evaluates the option of all possible combinations of flight formations, the problem leads to a NP-hard problem*, i.e., an intractable combinatorial optimisation problem. As a consequence of the NP-hard nature, led to a rise in complexity of the problem as the size of the problem increased which subsequently led to forbiddingly large computation times. For this reason, centralised approaches were not employed for in-flight planning and, instead, had to be used as a pre-flight formation strategy in order to accommodate the high computational time burdens. Particularly, this meant that the approach could not adapt to sudden schedule disruptions. This section presents the work of three main groups of leading authors that conducted research to find a solution to the inherent disadvantages in centralised formation flight planning.

The authors Bower et al. and Xu et al. laid the foundation for the centralised formation flight planning approach in 2009 [22] using the Nelder-Mead Simplex method to minimise the overall cost of formation flight. The authors investigated the potential net fuel savings for formation flight against solo flight. In order to assign aircraft into formations, within a reasonable time limit, Bower et al. used a heuristic method to filter out unfavourable formations, thereby reducing the number of possible combinations to be assessed. However, when they performed their analysis for 150 flights they encountered combinatorial explosions which significantly increased the computational resource time. This combinatorial explosion is governed by the number of aircraft and the number of aircraft in a formation. Using the binomial coefficient equation given by $\frac{N_a!}{n!(N_a-n)!}$ in which N_a is the number of aircraft and, n number of aircraft in formation (formation size), the number of all possible combinations can be computed. Using this equation, Bower et al. computed the following: for 150 aircraft, 11,117 possible two-aircraft formation combina-

*According in the book *Heuristic Search-theory and applications* by Edelkamp [33] the concept of NP-hard (non-deterministic polynomial) stems from Complexity Theory which deals with resources such as time required for computation in order to solve a problem.

tions exist and 551,300 three-aircraft possible formations combination exist. Since each possible combination needed to be enumerated, it dramatically increased the overall computation time [22]. The authors then used a heuristic filter for the respective two-aircraft and three-aircraft formations in order to obtain a clean set of 800 favourable formation combinations. Despite the use of a filter, the enumeration time still took 60 CPU hours on a standard 2.1 GHz personal computer used by Bower. This is a severe limitation in determining the potential fuel savings for centralised formation flights. The only option of maintaining a low computational time requirement is to decrease the number of aircraft seeking to join a formation and the formation size. However, this would not realise the full potential of formation flight operations for civil aviation.

In 2012, Xu et al. studied a similar research problem [24], [23]. The authors employed network optimisation, formulated by a Mixed Integer Linear Program (MILP) to determine an optimum network schedule against other combinations of optimal solo and formation missions. They used an external solver (CPLEX) to obtain an optimised formation flight schedule. However, the authors encountered the same combinatorial explosion problem. Evidently, it can be seen that the combinatorial nature of the problem prevents theoretical research into formation flight for network-wide applications for commercial aviation.

Inspired by the work of Bower et al. and Xu et al., the authors Kent & Richards developed a global routing and an assignment problem for commercial flight formation. They used an analytic geometric approach in order to speed up the calculation time for the formation flight routing problem. This allowed the authors to obtain the fuel cost (fuel burnt in kilo-grams) for all possible formation pairings. The set of flights and the possible formations were then optimally assigned to a cost minimising formation fleet. This optimal assignment of formation flights was solved using a MILP and optimised using the CPLEX solver. Since the authors required only an aircraft to be in either formation or to fly solo, binary variables were used. For this purpose, the constraints of the MILP were binary pairing matrices with the objective function being fuel cost. The authors indicated that the nature of the formulated problem was non-convex. This means that there are many possible local minima, therefore searching for the global minimum increased the amount of resources needed to solve the assignment problem. Furthermore, the authors also stressed the combinatorial impact of the problem while claiming it adversely affects two aspects of the problem. First, as the number of possible formation combination increase, the routing problem has to compute the cost for all these combinations. Second, the assignment problem will have an additional flight which adds a constraint and each combination adds a variable to the objective function. The authors further stated that the fast method developed for formation routing was diminished with the adverse combinatorial explosions. Kent & Richards conducted a case study for 210 eastbound transatlantic flights for a two/three-aircraft formation size. For the two-aircraft size formation, 21,945 possible two-aircraft formation combinations needed to be evaluated. Each combination required 0.5 micro seconds of enumeration time, which resulted to a total computational time of 11 seconds. However, when the authors increased the size of the formation from two-aircraft to three-aircraft formation, the number of combinations increased to 1,521,520 possibilities. Compared to the two-aircraft formation, the possible combinations increased by 70-fold while the computational time increased by 245-fold which caused a total enumeration time of 45 minutes. Evidently, it was seen that the routing of formations required majority of the enumeration time compared to the assignment problem. The authors recommended to keep the number of aircraft in a formation as low as possible since it has a significant impact on the complexity of the problem. This is a major limitation present in all the encountered centralised formation flight planning research work.

2.6 Concluding Remarks

The previous sections presented several challenges that need to be addressed in order to allow formation flight to be operationally feasible for civil aviation, as a means of fuel consumption reduction. It was seen that the trailing aircraft in formation flight was restricted to the position of the maximum upwash generated. However, research is still underway in order to find the exact location to position the trailing aircraft in a formation. It was also seen that flight speed in formation was limited to a 2.5% lower Mach number than the nominal cruise Mach number. Next to this, block time was also influenced by formation flight. Aircraft type and mix proved to be important since the total drag reduction in formation flight had a direct impact towards potential fuel savings. The current literature also showed that an invalid Breguet range equation was used frequently for fuel-burn modelling. The optimisation approaches used for the planning of formation flights showed several shortcomings. Decentralised formation flight planning allowed for fast computations, however it used a local optimisation-based approach and as a result, not all the fuel saving potential was captured. Next to this, the centralised formation flight plan, formulated by a MILP, produced global solutions, however, with the disadvantage of it being computationally more expensive due to the combinatorial nature of the problem. Therefore, it was hypothesised that the development of a cooperative optimisation-based method which holds the strengths of both centralised and decentralised approaches would significantly contribute to the scientific field and to the operational feasibility of formation flight for civil aviation.

3 Conceptual Model

The objective of this chapter is to outline the newly developed multi-stage formation flight planning (MS-FFLIP) model, for evaluating large number of possible formations for a (near)global optimisation of the overall fuel consumption, within an efficient computational time frame.

After the performed literature review, two key requisites for an attractive formation flight planning method were identified. The first requirement relates to the computational time required to evaluate a formation flight scenario for real-time applications. The second, relates to the ability of the method to show attractive fuel saving potential when operating formation flights. The key information needed for operating an attractive formation flight plan can be summarised as follows; *when, where and with whom* to fly in formation? The literature study indicated that in order to obtain the above information, a great amount of computational resources were required. So far, the decentralised approach was the only method able to provide fast computations for the purpose of formation flight planning. The decentralised approach, developed by Verhagen et al.[6] used a local optimisation-based method, thus providing local solutions. It was also observed that the decentralised approach was capable of assigning aircraft to large formation sizes. In spite of creating large formation sizes, the method failed to realise the majority of the potential fuel savings of formation flight due to its local optimisation method. Despite the decentralised approach satisfying the first requirement, it clearly failed to accomplish the second requirement. On the other hand, the centralised approaches seen in literature, partially realised the second requirement, but failed to realise the computational time requirement, hence enumeration times (depending on the number of aircraft and respective formation size) ranging from 11 minutes to 60 hours were observed. Under regular flight operating conditions, such computational time magnitudes are unrealistic for real-time/en-route operational use in civil aviation. Such high computational or enumeration times were a direct result of the combinatorial explosion (growth in the number of possible combinations) of the problem.

This chapter presents an overview of the developed MS-FFLIP model which demonstrated the potential fuel saving benefit of formation flight for commercial aviation. Moreover, inspired by the work of Kent & Richards, a single-stage centralised model was developed as a means to validate the newly developed MS-FFLIP model. Hence, this chapter will also give an overview of the developed validation model. Furthermore, the decentralised model developed by Verhagen et al., is used as a comparison model. This model which required a modification in order to allow a comparison against the MS-FFLIP model is also briefly outlined in this chapter.

The first section of the following chapter outlines the general structure of the MS-FFLIP model and its validation model. Next, in Section 3.2, the rationale for the developed MS-FFLIP model is presented, by introducing the combinatorial impact experienced by the conventional single-stage centralised models. Thereafter, Section 3.3, presents a general overview of the three models. Next, Section 3.4, presents the limitations and underlying assumptions of the MS-

FFLIP model. Then, for the purpose of modelling, a reference aircraft is identified in Section 3.5. Lastly, Section 3.6, gives an overview of the simulation set-up used for the purpose of this study.

3.1 Model Framework

This section presents an overview of the general framework for the developed model. Unless specified, the general framework is true for the developed MS-FFLIP model and the validation model as both models are essentially centralised formation flight planning approaches.

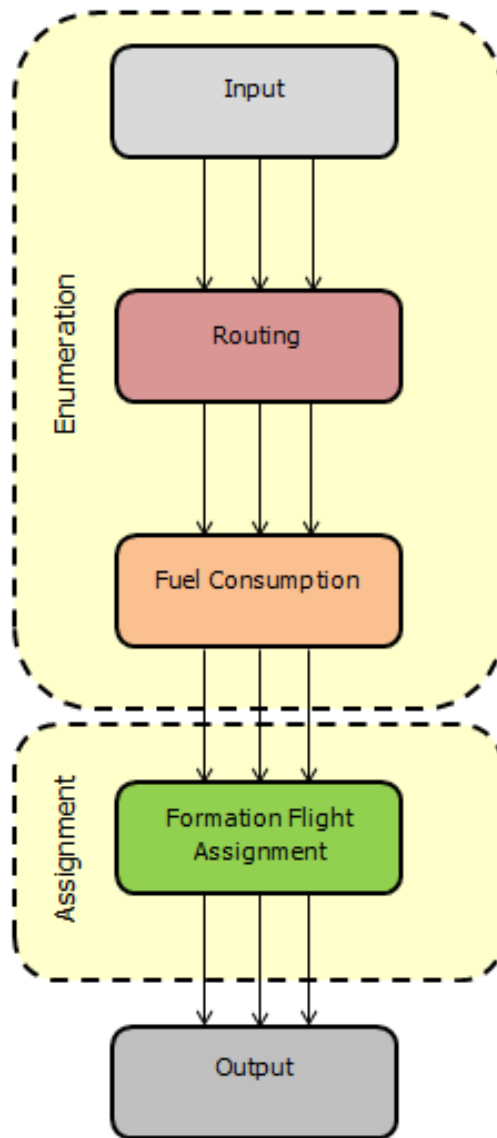


Figure 3.1: General framework of the MS-FFLIP model and the validation model

The key question (when, where and with whom to fly in formation?) presented in the introductory paragraph of this chapter requires the development of three complex interconnected

sub-models: routing model, fuel consumption model, and the formation flight assignment model. The routing model computes the optimal joining and splitting points for each possible formation flight. Second, the high-fidelity fuel consumption model estimates the potential fuel burnt for these possible formation flights. Then, the formation flight assignment model selects a set of compatible formation flights from those considered to achieve a (near)global optimal formation fleet that contributes to the overall reduction of the fuel use for the fleet. Figure 3.1, illustrates the arrangement of these models for the developed MS-FFLIP and the (single-stage) centralised validation model. The outcome of the model relies on the relevant input data and the required parameters. The main input data are the origin and destination locations which are represented by Cartesian 2-dimensional coordinates. The next input are the respective arc weights which represent the fuel-burn per formation segment, and the required formation size, n . These set of input are then loaded into the routing model. The model then immediately estimates the solo segment lengths for each flight between their respective origin and destination. Since the model operates with a 2-dimensional Cartesian coordinate system, the estimated lengths are based on straight lines between origin and destination points. Then, depending on the required formation size, n , the routing model creates combination sets with all the origin and destination locations. Subsequently, the routing model conducts a geometric routing optimisation in order to determine the 2-dimensional joining and splitting points for all the possible formation groupings. After obtaining the initial set of joining and splitting points, the model estimates the length of the formation segments, and based on this and together with the speed of flight, the model evaluates the possibility of synchronisation, i.e., the simultaneous merging into formation flights at the joining point. The synchronisation process then makes the necessary adjustments for efficient merging of formation flights at the desired joining point. The routing model updates the formation segment lengths taking into account the new joining and splitting points for efficient synchronisation. These formation segment lengths are then loaded into the fuel consumption model for which the potential fuel consumption per formation segment are estimated and also subsequently estimates the fuel consumption of the corresponding solo flights. Then, the model calculates the fuel consumption for every individual flight grouped within a formation. These aggregated fuel consumption estimates, yield cost coefficients per formation groupings to be used in the formation flight assignment problem. The formation flight assignment model, which is formulated as a MILP, uses the cost coefficients as variables in the objective function. The set of possible formation flights are used to construct the constraint matrices which contain of binary variables. The MILP runs the assignment of flights through an external optimiser in order to create a cooperative formation flight schedule. The output produced by the model are: CPU or computational time, potential fuel savings, flight time and the optimal formation flight schedule.

3.2 Combinatorial Explosion

The key impact of combinatorial explosion is manifested by the significant computational times which were observed in the current literature. This was the reason for which the conventional centralised methods failed to realise efficient real-time formation flight planning. Therefore, in order to enable real-time application of the MS-FFLIP tool, the model would need to escape the combinatorial explosions.

This is the primary goal of this study, avoiding the combinatorial explosion of centralised formation flight planning will provide a major leap-forward in formation flight operations. Therefore, before presenting the details of the developed models, it is important to grasp the magnitude of the combinatorial explosions and to conceptualise an intelligent method on tackling its effects

in order to reap the full benefits of centralised formation flight planning.

3.2.1 Combinatorial Problem

The combinatorial problem emerge when finding the possible number/combinations of aircraft in formations for a set of aircraft, N_a , to be grouped into a formation size, n . Therefore, given n , from a set of N_a , there exist a number of possible formation combinations, N_P . This number of possible combinations of aircraft in a formation can be computed using the binomial coefficient:

$$N_a \text{ Choose } n \equiv \binom{N_a}{n} = \frac{N_a!}{n!(N_a - n)!} \quad \text{for } 0 \leq n \leq N_a \quad (3.1)$$

The possible number of formations, given when using Equation 3.1, grows exponentially with either an increase to N_a or n . This can be seen by analysing the growth to the number of combinations with increase in n while keeping N_a fixed. An increase to n by 1 increases the number of possible combinations of aircraft in formation by a factor of $N_a/(n+1)$ [27], and a increase in n by 2, increases the possible combinations of aircraft in formation by a factor of $N_a^2/(n+1)(n+2)$ [27]. This substantial increase to the possible number of aircraft in formation dramatically increases the enumeration time.

Table 3.1: Number of possible combinations, N_P , with increase in number of aircraft, N_a and, formation size, n

n aircraft size formation	10 aircraft	50 aircraft	100 aircraft	267 aircraft	1,000 aircraft
2	45	1,225	4,950	35,511	499,500
3	120	19,600	161,700	3,136,805	166,167,000
4	210	230,300	3,921,225	207,029,130	41,417,124,75

One can observe from Table 3.1 the impact to the possible combinations of aircraft in a formation, when n or N_a is varied, hence the amount of resource required to solve the problem diverges drastically. For this reason, the validation model will be limited to assembling formation up to a four-aircraft formation size ($n \leq 4$) and with a fleet of 50 aircraft ($N_a = 50$). Thereby, allowing a reasonable computational time to gather the validation results. Since the possible combinations of aircraft in a formation is largely affected by n , the formation size n , will be fixed and therefore, the combinatorial impact of the problem can be substantially reduced. This means that once formations assemble at their respective joining points, the entire formation group will be considered as a “single entity” or “single flight”, hence allowing for joining of more aircraft in sequential stages and thus holding n at 2.

3.3 General Overview of The Models

This section presents a summary outline for the three formation flight planning models or tools used in order to attain the goal of this study. The motivation behind each model will also be explained in this section.

3.3.1 Validation Model: Single-Stage Centralised Formation Flight Planning

The (single-stage) centralised formation flight planning model was inspired by the work of Kent & Richards [27], [25], [26], [5] and was developed as a validation tool for the newly developed MS-FFLIP model. The centralised model is illustrated by Figure 3.1, which consist of three interconnected sub-models as explained in Section 3.1. This model is in exact form, hence it demonstrates the global fuel saving benefit of formation flight and conducts the optimisation in a single stage. Therefore, the routing model computes all the routes for all combinations of aircraft in each respective formation size. In other words, given a set of 50 aircraft, the validation model is capable of computing up to four-aircraft formation size, hence with reference to Table 3.1, there are 1,225, 19,600, and 230,300 possible number of aircraft combinations for the respective two, three and four-aircraft formation sizes. Therefore, excluding the 50 solo flights, there exist 251,125 total number of combinations and each requiring individual routing and fuel consumption estimations, hence the enumeration time will dramatically increase due to this combinatorial explosion. Furthermore, the assignment model featuring the MILP, has to optimise all the possible combinations for this non-convex problem*. Moreover, each aircraft will add an additional constraint and each combination will add an extra variable to the objective function of the MILP. For this reason, this validation model will be restricted to evaluating a 50 aircraft fleet with the ability to create formation sizes up to four-aircraft formation size. This is a sufficient validation set which will demonstrate the validity of the model and any larger validation example will lead to a waste of computational resource.

In addition to this model being a validation tool, it will support the purpose of the research question as well, by demonstrating the MS-FFLIP model has the essential advantage of a centralised model, in other words, to show a (near)global fuel saving potential.

3.3.2 Decentralised Model: Local Optimisation-Based Model

The decentralised formation flight planning model presented in this study, is based on the model developed by Verhagen et al [6]. This model is used as a comparison model against the developed MS-FFLIP tool.

As discussed in the literature study, the decentralised formation flight planning model incorporates a heuristic greedy algorithm in order to optimise formations in-flight. Since this decentralised model uses a local optimisation method compared to global optimisation used in the MS-FFLIP model, the results between the two models will provide interesting insights. Moreover, as explicitly stated in the research question, the MS-FFLIP model was essentially developed in order to demonstrate that it has the strength of the decentralised model, i.e., the ability to provide real-time computations for en-route formation planning applications and hence, the ability to adapt to sudden network disruptions. Therefore, this comparison will enable to answer a part of the research question. However, for the model to be eligible for a coherent comparison, the model input has to be compatible with that of the MS-FFLIP model. This includes the replacement of the fuel consumption model with the reintroduced Breguet range fuel burn approximation method, which allows for high-fidelity fuel-burn modelling.

*In principle linear constraints give rise to a convex problem, with a global optimum. The reason that it is still non-convex, in this context, is related to the fact that binary variables are used. A convex problem has to contain a convex feasible set for the problem to be convex – in this case the variables are not continuous hence, amounting to a non-convex problem

3.3.3 Multi-Stage Formation Flight Planning (MS-FFLIP) Model

The multi-stage formation flight planning model is developed in order to realise the objective of this study. The general structure of the MS-FFLIP model which is in principle is a centralised formation flight planning model and can be represented by Figure 3.1.

As mentioned, the current centralised framework is highly prone to combinatorial explosions due to the exceedingly high number of combinations required to be enumerated by the model. For this reason, an intelligent approach is required in order to escape combinatorial explosions.

As explained in Section 3.2, the combinatorial explosions mainly arise due to the increase in the formation size, n . By keeping the required formation size to a two-aircraft formation combination, i.e., at $n = 2$, the number of possible combinations can be kept relatively low as outlined in Table 3.1. In other words, the factor n will be fixed at $n = 2$ even for the requirement of large formation sizes. This can be achieved by conducting the optimisation for a two-aircraft formation size and then using the optimal solutions to attain larger formation sizes. Therefore, by considering the optimally assigned groups of two-aircraft formations to be single entities or “single flights”, these entities can then be combined with one another or other flights in order to increase the formation size and obtain higher fuel savings. Furthermore, the new origins of these entities will be represented by their respective optimal joining points and similarly, the new destination points will be represented by the respective optimal splitting points. The creation of entities are conducted using an event-driven simulation for which an event in this context is defined as a collection of all optimal formations or joining points. Subsequently, the event-driven simulation calls on the optimisation algorithm in order to proceed with the optimal assignment of the set of combinations of entities, or in this case the two/three/four-aircraft formation combinations. This process is conducted sequentially in stages until the maximum desirable formation size is attained and in each stage, n , is fixed at 2, thereby reducing the number combinations to enumerate and thus preventing combinatorial explosions.

The MS-FFLIP model consist of four stages and each stage contributes to the cumulative increase in formation sizes as demonstrated by Figure 3.2. Therefore, the maximum attainable formation size is a formation comprising of sixteen-aircraft. This innovative method of globally optimising formation flights will undoubtedly help to achieve the established objective of this study.

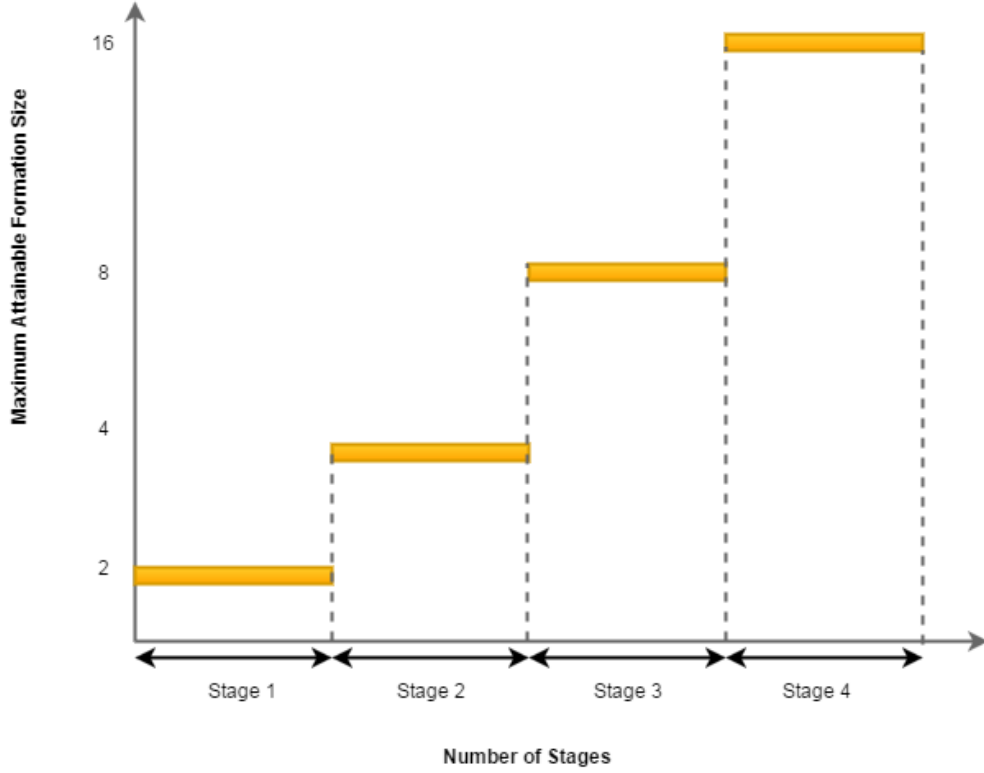


Figure 3.2: Illustration of maximum attainable formation size against the number of stages in the MS-FFLIP model

3.4 Governing Assumptions

The models developed incorporate several assumptions which directly translate into the quality of the obtained results. Therefore, the most important assumptions are presented in this section in order to define the scope of this thesis.

- All aircraft are of the same type.
- All flights depart at the same time.
- The effect of wind is not considered.
- The simulation is developed in a 2-dimensional Cartesian coordinate system.
- All solo great circle routes (shortest path over a sphere) are converted to a 2-dimensional flat space, hence, all solo aircraft fly in straight lines between their origin and destination.
- All aircraft are allowed to collaborate with each other.

- Formation flying results in a 10% fuel-burn reduction for trailing aircraft relative to the respective solo flight. The 10% in fuel-burn reduction applies for any aircraft in formation that is not leading the formation.
- Every aircraft flies at Mach 0.82 (2.5% lower than the cruise Mach number) while in formation in order to mitigate the compressibility effects.
- The minimum flight speed is set at the maximum endurance speed and the maximum flight speed is set at the maximum specific range speed.
- The International Standard Atmosphere (ISA) is valid in this study and the simulation is carried out at an $9km$ altitude.
- Aircraft are allowed to go into formation as soon as they take off from their respective origins.
- Flights proceeding into a formation experience instantaneous deceleration/acceleration and heading changes in order to fly at Mach 0.82 along the formation segment.
- The specific fuel consumption (c_t) is constant throughout the flight.
- Simulation takes place at steady and straight-and-level flight conditions. Therefore, thrust equals to drag and lift equals to weight ($T = D$ and $L = W$)

These assumptions clearly show that the produced results will be simplified compared to reality. The assumptions were imperative in order to arrive at a suitable answer to the thesis goal within the set time frame and research scope. Compared to past research work, these assumptions are not exclusive to the field of research. As a result, the work in this thesis will certainly be valid and would contribute to the current research field of formation flight operations.

3.5 Reference Aircraft

After presenting the overviews of the models and the governing assumptions, a reference aircraft still needs to be selected for the purpose of modelling formation flights. Therefore, in this section the reference aircraft is presented together with its performance data.

There are several aircraft that operate on long haul routes, e.g., B777, B767, A330, A380 [34] however, the choice of aircraft type, largely depends on the availability of the aircraft data. Moreover, as this research aims to compare the MS-FFLIP model against the decentralised model by Verhagen et al. [6], a B777 model is used in order ensure consistency of the comparison. Additionally, using a homogeneous set of aircraft or same type of fleet, reduces any further complexity to the already complex modelling process. Accordingly, the same aircraft type, B777, was used for all models throughout this study.

3.5.1 Aircraft Data: B777

The B777 has the desired performance and payload capability to perform long-haul flight operations. The properties for the B777 aircraft are shown in Table 4.1. The data presented in Table 4.1, is in line with aircraft data used in the work of Verhagen et al. [6]. The design range of $7,500km$ was used and the fuel consumption for this range was estimated using B777

performance manual [35] in order to provide verification for the developed fuel consumption model. Note that a 10% contingency fuel allowance was taken into account when modelling formation flights. This was required since formation flying is not always guaranteed, therefore one has to anticipate the possibility of formation flight and moreover account for the additional detours to be flown.

Table 3.2: B777 aircraft characteristics, Boeing B777 performance manual [35]

Aircraft type	B777
$S [m^2]$	427
AR	8.67
e	0.75
C_{Do}	0.018
$c_t [s^{-1}]$	1.56×10^{-4}
M_{cruise}	0.85
M_{max}	0.88
W_{start}	2.14×10^6 N
ZFTOW	1.67×10^6 N
Design range	7,500 km
Fuel burn on design mission	47,300 kg
Contingency fuel for formation flying	21,830 kg

3.6 Simulation Set-Up

This section presents the simulation set-up for the developed models. The program set-up is outlined, which elaborates on the experimental platform. Then, the details of the Coordinate Reference System is given and followed by the simulation scenario.

3.6.1 Program Set-Up

The MS-FFLIP model and the developed validation model were all implemented in MATLAB. The MILP used in the formation flight assignment is first formulated in MATLAB and optimised using the CPLEX solver. Two main script files for the MS-FFLIP and the validation model were developed while each of the sub-model was represented by a function file and then called onto the main script file. All stages of the simulation were implemented on the same machine: *ASUS Notebook – Windows 7, i7 processor with 6 GB of RAM*

3.6.2 Coordinate Reference System: Cartesian Coordinates

The initial flight data set represents the origin and destinations locations in longitude-latitude coordinates. As a result, the coordinates need to be transformed from their Spherical coordinates to a planar Cartesian coordinate system. For this purpose, the Azimuthal Equidistant [6] transformation method is used to transform Spherical coordinates to a Cartesian coordinate reference system. The Azimuthal Equidistant projection, according to [36], is adequate for great-circle projections and therefore, this projection method is suitable for the purpose of this research study.

The process of the transformation method is as follows: a sphere with radius equal to average radius of the earth is defined. Then a centre-point is defined by the average longitude and average latitude for each of the origin and destination points to be considered. This parameter represents the great circle distance to each of the considered origin and destination points from the predefined centre-point. Subsequently, the azimuth angle is defined between the horizontal plane and the great circle route, also from the centre-point to the origin and destination points. The transformation then projects the origin and destinations points, route lengths and headings onto a Cartesian coordinate system.

3.6.3 Simulated Scenario: Eastbound Transatlantic Flights

Formation flights are envisaged to be more attractive on long-haul flights, where it has the potential to show higher fuel savings [13]. This is also supported by the route similarities, such as flight heading, geographic origins and destination location and also departure times, allowing for maximum time to be spent flying in formation. Several flight markets exist where formation flight could be favourable, as shown in Figure 3.3. However, the air traffic along the transatlantic flight sector holds the highest percentage of global traffic density. A large traffic density is also experienced along Europe-Asia Pacific routes. Nonetheless, the majority of the literature on formation flight operations used flights over the transatlantic region for theoretical experiments. Such transatlantic flights are limited in radar coverage up to just the coastal areas [34] and, therefore a structure of highways in the sky known as the North Atlantic Tracks (NAT) has been assigned by Air Traffic Control (ATC) for the purpose of regular flight operations. The requirements of NAT allow for safe separation, hence this partially explains the reason why most formation flight theoretical studies were conducted within this region. In order to compare the result of the developed model with the current literature, the model will simulate flights over the transatlantic region. The developed model simulates aircraft that fly their great circle routes projected into a flat 2-dimensional space, i.e., they fly in straight lines between their origin and destination at a constant altitude of 9km (FL29). Within the context of this research, all flights depart at equal departure times. The assumption of equal departure times was made in order to demonstrate the efficiency of the developed MS-FFLIP model for the purpose of large-scale combinatorial optimisation.

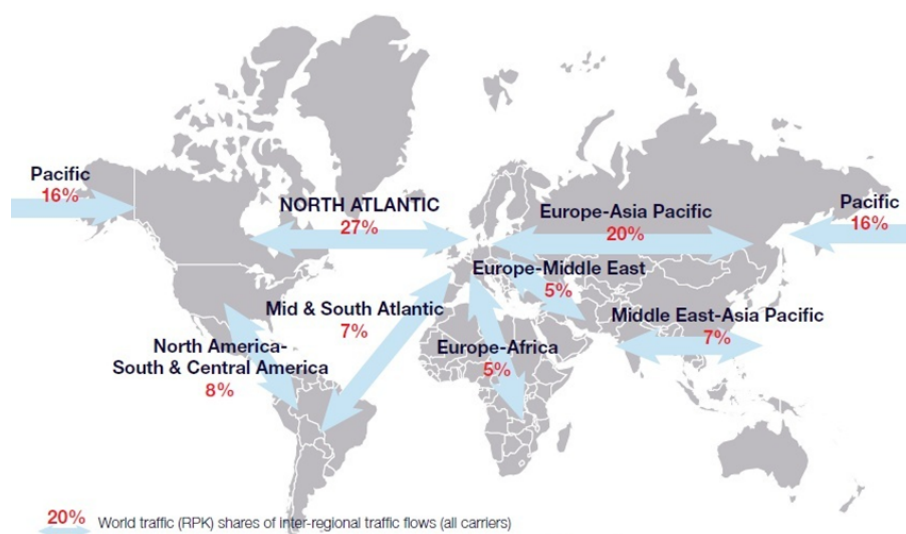


Figure 3.3: World aviation traffic flow [37]

4 Routing Model

This chapter introduces the route optimisation technique used in order to determine the optimal joining and splitting points for all possible aircraft combinations in a formation. Therefore, the objective of this chapter is to answer the following question: *when and where* should a pair of aircraft join and split from their respective formation flight? The route optimisation method used in order to generate the formation flight routes is based on the work of Kent & Richards [25], [26], [5], [27]. The authors used a geometric approach by minimising the weighted distance in order to obtain the optimal formation routes. This geometrical route optimisation approach allows for large scenarios of flights to be analysed faster than the route optimisation method presented by Bower et al [22]. However, Kent & Richards geometrical routing approach is only computationally efficient for two-aircraft size formations and, beyond this formation size the method becomes highly vulnerable to combinatorial explosions due to the NP-hard nature of the problem.

The first section of this chapter outlines the fundamental principles of the geometric approach and moreover, it highlights the combinatorial impact of the problem observed in the single-stage centralised formation flight planning approach or, the validation model of this study. After presenting the core principle of the routing approach, Section 4.2, illustrates the methodology of formation flight routing for larger formation sizes with the aid of examples. Then, in Section 4.3, the fuel consumption model is formulated for the purpose of estimating the fuel cost for the respective formation flight routes. Next, Section 4.4, studies the formation flight efficiency profiles, thus presenting the optimal airspeed to fly in formation. Last but not least, Section 4.5, examines the notion of synchronisation and presents three synchronisation methods for optimal sets of formation flights to rendezvous at the desired joining points at the exact same time.

4.1 The Geometric Routing Approach

This section introduces the geometric routing optimisation approach of Kent & Richards. For the purpose of explaining the route optimisation used in this study, a small scale scenario is used in order to clarify the concept.

Assuming that there is no airspace restriction, two flights, Flight *A* and Flight *B*, with origins *A* and *B*, fly at constant altitude towards a common destination point, *C*. Then, it can be assumed that there exist two solo flight legs extending from *A* and *B* towards a common joining point *P* and, a formation flight leg between point *P* and destination *C*, as depicted in the left side of Figure 4.1. Therefore, the routing problem can be defined by finding the segments *AP*, *BP* and *PC* such that it minimises the total fuel consumed.

The following sub-topics within in this section describe the fundamental principles used to

develop the routing model in this study.

4.1.1 The Notion of Arc Weights

In order to describe the notion of drag reduction benefits which arise from flying in formation, the authors Kent & Richards proposed the concept of arc weights [27]. Hence, the two flights A and B flying towards a joining point P and then to a final destination point C , consist of two solo legs, AP , BP , and one formation flight PC . Then the fuel burnt per unit distance flown on these segments can be represented by the arc weights w_A , w_B and w_C respectively. Since flight legs AP and BP are flown in solo flight, the flights do not experience any drag reductions and therefore, no reduction in the fuel cost. As a result, the respective arc weights (w_A, w_B) for these two solo flight legs are simply equal to 1, assuming that the aircraft type are equal or homogeneous. Subsequently, the two flights merge at point P and begin to fly in formation on segment PC while assuming that there is one flight trailing behind the leader aircraft for which this trailing aircraft experiences a drag reduction. As it is assumed that all trailing aircraft achieve a 10% reduction in fuel burn, the respective arc weight w_C for PC formation segment can be represented as follows:

$$w_C = (w_A + w_B - 1.0) \times \lambda + \mu \quad (4.1)$$

The parameter μ represents the number of leaders in a formation however, for the purpose of this study there is only one aircraft leading the formation, therefore $\mu = 1$. The fuel discount factor λ is equal to 0.9 which represents the assumed 10% reduction in fuel burnt for the trailing aircraft, while 1.0 represents the fuel burnt for the formation leader. Since it is assumed that the formation leader does not enjoy any fuel burn reduction benefit from flying in formation hence, 1.0 represents no savings for the leader.

This approach of arc weightings can be extended for larger formation sizes. As mentioned, for this study, it is assumed that all trailing aircraft irrespective of formation sizes, receive a 10% reduction in fuel flow rate. The reason for making such a conservative assumption is to ensure tractability of the problem for larger formation sizes.

Now that the notion of the arc weights have been explained, it remains to find the optimal location of point P . The authors Kent & Richards used a unique approach by adapting the Fermat point problem in order to find the optimal joining point.

4.1.2 The Fermat Point Problem

The Fermat point problem is a classical mathematical problem developed in the 17th century which states the following theory:

For a given triangle ABC on a 2-dimensional plane there exist a point P for which the sum of the distances $\|\mathbf{AP}\|$, $\|\mathbf{BP}\|$ and $\|\mathbf{CP}\|$ is minimum

The above theory, in principle means, if the arc weights are all equal, then point P is the optimal joining point [27]. The geometrical point P can be calculated by constructing a triangle ABC as shown on the left side of Figure 4.1. Subsequently, circles can be drawn from the outer vertex of each triangle. Then, the point of intersection of all three circles is the desired optimal joining point P as shown in Figure 4.1. Therefore, point P minimises the sum $\|\mathbf{AP}\| + \|\mathbf{BP}\| + \|\mathbf{CP}\|$. This method can be coupled with the concept of arc weights and thus extended to formation

flight routing.

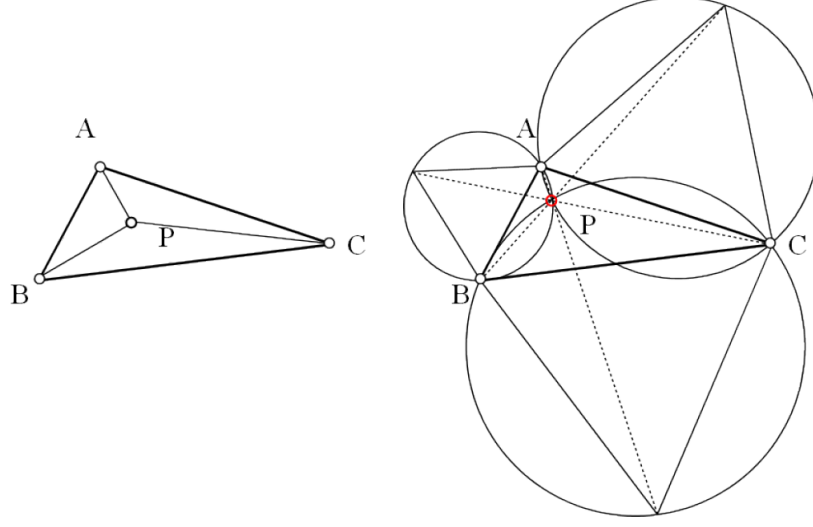


Figure 4.1: The geometric construction to determine the optimal point P [25]

4.1.3 Extending The Fermat Point Problem

The Fermat point problem can be extended to formation flight routing with the concept of arc weights representing the fuel burned per unit distance. The scalar arc weights w_A , w_B , w_C respectively, correspond to the fuel burnt on arc lengths of AP , BP , and PC .

It now remains to find the joining point P , such that the total weighted distance is minimised as shown below:

$$\text{Minimise : } f(P) = w_A \cdot \|\mathbf{AP}\| + w_B \cdot \|\mathbf{BP}\| + w_C \cdot \|\mathbf{CP}\| \quad (4.2)$$

The arc weights are adapted for the triangle ABC in Figure 4.1, giving the following vector equilibrium representation:

$$w_A \cdot \frac{\mathbf{AP}}{\|\mathbf{AP}\|} + w_B \cdot \frac{\mathbf{BP}}{\|\mathbf{BP}\|} + w_C \cdot \frac{\mathbf{CP}}{\|\mathbf{CP}\|} = 0 \quad (4.3)$$

Subsequently, the law of cosines is applied to Equation 4.3 in order to obtain the intersection angles $\angle APB$, $\angle APC$, and $\angle BPC$. The angle $\angle BPC$ represents the angle between the two solo flight legs AP and BP as depicted in Figure 4.2. This angle is an important parameter which is defined as the formation angle, θ_f , represented by Equation 4.4. It can be seen from Equation 4.4 that the magnitude of the angle only depends on the input of the scalar arc weights w_A , w_B , and w_C , therefore it is of paramount importance to ensure the correct arc weights are used in order to obtain an efficient formation flight route [27].

$$\theta_f = \cos^{-1}\left(\frac{-w_A^2 - w_B^2 + w_C^2}{2w_A w_B}\right) \quad (4.4)$$

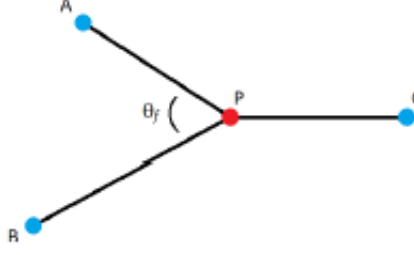


Figure 4.2: Formation angle $\angle BPC$ between AP and BP [25]

The method for finding the optimal joining points can be demonstrated for a scenario of two flights flying from distinct origins A , B , to distinct destination points C and D , as shown in Figure 4.3. Now it remains to locate the optimal joining and splitting points for the formation flight route. At these joining and splitting points, the condition for the formation angle needs to be satisfied, such that these respective points minimise the weighted distance and thus the fuel burnt along the formation segments. Next, the two arcs extending from A to B and C to D are constructed (see Figure 4.4) such that the formation angle remains constant. Subsequently, two equilateral triangles ABX_1 and ABX_2 are constructed using the relation that the sides of the triangles are in proportion to the arc weights, i.e., $w_A : w_B : w_C$ is equivalent to $\|AX_1\| : \|BX_1\| : \|AB\|$ and $\|AX_2\| : \|BX_2\| : \|AB\|$. As the length of AB is known, the remaining sides of the triangle can be found using the above relation. This creates two back vertices X_1 and X_2 as depicted in Figure 4.4. This procedure can be mirrored to the destination points C and D in order to obtain their respective back vertices, Y_1 and Y_2 . The optimal joining and splitting point J and S can then be found by constructing a straight line between the back vertex points X_1 and Y_2 . Therefore, the intersection of the line on arc AB yields the optimal joining point while the intersection of the line on arc CD yields the optimal splitting point as illustrated in Figures 4.4 to 4.6.

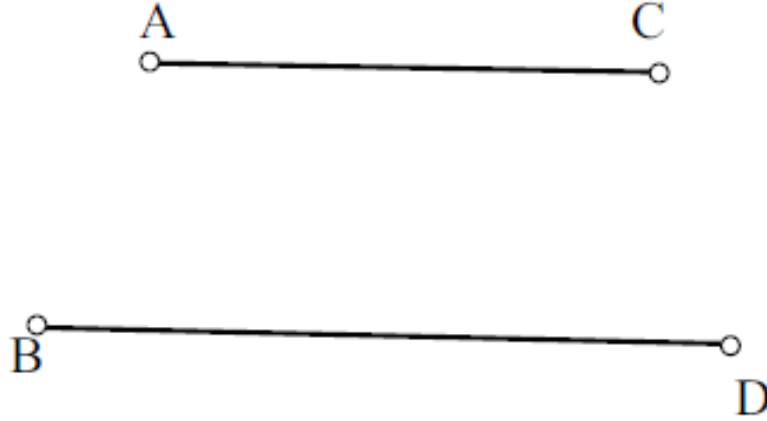


Figure 4.3: Two solo routes from A to C and B to D

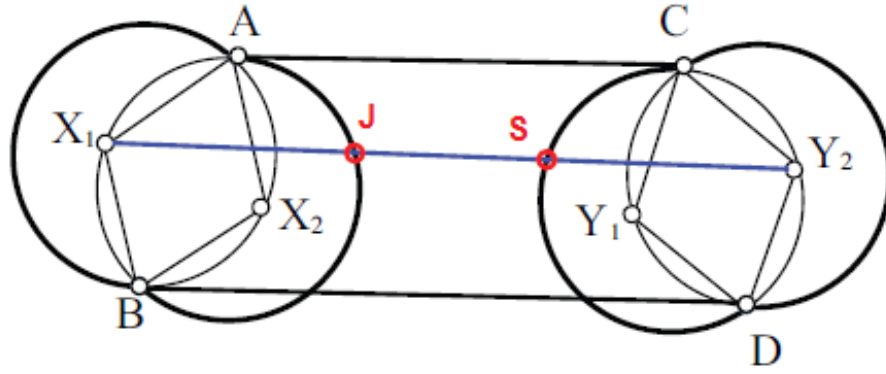


Figure 4.4: Flight trajectory between back vertexes X_1 and Y_2 for the origin and destination location [5]

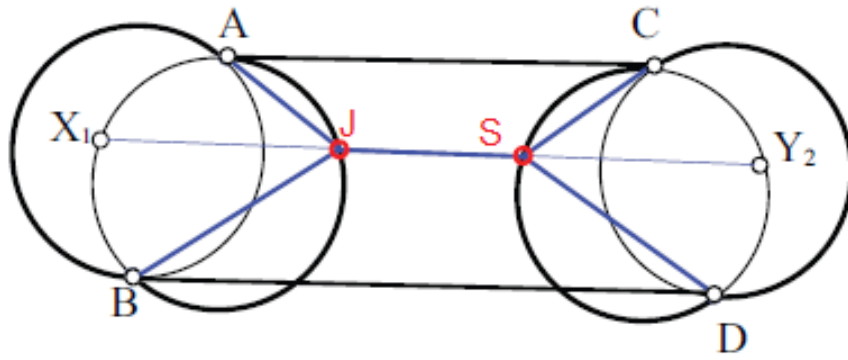


Figure 4.5: Optimal trajectory with intersection points at J and S [5]

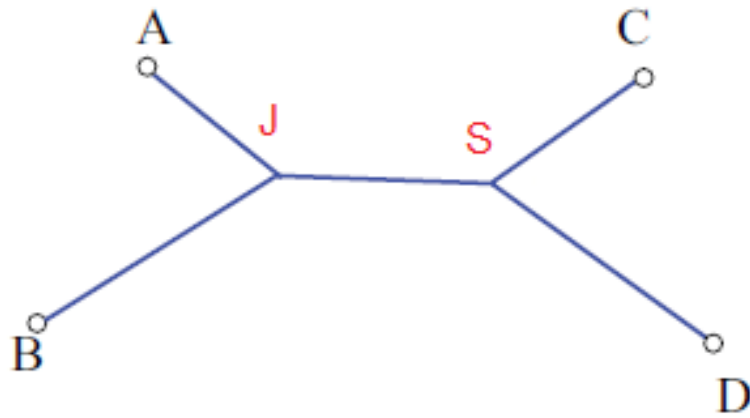


Figure 4.6: Typical optimal formation flight route with 4 solo segments and 1 formation segment

4.1.4 The Influence of Arc Weights on Optimal Formation Flight Assembly

Figure 4.7, demonstrates the influence of the respective arc weights towards the assembly of a formation flight route. When the arc weight is $w_A > w_B$ (green route) compared to $w_A = w_B$ (blue dashed route), it means that the green route is perceived to be more expensive per unit distance than the blue route. As a result, the arc weights aim to reduce the detour in order to conserve fuel and achieve a better potential fuel saving. In other words, the flight travelling on segment BJ are less expensive and therefore the flight will be incentivised to do most of the deviating as illustrated by the longer flight leg of BJ compared to AJ in Figure 4.7. This characteristic allows the routing model to easily adjust the geometry of the formation route in accordance with the number of aircraft in a formation and thereby allowing larger formations to be assembled without creating an inefficient formation flight route.

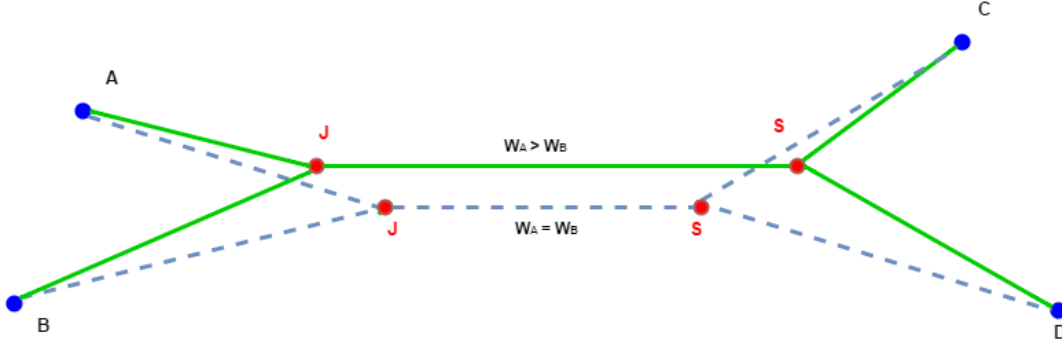


Figure 4.7: Geometry of formation flight route for; $w_A = w_B$ for the blue route and $w_A > w_B$ for the green route

4.1.5 Sub-Optimal Joining and Splitting Point Scenario

It was experimentally observed that the joining and splitting point, J and S were sometimes sub-optimally located, which resulted in an expensive formation route. Although, the formation route would have resulted in overall fuel savings, the total formation distance is not minimised, hence there is always a better solution. This undesirable effect is demonstrated by Figure 4.8, for which it can be seen that flight B will have to change its heading and fly opposite to its originally intended flight path. Therefore, a robust method is required in order to solve this issue.

An algorithm was developed which comprised of two adjustment measures which were used in order to correct for this sub-optimality: first, relocate joining point to one of the origin coordinates such that an aircraft is not assigned to fly in the opposite direction to its original flight path. Second, relocate the splitting point to one of the destination coordinates such that an aircraft does not overshoot its original destination point. Figure 4.9, demonstrates the performed adjustment in order to shift the sub-optimal joining point J to a new joining point J' on location B. Furthermore, flight combinations may depart from the same origin or arrive at the same destination. In such circumstances, the routing algorithm designates the origin to be the joining point, or the destination to be the splitting point. This is an advantageous and a logical approach to formation flying for non-distinct origins or destinations than having to perform sub-optimal flight routes.

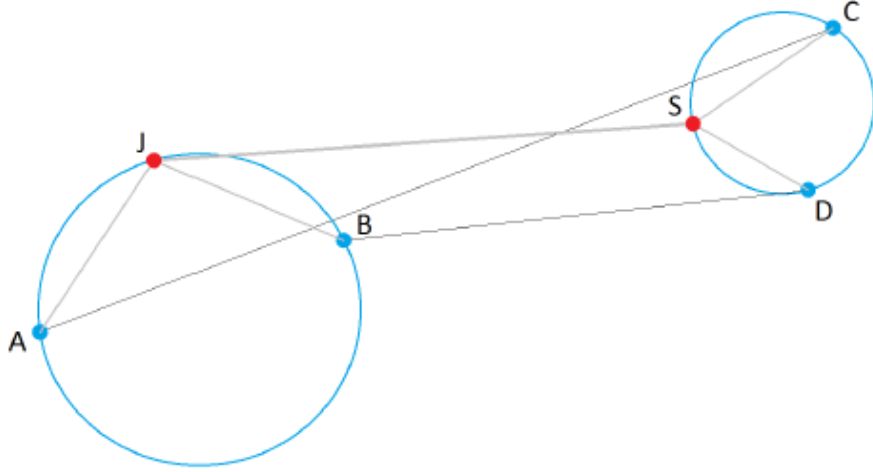


Figure 4.8: Typical sub-optimal formation route for which the joining point is located behind point B

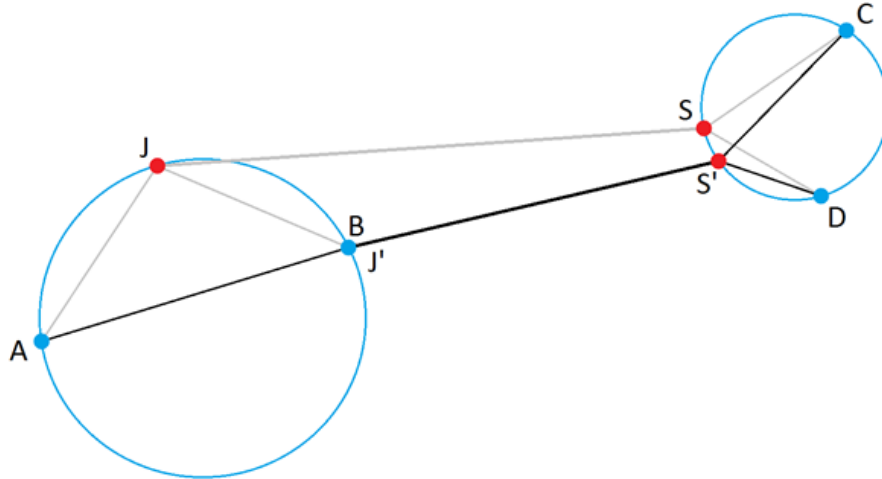


Figure 4.9: Grey: sub-optimal route, black: optimal route, old joining point, J is shifted to point B

4.2 Routing of Large Flight Formations: Single-Stage Centralised Approach

The geometric routing framework presented in Section 4.1, describes the methodology for the assembly of two-aircraft formation size routes. As it was demonstrated, the routing of formation flights depends only on the respective arc weights, therefore this routing method of Kent & Richards can be extended to larger formation sizes. This section presents the methodology for increasing the size of the formations for the (single-stage) centralised validation model.

In the work of Kent & Richards, the number of aircraft in a formation was limited to a three-aircraft formation size. In addition, the flights in their model were limited to the option of a two-aircraft formation size, or a three-aircraft formation size, or to fly solo. In this study, the

(single-stage) centralised model or validation model is extended to a four-aircraft formation size and the flights will have the option of selecting between a solo, two, three and a four-aircraft formation size. The reason for limiting this validation model to four-aircraft formation size is due to the combinatorial explosions, hence causing the enumeration time to diverge to dramatic numbers as explained in Section 3.2. Therefore, this section will highlight the combinatorial explosions even after limiting the size of the formations.

The procedure for creating larger formations is presented for a scenario of flights. Consider five ($N_a = 5$) flights $\{1, 2, 3, 4, 5\}$ (see Figure 4.10) that seek to fly in formation with three-aircraft formation ($n = 3$) and four-aircraft formation sizes ($n = 4$).

4.2.1 Route Optimisation For Three-Aircraft Formation Size

First, the three-aircraft formation size ($n = 3$) needs to be computed. Therefore, using the binomial coefficient Equation 3.1, the number of possible combinations of aircraft in a three-aircraft formation size can be determined as shown below:

The number of combinations for the three-aircraft formation size is:

$$5 \text{ Choose } 3 \equiv \binom{5}{3} = \frac{5!}{3!(5-3)!} = 10 \text{ possible combinations} \quad (4.5)$$

Ergo, there exist 10 possible formation flights, hence the combination sets for a three-aircraft formation size have the following 10 possible formation aircraft combinations: $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 2, 5\}$, $\{1, 3, 4\}$, $\{1, 3, 5\}$, $\{1, 4, 5\}$, $\{2, 3, 4\}$, $\{2, 3, 5\}$, $\{2, 4, 5\}$, $\{3, 4, 5\}$. Based on the order of these combination sets, the formation can be assembled by always starting with the assembly of a two-aircraft formation size. Consider the first combination set $\{1, 2, 3\}$ and begin assembling the two-aircraft formation with flights 1 and 2 taking into account their respective arc weights. Assuming the flights are two equal (homogeneous) aircraft for this first assembly, the arc weights are both equal to 1. Using Equation 4.1, gives the formation arc weight of 1.9, (i.e., $w_C = (1 + 1 - 1) \times 0.9 + 1 = 1.9$). Then, as shown in Figure 4.11, the joining point (J_{12}) for flights 1 and 2 is computed together with its respective projected back vertex, ($X_{\text{flight}12}$). This process is repeated for the destination points for flights 1 and 2 in order to find its respective splitting point (S_{12}) and its respective projected back vertex ($Y_{\text{flight}12}$). Therefore, this creates a so-called pseudo (“superficial”) flight comprising of flights 1 and 2. The back vertex $X_{\text{flight}12}$, becomes the pseudo coordinate for remaining flight 3. Thereafter, the process of finding the joining point for the pseudo flight and flight 3 is performed by the exact same steps as done for computing the joining point of flight 1 and 2 and, the only variable that needs to be updated is the arc weighting. The arc weights for pseudo flight will be equal to $w_A = 2$ and, $w_B = 1$ for flight 3. Using Equation 4.1, the formation arc weight is computed to be $w_C = 2.8$. Hence, the final joining point (J_{123}), for all three flights is then computed and the method is repeated for the destination points of the pseudo flight and flight 3 in order to find their respective splitting point S_{123} as illustrated in Figure 4.12. Thereafter, the optimal formation route for flights 1, 2 and 3 is created as shown in Figure 4.13.

The above optimal formation flight routing method is only demonstrated for the combination flight set of $\{1, 2, 3\}$, which is just one out of the possible 10 formations, the same procedure as presented above has to be conducted for the remaining 9 possible formation combination sets for this three-aircraft formation size. Therefore, one can imagine the amount of computational resource required to enumerate such formation routes for large number of aircraft. After the routing model has found all the respective joining and splitting points and thus formed the

optimal formation route for all combination sets, it still remains to enumerate the optimal formation routes for the four-aircraft formation size.

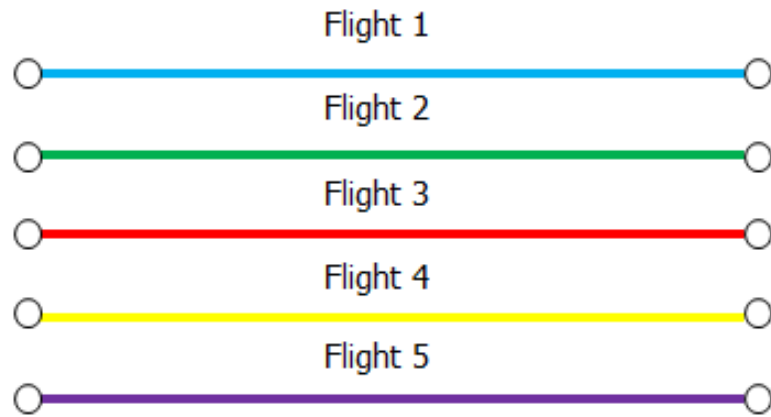


Figure 4.10: Solo flights 1, 2, 3, 4, 5

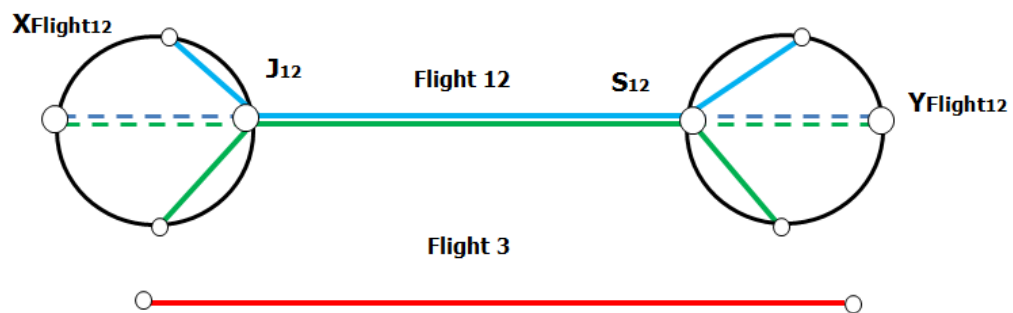


Figure 4.11: Flight 12 and its optimal route



Figure 4.12: Flight 3 added to pseudo flight 12

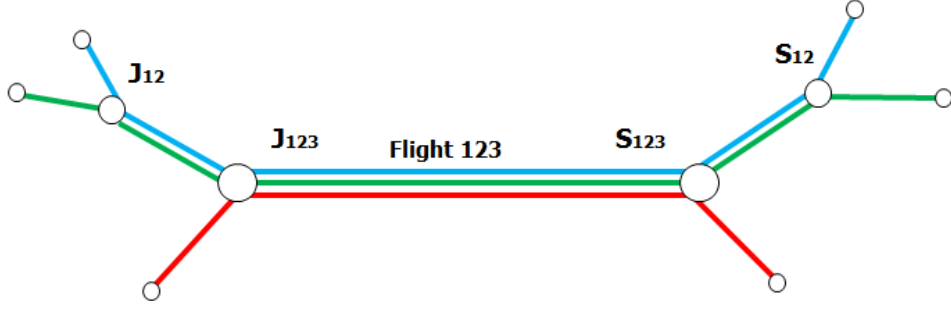


Figure 4.13: Optimal formation route for flight 1, 2 and 3

4.2.2 Route Optimisation For Four-Aircraft Formation Size

After the routing model has completed the route optimisation for the three-aircraft formation size, it begins to assemble the four-aircraft formation size ($n = 4$) optimal routes. First, the routing model assembles the 5 flights ($N_a = 5$) into combination sets comprising of four aircraft per set. Similar to the previous formation size, the binomial coefficient, represented by Equation 3.1 is used in order to find the combination sets for all possible formations for the four-aircraft formation size as shown below:

$$5 \text{ Choose } 4 \equiv \binom{5}{4} = \frac{5!}{4!(5-4)!} = 5 \text{ possible combinations} \quad (4.6)$$

The combinations for these 5 possible formation flights are: $\{1, 2, 3, 4\}$, $\{1, 2, 3, 5\}$, $\{1, 2, 4, 5\}$, $\{1, 3, 4, 5\}$, $\{2, 3, 4, 5\}$. Based on the order of the above combination flight set, the routing model starts to assemble the formation starting with the two-aircraft formation. The route optimisation procedure for the four-aircraft formation size of the first combination is explained in this section.

For the flight combination set $\{1, 2, 3, 4\}$, the routing model assembles the two-aircraft formation size route, taking into account their respective arc weights ($w_A = 1, w_B = 1$), the optimal formation route for the respective flights 1 and 2 are assembled as shown in Figure 4.14. Then, the two-aircraft route assembly process is carried out for the remaining flights 3 and 4, hence forming two pseudo flights 12 and 34 as shown in Figure 4.15. Accordingly, the two pseudo points ($X_{\text{flight12}}, X_{\text{flight34}}$) for the respective pseudo flights, together with the updated weights for flights 12 and 34 i.e., $w_C = (2 + 2 - 1) \times 0.9 + 1 = 2.7$ for the pseudo flights, are used to compute the final joining point for all four flights, i.e., joining point J_{1234} . This process is repeated for the destination of flight 1,2 and 3,4 in order to locate the splitting points as shown in Figure 4.16 and to form the optimal formation route as illustrated by Figure 4.17. This procedure is thereafter performed for the remaining four possible combination sets.

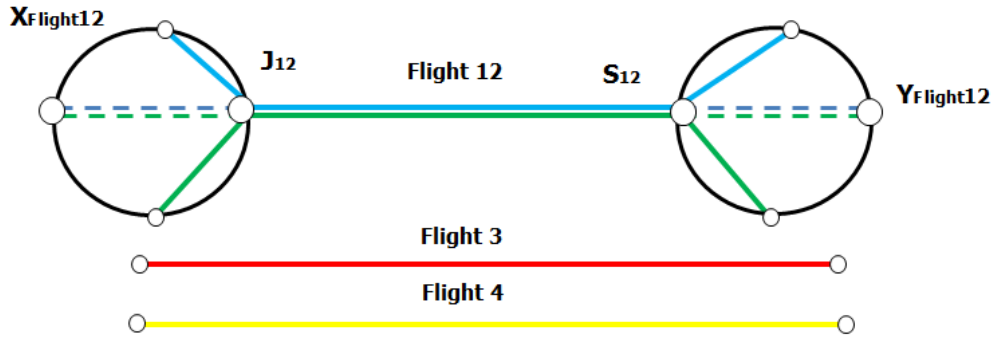


Figure 4.14: Pseudo flight 12 and solo flights 3,4

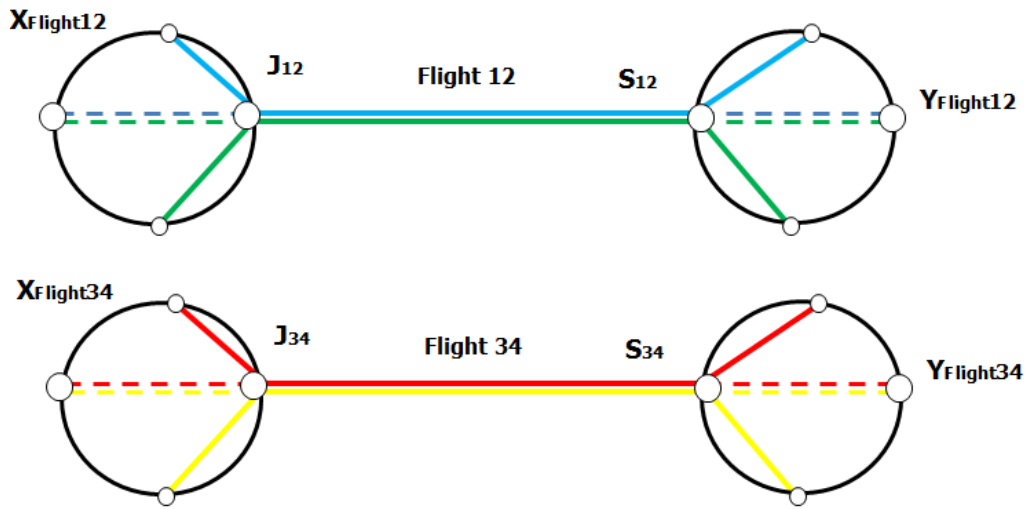


Figure 4.15: Pseudo flight 12 and pseudo flight 34

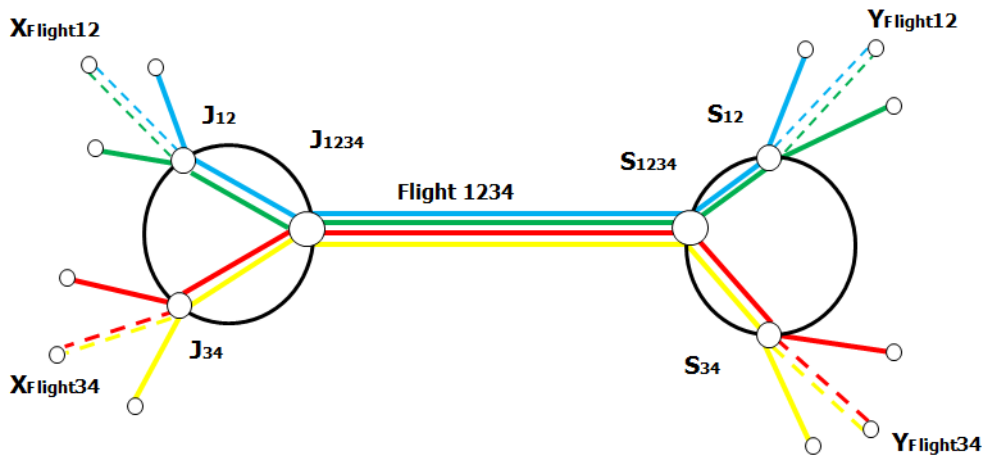


Figure 4.16: Flight 12 added to flight 34

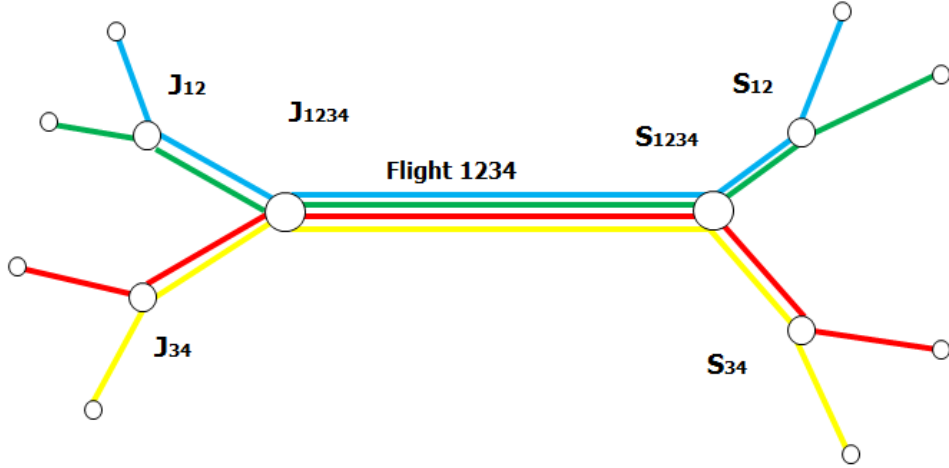


Figure 4.17: Optimal formation flight route for the four-aircraft formation size aircraft 1,2,3,4

The above discussed centralised formation flight routing method of Kent & Richards was presented on a small scale level for five flights and even though it is for a small scale scenario, the routing enumeration scheme is intensive. Moreover, through experimentation it was seen that the routing model consumes the majority of resource time compared to the MILP assignment problem. Therefore, when formation flight routing is to be considered for a larger fleet size, the above method of routing large formation sizes becomes intractable due to the combinatorial explosion of the problem. Thereby, the amount of computational resource requirement diverges dramatically to unrealistic computational time magnitudes. For that reason, the full potential of the centralised formation flight planning could not be realised.

The current centralised approaches cannot be used for en-route formation flight optimisation. Therefore, in order to solve this challenge faced by centralised formation flight planning methods, an innovative approach is required. Chapter 7, will be dedicated towards finding a solution to this problem and it will be seen that the routing approach is the exact same method as the one presented in this section with the only difference being the number of combinations enumerated.

4.3 Fuel Consumption Modelling

The previous section presented the methodology on finding the optimal formation flight routes using a geometric method, however it still remains to approximate the potential fuel consumption for the optimised formation flight routes. Therefore, this section presents the methodology for modelling the fuel consumption of formation flights.

Since this study assumes formation flight to take place at a constant altitude and at a constant airspeed per formation segment, a suitable fuel consumption approximation method is required. Several fuel consumption approximation methods were reported in the current literature, ranging from sophisticated aerodynamic analytical models for a mixed aircraft fleet and to simple Breguet range fuel burn approximation methods for equal aircraft fleet type. It was observed that a large majority of formation flight research work incorporated the Breguet range fuel consumption approximation method. The reason being that the approximation method re-

duces the complexity of the fuel burn modelling process, hence allowing for fast computations. However, the two recent formation flight research studies conducted by Kent & Richards [25], [26], [5], [27] and Verhagen et al. [6] employed a version of the Breguet range method for which their assumptions nullified the use of the particular version. This research study implements a Breguet range fuel burn approximation method for which it complies with the assumptions of constant altitude and constant airspeed. This fuel-burn approximation method has largely remained undetected in the aerospace literature. Through this research study, the fuel burn approximation method, which was originally conceived by Torenbeek [38], is reintroduced into the aerospace literature.

The elements presented in this section constitute to development of the fuel consumption model. Section 4.3.1, introduces the Breguet range fuel burn approximation method. Then, Section 4.3.2, presents the assumptions that govern the fuel burn approximation method. Next, Section 4.3.3, outlines the formulation of the Breguet range equation. Thereafter, Section 4.3.4, briefly explores the alternate forms of the Breguet range equations. Lastly, Section 4.3.5, compares the current fuel burn model against the fuel burn model used by Kent & Richards.

4.3.1 High-Fidelity Breguet Range Fuel Consumption Model

The Breguet range equation is a commonly used method in modelling fuel-burn for aircraft. The version of the Breguet range equation is dependent on the type of assumptions employed. However, all the versions describe a relation between start weight (W_i) and the end weight (W_e) of the aircraft and, the distance flown for a respective segment. The particular version used in this study allows for more realistic (high-fidelity) fuel-burn approximation [38]. In addition, the developed fuel-burn model is able to approximate the fuel consumed per formation flight leg.

As mentioned earlier, the fuel-burn model is a simple model, therefore it is able to compute the aircraft fuel burn for all possible combinations of formation flights and for each and every formation segment in a formation flight. Such a model is critical for such combinatorial optimisation problem as seen in this study. Furthermore, the fuel burn model is also used for the synchronisation of formation flights. As a result, the fuel burn model estimates the amount of fuel burnt when synchronising by varying the airspeed in order to efficiently merge at the respective joining point. This is one benefit for using such a fuel burn model compared to the fuel burn model used by Kent & Richards and Verhagen et al. It was also observed from the current literature that the work of Verhagen et al. employed a speed reduction factor in order to synchronise while flying between maximum and minimum speed, hence burning more fuel. As one of the goals of this thesis is to show the potential in fuel savings for flight formation relative to the greedy optimisation method, the fuel model explored in this study is also used in the greedy formation flight optimisation method of Verhagen et al. This allows to conduct a consistent comparison.

4.3.2 Assumptions

The following assumptions were made in order to derive the Breguet range fuel consumption estimation model used in this study.

- Airspeed (V) is constant throughout the flight
- Constant altitude, therefore the free stream density (ρ) is constant

- Oswald efficiency factor (e) is equal to 0.75 and is constant throughout the flight
- Aspect ratio (AR) is equal to 8.67
- The drag induced factor (K) is constant and is equal to $\frac{1}{\pi A Re}$
- The drag coefficient (C_D) is a function of lift coefficient (C_L) and the zero-lift drag coefficient (C_{D_o}) is equal to 0.018, hence the drag polar is given by $C_D = C_{D_o} + KC_L^2$
- Simulation takes place at steady and straight-and-level flight conditions. Therefore, $T=D$ and $L=W$.
- Cruise altitude is equal to 9km (Flight Level 29). A lower altitude is justified by the desire to have more excess power available for the purpose of manoeuvring to join formations.
- The specific fuel consumption (c_t) is constant throughout the flight
- Trailing aircraft experience a 10% reduction in fuel-burn, irrespective of formation size
- W_i is the aircraft weight at the beginning of a segment and W_e is aircraft weight at the end of the segment

4.3.3 Implemented Breguet range equation

Based on the above stated assumptions, the Breguet range fuel consumption model is derived. The following derivation was verified by [39]. Fuel consumption F for a commercial aircraft is defined as:

$$F = \frac{dW_F}{dt} = -\frac{dW}{dt} = c_t T = c_t D, \quad (4.7)$$

where the aircraft weight is given by W , total fuel load represented by W_F , Specific Fuel Consumption (SFC) c_t and the engine thrust T . The range, R , is defined as an horizontal straight-line flight distance between two points in cruising phase. The range is found by integrating the rate of change of distance as shown below:

$$R = \int_{t_i}^{t_e} V dt \quad (4.8)$$

Combining Equation 4.7 and 4.8 gives Equation 4.9. The equation shows an approximate analytic expression for range where V is the airspeed and t is the time. The subscripts i and e denote the initial (start) and end conditions. Equation 4.9, is integrated over aircraft weight.

$$R = -\frac{V}{c_t} \int_{W_i}^{W_e} \frac{dW}{D} \quad (4.9)$$

Then the total drag in Equation 4.9, can be substituted with $D = C_D \frac{1}{2} \rho V^2 S$ where C_D can be further substituted with the parabolic drag polar $C_D = C_{D_o} + KC_L^2$ in order to obtain Equation 4.10.

$$R = \frac{V}{c_t} \left(\frac{1}{qSC_{D_o}} \int_{W_i}^{W_e} \frac{-dW}{1 + a^2 W^2} \right) \quad (4.10)$$

Subsequently, to perform the integration of Equation 4.10 analytical, the equation is expressed as a standard integral of $\tan^{-1}$. Therefore, the terms within the integral can be replaced with Equation 4.11, as shown below for which q represents the dynamic pressure, S the wing surface area of the B777 aircraft and induced drag factor K .

$$a^2 = \frac{K}{q^2 S^2 C_{Do}} \quad (4.11)$$

Using the standard integral represented by Equation 4.12 [40], Equation 4.10 can be easily integrated to give Equation 4.13.

$$\frac{1}{a} \tan^{-1} \frac{x}{a} = \int \frac{dx}{a^2 + x^2} \quad (4.12)$$

$$R = \frac{2V(\frac{L}{D})_{max}}{c_t} \left(\tan^{-1}(2KC_{Li}(\frac{L}{D})_{max}) - \tan^{-1}(2KC_{Li}(\frac{L}{D})_{max} \frac{W_e}{W_i}) \right) \quad (4.13)$$

Now that Equation 4.10, has been integrated to give the range for a particular flight segment, it still needs to be rearranged such that the equation is represented by the weight at the end of the segment, W_e . The above equation can be rearranged using the standard trigonometry rule given by Equation 4.14 to find the final weight at the end of the segment.

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan(A)\tan(B)} \quad (4.14)$$

Equation 4.15, shows the final weight at the end of the segment flown.

$$W_e = W_i \frac{2KC_{Li}(\frac{L}{D})_{max} - \tan(\Psi)}{(2KC_{Li}(\frac{L}{D})_{max})^2 \tan(\Psi) + 2KC_{Li}(\frac{L}{D})_{max}} \quad (4.15)$$

The maximum lift to drag ratio $(L/D)_{max}$ and the lift coefficient C_{Li} are shown below. Furthermore, Ψ in Equation 4.15 is given by Equation 4.18, which is a function of SFC (c_t), range (R), airspeed (V) and maximum lift to drag ratio $(L/D)_{max}$.

$$(\frac{L}{D})_{max} = \sqrt{\frac{1}{4C_{Do}K}} \quad (4.16)$$

$$C_{Li} = \frac{W_i}{0.5\rho V^2 S} \quad (4.17)$$

$$\Psi = \frac{c_t R}{2V(\frac{L}{D})_{max}} \quad (4.18)$$

It is noteworthy to mention that the maximum lift to drag ratio remains constant throughout the flight. However, trailing aircraft are assumed a 10% reduction in fuel burn. In order for trailing aircraft to experience a 10% reduction in fuel burn, a numerical analysis was conducted to determine the magnitude of the maximum lift to drag ratio that would yield a 10% reduction in fuel burn. Accordingly, it was found that the maximum lift to drag ratio of trailing aircraft would require a factor of increase of 1.07373. This numerical factor successfully models the 10% decrease in fuel burn for trailing aircraft in formation flight. Moreover, this increase to

the maximum lift to drag ratio is in accordance with the decrease in drag experienced (which decreases the fuel flow) by trailing aircraft due to the local upwash generated by the leading aircraft.

Fuel burnt per flight segment can then be computed using, $W_i - W_e$, as illustrated by Equation 4.19. Since, it is assumed that airspeed and altitude are constant, it can be seen that as the aircraft burns fuel, its weight subsequently decreases. As a result, the lift coefficient will subsequently decrease. Additionally, the drag slightly decreases (induced drag decreases but zero-lift drag remains constant). This situation can be demonstrated by looking at the lift equation $L = W = qSC_L$, now as W decreases C_L will also decrease since the remaining parameters are constant.

$$\text{Fuel Burned Per Segment} = W_i - W_e = W_i \left(\frac{(2KC_{Li}(\frac{L}{D})_{max})^2 \tan(\Psi) + \tan(\Psi)}{(2KC_{Li}(\frac{L}{D})_{max})^2 \tan(\Psi) + 2KC_{Li}(\frac{L}{D})_{max}} \right) \quad (4.19)$$

4.3.4 Alternative Fuel Consumption Model

As previously mentioned, the current literature study reported various aircraft fuel burn models, ranging from simple to complex models. The most commonly used fuel burn model was the particular Breguet range equation version used in the work of Kent & Richards and also in Verhagen et al [27], [6]. The authors used a version that assumes constant altitude and constant angle of attack, expressed by Equation 4.20.

$$W_e = \left(\sqrt{W_i} - \frac{Rc_t}{2} \sqrt{\frac{\rho S}{C_L^{1/2}/C_D}} \frac{1}{C_L^{1/2}/C_D} \right)^2 \quad (4.20)$$

The $C_L^{1/2}/C_D$ ratio in Equation 4.20, can be represented by Equation 4.21 [41], [42].

$$\left(\frac{C_L^{1/2}}{C_D} \right)_{max} = \frac{3}{4} \left(\frac{1}{3KC_{Do}^3} \right)^{1/4} \quad (4.21)$$

The optimal airspeed is given by Equation 4.22. It can be observed that the airspeed varies with the weight of the aircraft, however the authors Kent & Richards and Verhagen et al. assumed a constant airspeed, hence violating the underlining assumptions used to derive Equation 4.20.

$$V = \sqrt{\frac{2W}{\rho SC_L}} \quad (4.22)$$

For the purpose of completeness, the alternate form of the Breguet range equation given by Equation 4.23, is also shown, which can be used for fuel burn modelling. The assumptions used to derive Equation 4.23 are constant angle of attack and constant airspeed.

$$W_e = W_i \times \exp\left(\frac{-c_t R}{V C_L / C_D}\right) \quad (4.23)$$

4.3.5 Comparison of Fuel Consumption Models

The two fuel consumption models represented by Equation 4.20 and 4.19, were used to compute the fuel burn on a Boeing 777 aircraft for the purpose of comparison of the two fuel burn results. The initial aircraft weight for formation flight is set at 2.35×10^6 [N]. The aircraft are flown on a range distance of 6,627km, between New York and Rome. For the purpose of this experiment, the aircraft is flown at Mach 0.85 (M_{cruise}) and at 9km altitude. Furthermore, the trailing aircraft in formation were assumed to experience a 10% reduction in fuel burn. The difference in the fuel burn (Δ_f) results between the two versions of the fuel models can be seen in Table 4.1. Version I represents the fuel model derived by Equation 4.20, while version II is represented by Equation 4.19. As predicted, the reintroduced fuel consumption model consumes relatively more fuel compared to the conventional fuel burn model given by Equation 4.20 and as already observed by Torenbeek, the version II model is a more realistic fuel burn model [38].

Table 4.1: Fuel burn for different Breguet range fuel model versions for two flights from JFK-ROM

Fuel Consumption Estimation	Version I	Version II	Δ_f
Solo	122,156 kg	123,266 kg	1,110 kg
Formation	116,086 kg	117,103 kg	1,017 kg

4.4 Formation Flight Efficiency

This section aims to answer the question, how fast a formation aircraft should fly? This is a critical topic to address since airlines look towards striving for flight efficiency as a means to widen their profit margins. Therefore, airlines pay close attention to achieving optimal flight profiles, i.e., operating minimal cost flight profiles, hence optimal horizontal, vertical and speed profiles. In the work of Visser, flight efficiency is defined as the difference between the optimal flight profile and actual flight profile or, to define it in monetary terms, the ratio between the theoretical operating cost and the actual cost [43]. Therefore, to ensure that formation flight is operational and sustainable, addressing the flight efficiency is imperative. This section presents the optimal airspeed to fly and the airspeed limitation for aircraft in formation.

4.4.1 Optimal Airspeed For Re-introduced Breguet Range Equation

The optimal airspeed is achieved by flying at the maximum specific range speed. Specific range is the distance flown by burning one unit of fuel mass [42]. The optimal airspeed can be derived by partially differentiating the specific range with respect to airspeed and then equating the term to zero. At this airspeed, for a given starting weight and amount of fuel, the aircraft achieves its maximum range as it burns the least amount of fuel and, hence it is a critical aircraft performance parameter. In this section the optimal airspeed is derived based on the work of Filippone [42].

The specific range (R_s) can be computed using Equation 4.24, where its dependent on the airspeed and fuel burn.

$$R_s = \frac{V}{F} = \frac{V}{c_t D} \quad (4.24)$$

Then substitute $D = C_D \frac{1}{2} \rho V^2 S$ and drag coefficient $C_D = C_{D_o} + K C_L^2$ into Equation 4.24. The lift coefficient C_L can be replaced by $C_L = \frac{W}{0.5 \rho V^2 S}$ which results in Equation 4.25.

$$R_s = \frac{V}{c_t (C_{D_o} + K \left(\frac{W}{0.5 \rho V^2 S} \right)^2) 0.5 \rho V^2 S} \quad (4.25)$$

To find the airspeed for which the specific range is maximum, the partial derivative of Equation 4.25 is taken with respect to the airspeed V .

The partial derivative $\frac{\partial R_s}{\partial V}$, is equated to zero in order to find the optimal airspeed.

$$\frac{\partial R_s}{\partial V} = 0 \quad (4.26)$$

Then, rearranging the partial derivative and equating it to zero, gives Equation 4.27.

$$V_{\text{optimal}} = \sqrt{\frac{2W}{\rho S}} \left(\frac{3K}{C_{D_o}} \right)^{1/4} \quad (4.27)$$

It can be seen that the optimal airspeed is dependent on the wing loading and the flight altitude. Moreover, the weight, W , in the Equation 4.27 can be represented by the segment initial weight W_i for out-of-formation flights. The optimal airspeed described by Equation 4.27 is used in Equation 4.18 in order to compute the fuel burn.

Alternately, the above optimal airspeed could be derived by finding the maximum range solution in Equation 4.21 and substituting the resulting coefficients, $C_L = \sqrt{\frac{C_{D_o}}{3K}}$ and $C_D = \frac{4}{3} C_{D_o}$, for maximum range, into Equation 4.22.

Although, this optimal speed leads to an efficient flight, there are limits to its use in flight formation and even in normal flight due to Air Traffic Control restrictions and limitations.

4.4.2 Speed Limitations of Formation Flight

According to the work of Ning et al., [14], [15], [10], formation aircraft are required to fly at a 2.5% lower Mach number than the cruise Mach number. This relatively lower Mach number mitigates compressibility effects which may impede aircraft safety, resulting in stalling the wing of the aircraft due to the flow separation caused by vortices generated by the leading aircraft. As a result, aircraft flying in formations need to fly at an airspeed not higher than Mach 0.82 (2.5% lower than M_{cruise}).

The inequality constraint given by Equation 4.28, represents the airspeed range for aircraft flying in formations need to strictly abide. Airspeed, V_{ff} , is defined as the formation flight airspeed and $V_{\text{max formation flight}}$ is the assigned speed which is related to the Mach number for flight formation that is allowed and is equal to Mach 0.82.

$$V_{\text{max endurance}} < V_{ff} \leq V_{\text{max formation flight}} \quad (4.28)$$

The maximum endurance speed or the speed at which drag is minimum, $V_{\text{max endurance}}$, is calculated using the relation given in the research presented in [44] and supported by [45] [19]:

$$V_{\text{max endurance}} = \frac{V_{\text{optimal}}}{3^{1/2}} \quad (4.29)$$

The optimal cruise speed was given by Equation 4.27. The rationale for choosing such an airspeed is logical to the actual representation in reality, which relates to the strict assertion by ATC on optimal flight profiles. Although, flying at sub-optimal airspeed burns more fuel, it is necessary to do so in order to abide by the ATC rules in congested airspace.

The literature indicated significant inefficiency of formation flight which was contributed by synchronisation of two aircraft when elected to join into formation. This research aims to minimise the inefficiencies sustained by synchronisation. The next section examines these matters further.

4.5 Synchronisation of Formation Flights

After addressing the question of where to fly in formation? It still remains to address the issue of when and how fast should a combination set of aircraft fly in order to enable them to fuse into a formation at their respective joining point at the same time. To address the merging of set of aircraft at their respective joining points, synchronisation is required. The important variable in the synchronisation process is the time lag between the two rendezvous flights. The time lag is defined as the time difference between the aircraft flying flight leg AJ and the aircraft flying the flight leg BJ , arriving at the joining point J , as illustrated in Equation 4.30.

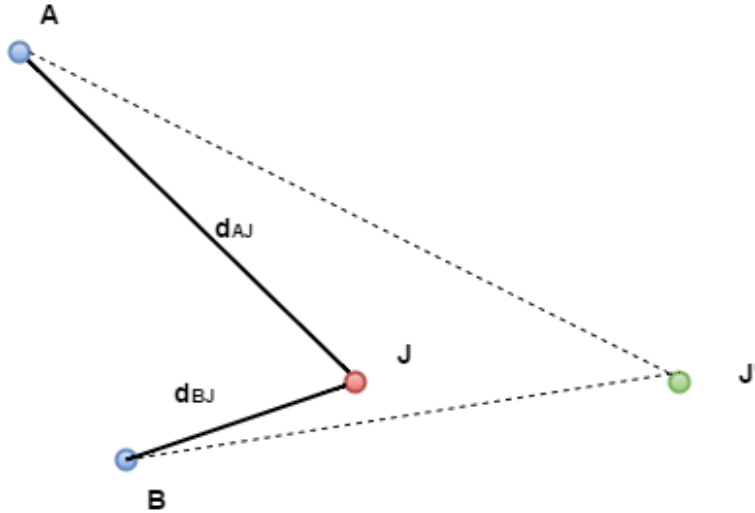


Figure 4.18: Synchronisation process

$$\text{Time lag} = \left| \frac{d_{AJ}}{V_{AJ}} - \frac{d_{BJ}}{V_{BJ}} \right| \quad (4.30)$$

$$V_{\text{max endurance}} \leq V_{AJ} \leq V_{\text{optimal}} \quad (4.31)$$

$$V_{\text{max endurance}} \leq V_{BJ} \leq V_{\text{optimal}} \quad (4.32)$$

The synchronisation mechanism developed for the purpose of this study endeavours to reduce the time lag to zero. As can be seen in Equation 4.30, there are just two variables that dictate

the magnitude of the time lag, i.e., airspeed on AJ and BJ and length of the flight legs AJ and BJ .

The following synchronisation mechanisms presented in this section address the ways in manipulating the two variables such that synchronisation is achieved. This section presents three synchronisation strategies: linear-hold, extending or shifting the joining and a flight detour strategy. The routing model contains algorithms for all three strategies, however the model aims for synchronisation using linear hold since it is comparatively more efficient than the remaining two strategies [43]. If linear-hold does not manifest an efficient synchronisation for a respective formation flight, then the model will opt for the synchronisation by way of extending the joining point. If this yields an inefficient synchronisation then, the model elects the flight path detour algorithm as its last resort for synchronising flights.

4.5.1 Linear-Hold

Linear-hold is a speed holding strategy that is able to accommodate the synchronisation of two flights arriving at their respective joining points. This strategy is performed by absorbing the respective time lags or time delay of the two flights arriving at the respective joining point. The absorption of the time lag is conducted by reducing the flight speed of the aircraft flying the shortest leg such that synchronisation is achieved at the desired joining point. As mentioned in the previous section, depending on the size of the formation, the flight speed is bounded by the maximum specific range speed (optimal speed) and the maximum endurance speed. This means that the linear-hold will have to obey these speed limits and, thus aim to achieve an efficient synchronisation.

The linear-hold process can be demonstrated for a scenario of two flights aiming to synchronise at a desired joining point. Using the aircraft data presented in Section 3.5 and together with Equations 4.27 and 4.29, the optimal airspeed (maximum specific range speed) and maximum endurance airspeed can be computed for a constant cruising altitude of 9km with an initial aircraft weight of 2.35×10^6 [N] (includes contingency fuel). This gives an optimal airspeed of $259m/s$ and maximum endurance speed of $149m/s$. These speeds are illustrated in Figure 4.19, which it presents the rate of fuel burn for a range of flight speeds. Figure 4.19, demonstrates a scenario in which linear hold sustains an efficient synchronisation process for two respective aircraft flying on leg $AJ = 850km$ and $BJ = 700km$. Since the two curves overlap in the feasible (unshaded) region, which is the allowable airspeed region for which the desired speed range limits (refer to Equations 4.31 and 4.32) for both flights are satisfied. Accordingly, the aircraft flying on flight leg BJ will have to reduce speed from its optimal speed of $259m/s$ to $212m/s$ (blue dot) in order to merge with the flight on AJ at a synchronisation time of 55 minutes. Moreover, for this particular scenario it can be seen from Figure 4.19, that only one flight sacrifices its optimal speed while the other aircraft (flying on leg AJ) remains flying at optimal airspeed (orange dot). Therefore, this synchronisation method is comparatively more efficient than the remaining two methods, and hence the reason why this synchronisation option is the first choice. However, the linear hold method does not always accommodate synchronisation of a pair of flights.

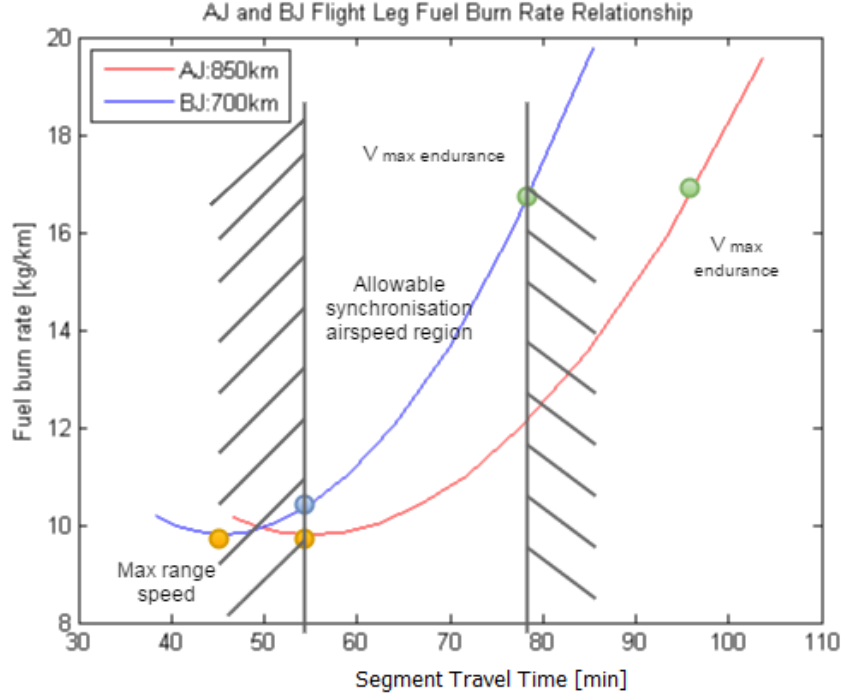


Figure 4.19: Fuel-burn rate vs segment travel time for two aircraft flying on AJ and BJ: B777, $W = 2.35 \times 10^6$ [N], $h = 9km$, Unshaded region indicates the allowable synchronisation range of speeds

Figure 4.20, demonstrates a scenario for which the linear-hold strategy fails to achieve an efficient synchronisation for the two respective flights on legs $AJ = 1,741km$ and $BJ = 746km$ towards a desired joining point. By inspecting Figure 4.20, it can be clearly seen that the two curves for the aircraft on flight legs on AJ and BJ do not overlap at the allowable unshaded regions. In other words, they do not satisfy the conditions presented by Equations 4.31 and 4.32 and violating these condition would create an inefficient synchronisation process or may impede the safety of flight. As a result of this inconvenience to accommodate the rendezvous flights, the routing model then opts for the extending joining point algorithm or the flight path detour in order to expedite the formation flight.

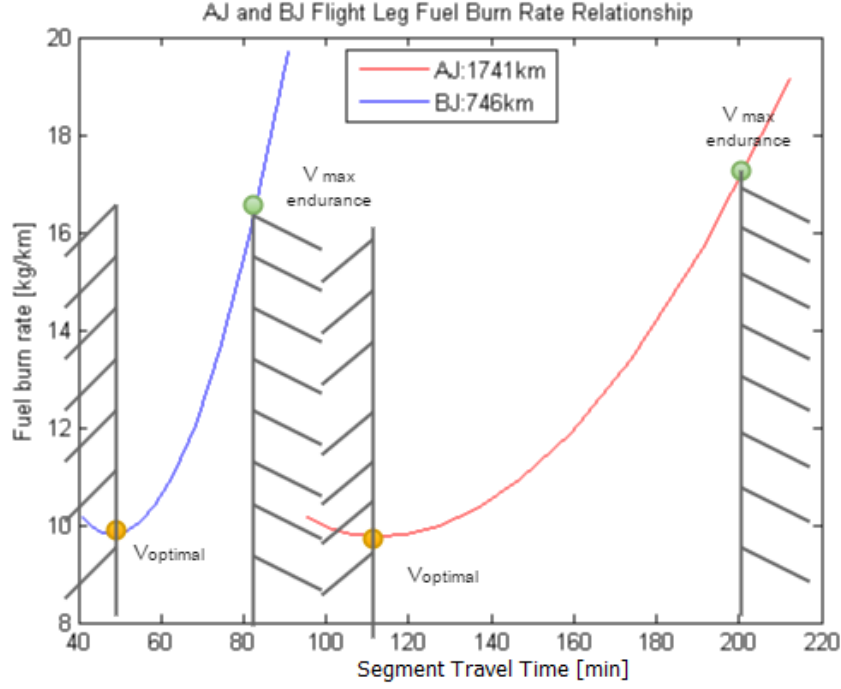


Figure 4.20: Fuel-burn rate vs time for two aircraft flying on AJ and BJ: B777, $W = 2.35 \times 10^6$ [N], $h = 9km$, Unshaded region indicates the allowable synchronisation airspeed

4.5.2 Extended Joining Point

The extended joining point method was developed in order to enable synchronisation of two flights that are unable to rendezvous using the linear hold method. Therefore, this synchronisation strategy is the second option to accommodate synchronisation of the combination set of flights. The following method re-locates the joining point towards a position closer to the splitting point as demonstrated by Figure 4.21.

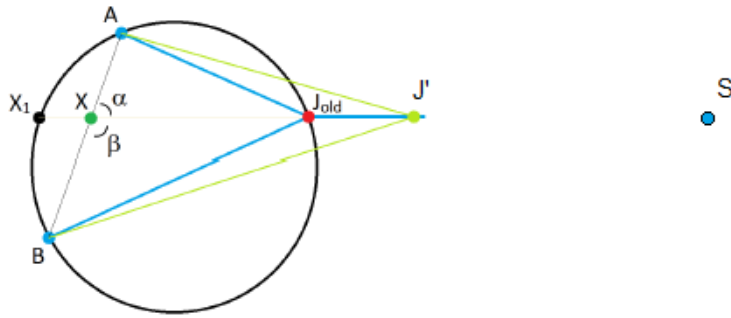


Figure 4.21: Re-location of the joining point to accommodate synchronisation of aircraft

This re-location of the joining point is conducted such that the time lag represented by Equation 4.30, is brought to zero. The segment length for AJ and BJ depend on the selected airspeed within its set ranges as presented by Equation 4.31 and 4.32.

The method developed, uses the law of cosines to arrive at an elegant analytically expression that constructs the new joining point as shown in Figure 4.21. Figure 4.21, shows the pseudo coordinate, X_1 , which is defined as the intersection point between the line extending out of the joining point and the line connecting the origins, as explained in Section 4.1.3. Once this is drawn as, illustrated in Figure 4.21, the triangles AXJ' and BXJ' can be constructed. Note, the only unknowns are: J' , α and β . The law of cosines can be used to find all the unknowns. Equations 4.33 and 4.34 represent the unknown angles.

$$\cos\alpha = \frac{(XA)^2 + (XJ_{old})^2 - (AJ_{old})^2}{2XAJ_{old}} \quad (4.33)$$

$$\cos\beta = \frac{(XB)^2 + (XJ_{old})^2 - (BJ_{old})^2}{2XBJ_{old}} \quad (4.34)$$

For triangle AXJ' , where J' is the new joining point, an expression given by Equation 4.35 is presented for the length of AJ' using the cosine rule.

$$(AJ')^2 = (XJ')^2 + (XA)^2 - 2XJ'XA \cdot \cos\alpha \quad (4.35)$$

Similarly, for triangle BXJ' , an expression given by Equation 4.36 is obtained as follows:

$$(BJ')^2 = (XJ')^2 + (XB)^2 - 2XJ'XB \cdot \cos\beta \quad (4.36)$$

For synchronisation to take place, both flights travelling on flight legs AJ and BJ have to arrive at the joining point at the same exact moment in time. For this purpose, the expression of time can be imposed using Equation 4.37.

$$t_{AJ'} = t_{BJ'} \quad (4.37)$$

Then the time expression given by Equation 4.37, can be related to distance of the flight leg and its respective airspeed flown by the aircraft on the flight leg. Subsequently, giving the expression represented by Equation 4.38.

$$\frac{AJ'}{V_{AJ}} = \frac{BJ'}{V_{BJ}} \quad (4.38)$$

Then, substituting the relation $BJ' = \frac{AJ'}{V_{AJ}} \cdot V_{BJ}$ into Equation 4.36, gives the expression represented by Equation 4.39 where the speed ratio, γ , is the ratio between the speed of the aircraft flying on AJ and the speed of the aircraft flying the BJ flight leg ($\gamma = V_{BJ}/V_{AJ}$).

$$(AJ')^2 \cdot \gamma^2 = (XJ')^2 + (XB)^2 - 2XJ'XB \cdot \cos\beta \quad (4.39)$$

Then Equation 4.35 can be inserted into Equation 4.39 which yields:

$$\left((XJ')^2 + (XA)^2 - 2XJ'XA \cdot \cos\alpha \right) \cdot \gamma^2 = (XJ')^2 + (XB)^2 - 2XJ'XB \cdot \cos\beta \quad (4.40)$$

Therefore, rearranging Equation 4.40 to give the expression represented by Equation 4.41:

$$(XJ')^2 \cdot \gamma^2 + (XA)^2 \cdot \gamma^2 - 2XJ'XA \cdot \cos\alpha \cdot \gamma^2 - (XJ')^2 - (XB)^2 + 2XJ'XB \cdot \cos\beta = 0 \quad (4.41)$$

The quadratic Equation 4.41 can finally be rearranged to give the expression;

$$(\gamma^2 - 1) \cdot (XJ')^2 + (XA)^2 \cdot \gamma^2 - (XB)^2 - 2XJ' \left(XA \cdot \cos\alpha \cdot \gamma^2 - XB \cdot \cos\beta \right) = 0 \quad (4.42)$$

For this particular case where $AJ_{old} < BJ_{old}$, the speed V_{AJ} is the $V_{\max \text{ endurance}}$ and speed V_{BJ} given by V_{optimal} can then be substituted into the quadratic expression of Equation 4.42. The quadratic expression given by Equation 4.42 is then used to solve for XJ' and since the position of X is known, the position of the new joining point can be computed by choosing the largest value of side XJ' . However, since the re-location of the joining depends on the length of legs AJ and BJ , it may not always be possible to efficiently re-locate the joining point. This is related to the fact that Equation 4.42 may not always produce feasible roots. For such a scenario, the routing model will select the flight path detour algorithm as a final measure to achieve a relatively efficient synchronisation.

4.5.3 Flight Path Detour

Extending of the joining point is not always a viable flight synchronisation strategy. An analysis revealed that for certain cases, the new joining point, J' , was re-located to a position beyond the splitting point or to a position before the origins (projected backwards) of the departing flights as shown in Figure 4.22. This is the inherent nature of the extended joining point algorithm. The algorithm positions the new joining point such that it only satisfies the time lag relation given by Equation 4.30, therefore it does not yield a minimum distance path and hence, it diminishes the potential fuel savings.

As the aim is to maximise formation flying, an alternative approach to prevent such an inefficient formation flight strategy was needed. Therefore, a flight path detour method is established for the purpose of synchronisation for the latter scenario. Accordingly, when the extending joining point algorithm fails to locate an optimal new joining point, the aircraft which requires to absorb the time lag is forced to fly a detour at maximum endurance speed for the flight distance represented by the time lag. This means that the detour distance will be:

$$\text{Detour Distance} = \text{Time Lag} \times V_{\max \text{ endurance}} \quad (4.43)$$

As illustrated in Figure 4.22, the flight path detour method simply elongates flight leg d_{BJ} and, hence the position of the original joining point, J , remains at its original location.

As mentioned earlier, this flight path detour synchronisation strategy is a last resort measure in which the linear hold and extended joining point strategies have failed to concretize an efficient synchronisation process.

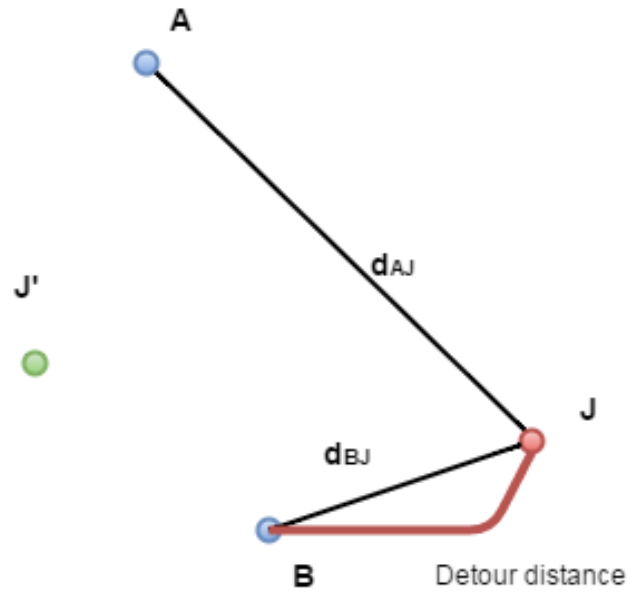


Figure 4.22: Flight path detour illustration for synchronisation

The elements presented here have successfully addressed the question posed in the introductory paragraph of this chapter.

5 Linear Program Formulation of Assignment Problem

After obtaining the optimal routes and the costs for all possible formation flight pairings, the following chapter elaborates on the methodology needed to optimally group aircraft in formations, i.e., it aims to answer the question of whom to fly in formation? Hence, this chapter gives a detailed description of all the components needed to solve this assignment problem. The chapter outlines the manner in which fuel cost estimates are incorporated into the optimisation problem by way of using the fuel cost as coefficients in the objective function. The goal of this research is to minimise fuel burn and, hence the objective function has been formulated such that it is optimised for global minimum fuel consumption while satisfying the defined constraints. The developed optimisation model implements a binary application of Mixed Integer Linear Program in order to minimise the fuel consumption. This MILP formulation was inspired by the work of Kent & Richards [27], [25], [26], [5]. However, the objective function and the constraints were extended in order to attain the goal for this study. Furthermore, the developed model uses the CPLEX optimiser as an external solver in order to optimise the MILP results. The sections in this chapter further explain the MILP and the formulation of the objective function, its constraints and the employed optimiser.

5.1 Mixed-Integer Linear Programming

Linear Programming (LP) is a class of mathematics that uses a mathematical model to describe a problem which requires optimisation [46]. Fundamentally, it aims to allocate scarce limited resources amongst competing activities in an optimal way. The word *linear* refers to the mathematical functions in the formulated problem which are required to be linear. Moreover, the word *programming* is a synonym for *planning*. Thus, LP, in principle deals with the *planning of linear activities* to obtain results which are profitable among other possible alternatives. LP is also a basic tool in the field of OR. In addition, the applications of LP are not restricted to the transportation domain. The use of LP applications are widely seen in many applications in finance and economics. However, linear programming is commonly seen in planning and assignment problems, due to its proven success. A single objective function is optimised in an LP problem while subjected to equality and/or inequality constraints. Given a feasible region within which to locate the optimum point, the LP strives to explore intersection points on the boundaries of the feasible region. The solutions obtained within the feasible region are known to be a feasible solution and if a solution exist beyond the feasible region then it is an indication of constraint violation, hence it is defined an infeasible solution. The general framework for the LP of the formation flight assignment is represented in Figure 5.1.

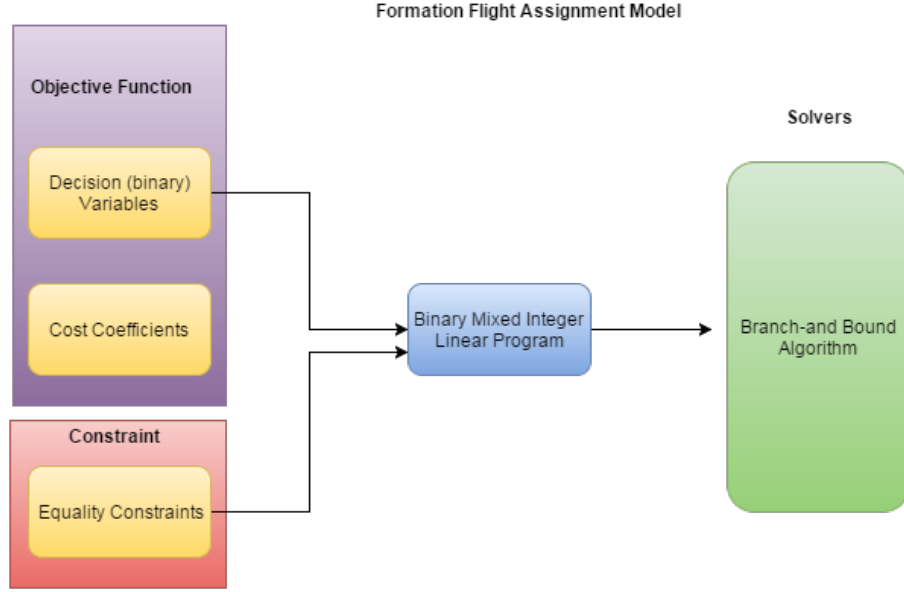


Figure 5.1: Linear programming framework for the flight formation assignment problem

The general format for a LP problem is as follows;

$$\text{maximise/minimise } C^T x \quad (5.1)$$

$$\text{Subject to: } Ax \leq b \quad (5.2)$$

$$\text{And } x \geq 0 \quad (5.3)$$

The objective function represented by Equation 5.1, is either maximised or minimised depending on the problem. The variable, x , in Equation 5.1, represents a vector of decision variables which will be used to find the most profitable solution. The term, C , in Equation 5.1, represents the cost coefficients (fuel costs) transposed to form a row vector. Moreover, A , is a matrix of constraints bounded by column vector, b (Equation 5.2). There can be three types of variables: integers, or integers with continuous variables or simply binary variables. If the variables are integers the optimisation problem then respectively takes the form of Integer Linear Program (ILP). If the variables are now integers with continuous variables the problem becomes a Mixed Integer Linear Program. Finally, if the variables are binary, the optimisation problem is then formulated as a Binary Mixed-Integer Linear Program (BMILP). A binary MILP uses x decision vectors comprising of binary values of 1's or 0's. In this case Equation 5.3, can either be $x = 1$ or $x = 0$. This type of optimising problem is used to solve the formation flight assignment problem, as it is also used in the work of Kent & Richards in order to optimally assign a set of aircraft to formations [27], [25], [26], [5].

5.2 Notation

This section presents the notations used in the objective function and constraints associated to the assignment problem. It is important to note that the following notations were extracted from the work of Kent & Richards [27]. An outline of the notation is listed below.

The pairing P between aircraft i and formation j is equal to the binary value 1 if and only if aircraft i is in formation j . Therefore, if aircraft i is indeed included in formation j there will be an associated incurred fuel cost, c_j . Comparing this formulation to the general LP formulation shown above one could replace A matrix with the matrix P containing binary values while C is a vector representing the fuel cost c_j for formations j .

$P_{j,i} = 1$, if and only if aircraft i is included in formation j

c_j is the incurred cost if formation j is used

x_j is the binary choice, if formation j is chosen then $x_j = 1$ else $x_j = 0$

5.3 Objective Function

The optimisation model for the formation flight allocation problem aims to allocate aircraft in the most profitable way in order to minimise the total fuel cost for the global network of fleet. Then, for a given aircraft N_a there exist possible formation combinations N_p of aircraft in formations of various sizes which result in fuel savings and, N_a solo flights which do not produce fuel savings.

$$\text{minimise } \sum_{j=1}^{N_p} c_j x_j \quad (5.4)$$

The objective function given by Equation 5.4, shows the minimisation of the aggregated fuel costs for each and every aircraft combination within a formation j with the decision variable $x_j = 1$ or $x_j = 0$. This means for each aircraft combination in a formation, a yes-no decision has to be made:

Should a certain aircraft be assigned to a certain formation combination?

Its decision variable = $\begin{cases} 1 & \text{if yes} \\ 0 & \text{if no} \end{cases}$

5.4 Constraints

The objective function is subject to N_a constraints (Equation 5.5), i.e., each aircraft in a formation will have an individual constraint, hence the number constraints increase as the size of the fleet increases, thereby increasing the computational burden of the problem. The constraints ensures that each aircraft is either assigned to just one formation or to a solo flight. The constraint ensures when an aircraft is selected by the optimisation solver, it is assigned to a specific formation without omitting any selected aircraft and without creating any redundant

aircraft in other formations.

$$\sum_{j=1}^{N_p} P_{j,i} x_j = 1 \quad \forall \quad i \in \{1, \dots, N_a\} \quad (5.5)$$

$$\text{where } x_j, \text{ binary} \quad \forall \quad i \in \{1, \dots, N_a\} \quad (5.6)$$

5.5 Optimiser

There are several optimisers available with varying complexity, for e.g. Gurobi and AMPL [27]. However, the optimisation solver used in this study is the CPLEX optimiser, as it is a widely used optimiser in Operations Research applications for major airlines and other industries. CPLEX makes use of the branch-and-bound algorithm along with cutting planes (i.e., reduces the feasible region), hence these sophisticated methods are employed in order to find the optimal solution [46]. This method is ideally suited for solving non-convex problems, i.e., problems that contain many possible local solutions. Therefore, the employed algorithm in CPLEX is able to search for the global solution amongst many possible local solutions.

Branch-and-bound is basically a technique that fundamentally uses the idea of *divide and conquer*. As the original problem is too large to solve in one go, the problem is divided in sub-problems. Then each sub-problem is solved until it is conquered. The branching or dividing of the problem is carried out by creating partitions from the set of feasible solutions. Thereafter, each partition creates smaller subsets of problems. Then fathoming or conquering is performed by bounding and testing the quality of the best solution in the subset. If the solution is not reasonable and does not contain an optimal solution to the original problem, the subset is disregarded. Below outlines the steps undertaken in the branch-and-bound algorithm for a BMILP.

5.5.1 Branch-and-Bound Algorithm Steps

First the algorithm is initialised by setting the objective function value to $-\infty$. Then, CPLEX conducts the bounding, fathoming and finally an optimality test to the entire problem. The steps for the first iteration are given below.

Steps for each iterations are:

1. *Branching:* Select one unfathomed sub-problem. Then branch from the node of the sub-problem to create two new sub-problems for the binary variables 1 and 0.
2. *Bounding:* Simplex method is applied to the new sub-problem and the LP is relaxed (deleting a constraint that has made the problem difficult to solve) in order to obtain an optimal solution. The objective function value is then rounded to an integer value if it was already not an integer. Then the integer value of the objective value becomes the bound for the sub-problem.
3. *Fathoming:* The three fathoming tests are applied to the new sub-problem in order for it to be fathomed. The tests are, if: its bound $\leq$ value of the objective function for the current incumbent (or best feasible solution so far). Or its LP relaxation results in no feasible solutions. Or the optimal solution is an integer for its LP relaxation.

Thereafter, a final optimality test is applied to check if there are no remaining sub-problems. If their exist another sub-problems and no incumbent solution then their are no feasible solutions for the given optimisation problem.

Chapter 6, further examines the elements of the LP problem in order to formulate the formation flight assignment model for this study.

6 Formation Flight Assignment Model

Chapter 5, presented the objective function, the constraints, and the optimisation solver for the MILP. In addition, the cost coefficients used in the objective function were explained together with the binary decision variables. The objective of this chapter is to explain the implementation of the assignment problem used in the MS-FFLIP model and the validation model. The concept description of the assignment model is given in Section 6.1, followed by a schematic flowchart of the developed assignment model in Section 6.2. Then, an example of the constraint matrix is demonstrated for the formulated MILP. Finally, Section 6.2 outlines the post-processing steps that is necessary to produce user-friendly results.

6.1 Concept Description

The formation flight assignment model developed, selects a compatible set of possible flight formations in order to achieve a global minimum fuel saving for the validation model and a (near)global fuel savings for the MS-FFLIP model. Depending on the constraints associated to each aircraft, the model generates an optimal formation flight schedule. The formation flight allocation model has been developed in a generic way. Therefore, the model is adaptable and can be easily scaled up to theoretically account for any number of possible aircraft. The output of the model generates an optimal formation flight schedule matrix, overall fuel savings, and percentage of total fuel savings.

6.1.1 Objective

In order to address the objective of the optimisation algorithm, a set of requirements for the algorithm was established as listed below:

1. Minimise total fuel consumption for a global fleet of aircraft.
2. Model is generic and is able to handle any number of aircraft and formation size.
3. The formulation of the model needs to be efficient in order to keep the computation time low, hence making it practical for real-time on-line application use.
4. Results can be easily understood.

6.2 Model Structure

In order for the formation flight assignment model to be functional, several key components need to be integrated into the model architecture. Figure 6.1, represents the schematic diagram of the developed assignment model. Each component is critical in achieving a reliable result. This model features the additional modification that was needed in order to implement a multi-stage formation flight assignment. This feature is represented by the *new event* decision algorithm and its feedback loop which is clearly shown on Figure 6.1. Removing these components prevent the sequential centralised stage process from occurring, thereby allowing for a single-stage formation flight plan which accounts for all possible combinations of aircraft in a formation, in essence this is the process undertaken for the single-stage validation model.

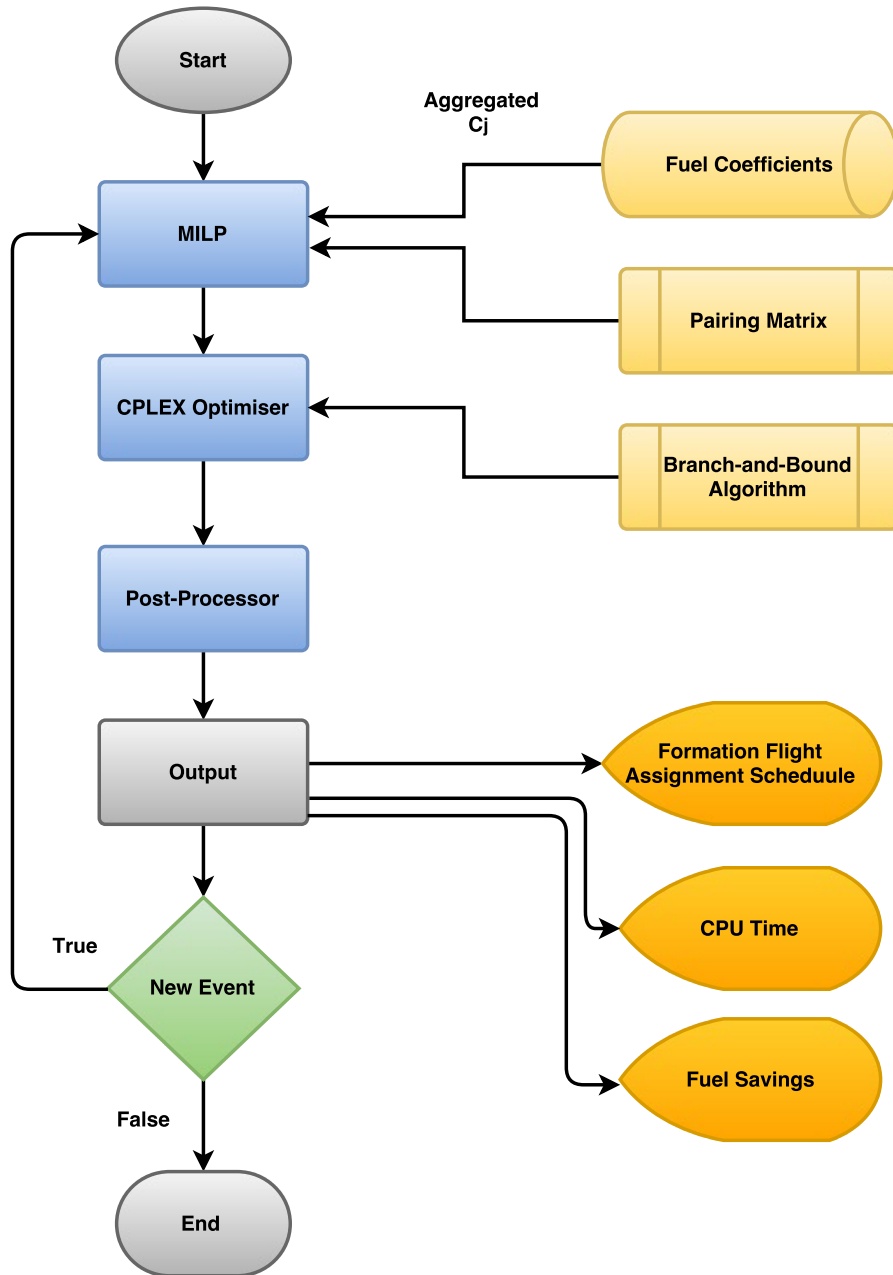


Figure 6.1: Formation flight assignment model architecture

6.2.1 Input

The MILP formulation of the assignment model is dependent on the correct input of the fuel coefficients estimated for each formation option and, the correct establishment of the pairing matrix $[P]$ (rows of the pairing matrix, represented by Equation 5.5). The pairing matrices form the A matrix while the b matrix comprises of a column vector of 1's. The number of rows of the A matrix are equal to the number of aircraft N_a and the number of columns of the A are equal to the number of possible formation combinations N_p .

Fuel Coefficients $[c_j]$

The fuel cost coefficient c_j is obtained by aggregating the fuel cost of each flight in a potential formation flight. For example this means that for a scenario of five possible formations there are five associated aggregated fuel cost coefficients (c_1, c_2, c_3, c_4, c_5). Hence, these fuel costs are assembled into the objective function with the respective binary decision variables.

Pairing Matrix $[P]$: Per Stage In the Multi-Stage Assembly Process

As mentioned, the pairing matrix P is a matrix with number of rows equal to aircraft number N_a and with columns represented by the number of possible formations, N_p . Then, P contains binary variables where the binary value 1 describes a respective aircraft allocated to fly in a formation and, 0 for a respective aircraft not eligible for formation flight.

The correct implementation of the pairing matrix is critical for obtaining an optimal formation assignment schedule. Even though the MS-FFLIP model conducts optimisation using a multi-stage process and assembling aircraft to large formations, the value of n in each stage of the formation assembly will remain equal to $n = 2$, thus preventing combinatorial explosions. However, the pairing matrices will still need to model the combinations in relation to formation sizes. The method for computing the number of combinations is described in detail in Section 3.2.

6.2.2 Pairing Matrix $[P]$: Demonstration of A Small-Scale Modelling Example

In order to demonstrate the method for setting up the pairing matrix for combinations of formation flights, this section presents an example scenario. A scenario of five aircraft, $N_a = 5$, that may join in a formation up to a five-aircraft formation, $n = 1, \dots, 5$, is presented. Each formation size, n , has a pairing sub-matrix with rows of aircraft i and columns of formations j , that amalgamate over all sizes, forms the pairing matrix P . In this example, the number of rows of all pairing sub-matrices is equal $i = 1, \dots, 5$ (five aircraft) where each row represents an aircraft i . However, it remains to calculate the number of formation combinations, j . The number of formation combinations, j , for each formation size for $n = 1, \dots, 5$ can be computed using Equation 3.1.

This section shows formulation for all pairing sub-matrices for this particular example scenario. First, the pairing sub-matrix for solo flights, $n = 1$, is given. Second, the pairing sub-matrix is given for the two-aircraft formation or $n = 2$. Next, the pairing sub-matrix is demonstrated for the three-aircraft formation. Finally, the pairing sub-matrices for four and five-aircraft formation sizes is outlined.

Solo Flight Pairing Sub-Matrix

First, the pairing sub-matrix for the solo flights is stated. This sub-matrix, for solo flights, is an identity matrix with 5 rows ($N_a = 5$) and 5 columns ($n = 5$), as shown below. Therefore, the number of rows equals to the number of aircraft, N_a , and the number of columns equals size of the formation, n .

$$\text{Sub-matrix, five solo flights} = P_{1-5} = \begin{matrix} & \begin{matrix} \xrightarrow{\text{Formation size } n} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5 \end{matrix} \\ \begin{matrix} \downarrow \text{aircraft } i \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Two-Aircraft Formation Pairing Sub-Matrix

The two-aircraft formation ($n = 2$) pairing sub-matrix is derived for the five flights ($N_a = 5$). Then using the binomial equation (Equation 3.1) one obtains 10 possible formation combinations as demonstrated below:

$$5 \text{ Choose } 2 \equiv \binom{5}{2} = \frac{5!}{2!(5-2)!} = 10 \quad (6.1)$$

All possible aircraft combinations for “5 Choose 2” are shown in set 6.2, which contains aircraft 1 grouped with aircraft 2, {1,2}, aircraft 1 grouped with aircraft 3, {1,3}, etc.

$$\begin{matrix} \{1, 2\} & \{1, 3\} & \{1, 4\} & \{1, 5\} \\ \{2, 3\} & \{2, 4\} & \{2, 5\} & \\ \{3, 4\} & \{3, 5\} & & \\ \{4, 5\} & & & \end{matrix} \quad (6.2)$$

Using the possible formation combinations in set 6.2, the pairing sub-matrix P_{6-15} can be formed as shown below for which the number of rows represent the number of aircraft i and the number of columns represented by the formations, j for which $j = 6, \dots, 15$.

$$\text{Sub-matrix, two-aircraft formation} = P_{6-15} = \begin{matrix} & \begin{matrix} \xrightarrow{\text{Formation } j} \\ 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \end{matrix} \\ \begin{matrix} \downarrow \text{aircraft } i \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \end{matrix}$$

Three-Aircraft Formation Pairing Sub-Matrix

Similarly, for the three-aircraft formation pairing sub-matrix with $N_a = 5$ and $n = 3$ there is 10 possible formation combinations as shown by Equation 6.3.

$$5 \text{ Choose } 3 \equiv \binom{5}{3} = \frac{5!}{3!(5-3)!} = 10 \quad (6.3)$$

The possible number of three-aircraft formation combinations are:

$$\begin{array}{llll} \{1, 2, 3\} & \{1, 2, 4\} & \{1, 2, 5\} & \{1, 3, 4\} \\ \{1, 3, 5\} & \{1, 4, 5\} & \{2, 3, 4\} & \\ \{2, 3, 5\} & \{2, 4, 5\} & & \\ \{3, 4, 5\} & & & \end{array} \quad (6.4)$$

After obtaining all the possible combination of aircraft in a formation for a three-aircraft formation size presented in set 6.6, the information is then translated into the pairing sub-matrix shown below, where the number of rows equal the number of aircraft and the number of columns equal to the number of formations, $j = 16, \dots, 25$.

$$\text{Sub-matrix, three-aircraft formation} = P_{16-25} = \begin{array}{c} \text{aircraft } i \\ \downarrow \end{array} \begin{array}{c} \text{Formation } j \\ \overrightarrow{\hspace{1.5cm}} \end{array} \begin{array}{c} 16 \quad 17 \quad 18 \quad 19 \quad 20 \quad 21 \quad 22 \quad 23 \quad 24 \quad 25 \\ \left(\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 5 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{array} \right) \end{array}$$

Four-Aircraft Formation Pairing Sub-Matrix

Similarly, for the four-aircraft formation using Equation 6.5, one obtains 5 (“5 Choose 4”) possible formation combinations.

$$5 \text{ Choose } 4 \equiv \binom{5}{4} = \frac{5!}{4!(5-4)!} = 5 \quad (6.5)$$

The number of possible aircraft combinations in a four-aircraft size formation are:

$$\begin{array}{lll} \{1, 2, 3, 4\} & \{1, 2, 3, 5\} & \{1, 2, 4, 5\} \\ \{1, 3, 4, 5\} & \{2, 3, 4, 5\} & \end{array} \quad (6.6)$$

Hence, the pairing constraint sub-matrix for the four-aircraft formation size is depicted below, where the number of columns, j , is represented by formations, $j = 26, \dots, 30$.

$$\text{Sub-matrix, four-aircraft formation} = P_{26-30} = \begin{array}{c} \text{aircraft } i \\ \downarrow \end{array} \begin{array}{c} \text{Formation } j \\ \overrightarrow{\hspace{1.5cm}} \end{array} \begin{array}{c} 26 \quad 27 \quad 28 \quad 29 \quad 30 \\ \left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 0 & 1 & 1 \\ 5 & 0 & 1 & 1 & 1 \end{array} \right) \end{array}$$

Five-Aircraft Formation Pairing Sub-Matrix

Lastly, for the five-aircraft formation size, there exist only 1 (5 Choose 5) possible combination.

$$5 \text{ Choose } 5 \equiv \binom{5}{5} = \frac{5!}{5!(5-5)!} = 1 \quad (6.7)$$

The only possible combination of aircraft in this particular formation ($j = 31$) is $\{1,2,3,4,5\}$ which gives the pairing sub-matrix:

$$\text{Sub-matrix, five-aircraft formation} = P_{31} = \begin{matrix} & \begin{matrix} \text{Formation } j \\ \longrightarrow \\ 31 \end{matrix} \\ \begin{matrix} \text{aircraft } i \\ \downarrow \end{matrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \end{matrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Pairing Matrix For $N_a = 5$ and $n = 1, \dots, 5$ Example Scenario

As observed, a total of 31 formations for all formation groupings were created for the five flights. As a result, the complete pairing matrix, P , will have 5 rows ($N_a = 5$) with 31 columns ($j = 1, \dots, 31$), i.e., a 5 by 31 matrix. The complete pairing matrix, P , can now be obtained by amalgamating all the pairing sub-matrices for each formation size including solo flights, as shown in Equation 6.8.

$$P_{5 \times 31} = [P_{1-5} \mid P_{6-15} \mid P_{16-25} \mid P_{26-30} \mid P_{31}] \quad (6.8)$$

6.2.3 Optimiser

An IBM ILOG CPLEX Optimisation Studio 12.5 version optimiser is used in this study. The optimiser makes use of the algorithms, as described in Section 5.5, in order to find optimal solutions. The essential components of the cost vector C , A and b matrices are fed to the CPLEX optimiser. Subsequently, the CPLEX optimiser provides an optimal solution set to the MATLAB environment. The solution set contains the (binary) values of the solution vector (x_j) which needs to be processed in order for the user to understand the results.

6.2.4 Post-Processor

Post-processing is carried out in order to extract the optimisation results from the raw data and to re-create the results in a user friendly manner. The post-processor uses the variables used in creating the combinatorial pairing matrices to produce the formation allocation schedule for each formation size and for each stage of the multi-stage process. In addition to the formation allocation schedule, the post processor also assesses the overall fuel savings and the fuel savings percentage. The overall fuel savings are computed by summing the fuel burn of all solo flights, even for the flights not included in formation and, subtracting the aggregated fuel burn for the assigned formation flights. As a result, there will always be a reference value in order to

be consistent when comparing results of the developed model to the results of the model of Verhagen et al. Equation 6.9, presents the formula used to compute the savings percentage. Note that the numerator represents the overall savings.

$$\text{Fuel Savings} = \left(\frac{\text{Total Fuel Burn Solo Flights} - \text{Total Fuel Burn Allocated Flights}}{\text{Total Fuel Burn Solo Flights}} \right) \times 100 \quad (6.9)$$

7 Multi-Stage Process

After developing the routing, fuel consumption and the formation flight assignment models, it still remains to demonstrate the efficiency of the technique that circumvent the adverse combinatorial explosion problem for centralised formation flight planning. This chapter presents the methodology by innovating the combinatorial optimisation problem in order to achieve the better fuel-saving potential for centralised formation flight planning, while still allowing for real-time formation planning. In order to demonstrate the better fuel-saving potential, the number of total aircraft and the number trailing aircraft within a formation will have to be increased. In order to do so, the experienced exponential computational burden needs to be solved. Therefore, Section 7.1, presents the enumeration routing scheme developed in order to reduce the computational time burden for a global optimisation of formation flights. Then, Section 7.2, demonstrates the formation flight staging process for a small-scale scenario of flights. Lastly, Section 7.3 provides a schematic flowchart that describes the MS-FFLIP model.

7.1 Staged Formation Flight Process

As mentioned in the introductory paragraph of this chapter, the critical aspect of the multi-stage formation flight planning approach is its capability of preventing enumeration time explosions, hence combinatorial explosions. When there exist many possible combinations for large formation sizes, enumerating all combinations is infeasible for real-time formation flight planning applications.

Through experimentation, it was observed that enumerating the two-aircraft formation size ($n = 2$) combinations required the least amount of computation time due its comparatively non-complex formation assembly process. Based on this observation, the assembly of larger formation sizes is performed by augmenting (building upon) the two-aircraft formation size solutions and, hence allowing the developed MS-FFLIP model to accommodate a higher number of aircraft within a formation.

As briefly mentioned in Section 3.3, the MS-FFLIP model consist of four stages, and each stage allows for the grouping of aircraft into larger formations of up to a sixteen-aircraft formation size. The first stage conducts the route optimisation for the two-aircraft formation size for which the total fuel cost of all possible formations are computed in order for the formation assignment model to optimally assign the combinations of aircraft into a cost minimised formation fleet. Now that the model has formulated the optimal formation fleet schedule for the two-aircraft formations (and solo flights), an event-driven simulation algorithm collects all the assigned two-aircraft formations and, if any, all solo flights and prepare these flights for the next stage of the simulation. The collection of all joining points of all the optimally assigned formations is defined as an event. If the flight assignment model does not find any cost minimising formation flights,

the simulation will end at its current respective stage, hence the reason it being an event-driven simulation. Subsequently, all the joining points of the two-aircraft formations in the 1st stage are updated to represent the “new origins” for the flights while the current position (position in time where the allocated formation flights join) of the solo flights become the new origins for the respective solo aircraft in the next stage. In addition, the optimal splitting points are updated to represent the “new destination” for the respective formation flights. This means that two-aircraft formations is now represented as a single entity in the next stage or, in other words a “single flight”. As a result, this allows for the respective two-aircraft formations to merge with solo flights and, to also merge with other respective two-aircraft formations in the 2nd stage of the process.

Since a two-aircraft formation is represented as a single entity, when it joins another single entity it can be considered as if two entities have joined together. Therefore, for any N-aircraft formation size ($N \leq 16$) it can always be represented as a single entity in order for it to create consecutive formations and, therefore to create combination sets of these entities. Since the MS-FFLIP model groups 2 entities, this means that binomial coefficient, n , in the binomial equation (Equation 3.1) is always equal to 2. This reduces the possible number of combinations needed to be enumerated. As a result, this approach transforms the factorial time problem into a polynomial time problem. As a consequence, the efficiency of the computation improves modestly, thereby allowing large formations to be created.

7.2 Staging Procedure Example

As mentioned, the developed multi-stage formation flight approach is designed to accommodate up to a sixteen-aircraft formation size and for any number of aircraft, therefore the routing and fuel consumption model have the capability to handle such large formation sizes.

This section describes the formation flight routing process undertaken within each stage with the aid of an example scenario of 20 flights. Note: For the purpose of this example, the illustration presented in this section is limited to: two/four/eight-aircraft formation sizes

7.2.1 Stage 1

As discussed in the previous section, the 1st stage of the multi-stage formation flight plan conducts an optimisation for the two-aircraft formation size ($n = 2$). Using Equation 3.1, there exist 190 (20 Choose 2) possible two-aircraft formation combinations for a scenario of 20 flights ($N_a = 20$). The routing model then conducts the formation route optimisation for all these 190 combinations. For the purpose of this exercise only two formation combination sets of aircraft $\{1,2\}$ and $\{3,4\}$ will be demonstrated. The fuel cost for the formation routes are then estimated and sent to the assignment model to be optimally assigned in a minimised fuel cost formation fleet.

Now assuming from the 190 possible combinations, only 5 groups of two-aircraft formations are optimally assigned, therefore creating an event, and the remaining 10 flights are assigned to fly solo. The event-driven simulation then assigns these 5 groups of two-aircraft formations as 5 entities (“5 single flights”) and also the 10 solo flights are assigned as 10 entities. This means the formation set $\{1,2\}$ and $\{3,4\}$ becomes pseudo flight 12 and flight 34 as shown in Figure

7.1. Subsequently, the respective joining points and splitting points are updated as their new origins and destinations and the new origins of the solo flights are updated to their current position (moment in time where the respective joining occurred). These 15 sets of entities are then processed in the 2nd stage of the MS-FFLIP model.

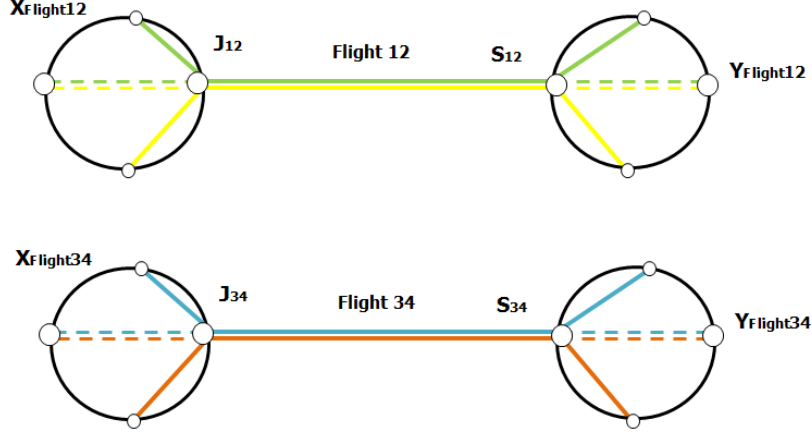


Figure 7.1: Two groups of two-aircraft formation ($\{1,2\}$, $\{3,4\}$) routes from a set of 190 combinations

7.2.2 Stage 2

In this 2nd stage, the 5 entities (5 groups of two-aircraft formation) is grouped pairwise to form new combination sets. Then, using the binomial coefficient of Equation 3.1 one obtains 10 (5 Choose 2) possible four-aircraft formation combinations. Furthermore, the 5 entities or 5 groups of two-aircraft formation are also eligible for grouping with the remaining 10 entities of solo flights, thus forming 50 three-aircraft formation combinations. The routing model then computes the optimal formation routes for the above combinations. Figure 7.2, demonstrates the entities, flight 12 and flight 34 have formed a four-aircraft formation size and it can be seen that the joining points J_{12} , J_{34} and the splitting points S_{12} and S_{34} are used as the new origins and destinations points in order to compute the four-aircraft formation route. The routing model assembles the remaining segments of two-aircraft formations and solo segments to form an optimal four-aircraft formation route as illustrated by Figure 7.3.

This enumeration scheme is the exact same scheme used in the centralised model which was presented in Section 4.2 and notably, the only difference between the two schemes is the number of combinations enumerated. If one were to conduct a four-aircraft formation route optimisation using the centralised model, it would have to enumerate 4,845 combinations (20 Choose 4) compared to just 10 combinations enumerated by MS-FFLIP model. As a result, the MS-FFLIP model reduces the enumeration of combinations by more than 500-fold, thereby decreasing the required computational time.

The assignment model then optimally assigns the flights to two/three/four-aircraft formations or, solo flights in this 2nd stage. Now assuming that out all the combinations only 2 groups of four-aircraft formations, 1 pair of two-aircraft formations and 10 solo flights, were optimally allocated by the assignment model. Then, similar to the previous stage, there exist 2 entities of four-aircraft formation size, 1 entity representing the single two-aircraft formation and 10 entities representing the 10 solo flights. The event-driven simulation thereafter detects the event,

it assigns the joining and splitting points as the new origins and destinations and prepares the event to be optimised in the 3rd stage.

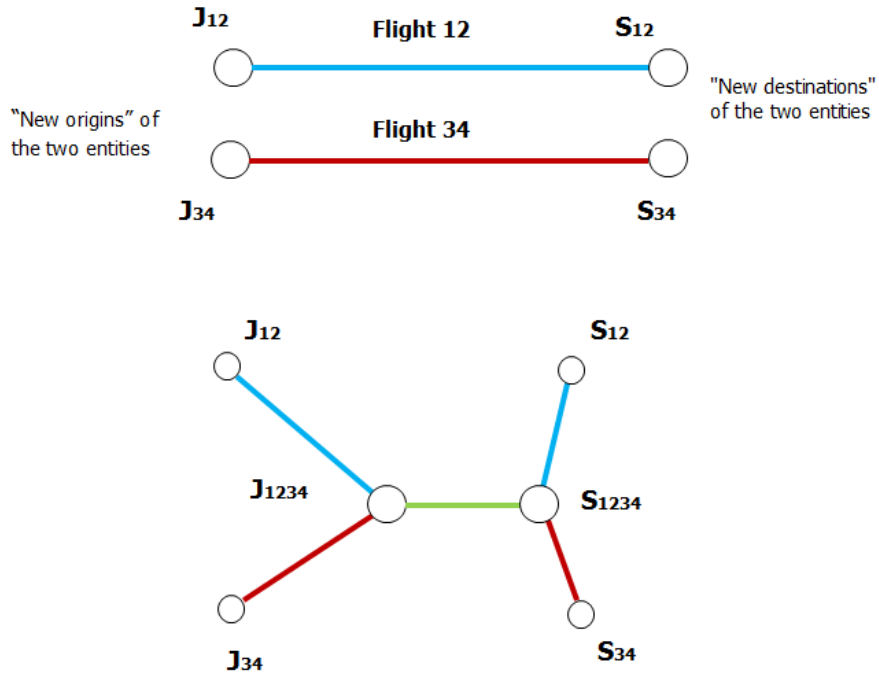


Figure 7.2: Two entities of two-aircraft formations and their optimal route

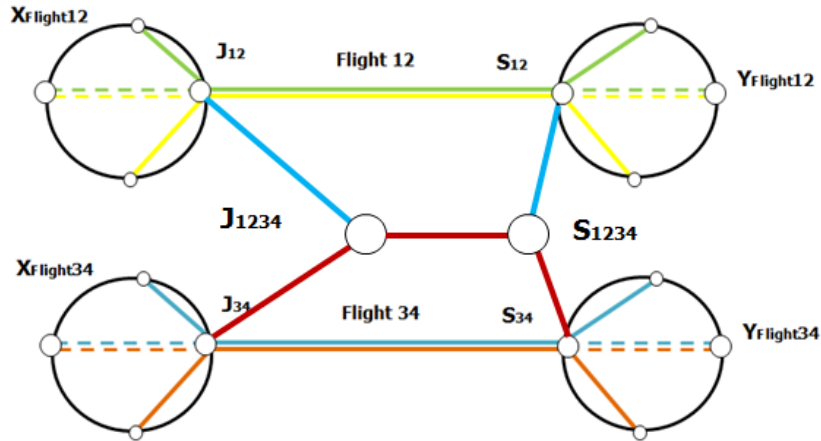


Figure 7.3: Optimal four-aircraft formation route for flights 1,2,3 and 4

7.2.3 Stage 3

In this 3rd stage, the optimally assigned flights in stage 2 is used to assemble large formations comprising of five, six and eight aircraft within a formation. The 2 entities of four-aircraft formation size create just one eight-aircraft formation (2 Choose 2). Moreover, these 2 entities also combine with the single entity of two-aircraft formation to form a six-aircraft formation, and the 10 entities of solo flights group with the 2 entities to form 20 groups of five-aircraft

formations. Figures 7.4 and 7.5, demonstrate the formation assembly for the 2 entities of four-aircraft formation i.e., groups $\{1,2,3,4\}$ and $\{5,6,7,8\}$ in creating a single eight-aircraft formation. After all combinations are enumerated the assignment model then optimally assigns the flights to the respective formation sizes.

Now assuming that the assignment model has optimally allocated flights to 2 groups of eight-aircraft formations and remaining 4 flights assigned to fly solo. The event-driven algorithm then assigns 2 groups of eight-aircraft formation as 2 entities and also the 4 solo flight as 4 entities. Then the respective joining and splitting points for the for 2 entities are defined as the new origins and destinations points for these two entities. Moreover, the current position of the solo flights are also updated. The optimisation of these optimal solutions are conducted in stage 4 of the MS-FFLIP process.

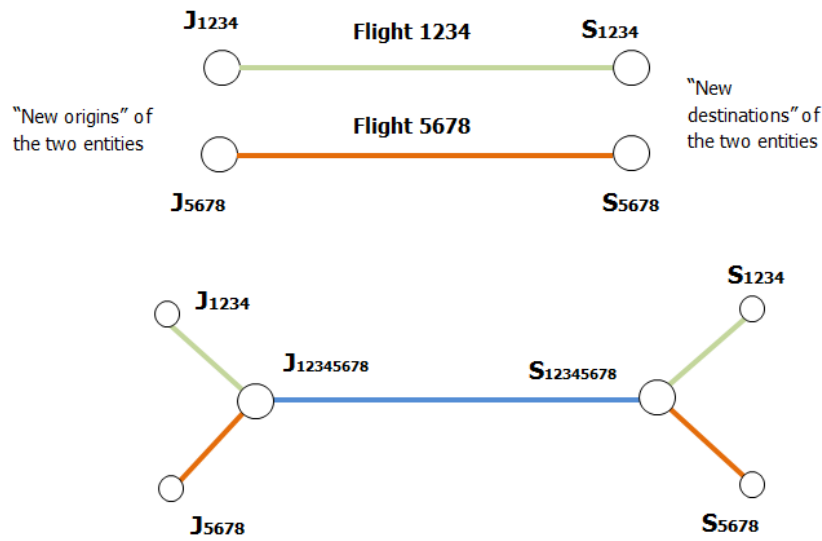


Figure 7.4: Optimal four-aircraft formation route for flights 1,2,3 and 4

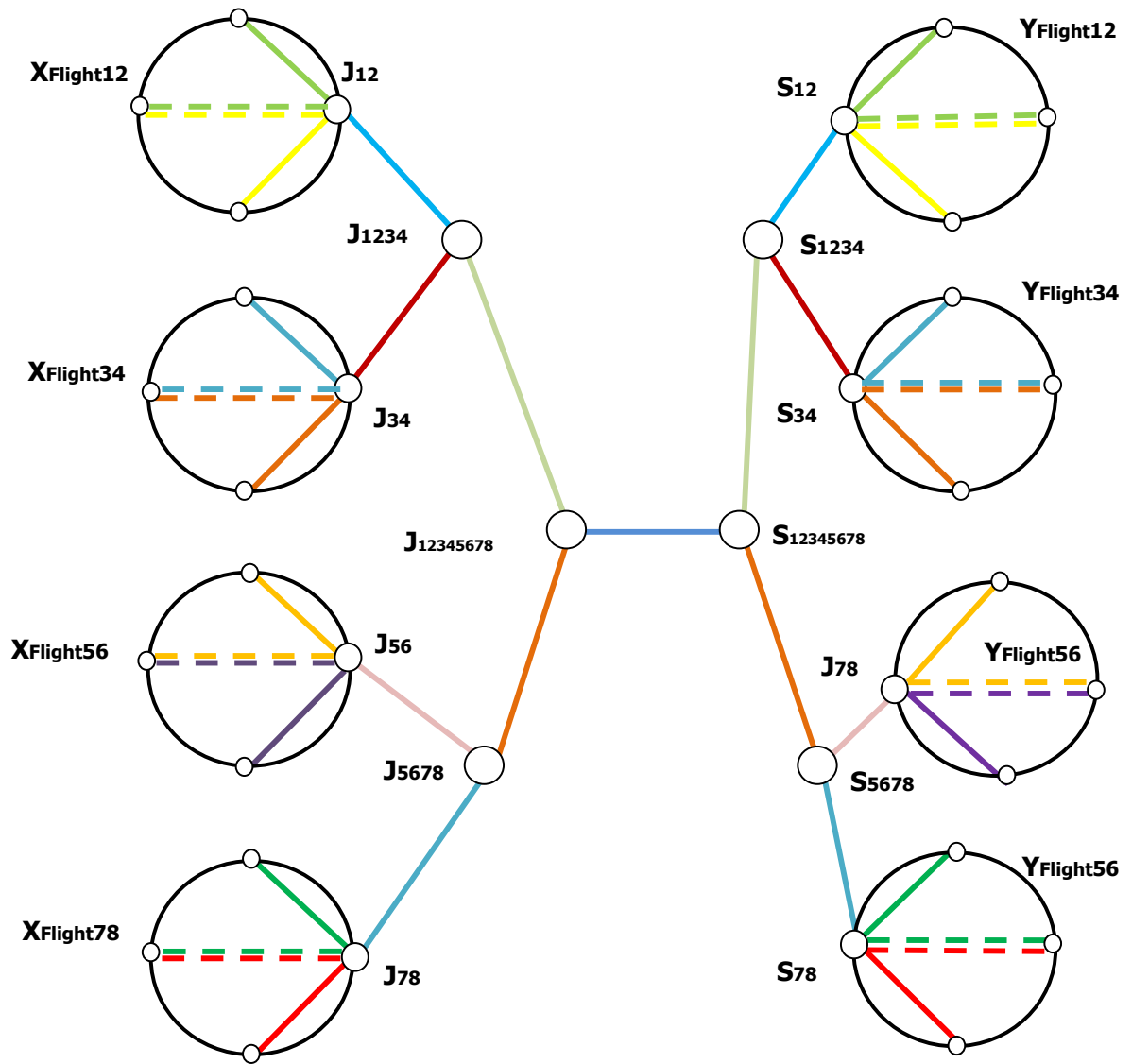


Figure 7.5: Optimal eight-aircraft formation route for flights 1,2,3,4,5,6,7,8

7.2.4 Stage 4

The 2 entities of eight-aircraft formation size and the 4 solo flights combine to form 8 groups of nine-aircraft formation combinations. Moreover, the 2 entities pair with each other to form single set of sixteen-aircraft formation combination. The routing model enumerates all the formation routes and subsequently estimates the associated fuel cost. Thereafter, the assignment model optimises the formation with respect to fuel cost to obtain an optimal set of formations. Finally, the assignment model assigned sixteen flights to fly in a single formation and 4 aircraft to fly solo.

7.3 Schematic Description of MS-FFLIP Model

The flow chart below provides an generic schematic description of the developed MS-FFLIP model.

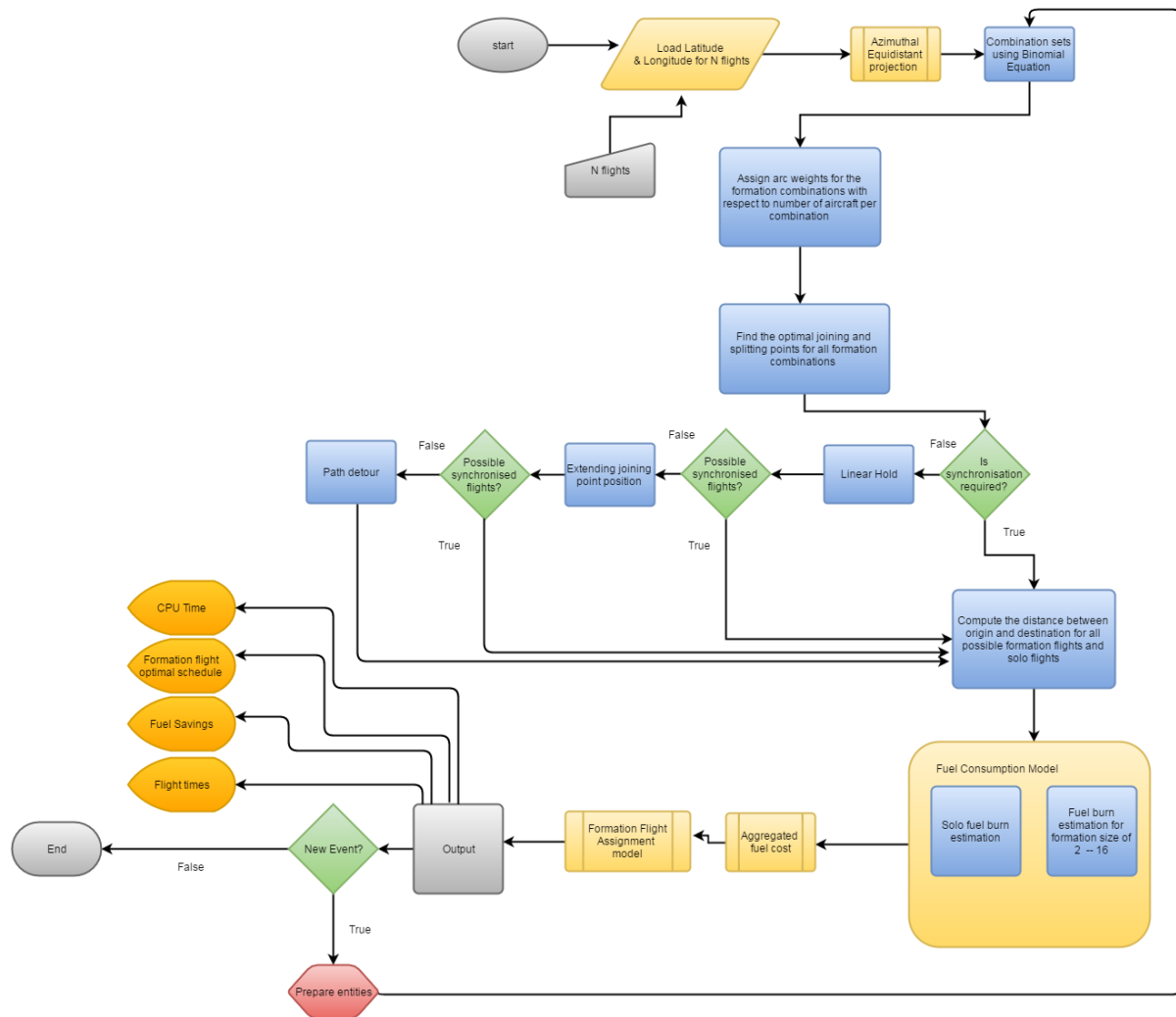


Figure 7.6: Flow chart for the developed MS-FFLIP model

8 Management of Formation Flight

After addressing the developed model for flight formation operations in the previous chapters, it still remains to investigate the management of formation flight for the current airspace system. An organisational structure for the developed MS-FFLIP model is imperative to the successful implementation in the aviation industry. Due to the flexibility of the developed MS-FFLIP tool, the tool can be used for both tactical and strategic planning phases. However, it is highly necessary to establish a governing body that would take responsibility for the actions of MS-FFLIP tool. Therefore, this chapter conceptualises an organisational structure and the establishment of a Formation Air Traffic Controller (FATCo).

It is crucial to see where formation flight planning can be incorporated into the current operations of commercial flying. For any flight to take place, flights need to be planned with ATC prior to departure. The flight route and fuel consumption need to comply to safety and ATC requirements. For airlines these flight plans must be submitted at least six months in advance [34]. This allows for airlines to properly schedule their flights and also gives ATC ample time to adequately plan such that they account for busy and congested flight routes. The main phases in ATC are: strategic, pre-tactical and tactical planning [27].

Strategic planning has a time frame from months to two days before a flight departure. This stage of planning is used by ATC to prepare and predict the traffic in an airspace, for instance, the transatlantic oceanic airspace. Once the strategic planning has taken place, a pre-tactical planning phase is carried out in order to fine tune or adjust and confirm the flight plan with the respective airline. This pre-tactical planning phase takes place within the last two days of the flight. The final phase is the tactical planning phase, which occurs on the day of flight where adjustments are carried out depending on the current air traffic flow situation [34].

Formation flight planning will need to address all three phases including a number of formation flight obstacles that concern its viability, such as the required large formation flight separation limits and inefficient communication links. The research work carried out by Kent & Richards focused on the strategic planning phase of flight formations [27]. This was mainly due to the high computation times caused by the adverse combinatorial complexity of the problem. On the other hand, the work of Verhagen et al. dealt on a tactical planning level [6]. This was because of the robust nature of their model and also the ease of performing swift computations at the expense of lower fuel savings. However, the MS-FFLIP model developed for achieving the goal of this thesis will be able to achieve results that capture the majority of the potential fuel savings while performing the computations relatively faster than the original centralised model of Kent & Richards.

8.1 Organisational Structure

This section presents the organisation structure that is required for the planning of centralised flight formations. This section aims to motivate the incorporation of centralised formation flight planning into the existing Air Traffic Management system with minimal requirements to infrastructure. Before discussing the rudiments of formation flight management, the current organisational system for oceanic flights is explained.

Within the context of this study, the embodiment of formation flight is to take place across the Atlantic on Organised Track Systems (OTS), with the assumption of constant altitude of 9km (FL29) for eastbound flights, designated as the Minimum Navigation Performance Specification (MNPS) airspace. The MNPS features thirteen flight levels: FL29 to FL410, nine of the Flight Levels ranging from FL310 to FL390 are allocated for westbound flights and, the remaining four Flight Levels are assigned for eastbound flights, which are optimised for the southerly jet streams. The control centre responsible for westbound transatlantic oceanic flights is the Oceanic Area Control Centre (OACC) which belongs to the Britain's National Air Traffic Services in Scotland [34]. On the other hand, eastern flights across the Atlantic comes under the purview of the Canadian OACC.

Airlines are responsible for choosing which routes to fly on. The best route is selected based on weather and flight performance characteristics for the selected aircraft. The airline then sends the selected route via a Preferred Route Message (PRM) to the respective OACC. This communication of PRM is available on a 14 hour time window up until the departure time of the respective flight [34]. The OACC then takes into account the airline's respective PRM together with airspace restrictions, expected traffic and predicted jet stream in order to design the OTS [34].

Since there is no radar coverage within the oceanic region, pilots are required to report their position at specific way-points [34]. These way-points are located at every 10 degree longitudinal and 2 degrees latitude [34] along the OTS. A typical way-point lies between 600km with five segments between way-points. The pilots positional information is recorded by two controller: Planning Controller (PCo) and the En-route Controller (RCo). The PCo, uses the aircraft Mach number, Flight Level and prevailing wind forecast to estimate the position of the aircraft between way-points, while the RCo monitors the progress of the flight. Currently, PCo requires 30 minutes of minimum request time in order to provide clearance for an aircraft [34].

8.2 Proposal For The Management of Formation Flight

It can be conceived that the concept of a Formation Air Traffic Controller or FATCo can be strategically positioned within the OACC and horizontally integrated amongst the PCo and RCo. However, to do so, critical assumptions have to be made in order to ensure the management of formation flight traffic. First, it is assumed that formation aircraft are equipped with formation auto-pilots which automatically seek the leading upwash extrema using feedback loops that monitor and make adjustments in order to maintain an optimal lateral and longitudinal separation between aircraft. Second, it is assumed that the current OACC is capable of processing space-based ADS-B data of aircraft relayed by a constellation of satellites which allow formation aircraft to constantly enable real-time communication with the respective OACC. Therefore, this will allow formation aircraft to receive clearance without the concern of delays. Then, the FATCo uses all the aircraft data from the ADS-B, i.e., aircraft position, Mach number

and flight level at the beginning of cruise flight and then load the data into the MS-FFLIP model in order to determine a potential fuel saving opportunity using formation flight. If a possibility to save fuel exist, the model displays the optimised formation flight schedule for all stages of the MS-FFLIP process. The FATCo then provides clearance and notifies the assigned aircraft for possible formation flight. The FATCo will then assist the allocated aircraft in assembling into formation and shall monitor the flight progress of the respective formation flights. In case an unexpected disruption occurs, the FATCo is be able to run the model for this new scenario and provide immediate solutions since the model has the ability to perform real-time computations. For aircraft not assigned into a formation even after complete stage enumeration, the respective aircraft will be allocated to fly solo. FATCo then handovers the respective solo flights to RCo in order to monitor and track the progress of the flights. Figure 8.1, summarises the conceptual method of formation flight communication with OACC using a global constellation of satellites to establish a real-time data link with oceanic flights.

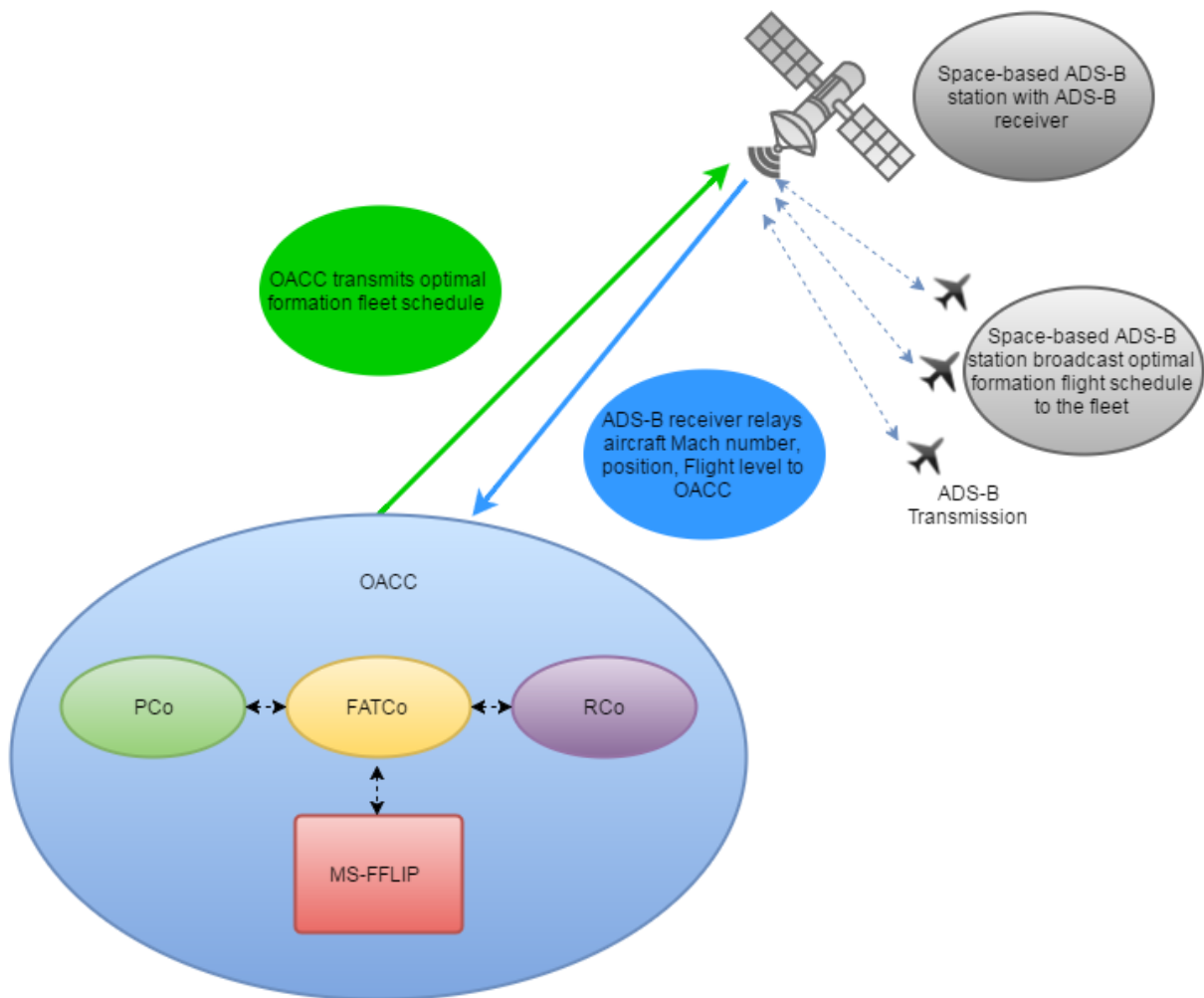


Figure 8.1: Space-based ADS-B communication for formation flight management

9 Case Study

For multi-stage formation flight planning to become a reality within the civil aviation sector, significant motivation for its use need to be presented. It is therefore, necessary to analyse the results for a scenario of 267 transatlantic flights for which formation flight may become a viable fuel saving option. The two primary objectives for this chapter is to observe the potential fuel saving benefit of formation flight as well as the required computational time for the MS-FFLIP model.

Before presenting the potential fuel savings of the MS-FFLIP model, the routing capability of the model is examined in Section 9.1. Next, Section 9.2 presents the results for the stages of the MS-FFLIP model followed by the results for the decentralised model for the same scenario of flights. Thereafter in Section 9.3, the results from the developed MS-FFLIP model is compared against the decentralised greedy optimisation method in order to demonstrate the recovered potential fuel savings by the MS-FFLIP model. In Section 9.4, the MS-FFLIP model is compared against the centralised model of Kent and Richards in order to reveal the model's ability of handling combinatorial problems. Finally, Section 9.5 provides an interpretation of the obtained results.

9.1 Formation Flight Routing Capability

The following section demonstrates the 2-dimensional formation flight routing geometry produced by the developed MS-FFLIP model. The model analyses for a scenario of aircraft flying from east to west on their transatlantic routes which happen to create a three, four and an eight-aircraft formation size. The objective of this section is to highlight the optimal locations for the joining and splitting points with respect to the origin and destinations, thus allowing for optimal routing.

9.1.1 Three-Aircraft Formation Flight Routing

Figure 9.1, presents the solo routes for three flights between their respective origin and destination locations. For this scenario, flight 1 and 2 depart from the same origin and fly towards their respective destination locations. Figure 9.1, shows that the two flights $\{1,2\}$ have similar flight routes and since all flights are assumed to have equal departure times, the two flights will require minimal detours for the purpose of creating formations. As a result, they become ideal formation flight candidates. The MS-FFLIP model then estimates the fuel cost for the possible combinations of $\{1,2\}$, $\{1,3\}$ and $\{2,3\}$, two-aircraft formation sizes for the 1st stage of the MS-FFLIP model. As anticipated, the formation flight assignment model only assigns the flight pair of $\{1,2\}$ into a two-aircraft formation size thereby creating an event. This ends the 1st stage of the multi-stage process and the model aims to create a three-aircraft formation with

the solo flight 3, i.e., to create the combination $\{1,2,3\}$. Therefore, flights $\{1,2\}$ are considered as a single entity or a single flight and their respective joining point and splitting point are updated as the new origin and destinations. Moreover, the current position (moment in time where joining of flight 1 and 2 take place) of the solo flights are updated as the new origin point. The routing model then updates the arc weights and computes the optimal formation route for this three-aircraft formation. Subsequently, the fuel cost for the formation segments are computed. Thereafter, the assignment model conducts an optimisation and determines the grouping of flight 3 with flights $\{1,2\}$ such that it minimises the overall fuel burn. As a result, an optimal three-aircraft formation is created as shown in Figure 9.2. The optimal joining (red dot) and splitting point (yellow dot) for this three-aircraft formation are also shown in the figure.

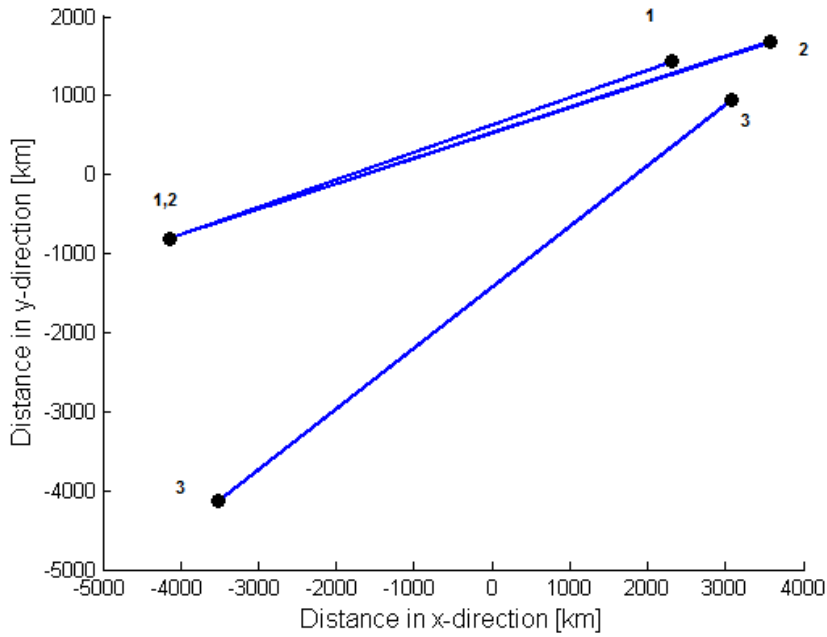


Figure 9.1: Solo routes for flights 1, 2, and 3 between their respective origin and destinations locations

A critical characteristic of the MS-FFLIP model is its ability to cooperate with all aircraft, thus creating an awareness amongst aircraft due to the global optimisation method. Consequently, this allowed for the possibility of optimal routing for the allocated formation flights and prevented possible sub-optimal routing as observed in the work of Verhagen et al. [6] which developed due to the authors use of the local optimisation method. Therefore, if the current scenario of three flights were to be optimised with a local optimisation technique, the flights would not have global awareness of all flights and as a result, the respective joining and splitting points would have been located in a sub-optimal manner. Allowing all flights to have non-restricted access to global-information allows the routing model to produce an elegant optimal formation flight route.

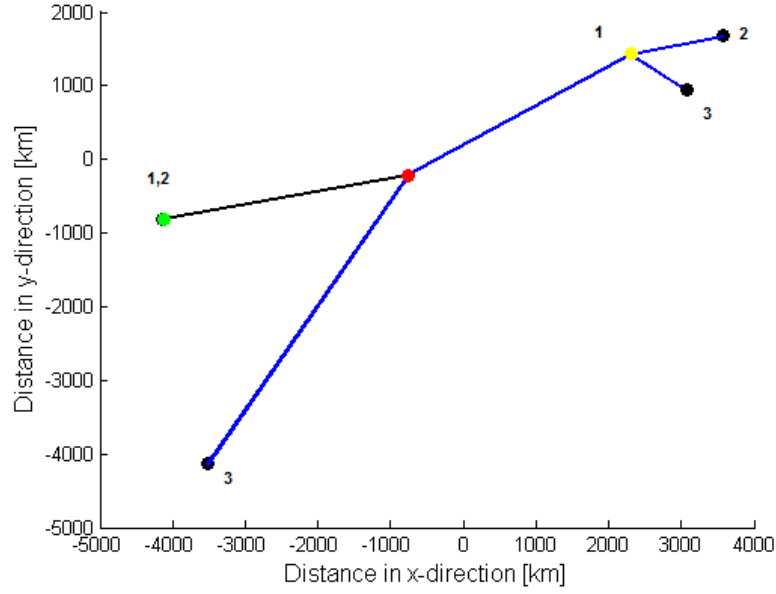


Figure 9.2: Illustration of formation routing of three flights $\{1,2,3\}$

9.1.2 Four-Aircraft Formation Routing

Figure 9.3, shows the solo routes of four flights flying from their respective origin to their respective destinations. The characteristics of similar routes between the flights as depicted by Figure 9.3, provide an ideal case for which formation flight would be an attractive fuel saving option.

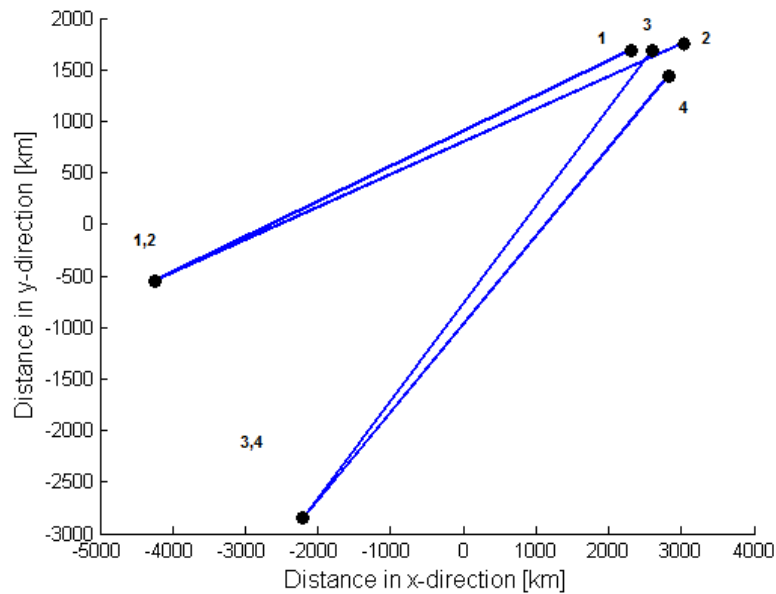


Figure 9.3: Illustration of solo routes flown by 4 flights between their respective origin and destination points

Figure 9.4, demonstrates the formation flight routes created for the 4 solo routes. The Figure shows flights 1,2, 3 and 4 flown in a two-aircraft formation size after conducting the 1st stage of the multi-stage formation flight plan. Therefore, the four flights are optimally grouped by the assignment model and assigned to fly in the following formation combination: {1,2} and {3,4}. Then, the two pairs of two-aircraft formations are defined as an event and become 2 entities for which their respective joining points and splitting points are updated as the new origin and destination points. The MS-FFLIP model, then seeks to optimally augment larger formations sizes to either a three/four-aircraft formation size. This is conducted in the 2nd stage of the MS-FFLIP model process and as a result, the assignment model at the 2nd stage, optimally assigns the two-aircraft formations to merge and thus create a four-aircraft formation. The figure shows the pair of the two-aircraft formation flying towards the optimal joining point (red dot) in order to assemble into a four-aircraft formation. Thereafter, the four flights fly in formation with one leader and three followers until they arrive at the optimal splitting point (yellow dot). The four-aircraft formation dis-assembles to a pair of two-aircraft formations and they fly with this two-aircraft formation to their second splitting points which was the original splitting points in the 1st stage (green dots). Finally, the two pairs of two-aircraft formation disperse to their respective destinations in solo flight.

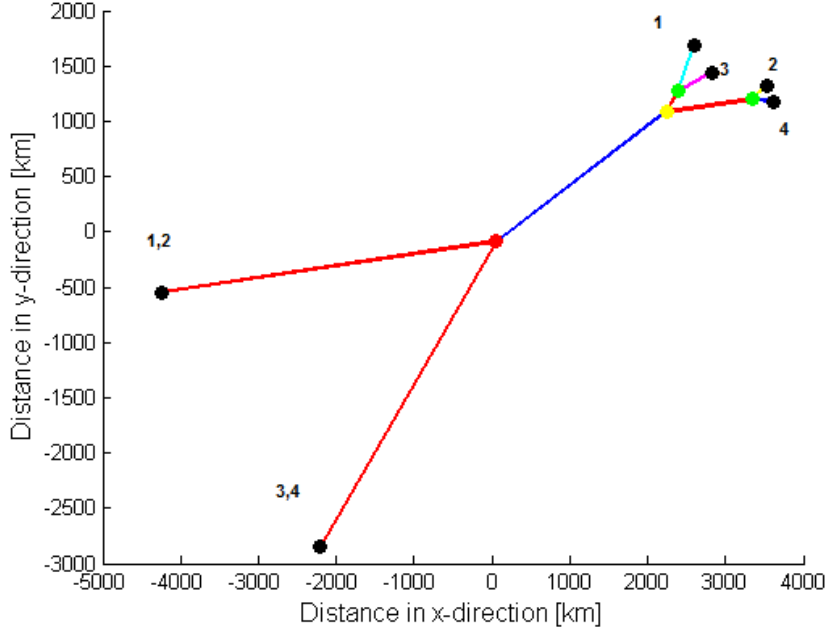


Figure 9.4: Illustration of 4 optimal formation routes for flights {1,2,3,4}

It is important to highlight the optimal dis-assembly process of this four-aircraft formation. Figure 9.5, shows an exploded view of the four-aircraft dispersing to their respective destinations. This optimal dis-assembly was achieved due to the optimisation algorithm which possesses the ability to anticipate formations beforehand, thereby allowing the routing model to conduct an optimal routing process. Moreover, the optimisation method is a global method, hence complete information is available. If the global optimisation method were to be replaced with a local method, the (dis)-assembly process would be sub-optimal. This was experienced by the work of Verhangen et al. [6] for which their decentralised greedy local optimisation method failed to anticipate formations and thus it led to a sub-optimal dis-assembly.

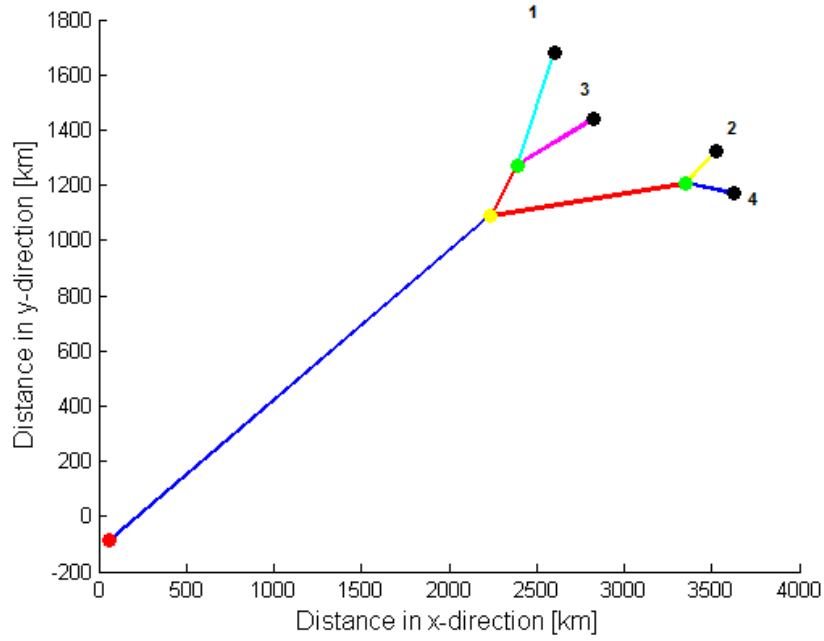


Figure 9.5: Illustration of optimal dis-assembly for four-aircraft formation scenario

Figure 9.6, shows the routes of four solo flights, with large spread in origin and destination locations. Intuitively, one would envision that it is in the best interest if the four flights try to group together and create formations in an attempt to save fuel. Therefore, Figure 9.7 presents the result of the MS-FFLIP model for the four solo routes.

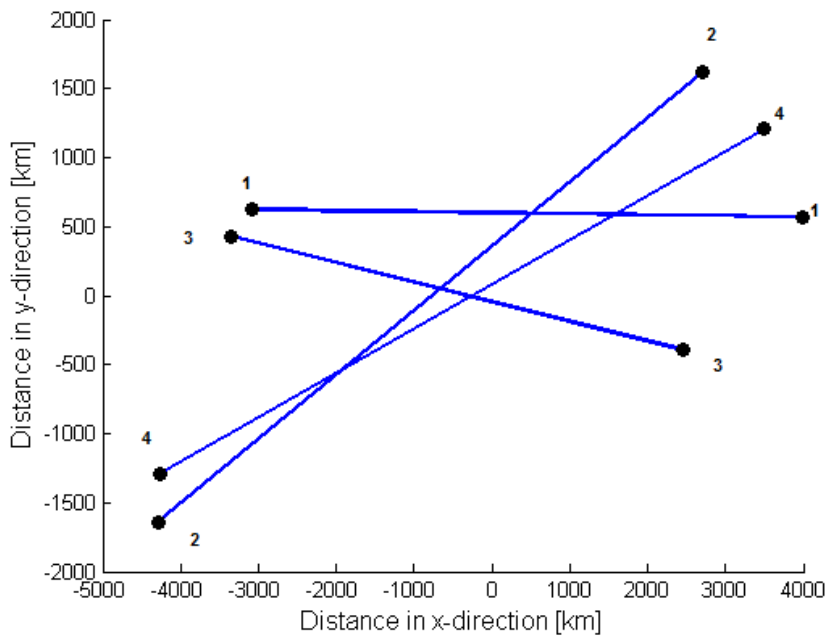


Figure 9.6: Illustration of four solo flights from widely spread origin and destination locations

As demonstrated in Figure 9.7, a four-aircraft formation was created by the MS-FFLIP model.

Flights {1,3} and {2,4} were first assigned to fly to in a two-aircraft formation during the 1st stage of the multi-stage process. Their merging of the flights at their respective joining points (east side green dots) created two events which were then categorised as two entities. The MS-FFLIP model then conducted the 2nd stage of the optimisation process, thus creating a four-aircraft formation.

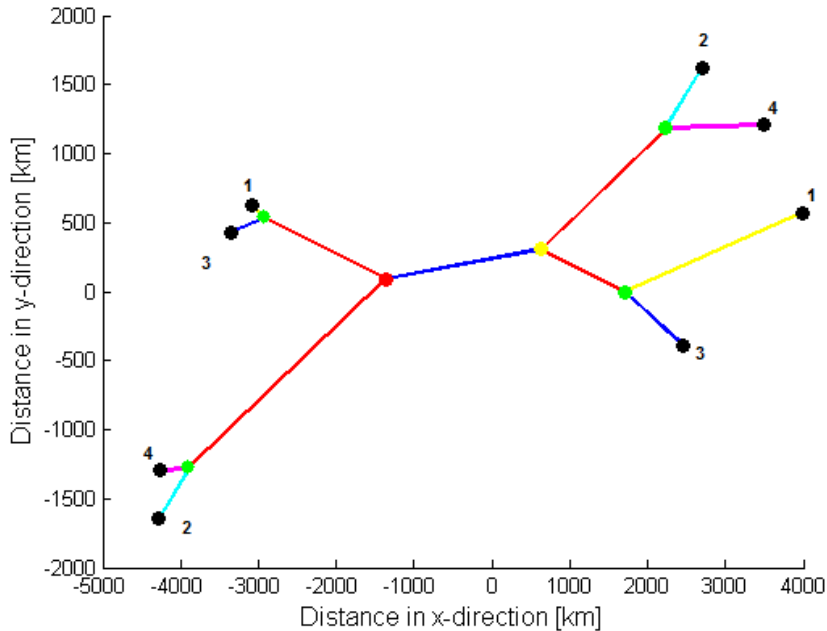


Figure 9.7: Four flights in formations with large origin and destination spread

Once the four-aircraft formation with one leader and three followers arrives at the splitting point, the formation splits into pairs of two-aircraft formations. The flights continued to fly in this two-aircraft formation until they reached their respective final splitting points (green dots on right side of figure). Thereafter, the aircraft fly to their respective destinations in solo flight. It is important to note the elegant formation routing patterns for all the examples shown in this section. Having complete information entails optimal decisions, thus optimal routing and better fuel savings.

9.1.3 Eight-Aircraft Formation Routing

Figures 9.8 to 9.10, demonstrate the capability of the MS-FFLIP model in dealing with relatively larger formations. For the purpose of illustrating the routing of formation flight by the MS-FFLIP model, this section presents the routing scenario of an eight-aircraft formation flight route. Figure 9.8, illustrates the routes for eight solo flights which depart from their respective origins and fly east towards their destination points. Moreover, Figure 9.8, depicts the similarities of the eight solo flight routes, for which two groups of four flights have the same routes. Therefore, this scenario presented an ideal case for which the application of formation flight would be beneficial in terms of fuel saving potential. At the 1st stage of the MS-FFLIP process, the flights create four pairs of two-aircraft formations. These four pairs become four entities and are eligible for grouping into larger formations in the 2nd stage. At the end of the 2nd

stage, two groups of four-aircraft formations were created, thus creating two entities in order to further augment the formation size in the 3rd stage. Therefore, at the end of the 3rd stage, the two entities (eight aircraft or two groups of four aircraft) were assigned to fly an eight-aircraft formation as shown in Figure 9.9. This eight-aircraft formation consist of one leader and seven followers that fly towards an optimal splitting point where they disassembled to fly towards their respective destinations in a four-aircraft formation size. This scenario highlights the model's capability of maximising the size of formations.

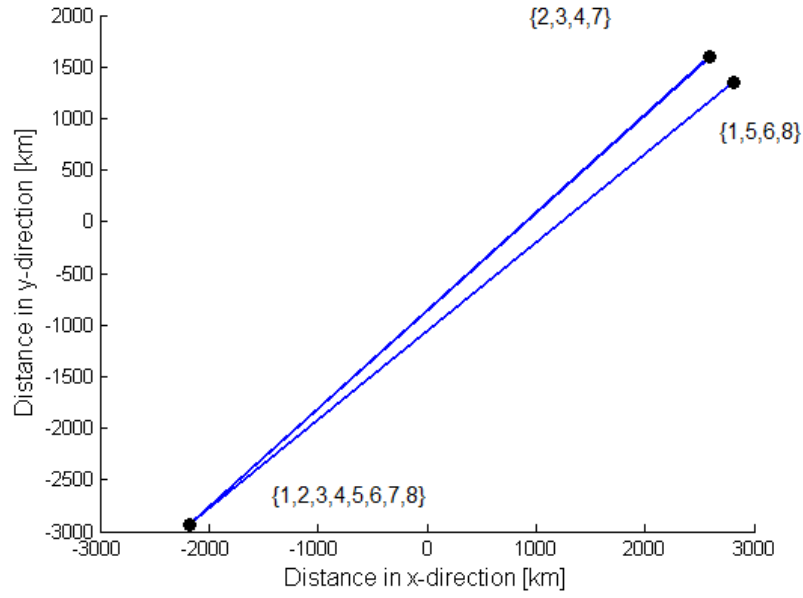


Figure 9.8: Illustration of 8 aircraft with 2 equal solo routes

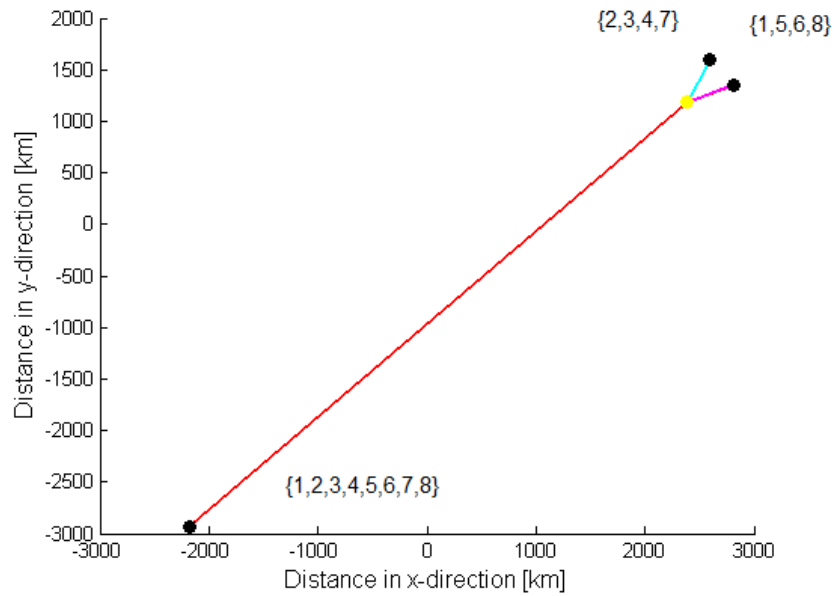


Figure 9.9: Illustration of 8 aircraft in formation flight

Figure 9.10, demonstrates the formation routing for eight random spread of origins and destination points. One can observe that a more fragmented use of formation flight routing was created that amalgamate to form an eight-aircraft formation size. Figure 9.10, shows flights $\{1,2\}$, $\{3,4\}$, $\{5,6\}$ and $\{7,8\}$ merge to create four pairs of two-aircraft formations in the 1st stage of the MS-FFLIP model. The purple dots on the left-side of the figure represent the joining points for the two-aircraft formations. In the 2nd stage of the MS-FFLIP model, the two-aircraft formations were optimally assigned to two groups of four-aircraft formations and subsequently, in the 3rd stage the two groups of four-aircraft formations combined to create an eight-aircraft formation size. The red dot in the figure represents the joining point for the eight-aircraft formation for which there are seven followers and one leader flying towards the optimal splitting point. At the splitting point, the eight-aircraft formation disperse into two pairs of two-aircraft formations and one four-aircraft formation as depicted in Figure 9.10. The green dot on the right side of the figure represents the splitting point for the four-aircraft formation. The four-aircraft formation further splits into two pairs of two-aircraft formations and then finally into solo flights. It can be seen that at all disperse instances, the MS-FFLIP model aims to optimise the routing of the flights in order to minimise the fuel burn. Furthermore, access to all information compliments the global routing of formation flights, thus preventing sub-optimal routing of formation flights.

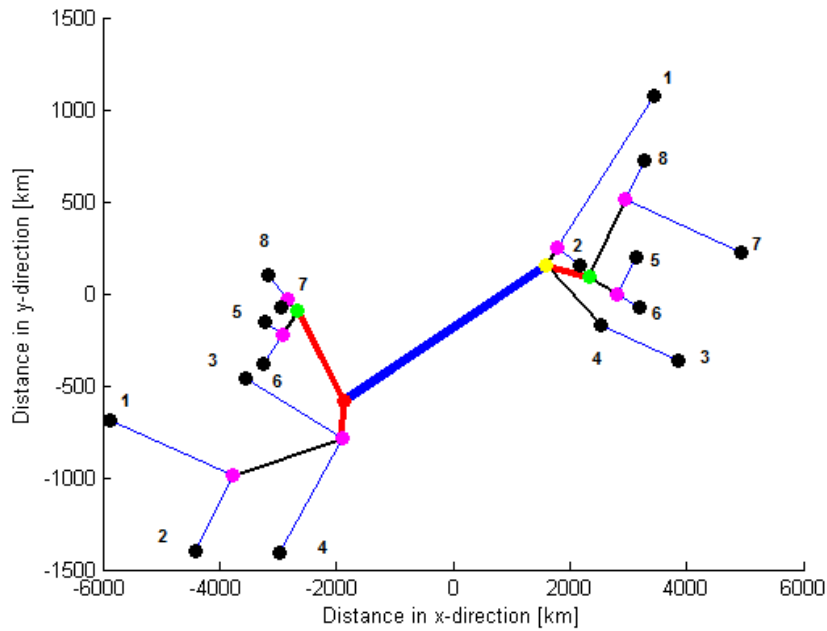


Figure 9.10: Illustration of an eight-aircraft formation for randomly spaced origins and destination points

9.2 Case Study: 267 Eastbound Transatlantic Flights

The case study scenario for this research work deals with the data set of 267 eastbound transatlantic flights between the United States and Europe as represented in Figure 9.11. It is assumed that all flights belong to a single alliance, such as Oneworld or SkyTeam, therefore ensuring full cooperation amongst aircraft members.

As discussed in Section 3.6.3, the transatlantic flights scenario is an ideal scenario for the utilisation of formation flights. Therefore, the results presented in this section will provide a strong motivation towards formation flight implementation in commercial aviation. In addition, the 267 flights are considered a large enough data set to observe intriguing results pertaining to the potential fuel savings and the computation time of the MS-FFLIP model. Moreover, the same scenario is simulated with the decentralised local optimisation method, in order to highlight the potential fuel saving benefits between the two models. It is important to note that all trailing aircraft achieve a 10% reduction in fuel burn. Furthermore, to demonstrate the capability of the model in being able to handle combinatorial problems, all flights are assumed to depart at the same time. This provides a larger data set for the combinatorial optimisation problem.



Figure 9.11: Projection of 267 great circle routes [47]

9.2.1 MS-FFLIP Results

The multi-stage formation flight planning method consist of four stages, therefore, the results obtained for the 267 transatlantic scenario at each stage will be presented in this section.

Stage 1

The 1st stage of the MS-FFLIP model is inherently limited to form a two-aircraft formation size and therefore, all possible two-aircraft formation combinations are enumerated during this 1st stage of the MS-FFLIP process. As a result, the 1st stage conducts the enumeration of all 267 solo flights and all 35,511 (267 Choose 2) combinations and since the MS-FFLIP model uses a global optimisation method, the solutions obtained in this 1st stage are global solutions.

In addition, a close examination of the stage 1 process demonstrated that only 38% of the possible 35,511 combinations were favourable in terms of fuel saving potential. Notably, a heuristic “filter” was not employed in this study since it did not lead to any improvement in computational performance. This is because, more than 95% of the computation time was required for computing the optimal routes. This statement is also supported by the work of Xu et al. and Kent & Richards, in which the authors claimed that filtering unfavourable formation combinations does not improve the computational time performance of the simulation [24], [27].

Table 9.1, presents the results obtained at the end of the 1st stage. As shown, a 4.80% reduction in overall fuel consumption was achieved, thereby amounting to 834,448kg of total fuel savings. Furthermore, the computational time (CPU time) of just over 2 minutes was noted. In addition, a high formation participation rate of 98% (133 pairs of two-aircraft formations) was seen which translates to the fact that almost all 267 flights were optimally grouped into two-aircraft formations, hence the reason for such high fuel savings. If the flights adhered to their actual flight schedule the formation participation rate would be estimated to be approximately 70%.

Table 9.1: Fuel saving against corresponding solo flights for stage 1 of MS-FFLIP model

Formation sizes	Combinations	Computation time (mm:ss)	Fuel saving	Fuel saved
2	35,511	02:21	4.80%	834,448 kg

Additionally, it is important to analyse the average fraction of total time spent in solo and in a two-aircraft formation. The formation flight times observed in the current literature were based on the distance covered by the formation flights [23]. Therefore, in order to allow for a comparable analysis of this study against the literature, similar flight time indicators were used. However, since this study accounted for synchronisation, using the distance covered would overestimate the time spent in formations. As a result, the fraction of time spent in formations were computed using the respective flight speeds per segment and the flown distance. Table 9.2, presents the results obtained for the average fraction of total time spent in a formation and in solo flight for the 1st stage. An average of 35% of total flight time was spent in a two-aircraft formation which was also in accordance with the work of Xu et al [23]. Further examination of the time spent in formation indicated that around 15 pairs of two-aircraft formations spent more than 60% of the total time flying in formations while 7 pairs of two-aircraft formations spent less than 10% of their total time in formation flight.

Table 9.2: Stage 1 average fraction of time spent in solo and in a two-aircraft formation size relative to the total flight time

	Solo	Two-aircraft formation
Average fraction of total time	0.651	0.349

Stage 2

After obtaining the optimal two-aircraft formation combinations from the 1st stage, the solutions are used in the 2nd stage in order to increase the formation size. Hence, this increases the number of trailing aircraft which subsequently increases the overall fuel saved without being adversely affected by combinatorial explosions. Therefore, in this 2nd stage of the MS-FFLIP process, each optimally assigned 133 pairs of two-aircraft formations from the 1st stage, defined

as 133 entities, create formations with the single solo flight and also create pairwise combination sets with each other (133 Choose 2). This allows for three and four-aircraft formations to be formed in this 2nd stage of the MS-FFLIP process. Table 9.3 presents the results for this 2nd stage. As seen, a 6.29% reduction in fuel savings was obtained, hence indicating that the respective formations have increased in number of trailing aircraft. Moreover, the total number of possible combinations have increased relative to the 1st stage. This increase in number of possible combinations is the cumulative number of combinations of two, three and four-aircraft formations: 35, 511 two-aircraft formations, 133 three-aircraft formations and 8, 778 (133 Choose 2) four-aircraft formations combinations. Therefore, this increase in the number of combinations explains the increase in the computational time as all combinations need to be enumerated.

Additionally, the analysis of the results indicated that 240 aircraft were grouped into groups of four-aircraft formations, forming 60 optimally allocated four-aircraft formation groups. Furthermore, 11 pairs of two-aircraft formations were optimally assigned while only a single three-aircraft formation group was formed as shown in Table 9.4. The remaining flights were assigned to fly solo as they did not contribute to the overall fuel savings.

Table 9.3: Fuel saving against corresponding solo flights for stage 2 of MS-FFLIP model

Formation sizes	Combinations	Computation time (mm:ss)	Fuel saving	Fuel saved
2,3,4	44,422	05:36	6.29%	1,090,790 kg

Table 9.4: Number of optimally assigned formation pairs in stage 2

Formation size	Pairs/groups
One-aircraft	2
Two-aircraft	11
Three-aircraft	1
Four-aircraft	60

Furthermore, the fraction of total time spent in the two, three and four-aircraft formations are depicted by Figure 9.12. The figure clearly shows that the majority of the flight time was spent flying in four-aircraft formations. Therefore, this explains the 24% (263,452kg) increase of in fuel saved relative to stage 1. This increase in fuel savings is also attributed to the higher number of follower aircraft than leading aircraft compared to the previous stage.

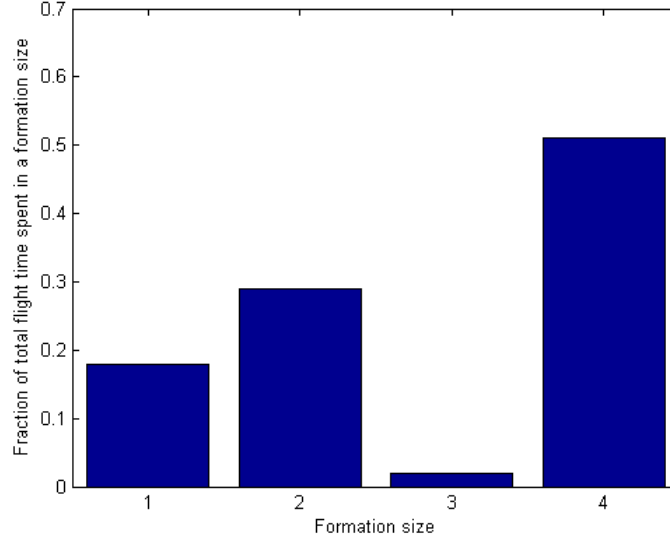


Figure 9.12: Fraction of total time in formation size for stage 2

Stage 3

In this 3rd stage of the MS-FFLIP simulation process, the solutions obtained from stage 2 are used in order to assemble larger formation sizes. The optimal formation groups in stage 2 are assigned as single entities, i.e., the 60 optimally assigned four-aircraft formation groups were defined as 60 entities that become eligible for pairing with each other. In other words, for the 60 entities there exist 1,770 (60 Choose 2) eight-aircraft possible formation combinations. Similarly, the 11 optimal formation groups of two-aircraft formations (11 entities) merge with the 60 entities in order to form 660 possible six-aircraft formation combinations. Moreover, the group of two-aircraft formations or 11 entities, pairs with the single three-aircraft entity in order to form 11 possible combinations of five-aircraft formations. In addition, the 60 entities of four-aircraft formations are also eligible for pairing with the 2 solo entities flights, thus forming 120 possible five-aircraft formations. To form a seven-aircraft formation size, the 60 four-aircraft entities were paired with the single three-aircraft formation entity in order to create 60 possible seven-aircraft formation groups. Therefore, together with stage 2, the total number of possible combinations amounts to 47,043. All these possible formation combinations with respect to each others formation sizes were enumerated by the MS-FFLIP model.

Table 9.5, presents the results for stage 3. The analysis of the results indicated that two,three,four, six and eight-aircraft formations were optimally assigned by the assignment model as shown in Table 9.6. However, five and seven-aircraft formations were not selected as they did not contribute to the overall fuel savings of the fleet. This stage achieved a 6.74% fuel consumption reduction for the 267 flights against the corresponding solo flights. Moreover, the computation time has now risen by 30% (3 minutes) compared to stage 2. This increase in computation time is caused by the greater enumeration time required for the larger formation size routing calculations.

Table 9.5: Fuel saving against corresponding solo flights for stage 3 of MS-FFLIP process

Formation sizes	Combinations	Computation time (mm:ss)	Fuel saving	Fuel saved
2 -8	47,043	08:20	6.74%	1,177,000 kg

Table 9.6: Number of optimally assigned formation pairs in stage 3

Formation size	Pairs/groups
One-aircraft	2
Two-aircraft	14
Three-aircraft	1
Four-aircraft	29
Six-aircraft	1
Eight-aircraft	14

Figure 9.13, illustrates the fraction of total time spent in formation during stage 3. Similar to stage 2, the majority of the flight time was spent in four-aircraft formation sizes. Therefore, this means that the majority of the eight-aircraft formations were spent assembling into formations, i.e., spending majority of their flight time in two and four-aircraft formations in order to assemble the eight-aircraft formations. Hence, this explains the relatively less time spent in the eight-aircraft formation. This smaller fraction of the time spent in eight-aircraft formations is also represented by moderate increase in total fuel saved of 7.3% (86,210kg) for this 3rd stage compared to 2nd stage.

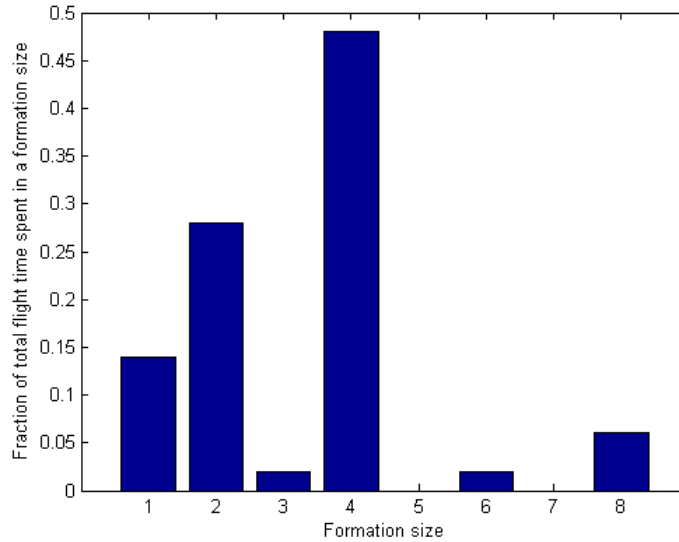


Figure 9.13: Fraction of total time in formation sizes for stage 3

Stage 4

The 4th stage was the final stage of the MS-FFLIP model. At this stage, flights were able to create formations comprising of maximum sixteen-aircraft within a formation. Similar, to the

previous stages, the solutions obtained from the 3rd stage was used in this 4th stage in order to concatenate larger formation sizes.

The 14 optimally assigned groups of eight-aircraft formations, now defined as entities create pairwise combination sets. Hence forming 91 (14 Choose 2) possible sixteen-aircraft formation combinations. Moreover, the 14 entities of eight-aircraft also merge with the optimally assigned single six-aircraft formation, thereby forming 14 groups of fourteen-aircraft formations. Next, the 29 entities of four-aircraft formations were forced to join with the 14 entities of eight-aircraft formation sizes in order to give 406 possible twelve-aircraft formations. Then, the 14 entities of eight-aircraft formation sizes were paired with the single entity of the three-aircraft formation in order to form 14 groups eleven-aircraft formation. Next, the 14 eight-aircraft formation entities and were forced to merge with the 14 entities of two-aircraft formations, thus forming 196 ten-aircraft formation combinations. In the same token, the single six-aircraft formation was merged with the 29 four-aircraft entities to create 29 ten-aircraft formations, hence a total of 225 ten-aircraft formations are created. Subsequently, 28 combinations of nine-aircraft formations were formed by combining the entities of the 14 eight-aircraft formations and the 2 solo flight entities. Similarly, the single entity of six-aircraft formation joined with the three-aircraft formation entity to form a single nine-aircraft formation as well, hence a total of 30 nine-aircraft formation combinations.

Notably, formation sizes thirteen and fifteen were not able to assemble since five and seven-aircraft formations sizes were not optimally assigned in the 3rd stage. Therefore, together with stage 3 combinations, 47,822 combinations were enumerated in stage 4.

The assignment model then optimally allocated the flights to their respective formation sizes. It was observed that only two, three, four, six, eight and sixteen-aircraft formation sizes were optimally selected and as presented in Table 9.8. The assignment algorithm allocated 116 flights into groups of four-aircraft formations, subsequently, 48 flights were assigned to groups of eight-aircraft formations followed by 64 flights allocated to sixteen-aircraft formation sizes. Compared to the 3rd stage, the 4th stage only achieved an overall fuel saving increase 2% (25,000kg), thereby contributing to a cumulative fuel savings of 1,202,000kg or a cumulative reduction of fuel burn of 6.88%.

This marginally fuel savings of 25,000kg compared to stage 3 is explained by low flight time spent in the larger formation sizes as depicted by Figure 9.14. Even with the required enumeration of large formation sizes, the computational time was under 10 minutes. If original centralised algorithm of Kent & Richards was used, in lieu of the MS-FFLIP model, the routing model would have had to enumerate 2×10^{25} (267 Choose 16) possible sixteen-aircraft formation combinations instead of just 91 combinations and clearly it is unrealistic to compute all these combinations. Therefore, the MS-FFLIP model has achieved a major breakthrough in centralised formation planning.

Table 9.7: Fuel saving against corresponding solo flights for stage 4 of MS-FFLIP model

Formation sizes	Combinations	Computation time (mm:ss)	Fuel saving	Fuel saved
2 -16	47,822	09:24	6.88%	1,202,000 kg

Table 9.8: Number of optimally assigned formation pairs in stage 4

Formation size	Pairs/groups
One-aircraft	2
Two-aircraft	14
Three-aircraft	1
Four-aircraft	29
Six-aircraft	1
Eight-aircraft	6
Sixteen-aircraft	4

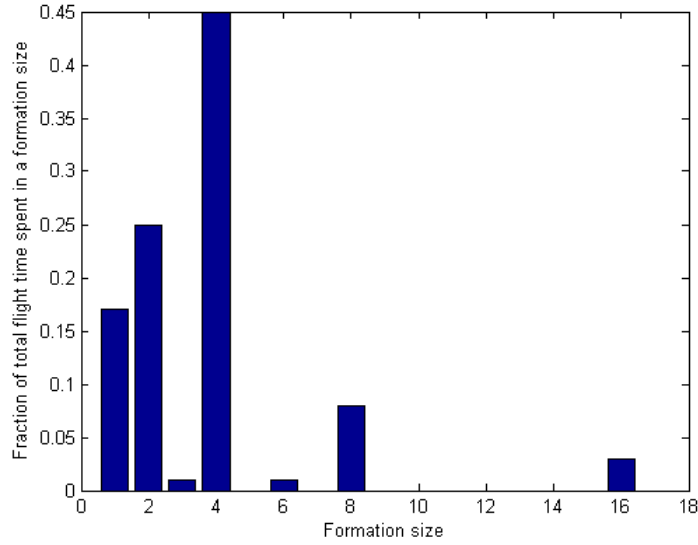


Figure 9.14: Fraction of total time in formation sizes for stage 4

9.2.2 Decentralised Optimisation Planning Method: Greedy Model

In order to further motivate the implementation of the developed MS-FFLIP model, it needs to be compared against a local optimisation-based approach in order demonstrate the higher fuel savings. This section presents the results for the performed decentralised local optimisation-based formation planning method for the same scenario as conducted by the MS-FFLIP model.

Before presenting the results, the model configuration presented in Table 9.9 will be discussed. The simulation of this decentralised formation flight planning method is conducted using a discrete time simulation with time steps consisting of 5 minute intervals. This means that the optimisation of formation flights performs a local search at each 5 minute time intervals. Furthermore, the communication radius for this optimisation is set to $250km$, hence only flights within this radius are considered for flight formation. In addition, the allowed delay time per formation decision is set to infinity in order to keep the analysis to be purely based on fuel saving potential. Moreover, the simulation of the 267 transatlantic flights was performed with the assumption that all flights depart from their respective airports at the same time and arrive at their cruising altitude of $9km$. This was done in order to have a consistent comparison with the developed MS-FFLIP model.

Table 9.9: Greedy model configuration parameters

Parameter	Value
Time step	5 minutes
Communication range	250km
Decision time per formation	∞
Departure time	Equal

Table 9.10: Legend descriptions for plots 9.15 to 9.18

Dot colour	Representation
Blue	Origin/Destination
Green	Solo flight
Cyan	Engaged flight
Purple	Formation flight

The results for the local optimisation model will now be presented by way of screen-shot illustrations of the conducted simulation.

Figures 9.15 to 9.18 show screen-shots of the formation flight results for a range of time steps during the simulation process. The blue markings on the figures represent the respective origins (right side) and destinations (left side) of the flights. Since all flights are imposed with equal departure times, they all emerge (“pop up”) together and fly solo (green dots) while communicating with potential partners in order to negotiate a possible formation. Once a formation is negotiated, the pairing flights become engaged (cyan dots), they change flight plans and fly towards their joining points in order to initiate formation flight as represented by the purple dots in Figure 9.15. Since all flights depart at the same time, a large majority of the flights start to engage and negotiate possible formations as depicted by the cyan dots in Figure 9.15.

At the 40th time step demonstrated by Figure 9.15, only two and three-aircraft formations sizes were created. However, as represented by the diameter of the purple dots in Figure 9.16, the formation sizes have increased in size half way through the flight leg.

At the 80th time step, the largest formation size comprising of sixteen-aircraft were formed. Having to wait for the creation of larger formation is a disadvantage since it causes a missed opportunity to save fuel. Furthermore, one can observe that there are still some solo flights that have not found suitable partners. This might be because they either do not contribute to any fuel savings or they are outside of the local search area (communication range).

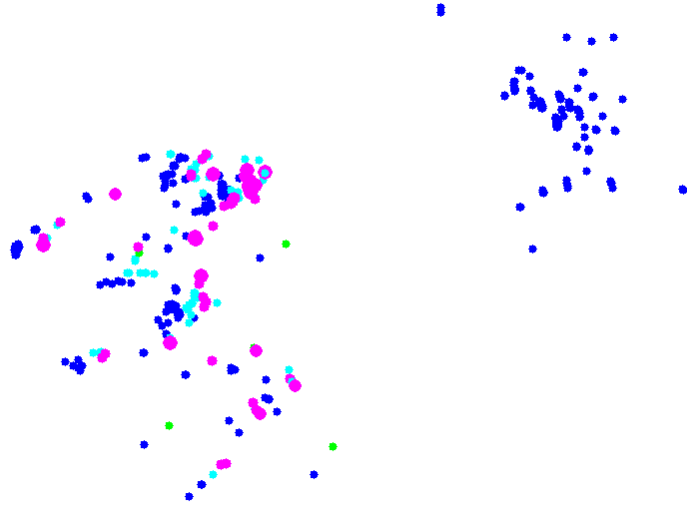


Figure 9.15: Simulated screen shot of 267 flights at 40^{th} time step for the Greedy model

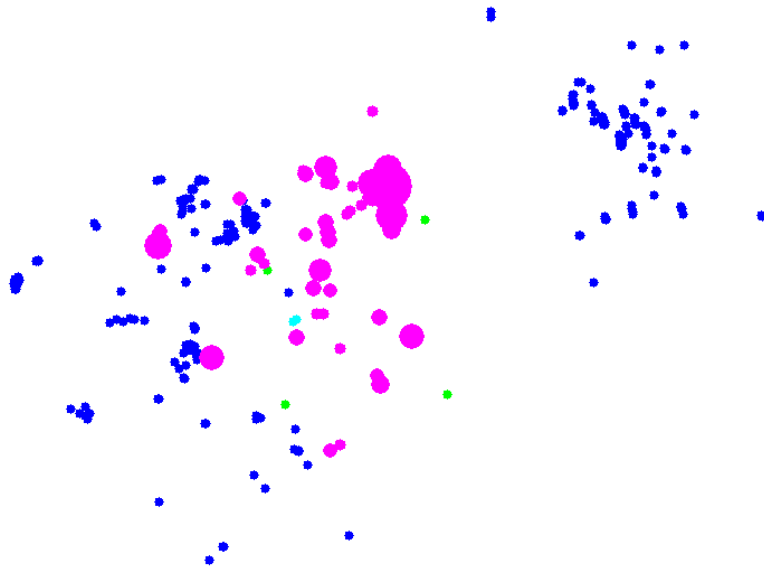


Figure 9.16: Simulated screen shot of 267 flights at 60^{th} time step for the Greedy model

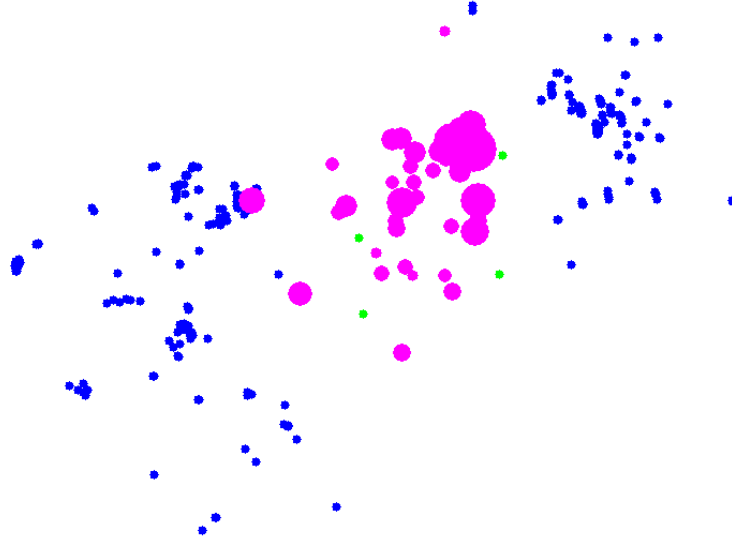


Figure 9.17: Simulated screen shot of 267 flights at 80^{th} time step for the Greedy model

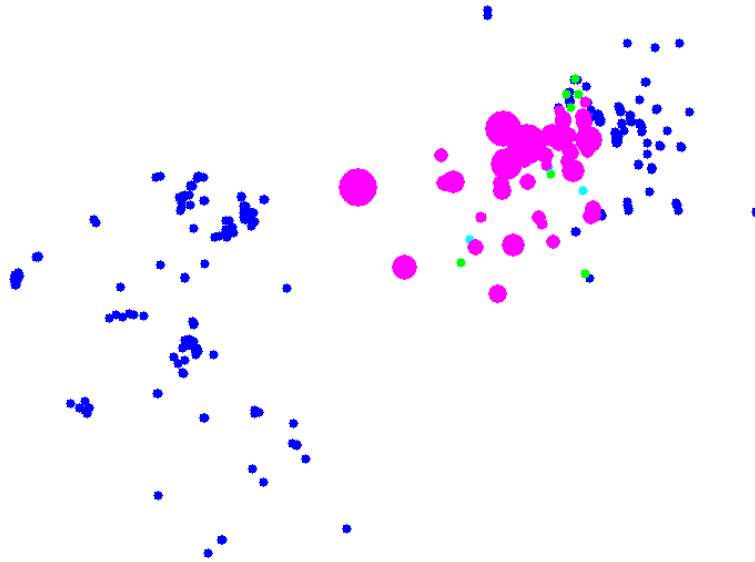


Figure 9.18: Simulated screen shot of 267 flights at 125^{th} time step for the Greedy model

Figure 9.18 shows the final stages of the simulation, where it can be seen that the large formations have now disaggregated into relatively smaller formations when nearing the end of the transatlantic flight journey. Despite the flights approaching their final destinations, a few flights were still seen negotiating possible formations, as shown by the cyan dots towards the end of the flight journey. These engaging aircraft demonstrate that the decentralised model is constantly seeking for partners to create formations. However, creating formations towards the end of the flight journey is less beneficial compared to creating formations at the beginning of the flight. Moreover, the practicality of creating formations at the end of the flight journey can be questioned, since it can complicate arrival procedures for both ATC and the pilots. This could also

cause cognitive strain for the pilots and ATC thus, creating a hazardous situation. Therefore, the formation of flights at the end of the transatlantic journey can be deemed unrealistic.

As mentioned before, this an inherent disadvantage of the optimisation algorithm used, therefore if aircraft were able to anticipate better fuel saving opportunities at the start of the flight, greater fuel savings would have been attained.

Table 9.11, summarises the results obtained after conducting the simulation. Based on the assumed 10% fuel flow reduction for trailing aircraft, an overall fuel saving of 681,200kg was obtained, hence resulting in an total fuel reduction of 3.90% with respect to the cumulative fuel burned for all 267 solo flights.

Table 9.11: Results for Greedy Simulation Model

Formation sizes	Computation time (mm:ss)	Fuel saving	Fuel saved
2 -16	3:23	3.90%	681,200 kg

The data presented by Figure 9.19 indicate that the majority of formations comprised of two-aircraft in a respective formation flight. The figure also indicates that most of the formations were inclined towards smaller sizes. In addition, the data presented in Figure 9.20, shows the percentage of total flight time spent in a respective formation size. It can be seen that only a relatively small fraction of the total flight time is spent flying in larger formation sizes, while the majority of total flight time is dominated by solo and two/three/four-aircraft formation sizes. This explains the relatively low fuel saving performance. If flights spent the majority of their flight time in large formations, they would have accumulated higher savings. Such conclusions were also seen in the work of Blake et al [4].

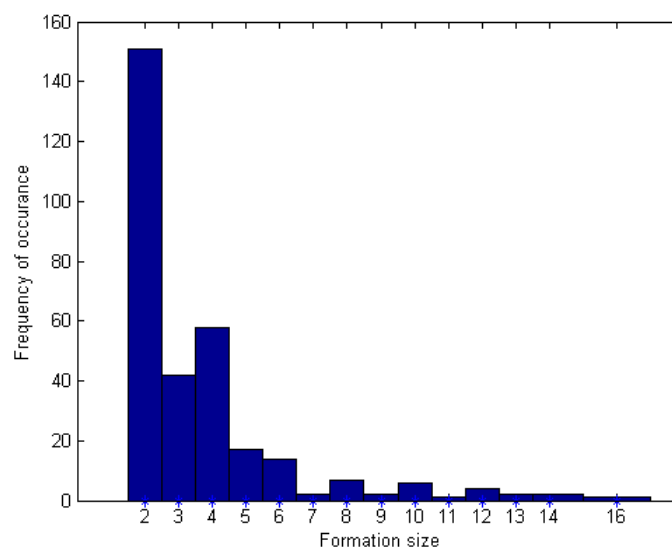


Figure 9.19: Frequency encounter for the respective flight formation sizes

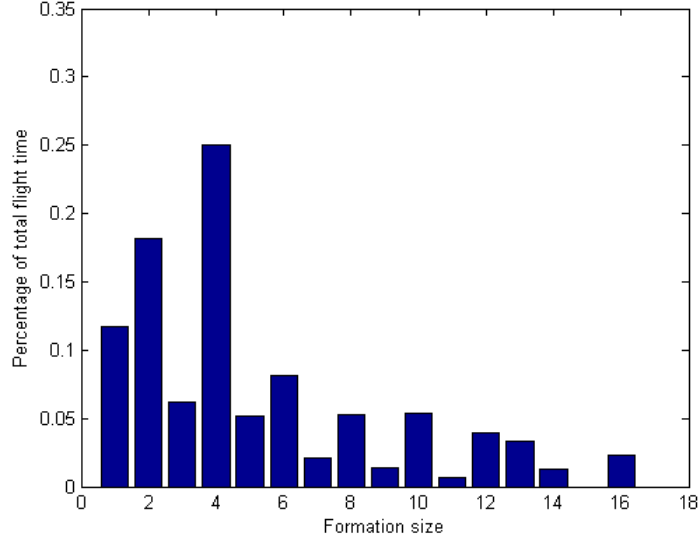


Figure 9.20: Formation size usage rate for 267 flights

9.3 Comparison: MS-FFLIP Model Against Greedy Model

The objective of this section is to compare the fuel saving potential and the computational times between the local optimisation method against the MILP global optimisation for the 267 transatlantic flight case study. As one would expect, the decentralised formation flight planning method perceived the least amount of computational time of just over 3 minutes. This was because the greedy algorithm only optimised a pair of flights for any given time, therefore it did not have to deal with the combinatorial aspect of the formation optimisation problem. On the contrary, the MS-FFLIP model performed a (near)global optimisation of the 267 flights in just 9 minutes, hence the MS-FFLIP model can be deemed viable for real-time application of formation flight operations.

Moving onto the potential fuel savings, the MS-FFLIP model achieved a 6.88% reduction in overall fuel consumption against the corresponding solo flights while the decentralised greedy algorithm model obtained a 3.90% in overall fuel reduction. Therefore, the MS-FFLIP model was able to recover 520,800kg of potential fuel savings which also translates to approximately 1.6M kg in CO_2 savings emissions. The decentralised model showed relatively lower fuel savings due to the greedy algorithm which performed a pairwise optimisation based on an aircraft individual fuel saving benefits within a local search area of 250km. On the other hand, the MS-FFLIP conducted global optimisation formulated by the MILP which optimised the fuel metric based on the overall benefit of the entire fleet. From this comparison it can be stated that the MS-FFLIP model not only recovered the majority of the potential fuel savings, it also saved a significant amount of CO_2 emissions as well. Therefore, the MS-FFLIP model is a far better formation flight optimisation tool than the decentralised model.

9.4 Comparison: Validation Model Against MS-FFLIP Model

This section presents the critical comparison between the MS-FFLIP model and the validation model which is the (single-stage) centralised model of Kent & Richards. The objective for this comparison is to demonstrate that the MS-FFLIP model is able to perform computations relatively faster than the validation model, while achieving fuel savings deviation not higher than 10%. Since the validation model is highly prone to combinatorial explosions, a smaller scenario of 50 transatlantic flights was used, with the option of creating formations comprising of two, three or four-aircraft per formation. The simulation was conducted assuming all 50 flights depart at the same time. Although this assumption is unrealistic, the assumption had to be made in order to demonstrate the combinatorial complexity of the combinatorial optimisation problem.

The simulation for the validation model was performed for the 50 transatlantic flights with the assumption of equal departure times. The results revealed a 5.72% overall fuel burn reduction against the corresponding solo flights, hence resulting in 184,381kg in overall fuel saved. Furthermore, as the model had to perform the routing for two, three and four-aircraft formation sizes, all the respective possible combinations were enumerated. The number of possible combinations were computed using the binomial equation (Equation 3.1). Therefore, for $N_a = 50$ and $n = 2$ (two-aircraft formation size) resulted in 1,225 possible two-aircraft formation combinations. Similarly, for the three-aircraft formation size, 19,600 possible combinations and, 230,300 possible four-aircraft formation combinations were computed. As a result, a total number of combinations of 251,125 were enumerated. Due to the large number of possible combinations for the three and four-aircraft formation sizes, the enumeration time dramatically increased, hence causing the computational time to rise to 58 minutes.

Next, the simulation of the 50 transatlantic flights was conducted using the MS-FFLIP model. The simulation was performed for two stages in order to obtain three/four-aircraft formation sizes. The 1st stage of the MS-FFLIP simulation obtained 24 pairs of two-aircraft formations and 2 solo flights while achieving a 4.54% in fuel savings against the respective solo flights for stage 1. Moreover, a computation time of 6.3 seconds was recorded at stage 1. Subsequently, the 24 pairs were defined as single entities, hence forming 24 entities. These 24 entities, then paired with the 2 solo flights in order to form 48 possible three-aircraft formation combinations. In addition, the 24 entities were forced to form pairs with each other, therefore creating 276 (24 Choose 2) possible four-aircraft formation combinations. As a result, the total number of enumerated combinations in stage 1 and 2 was estimated at 1,549. This relatively low number of combinations resulted in 14 seconds of required computational time. Therefore, at the end of this 2nd stage, a fuel burn reduction of 5.66% was recorded, as shown in Table 9.12.

In comparison to the single stage centralised validation model, the MS-FFLIP model is clearly more efficient in tackling combinatorial optimisation problems, as it is able to prevent combinatorial explosions from occurring. Table 9.12, presents the potential fuel savings of the two models for which the validation model saves 1,901kg more fuel relative to the MS-FFLIP model. This is a 1.1% margin in fuel saving capability and therefore, this result is well within the 10% deviation limit. For this reason, it can be conjectured that the MS-FFLIP model will maintain this same performance in fuel savings for larger number of flights as well.

Table 9.12: Fuel saving and computational time difference between centralised validation model and MS-FFLIP model

Model	Combinations	Computation time (mm:ss)	Fuel savings	Fuel saved
Validation	251,125	58:43	5.72%	184,381 kg
MS-FFLIP	1,549	00:14	5.66%	182,480 kg

9.5 Interpretation of Results

A multi-stage model has been developed that focuses on a more efficient approach towards centralised formation flight operations. The objective of this section is to highlight the value of the conducted study and the developed model by using the current literature to draw conclusions towards the credibility of the results.

The current literature on formation flight operations provides extensive research into the potential fuel saving benefits of operating formation flight for commercial airliners. However, the literature underlined several key challenges that prevent the implementation of formation flight for commercial aviation. The major challenge was the inflexibility of the models due to the high computational time burdens. Second, the models that achieved flexibility (low computational times) were not able to reap the full benefits of the potential fuel savings. The developed MS-FFLIP model, addresses all of the challenges faced by formation flight operations optimisation. However, in the MS-FFLIP model one limitation remained, i.e., all flights are assumed to have equal departure times. This assumption had to be made in order to demonstrate the capability of the model in handling combinatorial optimisation problems. This limitation can be easily removed by incorporating a realistic schedule and developing a flight generator to simulate flights departing according to their scheduled departure times.

Based on the results provided in the previous sections. The MS-FFLIP model has established the capability in preventing combinatorial explosions. This was a major drawback in coordinating centralised formation flights. Additionally, the model's fuel consumption results are more closer to reality due to the reintroduced high-fidelity Breguet range fuel-burn approximation method. Furthermore, the centralised model of Kent & Richards was extended to account for the option of larger formation sizes. This extension allowed the MS-FFLIP model to be validated for a smaller fleet size and thereby proving the model's ability in escaping enumeration time explosions while providing viable fuel saving results. These breakthroughs are adverts not just for the value of centralised formation flight planning, but for decentralised formation flight operations as well since the MS-FFLIP framework can be used in place of the greedy algorithm in the work of Verhagen et al. Before elaborating further, the properties of the MS-FFLIP model will be presented.

The MS-FFLIP model has the following properties:

- **Flexibility:** The MS-FFLIP model has the ability to handle large number of flights. Moreover, the MS-FFLIP framework is independent of the type of routes and, origin and destination locations. In other words, the model is not confined to transatlantic routes and as a consequence, the model can be applied to any route network.
- **Reliability:** The MS-FFLIP model uses a global optimisation method formulated by a MILP. Hence, the results are (near)global and as a result, the potential fuel savings are

maximised. Furthermore, the ability to handle numerous flights compliments the above statement.

- **Adaptability:** The MS-FFLIP model has the ability to conduct fast computations, thus allowing the model to be used for real-time formation flight planning. In addition, the fast computations allow for the model to account for any unexpected schedule disruptions by simply allowing the model to reiterate, in the likely event of schedule changes.
- **Operational use:** The computational time requirement of the MS-FFLIP model does not cause any disruptions to the normal functioning of flight operations. The model only requires a centralised formation controller or the Formation Air Traffic Controller for assembly of formation flights. Moreover, the MS-FFLIP model will compliment future advances in space-based ADS-B, thus allowing for reduced flight separation spacings and real-time communication links on oceanic routes.

The performed transatlantic scenario was a commonly seen case study used in centralised formation flight approaches of Kent & Richards [5] and Xu et al. [24]. Therefore, the results can be deemed comparable to a certain extent.

Before the fuel saving estimations are compared to the values seen in literature, one should bear the following limitations of the model in mind.

- The model assumes equal departure times for all flights. However, this assumption was an inherent requirement in order to demonstrate combinatorial explosion escapability.
- A 10% fuel-burn reduction for all trailing aircraft, independent of formation size was assumed. Therefore, there is no increase in fuel flow reduction with increase in formation size.
- The 10% fuel-burn reductions were based on flight test conducted on a military C-17 aircraft. This aircraft has different performance characteristics compared to the B777 commercial airliner model used for the purpose of this study. For this reason, the 10% reduction in fuel burned for trailing aircraft cannot be directly translated to commercial jets.
- Flights that are not grouped into formations are included when computing the overall fuel savings.

Notably, not all of these assumptions are new to formation flight research. The equal departure times does provide an optimistic result on potential fuel savings. However, the remaining conservative assumptions, such as equal reductions in fuel burn for all trailing aircraft do compromise the results. Furthermore, the obtained fuel consumption reduction of 6.88% for the transatlantic case study is in the same order of magnitude as in the case study of Xu et al. [24] and Kent and Richards [5].

The current case study outlined that the MS-FFLIP model produced large formations and majority of the total flight time was spent flying in four-aircraft formation sizes. If the MS-FFLIP model were to be replaced with a (single-stage) centralised model, the estimated enumeration time would be 16 days to compute a four-aircraft formation size for the 267 flights in lieu of just 9 minutes. Therefore, this is a great achievement in reducing the required enumeration time for global optimisation of formation flight operations and thus making it viable for real-time applications.

Moreover, the 50-flights transatlantic case study conducted, provided validation of the MS-FFLIP model. A 1.1% difference in potential fuel saving between the single-stage centralised validation model demonstrated that the MS-FFLIP tool models the centralised allocation process accurately. Therefore, it was conjectured that the same difference in potential fuel savings will hold true for large flight scenarios such as the current 267 transatlantic scenario.

To make better use of the MS-FFLIP model, the model can be extended for the use of decentralised formation flight operations. This can be done by running the model when each joining event occurs in lieu of waiting for all formations to occur. Moreover, instead of an event being only a joining of formations, it can also be extended to include departures, splittings, and landings as events. Therefore, the model will be able to adapt to these changes and thus have a higher degree of flexibility in modelling formation flights.

The MS-FFLIP model has revealed considerable breakthroughs in avoiding or dealing with the combinatorial explosions, which until now remained a significant obstacle for global optimisation of formation flight operations. As a result, this study provided a major leap forward towards a more adaptive formation flight planning method to be developed in the future.

10 Conclusion

In this chapter, the summary of the research contribution of this study is presented. The research contribution summary focuses on highlighting the value of the conducted research to the field of formation flight operations. Next, conclusions are drawn for the newly developed MS-FFLIP model based on the conducted research findings and case study results. This chapter will end with a comprehensive and inclusive set of recommendations for future research efforts for formation flight operations in attempts to further develop the research.

10.1 Summary of Research Contribution

The work explored in this study contributes to three main notions that excel the research on commercial formation flight planning. The third, is an efficient fuel-burn model which compliments the first two notions. The following outlines the contributions of this study:

1. **A computationally efficient method in preventing combinatorial explosions**

A multi-stage centralised formation flight planning method was developed that prevented combinatorial explosions associated to NP-hard problems, hence enabling large aircraft sets with large aircraft formation sizes to be analysed in en-route airspace, thereby allowing for a (near)global optimum of potential fuel savings.

2. **An extension of the centralised formation flight planning method**

The centralised formation flight planning framework developed by Kent and Richards was extended with the option to assign aircraft to either one/two/three/four-aircraft formation sizes. This extended centralised method was subsequently used as a validation and benchmark model.

3. **Reintroduction of the high-fidelity Breguet range fuel consumption method**

The particular form of the Breguet range fuel-burn approximation method employed herein has largely remained undetected in literature. The reintroduced Breguet range approximation method allows a higher level of fidelity in aircraft fuel consumption modelling.

10.2 Research Conclusions

This study was set out to develop an optimisation-based cooperative planning method using a MILP approach with the aim to reduce the overall fuel consumption of formation flight. This study posed the following research question:

How much potential fuel could be saved when using formation flight in civil aviation by combining the strengths of both centralised and decentralised strategies?

A multi-stage formation flight planning framework was developed which showed (near)global optimal fuel saving potential with efficient real-time computational ability.

The following conclusions lay out a set of clear research observations for proving that the developed model has indeed achieved the stated research objective and research question.

- The developed model is capable of demonstrating similar overall fuel savings to the savings obtained via the (single-stage) validation centralised formation model. Hence, proving it holds the strength of the centralised formation flight strategy.
- The developed model was able to perform event-driven simulations which led to the augmentation of consecutive formation assemblies, avoiding the occurrence of a combinatorial explosions. The MS-FFLIP model was not affected by the NP-hard inherent characteristics of the problem, and relatively low computational times were experienced.
- The developed model showed significant operational advantages in comparison to the centralised approaches seen in the work of Xu et al. and Kent and Richards, in terms of its ability to deal with schedule disturbances, and hence the possibility of conducting real-time planning. Furthermore, the MS-FFLIP model showed that it was able to assemble large consecutive formation sizes under an efficient computation time-frame. Hence, demonstrating that it holds the strength of the decentralised formation flight strategy as well.
- Similar to conventional centralised approaches, the MS-FFLIP model also made use of complete information in its optimisation process, hence providing better fuel saving potential relative to decentralised, limited-information approaches. This also meant that the MS-FFLIP model conducted optimisation on the entire search space, therefore resulting in (near)global optima.
- The model was capable of evaluating formations greater than three-aircraft formations, resulting in larger potential fuel savings.
- Similar to what was observed in the literature, the developed model was able to spend the majority of total flight time in four-aircraft formations, resulting in significant improvements in fuel consumption.
- The developed model was able to form large formations already at an early stage of the flight, hence maximising fuel saving capabilities.
- The number of flights considered in this research was much larger than in any other centralised approach observed in the current literature. Given the model's tractability, and computational efficiency, the large-scale potential of the MS-FFLIP model was exposed.
- Last but not least, the developed MS-FFLIP tool incorporated a high-fidelity fuel burn model which allowed for airspeed manipulation, hence reducing sub-optimality during synchronisation.

The fuel savings gathered in this research were obtained via a global optimisation technique on an entire network formulated by a MILP. Naturally, the fuel savings were significantly greater than the local optimisation methods used. However, it is predicted that the obtained fuel

savings could reduce with the introduction of a realistic schedule since there will be less aircraft for the respective formation groupings.

In the transatlantic case study conducted for 267 flights, fuel savings of 6.88% were recorded for the MS-FFLIP model. Furthermore, the MS-FFLIP model was validated with a fleet of 50 aircraft by way of using the developed validation (single-stage) centralised model of Kent & Richards. Notably, the fuel savings obtained for the developed model are in agreement with the encountered literature present in Chapter 2. The ability of the developed model to assemble large number of formation sizes, leads to the experienced high fuel savings. Unlike the work seen in literature, no other centralised approach was able to achieve large formations due to explosions of enumeration time. Therefore, given the employed conservative assumptions, the model showed high fuel saving potential for formation flight operations and escaped combinatorial explosions.

The work carried out in this research underlines the potential of the centralised formation flight approach and, hence it proves the approach is not limited by the NP-hard computational complexity. The MS-FFLIP model has demonstrated efficient computations, thus proving it is capable of performing in real-time applications. Therefore, the MS-FFLIP model is in principle not affected by delays since the model can be reiterated to adapt to the changing circumstances.

The developed MS-FFLIP model will further extend the research of formation flight operations due to its demonstration in potential fuel savings and efficient computational times. Its uniqueness in combining the strengths of both formation flight approaches, can be further refined by incorporating the following recommendations presented in the next section.

10.3 Recommendation for Future Work

The following set of recommendations can be used to extend the MS-FFLIP model and to see to its success in real world applications. The multi-stage formation flight planning took place under event-driven simulations and it is expected that enhancements to the event-driven simulation would lead to maximum fuel savings while offering the required adaptability to accommodate schedule delays.

- Study the effect of using a realistic flight schedule.
- Study the effect of a mixed fleet composition.
- Study the effect of incorporating trajectory-based optimisation in lieu of the geometric routing method.
- Study the effect of incorporating wind.
- Incorporate the ATM structure, taking into account present inefficiencies in vertical and horizontal flight profiles.
- Study the flexibility of flight profiles on transatlantic routes.

Bibliography

- [1] Association, I. A. T., *Economic Performance of The Airline Industry*, IATA, 2016, <http://www.iata.org/whatwedo/Documents/economics/IATA-Economic-Performance-of-the-Industry-mid-year-2016-report.pdf>.
- [2] Economist, *The Air-Leasing Sector May Soon Face Harder Times*, Economist Magazine - Aircraft Finance, 2016, <http://www.economist.com/news/business-and-finance/21699893-cheaper-oil-has-helped-many-parts-aviation-industry-not-leasing>.
- [3] Haissig, C., “Military formation flight as a model for increased capacity in civilian airspace,” *The 23rd Digital Avionics Systems Conference*, , No. IEEE Cat. No. 04CH37576, 2004, pp. 1.C.4–1.1–9.
- [4] Blake, W. B. and Flanzer, T. C., “Optimal Routing for Drag-Reducing Formation Flight: A Restricted Case,” *Journal of Guidance, Control, and Dynamics*, February 2015, pp. 1–4.
- [5] Kent, T. and Richards, A., “Analytic Approach to Optimal Routing for Commercial Formation Flight,” *JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS*, Vol. 38, 2015.
- [6] Verhagen, C., Visser, H., and Santos, B., *Formation Flight in Civil Aviation: Development of A Decentralised Approach to Formation Flight Routing*, Master thesis, Delft University of Technology, Delft, The Netherlands, June 2015.
- [7] Lissaman, P., “Formation flight of birds,” *Nature, California Institute of Technology*, 1970.
- [8] “Energy saving in flight formation.” *Nature*, Vol. 413, No. 6857, 2001, pp. 697–8.
- [9] Gill, V., *Fly like a bird: V-formation finally explained*, Science & Environment, 2014.
- [10] Ning, S. A., *Aircraft drag reduction through extended formation flight*, Ph.D. thesis, Stanford University, CA, United States of America, August 2011.
- [11] Veldhuis, L., Voskuil, M., and Fransen, B., “Formation Flight - Fine-tuning of Theoretical Performance Prediction,” *51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, January 2013, pp. 1–17.
- [12] Nangia, K. and Palmer, M., “Formation Flying of Commercial Aircraft – Assessment,” *AIAA Journal*, January 2007, pp. 1–16.
- [13] Herinckx, L. E., Dijkers, H. P. A., Nunen, R. V., Bos, D. A., Gutleb, T. L. M., Radfar, H., Rompuy, V. E., Sayin, S. E., Wit, D. J., and Blokland, V. B., “Formation flying as an innovative air transportation system for long-haul commercial flight: A focus on operational

- feasibility and potential gain,” *11th AIAA Aviation Technology Integration and Operations ATIO Conference*, September 2011, pp. 1–11.
- [14] Ning, S. A., Flanzer, T. C., and Kroo, I. M., “Aerodynamic Performance of Extended Formation Flight,” *Journal of Aircraft*, Vol. 48, March 2011, pp. 855–865.
 - [15] Ning, S. A., Kroo, I., Aftosmis, M. J., Nemec, M., and Kless, J. E., “Extended Formation Flight at Transonic Speeds,” *Journal of Aircraft*, Vol. 51, May 2014, pp. 1501–1510.
 - [16] Ning, S. A. and Kroo, I., “Compressibility Effects of Extended Formation Flight,” *29th AIAA Applied Aerodynamics Conference*, June 2011, pp. 1–24.
 - [17] Blake, W. B., “Formation Flight Interference Effects,” *Journal of Aircraft*, Vol. 41, February 2004.
 - [18] Okolo, W., Dogan, A., and Blake, W., “Effect of Trail Aircraft Trim on Optimum Location in Formation Flight,” *Journal of Aircraft*, Vol. 52, April 2014, pp. 1–13.
 - [19] Ruijgrok, G., *Elements of Airplane Performance*, Vol. 1, VSSD, Delft, The Netherlands, 2nd ed., 2009.
 - [20] Xue, M. and Hornby, G. S., “An Analysis of the Potential Savings from Using Formation Flight in the NAS,” *AIAA Guidance, Navigation, and Control Conference*, , No. August, 2012, pp. 1–12.
 - [21] Ray, R., Cobleigh, B., Vachon, M. J., and St. John, C., “Flight Test Techniques used to Evaluate Performance Benefits During Formation Flight,” *AIAA Atmospheric Flight Mechanics Conference*, , No. AIAA Paper 2002-4492, 2002.
 - [22] Bower, G. C., Flanzer, T. C., and Kroo, I. M., “Commercial Formation Flight,” *AIAA Journal*, June 2009.
 - [23] Xu, J., Ning, S. A., and Bower, G., “Aircraft Route Optimization for Heterogeneous Formation Flight,” *53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, April 2012.
 - [24] Xu, J., Andrew Ning, S., Bower, G., and Kroo, I., “Aircraft Route Optimization for Formation Flight,” *Journal of Aircraft*, Vol. 51, February 2014, pp. 490–501.
 - [25] Kent, T. E. and Richards, A. G., “A geometric approach to optimal routing for commercial formation flight,” *AIAA Guidance, Navigation, and Control Conference 2012*, August 2012, pp. 1–17.
 - [26] Kent, T. E. and Richards, A. G., “On Optimal Routing For Commercial Formation Flight,” *AIAA Guidance, Navigation, and Control (GNC) Conference*, , No. AIAA Paper 2013-4889, 2013, pp. 1–13.
 - [27] Kent, T., *Optimal Routing and Assignment for Commercial Formation Flight*, Ph.D. thesis, University of Bristol, Bristol, United Kingdom, April 2015.
 - [28] Marks, T., Linke, F., and Gollnick, V., “Evaluation of potential fuel savings by introducing formation flight on a North Atlantic scenario,” 2015, pp. 1–10.
 - [29] Belobaba, P., Odoni, A., and Barnhart, C., *The Global Airline Industry*, John Wiley and Sons, West Sussex, 1st ed., 2009.

- [30] Nehrbass, J. G., Frommer, J. B., Garison, L. A., Loffing, D. N., and Crossley, W. A., "Study Including Formation Flight and Morphing," *AIAA 4th Aviation Technology, Integration and Operations (ATIO) Forum*, September 2004, pp. 1–9.
- [31] Ribichini, G. and Frazzoli, E., "Efficient coordination of multiple-aircraft systems," *42nd IEEE International Conference on Decision and Control (IEEE Cat. No.03CH37475)*, Vol. 1, December 2003.
- [32] Ribichini, G. and Frazzoli, E., "Energy-Efficient Coordination of Multiple-UAV systems," *AIAA Guidance, Navigation, and Control Conference and Exhibit*, August 2003.
- [33] Edelkamp, S. and SchrodL, S., *Heuristic Search: Theory and Application*, Morgan Kaufmann, Waltham, MA, 1st ed., 2012.
- [34] Ruis, M. and Visser, H., *Scheduling Flights For Oceanic Track Entry: A sliding window Mixed Integer Linear Programming algorithm for altitude, speed and arrival time assignment*, Master thesis, Delft University of Technology, Delft, The Netherlands, December 2011.
- [35] Boeing, *B777 Flight Manual*, Continental Airlines, 2002.
- [36] Maling, D., *Coordinate Systems and Map Projections*, Vol. 1, Pergamon Press, University of Wales, United Kingdom, 2nd ed., 1992.
- [37] IATA, *Economic Performance of The Airline Industry*, International Air Transport Association, 2014, <https://www.iata.org/whatwedo/Documents/economics/IATA-Economic-Performance-of-the-Industry-mid-year-2015-report.pdf>.
- [38] Torenbeek, E., "Cruise Performance and Range Prediction Reconsidered," *Progress in Aerospace Sciences*, Vol. 33, No. 96, 1997, pp. 285–321.
- [39] Maido, S., *Aircraft Performance*, John Wiley and Sons, New Jersey, 2007.
- [40] Rama, V. and H, D., *Fundamentals of Engineering Mathematics*, Vol. 1, S.Chand & Company, New Delhi, 1st ed., 2008.
- [41] John, A., *Aircraft Performance and Design*, McGraw Hill, 1999.
- [42] Filippone, A., *Advanced Aircraft Flight Performance*, Cambridge University Press, New York, 2012.
- [43] Visser, H., *Terminal Area Traffic Management*, Delft University of Technology, Faculty of Aerospace Engineering, 1992, <http://adsabs.harvard.edu/abs/1991PrAeS..28..323V>.
- [44] Russ, E., *Maximum Endurance, Maximum Range and Maximum Cruise Speeds*, Experimental Aircraft Association, 1993, http://www.eaa1000.av.org/technicl/perfspds/perfspds.htm#Optimum_Cruising_Speed.
- [45] Francis, H., *Introduction to Aircraft Performance selection and Design*, John Wiley and Sons, New York, 1st ed., 1984.
- [46] Hillier, F. and Lieberman, G., *Introduction To Operations Research*, Mc Graw Hill, New York, 9th ed., 2010.
- [47] Swartz, K., *Map For Transatlantic Flights*, Great Circle Mapper, 2016, <http://www.gcmap.com/>.