Course ME2110

Literature Assignment

Fatigue reliability of belt conveyor accounting for Bayesian updating

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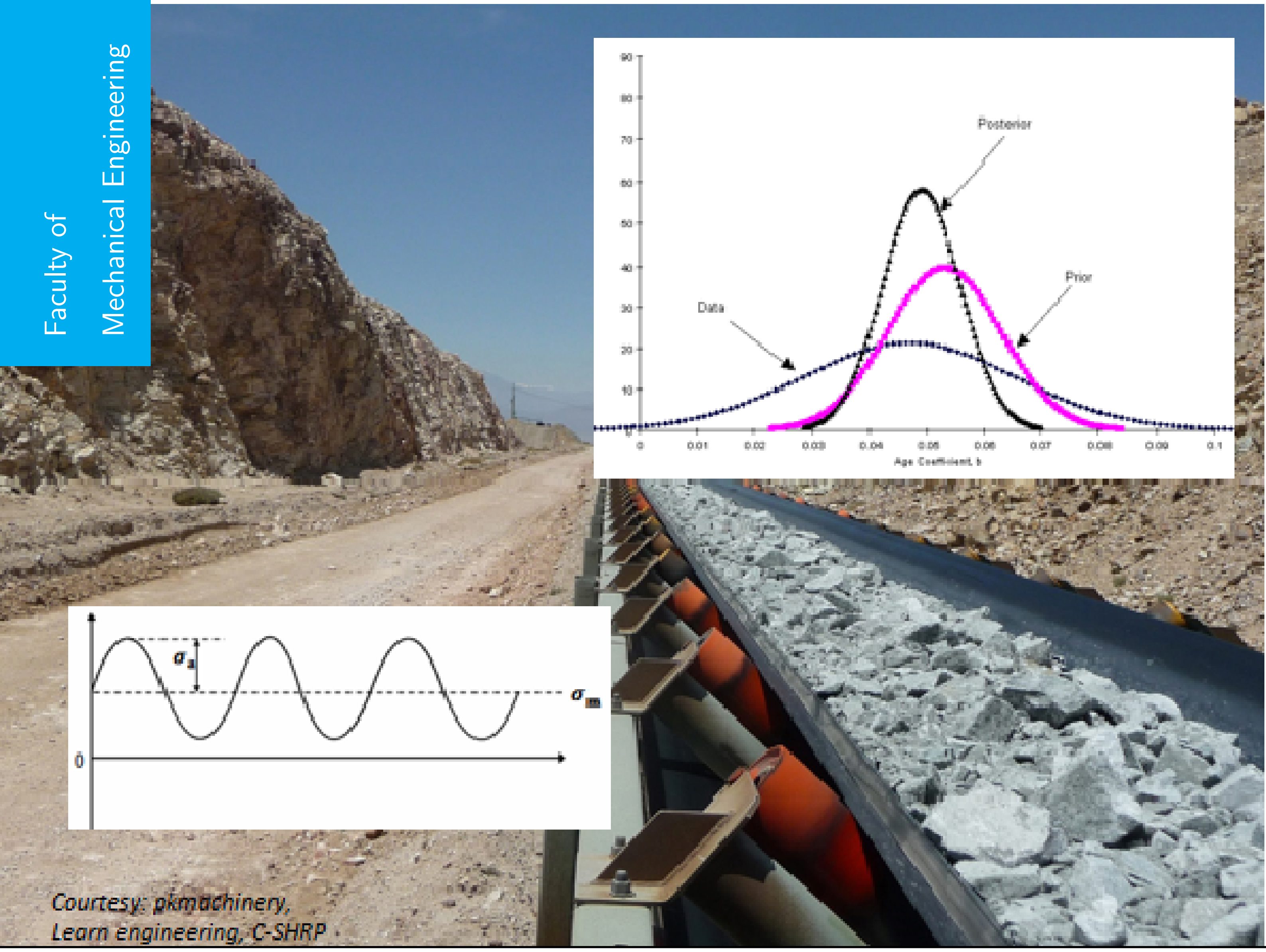

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Preface

It is a well know fact nowadays that metal structures are vulnerable to cyclic loading and fail during service. It is also being studied that these cyclic loads need not have higher magnitudes and cause structures to fail without providing any visible indication. For a century scientists have wondered as to what is the source of the phenomena because if these small loads act on their own under static conditions, structure remain unharmed [13]. This type of failure in which first a crack is formed and then it propagates to eventually lead to failure of the structure, came to be known as 'Fatigue' [13]. To prevent such failure in belt conveyor system proper maintenance strategies have to be implemented in coordination with efficient design.

Fatigue has been one of the major concerns for manufactures as predication of life of a particular component is very difficult and time consuming. Through certain software's which use multi-body dynamics, a calculated guess can be made regarding the life of components under loading. Although a lot of progress has been made to identify the area of concern and to predict the life of components situated around these areas, there is still some uncertainty while scheduling maintenance and replacement activities. These uncertainties includes environmental effects, dimensional tolerances, material properties and maintenance processes [10].

To account for these uncertainties safety factors are induced which are form of conservative estimates and thus using them leads to disagreement between field data and analytical predictions [10]. Due to evolution in sensing technologies, the health of an infrastructure can be easily monitored. All these data can then be used to predict the life of various components, all in real time. To predict this information correctly, we do need a method or methodology that can evaluate and interpret results from this huge accumulation of data. The best methodology for implementing this procedure is the use of Bayesian theorem. Bayesian theorem is used to calculate the conditional probability of the likelihood of an event happening based on probability of the events that have already happened. Also information from inspection using non-destructive techniques can be fed into Bayes theorem to make life predictions more accurately [11]. In this paper will be discuss about the possibility of implementing Bayes theorem to predict reliability of belt conveyors.

Since there has been no evidence of implementation of Bayesian theorem in predicting failures and scheduling maintenance activities in belt conveyors, this paper will discuss the areas which have implemented them and also benefited from this. Chapter 2 will discuss about the various parameters related to fatigue assessment in belt conveyors. This will be followed by Chapter 3 which will discuss the basic theory of Bayesian and discuss ways in which it has been implemented for bridges. Chapter 4 will discuss upon possible implementation of Bayesian theorem in belt conveyors. Finally Chapter 5 we will concluded by suggesting future research that can be conducted in this field.

1 Introduction

Belt conveyors are one of the most important transport infrastructure developed by mankind. It is also considered as one of the most complicated electromechanical equipment because of the amount of components that are involved. Calculating and maintaining the reliability of such an important equipment is also a difficult task owing to the number of components it comprises. Like every transport infrastructure, belt conveyors are subjected to transient loading which leads to transient stresses acting on different components of the system. Stresses of transient nature have a tendency to drastically decrease the reliability of a structure by inducing fatigue failure. Before we start to pinpoint the major assessment criteria and critical parameters of belt conveyors for fatigue, we will touch upon meaning and causes of fatigue in general.

1.1 Fatigue

While designing a component or a structure following three conditions must be satisfied [13]:

1. Performance should be as efficient as possible.
2. Fabrication should be as easy as possible.
3. Service life should be as adequate as possible.

As it can be seen from the above mentions conditions mostly the first two conditions do not compliment the third condition. This means, to achieve efficient performance and ease in fabrication components and structures have to weigh less. This means no excess thickness, less factor of safety which leads to shortfall in meeting the third condition. Such problems are encountered and seen in components and structures failing due to fatigue. An estimated, 90% of structure failure happens due to fatigue [13].

Fatigue failures in metals can be defined as formation of crack or cracks caused by a continuous damage of material which is subjected to cyclic loads each of which is insufficient to cause any damage under 'static conditions' [3]. Figure 1.1 shows the two main stages of fatigue along with the phases in each of these stages. It also points the important factors which help in calculating the effective stresses in each of these phases. In the early 1950s many researchers were able to find out microscopic cracks which made it clear that fatigue has two major stages namely, crack initiation and crack growth [1].

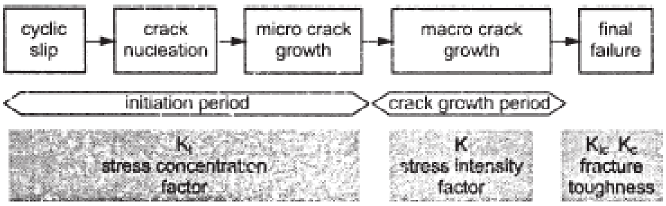


Figure 1.1: Different phases in fatigue life [1]

1.2 Characteristics of Fatigue

When a ductile metal is loaded statically like the testing for plotting Hooke's law, increasing the load from zero to maximum it is clearly seen that the material deforms leading to neck formation and eventual complete failure of the material. Figure 1.2 gives more clarity on the different aspects of failure of a ductile material.

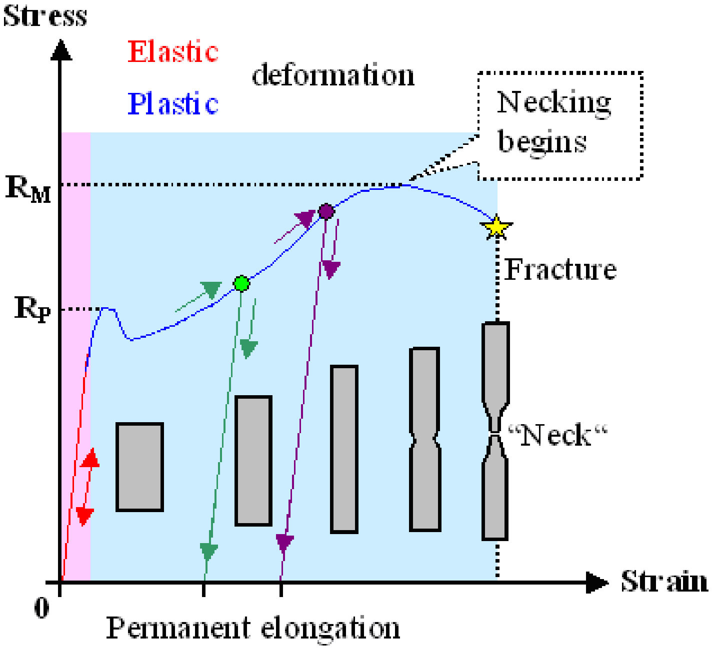


Figure 1.2: Stages of ductile material failure [2]

However, if the same material is subjected to fatigue loading, cracks may appear at stresses far lower than the yield limit of the material without any visual indication [13]. As there is no deformation or any indication, fatigue cracks are really hard to see, specially in the early stages even at likely predicted places. A fatigue fracture surface has a characteristic appearance and can be seen on the surface of components. Although, it can be said that in welded joints the crack can be easily generated due to defects in the weld itself [13].

Fatigue fracture is usually normal to the direction of principal tensile stress [3]. It generally occurs on the place of stress concentration,a smooth region, a rough area related to the failure area in a ductile manner. These places can also be seen in Figure 1.3 which provides all these regions schematically. It is important to know that these region shown below are ideal and in reality this can differ. Sometimes you might find that these regions are not easily distinguishable and also there could be more than one crack initiation points on the component [3].

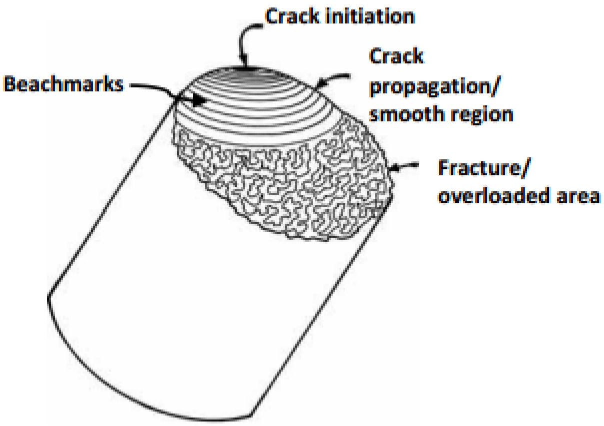


Figure 1.3: Fatigue fracture and its parts [3]

Fatigue induced in structural components leads to crack or cracks which initiates in specific sites that are suitable for them to grow. These stresses then coalescence and propagate by plastic deformation blunting [3].

1.3 Classical Fatigue analysis methods

X.W.Ye discusses in the article [14] about some of the classical analysis methods for fatigue, pointing out three major methods, stress life method, fracture mechanics and strain life method. Though these methods employ different strategies to understand fatigue process, they do have some degree of overlap between them. These methods are discussed in brief below [14]:

Using Stress-life method:

Mostly suited for HCF (High-Cycle fatigue) where both stress and strain are elastic. This method represents a relationship between stress range and fatigue failure using S-N curve. This curve is obtained from testing specimens at constant amplitude stress and are observed until cracking occurs in these specimens. W$\ddot{o}$hler was the pioneer researcher in this field who quantified fatigue strength in accordance to testing results. Basquin later represented finite life region as log N on abscissa and log S on ordinate. Mathematically the function is represented as shown in Equation 1.1 below.

$$NS^m = A \tag{1.1}$$

where 'm' and 'A' are positive empirical constants that depend upon the material. The stress-life methods usually can be divided into different categories depending upon structural stress analysis into either hot-spot method or nominal stress method. These methods are not discussed here but more information is available in article [14].

Using Fracture Mechanics:

The method is usually applied to predict the propagation life from an initial crack. This method is a linear elastic fracture mechanics which relates growth with a crack size 'a' to number of fatigue cycle 'N'. This is given by Paris's rule which is expressed in Equation 1.2 below:

$$\frac{da}{dN} = C(\Delta K)^m \tag{1.2}$$

where ’C’ and ’m’ are material related parameters and range of stress intensity factor ΔK can be determined using below equation

$$\Delta K = SY(a)\sqrt{\pi a}$$

where $Y(a)$ is the function of crack and ’S’ is stress range.

Using Strain-Life method:

This method is mostly concerned with the crack initiation stage. In this methodology the strain is considered to have gone in the plastic region rather than staying in the elastic region as in above methods. The majorly faced issue with this approach is that not a lot of research has be conducted on this method as it deals with the non-linear region of material which is quite difficult to understand even today. Another reason maybe the application of this method is usually in cases of LCF (Low cycle fatigue).

1.4 Fatigue reliability

Structural design against fatigue has always been a tedious task. Figure 1.4 shows the various aspects considered as inputs to predict life of components and their other outputs. Below figure clearly illustrates that a fatigue problem can be quite elaborate depending on the structure which is being evaluated. The prediction of fatigue involves many small steps along with some plausible assumptions. So, it can be considered that the outputs obtained are not as accurate. The accuracy of the outputs also depend upon the engineering judgement, experience and intuition [1].

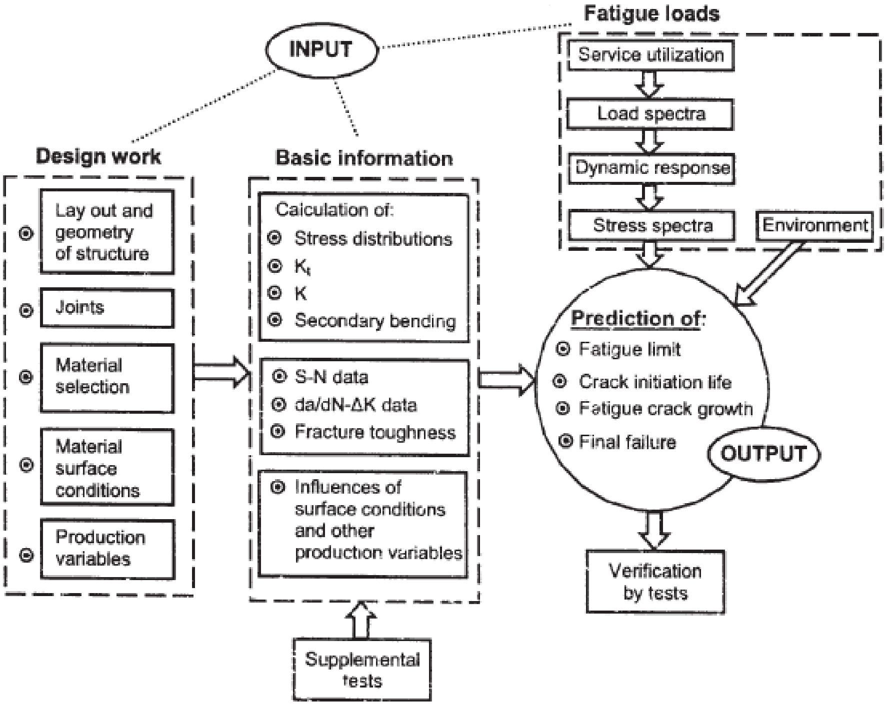


Figure 1.4: Various aspects of fatigue of structures [1]

Probabilistic approaches are more reliable and may serve as a possible way to acquire this knowledge. This approach enables the uncertainties of different parameters involved in the fatigue analysis to be propagated to the mechanical response of structure [15]. Such analysis employ a failure scenario of the structure which is mathematically represented called as performance function. This is dependent on the on random vector, let's say 'X', which deals with the uncertainties like material properties, fatigue loads and its behavior. The objective of this reliability analysis is then to assess the failure probability as given in [15]:

$$P_f \;=\; Prob(G(x) \leq 0) \tag{1.3}$$

where $G(x)$ is the performance function.

Most of the reliability based fatigue analysis is mostly focused on steel bridges [14]. The general approach that is being followed is essentially building up a mathematical model using either basics of mechanics or extensive observation of the phenomena [14]. Then the probabilistic and statistical analysis will be performed in this framework. This assessment of fatigue reliability can then be done through either Stress life method or fracture mechanics. More literature can be found in X.W.Ye's paper on "State-of-the-art review on fatigue life assessment of steel bridges" [14].

2 Fatigue Assessment in Belt Conveyor

2.1 Failures in Belt conveyors

A typical belt conveyor system consists of following main components:

1. Belt.
2. Idler rollers (Carry and return side).
3. Pulleys (Drive, tail and snub).
4. Drive unit.

Figure 2.1 shows an example of a simple belt conveyor system consisting of above mentioned parts. We will now look into the various types of failures in each of these components.

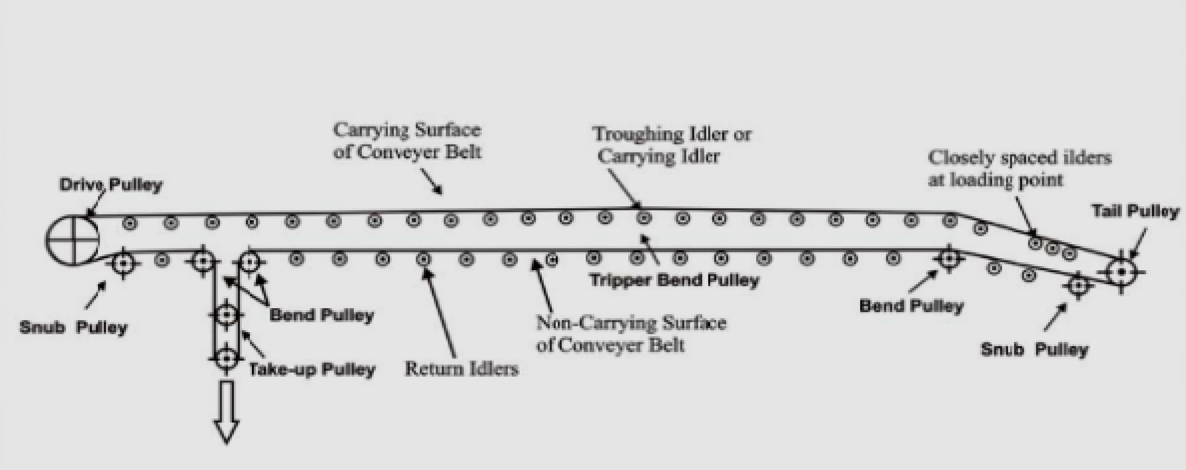


Figure 2.1: Simple belt conveyor system [4]

Belt:
There are two major reasons for belt failure according to [4] namely, belt failing to turn and belt deviation. The first one happens due to insufficient tension in the system and/or due to insufficient friction between between belt and the pulley surface.

(a) Belt Deviation [4]

(b) Belt Damage [4]

Figure 2.2: Belt Failures

Belt deviation is also a major failure of belt in conveyors as can be seen from Figures 2.2a and 2.2b. There can be many reasons for belt deviating from its original path. Some these reasons can be either loading position of the conveyor is not at the center, idler rollers on the carry side are not installed keeping the central axis on the same line. As seen from Figure 2.2b this can lead to severe belt damage and in turn leads to significant loss of productivity to the company. As it can be seen that the reasons for belt failures have no connection to fatigue, these should not be included while analyzing the fatigue strength of the system.

Idler rollers:
Idler rollers are used to support the belt on both carry and return side. They consist of shaft and a bearing and are very similar to pulley [4]. Except in case of idler rollers the shaft is not rotating. There are minimum three rollers supporting the carry side which leads to uneven load distribution on each of the idlers. The rollers placed on the sides are more susceptible to failure as can be seen from Figure 2.3.

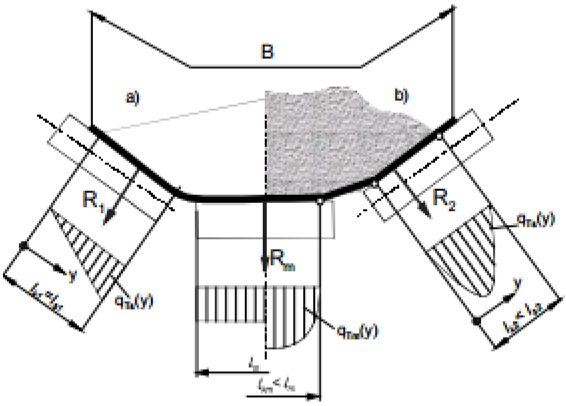


Figure 2.3: Load distribution on carry side rollers [5]

Failures in rollers leads to excessive power consumption and if rollers reach high temperatures, it can also lead to damage to the belt. A damaged idler roller due to excessive wear is shown in Figure 2.4 below.

Figure 2.4: Idler roller wear [4]

Pulley:
Conveyor pulley is a rotating device which has dynamic load acting on it which causes each of its parts to undergo reversal load after every revolution [16]. This will eventually lead to fatigue in various components of pulley. Figure 2.5 shows the different parts of a standard conveyor pulley.

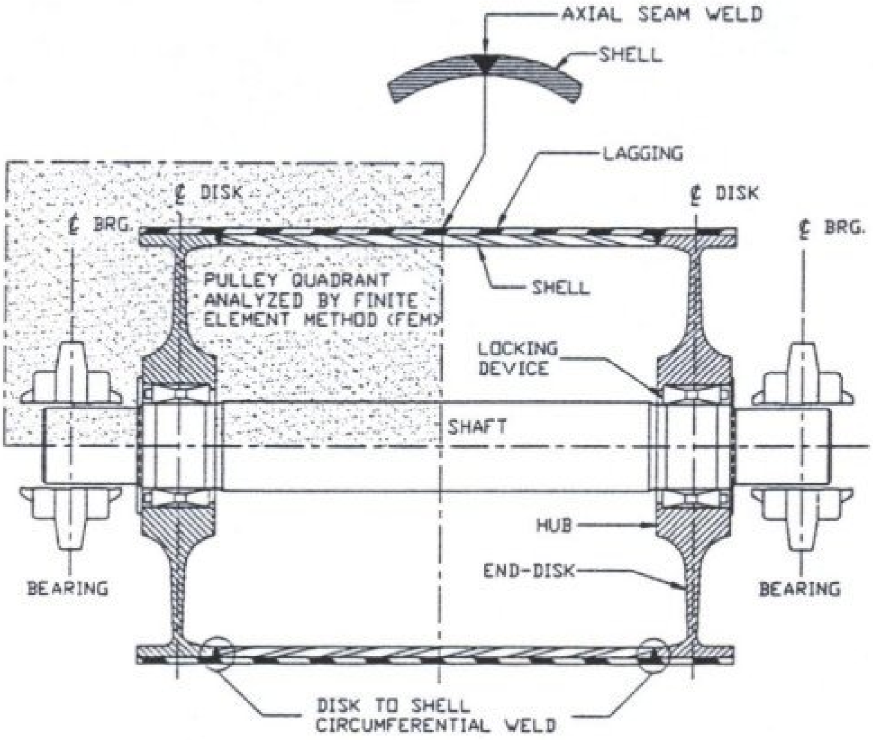


Figure 2.5: Standard conveyor pulley [6]

The main criteria on which a conveyor pulley is designed is strength and rigidity as the main components like diaphragm, hub and shafts are subjected to bending stress. The drum of the pulley is welded which could also suffer fatigue damage due to application of dynamic loads due to belt. The major part which is affected by transient loading in pulley is the pulley shaft. There are changes in diameter of the shaft along its length to facilitate fixing of hubs and bearings. Also the shaft is fixed to the pulley using key, which induces high notch stresses during transient operation. Also the pulley drum is welded along its length which can lead to fatigue failure due to belt pressure. State of the art design of these two major parts of pulley will be discussed further in Sections 2.2 and 2.3 respectively.

Drive Unit:

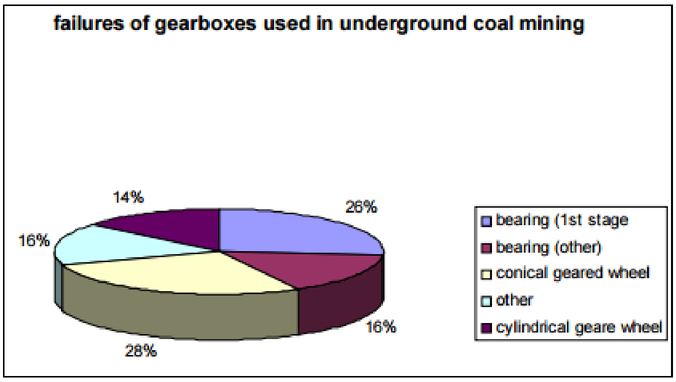


Figure 2.6: Different failures in gearboxes [5]

The drive unit consist of electric motor, coupling units, gearbox and a coupling unit to connect to the pulley shaft. The main failure point in this system is the gearbox, according to [5] as close to 14% of gearboxes can be changed in an open cast mine. The failures can be related to wearing, tooth failure to bearing damages. Analysis shown in [5] a total of 50% of conveyors problems are related to the input stage that is the bevel stage. Figure 2.6 shows various failures of gearboxes found in underground mining.

Out of all the major components of belt conveyors we have discussed above it is clearly visible that the pulley shaft and the pulley drum are more susceptible to fatigue damage than other. Failure of other components cannot be specifically assumed to be caused because of fatigue. For example, idler rollers fail due to abrasion between belt and outer cover of rollers. The shaft of idler rollers does not rotate, hence does not undergo any fluctuating loads. The drive system of the belt conveyor although is susceptible to fatigue failure is not considered in this paper as their design is governed by more specific standards and also because these are considered as bought out components. More specifically, their structural health is of no concern to a belt conveyor manufacturer as these components are merely used in the system and no designed by them. We will be discussing the state of the art design of pulley shaft and pulley drum in this chapter.

2.2 Failures in Pulley shaft

There are no specific standards available for conveyor belt application in particular, but there are few standards like the Australian standard AS1403 and German standard DIN743 which can be used for rotating steel shafts. Although these standards are not freely available to study, there are some references in articles which reflect on the advantages and disadvantages of using these standards. According to Adams Mayers [7] shaft design is of utmost critical as its failure can lead to large repair cost in addition to subsequent downtime of the system. The article [7] also suggests that a rotating shaft should be designed considering the following main criterion.

1. **Design for Strength:** During operation of equipments there could be a high load acting for a low number of cycle like starting up or stalling which are not a fatigue concern. But the shaft should be able to counter the effects of such loads. Standards like AS3990 provide a proper design methodology to design shafts based on allowable stress. A suggestion would be to use higher strength steel if this approach is to be used. Care should be taken that as mass of the shaft increases due to use of higher strength steel, the natural frequency of shaft decreases which could reduce the operational speed of shaft.

2. **Design for Deflection:** Deflection is an important parameter that should be considered while designing a rotating shaft as its coupling with the drive unit could get effected by excessive deflection. Basic thumb rule is that the maximum deflection of shaft should not be more than overall length/2000 or not more than 8 minutes of angular deflection on bearings. Deflection of shafts can be calculated using traditional beam formulae with respect to inertia, length and material of the shaft. To reduce deflection of shaft the stiffness should increase which could done by reducing length of shaft or increasing the size of shaft diameter. One particular thing should be noted that changing the material will have no to little effect on stiffness of shaft.

3. **Design for critical speed:** Critical speed is mainly dependent on the natural frequency of the shaft. It should be considered that the natural frequency should be as high as possible. The critical speed is dependent on stiffness and mass of the shaft.

4. **Design for Inspection:** One of the most important parameter for shaft design is toughness of component. Toughness can be used to determine maximum tolerable crack sizes at different locations of shaft if a fracture mechanics mechanism is used to design. It is important to design a shaft along with all its components such that it will be easy to inspect.

One of the more important consideration while designing a rotating shaft is design based on 'Fatigue'. The methodology of standards like AS1403 is to design a shaft ensuring stress is calculated from combining axial, bending and torsional loads [7]. This combine effects of these loads should be within the fatigue limit of the selected design material. Stress raising factors can be used to increase the stress in each axial location to account for any keyholes, notches, interference fits etc. But the article [7] describes that there are certain disadvantages to the methods described in AS1403, these are listed below.

1. AS1403 makes no differentiation between stress concentration factors between torsional and bending, this leads to inadequate comparison between these two factors as these can vary based on shaft's application. The article also provides an example of press fitted components.

2. The standard has no specific allowances for stress relieving modifications incorporated during the design of the shaft. Figure 2.7 shows different type of stress relieving modifications that can be used on a shaft. Also the standard excludes the effect of surface finish of shaft.

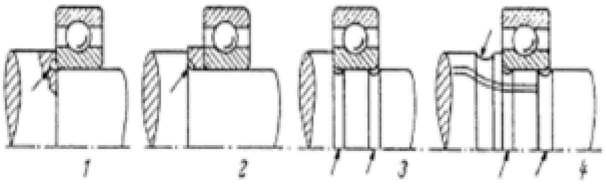


Figure 1. Examples of Stress Relieving Features

For a shaft step with a bearing seat:

1. Rounded engraved groove at step.
2. Rounding out with a spacer ring.
3. Rounded out discharge notches.
4. Additional discharge notches.

Figure 2.7: Stress relieving modifications [7]

3. AS1403 is also misleading on the effects of shrinkage fits in shafts. The standard provides safety factors based on the type of fit rather than the contact pressure between them. In article, it is suggested that the contact pressure between fits plays an important role in

stress concentration rather than the type of fit between shaft and hub. It is also suggested that care should be taken while designing the shaft as per shrinkage and manufacturer guidelines should be followed rather than taking safety factors from standards.

2.3 Failures in Pulley Drums

There is no specific standard which deals with fatigue design of conveyor pulley, but as it is a steel structure, there are lot of related standards like EN 1993-1-9 eurocode, British standard BS:7608:2014 also the NORSOK standard N-004 which look into the aspect of fatigue design of steel structure. We will discuss in brief about the EN standard and the NORSOK standard, about the methodology to employ when designing a structure for infinite fatigue life.

EN-1993-1-9:

Euro code provides extensive knowledge about the consideration that should be taken while designing a pulley. EN1993-1-9 deals with the stresses that arise from fatigue, the modification that can be done to improve the fatigue strength of components and finally the verification of fatigue strength. The standard specifies that the fatigue assessment can be done using two different methodologies namely, damage tolerant method and a safe life method. Damage tolerant method is to assess the structure or component to predict its life provided that proper maintenance and correction of fatigue damage have been done throughout its design life [8]. The safe life method should be used to predict reliability of any structure without the need of regular inspection of fatigue damage. It is also suggested in [8] that the safe life method should be utilized only in cases where the crack initiation could lead to failure of the component.

EN1993 provides assessment of fatigue resistance of member, connections and joints that undergo fatigue loading [8]. Thus it can be seen that the use of this standard for pulley drums as the drums undergo fatigue loading at the joint where it is welded. The welding in pulley drums is usually done along its length. This type of welding should be able to sustain the fatigue loading subjected on the drums during the operation of belt conveyor. There is a constant change in tension in the belt depending upon the material weight on the belt which leads to uneven load distribution on the surface of pulley drum. This eventually leads to fatigue loading acting on its surface which might lead to failure of welded joint on the drum. The different types of assessment talked before is used to for providing recommended values of partial factors for fatigue strength, Table 3.1 in [8] provides the values. These values should be chosen based on the consequences of the failure of member connection or joint.

The most appropriate and safe method of assessment that should be implemented is the damage tolerant method, as it offers to monitor the condition of the member. Pulley drum is an important component of the belt conveyor system and its failure could lead to shutdown of the system, which is hazardous to the company from cost point of view. The procedure followed in EN1993 is that the nominal stresses are calculated using the formulae provided in the standard and these stresses should be less then the strength limit of welds. The strength limits of various welds are provided in Tables 8.1 to 8.10 in [8] which are subjected to the fact that welding grade is as suggested as in the standard. It also provides some literature on how to calculate stress ranges, cycle counting and use Miner's rule of cumulative damage to find the damage accumulation of the member. This process will be explained in brief below.

Firstly, it is important to calculate the nominal stresses for potential fatigue initiations. The relevant stresses that should be calculated are the nominal direct stress (σ) and nominal shear stress (τ). There are two possible stresses which act on the weld, normal stress transverse to the axis of weld (σ_{wf}) and shear stress in longitudinal to the axis of the weld (τ_{wf}) [8]. Figure 2.8 shows the directions in which the these two stresses act on a fillet weld.

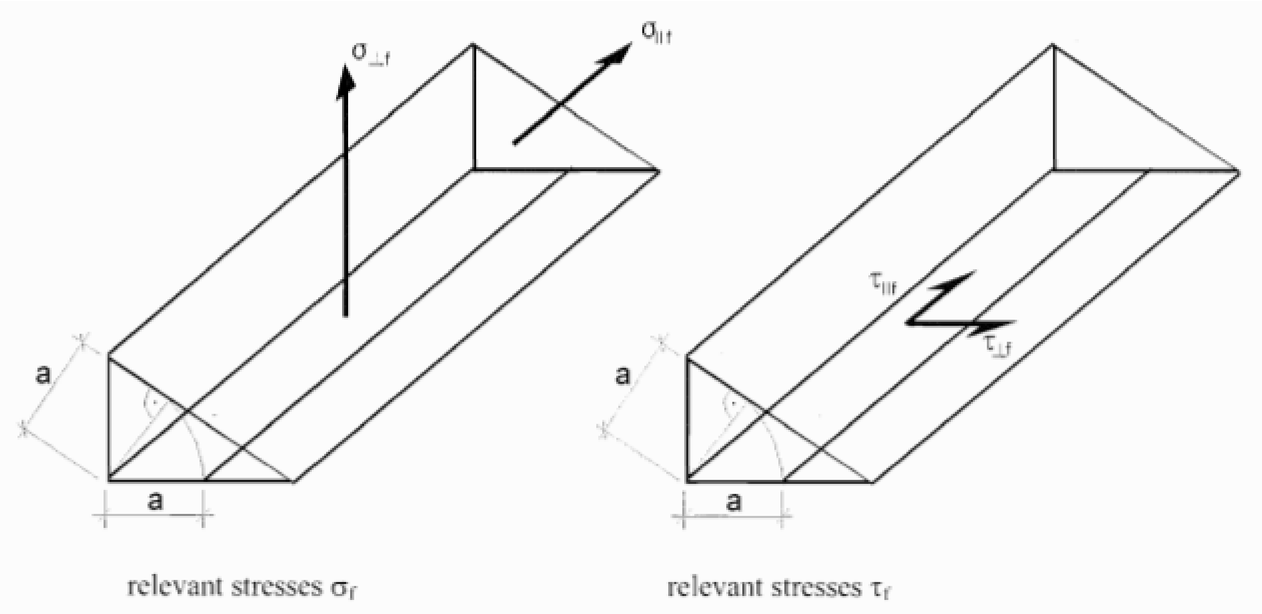
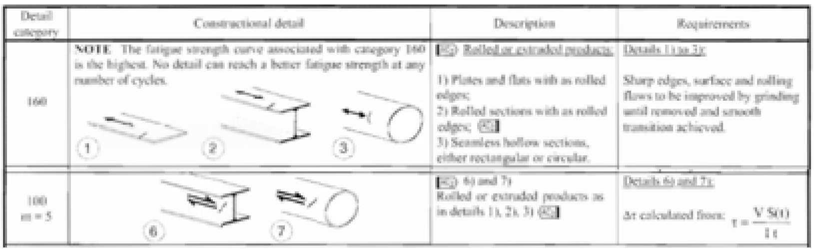


Figure 2.8: Relevant stresses in a fillet weld [8]

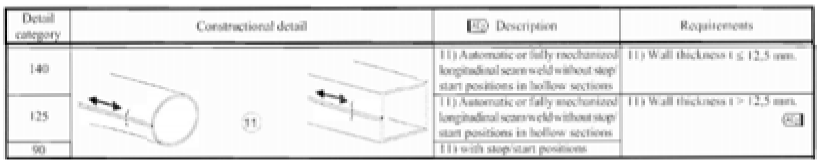
The main criteria to check the fatigue strength of the weld is to calculate the stress of material using detail category provided in the standard. The detail category is based on the type of weld which is done on the component. The standard provides different types of detail categories for a particular type and direction of weld. As it can be seen from Figures 2.9 and 2.10 the different detail category that are applicable for a pulley drum.



Detail category	Constructional detail	Description	Requirements
160	**NOTE** The fatigue strength curve associated with category 160 is the highest. No detail can reach a better fatigue strength at any number of cycles. ① ② ③	Rolled or extruded products: 1) Plates and flats with as rolled edges; 2) Rolled sections with as rolled edges; 3) Seamless hollow sections, either rectangular or circular.	Details 1) to 3): Sharp edges, surface and rolling flaws to be improved by grinding until removed and smooth transition achieved.
100 m = 5	⑥ ⑦	6) and 7) Rolled or extruded products as in details 1), 2), 3)	Details 6) and 7): Δτ calculated from: $\tau = \frac{V\,S(t)}{I\,t}$

Figure 2.9: Plain members and fastened joints [8]

Figure 2.9 provides two different detail categories for hollow circular section which are 160 and 100. These categories can be used to determine the maximum strength of the weld, this means 160 and 100 is the strength in MPa depending upon type of weld used. Detail category 160 is the best possible category and should only be used when the weld type is K1 or higher. More realistic and conservative value should be chosen as the weld quality can not be high at all times, due to cost reduction so detail category 100 can be chosen for such applications.



Detail category	Constructional detail	Description	Requirements
140	⑪	11) Automatic or fully mechanized longitudinal seam weld without stop/start positions in hollow sections	11) Wall thickness t ≤ 12,5 mm.
125		11) Automatic or fully mechanized longitudinal seam weld without stop/start positions in hollow sections	11) Wall thickness t > 12,5 mm.
90		11) with stop/start positions	

Figure 2.10: Welded built-up sections [8]

The welded built-up sections detail category differentiates between longitudinal welds which have start and stop points and which don't. The important thing to note in here is that the strength of the overall component is decreased if start/stop is used and corresponding detail category will be applicable. The strength analysis should be done in consideration with the weld strength.

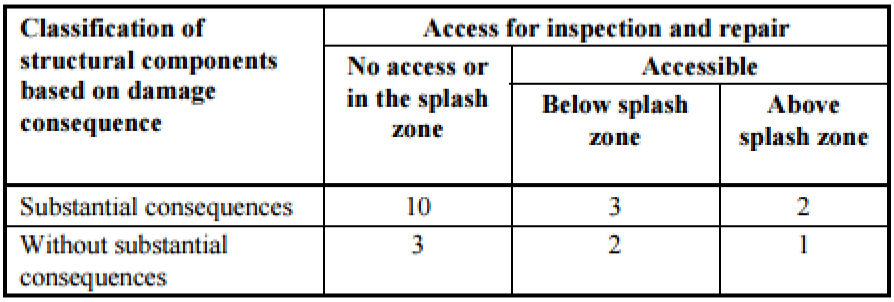
NORSOK N-004:

The NORSOK standard deals with the overall design of offshore components right from the selection of material and the requirements of non destructive testing to covering all the limit states of failures [9]. For the purpose of this research we will be only discussing on the fatigue limit state of the standard. The most important part said in the standard is the general safety format should be satisfied. Equation 2.1 provides the equation which should always be satisfied while designing [9].

$$S_d \leq R_d \tag{2.1}$$

Where S_d is the design action effect which loosely can be translated as the stress acting on the particular component due to force acting on it. R_d is the design resistance of component which is the strength of the material. It is to be noted that the terms already consider the respective safety factors both for design action and resistance.

According to NORSOK [9] the aim of fatigue design is to ensure that the structure has an adequate fatigue life which could also form basis for efficient inspection programmes during fabrication and operational life of the component or structure. It continues by saying that the design fatigue life should be decided by the operator and if no such data is provided than structure should be constructed for a service life of 15 years. NORSOK [9] also points out the most crucial areas which are the most susceptible to fatigue like welded joints, attachments and any form of stress concentration member is a potential source of fatigue crack. To design a member with all things considered a factor should be applied to the number of load cycles depending upon the classification shown in Figure 2.11.



Classification of structural components based on damage consequence	Access for inspection and repair		
	No access or in the splash zone	Accessible	
		Below splash zone	Above splash zone
Substantial consequences	10	3	2
Without substantial consequences	3	2	1

Figure 2.11: Design fatigue factors [9]

Fatigue analysis should be done using S-N curve either determined by fatigue testing or the linear damage hypothesis [9]. Alternatively it is suggested by NORSOK to use fracture mechanics to understand the behavior of structure under load. It is also suggested that while using fracture mechanics, it should be noted that the in-service inspection accommodates sufficient time interval between time of crack detection and the time it reaches to an unstable crack. The long term distribution of stress ranges should be determined using spectral analysis.

Having gone through the state of the art fatigue design of pulley shaft and drum, we can conclude that there are uncertainties in the design which are accommodated using safety factors defined in the standards. Also it should be noted that there is no particular standard specified for pulley shaft design but a general standard is usually applied. Having factor all these points, use of probabilistic theories to predict fatigue failure can serve as a useful procedure. The probabilistic theory discussed in this research is Bayesian theory. This is explained in next chapter along with the advantages and limitation of this approach.

3 Bayesian Approach

It is clear from the above chapters that the fatigue phenomena is very complex as we are dealing with a lot of uncertainties which are a inherent part of it. For example, uncertainties related to material properties, loading imposed on structure and also geometrical properties [15]. Considering this, safety factors that can be induced on each and every parameter to improve structure reliability is an extremely significant knowledge for the designer. This is very promising for designer's who have to deal with random fatigue loading on their structures. In today's world, there is still a varied amount of uncertainties faced while designing high reliable structures as discussed in the Chapter 2. Probabilistic approaches maybe a good solution not only in minimizing these uncertainties but at the same time predicting the future outcomes and providing necessary actions to deal with them.

3.1 Probability approach to fatigue analysis

There are various papers dealing with the issue of fatigue in structures and two main approaches can be distinguished. B.Echard in [15] states these two approaches and discusses about their advantages and disadvantages. The first one is a frequency domain approach modelling stress-time variations with respect to a random process defined by its Power spectral density (PSD). Paper [15] suggests that this is a very simple and efficient method at the early design stage but it is not helpful in assessing the time of expected structural failure. Also, only linear mechanical behavior can be calculated because simple PSD can only be implemented.

The second one is more towards time-domain method based on probabilistic concepts on designing new structures with a higher reliability objective or to assess failure probability of an existing structure [15]. The main methodology is that there are mainly two PDF (Probability density function), the stress and strength hence the method is also called as 'Stress-Strength approach' [15]. B.Echard in [15] points out the limitation in this approach is the failure probability maybe very sensitive to the PDF's as there can be many parameters to evaluate and that would increase the processing or computational time. Also, influence of each variable on the failure probability is hard to determine.

Bayesian theorem is a much more advance version of Stress-Strength approach as it utilizes field data to constantly update its prior knowledge and hence should effectively help in better analysis of structure. What is Bayesian theorem and how it can help in predicting fatigue behavior of system will be discussing in Sections 3.2 and 3.3.

3.2 Bayesian Theorem

Burno points out in a simplified manner in [17] that Bayes theorem is not more than a manipulation of conditional probabilities. The joint probability of two events 'A' and 'B' can be expressed very simply as follows [17]:

$$P(AB) = P(A|B)P(B) = P(B|A)P(A) \tag{3.1}$$

where 'A' and 'B' are two random events. Bayesian probability can be used to verify hypothesis based on data which would help to judge the relative truth in the hypothesis. For assessing the fatigue damage of components, Equation 3.1 can be modified to 3.2 [17]:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)} \tag{3.2}$$

In Equation 3.2, 'H' is the event whose probability needs to be assessed and 'D' is the event which has already been occurred. For example, to assess the additional damage induced in a particular component, the effect of loading can be taken as an event 'H' and the cause of these particular damage is taken as an event 'D'. Now the combined effect on the fatigue damage of the components due to event 'D' is calculated using the above equation.

$P(D|H)$ is called as the 'likelihood function' and is said to be known beforehand as it expresses ones expert knowledge how to expect the data if the hypothesis is true. P(H) is called the 'prior' as it is said to reflect knowledge before data is considered. P(D) is usually a normalizing constant found by integrating $P(D|H)P(H)$ over all H. $P(H|D)$ is called as the 'posterior' which reflects the probability of hypothesis after considering the data. This can thus keep on updating to prove the correctness of hypothesis. Now we will be discussing on the advantages and disadvantages of this approach [18].

Advantages:

1. It allows the inclusion of past data of the component to assess the fatigue damage of a component.
2. It provides interpretable answers for the parameters like, probability of 95% failure of component.
3. Provides a convenient setting which can be used by variety if models like, MCMC, numerical models which makes computational quite parametrically.

Disadvantages:

1. Output highly depends upon the selection of prior data and there is no correct way to chose it.
2. It often has high computational cost which are dependent on the parameters used. In addition to this, slight contradiction is also observed in the results if different seeds are used.

3.3 Bayesian theorem in Fatigue

As it is seen from Section 3.2, there are two important things required which are prior PDF and a likelihood function. As mentioned in [10] for fatigue, prior distribution can be obtained from numerical models or laboratory results. On the other hand, likelihood function can be obtained from field data as it needs to keep on updating. Another reason for using field data is that it represents reality which includes various uncertainties like environmental, measurement and loading conditions [10].

A standard approach to using Bayesian theorem for fatigue analysis is to build the field data to use as a likelihood function. Since a distribution cannot be made from the initial field data an assumption is made and Bayesian theorem is modified accordingly [10]. The assumption made is that the fatigue life distribution is already known and out of all the known distribution the most conservative is chosen [10]. Based on the chosen distribution, related parameters are selected for example, in case of normal distribution a mean and standard deviation need to be identified.

Let's say the field data consists of number of hours of operation until inspection (N_f) and let the number of defects be 'r' out of the total number of components 'n'. The field data is always in terms of N_f and 'n', the likelihood function can only be represented in 'r'. But the number of defective components can never be a distribution as it can only be an integer. The likelihood for a distribution X(μ, σ) can be represented by probability mass function [10] in Equation 3.3

$$f_Y(y|X = (\mu, \sigma)) \;=\; \frac{n!}{r!(n-r)!}\,(p_f)^r\,(1-p_f)^{n-r} \tag{3.3}$$

where f_Y is the likelihood function in PDF and p_f is the probability of defects at a given N_f or a particular variable 'X' which lets say has a normal distribution with mean 'μ' and standard deviation 'σ'. This likelihood function can be updated regularly with new information in variable 'X' and similarly the probability distribution can be updated. Figure 3.1 shows the PDF of fatigue life which can be used to find the probability of defects in a particular sample.

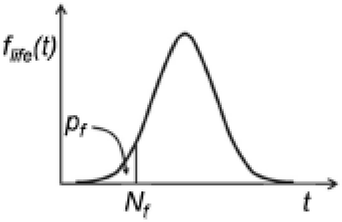


Figure 3.1: Probability of defects calculated from fatigue distribution [10]

According to Ruoxue Zhang in [11] for a particular fatigue problem in principle it should have only one mathematically model to calculate its fatigue life. But it is also discussed that in matter of uncertainty one should include all possible models as we should minimize the expected loss that would arise from choosing a wrong model. Also it has been demonstrated that if all available information is implemented through Bayesian predicting it leads to a smaller variability with the field result than using individual model. Ruoxue also goes on to say that the Bayesian approach is extremely valuable in cases that deal with rare events. Thus in situations where physical, model and statistical uncertainties are all potentially important, a more logical approach by using a weighted average method be employed. This method can be applied to the whole model which is like a single layered model or this weighted average method can also be applied to various parameter distribution of each variable which makes it a multi-layered model.

Ruxoue Zhang continues on saying that the Paris law only considers stress range but in turn ignores stress intensity ratio (R). Paris law is considered to be quite simple and not all times universally acceptable. Figure 3.2 shows the typical crack growth in metals in terms of stress range (ΔK) which shows that stress range growth is sigmoidal rather than linear when plotted over a longer stress range. In addition to this, fatigue crack growth rate also shows signs of involvement of stress intensity ratio. Keeping both of these considerations, a modified relationship of Paris Law is provided by 'Foreman' in Equation 3.4 below.

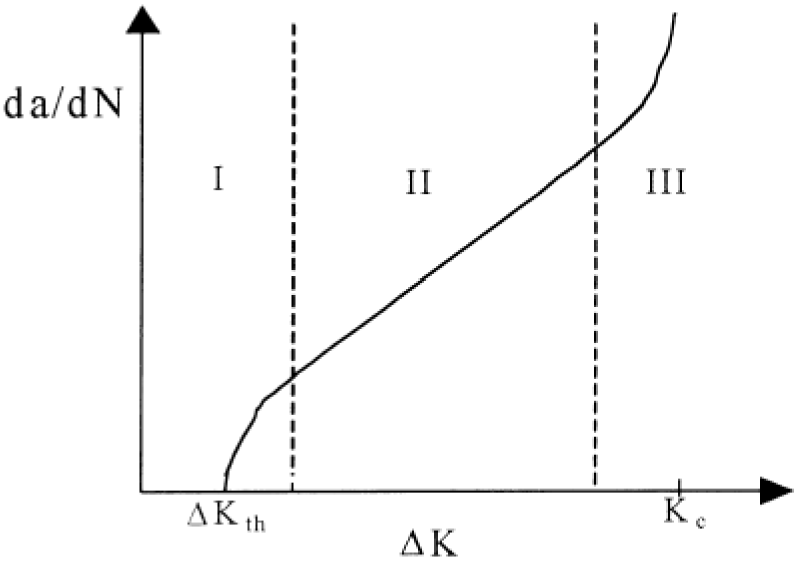


Figure 3.2: Typical crack growth in metals [11]

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_{crit} - \Delta K} \tag{3.4}$$

where 'C' and 'm' are not the same constants as described in Paris law in Equation 1.2. K_{crit} is the fracture toughness of the material under consideration. Although, there is an other proposed equation proposed by 'Weertman' which can be seen below in Equation 3.5

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{K_{crit}^2 - K_{max}^2} \tag{3.5}$$

Both Equations 3.4 and 3.5 include the effect of stress intensity ration (R) on the fatigue life of a material. Together they can be represented by Equation 3.6

$$\frac{da}{dN} = f(\Delta K, R) \tag{3.6}$$

This equation can then be integrated from initial crack size 'a_0' to 'a' to find the number of cycles it would require to achieve this final crack size. It should be noted that this is still an estimation of the fatigue life of the material.

$$\int_{a_0}^{a} \frac{da}{f(\Delta K, R)} = N$$

Performing reliability which involves uncertainties, the first thing is to define and quantify each of these uncertainties. Sources of these uncertainties are provided in [11] which are given below:

1. Selection of crack growth model, corresponding to limit state function.
2. Random parameters associated within each crack growth model.

The two models given in Equations 3.4 and 3.5 have never been actually compared and there is no literature available on this so it is difficult to decide which of these models should be chosen to obtain realistic results. The prior weight can be assigned to each of these models and the final failure probability can be given by Equation 3.7 below [11]:

$$P_f = P(M_1)P_{f1} + P(M_2)P_{f2} \tag{3.7}$$

where $P(M_1)$ and $P(M_1)$ are weighted average taken as per experience and P_{f1} and P_{f2} are the failure probabilities calculated using Equations 3.4 and 3.5 respectively. Also this concept can be extended to distribution type of variables associated with each of the models. This in turn will develop into a multi-layer model. The reliability analysis can then be performed with a multi-layer Bayesian description in the same way as a single layer model [11].

3.4 Fatigue reliability in Bridges

Most common infrastructure which uses this method of analysis is steel bridges. Bridges have been studied a lot and literature is available on the various methods that are utilized to keep the infrastructure as healthy as possible. Significant amount of research has been carried out to utilize Bayesian approach to predict fatigue reliability in bridges. It is quite unfortunate that the same research has not yet done in the field of belt conveyors. We can assume that the major cause of fatigue in structures are due to wielding of components which would help us in understanding the failure explained here for bridges. To update Bayesian models of uncertainty, non destructive inspections are done on various critical sections of the structure and the model is updated accordingly. Most common non destructive inspection is the ultrasonic inspection to inspect wielding joints in structures [19].

Zhao points out in [19] that each NDI (Non-destructive testing) method has its own characteristics and output from these tests are bound to have some kind of uncertainty. The article continues to say whether or not a crack or cracks are detected, each inspection provides essential information and eventually results in changes in prior estimated reliability and uncertainty in the basic variables. Ultrasonic inspection has certain advantages over other forms of testing techniques and has been regarded as the best method for inspecting wielding joints by ASTM [19]. These advantages are given below [19]:

1. The relative ease of penetration into materials like steel and aluminum.
2. The ability to test from only one surface and to detect defects at substantial depths.
3. Sensitivity of the technique is quite good and compared to other techniques, accuracy is higher.
4. The presence of no significant radiation hazard requiring operational precautions.

Two sources of uncertainty in ultrasonic NDI are given below, these have to be taken care in the mathematical model [11]:

1. **Detectability:** which refers to the fact there is always a crack size for a given NDI which cannot be detected by the technique.
2. **Accuracy:** which refers to the measured crack size may not represent the actual size of the crack due to measurement error.

Although NDI contains itself certain uncertainties and whether or not a crack is detected during inspection, it provides additional information which can lead to changes in prior reliability as well as uncertainty in the model and basic random variables [11]. The Bayesian method is then utilized to update the information on model weights and statistical characteristics of random variable to calculate the new reliability index [11].

From this chapter we now have a better understanding as to the importance of Bayesian theorem and its applicability in predicting fatigue failure of components. It is also be noted that there is a lack of literature on implementation of Bayesian theorem in predicting failures in belt conveyors, but it has been extensively investigated in bridges. Using the general principles for prediction of fatigue damage in bridges, broadly same principles can be implemented for belt conveyors. In the next chapter we will be looking at the various sensors which can be implemented in the belt conveyor system to provide information regarding the health status of various components. This information can then be implemented to predict the fatigue damage of these components.

4 Implementation in Belt conveyors

As there is little to no literature available on indication of use of Bayesian theorem in belt conveyors, this provides us with the advantage of implementing the theory including all the various non destructive inspection techniques available for use. Below a few different techniques will be discussed which can be used to assess the health of the system and provide necessary data to predict reliability of the system along with its components. It is briefly discussed in Chapter 3, Section 3.4 about the implementation of Bayesian theorem in Steel bridges which can be assumed to be more reliable than the current methodology. Belt conveyor is a complicated system and use of Bayesian theorem to evaluate system reliability would help in scheduling maintenance activities and help in maintaining adequate inventory.

4.1 Inspection Techniques

This section is about the various non-destructive inspection techniques available for belt conveyors. Specifically in the testing of opencast mines, some of these techniques are already been used while others are still being under improvement in laboratories. Ryszard discusses in [12] that the conditional monitoring of belt conveyor system is a complex problem with no universal solution. The solution mainly depends upon belt type, operational condition and material type and can either be a simple and inexpensive one or a complex and costly one [12].

4.1.1 Magnetic Inspection Systems

These techniques has been used for past 30 years but unfortunately cannot be used on a daily basis as it is very costly. Although, there has been some development in this field and a unique software has been developed by a Polish mining company based on Labview environment. It provides automatic data analysis tools, that might be able to detect and localize damage. The system consists of two parts [12]:

1. A portable module for data acquisition which can do some basic analysis while on the move.
2. A more advanced software installed on the desktop for more in depth analysis, visualization and measurements.

Figure 4.1: Steel cord belt conditional measurements [12]

Figure 4.1 provides clarity on how the magnetic inspection equipments are installed on the conveyor system. The magnetic properties of ferromagnetic materials change under fatigue and applied loads [20]. These changes in the magnetic properties could be used as indicators of stress state of materials which could help in great extent to predict the remaining fatigue life [20]. The article by Devine [20] discusses on a field usable instrument that was developed called as Magnescope, further details can be studied from the article.

4.1.2 Machine Vision Diagnostic System

Assessment of the top cover of the belt is very difficult as it is covered with materials most of the time. The materials can have rough and sharp edged lumps which can effect the belt material. Typical belt cover damages are length-wise cuts, abrasion, through cut and groves [12]. Machine vision system is suggested to be used to avoid these issues. The idea is simple and explained in [12], use of specialized linear cameras images line by line. The specialized software is used to read and enhance the images which can help in detecting damages which can be interpreted from the images. The defect is detected and stored in a database, the software then supports decision making process by suitable statical analysis [12]. Figure 4.2 shows the appropriate points where these cameras are placed giving the idea of machine vision diagnostic system.

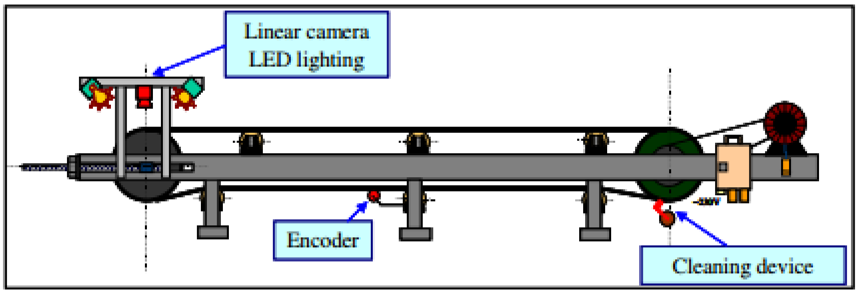


Figure 4.2: Idea of machine vision diagnostic system [12]

4.1.3 Infrared Thermography

A novel procedure consist of images from infrared thermography is introduced. These are used in order to detect damage to the belt covers and steel cords. Any change in their condition will modify the energy levels which could indicate increase in friction or bending resistance [12]. This means the energy is transformed into heat which is clearly detectable using infrared thermography. It is said in [12] that advance processing techniques have to be utilized in order to make the images more clearer.

4.1.4 Ultrasonic Inspection

Jack Blitz discussed in his book on “Ultrasonic methods of non-destructive testing” [21] that ultrasonic waves have a wide variety of applications over an extended range of intensity including cutting, cleaning and fault detection. This type of inspection consists of effectively propagation of low amplitude waves through a material to measure either both time of travel and change of intensity for a given distance [21]. He continues to discuss on the main advantages of using ultrasonic for inspection rather than using audible frequencies [21]:

1. Wavelengths decrease inversely with frequency making in advantageous for testing smaller samples which have dimensions almost same as the wavelength being used. The use of short wavelengths helps in employment of shorter pulses resulting in higher degrees of resolution of defect detection.

2. Testing can be carried out from a single surface.

3. Higher degree of penetration is possible which can be applied in cases where radiology cannot be used due to lower degree of penetration.
4. High accuracy in locating and measuring defects.
5. Ability to detect smaller cracks is quite high as compared to other techniques.
6. It is also compatible with automatic scanning devices and with microprocessors and computers. Data received can be directly utilized and analyzed.

Although there are many advantages of using ultrasonic methods, there are some principal drawbacks which are being worked on [21]:

1. Operators have to be properly trained and should be experienced and possess a high degree of reliability and integrity. However, automatic testing requirements are coming up and in future this won't be much of a problem.
2. manual operation over a larger area is difficult as only a small part can be scanned at one time, but this can be solved with introduction of transducer arrays.
3. A high degree of coupling should be available between transducer and surface, though much progress is being made with the development of non-contact transducer.

Of the commonly used methods, only ultrasonic and radiological techniques can detect internal flaws with a high degree of reliability.

4.2 Implementation in Belt conveyor

There is no available literature on the application of Bayesian updating to predict fatigue damage in belt conveyors. In this section we will discuss about how the general principles of Bayesian updating in steel bridges can be used to predict fatigue damages in belt conveyor system especially for pulley shaft. As the general formulation of fatigue damage of pulley shaft is discussed here, it can also be implemented for other components of the system if it is required. This means that the formulation to predict the fatigue damage remains the same, the changes can be observed in terms of loading and calculation of stresses. The material properties which are going to change are taken as parametric values which can be changed and utilized depending upon the material of the component under consideration. It should be noted that there is no consideration on how the stress ranges acting on the component is calculated as it is not in scope of this assignment. It is assumed that stress ranges and material data is already available and based on this data fatigue life of the component is predicted. Also the steps that should be taken once the fatigue life is predicted are also discussed as Bayesian is a continuous process.

To incorporate all uncertainties the deterministic model is extended to a reliability model [19]. The advanced first order Second moment (ASM) can be used for this purpose but this requires a limit state function that will provide a relationship between basic variables and respective requirement. For this particular task a reliability index (β) is given which can be defined as distance from the origin to the design point in a reduced variable space [19]. This design point is found using ASM which locates the most probable failure point on the limit state function [19]. As there are non linear functions that are being used these can be verified using Monte Carlo simulation technique. The limit state function is given by Equation 4.1 below [19]:

$$g(Z) \;=\; \psi(\alpha_c, \alpha_0) - C\bar{S}^m(N - N_0) \leq 0 \tag{4.1}$$

The limit state function in Equation 4.1 states that the crack size α_0 at N_0th cycle has propagated to crack size α_c at the Nth cycle. The main aim in this analysis is to keep the reliability index above a preassigned value during service life of system [19]. The reliability index is decided by the designer for different components of the belt conveyor system on the basis of experience.

The capability of NDI is detectability and accuracy, but these have added uncertainties which should be incorporated while developing a mathematical models [19]. Based on experiment results obtained from for ultrasonic method suggested that the probability of detection of crack size follows a lognormal distribution and can be expressed below in Equation 4.2 below [19]:

$$P_D(\alpha \leq \alpha_d) = \phi\left(\frac{ln\alpha_d - ln\alpha_{50}}{\sigma_{ln\alpha}}\right) \tag{4.2}$$

where α and α_d are the crack sizes at different stress cycle, $\sigma_{ln\alpha} = 1/\mu\sqrt{2}$ and μ is a statistical parameters. The accuracy of the measurement can also be quite uncertain and needs to be statistical. It is also observed that the coefficient of variation (COV) of the crack size does not effect the updated reliability index [19]. Only the accuracy of ultrasonic inspection technique is discussed over here as it is concluded from Subsection 4.1.4 that it is the most common method employed for detection of cracks in welded components.

There are three basic cases with respect to detection of crack and model is to be updated accordingly after the inspection. These three cases are discussed below [19]:

1. **No crack detection:**

   The event of no crack detection after the structural element has been subjected to N stress cycles can be expressed as :

   $$I = C\bar{S^m}(N - N_0) - \psi(\alpha_d, \alpha_0) \leq 0 \tag{4.3}$$

   where α_d is the detectability at time of inspection. The updated probability can then be obtained using Bayesian approach and probability of failure can be given as:

   $$P_{f,up} = \frac{P(L \leq 0 \bigcap I \leq 0)}{P(I \leq 0)} \simeq \frac{\phi_2(-\beta_L, -\beta_I, \rho)}{\phi(-\beta_I)} \tag{4.4}$$

   where β_L is the reliability index found from Equation 4.1 and β_I is the reliability index corresponding to Equation 4.4, ρ is the correlation between β_L and β_I. ϕ_2 is the cumulative density function of two dimensional normal distribution. $P_{f,up}$ is the updated probability of failure and update reliability index is given by $\beta_{up} = \phi^{-1}(1 - P_{f,up})$ [19].

2. **Crack detection without size measurement:**

   This event is quite simple as it is the complete opposite of event of no crack detection. The event 'D' is given as below and updated probability of failure is also given below:

   $$D = -I = \psi(\alpha_d, \alpha_0) - C\bar{S^m}(N - N_0) \leq 0 \tag{4.5}$$

   Similarly, the updated probability is same as that for no crack detection except that β_L is now the reliability index corresponding to Equation 4.5 instead of Equation 4.4.

3. **Crack detection with size measurement:**

   Lets say a crack is detected by a non-destructive inspection technique and let the crack size is 'A'. This event can be expressed by Equation 4.6

$$M \;=\; \psi(A, \alpha_0) - CS^{\bar{m}}(N - N_0) \;=\; 0 \tag{4.6}$$

   The updated probability of failure can then be calculated and represented as in Equation 4.7

$$P_{f,up} \;=\; P(L \leq 0 | M = 0) \;=\; \phi(-\beta_{up}) \tag{4.7}$$

   where β_{up} is the update reliability index and is given by Equation 4.8

$$\beta_{up} \;=\; \frac{\beta_L - \rho_{LM}\beta_M}{\sqrt{1 - \rho_{LM}^2}} \tag{4.8}$$

   β_L is reliability index from event given in Equation 4.1, β_M is the reliability index for event in Equation 4.6 and ρ_{LM} is the correlation coefficient.

The updated reliability index can then be utilized to make decisions based on its value. As discussed earlier, a preassigned reliability index is used for comparison with the updated value and based on this following three decisions can be made [19].

1. **To do nothing:** If the updated reliability index is at a safe value as compared to the preassigned reliability index then the best thing is to do nothing and keep on monitoring the system and its components.
2. **Reschedule inspection to earlier date:** If there are signs that the reliability of the component is decreasing faster than before than it is advised to move up the date of inspection in the maintenance chart for that particular component.
3. **Repair or replace:** If the reliability index is very close to the preassigned reliability index, then its item to either repair or replace the particular component. Although, it is advisable to inspect the component more thoroughly before making the final decision.

This proposed method refines the information including uncertainty filling in measurement of crack size and uncertainties in the basic design variables merging together to calculate reliability index [19]. If then a repair is being done, then the reliability index needs to be recalculated and that event is represented in Equation 4.9 below [19]

$$L_{new} \;=\; \psi(\alpha_c, \alpha_{0,rep}) - C_{new} S_{new}^{\bar{m}_{new}}(N - N_{rep}) \;=\; 0 \tag{4.9}$$

where α_c is the critical crack size, C_{new} and m_{new} are the new material properties and S_{new} is the new stress range.

This chapter discusses on the various non-destructive techniques which are used for inspection in belt conveyor which can provide the status of selected components. These selected components depends on the consideration of designer and customer. Statistically, pulley drum and pulley shaft are prone to fatigue damage and hence it is suggested to monitor the health of these components. Proper monitoring would avoid any unnecessary under-design or over-design of these components.

5 Conclusion & Future Research

The literature study is done on the application of Bayesian theorem on predicting fatigue life of belt conveyor system. As there has been little to no progress in this field, literature was done on the implementation of Bayesian theory for steel bridges. For structural health monitoring of bridges, the fatigue life of the bridge and its components is predicted using Bayesian theory. There has been lot of research conducted on this particular topic and this has been presented in this discussion. It can been concluded that the methodology used in steel bridges can be used as the starting point to begin with actual research and eventual implementation of Bayesian theory in belt conveyors. The basic principles of fatigue damage prediction can be assumed to be the same as the damage depends upon the stress range and material properties which are parametrized.

The current methods to design the fatigue susceptible components like conveyor pulley shaft and pulley drum are discussed. Although, there is no direct standard of predicting fatigue life of its components but, some standards and guidelines can be used to design these components so that fatigue damage is not induced. Predictive monitoring techniques should be applied to predict life of components but it needs lot of on field data. All the field data can be found using non-destructive inspection techniques which are also presented in this assignment. Also it is discussed that the best inspection method is ultrasonic testing which can monitor the health of welded components efficiently.

The data after the inspection is then compared to the existing health of the component to predict the remaining fatigue life. A rudimentary mathematical model is suggested in this discussion which are inspired from prediction of fatigue life in bridges. Although, individual details regarding loading conditions and stress range calculations for each parameter has to be studied and developed further. B.Echard discussed in [15] proposes in his paper about a more general and robust approach that will be able to accurately assess the failure probability. The probability failure detection has to be assess through Monte Carlo simulation. But this method has certain disadvantages because of which it is suggested in article [15] to use Metamodels to handle large number of mechanical models. Metamodels like Quadratic response surface, Polynomial chaos and Support vector machine are better alternatives than Monte Carlo simulation [15].

If further research is conducted on the basis of this assignment, importance should be given on welded components and proper mathematical model have to be derived for calculation of stress ranges for each component. Then the output of the stress ranges can be used with the limit state function in Equation 4.1 to calculate and compare the reliability index considering the uncertainties in inspection techniques as described in Equation 4.2. Based on the case of whether or not crack is detected in the component, the probability of failure has to be checked using Equations 4.3 to 4.7. If the component is being replaced after the calculation of its probability of failure, it should be taken care that the reliability index is then updated as per Equation 4.9. This procedure is run continuously and the reliability index is updated regularly to predict the probability of failure of component. The computational time is very high for such a analysis so care has to be taken that only the critical components of the system is analyzed so that their structural health can be maintained efficiently.

Bibliography

[1] J. Schijve. Fatigue of structures and materials in the 20^{th} century and the state of the art. International Journal of Fatigue, 25:679–702, 2003.

[2] H. Foll. A bit more about tensile testing. `http://www.tf.uni-kiel.de/matwis/amat/iss/kap_3/backbone/r3_1_4.html`. [Online; 02/01/2017].

[3] Ceferino Steimbreger. Fatigue of welded structures. Technical report, Lulea University of Technology, 2014.

[4] Raghvendra Singh Gurjar. Failure analysis of belt conveyor system. International Journal of Engineering and Social Science, 2, 2012.

[5] Robert KROL Radoslaw ZIMROZ. Failure analysis of belt conveyor systems for condition monitoring purposes. 2009.

[6] Vinit Sethi and Inc.) Lawrence K. Nordell.(Conveyor Dynamics. Modern pulley design techniques and failure analysis methods. `http://www.ckit.co.za/secure/conveyor/papers/troughed/modern/modern.htm`. [Online; 24/03/2017].

[7] Peter Ford Adam Mayers, Gary James. Rotating steel shafts. ASPEC, 2011.

[8] The European Union. Eurocode 3: Design of steel structures - part 1-9: Fatigue. Technical report, EUROPEAN COMMITTEE FOR STANDARDIZATION, May 2005.

[9] Standards Norway. Design of steel structures. Technical report, Oil industry association & Federation of Norwegian Manufacturing Industries, October 2004.

[10] Nam. H Kim Sriram Pattabhiraman Dawn An, Joo-Ho Choi. Fatigue life prediction based on bayesian approach to incorporate field data into probability model. Structural Engineering and Mechanics, 37:427–442, 2011.

[11] Sankaran Mahadevan Ruozue Zhang. Model uncertainty and bayesian updating in reliability based inspection. Structural Safety, 22:145–160, 2000.

[12] Leszek Jurdziak Monika Hardygora Witold Kawalec Ryszard Blazej, Radoslaw Zimroz. Conveyor belt condition evaluation via non-destructive testing techniques. Mine planning and Equipment Selection, pages 1119–1126, 2014.

[13] T. R. Gurney. Fatigue of Welded Structures - Second edition. Cambridge University Press, 1979.

[14] J.P.Han X.W.Ye, Y.H.Su. A state-of -the-art review on fatigue life assessment of steel bridges. Mathematical problems in engineering, 2014, 2014.

[15] A. Bignonnet B. Echard, N. Gayton. A reliability analysis method for fatigue design. International Journal of Fatigue, 59:292–300, 2014.

[16] Prof. Shashank P Joshi Tapan R Patel, Mr.Ronak R Patel. Conveyor pulley failure analysis. International Journal of Engineering Development and Research, 2:445–447, 2014.

[17] Bruno A. Olshausen. Bayesian probability theory. `http://redwood.berkeley.edu/bruno/npb163/bayes.pdf`. [Online; 04/01/2017].

[18] SAS institute Inc. Bayesian analysis: Advantages and disadvantages. `https://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm#statug_introbayes_sect006.htm`. [Online; 20/03/2017].

[19] Achintya Haldar Zhengwei Zhao. Bridge fatigue damage evaluation and updating using non destructive inspections. Engineering Fracture mechanics, 53:775–788, 1994.

[20] L.J.H.Brasche D.C.Jiles M.K.Devine, S.Hariharan. Development of magnetic inspection techniques for evaluation of fatigue damage and stress in low alloy steels. Review of progress in Quantitative Nondestructive Evaluation, 10B:2021–2028, 1991.

[21] Geoff Simpson Jack Blitz. Ultrasonic methods of non-destructive testing - First Edition. Chapman & Hall, 1996.