



Delft University of Technology

Document Version

Final published version

Licence

Dutch Copyright Act (Article 25fa)

Citation (APA)

Xia, D., Ma, J., & Azadeh, S. S. (2026). Integrated Timetabling and Scheduling of Modular Autonomous Vehicles Under Uncertainty. *Transportation Science*, 60(2), 284-315. <https://doi.org/10.1287/trsc.2025.0116>

Important note

To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright

In case the licence states "Dutch Copyright Act (Article 25fa)", this publication was made available Green Open Access via the TU Delft Institutional Repository pursuant to Dutch Copyright Act (Article 25fa, the Taverne amendment). This provision does not affect copyright ownership.
Unless copyright is transferred by contract or statute, it remains with the copyright holder.

Sharing and reuse

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy

Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.

Integrated Timetabling and Scheduling of Modular Autonomous Vehicles Under Uncertainty

Dongyang Xia,^{a,b} Jihui Ma,^b Shadi Sharif Azadeh^{a,*}
^aDepartment of Transport & Planning, Delft University of Technology, 2628 CN Delft, Netherlands; ^bKey Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Ministry of Transport, Beijing Jiaotong University, Beijing 100044, China

*Corresponding author

Contact: d.xia@tudelft.nl,  <https://orcid.org/0000-0002-5753-480X> (DX); jhma@bjtu.edu.cn,  <https://orcid.org/0000-0003-4812-9295> (JM); s.sharifazadeh@tudelft.nl,  <https://orcid.org/0000-0003-4181-2083> (SSA)

Received: October 31, 2023

Revised: July 3, 2024; March 20, 2025

Accepted: December 15, 2025

Published Online in Articles in Advance:
February 5, 2026

Area of Review: Cross-Cutting & Emerging

<https://doi.org/10.1287/trsc.2025.0116>

Copyright: © 2026 INFORMS

Abstract. Addressing the integrated timetabling and vehicle scheduling (TTVS) problem is important for improving transit operations. Recently, the emerging modular autonomous vehicles composed of modular autonomous units have made it possible to dynamically adjust onboard capacity to better match space-time imbalanced passenger flows. This paper introduces an integrated framework for the TTVS problem in a dynamically capacitated and modularized bus network considering time-varying and uncertain passenger demand. In this network, units can be (de-)coupled and rerouted across different lines within the network at various times and locations, providing passengers with the opportunity to make in-vehicle transfers—that is, to transfer between lines while remaining on board. We formulate a stochastic programming model to jointly determine the optimal robust timetable, dynamic formations of vehicles, and cross-line circulations of units, aiming to minimize the weighted sum of operators’ and passengers’ costs. To solve realistic instances, we propose a tailored integer L-shaped method to solve the formulated model dynamically through a rolling-horizon (RH) optimization algorithm. Furthermore, we extend our approach into a novel learning-based real-time decision-making framework that fine-tunes timetables and reoptimizes vehicle schedules in response to evolving and new demand realizations during practical operations. At its core is a scenario-retention method that selects a representative subset of scenarios using a machine learning model trained on scenario-level features. This subset is then incorporated into the optimization, ensuring both computational scalability and solution quality. To validate the effectiveness of our methods on realistic instances, we conduct experiments based on the Beijing bus network involving two bidirectional lines, 89 stops, up to 50 trips, and a four-hour operational horizon. Our integrated optimization method outperforms the sequential approach. Compared with fixed-formation vehicles, our approach generates timetables and vehicle schedules that require fewer units. Additionally, the learning-based real-time decision-making framework outperforms benchmark algorithms in solution quality within a one-minute computation time limit.

Funding: This work was supported by the National Natural Science Foundation of China [Grant 72288101].

Supplemental Material: The online appendix is available at <https://doi.org/10.1287/trsc.2025.0116>.

Keywords: modularized bus network • cross-line circulation • rolling-horizon framework • integer L-shaped method • machine learning

1. Introduction

In the operations of a bus system, timetabling and vehicle scheduling link the quality of services with operational costs (Ibarra-Rojas, Giesen, and Rios-Solis 2014). Passenger demand for bus networks is time varying and uncertain because of factors such as time of day, day of the week, season, special events, and unexpected disruptions. Within this context, the emerging technology of modular autonomous vehicles (MAVs) provides an innovative solution in bus operations, enhancing flexibility in catering to spatiotemporally varied passenger demand.

Specifically, MAVs represent a new generation of vehicles composed of interchangeable modular autonomous units (MAUs). The number of MAUs forming an MAV is referred to as its *formation* in this study. For example, MAVs developed by Next Future Transportation Inc. with formations of one and two are shown in Figure A.1 in Online Appendix A (Next Future Inc. 2023b). A key feature of MAVs is their *flexible formations*, allowing for dynamic composition adjustments en route and at depots through (de-)coupling MAUs. Bus operations can benefit from the flexibility of MAVs by dynamically adjusting their capacity through

(de-)coupling units both en route and at depots and rerouting decoupled units to high-demand lines. We refer to this capacity adjustment as *dynamic capacity allocation*. Additionally, MAVs feature an open, bus-like space between coupled MAUs, allowing passengers to move freely between units (Next Future Inc. 2023b). This design enables *in-vehicle transfers* of passengers, which refers to the feature allowing passengers to transfer between lines by walking from one onboard MAU to another without having to disembark, walk to other platforms, and board again. Several pilot studies have demonstrated promising results for modularized transit networks (Next Future Inc. 2023a).

Timetabling at the network level has been proven to be NP hard (Caprara, Fischetti, and Toth 2002). Integrating the vehicle scheduling decisions increases this computational challenge significantly. The literature has mostly explored the timetabling problem and the vehicle scheduling problem independently and sequentially, typically determining vehicle schedules only after timetables have been optimized. However, this sequential approach often results in suboptimal solutions, requiring more vehicles to realize the optimized timetables, and in extreme cases, generating timetables that exceed available fleet capacities. This limitation has already been highlighted in Laporte et al. (2017).

Therefore, the integration of timetabling and vehicle scheduling has gained attention in the railway domain. Van Lieshout (2021) demonstrated that timetables generated by the sequential optimization approach are strongly inferior to those produced through the integrated optimization method in realistic instances. This is because sequential approaches design timetables under the assumption of unlimited resources, often resulting in misalignment with practical operations and leading to inefficiencies. This issue is even more critical for MAVs as their decision-making time windows are significantly shorter than those of railways. Moreover, overestimating or underestimating the required number of vehicles to meet an optimized timetable has a greater impact at the network level, further underlining the necessity of integrated optimization in modularized bus networks with MAVs.

However, even research addressing the timetabling and vehicle scheduling problem with fixed-formation vehicles at the network level remains limited, despite focusing on static passenger demand. Existing studies typically assume fixed-formation vehicles and have not fully utilized the advantages of MAVs that enable dynamic and flexible unit allocation at various locations and times and across multiple lines. This flexibility allows for better alignment of supply with dynamic passenger demand. Recently, Xia et al. (2023) explored this framework on a single bus line, and Xia, Ma, and Sharif Azade (2024) extended it to an intermodal urban

transit network with fixed-line and demand-responsive transport services. However, neither study accounts for in-vehicle passenger transfers or vehicle scheduling in a multiline network. Additionally, their fleet sizing assumptions are overly optimistic as they allow MAUs to appear or disappear at any station.

In this paper, we explore the integrated timetabling, vehicle scheduling, and dynamic capacity allocation (TT-VS-DCA) problem in a network with MAVs under time-varying and uncertain demand with explicit consideration of both the tactical level and the operational level. Given that the aforementioned timetable and vehicle schedule are obtained from optimization under historical demand scenarios, we further introduce a learning-based real-time decision-making framework to fine-tune them in response to newly emerging and time-varying demand realizations during practical operations, aiming to maintain high service quality. In the integrated optimization approaches, we examine the trade-off between passengers' costs and operational costs (fleet size and dynamic capacity usage of MAUs). Passengers' costs are measured as a sum of waiting costs at origin stops and transfer waiting costs at transfer corridors incurred during initial boarding and conventional transfers that require alighting, walking to change platform, and reboarding. The transfer waiting costs serve as the transfer penalties, reflecting the inconvenience associated with conventional transfers.

The key idea of our integrated optimization approach is to jointly determine robust timetables, fleet sizes, and dynamic capacity allocations at different times and stops while enabling *cross-line vehicle circulations* to flexibly accommodate uneven spatiotemporal demand across the network. A *vehicle circulation* in our study is defined as the complete working sequence of an MAU within the studied time horizon. A *cross-line vehicle circulation* refers to a circulation in which an MAU serves trips on multiple lines, which are made possible by coupling, decoupling, and rerouting both at depots and at transfer stops. In comparison, traditional *one-line circulations* can be defined as restricting each MAU to operate on a single bidirectional line and to return to the same depot in this study. This vehicle scheduling strategy can be seen as a special case of our cross-line method. A second special case is *depot-based cross-line circulation*, where an MAU may be reassigned to a different line but only at depots. Our cross-line circulation enables *in-vehicle transfers*, where passengers remain on board while transferring between lines. This avoids the inconvenience of exiting, walking, and reboarding, thereby improving the passengers' traveling experience. By leveraging the flexible (de-)coupling and rerouting capabilities, our approach supports more flexible fleet operations and enhances service responsiveness across the network.

To model this, we first formalize and analyze the integrated optimization problem, explicitly incorporating the reconfigurability of MAUs and cross-line vehicle circulation, two features that have received limited attention in existing studies on timetabling and vehicle scheduling with fixed-formation vehicles. We then propose a stochastic mixed-integer linear programming (MILP) formulation that accommodates scenarios with nonuniform probabilities. From the operator's perspective, our formulation enables MAVs to change their formations both at depots and at transfer stops en route, and it allows MAUs to execute trips across multiple lines at both depots and transfer stops through coupling and decoupling operations. From the passenger's perspective, the model facilitates in-vehicle transfers by taking advantage of the modularity of MAUs, thereby avoiding the need to exit, walk to change platforms, and reboard when making transfers.

The investigated TT-VS-DCA problem in this study is as hard as the traditional timetabling and vehicle scheduling problem with fixed-formation vehicles at its minimum because it involves the joint design of timetables and vehicle schedules with flexible formations and cross-line circulations at various locations and times. Different from existing studies, the decoupling, coupling, and rerouting of MAUs at depots and en route are all allowed in our problem, requiring additional decision variables and coupling constraints. Thus, the solution approaches from previous studies cannot be directly applied to the TT-VS-DCA problem. To solve this problem, we develop an integer L-shaped method (IL method) with tailored valid inequalities to solve the model. However, results show that the proposed exact method is ineffective for solving large-scale real-world instances. To address this computational challenge, we further design a rolling-horizon (RH) algorithm that solves the problem dynamically. In addition, we incorporate the proposed integer L-shaped method into the rolling-horizon optimization framework after finding that GUROBI, despite using the rolling-horizon approach, is unable to solve the TT-VS-DCA problem at the network level. As a result, instead of solving a large MILP, the developed algorithm solves a series of smaller MILPs.

Furthermore, as operational conditions evolve (e.g., because of evolving passenger demand patterns that may fall outside the historical demand scenarios), the optimized timetables and vehicle schedules derived from the historical demand scenario set may become suboptimal, resulting in inefficient resource utilization or service shortages. To address this, we extend our approach to the real-time control level by proposing a novel learning-based real-time decision-making framework that fine-tunes tactical timetables within a small adjustment range and reoptimizes vehicle schedules accordingly under strict computational time limits. The effectiveness of

our learning-based real-time decision-making algorithm stems from introducing a new and fundamental idea that carefully selects and incorporates a subset of scenarios into optimization. The first key concept of this algorithm involves a learning-based scenario-retention method, which selects a representative subset of scenarios using machine learning (ML) models trained on scenario-level features. This allows high-quality solutions to be generated in real time. The second key concept is the identification of features, which determines the most relevant scenario characteristics for predicting their criticality. Our learning-based real-time decision-making algorithm is novel because it provides a thorough method to construct a scenario subset to be integrated into the optimization of the stochastic programs to improve the scalability of the algorithm, and it provides a framework that can be generalized or extended in broader real-time optimization contexts. We conduct extensive computational experiments on real-world instances to validate the effectiveness, robustness, and scalability of our proposed methods. The results highlight the practical benefits of our integrated optimization approach, including the (de-)coupling and cross-line vehicle circulation strategies at both depots and transfer stops, and they demonstrate the superior solution quality, scalability, and real-time applicability of our algorithms compared with benchmark approaches.

To summarize, the main contributions of this paper are (i) conceptualizing the TT-VS-DCA problem within a dynamically capacitated and modularized bus network that enables in-vehicle passenger transfers and cross-line unit circulations; (ii) developing a stochastic MILP model; (iii) designing effective solution methods; (iv) developing a learning-based real-time decision-making algorithm to fine-tune schedules in response to new demand realizations during practical operations; and (v) illustrating that considerable savings in operational costs can be achieved by allowing flexible formations of MAVs and cross-line circulations of MAUs and showing the benefits of our approaches.

The remainder of this paper is structured as follows. In Section 2, we summarize related literature. In Section 3, we formalize the investigated problem. In Section 4, we describe the mathematical formulation. In Section 5, we analyze the properties of the proposed model and present the tailored valid inequalities. In Section 6, we describe the integer L-shaped algorithm, the rolling-horizon optimization approach, the integrated solution method, and the learning-based real-time decision-making framework. In Section 7, we discuss the results of the computational experiments. Finally, in Section 8, we summarize our findings.

2. Literature Review

In this section, we discuss research on integrated timetabling and vehicle scheduling (TTVS) with fixed-

formation vehicles, timetabling of MAVs, and real-time optimization under uncertainty.

2.1. Integrated Timetabling and Vehicle Scheduling with Fixed-Formation Vehicles

The traditional planning procedure for public transportation networks as outlined by Huisman, Freling, and Wagelmans (2004) can be divided into the following sequential steps. First, the line plans and the corresponding departure frequency are designed. Second, timetables are developed to minimize passengers' travel time (or waiting time). Subsequently, vehicle schedules are determined to minimize operational costs, and finally, crew schedules are planned. The disadvantages of this sequential planning are mainly two-fold. First, the main components of operational costs (i.e., the number of used vehicles and crews) are only determined at the final stage of the planning process (Michaelis and Schöbel 2009), which may result in generating inefficient operational plans or suboptimal solutions. Second and perhaps more critically, a solution for the timetabling problem can turn out to be infeasible for the vehicle scheduling problem.

Models for the integrated timetabling and vehicle scheduling problem with fixed-formation vehicles focus on minimizing the weighted sum of operational and passengers' costs measured by the number of vehicles required to satisfy the optimal timetable, passengers' waiting time, and transfer penalties. The first category of the integrated TTVS problem is dedicated to the one-line circulation of vehicles with static demand (e.g., Michaelis and Schöbel 2009, Ibarra-Rojas, Giesen, and Rios-Solis 2014). Considering one-line vehicle circulation, Laporte et al. (2017) presented an integrated optimization approach to jointly optimize timetables and vehicle schedules while incorporating information about users' preferences. The objectives include the minimization of users' inconvenience and the operator's costs. For the TTVS problem with fixed-formation vehicles, depot-based cross-line circulations of vehicles, and static passenger demand, Van Lieshout (2021) proposed an MILP model to jointly optimize the periodic timetable and the vehicle schedule, where vehicles are allowed to be scheduled across lines at depots. The depot-based cross-line circulations of fixed-formation buses are further addressed by Van Lieshout et al. (2021). Recently, Amberg and Amberg (2023) presented a flow-based formulation for the integration of vehicle scheduling and crew scheduling in a bus network, which incorporates the controlled trip-shifting strategy.

To sum up, only a few studies address the TTVS problem, and most of them consider fixed-formation vehicles and static and deterministic passenger demand. In particular, only Van Lieshout (2021) and Van Lieshout et al. (2021) consider depot-based cross-line circulation,

assuming that each trip can be served by a single vehicle and that passenger demand is both static and deterministic. We advance this research stream by integrating cross-line vehicle circulation into the TTVS problem under time-varying and uncertain demand. Our model allows each MAU to change its serving lines both during and after trips, and it enables each MAV to adjust formations at both depots and transfer stops. These capabilities also support in-vehicle transfers, allowing passengers to transfer between lines without alighting and reboarding.

2.2. Timetabling of MAVs with Flexible Formations

With technological advancements, scheduling MAVs with flexible formations has attracted increasing attention in the timetabling literature for public transportation systems. In such systems, one benefit of MAVs is the reduction in the number of MAUs required for operations enabled by flexible formation adjustments.

The majority of existing contributions consider the flexible formation of an MAV at the temporal level on a one-way line, allowing MAUs to be (de-)coupled at depots while taking into account deterministic passenger demand (e.g., Chen, Li, and Zhou 2019, 2020; Shi and Li 2021; Xia et al. 2023). To further close the substantial gap between passenger demand and vehicle capacity, a few recent studies have begun to focus on the timetabling of MAVs with station-wise flexible formation (i.e., an MAV can change its formation at any station on a single line) (e.g., Chen, Li, and Qu 2022, Tian, Lin, and Wang 2023, Xia, Ma, and Sharif Azadeh 2024). Considering the uncertain passenger demand, related studies have employed stochastic programming, robust optimization, and distributionally robust optimization (DRO) approaches, aiming to improve the robustness of timetables and vehicle schedules (e.g., An 2020). In the context of the timetabling of MAVs, Xia et al. (2023) proposed a DRO model on a modularized bus line and designed an exact solution method to solve it. They do not consider the vehicle scheduling aspect and assume that the fleet sizing is fixed. Xia, Ma, and Sharif Azadeh (2024) addressed the timetabling and vehicle scheduling problem within an integrated fixed-line and demand-responsive transport system. Despite considering the scheduling of units, they focus on fixed-line services on a bus line and direct services between two specific stops on this line, considering vehicle scheduling within a bus network and passengers' in-vehicle transfers out of scope.

In summary, the existing literature considering MAVs has conceptually demonstrated that flexible coupling and decoupling strategies can reduce both the number of used units and the associated operational costs. However, most of these studies focus on timetabling and neglect vehicle scheduling, resulting

in an overly optimistic estimation of the potential reduction in fleet size. There is a noticeable gap concerning the joint optimization of timetabling and vehicle scheduling with flexible formations of MAVs and cross-line circulations of units to further reduce operational costs at the network level, which is significantly more challenging than at the line level because of the complexity of additional variables and the coupling constraints among timetables, vehicle schedules, and MAV formations within the entire network.

2.3. Real-Time Optimization Under Uncertainty

To better position our study, we briefly review two major streams of real-time optimization under uncertainty in broader domains, such as public transport and ride hailing. Robust model predictive control (RMPC) and stochastic model predictive control (SMPC) are the widely used approaches, repeatedly solving short-horizon problems within the rolling-horizon solution framework while accounting for uncertainty and operational constraints (e.g., Ma et al. 2021, Carli et al. 2022, Wang et al. 2022). For example, Ma et al. (2021) proposed an RMPC framework for bus regulation under demand uncertainty, where timetable adjustments are made in a rolling-horizon fashion using CPLEX.

Beyond RMPC and SMPC, another line of studies explores online algorithms that make per-epoch decisions with performance guarantees, such as competitive ratios or regret bounds (e.g., Cho and Papavasiliou 2023, Lyu et al. 2024). For instance, Lyu et al. (2024) developed an adaptive-weighting method that solves a single weighted-sum subproblem each epoch and uses a randomized policy across scenarios and time.

We contribute to this research line by proposing a learning-based decision-making framework for solving real-time optimization problems under uncertainty. In this framework, we develop a learning-based scenario-retention method that ranks all stochastic scenarios based on their criticality using machine learning and carefully selects a representative subset for optimization. This framework greatly enhances computational efficiency while maintaining high solution quality under strict real-time limits. This framework is broadly applicable to real-time stochastic optimization problems beyond the timetabling and vehicle scheduling problem studied in this study.

2.4. Contributions Against the State-of-the-Art Studies

Table B.1 in Online Appendix B overviews closely related studies and highlights the current research gaps that motivate our study and contributions. In the existing literature on MAVs, most studies focus on timetabling for a single line, whereas vehicle scheduling, particularly at the network level, has received

limited attention. The broader TTVS literature for fixed-formation fleets is likewise limited and typically assumes single-line circulations under static, deterministic demand. When depot-based cross-line circulation is considered, line changes of vehicles are usually restricted to depots; midtrip line changing at transfer stops of units and passengers' in-vehicle transfers remain largely unexplored. Besides, solution approaches from previous studies are tailored for TTVS with fixed-formation vehicles and cannot address the unique operational features of MAVs, such as in-vehicle transfers and cross-line circulation of MAUs. Our main contributions are as follows.

- We tackle the TT-VS-DCA problem in a dynamically capacitated and modularized bus network with time-varying and uncertain demand. Our approach allows MAVs to dynamically adjust formations and enables MAUs to be (de-)coupled and rerouted both midtrip and after trips, supporting each MAU in changing its serving line at depots and transfer stops. This operational flexibility enables in-vehicle transfers of passengers, allowing passengers to transfer without disembarking and reboarding. In contrast to existing methods that typically assume fixed formations of vehicles, restrict line changes of vehicles to depots, or do not allow every vehicle to change its serving line, our approach enables more efficient utilization of MAUs across the entire network.

- We extend the formulations proposed by Xia et al. (2023) and Xia, Ma, and Sharif Azadeh (2024) to a stochastic MILP model at the network level, explicitly incorporating in-vehicle transfers of passengers and cross-line vehicle circulation of MAUs. Key decisions include fleet sizing, departure times, buffer times at stops, the optimal number of MAUs comprising each MAV as it leaves the depot and each transfer stop, (de-)coupling, and rerouting of units to other lines at transfer stops. We also formulate strict capacity constraints, coupling constraints between MAVs and passengers, and vehicle scheduling constraints.

- We develop an integer L-shaped method with tailored valid inequalities grounded in the theoretical properties of the proposed model, outperforming GUR-OB1 on moderate-size instances. To tackle real-world instances with time-varying and uncertain passenger demand, where the flexibility of MAUs introduces high operational complexity, we propose a rolling-horizon optimization approach within a double-decomposition framework, integrating the integer L-shaped method. This framework first decomposes the planning horizon into smaller decision-making stages and then incorporates the integer L-shaped method within each stage. In this framework, RH is used to bring dynamism into the solution process. By decomposing the problem into a series of smaller MILPs, our approach improves tractability compared with solving a large MILP with GUR-OB1 or the integer L-shaped method alone.

- To handle the evolving operational conditions in practical operations, we extend our approach to a novel learning-based real-time decision-making algorithm embedded in an RH framework with the integer L-shaped method. At each decision-making stage, this framework fine-tunes tactical timetables within a small adjustment range and reoptimizes vehicle schedules based on real-time conditions (e.g., time-varying passenger demand). To ensure computational tractability, we solve a stochastic MILP using the integer L-shaped method, incorporating a representative subset of scenarios. This subset is carefully selected through our proposed learning-based scenario-retention procedure, which ranks all scenarios based on predefined features using machine learning techniques and selects the top k -most representative ones. By limiting the number of scenarios and ensuring that only the most informative ones are retained in the optimization at each decision-making stage, our framework achieves high solution quality while satisfying real-time computational requirements.

- Lastly, we validate our model and solution methods using real-world instances from the Beijing bus network. Extensive experiments demonstrate the effectiveness of our algorithms and the practical advantages of the integrated optimization approach over a sequential optimization method. Moreover, our proposed approach with flexible (de-)coupling and cross-line circulation achieves considerable savings in the number of required MAUs compared with approaches using fixed-formation vehicles or those allowing (de-)coupling only at depots and depot-based cross-line circulation. Post hoc evaluations further show the robustness of our integrated optimization method in handling out-of-sample scenarios as well as the adaptability of the fine-tuning strategy under demand surges and redistributions. In addition, we show that the learning-based real-time decision-making framework consistently produces high-quality solutions within one minute per decision-making stage. It outperforms both full scenario enumeration and the greedy scenario-retention method, demonstrating its scalability for real-time and large-scale applications.

3. Problem Description

To provide a formal definition of the TT-VS-DCA problem from a mathematical perspective, we describe the network structure and operational framework within this network in Section 3.1. This is followed by the introduction of the time-dependent and uncertain passenger demand in Section 3.2. The operators' decisions and costs are summarized in Section 3.3. Lastly, the assumptions adopted in our model are introduced in Section 3.4.

3.1. Network Structure and Operational Framework

We first introduce the bus network. The structure of lines and trips within the network is then defined

followed by an introduction of MAVs. Lastly, we describe the operations of MAVs, including timetabling, vehicle scheduling, and dynamic capacity allocation.

3.1.1. Network Structure. We consider a bus network represented as a directed graph $\mathcal{G} = \{\mathcal{S}, \mathcal{A}\}$, where $\mathcal{S}$ represents the set of line-dependent stops and $\mathcal{A}$ represents the set of directed links between stops. The set $\mathcal{A}$ consists of two types of directed connections: *sections* and *transfer corridors*. Sections denoted as $\tilde{\mathcal{A}}$ represent directed links between two consecutive stops along the same line. Transfer corridors denoted as $\hat{\mathcal{A}}$ connect stops that correspond to the same physical location but belong to different lines, facilitating passenger transfers. Formally, we have $\mathcal{A} = \tilde{\mathcal{A}} \cup \hat{\mathcal{A}}$.

The network includes multiple lines denoted as $\mathcal{L}$. Each line $l \in \mathcal{L}$ is defined as an ordered sequence of line-dependent stops and directed sections that follows a specific operational direction. Each line is represented as a directed subgraph $\mathcal{G}_l = \{\mathcal{S}_l, \tilde{\mathcal{A}}_l\}$, where $\mathcal{S}_l$ denotes the set of stops on line l and $\tilde{\mathcal{A}}_l$ represents the directed sections along the line. The set of stops on line l is indexed as $\mathcal{S}_l = \{s_1, s_2, \dots, s_{|\mathcal{S}_l|}\}$, ensuring that each stop is uniquely associated with its respective line. The set of sections along line l is given by $\tilde{\mathcal{A}}_l = \{(l, i, l, i+1) : i, i+1 \in \mathcal{S}_l, l \in \mathcal{L}\}$.

Stops in the network are categorized into *transfer stops* and *nontransfer stops*. A transfer stop is a stop that appears on at least two different lines, allowing passengers to transfer between lines. The set of transfer stops on line l is denoted as $\mathcal{R}_l$, whereas the set of nontransfer stops is denoted as $\hat{\mathcal{R}}_l$. Thus, the set of all stops on line l satisfies $\mathcal{S}_l = \mathcal{R}_l \cup \hat{\mathcal{R}}_l, \forall l \in \mathcal{L}$. At transfer stops, there are transfer corridors to facilitate passenger transfers. The set of transfer corridors is denoted as $\hat{\mathcal{A}} = \{g : g = (l, i, l', i'), i \in \mathcal{R}_l, i' \in \mathcal{R}_{l'}, l \neq l', l, l' \in \mathcal{L}\}$. In addition, the network includes depots, where MAVs are stored or (de-)coupled. The set of depots is denoted as $\mathcal{M}$. Each bidirectional line is associated with two depots located adjacent to its terminal stops. These depots are shared by the corresponding pair of unidirectional lines used to represent the bidirectional line in our model.

3.1.2. Trips and MAVs. A *trip* is defined as a scheduled movement of an MAV along a specific line $l \in \mathcal{L}$, starting at a depot and ending at the other depot. The set of trips on line l is denoted as $\mathcal{K}_l = \{1, 2, \dots, |\mathcal{K}_l|\}$. Each trip is executed by a single MAV, whose formation can be changed at both depots and transfer stops.

An MAV consists of a specific number of MAUs, each with a fixed-capacity CAP. The MAV formation for a trip depends on the time-varying and uncertain passenger demand and the trip's departure time. As a

result, the relationship between MAVs and their formations plays a crucial role in operational planning.

3.1.3. Timetabling, Vehicle Scheduling, and Dynamic Capacity Allocation. The TT-VS-DCA problem studied here integrates tactical-level timetabling with operational-level vehicle scheduling and dynamic capacity allocation to generate optimal operating plans for a bus network using MAVs. The timetable specifies the departure and arrival times of MAVs at all stops on their assigned trips. Vehicle scheduling and dynamic capacity allocation determine at each stop the formation of the operated MAV given passenger demand and operational constraints. At the start of any trip, an MAU operates as part of an MAV. From this perspective, two operational patterns arise. (i) An MAU may complete an entire trip from the first stop to the last stop as part of an MAV; after the trip, the MAU may be stored in a depot or reassigned to another trip on the same line or a different line. (ii) An MAU may operate from the first stop to a transfer stop on the same line, after which it is decoupled from its original MAV and rerouted to another line, enabling efficient cross-line circulation of units throughout the network.

We allow each MAU to change its serving line at depots and transfer stops through (de-)coupling and rerouting. We do not allow deadheading, so MAUs never move empty between depots or between a depot and a transfer stop. Rerouting at transfer stops is not limited to lines that share a depot; an MAU decoupled at a transfer stop may be rerouted to any connected line, even if this line has different depots. Example C.1 in Online Appendix C illustrates vehicle scheduling and dynamic capacity allocation with our proposed cross-line circulations of MAUs.

3.2. Time-Dependent and Uncertain Path-Based Passenger Demand

It is well known that passenger demand in bus networks is typically time dependent and uncertain. Hence, we use the historical data to construct a set of stochastic passenger flow scenarios to capture the passenger demand in real-world operating environments. To facilitate modeling, we first discretize the studied time horizon into a finite set of time intervals with a duration Δ denoted by $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$ and indexed by t . The set of stochastic passenger flow scenarios is represented as $\mathcal{W} = \{1, 2, \dots, w, \dots, |\mathcal{W}|\}$, where ρ_w denotes the probability of each scenario $w \in \mathcal{W}$.

In each scenario, passenger demand is time dependent. To describe the dynamic evolution of passengers in the bus network, we adopt path-based passenger groups following Lee et al. (2022). Each *passenger group* p belongs to the set $\mathcal{P}_w$ under scenario $w \in \mathcal{W}$ and consists of n_p passengers who arrive at the origin stop i_p^o at time u_p and travel to the destination stop i_p^d following

the same sequence of lines and transfer facilities. We denote the ordered sequence of lines used by group p as $\mathcal{L}_p = (l_0, l_1, \dots, l_{|\mathcal{L}_p|}) \subseteq \mathcal{L}$ and the ordered sequence of transfer corridors as $\mathcal{R}_p = (g_1, g_2, \dots, g_{|\mathcal{R}_p|})$, where each g belongs to the set of transfer corridors $\mathcal{A}$.

We define a *journey* as the complete movements of passenger group p from its origin i_p^o to its destination i_p^d . The journey consists of one or more *legs*, where each leg is the travel along a single line between two consecutive transfers (or between the origin/destination and the nearest transfer). If no transfer is required, the journey has a single leg, and the set of the ordered sequence of transfer corridors (i.e., $\mathcal{R}_p$) is empty. If transfers occur, the first leg starts at the origin and ends at the first transfer stop, subsequent legs connect successive transfer stops, and the final leg ends at the destination. The elements of $\mathcal{R}_p$ provide the links between consecutive legs of the journey.

Passenger movements within a modularized bus network consist of four phases. Passengers first arrive at their origin stop and wait for an MAV to begin their first leg. Once an MAV arrives, they board. Upon reaching the destination stop of their final leg, they alight. If a transfer is required, passengers alight at the designated transfer stop and begin the next leg of their journey by boarding another MAV. Unlike conventional bus systems, where passengers must disembark and wait for another scheduled vehicle, the modularized system allows for *in-vehicle transfers*. If an MAU within the passenger group's onboard MAV is scheduled to reroute to their transfer-in line, the transfer can occur without requiring passengers to disembark and reboard, thereby improving transfer convenience.

3.3. Decisions and Costs of Operators

Bus operators make three key decisions: timetabling, vehicle scheduling, and dynamic capacity allocation. These decisions are highly interdependent and must adapt swiftly to the time-dependent and uncertain nature of passenger demand while also accounting for the dynamics of passengers within the network.

3.3.1. Timetabling Decisions. To provide passengers with a reliable and easy-to-remember schedule, operators must determine a robust timetable that remains consistent across all passenger flow scenarios occurring on different operational days. The timetable specifies arrival, departure, and dwell times at each stop for every trip on all lines.

3.3.2. Vehicle Scheduling and Dynamic Capacity Allocations Decisions. Operators have to make decisions on the formation of the MAV assigned to each trip leaving the depot and each transfer stop, whether the MAV assigned to each trip is decoupled or coupled at

transfer stops, and how the decoupled MAUs should be rerouted to serve new trips and lines.

3.3.3. Operational Costs. The costs of purchasing MAUs are formulated as the budget constraints in this study. Additionally, operational costs are formulated as a *cost function of operators*, which correlates positively with the increase in the number of used MAUs assigned to execute all trips. We also explore the impact of the fleet size on the trade-off between operational costs and passengers' costs.

We can now formally define the investigated TT-VS-DCA problem as follows.

Definition 1 (The TT-VS-DCA Problem). Given a bus network $\mathcal{G} = (\mathcal{S}, \mathcal{A})$, the stochastic scenario set $\mathcal{W}$, the set of time-dependent and uncertain passenger groups $\mathcal{P}_w$ under scenario w , and the group of trips $\mathcal{K}_l$ on line $l \in \mathcal{L}$, the goal of the TT-VS-DCA problem is to determine a robust timetable that specifies the arrival time and the departure time of each trip on every stop along each line (determined by $z_{k,i,t}^l$, which indicates whether an MAV assigned to trip k on line l departs stop i at time interval t). Additionally, the problem involves determining the formation of the MAV assigned to each trip k at each stop i on line l in scenario w (denoted as $x_{k,i}^l(w)$), the cross-line circulation of decoupled MAUs at transfer corridor g (denoted as $h_{k,k'}^g(w)$), and the number of MAUs stored at each depot m at the beginning of the studied time horizon κ_m . The goal is to minimize the weighted sum of passengers' costs and operational costs while satisfying strict capacity constraints.

3.4. Assumptions

To formulate the investigated problem, we adopt the following assumptions without loss of generality.

Assumption 1. We assume that the formation of an MAV can be reconfigured at depots and transfer stops through the coupling and decoupling of MAUs. These operations are assumed to require minimal time and are fully accommodated within the dwell time at transfer stops and the preparation time at depots without introducing any additional time. Both dwell time and preparation time are explicitly incorporated into the model.

Assumption 2. We assume that when coupled, MAUs form an open, bus-like interior space that allows passengers to stand and move freely between units.

Assumption 3. We assume that in-vehicle transfers are completed before the MAV arrives at transfer stops guided by onboard announcements. According to the operational plan, eligible passengers are informed in advance and can relocate to the appropriate MAU within the same coupled MAV. Upon arrival at the transfer stop, the MAV

decouples, and the appropriate MAU with passengers will be rerouted to a different line.

Assumption 1 has been widely adopted in existing studies (e.g., Chen and Li 2021, Chen, Li, and Qu 2022, Xia, Ma, and Sharif Azadeh 2024). Assumption 2 reflects the practical design features of emerging MAU technologies (e.g., Next Future Inc. 2023c). As for Assumption 3, unlike conventional transfers where passengers must alight, walk to another platform, wait for a different MAV, and board again, in-vehicle transfers involve no such actions. From the passenger's perspective, the rerouting of the MAUs to another line feels no different from a typical stop at a nontransfer bus stop. They remain seated throughout the decoupling and coupling process. It is important to note that during a regular stop at a nontransfer bus stop, passengers also remain on board while the MAV dwells at this bus stop. This period is also not considered part of the passenger's waiting time in our objective function. In the same spirit, the period associated with in-vehicle transfers is also excluded from the passenger's waiting time in the objective function as it does not impose additional effort on passengers unlike conventional transfers. In contrast, conventional transfers incur waiting time because of the inconvenience of alighting, walking, and reboarding, which we account for as a transfer penalty. This assumption highlights the superior comfort of in-vehicle transfers and supports the efficient design of timetables and (de-)coupling strategies of MAUs.

4. Mathematical Formulation

In this section, we propose a stochastic programming formulation for the TT-VS-DCA problem. We begin with a concise formulation to outline the model framework in Section 4.1. Section 4.2 presents the detailed formulations of the objective function. In Section 4.3, we delve into the timetabling constraints. In Section 4.4, we formulate constraints on interactions between passengers' dynamics and MAVs' movements. Section 4.5 presents the formulations related to vehicle scheduling. Finally, formulations for dynamic capacity allocations are provided in Section 4.6. All notations are summarized in Tables D.1 and D.2 in Online Appendix D.

4.1. Concise Formulation of the Stochastic Programming Model for the TT-VS-DCA Problem

Because of the complexity of the TT-VS-DCA problem, a large number of constraints are incorporated into the developed stochastic programming model. These constraints cover various aspects, including passengers' and operational costs (PO constraints), timetabling constraints (TT constraints), interactions between passengers' dynamics and MAVs' movements (PM

constraints), vehicle scheduling constraints (VS constraints), and interactions between vehicle scheduling and dynamic capacity allocation (VD constraints). To provide readers with a clear overview of the model structure, we first present the following concise formulation, which outlines the types of constraints and decision variables to illustrate the modeling framework:

$$\text{minimize}_{\mathbf{z}, \mathbf{h}, \mathbf{x}, \boldsymbol{\kappa}} \quad \varphi_1 \Psi^{\text{pass}} + \varphi_2 \Psi^{\text{oper}} \quad (1a)$$

$$\text{subject to} \quad (\text{PO constraints (2)–(5)}), \quad (1b)$$

$$(\text{TT constraints (7)–(13)}), \quad (1c)$$

$$(\text{PM constraints (14)–(21)}), \quad (1d)$$

$$(\text{VS constraints (22)–(25)}), \quad (1e)$$

$$(\text{VD constraints (26)–(33)}), \quad (1f)$$

$$z_{k,i,t}^l \in \{0,1\} \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l, t \in \mathcal{T}, \quad (1g)$$

$$h_{k,k'}^g(w) \in \{0,1\} \quad \forall w \in \mathcal{W}, g = (l, i, l', i') \\ \in \dot{\mathcal{A}}, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}, \quad (1h)$$

$$x_{k,i}^l(w) \in \mathbb{N} \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l, \quad (1i)$$

$$\kappa_m \in \mathbb{N} \quad \forall m \in \mathcal{M}. \quad (1j)$$

The first term in objective function (1a), Ψ^{pass} , represents passengers' costs in monetary units. Passengers' costs include the waiting costs at origin stops and the transfer waiting costs. The transfer waiting costs account for the inconvenience of conventional transfers, such as disembarking, walking, waiting, and reboarding. The second term, Ψ^{oper} , denotes operational costs, which are also expressed in monetary units. The coefficients φ_1 and φ_2 serve as weighting factors. Constraints (1g)–(1j) define the domains of the decision variables. In Sections 4.2–4.6, we provide detailed formulations of Constraints (1b), (1c), (1d), (1e), and (1f), respectively.

4.2. The Objective Function

The goal of this study is to minimize the weighted sum of passengers' costs and operational costs from a system-wide perspective. Below, we introduce the specific formulations for each cost component. First, from the perspective of passengers, the objective is to minimize the expected value of waiting costs (denoted as Ψ^{pass}) both at the origin stop of a passenger group p in scenario w (denoted as $\Psi_p^1(w)$) and at the transfer corridors (denoted as $\Psi_p^2(w)$), which can be expressed as follows:

$$\Psi^{\text{pass}} = \sum_{w \in \mathcal{W}} \sum_{p \in \mathcal{P}_w} n_p \cdot \rho_w \cdot \vartheta^T (\varepsilon_1 \cdot \Psi_p^1(w) + \varepsilon_2 \cdot \Psi_p^2(w)), \quad (2)$$

where ε_1 and ε_2 are weighting coefficients. ϑ^T

represents the equivalent monetary value per unit of passengers' waiting costs (unit: dollars). ρ_w and n_p indicate the probability of each scenario w and the number of passengers within the passenger group p , respectively.

We can now formulate the specific mathematical expressions of $\Psi_p^1(w)$ and $\Psi_p^2(w)$. Let a binary variable $\chi_{p,k}^l(w)$ denote whether passenger group p boards the MAV assigned to trip k on line l at their origin stop in scenario w . If yes, $\chi_{p,k}^l(w) = 1$; otherwise, $\chi_{p,k}^l(w) = 0$. The waiting time $\Psi_p^1(w)$ at the origin stop i_p^o of passenger group $p \in \mathcal{P}_w$ for all $w \in \mathcal{W}$ is formulated as follows:

$$\Psi_p^1(w) = \sum_{k \in \mathcal{K}_l} \chi_{p,k}^l(w) (d_{k,i}^l - u_p) \\ \forall w \in \mathcal{W}, p \in \mathcal{P}, l = l_0 \in \mathcal{L}_p, i = i_p^o, \quad (3)$$

where u_p represents the arrival time of passenger group p and $d_{k,i}^l$ denotes the departure time of trip k at stop i on line l .

To compute the waiting costs because of conventional transfers, we define two binary variables $\zeta_{p,k,k'}^g(w)$ and $h_{k,k'}^g(w)$. The variable $\zeta_{p,k,k'}^g(w)$ indicates whether passenger group p transfers from the MAV assigned to trip k to trip k' at transfer corridor g in scenario w . The variable $h_{k,k'}^g(w)$ represents whether MAUs on the MAV assigned to trip k on line l will be decoupled at transfer corridor $g = (l, i, l', i') \in \dot{\mathcal{A}}$ and rerouted to execute the k' th trip on line l' in scenario w . The waiting costs at transfer corridors depend on the type of transfer, which can be categorized as either an in-vehicle transfer or a conventional transfer that requires alighting and reboarding.

1. *In-vehicle transfers.* An in-vehicle transfer occurs when the MAUs on the MAV assigned to trip k on line l are scheduled to be decoupled at transfer corridor g and rerouted to serve the k' th trip on line l' (i.e., when $h_{k,k'}^g(w) = 1$) and there exists an onboard passenger group p requiring a transfer to that line (i.e., $\zeta_{p,k,k'}^g(w) = 1$). In this case, passenger group p can make an in-vehicle transfer, benefiting from its convenience, with zero transfer waiting cost.

2. *Conventional transfers requiring alighting and reboarding.* If either of the aforementioned conditions is not met, passengers must alight from the MAV assigned to trip k on line l , walk to and wait at the transfer corridor g , and board another MAV assigned to trip k' on line l' . In this case, the waiting time in the transfer corridor is given by the difference between the departure time of the transfer-in MAV assigned to trip k' (i.e., $d_{k',i'}^{l'}$) and the arrival time of the transfer-out MAV assigned to trip k (i.e., $a_{k,i}^l$) formulated as $d_{k',i'}^{l'} - a_{k,i}^l$.

To sum up, transfer waiting costs can be formulated in a single equation as follows:

$$\Psi_p^2(w) = \sum_{g=(l,i,l',i') \in \mathcal{R}_p} \sum_{k \in \mathcal{K}_l} \sum_{k' \in \mathcal{K}_{l'}} \zeta_{p,k,k'}^g(w) \cdot (1 - h_{k,k'}^g(w)) \cdot (d_{k',i'}^{l'} - d_{k,i}^l) \quad \forall w \in \mathcal{W}, p \in \mathcal{P}_w, \quad (4)$$

where the terms $\zeta_{p,k,k'}^g(w)(1 - h_{k,k'}^g(w)) \cdot d_{k',i'}^{l'}$ and $\zeta_{p,k,k'}^g(w)(1 - h_{k,k'}^g(w)) \cdot d_{k,i}^l$ are both nonlinear. The linearized formulations of the nonlinear Constraints (3) and (4) are provided in Online Appendix E corresponding to Constraints (47) and (48), respectively.

On the other hand, from the perspective of operators, the objective is to minimize operational costs. These costs are associated with the number of MAUs serving each section between any two adjacent stops on every line in each scenario. We define a binary decision variable $y_{k,i,q}^l(w)$ to indicate whether the number of MAUs composed in the MAV assigned to trip k serving the section between stops i and $i+1$ on line l in scenario w is $q \in \mathcal{Q}$ or not, where $\mathcal{Q}$ denotes the set of number of MAUs that can be contained in an MAV. A cost-related parameter associated with using q MAUs when an MAV departs from stop i is denoted by $\vartheta_{q,i}$. The formulation for the expected value of operational costs among all scenarios (denoted as Ψ^{oper}) can be presented as follows:

$$\begin{aligned} \Psi^{oper} &= \sum_{w \in \mathcal{W}} \rho_w \Psi_{op}(w) \\ &= \sum_{w \in \mathcal{W}} \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{S}_l} \sum_{k \in \mathcal{K}_l} \rho_w \cdot \sum_{q \in \mathcal{Q}} \vartheta_{q,i} \cdot y_{k,i,q}^l(w), \end{aligned} \quad (5)$$

where $\Psi_{op}(w)$ refers to the operational costs in each scenario $w \in \mathcal{W}$.

To facilitate the trade-offs between operators and passengers, the aforementioned objective functions are reformulated as a single one through the weighting coefficients φ_1 and φ_2 , shown as

$$\Psi = \varphi_1 \Psi^{pass} + \varphi_2 \Psi^{oper}. \quad (6)$$

4.3. Timetabling Constraints

The timetable specifies the arrival and departure times of each trip at every stop along each line. We introduce the parameters $\alpha_{l,k}^{min}$ and $\alpha_{l,k}^{max}$ as the lower and upper bounds of the shifting time at the first stop of trip k on line l . Similarly, $\beta_{l,k,i}^{min}$ and $\beta_{l,k,i}^{max}$ define the dwell time limits of trip k on line l at stop i , whereas h_{min}^l and h_{max}^l constrain the headway between two consecutive trips on line l . The running time between stops $i-1$ and i on line l for trip k is denoted as $r_{k,i-1}^l$.

To model timetabling decisions, we define the binary decision variable $z_{k,i,t}^l$ to indicate whether an MAV assigned to trip k on line l departs stop i at time interval t .

$z_{k,i,t}^l = 1$ indicates that trip k on line l has not departed from stop i ; otherwise, $z_{k,i,t}^l = 0$. The real-valued variables $a_{k,i}^l$ and $d_{k,i}^l$ represent the arrival and departure times at stop i , respectively. Additionally, $\alpha_{k,1}^l$ captures the shifting time at the first stop, while $\beta_{k,i}^l$ denotes the dwell time at stop i . The timetable is governed by the following constraints:

$$z_{k,i,t+1}^l \leq z_{k,i,t}^l \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l, t \in \mathcal{T} \setminus \{T\}. \quad (7)$$

$$d_{k,i}^l = \Delta \left(1 + \sum_{t \in \mathcal{T}} z_{k,i,t}^l \right) \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \quad (8)$$

$$a_{k,i}^l = \begin{cases} o_{k,1}^l + \alpha_{k,1}^l & \text{if } i = 1 \\ d_{k,i-1}^l + r_{k,i-1}^l & \text{otherwise} \end{cases} \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l. \quad (9)$$

$$\beta_{k,i}^l = d_{k,i}^l - a_{k,i}^l \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \quad (10)$$

$$\alpha_{l,k}^{min} \leq \alpha_{k,1}^l \leq \alpha_{l,k}^{max} \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l. \quad (11)$$

$$\beta_{l,k,i}^{min} \leq \beta_{k,i}^l \leq \beta_{l,k,i}^{max} \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \quad (12)$$

$$h_{min}^l \leq d_{k+1,i}^l - d_{k,i}^l \leq h_{max}^l \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l \setminus \{|\mathcal{K}_l|\}, i \in \mathcal{S}_l. \quad (13)$$

Constraints (7) ensure the monotonically nonincreasing property of the binary variable related to timetabling. Constraints (8) define the relationship between the binary variable related to timetabling and the real-valued departure times. Constraints (9) calculate the arrival time at stop i of trip k on line l . Constraints (10) are formulated to track the dwell time of trip k at stop i on line l denoted by $\beta_{k,i}^l$. This dwell time is defined as the difference between the departure time $d_{k,i}^l$ and the arrival time $a_{k,i}^l$ at stop i on line l for trip k . Additionally, Constraints (11), (12), and (13) set the lower and upper limitations on shifting times, dwell times, and headway, respectively.

4.4. Constraints on the Interactions Between Passengers' Dynamics and MAVs' Movements

Passenger dynamics in the bus network are closely linked to MAV movements and capacity constraints. These interactions are formulated in two subsections; the first subsection presents interactions between dynamics on different legs of the passengers' journey and MAVs, whereas the second subsection formulates interactions between dynamics of passengers and capacity of MAVs.

4.4.1. Interactions Between Dynamics on Different Legs of the Passengers' Journey and MAVs.

Theoretically, a passenger can board all trips arriving after his arrival at the origin station. In real-world operations, a passenger waiting at a stop usually chooses the first available MAV capable of completing their journey. To model relations between passengers' dynamics and movements of MAVs, we define a binary variable $\xi_{p,k}^l(w)$ that indicates whether passenger group p can board the MAV assigned to trip k on line l in scenario w or not for the first leg of their journey. Specifically, this variable takes a value of one if the MAV assigned to trip k on line l departs from the origin stop after the arrival time of passenger group p in scenario w and zero otherwise.

In the first leg of the journey, before any transfers, passenger group p has the opportunity to board any trip k on line l whose departure time d_{k,i_p}^l is later than the group's arrival time u_p at the origin stop i_p^o . That is,

$$M_1(\xi_{p,k}^l(w) - 1) \leq d_{k,i_p}^l - u_p \leq M_1\xi_{p,k}^l(w) - \epsilon_1 \quad \forall w \in \mathcal{W}, p \in \mathcal{P}_w, l = l_0, k \in \mathcal{K}_l. \quad (14)$$

Thereafter, if passenger group p needs to transfer from the MAV assigned to trip k on line l to trip k' on line l' at transfer corridor g , the departure time $d_{k',i'}^{l'}$ of the transfer-in trip has to be later than the arrival time $a_{k,i}^l$ of the transfer-out trip. To model this, we introduce a binary variable $\pi_{k,k'}^g$, which indicates whether passengers have the opportunity to transfer from the MAV assigned to trip k to the other MAV allocated to trip k' at transfer corridor g . This condition is formulated by the following constraints:

$$M_2(\pi_{k,k'}^g - 1) \leq d_{k',i'}^{l'} - a_{k,i}^l - \underline{\theta}_g \leq M_2\pi_{k,k'}^g - \epsilon_2 \quad \forall g = (l, i, l', i') \in \mathcal{A}, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}, \quad (15)$$

where the parameter $\underline{\theta}_g$ represents the minimum time required for a transfer at transfer corridor g .

Note that Constraints (15) capture all possible transfer options for a passenger group p arriving at a transfer corridor g . However, because we assume that passengers always board the first available trip, additional constraints should be introduced to enforce this selection. To do so, we formulate nonlinear Constraints (16) to link passenger group p and its actually boarded trip. Specifically, passenger group p actually transfers from the MAV assigned to trip k to trip k' at transfer corridor g in scenario w ($\zeta_{p,k,k'}^g(w) = 1$) if and only if this group was on trip k on line l (i.e., $\chi_{p,k}^l(w) = 1$), trip k' on line l' has a later departure time than the arrival time of trip k , and this is the closest among all possible transfer options on line l' (i.e., $\pi_{k,k'}^g - \pi_{k,k'-1}^g = 1$). As a result, passenger group p actually transfers from trips k to k'

at the transfer corridor g . The linearized formulation of this set of constraints is presented in Online Appendix E denoted as Constraints (49).

$$\begin{aligned} \zeta_{p,k,k'}^g(w) &= \chi_{p,k}^l(w) \cdot (\pi_{k,k'}^g - \pi_{k,k'-1}^g) \\ \forall w \in \mathcal{W}, p \in \mathcal{P}_w, g = (l, i, l', i') \in \mathcal{R}_p, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}. \end{aligned} \quad (16)$$

Additionally, Constraints (17) are proposed to track the MAV assigned to trip k that passenger group p boards when transferring from trip k' on line l' to trip k on line l in scenario w . If line l is the line where the origin stop of passenger group p 's journey is located (i.e., $l = l_0$), $\chi_{p,k}^l(w)$ indicates whether the group boards trip k in the first leg. If line l is a transfer-in line within the journey, the variable determines whether the group successfully transfers to trip k after departing from trip k' . The corresponding formulation is as follows:

$$\chi_{p,k}^l(w) = \begin{cases} \xi_{p,k}^l(w) - \xi_{p,k-1}^l(w), & \text{if } l = l_0 \\ \sum_{k' \in \mathcal{K}_{l'}} \zeta_{p,k',k}^g(w), & \text{if } l \neq l_0, \\ & g = (l', i', l, i) \in \mathcal{R}_p \end{cases} \quad \forall w \in \mathcal{W}, p \in \mathcal{P}_w, l \in \mathcal{L}_p, k \in \mathcal{K}_l. \quad (17)$$

4.4.2. Interactions Between Dynamics of Passengers and Capacity of MAVs.

Let variables $b_{k,i}^l(w)$ and $c_{k,i}^l(w)$ represent the numbers of passengers boarding and alighting trip k at stop i on line l in scenario w , respectively. The variable $v_{k,i}^l(w)$ denotes the number of in-vehicle passengers when the MAV assigned to trip k departs from stop i on line l in scenario w . We define an integer variable $x_{k,i}^l(w)$ to indicate the real-valued number of MAUs composed in the MAV allocated to trip k serving the section between stops i and $i + 1$ on line l . We then formulate constraints related to passenger movements as follows:

$$b_{k,i}^l(w) = \sum_{p \in \mathcal{PO}_i} n_p \cdot \chi_{p,k}^l(w) \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \quad (18)$$

$$c_{k,i}^l(w) = \sum_{p \in \mathcal{PD}_i} n_p \cdot \chi_{p,k}^l(w) \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \quad (19)$$

$$v_{k,i}^l(w) = \sum_{j \leq i} (b_{k,j}^l(w) - c_{k,j}^l(w)) \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i, j \in \mathcal{S}_l. \quad (20)$$

$$v_{k,i}^l(w) \leq \text{CAP} \cdot x_{k,i}^l(w) \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \quad (21)$$

Constraints (18) compute the number of passengers boarding the MAV assigned trip k when it arrives at stop i , where set $\mathcal{PO}_i$ consists of all of the passengers who board at stop i . Similarly, Constraints (19) are formulated to compute the number of passengers alighting at stop i , where the set $\mathcal{PD}_i$ includes all of the passengers who alight at stop i . Constraints (20) compute the number of in-vehicle passengers when the

MAV assigned to trip k leaves stop i on line l , which equals the difference between the cumulative number of boarding and alighting passengers. Constraints (21) ensure that the number of in-vehicle passengers cannot exceed the capacity.

4.5. Constraints Associated with Vehicle Scheduling

In this study, we assume that MAVs allocated to trips can be decoupled and coupled at transfer stops and depots. Thus, the number of usable MAUs stored at the depots at each time interval during operations should be sufficient to perform trips. We introduce two variables $AV_{m,t}(w)$ and $DV_{m,t}(w)$ to track the cumulative inflow and outflow of MAUs at depot $m \in \mathcal{M}$ at time t in scenario w , respectively. The decision variable κ_m is defined to indicate the number of MAUs stored at each depot $m \in \mathcal{M}$ at the beginning of the study time horizon. We have the following constraints:

$$\sum_{m \in \mathcal{M}} \kappa_m \leq B. \quad (22)$$

$$DV_{m,t}(w) \leq \kappa_m + AV_{m,t}(w) \quad \forall m \in \mathcal{M}, t \in \mathcal{T}, w \in \mathcal{W}. \quad (23)$$

$$AV_{m,t}(w) = \sum_{l \in LL_m} \sum_{k \in \mathcal{K}_l} (1 - z_{k,|S_l|,t-t_m^{pre}}^l) \cdot x_{k,|S_l|}^l(w) \quad \forall m \in \mathcal{M}, t \in \mathcal{T}, w \in \mathcal{W}. \quad (24)$$

$$DV_{m,t}(w) = \sum_{l \in FL_m} \sum_{k \in \mathcal{K}_l} (1 - z_{k,1,t}^l) \cdot x_{k,1}^l(w) \quad \forall m \in \mathcal{M}, t \in \mathcal{T}, w \in \mathcal{W}. \quad (25)$$

Constraints (22) set the upper bound of MAUs used to cover all trips, where the parameter B represents the maximum number of available MAUs. Constraints (23) are imposed to ensure that the cumulative number of MAUs exiting the depot m cannot exceed its own storage and inflow. Constraints (24) and (25) are formulated to compute the inflows and outflows of MAUs at time interval t and depot m in scenario w , respectively, where FL_m represents the sets of lines whose starting stop is depot m and LL_m refers to the sets of lines for which the ending stop is depot m . The parameter t_m^{pre} is the preparation time at depots of each MAV to complete a trip before it is reallocated by the next trip. The linearization of the nonlinear Constraints (24) and (25) is presented in Online Appendix E as Constraints (50).

4.6. Constraints on the Interactions Between Vehicle Scheduling and Dynamic Capacity Allocation

We assume that rerouting operations are performed at the transfer stops only when the time window requirements are met. The reason for this is twofold. The above operations take a certain length of time to perform; on the other hand, if the decoupled units are stored for a considerable period of time in a transfer

stop without being able to be rerouted, their utilization rate will be lower, and they will take up the limited space in the transfer stop, becoming inconvenient for both operations and passengers. We define the variable $e_{k,k'}^g$ to indicate whether units assigned to trip k can be decoupled and coupled to the MAV executing trip k' at the transfer corridors g or not: in other words, whether the decoupling and coupling behaviors are feasible in the time dimension. Recall that we have defined the variable $h_{k,k'}^g(w)$ to indicate whether MAUs on the MAV assigned to trip k on line l will be decoupled at transfer corridor $g = (l, i, l', i') \in \dot{\mathcal{A}}$ and rerouted to execute the trip k' on line l' in scenario w . We formulate the following constraints to describe the decoupling and coupling operations at transfer stops:

$$e_{k,k'}^g = \begin{cases} 1 & \text{if } \underline{\theta}_g \leq d_{k',i'}^{l'} - a_{k,i}^l \leq \bar{\theta}_g \\ 0 & \text{otherwise} \end{cases} \quad \forall g = (l, i, l', i') \in \dot{\mathcal{A}}, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}. \quad (26)$$

$$h_{k,k'}^g(w) \leq e_{k,k'}^g \quad \forall w \in \mathcal{W}, g = (l, i, l', i') \in \dot{\mathcal{A}}, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}. \quad (27)$$

$$h_{k,k'}^g(w) + h_{k',k}^{g'}(w) \leq 1 \quad \forall w \in \mathcal{W}, g = (l, i, l', i') \in \dot{\mathcal{A}}, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}. \quad (28)$$

Constraints (26) define the temporal bounds $[\underline{\theta}_g, \bar{\theta}_g]$ for rerouting the (de-)coupling MAUs across lines, which ensure that rerouting the decoupled MAUs from trip k on line l to serve trip k' on line l' is only possible if the time difference between the departure time of trip k' (i.e., $d_{k',i'}^{l'}$) and the arrival time of trip k (i.e., $a_{k,i}^l$) is within the predetermined time window. This set of nonlinear constraints is linearized in Online Appendix E as Constraints (51). Constraints (27) are formulated to ensure that this operation is practicable only after the aforementioned temporal conditions of decoupling and rerouting are met. Constraints (28) guarantee the unidirectional rerouting and coupling of the decoupled MAUs. The reason for developing Constraints (28) is that if MAUs composed in the MAV assigned to trip k on line l are decoupled, rerouted to line l' , and coupled on the MAV serving trip k' , this indicates that additional MAUs are required to successfully complete trip k' . Consequently, this also indicates that further decoupling of the MAV assigned to trip k' is not necessary.

Finally, we formulate the following constraints to model the dynamics of the formations of MAVs during operations and to characterize the connections between the formations and the en route (de-)coupling operations at transfer stops. The binary variable $y_{k,i,q}^l(w)$ is defined to indicate whether the MAV allocated to perform the trip k at stop i on line l in scenario w is comprised by q MAUs. If yes, we have $y_{k,i,q}^l(w) = 1$;

otherwise, it equals to zero.

$$x_{k,i}^l(w) = \sum_{q \in \mathcal{Q}} q \cdot y_{k,i,q}^l(w) \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \quad (29)$$

$$x_{k,i-1}^l(w) = x_{k,i}^l(w) \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i-1, i \in \hat{\mathcal{R}}_l. \quad (30)$$

$$\begin{aligned} x_{k,i}^l(w) - x_{k,i-1}^l(w) + 1 &\leq M_3(1 - h_{k,k'}^g(w)) \\ &\quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, \\ &\quad k' \in \mathcal{K}'_l, i \in \mathcal{R}_l, g = (l, i, l', i') \in \dot{\mathcal{A}}. \end{aligned} \quad (31)$$

$$\begin{aligned} x_{k,i}^l(w) &= x_{k,i-1}^l(w) + \sum_{l'} \sum_{k' \in \mathcal{K}'_{l'}} (h_{k',k}^{g'}(w) \\ &\quad - h_{k,k'}^g(w))(x_{k',i'-1}^{l'}(w) - x_{k',i'}^{l'}(w)) \\ &\quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{R}_l, g \\ &\quad = (l, i, l', i') \in \dot{\mathcal{A}}, g' = (l', i', l, i) \in \dot{\mathcal{A}}. \end{aligned} \quad (32)$$

$$y_{k,i,q}^l(w) \in \{0, 1\} \quad \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l, q \in \mathcal{Q}. \quad (33)$$

Constraints (29) are imposed to compute the real-valued number of MAUs comprising this MAV at stop i on line l (i.e., $x_{k,i}^l(w)$). Constraints (30) are developed to guarantee that the formation of an MAV executing trip k remains unaltered upon arrival and departure at stops other than transfer stops. Moreover, given that MAVs allocated to trips have the flexibility to be decoupled or coupled at transfer stops, the formation of MAVs can potentially change as they depart from transfer stops. Following this, Constraints (31) link routing and decoupling operations at the transfer stop (i.e., when $h_{k,k'}^g(w) = 1$) with modifications to the number of MAUs that make up the MAV to be decoupled. Nonlinear Constraints (32) are formulated to calculate the number of MAUs included in the MAV that is assigned to execute trip k upon departing from transfer stop i on line l . We derive the linearized form of this set of nonlinear constraints in Constraints (52) presented in Online Appendix E. Constraints (33) define the domain.

In summary, we propose the following MILP model for the TT-VS-DCA problem in the modularized bus network with MAVs:

$$\begin{aligned} &\text{minimize}_{\mathbf{z}, \mathbf{h}, \mathbf{x}, \mathbf{K}} \quad \varphi_1 \Psi^{\text{pass}} + \varphi_2 \Psi^{\text{oper}} \\ &\text{subject to} \quad (1g)-(1j), (2), (5), (7)-(15), (17)-(23), \\ &\quad (27)-(31) \text{ and } (33) \text{ and } (47)-(52) \\ &\quad \text{in Online Appendix E.} \end{aligned} \quad (34)$$

5. Theoretical Properties of the TT-VS-DCA Model

In this section, we establish propositions based on the mathematical properties of the proposed model that

serve as the foundation for the solution methods introduced in the following section. The TT-VS-DCA problem studied in this work is NP hard. As the problem size increases, the number of combinations across line $l \in \mathcal{L}$, trip $k \in \mathcal{K}_l$, stop $i \in \mathcal{S}_l$, and discretized time intervals $t \in \mathcal{T}$ grows exponentially, substantially increasing the computational complexity of the problem. To address this challenge, we propose the following propositions to strengthen the model.

Specifically, Proposition 1 introduces a time window-based reduction that restricts the feasible domain of timetabling variables. Proposition 2 encodes boarding feasibility by eliminating logically invalid boarding times. Proposition 3 ensures correct temporal sequencing for passenger transfers. Proposition 4 provides a compact temporal window that enables the decoupling of an MAV at a transfer stop and its subsequent reassignment to another line, ensuring the temporal feasibility of its cross-line circulation.

Proposition 1 (Time Window-Based Search Space Reduction). *Constraints (35) are valid for the TT-VS-DCA model (34):*

$$z_{k,i,t}^l = \begin{cases} 0, & \frac{\bar{\varsigma}_{k,i}^l}{\Delta} \leq t \leq |\mathcal{T}| \\ 1, & 1 \leq t \leq \frac{\underline{\varsigma}_{k,i}^l}{\Delta} - 1, \end{cases} \quad \forall l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l, \quad (35)$$

where $\underline{\varsigma}_{k,i}^l$ and $\bar{\varsigma}_{k,i}^l$ represent the earliest and latest departure times of the MAV assigned to trip k leaving stop i of line l , respectively. The specific values of $\underline{\varsigma}_{k,i}^l$ and $\bar{\varsigma}_{k,i}^l$ can be determined previously before optimization.

Proof. See Online Appendix F. $\square$

Proposition 2 (Passenger Boarding-Related Inequality). *Constraints (36) are a family of valid inequality for the TT-VS-DCA model related to the boarding process (34):*

$$\begin{aligned} \bar{\varsigma}_{k,i}^l - u_p &\geq |\mathcal{T}|(\xi_{p,k}^l(w) - \xi_{p,k-1}^l(w) - 1) \\ &\quad \forall w \in \mathcal{W}, p \in \mathcal{P}_w, l = l_p^1 \in \mathcal{L}_p, k \in \mathcal{K}_l. \end{aligned} \quad (36)$$

Proof. See Online Appendix F. $\square$

Proposition 3 (Transfer-Related Inequality). *Constraints (37) are a family of valid inequality for the TT-VS-DCA model (34) associated with transfers:*

$$\begin{aligned} \bar{\varsigma}_{k',i'}^{l'} - \underline{\varsigma}_{k,i}^l + \beta_{l,k,i}^{\min} - \underline{\theta}_g &\geq |\mathcal{T}|(\pi_{k,k'}^g - \pi_{k,k'-1}^g - 1) \\ &\quad \forall g = (l, l', i, i') \in \dot{\mathcal{A}}, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}. \end{aligned} \quad (37)$$

Proof. See Online Appendix F. $\square$

Proposition 4 (Coupling-Related Inequality). *Constraints (38) are a family of valid inequality for the TT-VS-DCA*

model (34):

$$\begin{cases} \bar{c}_{k',i'} - \underline{c}_{k,i} + \beta_{l,k,i}^{min} - \theta_g \geq |\mathcal{T}|(c_{k,k'}^g - 1) \\ \bar{\theta}_g - \underline{c}_{k',i'} + \bar{c}_{k,i} - \beta_{l,k,i}^{max} \geq |\mathcal{T}|(c_{k,k'}^g - 1) \end{cases} \quad \forall g = (l, l', i, i') \in \mathcal{A}, k \in \mathcal{K}_l, k' \in \mathcal{K}_{l'}. \quad (38)$$

Proof. See Online Appendix F. $\square$

6. Solution Method

In this section, we design the solution approaches to solve the TT-VS-DCA problem. First, we develop an integer L-shaped method with tailored valid inequalities in Section 6.1. However, through numerical experiments based on real-world operational data from the Beijing bus network with a three-hour planning period, we observe that the proposed exact solution method still faces computational challenges. To address this, in Section 6.2, we introduce a rolling-horizon optimization framework that dynamically incorporates the exact solution method, enabling a double decomposition of the problem. Finally, recognizing the need for real-time adaptability in practical operations with evolving passenger demand, we propose a learning-based real-time decision-making framework as detailed in Section 6.3.

6.1. The Integer L-Shaped Algorithm with Tailored Valid Inequalities

Because our proposed model (34) is an MILP model, one way to find the optimal solution is to adopt the solution methods designed for linear programs, such as the built-in branch-and-bound or cutting-plane algorithms available in commercial software. Nevertheless, the quality of the obtained solution falls short of expectations when these methods are applied to a real-life instance based on a Beijing bus subnetwork that consists of two bidirectional lines, and the gap is greater than 42% even after a 10-hour computation.

Another approach is to decompose the model using the Benders decomposition method or the integer L-shaped method. Specifically, the integer L-shaped method is a powerful extension of the basic Benders decomposition approach, which is specially designed for addressing stochastic integer programming problems. In problems where there are binary first-stage variables and integer second-stage variables within a two-stage stochastic program, the integer L-shaped method has proven to be remarkably effective. Recall that our model has an endogenous sequential solution property (i.e., the optimization of vehicle scheduling and dynamic capacity allocation can only be achieved if timetables are determined). Therefore, decomposing our problem and solving it with the integer L-shaped algorithm promise to improve the solution efficiency and solution quality.

We denote $\Psi_{wait}^1 = \sum_{w \in \mathcal{W}} \sum_{p \in \mathcal{P}_w} n_p \cdot \rho_w \cdot \varepsilon_1 \cdot \vartheta^T \cdot \Psi_p^1(w)$ and $\Psi_{wait}^2(w) = \sum_{p \in \mathcal{P}_w} n_p \cdot \rho_w \cdot \varepsilon_2 \cdot \vartheta^T \cdot \Psi_p^2(w)$. To implement the integer L-shaped method, we first

reformulate the proposed model (34) (denoted as MP) as follows:

$$[\text{MP}] \begin{cases} \min_{\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi}} \Psi_{wait}^1 \\ \text{s.t.} & (1g), (7)-(15), (17), (35) \\ & -(37) \text{ and } (47) \text{ and } (49), \\ & \text{in Online Appendix E.} \\ & \eta \geq F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi}), \end{cases} \quad (39)$$

where $F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi})$ is real-valued function of variables $\mathbf{z}$, $\boldsymbol{\chi}$, and $\boldsymbol{\xi}$, which can be formulated as follows:

$$\begin{aligned} F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi}) &= \sum_{w \in \mathcal{W}} \rho_w F^w(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi}) \\ &= \sum_{w \in \mathcal{W}} \rho_w [\min_{\mathbf{x}, \mathbf{h}} \{ \Psi_{wait}^2(w) + \Psi_{op}^w | (1h)-(1j), \\ &\quad (5), (18)-(23), (27)-(31), (33), (38) \\ &\quad \text{and } (48) \text{ and } (50)-(52) \\ &\quad \text{in Online Appendix E} \}]. \end{aligned} \quad (40)$$

In this study, we denote $F^w(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi})$ as the value function of the subproblem with respect to the scenario w . Given the timetables and the state of the passengers, the subproblem $F^w(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi})$ is to determine the optimal vehicle schedule and dynamic capacity allocations.

Lemma 1. *There exists a finite lower bound $L = \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{S}_l} \sum_{k \in \mathcal{K}_l} \varphi_2 \vartheta_{1,i}$ that satisfies $F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi}) \geq L$.*

Proof. See Online Appendix F. $\square$

As outlined in Laporte and Louveaux (1993) and Angulo, Ahmed, and Dey (2016), the core idea of the integer L-shaped method is to relax the constraint $\eta \geq F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi})$. This approach entails adding cuts to more accurately approximate the shape of $F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi})$. This iterative process continuously refines the solution space, providing an optimal solution that is incrementally tightly fitted. Specifically, the optimality cut proposed by Laporte and Louveaux (1993) can be formulated for the explored problem as follows:

$$\begin{aligned} \eta \geq (F(\mathbf{z}^*, \boldsymbol{\chi}^*, \boldsymbol{\xi}^*) - L) &\left(\sum_{z_{k,i,t}^l \in H(z^*)} z_{k,i,t}^l + \sum_{\chi_{p,k}^l(w) \in G(\chi^*)} \chi_{p,k}^l(w) \right. \\ &+ \sum_{\zeta_{p,k,k'}^g(w) \in V(\zeta^*)} \zeta_{p,k,k'}^g(w) \\ &- \sum_{z_{k,i,t}^l \notin H(z^*)} z_{k,i,t}^l - \sum_{\chi_{p,k}^l(w) \notin G(\chi^*)} \chi_{p,k}^l(w) \\ &- \sum_{\zeta_{p,k,k'}^g(w) \notin V(\zeta^*)} \zeta_{p,k,k'}^g(w) - |H(z^*)| \\ &\left. - |G(\chi^*)| - |V(\zeta^*)| \right) + F(\mathbf{z}^*, \boldsymbol{\chi}^*, \boldsymbol{\xi}^*), \end{aligned} \quad (41)$$

where η represents the estimated value of $F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\xi})$ and where $H(z^*) = \{z_{k,i,t}^l | z_{k,i,t}^l = 1\}$, $G(\chi^*) = \{\chi_{p,k}^l(w) | \chi_{p,k}^l(w) = 1\}$,

$V(\zeta^*) = \{\zeta_{p,k,k'}^g(w) | \zeta_{p,k,k'}^g(w) = 1\}$. Moreover, L represents the lower bound of $F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\zeta})$, whose values can be set based on Lemma 1. Because the aforementioned optimality cuts are not tight for solutions other than $(\mathbf{z}^*, \boldsymbol{\chi}^*, \boldsymbol{\zeta}^*)$, we employ the following continuous optimality cut the same as Angulo, Ahmed, and Dey (2016):

$$\eta \geq \bar{\mathbf{t}}(\mathbf{z} - \mathbf{z}^*) + \hat{\mathbf{t}}(\boldsymbol{\chi} - \boldsymbol{\chi}^*) + \tilde{\mathbf{t}}(\boldsymbol{\zeta} - \boldsymbol{\zeta}^*) + F_{LP}(\mathbf{z}^*, \boldsymbol{\chi}^*, \boldsymbol{\zeta}^*), \quad (42)$$

where $\bar{\mathbf{t}}$, $\hat{\mathbf{t}}$, and $\tilde{\mathbf{t}}$ represent the subgradient vectors of $F(\mathbf{z}, \boldsymbol{\chi}, \boldsymbol{\zeta})$ at $\mathbf{z}^*$, $\boldsymbol{\chi}^*$, and $\boldsymbol{\zeta}^*$, respectively. $F_{LP}(\mathbf{z}^*, \boldsymbol{\chi}^*, \boldsymbol{\zeta}^*)$ represents the continuous relaxation of $F(\mathbf{z}^*, \boldsymbol{\chi}^*, \boldsymbol{\zeta}^*)$ that resets the binary variable $y_{k,i,q}^l(w)$ as a continuous variable. The subgradient optimality cut (42) is not necessarily required for the convergence of the proposed method; however, it enhances the computing performance of this algorithm.

To ensure the feasibility of the solution, we introduced the following combinatorial cut:

$$\begin{aligned} & \sum_{z_{k,i,t}^l \in H(\mathbf{z}^*)} (1 - z_{k,i,t}^l) + \sum_{\chi_{p,k}^l(w) \in G(\boldsymbol{\chi}^*)} (1 - \chi_{p,k}^l(w)) \\ & + \sum_{\zeta_{p,k,k'}^g(w) \in V(\boldsymbol{\zeta}^*)} (1 - \zeta_{p,k,k'}^g(w)) \\ & + \sum_{z_{k,i,t}^l \notin H(\mathbf{z}^*)} z_{k,i,t}^l + \sum_{\chi_{p,k}^l(w) \notin G(\boldsymbol{\chi}^*)} \chi_{p,k}^l(w) \\ & + \sum_{\zeta_{p,k,k'}^g(w) \notin V(\boldsymbol{\zeta}^*)} \zeta_{p,k,k'}^g(w) \geq 1. \end{aligned} \quad (43)$$

Finally, we reformulate the MP based on the above analyses and construct the current problem (CP) at each node of the branch-and-cut search tree as

$$\left\{ \begin{array}{l} \min \Psi^{pass} + \eta \\ \text{s.t.} \quad \text{Optimality cuts (41),} \\ \quad \text{Subgradient optimality cuts (42),} \\ \quad \text{Feasibility cuts (43),} \\ \quad \text{Constraints (1g), (7)–(15), (17), (35)–(37)} \\ \quad \text{and (47) and (49) in Online Appendix E.} \end{array} \right. \quad (44)$$

In summary, the main procedure of our designed decomposition-based integer L-shaped algorithm for the TT-VS-DCA problem can be presented as follows. (1) Relax CP (44) by discarding Constraints (1g) and solving it to the optimum. It is worth noting that no optimality cut or subgradient cut has been generated yet. (2) Decide whether to proceed with the computation. If the stopping criteria are met, terminate the algorithm. Otherwise, we select a $z_{k,i,t}^l \notin \{0,1\}$ and then branch at this node. (3) Repeat the previous step until an integer solution $\mathbf{z}^*$ is obtained. In this process, it is

necessary to determine whether to add optimality cuts (41) and (42) or feasibility cuts (43).

In the specific implementation, we construct a branch-and-cut search tree using the state-of-the-art commercial solver GUROBI. The optimality cuts (41) and (42) as well as the feasibility cuts (43) are added to the CP step by step using the lazy constraint callback function. During the search process, the callback function is executed at the node where the relaxed CP is solved to the optimum or a new mixed-integer programming (MIP) incumbent is found. The pseudocode of this algorithm is presented in Algorithm 1 in Online Appendix G. When the timetable is fixed, the vehicle schedule and dynamic capacity allocation plan in each scenario can be optimized in parallel.

6.2. A Rolling-Horizon Optimization Algorithm Based on a Double-Decomposition Framework

The TT-VS-DCA problem is NP-hard, making the integer L-shaped method from Section 6.1 inefficient for large-scale instances. A promising approach to address this challenge is to design a rolling-horizon optimization algorithm, which has been widely used to solve various traffic problems as detailed in Sánchez-Martínez, Koutsopoulos, and Wilson (2016). This algorithm follows a progressive, periodically updated decision-making process, adding dynamism to the solution approach.

The rolling-horizon optimization algorithm decomposes the TT-VS-DCA problem in both spatial and temporal dimensions, allowing for the solution of a series of smaller MILPs instead of a single large MILP. Specifically, in the temporal dimension, the algorithm splits the entire study time horizon into multiple sequential stages, with each treated as a separate subproblem. In the spatial dimension, it strategically considers only the stops involved within a specific stage during the optimization process. Furthermore, our rolling-horizon optimization algorithm is built upon a double-decomposition framework by incorporating the integer L-shaped method. It first decomposes the decision period into sequential stages with shorter time durations, and within each stage, the integer L-shaped method further decomposes the problem to improve computational efficiency.

Thereafter, we introduce this algorithm tailored to the TT-VS-DCA problem utilizing a rolling-horizon optimization framework in detail. The algorithm segments the studied time horizon into equal-length decision-making stages, each with two time domains: the operation control horizon (CH) and the information prediction horizon (PH). Both start simultaneously, but the PH, which is longer than the CH, allows for a broader scope of decisions and external data. Each PH intersects with CHs from adjacent

stages, facilitating real-time updates. Within each phase, we optimize the TT-VS-DCA problem for the entire PH but employ solutions only within the CH. Information is sequentially transferred between stages, streamlining optimization and boosting algorithm efficiency. For clarity, we encapsulate the functions of the algorithm into the following modules.

Initialization. Input the network topology, the set of all passengers groups $\mathcal{P}_w$, the control parameters of the system, and the model parameters. Initialize the time duration of each CH and PH. Also, the empty sets $\tilde{\mathcal{P}}$, and $\bar{\mathcal{P}}$ are initialized to store the set of passengers who do not complete transfers and the set of passengers who do not get off MAVs, respectively.

Prediction modular. We incorporate a prediction module into the algorithm to enable its extension to dynamic optimization. In this module, the travel time of vehicles between adjacent stops is predicted based on detailed road traffic conditions within a specific time horizon. Additionally, the module estimates passenger demand at each stop using data from monitoring equipment during the same period.

Timetable simulation module. Update the travel time of the vehicle between two adjacent stops. Simulate the vehicle arrival and departure times at each stop based on the control strategy imposed during the CH.

Select module. The timetable simulation module is first called to simulate the arrival and departure times of trips. Then, both MAVs assigned to trips and stops that are within the PH are selected. Denote the selection vehicles set as $\mathcal{K}^n \in \mathcal{K}$, and denote the selection stops set as $\mathcal{S}^n \in \mathcal{S}$. For each scenario w , we select all passengers whose arrival time is earlier than the ending time of PH from the set $\mathcal{P}_w$ and form the new set $\mathcal{P}_w^n$. Then, the model of the TT-VS-DCA problem for the current decision stage is created.

Status updating module. First, we check the status of each passenger group in each scenario w . Specifically, if the passenger group has completed all trips, then the group from $\mathcal{P}_w$ will be removed. If the passenger group has boarded but has not completed all transfers, then we place it in set $\tilde{\mathcal{P}}_w$. If the passenger has completed all transfers but has not yet alighted from the last leg of the trip, then we place it in $\bar{\mathcal{P}}_w$. In particular, for passengers who require placement in $\tilde{\mathcal{P}}_w$, we identify their completed legs, record the set of all vehicles that they have boarded, and then, remove the completed trips from the passenger group attribute. In the next optimization, by identifying the current state of passengers, the variables describing passengers' state (i.e., $\xi_{p,k}^l(w)$, $\zeta_{p,k,k'}^g(w)$, and $\chi_{p,k}^l(w)$) can be precisely constructed to complete the information transfer between different optimization time domains. Then, the number of MAVs arriving and alighting at each stop, the number of MAVs in operations, the formation of each operated MAV, and the number of MAVs

available at each depot and time are all recorded. Lastly, we update the time range covered by CH and PH.

Optimization module. Input $\mathcal{K}^n, \mathcal{S}^n, \mathcal{P}_w^n, \tilde{\mathcal{P}}_w$, and $\bar{\mathcal{P}}_w$. For passenger groups in $\bar{\mathcal{P}}_w$, because they have already completed their first boarding, we do not construct variable $\chi_{p,k}^l(w)$ to track their waiting time. However, we still need $\bar{\mathcal{P}}_w$ to accurately calculate the number of people alighting from the vehicle at each stop. For the passenger in $\tilde{\mathcal{P}}_w$, we selectively construct variable $\zeta_{p,k,k'}^g(w)$ to track the transfer possibly completed at that stage. For the passengers in $\mathcal{P}_w^n$, we leave out any special treatment and construct the corresponding variables according to the model requirements. Then, we run Algorithm 1 in Online Appendix G and output the optimization results (i.e., timetables, vehicle schedules, and the dynamic capacity allocation strategy in the current CH).

To sum up, the procedure of the overall algorithm is presented in Figure H.1 in Online Appendix H. This procedure begins by introducing the initial system settings. The next step is to check that the start of the current iteration and the end of the entire optimization period coincide. If they align, the process is stopped, and the optimization results are obtained. Otherwise, the prediction module is started as a precautionary measure to update the travel time between subsequent stops and the passenger demand at each stop. Subsequently, the timetable simulation module is utilized to obtain a partially regulated timetable including all of the CH cycle control strategies from the previous sequence. Next, the selection module is run to select the stops and MAVs assigned to trips involved in this iteration. After that, the optimization module is run to get the current CH control strategy and the vehicle schedules. Further, we run the state update module to accurately identify the states of passengers and vehicles, and we update the corresponding information, which is the input for the next optimization iteration.

6.3. Learning-Based Real-Time Decision-Making Framework

As passenger flows evolve over time, timetables and vehicle schedules derived from historical demand scenarios may become suboptimal when faced with real-time operational conditions. Moreover, as discussed in Section 1, MAVs are operated under a short decision-making time window. To address this and better respond to time-varying demand that may fall outside the historical passenger demand scenarios, we propose a real-time decision-making framework that fine-tunes the tactical timetable within a small adjustment range using real-time information and reoptimizes the vehicle schedule accordingly. The framework follows an RH approach, partitioning the studied time horizon into multiple decision-making stages with equal-length short durations. At each stage, a stochastic

MILP is solved using the integer L-shaped method, where the timetable obtained from tactical-level planning is allowed to shift within an adjustment range (e.g., one or two minutes) and the vehicle schedules can be reoptimized.

In stochastic programming, incorporating more scenarios can improve solution quality, but it can greatly increase computational difficulty (as will be shown in Section 7.8). However, real-time operations require finding solutions efficiently while maintaining high quality. To address this challenge, we design a learning-based scenario-retention method and embed it into our real-time decision-making framework. Our proposed learning-based real-time decision-making framework is illustrated in Figure I.1 in Online Appendix I. At each decision-making stage, (i) the real-time information is updated. (ii) Using predefined features, a trained machine learning model is used to rank all new demand scenarios that are out of the historical demand scenario set. (iii) We retain the top k -most representative scenarios into a scenario subset. (iv) We solve the stochastic MILP using the integer L-shaped method over this scenario subset to optimize deviations of the tactical timetable and the vehicle schedule. This process is repeated throughout the entire studied horizon, ensuring that timetables and vehicle schedules in all decision-making stages remain consistent with real-time dynamics. By avoiding incorporating all demand scenarios into the integer L-shaped method at each stage and focusing computation over the most representative scenarios, the framework meets real-time computational requirements while preserving solution quality.

We now introduce our proposed learning-based scenario-retention method in detail. Table 1 summarizes the features used to characterize each demand scenario, which explicitly capture interactions between supply and demand. Intuitively, one might assume that scenarios with higher total demand are more critical.

However, this criterion can be misleading. Consider the following illustrative example. Suppose there are two stations. In scenario 1, the total demand is 50, and it is evenly distributed with 25 passengers at each station. In scenario 2, the total demand is slightly lower at 45, but it is highly imbalanced, with 40 passengers at the first station and only 5 passengers at the second station. A timetable that is optimized solely based on scenario 1 may fail to provide sufficient capacity for scenario 2, resulting in large passengers' costs or even infeasibility. This example highlights that the criticality of a scenario depends not only on the aggregate demand level but also, on the distribution of demand, the alignment of supply and demand, and their interactions. These relationships are highly nonlinear and difficult to capture analytically in complex and uncertain operating environments. Motivated by the strength of ML methods in capturing nonlinear patterns, we adopt an ML-based approach to learn the mapping between scenario features and their impact on system performance, thereby identifying the most critical scenarios for real-time optimization.

In the learning-based scenario-retention method, we perform an *offline training* procedure using historical data. In the offline training framework, passenger flow fluctuations are first simulated by perturbing station-level demand within historical demand scenarios, thereby generating a rich set of stochastic passenger flow scenarios that reflect possible real-time variations. Given that our proposed optimization method follows an RH framework (see Section 6.2), the training data generation process is embedded within this RH structure to ensure compatibility between the learning model and the underlying optimization procedure. Thereafter, at each decision-making stage, we randomly select a subset of scenarios and run the integer L-shaped method with this subset. At the beginning of each decision-making stage, we simulate the current operational plan and estimate station-level supply (i.e.,

Table 1. Feature Definitions and Formulations Used in the Learning-Based Scenario-Retention Method

Feature name	Description	Formula
total_waiting	Total passengers waiting in the system	$\sum_{p \in P_w} n_p$
max_waiting	Maximum passengers waiting at a single stop	$\max_{i \in S} \{\text{demand}_i\}$
avg_gap	Average of supply-demand gap across stops	$\frac{1}{ S } \sum_{i \in S} \text{diff}_i$
worst_gap	Most negative (worst-case) supply-demand gap	$\min_{i \in S} \{\text{diff}_i\}$
gap_std	Standard deviation of supply-demand gap	$\sqrt{\frac{1}{ S } \sum_i (\text{diff}_i - \overline{\text{diff}})^2}$
demand_std	Standard deviation of demand across stops	$\sqrt{\frac{1}{ S } \sum_i (\text{demand}_i - \overline{\text{demand}})^2}$
supply_std	Standard deviation of supply across stops	$\sqrt{\frac{1}{ S } \sum_i (\text{supply}_i - \overline{\text{supply}})^2}$

Notes. $\text{demand}_i = \sum_{p \in P_O} n_p$ is the demand at stop i . $\text{supply}_i = \text{CAP} \cdot \hat{x}_{k,i}^l(w) - \hat{v}_{k,i}^l(w)$ is the supply at stop i . $\text{diff}_i = \text{supply}_i - \text{demand}_i$ is the supply-demand gap at stop i . $\overline{\text{diff}} = \frac{1}{|S|} \sum_{i \in S} \text{diff}_i$ is the average supply-demand gap across stops. $\overline{\text{demand}} = \frac{1}{|S|} \sum_{i \in S} \text{demand}_i$ is the average demand across stops. $\overline{\text{supply}} = \frac{1}{|S|} \sum_{i \in S} \text{supply}_i$ is the average supply across stops.

available capacity). We then extract scenario-level features as defined in Table 1. Next, we randomly choose one scenario from the selected subset as the input of the optimization procedure, solve the problem under this scenario, and evaluate the solution's performance across the other selected scenarios. The resulting *performance gap* between the pre- and postoptimization objectives is used as the ground-truth label for the supervised learning. To calibrate the learning model, we adopt the XGBoost algorithm. The main reasons are threefold. (i) XGBoost captures complex nonlinear relationships between scenario features and solution quality; (ii) it supports efficient training with large-scale data sets and delivers high predictive performance; and (iii) it provides interpretable feature importance metrics, helping us understand the influence of each feature on scenario criticality.

7. Numerical Experiments

In this section, we conduct a comprehensive set of numerical experiments to evaluate the performance and practical value of the proposed models and solution methods. The experiments are organized as follows. We begin by describing the test instances used throughout the experiments in Section 7.1. We then analyze the benefits of incorporating dynamic capacity allocations and cross-line circulations in Section 7.2, and we evaluate the impact of prioritizing passenger interests on both operational costs and the proportion of in-vehicle transfers in Section 7.3. Next, we evaluate the benefits of our proposed integrated optimization method compared with a sequential approach (Section 7.4) followed by a comparison of the performance of various solution methods, including our exact algorithm, the rolling-horizon framework, and GUROBI (Section 7.5).

To evaluate the robustness of our methods, we conduct two post hoc analyses: one testing the timetable and vehicle schedule and doing a sensitivity analysis of overload allowance in Section 7.6 and another testing the effectiveness of fine-tuning strategies embedded in our learning-based real-time decision-making framework in Section 7.7. We also investigate the value of the stochastic programming approach compared with deterministic models (Section 7.8). Finally, we evaluate the effectiveness of our learning-based real-time decision-making framework in Section 7.9, and we analyze the trade-offs between operational costs and passengers' costs in Section 7.10. All experiments were implemented in Python and executed on a Windows 11 personal computer equipped with a 12th Gen Intel Core i7-12700H processor and 64 GB RAM.

7.1. Instances

The instances used for the experiments are derived from a virtual network and a part of the Beijing bus network,

and they are depicted in Figure J.1 in Online Appendix J. The virtual network comprises three bidirectional lines. The Beijing bus subnetwork contains two bidirectional lines and 89 stops, where passenger demand, network structure, and running times between stops are all derived from real-world operating data. Based on these two networks, we generate 24 sets of instances with varying durations of time periods and numbers of scenarios to test the effectiveness of the proposed formulation and algorithms. Detailed information on each instance is provided in Table J.1 in Online Appendix J. In addition, the five parameters used in objective function (1a) are set as follows. In all experiments, the equivalent monetary value per unit of passengers' waiting costs, ϑ^T , is set to 0.8 (dollars). The coefficients ε_1 and ε_2 , which represent the weights on waiting time at origins and during transfers, respectively, are set to 1 and 1.5, respectively. The coefficients φ_1 and φ_2 , representing the weights on passengers' costs and operational costs, respectively, are both set to one in the experiments from Sections 7.2–7.9. A sensitivity analysis of these two coefficients is provided in Section 7.10.

7.2. Benefits of Incorporating Dynamic Capacity Allocations and Cross-line Circulations

To gain insights into the effects of dynamic capacity allocations and cross-line circulations on the passengers' and operational costs, we perform a series of experiments. We define three operational strategies: (1) *fixed capacity*, which refers to executing all trips using MAVs at their maximum capacity to ensure service quality; (2) *partially flexible capacity*, which permits MAVs to be coupled or decoupled only at depots, maintains consistent en route formations, and allows MAUs to be dispatched to different lines at depots (i.e., depot-based cross-line circulations); and (3) *completely flexible capacity*, which allows MAVs to be flexibly coupled or decoupled at depots and transfer stops as well as cross-line circulations of MAUs.

In Table 2, we present the results of adopting the aforementioned three operational strategies. We can observe that compared with the fixed-capacity operational strategy, both the partially flexible-capacity and completely flexible-capacity strategies significantly reduce operational costs and the number of used MAUs. For example, for *instance_rp_2*, the fixed-capacity strategy requires 72 MAUs, whereas partial flexible-capacity and flexible-capacity strategies only require 48 and 36 MAUs, respectively, resulting in reductions of 33.3% and 50.0%, respectively. We also observe that the completely flexible-capacity strategy slightly reduces total passengers' costs compared with the fixed-capacity strategy. This is mainly because the flexible-capacity strategy allows for in-vehicle transfers, which do not incur transfer waiting costs that are included in the passengers' cost term of the objective

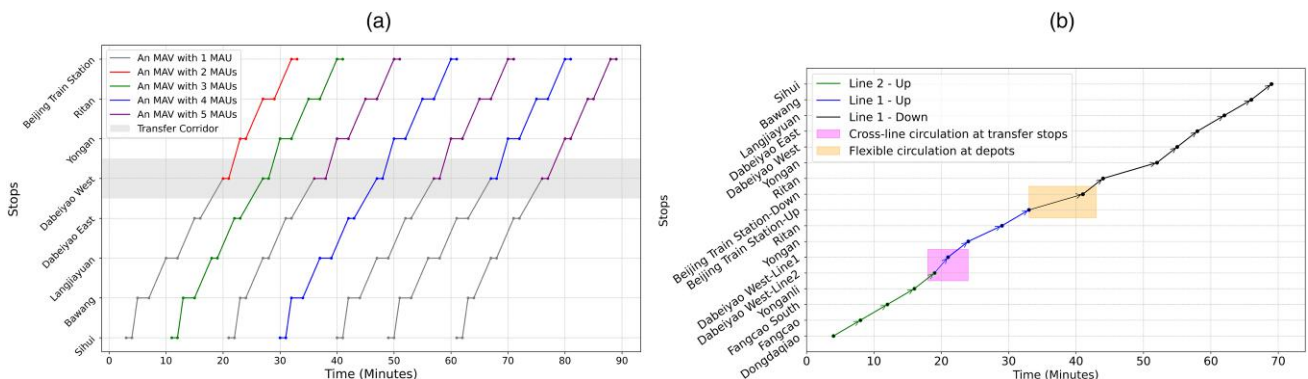
Table 2. Total Passengers' Costs, Operational Costs, and Number of Used MAUs Obtained by Adopting Various Operational Strategies

Instance	Operational strategy	Total passengers' costs (\$)	Total operational costs (\$)	No. of used MAUs
<i>instance_rp_1</i>	Fixed capacity	1,541.8	198.9	48
	Partially flexible capacity	1,545.7	115.0	25
	Completely flexible capacity	1,539.9	100.2	21
<i>instance_rp_2</i>	Fixed capacity	4,106.3	300.4	72
	Partially flexible capacity	4,122.4	197.4	48
	Completely flexible capacity	4,047.1	163.3	35
<i>instance_rp_3</i>	Fixed capacity	7,042.6	403.3	96
	Partially flexible capacity	7,064.5	283.0	71
	Completely flexible capacity	6,981.1	227.8	51
<i>instance_rp_4</i>	Fixed capacity	10,413.0	506.2	96
	Partially flexible capacity	10,459.2	372.3	79
	Completely flexible capacity	10,472.1	310.5	58
<i>instance_rp_5</i>	Fixed capacity	15,614.7	610.4	96
	Partially flexible capacity	15,682.7	461.3	84
	Completely flexible capacity	15,666.8	380.2	61
<i>instance_rp_6</i>	Fixed capacity	19,507.2	718.8	96
	Partially flexible capacity	19,578.0	549.3	85
	Completely flexible capacity	19,555.0	458.6	69

Note. Bold indicates the best results among the compared strategies.

function. In contrast, conventional transfers under the fixed-capacity strategy involve inconvenience, leading to positive transfer waiting costs. A third observation is that the completely flexible-capacity strategy outperforms the partially flexible-capacity strategy with respect to the number of used units among all instances. The largest relative savings compared with the partially flexible-capacity operational approach occur in the *instance_rp_3*, where the completely flexible-capacity strategy saves over 28.1% of units. In conclusion, the completely flexible-capacity operational strategy is beneficial to operators in terms of lowering operational costs and the number of required MAUs thanks to its ability to dynamically allocate capacity at transfer stops and schedule cross-line MAU circulations during trips.

Figure 1 delves into the solution for the *instance_rp_6*. Figure 1(a) presents the optimal robust timetable and formations of MAVs designated to each trip for the east-west line, which is termed line 1, using the flexible-capacity operational strategy. Figure 1(b) displays the complete trajectory of an unit departing from the first stop in the upstream direction of the north-south line, which is denoted line 2. We can observe that the formation of the MAV assigned to the first trip is one unit before reaching the transfer stop Dabeiyao West because of low passenger demand. After the transfer stop, because of a marked surge in demand, a unit is insufficient. A vacant unit on line 2 is coupled to the MAV assigned to this trip on line 1 as shown in Figure 1(b). After completing the trip and returning to the depot, this MAV is reallocated to a trip in the opposite direction. To

Figure 1. (Color online) The Optimized Timetable, Formations of MAVs Assigned to Trips, and Cross-line Schedules of MAUs of the *instance_rp_6*

Notes. (a) The timetable and formations of MAVs. The shaded area indicates the transfer stop. (b) Movements of an MAU.

sum up, the timetable, formations of MAVs at different stops and times, and cross-line circulations of MAUs are optimized according to the time-varying and unevenly distributed demand.

7.3. Impact of Valuing the Interests of Passengers

We next perform a series of experiments based on *instance_g_1* and *instance_g_2* to assess the impact of valuing interests of passengers on passengers' costs and operational costs as well as the proportion of in-vehicle transfers among all transfers. First, we set the weighting coefficient related to passengers' costs in the objective function, φ_1 , to zero and the weight associated with operational costs φ_2 to one to obtain the solution with the minimal operational costs. Thereafter, we incrementally increase the value of φ_1 to explore the effects of valuing passenger interests.

As the value of φ_1 increases, the operational costs grow, and passengers' costs decrease. Figure 2 illustrates the increase in operational costs and decrease in passengers' costs for all Pareto-efficient solutions relative to the solution with minimal operational costs. It can be observed that an increase in operational costs by approximately 5% correlates with a reduction in total passengers' costs by about 30% in both instances. However, as operational costs continue to increase, the subsequent reduction in total passengers' costs becomes marginal. These findings suggest that although high values only have a little positive impact, reasonable attention in the interests of passengers can significantly increase the quality of the service.

Another interesting aspect is the impact of prioritizing passengers' interests on the proportion of in-vehicle transfers among all passengers' transfers in the solution and the number of required MAUs. The obtained solutions are presented in Figure 3. In both instances, there is a clear trend indicating that greater proportions of

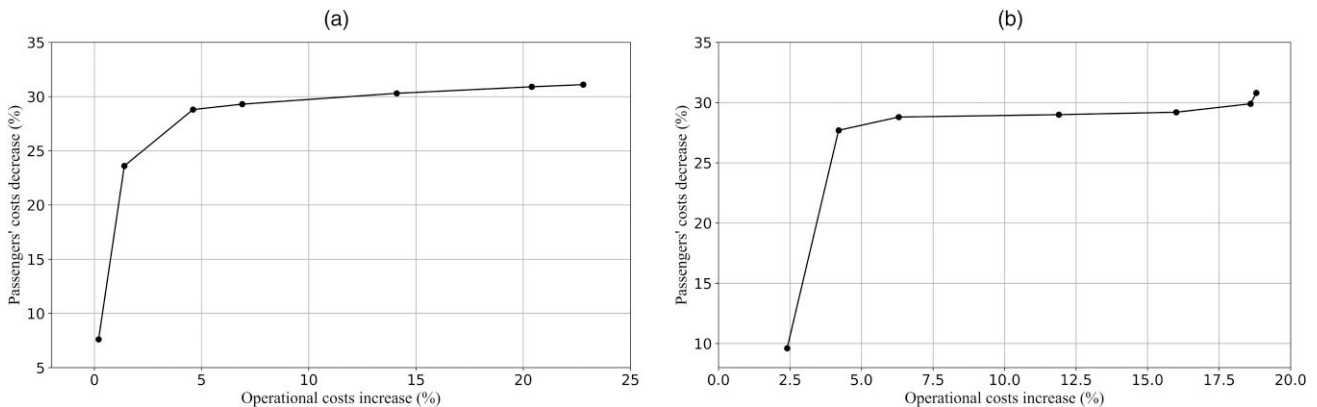
in-vehicle transfers are closely associated with higher operational costs and more utilized MAUs. Specifically, as the proportion of in-vehicle transfers increases from 0% to 50%, there is a consistent and significant increase in operational costs—up to 25% in *instance_g_1* and 20% in *instance_g_2*. Furthermore, in *instance_g_1* and *instance_g_2*, the numbers of used MAUs expand from 26 to 45 and from 34 to 52, respectively, with relative percentage increases of almost 73% and 53%, respectively.

This trend can be explained by the role of transfer waiting time in passenger costs. In-vehicle transfers are not penalized, whereas conventional transfers incur additional penalties because of the inconvenience of alighting, walking between platforms, and reboarding. These penalties are explicitly incorporated into passengers' costs through weighted transfer waiting time as defined in Constraints (4). As greater weight is placed on passenger interests, the model naturally favors in-vehicle transfers to reduce these costs. In summary, prioritizing passengers' interest increases the share of in-vehicle transfers and enhances transfer convenience, but this comes at the expense of higher operational costs and greater MAU usage.

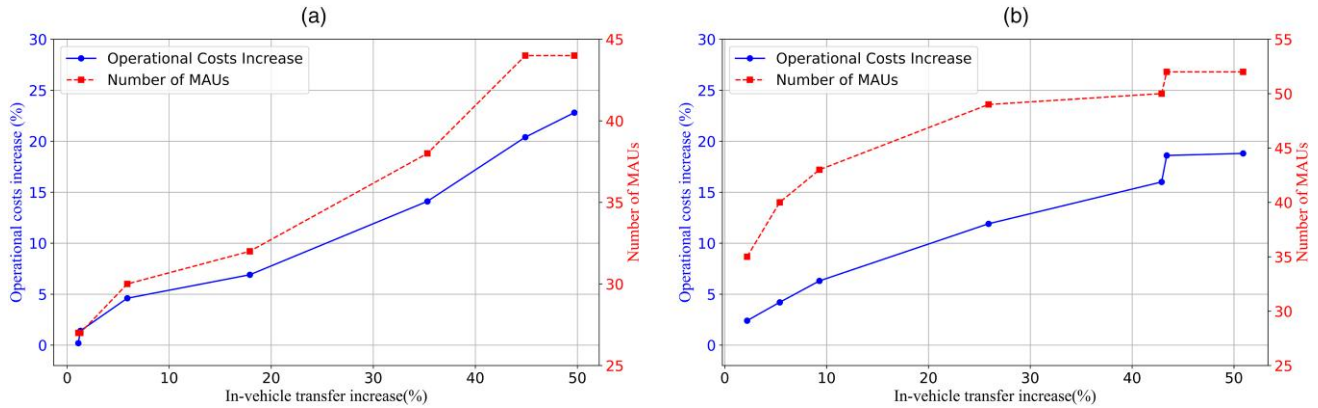
7.4. Benefits of the Proposed Integrated Optimization Method

This section assesses the benefits of integrating timetabling and vehicle scheduling decisions. We construct a set of experiments based on six instances from *instance_rp_1* to *instance_rp_6*, where we use the sequential decision-making approach as a benchmark. Specifically, we propose a two-step sequential method that first determines the timetable. Then, the derived optimal timetable is treated as an input to optimize the vehicle schedule. We use GUROBI to solve our integrated optimization model (34) and the sequential optimization formulation.

Figure 2. Results of Passengers' and Operational Costs When the Weighting Coefficient Related to Passengers' Costs in the Objective Function Increases



Notes. (a) *instance_g_1*. (b) *instance_g_2*.

Figure 3. (Color online) Results of the Proportion of In-Vehicle Transfers Among All Transfers and the Used MAUs When the Weighting Coefficient Related to Passengers' Costs in the Objective Function Increases

Notes. (a) *instance_g_1*. (b) *instance_g_2*.

In Table 3, we present the objective function values, passengers' costs, relative deviations in passenger costs (denoted as Dev^p), and number of utilized MAUs obtained from the two optimization methods. It can be observed that (i) for smaller-scale instance, such as *instance_rp_1*, these two methods achieve the same optimal solutions; however, as the scale of the instances slightly increases (e.g., *instance_rp_2* and *instance_rp_3*), the sequential optimization method attains suboptimal solutions with a higher objective value compared with our proposed integrated optimization method. (ii) When the scale of instances is further enlarged (i.e., *instance_rp_4*, *instance_rp_5*, and *instance_rp_6*), the sequential method fails to find feasible solutions, which indicates that a given number of units cannot fully execute the timetable obtained from the optimization in the first step. In contrast, the proposed integrated optimization method is able to find the optimal timetable and corresponding vehicle schedule for each instance. In these larger instances, the numbers of MAUs used are 58, 61, and 69, respectively. (iii) When the sequential optimization method yields

feasible solutions, the passenger costs obtained by the integrated optimization method are slightly higher, with deviations up to 0.12%. These observations highlight the necessity for incorporating an integrated optimization approach to efficiently plan timetables and vehicle schedules in large-scale networks.

On the other hand, sequential optimization has its own advantages. Specifically, the sequential method is advantageous in scalability and modeling flexibility, notably by allowing for the flexible modeling of oversaturation scenarios, where passengers unable to board the first available MAV may wait for subsequent ones. Although oversaturation did not occur in our computational results, it remains a possibility in practice. In contrast, the integrated optimization method excels in providing high-quality solutions, especially concerning fleet sizing, because of the inherent feedback between vehicle scheduling and timetabling. Nevertheless, incorporating these considerations into the integrated optimization framework significantly increases model complexity and computational burden, making it impractical at large scales. Therefore, the proposed RH optimization approach is

Table 3. Performance Comparison Between Integrated and Sequential Timetabling and Vehicle Scheduling for MAVs

Instance	Optimization method	Objective value (\$)	Passengers' cost (\$)	Dev^p (%)	No. of used MAUs
<i>instance_rp_1</i>	Sequential	3,544.32	2,121.12	—	21
	Integrated	3,544.32	2,121.12	0	21
<i>instance_rp_2</i>	Sequential	7,349.31	5,049.79	—	36
	Integrated	7,313.58	5,053.34	0.07	35
<i>instance_rp_3</i>	Sequential	11,633.17	8,791.77	—	59
	Integrated	11,538.13	8,802.32	0.12	51
<i>instance_rp_4</i>	Sequential	—	—	—	—
	Integrated	16,682.10	12,513.17	—	58
<i>instance_rp_5</i>	Sequential	—	—	—	—
	Integrated	23,270.80	18,783.85	—	61
<i>instance_rp_6</i>	Sequential	—	—	—	—
	Integrated	28,727.00	22,727.0	—	69

motivated and useful, effectively balancing model complexity and scalability while retaining the benefits of the integrated optimization framework.

7.5. Performance Comparison Among Various Solution Methods

To assess the effectiveness of the proposed solution methods, we perform the following two experiments. First, we construct 12 instances based on the virtual network to compare the performance between GUROBI and the integer L-shaped method. Second, based on the Beijing bus subnetwork, we build six instances to evaluate the efficiency and solution quality of both the integer L-shaped and the rolling-horizon solution methods by varying the control and predict horizons.

7.5.1. Performance Comparison Between GUROBI and the Integer L-Shaped Method. In this experiment, we compare the performance between methods for solving the TT-VS-DCA model: a direct approach utilizing GUROBI and the integer L-shaped method with valid inequalities. Our termination criteria include (i) achieving an optimality gap tolerance of 5% and (ii) reaching a two-hour computational time limit.

In Table 4, we present the results of 12 instances, stating the objective value, the computational time, and the relative optimality gap. It can be observed that the proposed IL method obtains solutions with smaller objective function values in all instances with respect to the solutions obtained by GUROBI. Although GUROBI demonstrates smaller optimality gaps in smaller instances, such as *instance_g_1* and *instance_g_7*, it requires longer computational times. As the size of instances increases, however, the superiority of the IL method over GUROBI becomes more evident. For example, when solving *instance_g_5* and *instance_g_10*, GUROBI struggles to find a good-quality solution within two hours. Although the IL method cannot achieve an optimality gap of less than 5% for *instance_g_6*, *instance_g_11*, and *instance_g_12* within two hours, it produces solutions with lower objective values than those generated by GUROBI in the same computational time. Overall, our proposed exact solution method has a significant advantage over GUROBI when addressing medium-scale problems.

7.5.2. Performance Comparison Between the Integer L-Shaped Method and the Rolling-Horizon Algorithm Under Various Settings. In this experiment, we compare the integer L-shaped method and the rolling-

Table 4. Performance Comparison Between GUROBI and the Designed Exact Solution Algorithm

Instance	Solution method	Objective value (\$)	Computational time (seconds)	Gap (%)
<i>instance_g_1</i>	GUROBI	3,075.2	452.7	4.3
	IL	2,990.2	344.1	4.8
<i>instance_g_2</i>	GUROBI	4,985.6	1,065.7	2.3
	IL	4,976.8	588.5	4.7
<i>instance_g_3</i>	GUROBI	7,254.3	2,681.3	4.6
	IL	7,105.1	1,186.7	5.0
<i>instance_g_4</i>	GUROBI	9,663.5	5,021.3	4.9
	IL	9,450.6	1,917.3	4.9
<i>instance_g_5</i>	GUROBI	12,639.9	7,200.0	8.8
	IL	11,915.6	4,602.8	5.0
<i>instance_g_6</i>	GUROBI	19,023.6	7,200.0	24.5
	IL	15,035.9	7,200.0	7.9
<i>instance_g_7</i>	GUROBI	2,727.8	668.3	1.7
	IL	2,708.8	564.4	4.4
<i>instance_g_8</i>	GUROBI	4,636.0	3,046.2	4.7
	IL	4,523.3	756.8	5.0
<i>instance_g_9</i>	GUROBI	6,477.7	6,799.3	3.7
	IL	6,388.7	1,670.9	5.0
<i>instance_g_10</i>	GUROBI	11,149.1	7,200.0	25.7
	IL	8,526.1	5,453.4	4.7
<i>instance_g_11</i>	GUROBI	15,134.3	7,200.0	30.9
	IL	10,819.7	7,200.0	5.2
<i>instance_g_12</i>	GUROBI	18,615.1	7,200.0	30.7
	IL	13,568.2	7,200.0	7.3

Note. Bold indicates the best result for each instance across the compared methods.

horizon algorithm combined with this exact solution method (denoted as RH + IL). We consider six different scales of instances based on the Beijing bus subnetwork. For each instance, we vary the lengths of both the control horizon and the predict horizon to explore the impact of increasing the length of each optimization span on the quality of the final solution. It is worth mentioning that the RH + IL algorithm is equivalent to the IL algorithm when the CH length equals the length of the entire investigated planning period. Besides, we use the formula $Opt. Gap = \frac{(\text{Objective value} - \text{Best-known objective value})}{(\text{Objective value})} \times 100 (\%)$ to calculate the optimality gap for the rolling-horizon algorithm.

The results are presented in Table 5. We obtain the following observations. (1) For small instances, the setting with $CH = PH$ spanning the entire studied time horizon attains the best or comparable gaps while requiring the highest computational time. However, as instance size grows, this setting becomes the worst performer within the time limit (e.g., 7,200 seconds), yielding the largest optimality gaps because of the much larger problem size. (2) The quality of solutions generated by the RH + IL algorithm improves as the durations of CH and PH increase, especially when the scale of instances is relatively small. (3) For large-scale

instances, such as *instance_r_5* and *instance_r_6*, obtaining high-quality solutions within a reasonable computational time using only the integer L-shaped algorithm proves challenging. (4) Even with a shorter optimization period (i.e., $CH = 20$, $PH = 40$), the gap of the solution obtained by RH + IL with respect to the optimal solution (maximum of not more than 7% in all of the instances) is acceptable; the gap of the resulting solution is further reduced to not more than 5% after increasing to $CH = 80$ and $PH = 100$. This observation indicates the high quality of the solution obtained in our proposed rolling-horizon algorithm. If real-time passenger demand is available, the proposed algorithm can optimize decisions in real time. It provides high-quality solutions with a short optimization window and runs fast enough for real-time control.

7.6. Post Hoc Robustness Evaluation and Sensitivity Analysis of Overload Allowance

In this section, we assess the robustness of the optimized timetable and vehicle schedule on out-of-sample demand scenarios and evaluate the potential of capacity relaxation. We first conduct a post hoc evaluation using new demand scenarios to examine how the

Table 5. Objective Value, CPU Time, and *Opt. Gap* That Are Obtained by the RH + IL Algorithm Under Different Lengths of CH and PH

Instances	Clairvoyant ^a (\$)	Solution method	Objective value (\$)	Computational time (seconds)	<i>Opt. Gap</i> (%)
<i>instance_r_1</i>	958.5	RH with $CH = 20$ minutes and $PH = 40$ minutes	967.9	59.7	1.0
		RH with $CH = 40$ minutes and $PH = 60$ minutes	966.7	71.7	0.9
		RH with $CH = 80$ minutes and $PH = 100$ minutes	963.7	356.4	0.5
		RH with $CH = 120$ minutes and $PH = 120$ minutes	963.4	977.1	0.5
<i>instance_r_2</i>	1,965.7	RH with $CH = 20$ minutes and $PH = 40$ minutes	2,011.5	227.4	2.3
		RH with $CH = 40$ minutes and $PH = 60$ minutes	2,052.3	516.9	4.4
		RH with $CH = 80$ minutes and $PH = 100$ minutes	1,996.4	1,800.1	1.6
		RH with $CH = 130$ minutes and $PH = 130$ minutes	1,983.1	7,200	0.9
<i>instance_r_3</i>	3,055.2	RH with $CH = 20$ minutes and $PH = 40$ minutes	3,172.7	317.8	3.9
		RH with $CH = 40$ minutes and $PH = 60$ minutes	3,187.7	1,745.0	4.3
		RH with $CH = 80$ minutes and $PH = 100$ minutes	3,176.9	3,605.1	4.0
		RH with $CH = 140$ minutes and $PH = 140$ minutes	3,161.1	7,200.0	3.5
<i>instance_r_4</i>	5,130.5	RH with $CH = 20$ minutes and $PH = 40$ minutes	5,261.5	1,379.6	2.6
		RH with $CH = 40$ minutes and $PH = 60$ minutes	5,264.8	3,105.5	2.6
		RH with $CH = 80$ minutes and $PH = 100$ minutes	5,247.2	7,210.0	2.3
		RH with $CH = 150$ minutes and $PH = 150$ minutes	5,636.6	7,200.0	9.9
<i>instance_r_5</i>	5,934.7	RH with $CH = 20$ minutes and $PH = 40$ minutes	6,353.2	1,551.0	7.0
		RH with $CH = 40$ minutes and $PH = 60$ minutes	6,239.0	3,332.0	5.1
		RH with $CH = 80$ minutes and $PH = 100$ minutes	6,131.6	7,211.4	3.3
		RH with $CH = 150$ minutes and $PH = 150$ minutes	6,688.2	7,200.0	12.7
<i>instance_r_6</i>	8,622.4	RH with $CH = 20$ minutes and $PH = 40$ minutes	9,158.2	1,687.3	6.2
		RH with $CH = 40$ minutes and $PH = 60$ minutes	9,322.0	1,978.0	8.1
		RH with $CH = 80$ minutes and $PH = 100$ minutes	9,033.4	3,614.1	4.7
		RH with $CH = 240$ minutes and $PH = 240$ minutes	9,677.7	7,200.0	12.2

Notes. CH refers to the control horizon of RH. PH refers to the prediction horizon of RH.

^aThe clairvoyant is obtained by GUROBI, with a termination condition of 168 hours of computation time or a gap value of less than 0.1%.

solution performs under both demand decreases and increases. We then perform a sensitivity analysis by gradually relaxing the capacity constraints through an overload allowance parameter λ to investigate how small relaxations can help absorb demand surges without requiring additional resources.

7.6.1. Out-of-Sample Robustness Under Demand Perturbations. We assess the robustness of the optimized timetable and vehicle schedule obtained from our proposed integrated optimization method through post hoc evaluation using new demand realizations. Based on the original demand scenario set of *instance_rp_6* used during optimization, we generate five new stochastic scenario sets by reducing total passenger demand by 20%, 15%, 10%, and 5% and by increasing it by 5%.

To evaluate the number of overloaded passengers, we relax the capacity Constraints (21) and instead, incorporate an overload penalty term $\sum_{w \in \mathcal{W}} \rho_w \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}_l} \sum_{i \in \mathcal{S}_l} \max(0, v_{k,i}^l(w) - \text{CAP} \cdot x_{k,i}^l(w))$ into the objective function, where $v_{k,i}^l(w)$ denotes the number of in-vehicle passengers, $\text{CAP} \cdot x_{k,i}^l(w)$ represents the available capacity of the MAV operating section $(i, i + 1)$ on trip k and line l , and ρ_w is the probability of scenario w . Because overloading is not allowed during robustness evaluation, these overloaded passengers are interpreted as unserved. We then input the optimized timetable and vehicle schedule of *instance_rp_6* into Formulation (34) with the modified objective function and without the original capacity constraints.

Table 6 summarizes the robustness of the optimized solution for *instance_rp_6* under five new demand scenario sets. The first column in Table 6 indicates the tested demand scenario set, the second column in Table 6 reports the expected number of overloaded passengers (i.e., unserved passengers), and the third column in Table 6 reports the percentage of overloaded situations. This percentage is calculated as the number of overloaded MAV-section traversals divided by the total number of such traversals, where each MAV-section traversal refers to an MAV operating over a specific section. From the results shown in Table 6, we

Table 6. Post Hoc Robustness Evaluation of the Optimized Timetable and Vehicle Schedule Under New Demand Scenario Sets

Demand scenario set	Expected no. of overloaded (unserved) passengers	Overloaded MAV-section traversals (%)
20% decrease	0	0
15% decrease	0	0
10% decrease	0	0
5% decrease	0	0
5% increase	147.00	0.72

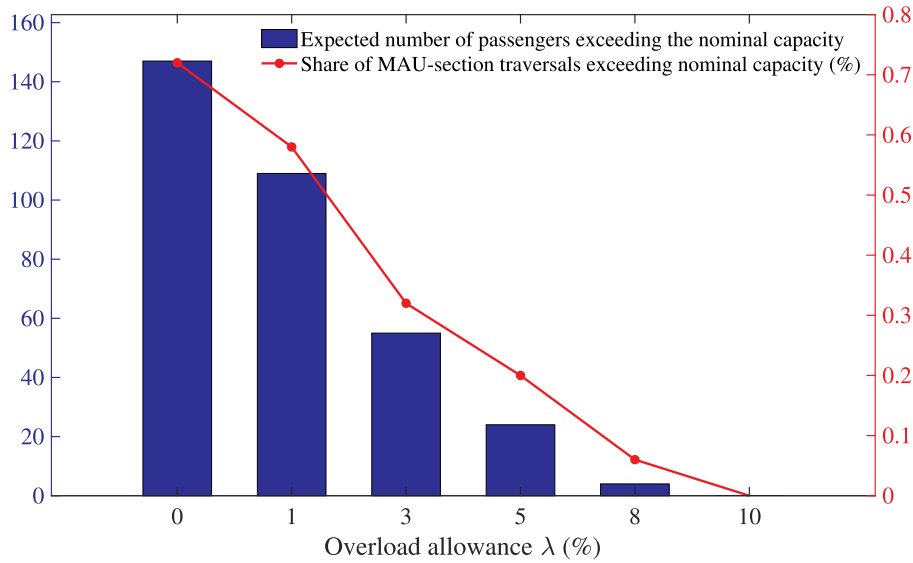
observe that under scenario sets with decreased passenger demand, the solution remains feasible with no unserved passengers. When the demand increases by 5%, 0.72% of MAV-section traversals become overloaded, leading to an expected number of 147 overloaded (i.e., unserved) passengers.

7.6.2. Sensitivity Analysis to Overload Allowance. To address the potential unserved issues, one possible method is to relax the capacity limitation and allow for overloading. Therefore, we conduct a sensitivity analysis on overload allowances under the new stochastic scenario set with a 5% increase in passenger demand. To do so, we relax the original capacity Constraints (21) by introducing a soft overload threshold. Specifically, we define an overload factor $\lambda \geq 1$, which allows for a certain level of overloading. Recall that $v_{k,i}^l(w)$ denotes the number of in-vehicle passengers when the MAV assigned to trip $k \in \mathcal{K}_l$ departs from stop $i \in \mathcal{S}_l$ on line $l \in \mathcal{L}$ in scenario $w \in \mathcal{W}$. Let $\text{CAP} \cdot x_{k,i}^l(w)$ denote the nominal capacity of the MAV with the formation of $x_{k,i}^l(w)$ assigned to that trip. We allow $v_{k,i}^l(w)$ to exceed the nominal capacity by up to a factor of λ . The modified capacity constraints can be expressed as

$$\begin{aligned} v_{k,i}^l(w) &\leq \lambda \cdot \text{CAP} \cdot x_{k,i}^l(w), \\ \forall w \in \mathcal{W}, l \in \mathcal{L}, k \in \mathcal{K}_l, i \in \mathcal{S}_l. \end{aligned} \quad (45)$$

Here, if $\lambda = 1$, the capacity is the nominal capacity without allowing overloading.

Figure 4 shows the effect of the overload allowance λ on two metrics, which are both evaluated against the nominal capacity (i.e., $\lambda \cdot \text{CAP} \cdot x_{k,i}^l(w)$ with $\lambda = 1$): (i) the expected number of passengers exceeding the nominal capacity (i.e., $\sum_{w \in \mathcal{W}} \rho_w \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}_l} \sum_{i \in \mathcal{S}_l} \max(0, v_{k,i}^l(w) - \text{CAP} \cdot x_{k,i}^l(w))$) and (ii) the proportion of overloaded MAV-section traversals, where the number of in-vehicle passengers exceeds the nominal capacity (percentage). The results demonstrate a clear monotonic improvement. Even modest overload allowances significantly reduce capacity violations. As λ increases from 0% to 5%, the expected number of overloaded passengers drops from 147 to 24, whereas the proportion of overloaded traversals declines from 0.72% to 0.20%. At $\lambda = 10\%$, both indicators reach zero. These findings suggest that a 10% overload allowance offers sufficient flexibility to absorb a 5% demand surge without requiring additional capacity over the nominal capacity. Conversely, if strict adherence to nominal capacity is enforced, the results can guide the required capacity increase for each MAU to fully serve all passengers under such demand variations based on the optimized timetable and vehicle schedule obtained from our proposed integrated optimization method.

Figure 4. (Color online) Effect of Overload Allowance λ on Violations of the Nominal Capacity

7.7. Post Hoc Robustness Evaluation of Fine-Tuning Strategies Under Demand Surges and Redistributions

We now design two fine-tuning strategies to address capacity shortages under demand surges and conduct a post hoc robustness evaluation of them, aiming to provide practical and resilient solutions for real-world operations. The first strategy, fix timetable and adjust vehicle schedule (AVS), retains the optimized timetable generated from our integrated optimization method based on the in-sample demand scenario set, and it reoptimizes only the vehicle scheduling decisions. The second strategy, fine-tune timetable and adjust vehicle schedule (FTTAVS), uses the optimized timetable as the baseline but permits each departure time to shift within a narrow range of ± 1 minute, allowing for joint reoptimization of the timetable and vehicle schedule.

To evaluate these two strategies, we construct 48 new stochastic scenario sets based on *instance_rp_6*. Specifically, for each of the four levels of demand increase (5%, 10%, 15%, and 20%), we apply three distinct redistribution strategies and generate 12 scenario sets. (i) The uniform (U) strategy proportionally increases demand across all origin-destination (OD) pairs, reflecting a general system-wide growth. (ii) The head-biased (HB) strategy concentrates the increase on the top 10% busiest OD pairs, simulating scenarios where the busiest corridors become even more popular. (iii) In contrast, the tail-biased (TB) strategy amplifies demand in the bottom 10% least-used OD pairs, capturing emergent or previously underutilized travel patterns. These profiles allow us to test robustness under a broad range of practically relevant demand surges and spatial redistributions. We test the two strategies across these 48 out-of-sample scenario sets.

Each instance is denoted as *Dk_profile_strategy*, where k represents the level of demand increase, “profile” refers to the redistribution strategy (U, HB, or TB), and “strategy” indicates the fine-tuning method (AVS or FTTAVS). For example, *D10_HB_FTTAVS* refers to the scenario with a 10% head-biased demand surge evaluated using the FTTAVS strategy.

Table 7 summarizes the key robustness metrics across all demand surge and redistribution scenarios. The second column in Table 7 reports the objective value, including passengers’ costs and operational costs. The third column in Table 7 presents the expected number of unserved passengers across all scenarios—that is, passengers assigned to overloaded MAVs under the soft capacity limitation, which are penalized heavily and thus, regarded as unserved. The fourth column in Table 7 (“Top 5% average load ratio (%)”) measures the average utilization of the most crowded 5% of sections, where utilization $r_{k,i}^l(w)$ is computed as $r_{k,i}^l(w) = v_{k,i}^l(w) / (x_{k,i}^l(w) \text{CAP})$, with $v_{k,i}^l(w)$ denoting the number of in-vehicle passengers when the MAV assigned to trip k leaves stop i on line l in scenario w , $x_{k,i}^l(w)$ representing the number of MAUs composing the MAV assigned to the section between stops i and $i + 1$, and CAP indicating the capacity per MAU. The fifth column in Table 7 (“MAU assignment shift (%)”) measures how widely the MAU formation assignments deviate from the baseline. It is calculated as the percentage of MAV-section pairs where the number of assigned MAUs differs from the baseline solution. The numerator is the total number of mismatched MAV-section assignments, and the denominator is the total number of all MAV-section assignments across the network. Finally, the last column in Table 7 reports the total number of used MAUs.

Table 7. Post Hoc Robustness of Fine-Tuning Strategies Under Demand Surges and Redistributions

Instance	Objective value (\$)	Expected no. of unserved passengers	Top 5% average load ratio (%)	MAU assignment shift (%)	Total no. of used MAUs
<i>D5_U_AVS</i>	9,532.88	0	—	7.6	112
<i>D5_U_FTTAVS</i>	9,216.68	0	0.89	13.7	109
<i>D5_HB_AVS</i>	9,575.65	0	0.87	10.3	115
<i>D5_HB_FTTAVS</i>	9,291.48	0	0.89	16.2	115
<i>D5_TB_AVS</i>	9,511.42	0	0.87	7.4	117
<i>D5_TB_FTTAVS</i>	9,195.74	0	0.89	14.3	111
<i>D10_U_AVS</i>	9,915.67	0	0.86	17.7	118
<i>D10_U_FTTAVS</i>	9,585.19	0	0.88	21.4	118
<i>D10_HB_AVS</i>	9,933.14	0	0.88	16.5	117
<i>D10_HB_FTTAVS</i>	9,642.35	0	0.89	25.3	116
<i>D10_TB_AVS</i>	9,864.62	0	0.88	14.2	115
<i>D10_TB_FTTAVS</i>	9,530.31	0	0.89	20.0	113
<i>D15_U_AVS</i>	10,239.50	0	0.88	21.3	120
<i>D15_U_FTTAVS</i>	9,929.97	0	0.88	26.7	124
<i>D15_HB_AVS</i>	10,291.87	0	0.90	22.7	127
<i>D15_HB_FTTAVS</i>	10,024.40	0	0.90	33.8	128
<i>D15_TB_AVS</i>	10,229.90	0	0.88	22.6	120
<i>D15_TB_FTTAVS</i>	9,905.54	0	0.88	25.8	119
<i>D20_U_AVS</i>	10,629.34	8	0.89	26.7	128
<i>D20_U_FTTAVS</i>	10,328.20	0	0.89	36.4	131
<i>D20_HB_AVS</i>	10,660.00	12	0.90	29.4	128
<i>D20_HB_FTTAVS</i>	10,356.40	0	0.90	34.8	133
<i>D20_TB_AVS</i>	10,614.61	0	0.89	29.3	126
<i>D20_TB_FTTAVS</i>	10,257.40	0	0.89	30.7	129

From the results shown in Table 7, we observe that across all instances, the FTTAVS strategy consistently yields lower objective values than the AVS strategy. Under severe surges, AVS fails to serve all passengers (e.g., 8 unserved passengers in *D20_U_AVS* and 12 unserved passengers in *D20_HB_AVS*), whereas FTTAVS accommodates full demand. A second observation is that the top 5% average load ratio remains stable between 86% and 90% as demand increases from 5% to 20%, suggesting that both strategies effectively adapt vehicle schedules to the changing demand. Notably, FTTAVS achieves slightly higher top 5% load ratios than AVS, indicating a tighter capacity-demand alignment made possible by jointly fine-tuning the timetable and reoptimizing vehicle schedules. These gains are accompanied by broader reallocations; the MAU assignment shift is consistently larger under FTTAVS, reflecting more extensive reoptimization of vehicle schedules compared with AVS. Additionally, for demand surges of 5%–10%, FTTAVS uses the same or fewer MAUs than AVS (e.g., *D5_U*: 109 versus 112, *D10_HB*: 116 versus 117, and *D10_TB*: 113 versus 115). However, when surges reach 15%–20%, FTTAVS typically employs more MAUs (e.g., *D20_U*: 131 versus 128 and *D20_HB*: 133 versus 128). This shift reflects the trade-off embedded in the objective function. Under small surges, operational costs dominate, incentivizing fleet conservation; beyond a 15% increase, passengers' costs become more prominent, and increasing fleet size becomes cost effective to improve service quality. In

conclusion, these findings suggest that both strategies exhibit a degree of robustness to demand surges. Jointly fine-tuning the timetable and reoptimizing the vehicle schedule consistently yield superior performance, particularly under more substantial surges.

7.8. Value of the Stochastic Programming Approach

In this section, we vary the number of scenarios to explore the effects of the number of scenarios on the efficiency of the algorithm and the quality of solutions. We generated 30 passenger flow scenarios based on real-world operational data, and we randomly selected 1, 2, 4, 6, 8, 12, 15, or 20 demand scenarios from them to form eight instances. Next, each of these instances is solved using the proposed RH + IL algorithm. Then, the obtained timetables are fed into the 30 scenarios to evaluate the expected passengers' costs and operational costs.

Table 8 shows the average performance of the above seven solutions over 30 scenarios. It can be seen that as the number of scenarios increases, the average performance of the resulting solution over the entire sample increases accordingly, but its solution time also tends to increase. Specifically, when the number of scenarios is very small (e.g., one and two), although the solution time is quite short, too few scenarios do not reflect the general pattern of passenger flow, and therefore, the average performance of the obtained timetable is poor. In constant, when the number of scenarios is

overly abundant, the solution quality improves accordingly, but the solution time also becomes dramatically longer. Choosing an appropriate number of scenarios (i.e., four, six, or eight) improves the solution quality by at least 6.5% relative to a deterministic optimization; meanwhile, it considerably improves the computational efficiency of solving the TT-VS-DAC problem relative to an excessive number of scenarios (i.e., 12, 15, or 20).

7.9. Performance of the Learning-Based Real-Time Decision-Making Framework

To evaluate the effectiveness of the proposed learning-based real-time decision-making framework and the embedded fine-tuning strategy, we conduct two sets of experiments based on the Beijing bus subnetwork. First, we build 12 instances to compare the performance of our framework against benchmark methods. Second, we construct six instances to investigate the impact of four levels of reoptimization flexibility within the real-time decision-making process.

7.9.1. Performance Comparison of the Learning-Based Scenario-Retention Method and Benchmarks. To validate the effectiveness of our proposed learning-based scenario-retention method, we compare it against two benchmark approaches: the *all scenario* scenario-retention method, which incorporates the entire stochastic demand scenario set at each decision-making stage, and the *greedy* scenario-retention method, which ranks all scenarios in descending order of the number of total passenger demand and selects the top k scenarios for optimization. For all methods, we impose a one-minute time limit per stage to ensure real-time applicability. If no feasible solution is found within this time limit, the process is terminated. This set of experiments is based on *instance_r_6* with newly generated stochastic demand scenario sets. Specifically, we construct scenario sets containing 8, 15, 20, and 30 scenarios, respectively, and we generate three distinct sets for each scenario size. These scenario sets are newly generated

to reflect diverse demand realizations for evaluation purposes. In addition, the value of k is set as three for both *greedy* and our learning-based scenario-retention methods. Each departure time is allowed to shift within a narrow adjustment range of ± 2 minutes.

The results are summarized in Table 9. In the instance column in Table 9, the notation $|W|$ refers to the number of stochastic demand scenarios included in the scenario set, and C1, C2, and C3 represent three distinct sets generated independently for each scenario size. The *Dev* (%) (total costs) column in Table 9 reports the relative difference in total passengers' costs and operational costs compared with the greedy method. A negative *Dev* value indicates that the corresponding scenario-retention method yields lower total costs (i.e., superior solution quality), whereas a positive value suggests inferior performance. Footnote a in Table 9 indicates that the method failed to produce a feasible solution within the time limit, whereas footnote b in Table 9 denotes model infeasibility.

From the results shown in Table 9, we can observe that the all scenario scenario-retention method fails to find feasible solutions within the one-minute computational time limit in 9 of 12 instances. This outcome is mainly because of the excessive computational burden associated with incorporating the entire scenario set into the optimization within the one-minute solution time limitation. In contrast, our proposed learning-based scenario-retention method generates feasible solutions across all instances, demonstrating its suitability for real-time optimization. Moreover, it consistently outperforms the *greedy* benchmark in terms of total passengers' costs and operational costs. Across the 12 tested instances, our method yields lower total costs in 11 cases, with relative cost reductions ranging from 0.3% (for $W = 15$, C3) to 7.9% (for $W = 15$, C1). In several large-scale instances (e.g., $W = 30$, C2 and C3), the *greedy* method fails to return feasible solutions. This is largely because of its strategy of selecting scenarios solely based on the passenger demand, which overlooks scenarios where local capacity constraints are more binding. These findings highlight the superior

Table 8. Results Among the Various Numbers of Stochastic Scenarios

No. of scenarios	Total passengers' costs (\$)	Total operational costs (\$)	No. of vehicles	Objective function value (\$)	Computational time (seconds)
1	8,511.2	2,314.1	112	10,825.3	106.2
2	8,554.6	2,334.3	113	10,589.4	253.5
4	7,791.3	2,324.2	112	10,115.5	524.4
6	7,599.1	2,321.6	115	9,920.7	1,047.7
8	7,591.9	2,316.4	112	9,908.2	1,896.3
12	7,468.8	2,313.9	113	9,782.6	5,160.3
15	7,381.6	2,316.0	110	9,697.5	7,202.1
20	7,381.6	2,314.1	112	9,696.1	9,711.2

robustness and efficiency of our learning-based scenario-retention method in delivering high-quality solutions under real-time computational requirements.

7.9.2. Impact of Reoptimization Flexibility in the Learning-Based Real-Time Decision-Making Framework. Furthermore, to assess the impact of reoptimization flexibility within the learning-based real-time decision-making framework, we compare four strategies. (i) *TT + VS* indicates that both the timetable and vehicle schedules can be dynamically reoptimized. (ii)

TT + VS (partial) is same as (i) but further indicates that boarding commitments from the previous stage are preserved, ensuring that passengers already on board are not rerouted. (iii) *VS* indicates that the timetable is fixed and that only vehicle schedules are dynamically reoptimized. (iv) *Fixed* indicates that no reoptimization is allowed. In this set of experiments, we focus on *instance_r_6*. The timetable and vehicle schedule obtained from our integrated optimization approach are used as the baseline solution. Each departure time is allowed to shift within a narrow window of ± 2

Table 9. Performance Comparison Between the Different Scenario-Retention Methods

Instance	Scenario-retention method	Passengers' cost	Operating cost	Dev (%)
$\mathcal{W}$ = 8, C1	Greedy	7,104.89	2,265.41	—
	All scenario	11,490.49	2,176.78	31.4
	Learning based	6,863.18	2,260.95	−2.6
$\mathcal{W}$ = 8, C2	Greedy	7,411.18	2,287.62	—
	All scenario	12,547.57	2,201.11	52.1
	Learning based	6,975.31	2,292.77	−4.4
$\mathcal{W}$ = 8, C3	Greedy	7,119.63	2,247.40	—
	All scenario	11,591.99	2,176.96	47.0
	Learning based	6,698.05	2,248.48	−4.5
$\mathcal{W}$ = 15, C1	Greedy	6,229.74	2,225.63	—
	All scenario	a	a	—
	Learning based	5,566.56	2,217.73	−7.9
$\mathcal{W}$ = 15, C2	Greedy	6,206.62	2,220.60	—
	All scenario	a	a	—
	Learning based	5,931.93	2,224.79	−3.2
$\mathcal{W}$ = 15, C3	Greedy	6,398.02	2,262.52	—
	All scenario	a	a	—
	Learning based	6,375.61	2,259.27	−0.3
$\mathcal{W}$ = 20, C1	Greedy	6,520.29	2,258.80	—
	All scenario	a	a	—
	Learning based	6,385.62	2,230.76	−1.9
$\mathcal{W}$ = 20, C2	Greedy	5,930.95	2,215.54	—
	All scenario	a	a	—
	Learning based	5,963.28	2,216.69	0.4
$\mathcal{W}$ = 20, C3	Greedy	6,140.54	2,227.32	—
	All scenario	a	a	—
	Learning based	5,914.03	2,217.00	−2.8
$\mathcal{W}$ = 30, C1	Greedy	6,960.45	2,234.45	—
	All scenario	a	a	—
	Learning based	6,528.14	2,236.00	−4.7
$\mathcal{W}$ = 30, C2	Greedy	b	b	—
	All scenario	a	a	—
	Learning based	6,888.64	2,267.09	—
$\mathcal{W}$ = 30, C3	Greedy	b	b	—
	All scenario	a	a	—
	Learning based	6,965.53	2,259.40	—

Notes. | $\mathcal{W}$ | refers to the number of stochastic demand scenarios included in the scenario set, and C1, C2, and C3 represent three distinct sets generated independently for each scenario size. Dev(%) represents the relative difference in total passengers' and operational costs compared with the greedy method.

^aNo feasible solution was found within the allowed computation time.

^bThe model is infeasible.

Table 10. Performance Comparison Among Various Reoptimization Strategies

Instance	Strategy	Dev.(%) (total costs)	Unserved passengers	Total no. of used MAUs
C1	Fixed		No	85
	VS	−1.44	No	85
	TT + VS (partial)	−3.40	No	87
	TT + VS	−3.92	No	86
C2	Fixed		Yes	91
	VS	−1.68	No	89
	TT + VS (partial)	−4.43	No	89
	TT + VS	−8.14	No	86
C3	Fixed		Yes	93
	VS	−1.87	No	88
	TT + VS (partial)	−2.79	No	91
	TT + VS	−5.38	No	90
C4	Fixed		No	101
	VS	−0.81	No	99
	TT + VS (partial)	−2.19	No	96
	TT + VS	−3.49	No	97
C5	Fixed		Yes	106
	VS	−1.12	No	101
	TT + VS (partial)	−1.48	No	104
	TT + VS	−3.15	No	101
C6	Fixed		Yes	113
	VS	−3.47	No	105
	TT + VS (partial)	−3.60	No	99
	TT + VS	−5.48	No	102

minutes. To evaluate the learning-based real-time framework under the four reoptimization strategies, we construct six new demand scenario sets, each containing eight scenarios that follow the characteristics of *instance_r_6*.

Table 10 summarizes the results, reporting the relative deviations (percentages) in total passengers' costs and operational costs compared with the *fixed* strategy, the presence of any unserved passengers, and the number of used MAUs. The negative value of *Dev.* represents the reduction of the total costs. We can observe that fixing the timetable and vehicle schedule with no reoptimization leads to unserved passengers in four of six instances (i.e., C2, C3, C5, and C6). The reason is that the space-time distribution of passenger demand changes in the new scenario sets, while the operational plans are frozen and overloading is disallowed; therefore, some passengers can no longer board.

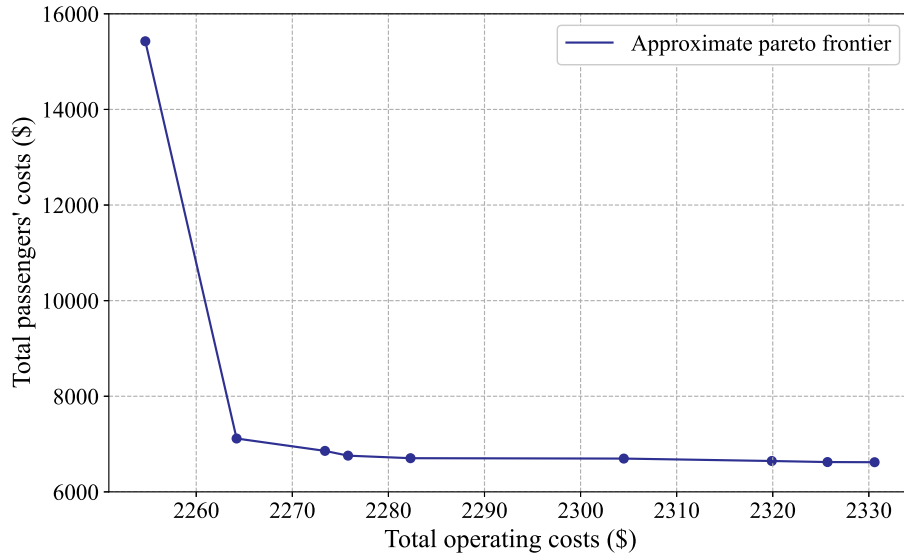
A second observation is that allowing only vehicle rescheduling resolves all infeasibilities and lowers total costs relative to the *fixed* strategy in every case from

0.81% to 3.47%. It also reallocates capacity more efficiently, typically using fewer MAUs than the *fixed* strategy (e.g., C2: 89 versus 91, C3: 88 versus 93, C5: 101 versus 106, and C6: 105 versus 113). In addition, TT + VS (partial) yields additional gains over VS in most instances. To honor already-boarded passengers, this policy occasionally needs a slightly larger fleet than VS (e.g., C3 and C5), but it maintains passenger continuity and remains feasible throughout. Granting full flexibility to fine-tune the timetable and reoptimize vehicle schedules (i.e., TT + VS) delivers the best performance overall, with the largest cost reductions in every instance from 3.15% to 8.14% and with a fleet size that is comparable with the other flexible strategies. In conclusion, relying on vehicle schedules and timetables optimized based on historical demand scenarios poses operational risks when facing real-time information in practical operations. Reoptimizing only the vehicle schedule can improve service quality by ensuring that all passengers are served. Preserving onboard commitments enhances the passenger experience, although it typically results in slightly higher total

Table 11. Total Passengers' Costs, Operational Costs, and Number of Used MAUs Obtained by Employing the Fixed-Capacity and Flexible-Capacity Strategies in the Real-World Instance

Operational strategy	Total passengers' costs (\$)	Total operational costs (\$)	No. of used MAUs
Fixed capacity	6,861.3	4,381.2	264
Flexible capacity	6,757.5	2,275.8	112

Figure 5. (Color online) Approximate Pareto Frontier of the TT-VS-DCA Model



costs compared with the strategy that does not account for such commitments. Among these four strategies, the approach that dynamically reoptimizes both the timetable and vehicle schedules (i.e., TT + VS) consistently achieves the lowest total cost.

7.10. Analysis of the Benefits in the Practical Instance and the Trade-offs

In this section, we explore the benefits that the proposed approach can provide to real-world operations by analyzing the results of *instance_r_6*. We first explore the benefit of adopting our proposed flexible-capacity operational strategy. The results are presented in Table 11. It can be observed that adopting the flexible-capacity operational strategy can significantly reduce operational costs compared with the fixed-capacity operational strategy. Specifically, the implementation of the proposed flexible-capacity strategy approximately results in a 48% reduction in operational costs and a 57% decrease in the number of utilized MAUs. At the same time, passengers' costs also decrease from 6,861.3 to 6,757.5. The main reason is that our proposed method provides passengers with the convenience to make in-vehicle transfers, which reduces the waiting time for transfers in transfer corridors.

Next, we use the weighted sum approach to analyze the trade-off between operational costs and passengers' costs. The objective function (denote as Ψ here) is reformulated as follows:

$$\Psi = \omega_1 \frac{\Psi^{pass} - \underline{\Psi}^{pass}}{\overline{\Psi}^{pass} - \underline{\Psi}^{pass}} + \omega_2 \frac{\Psi^{oper} - \underline{\Psi}^{oper}}{\overline{\Psi}^{oper} - \underline{\Psi}^{oper}}. \quad (46)$$

To be specific, ω_1 and ω_2 are weighting coefficients designed to control the magnitude of the impact that

normalized passenger travel costs and normalized bus operational costs have on the change in objectives, respectively. $\underline{\Psi}^{pass}$ is the lower bound on the total passenger travel cost, and $\overline{\Psi}^{pass}$ is the upper bound on the total passenger travel cost. $\underline{\Psi}^{pass}$ is theoretically minimized to zero, and $\overline{\Psi}^{pass}$ can be calculated by taking the maximum headway for both waiting time per passenger and transfer waiting time. Similarly, $\underline{\Psi}^{oper}$ is the lower bound on the total operating travel cost, and $\overline{\Psi}^{oper}$ is the upper bound on the total operating travel cost. $\underline{\Psi}^{oper}$ can be calculated according to Lemma 1, and $\overline{\Psi}^{oper}$ can be computed by assuming that the maximum fleet is dispatched for each trip and does not allow for decoupling and coupling en route.

To approximate the Pareto optimal point, we incrementally increase ω_1 from 0 to 1 while simultaneously decreasing ω_2 from 1 to 0 in increments of 0.1. The results obtained through the weighted sum method are depicted in Figure 5. Upon analyzing the results, two key observations emerge. (1) Excessive emphasis on reducing operational costs can precipitate a significant decline in service quality, thereby leading to a substantial rise in passengers' costs. (2) Reducing operational costs with little to no negative impact on service quality can be achieved with a slight increase in the weight assigned to operational costs.

8. Conclusions

We present a novel modeling approach for the TT-VS-DCA problem with time-varying and uncertain passenger demand. Our approach explicitly considers the trade-off between passengers' costs and operational costs. MAUs can be flexibly (de-)coupled at depots and transfer stops, enabling efficient circulations and

rerouting across lines. To solve this integrated optimization problem, we propose an exact solution method with tailored valid inequalities and embed it in a rolling-horizon framework to enhance computational efficiency. To facilitate real-time adjustments under evolving operational conditions, we develop a real-time decision-making framework to dynamically fine-tune timetables that are obtained based on historical passenger flow scenarios and optimize vehicle scheduling.

Computational experiments on both a virtual network and the Beijing bus subnetwork demonstrate the effectiveness of our approach. Several key takeaways are summarized as follows. (i) Our exact method outperforms GUROBI for medium-sized instances, whereas the rolling-horizon optimization framework combined with the integer L-shaped approach yields superior results for large-scale instances. (ii) Compared with the sequential optimization method, the proposed integrated approach consistently performs better across instances of varying scales. For smaller instances, it results in fewer required MAUs. As the network size increases, it remains capable of generating feasible timetables and vehicle schedules, whereas the sequential method often fails to find feasible solutions. (iii) Allowing MAVs to (de-)couple and be rerouted at transfer stops in addition to depots further reduces both fleet size and operational costs. (iv) The optimized timetable and vehicle schedule generated from historical demand scenarios using our integrated optimization approach are robust under demand decreases in out-of-sample scenarios. Moreover, allowing overloading can handle a 5% demand surge without requiring additional capacity resources. (v) Both the strategy that fixes the timetable while reoptimizing the vehicle schedule and the strategy that jointly fine-tunes these two schedules can handle the demand surges and redistributions and exhibit robustness. (vi) The proposed learning-based real-time decision-making framework outperforms benchmark methods thanks to the novel scenario-retention approach using the machine learning model.

References

- Amberg B, Amberg B (2023) Robust and cost-efficient integrated multiple depot vehicle and crew scheduling with controlled trip shifting. *Transportation Sci.* 57(1):82–105.
- An K (2020) Battery electric bus infrastructure planning under demand uncertainty. *Transportation Res. Part C Emerging Tech.* 111:572–587.
- Angulo G, Ahmed S, Dey SS (2016) Improving the integer L-shaped method. *INFORMS J. Comput.* 28(3):483–499.
- Caprara A, Fischetti M, Toth P (2002) Modeling and solving the train timetabling problem. *Oper. Res.* 50(5):851–861.
- Carli R, Cavone G, Pippia T, De Schutter B, Dotoli M (2022) Robust optimal control for demand side management of multi-carrier microgrids. *IEEE Trans. Automation Sci. Engrg.* 19(3):1338–1351.
- Chen Z, Li X (2021) Designing corridor systems with modular autonomous vehicles enabling station-wise docking: Discrete modeling method. *Transportation Res. Part E Logist. Transportation Rev.* 152:102388.
- Chen Z, Li X, Qu X (2022) A continuous model for designing corridor systems with modular autonomous vehicles enabling station-wise docking. *Transportation Sci.* 56(1):1–30.
- Chen Z, Li X, Zhou X (2019) Operational design for shuttle systems with modular vehicles under oversaturated traffic: Discrete modeling method. *Transportation Res. Part B Methodological* 122:1–19.
- Chen Z, Li X, Zhou X (2020) Operational design for shuttle systems with modular vehicles under oversaturated traffic: Continuous modeling method. *Transportation Res. Part B Methodological* 132:76–100.
- Cho J, Papavasiliou A (2023) Pricing under uncertainty in multi-interval real-time markets. *Oper. Res.* 71(6):1928–1942.
- Huisman D, Freling R, Wagelmans APM (2004) A robust solution approach to the dynamic vehicle scheduling problem. *Transportation Sci.* 38(4):447–458.
- Ibarra-Rojas OJ, Giesen R, Rios-Solis YA (2014) An integrated approach for timetabling and vehicle scheduling problems to analyze the trade-off between level of service and operating costs of transit networks. *Transportation Res. Part B Methodological* 70:35–46.
- Laporte G, Louveaux FV (1993) The integer L-shaped method for stochastic integer programs with complete recourse. *Oper. Res. Lett.* 13(3):133–142.
- Laporte G, Ortega FA, Pozo MA, Puerto J (2017) Multi-objective integration of timetables, vehicle schedules and user routings in a transit network. *Transportation Res. Part B Methodological* 98:94–112.
- Lee K, Jiang Y, Ceder AA, Dauwels J, Su R, Nielsen OA (2022) Path-oriented synchronized transit scheduling using time-dependent data. *Transportation Res. Part C Emerging Tech.* 136:103505.
- Lyu G, Cheung WC, Teo CP, Wang H (2024) Multiobjective stochastic optimization: A case of real-time matching in ride-sourcing markets. *Manufacturing Service Oper. Management* 26(2):500–518.
- Ma Q, Li S, Zhang H, Yuan Y, Yang L (2021) Robust optimal predictive control for real-time bus regulation strategy with passenger demand uncertainties in urban rapid transit. *Transportation Res. Part C Emerging Tech.* 127:103086.
- Michaelis M, Schöbel A (2009) Integrating line planning, timetabling, and vehicle scheduling: A customer-oriented heuristic. *Public Transport* 1(3):211–232.
- Next Future Inc. (2023a) Dubai experiments the future of transportation, with NEXT. Accessed July 9, 2023, <https://www.next-future-mobility.com/post/dubai-with-next-experiments-with-the-future-of-transportation>.
- Next Future Inc. (2023b) NEXT announces a new investment and partnership in UAE. Accessed October 10, 2023, <https://www.next-future-mobility.com/post/next-announces-a-new-investment-and-partnership-in-uae>.
- Next Future Inc. (2023c) One vehicle, many use cases. Accessed September 19, 2025, <https://www.next-future-mobility.com/maas>.
- Sánchez-Martínez G, Koutsopoulos H, Wilson N (2016) Real-time holding control for high-frequency transit with dynamics. *Transportation Res. Part B Methodological* 83:1–19.
- Shi X, Li X (2021) Operations design of modular vehicles on an oversaturated corridor with first-in, first-out passenger queuing. *Transportation Sci.* 55(5):1187–1205.
- Tian Q, Lin YH, Wang DZ (2023) Joint scheduling and formation design for modular-vehicle transit service with time-dependent demand. *Transportation Res. Part C Emerging Tech.* 147:103986.
- Van Lieshout RN (2021) Integrated periodic timetabling and vehicle circulation scheduling. *Transportation Sci.* 55(3):768–790.
- Van Lieshout RN, Bouman PC, Van den Akker M, Huisman D (2021) A self-organizing policy for vehicle dispatching in public transit systems with multiple lines. *Transportation Res. Part B Methodological* 152:46–64.

- Wang Y, Zhu S, Li S, Yang L, De Schutter B (2022) Hierarchical model predictive control for on-line high-speed railway delay management and train control in a dynamic operations environment. *IEEE Trans. Control Systems Tech.* 30(6): 2344–2359.
- Xia D, Ma J, Sharif Azadeh S (2024) Integrated timetabling and vehicle scheduling of an intermodal urban transit network: A distributionally robust optimization approach. *Transportation Res. Part C Emerging Tech.* 162:104610.
- Xia D, Ma J, Sharif Azadeh S, Zhang W (2023) Data-driven distributionally robust timetabling and dynamic-capacity allocation for automated bus systems with modular vehicles. *Transportation Res. Part C Emerging Tech.* 155:104314.

Copyright of Transportation Science (INFORMS) is the property of INFORMS: Institute for Operations Research & the Management Sciences and its content may not be copied or emailed to multiple sites without the copyright holder's express written permission. Additionally, content may not be used with any artificial intelligence tools or machine learning technologies. However, users may print, download, or email articles for individual use.