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On the propagation of a disturbance in the bed composition of an open channel

K. SUZUKI

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by

Koichi SUZUKI

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1. General

The transport of a sand mixture is more complicated than that of uniform sand, because the transport velocity of a grain is varying with the grain size. And in case of sand mixture, the change of composition of mixture in addition to the change of bed height according to the change of boundary conditions occurs.

It is very important from the point of river engineering to know how the bed level or the composition of mixture responds to a change of boundary conditions. For example the change of the water level caused by the construction of a dam causes a change of bed level and of composition of bed sand both up- and down-stream of the dam.

In this report, some aspects of this reponse will be discussed.

2. Theoretical analysis

2.1. General

The morphological phenomena of river bed can be solved theoretically if the equations of motion and continuity of both water and sand are known. But even if these equations are known, it is difficult to find out the solutions on some boundary and initial conditions.
After some consideration of these equations, propagation phenomena of small disturbance of bed height and composition of sand mixture will be discussed.

2.2. Equations of motion and continuity of sand mixture

Equation of continuity of sand mixture

In fully developed dunes, we may assume, that the average dune height (= ΔH) is constant with time.
In this study the micro behaviour of individual grains is neglected; therefore the dune layer may be considered as a continuous moving layer with a thickness δ.
And if we assume that the dune shape is triangular δ becomes a half of the dune height (Fig. 2-1).

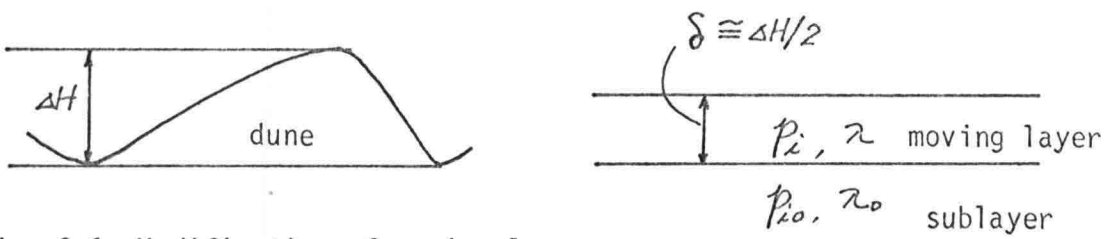


Fig. 2-1 Modification of moving layer.

If we consider the continuity of sand on a small distance in the bed, the accumulated amount of sand di in Δx and during a small time Δt is

$$-(\partial q_{si}/\partial x)\,\Delta x\,\Delta t \qquad \text{(a)}$$

where q_{si} is the discharge of sediment with diameter di per unit time and unit width. The accumulated amount of sand di in moving layer δ in Δx and Δt is

$$-\frac{\partial}{\partial t}\{(1-\lambda)\,\delta\cdot p_i\}\,\Delta x\cdot\Delta t \qquad \text{(b)}$$

where λ : porosity of moving layer, and

p_i : frequency of sand di in moving layer

The accumulated amount of sand di in sublayer can be written as follows:

$$\frac{\partial}{\partial t}\{(z-\delta)(1-\lambda)\bar{p}_{is}\}\Delta x\,\Delta t$$

where z is the bed level and $\bar{p}_{is}$ is the frequency of sand di in sublayer. But $\bar{p}_{is}$ cannot be calculated because $\bar{p}_{is}$ is the mean frequency of the sublayer. But if we consider that only in the hatched part of the sublayer (Fig. 2-2) the frequency of sand di changes, we can consider the accumulated amount of sand di in sublayer by considering the frequency of the hatched part instead of $\bar{p}_{is}$.

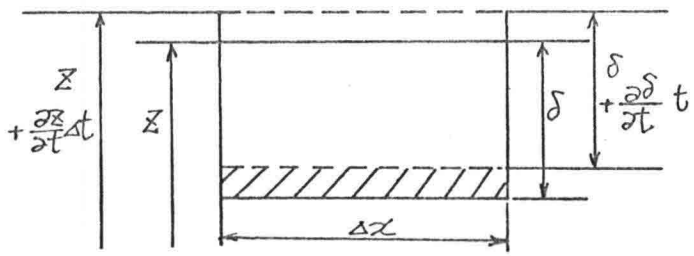


Fig. 2-2 Control section

The volume of the hatched part is:

$$\frac{\partial}{\partial t}(z-\delta)\,\Delta x\,\Delta t$$

and for the degradation of bed the porosity and frequency of the hatched part are λ_0 and p_{io} respectively, where suffix o means initial value of sublayer.

But in case of aggradation of the bed, the frequency of the top of the hatched part is $p_i + \frac{\partial p}{\partial t}\Delta t$ and the frequency of bottom of the hatched part is p_i, so we can write the mean frequency of the hatched part, using the frequency of moving layer.

$$p_i + \frac{1}{2}\frac{\partial p_i}{\partial t}\Delta t$$

and as to the mean porosity

$$\lambda + \frac{1}{2}\frac{\partial \lambda}{\partial t}\Delta t$$

So, the accumulated amount of sand di in sublayer can be written as follows:

for degradation, if π_0 and p_{io} are constants,

$$\frac{\partial}{\partial t}(z-\delta)\Delta x \Delta t \times (1-\lambda_0) p_{io} \tag{c}$$

for aggradation,

$$\frac{\partial}{\partial t}(z-\delta)\Delta x \Delta t \times (1-\lambda) p_i + \frac{1}{2}\left\{(1-\lambda)\frac{\partial p_i}{\partial t}\Delta t - p_i \frac{\partial \lambda}{\partial t}\right\}\Delta x \Delta t \tag{c'}$$

(a) should be sum of (b) and (c) (or (c')). After dividing this equation ((a) = (b) + (c)) by $\Delta x \Delta t$ and putting Δx, $\Delta t \to 0$, the continuity equation for sand di reads:
for degradation of bed

$$\frac{\partial}{\partial t}\left\{\delta(1-\lambda) p_i + (z-\delta)(1-\lambda_0) p_{io}\right\} + \frac{\partial q_{si}}{\partial t} = 0 \tag{2-1}$$

for aggradation of bed

$$(1-\lambda) p_i \frac{\partial}{\partial t}(z-\delta) + \frac{\partial}{\partial t}\left\{\delta(1-\lambda) p_i\right\} + \frac{\partial q_{si}}{\partial x} = 0 \tag{2-2}$$

The total amount of transported sand per unit time and unit width q_{st} is

$$q_{st} = \sum_i q_{si} \tag{2-3}$$

and eq. (2-1) and (2-2) become for the total transport as follows, respectively,
for degradation of bed

$$\frac{\partial}{\partial t}\left\{\delta(1-\lambda) + (z-\delta)(1-\lambda_0)\right\} + \frac{\partial q_{st}}{\partial x} = 0 \tag{2-4}$$

for aggradation of bed

$$(1-\lambda)\frac{\partial}{\partial t}(z-\delta) + \frac{\partial}{\partial t}\left\{\delta(1-\lambda)\right\} + \frac{\partial q_{st}}{\partial x} = 0 \tag{2-5}$$

Equation of motion of sand

The transport q_{si} of sand with diameter di should satisfy the conditions:

1. when $p_i = 0$; $q_{si} = 0$
2. when $p_i = 1$; $q_{si} = f_i$

where f_i is the transport function of sand di which coincides with the transport equation of uniform sand when $p_i = 1$.
The most simple form to satisfy the conditions 1 and 2 is

$$q_{si} = p_i f_i \tag{2-6}$$

Eq (2-6) should be studied further but for the time being we use eq. (2-6) for transport equation of sand di.
And as to f_i, we can use a modified sediment transport formula of uniform sand.

Equations of motion and continuity of water

The equations of motion and continuity of water for one dimensional flow can be written as follows

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z}{\partial x} = -\frac{\tau_b}{\rho_w R_b} \tag{2-7}$$

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0 \tag{2-8}$$

where u : velocity in the x-direction
h : waterdepth
g : acceleration of gravity,
z_b : bed shear stress
z : bed height
f_w : density of water
R_b : hydraulic radius

2.3. Propagation velocities of small disturbance of bed height and composition of sand mixture

The response of the sand movement to a change of boundary conditions is very slow as compared with the response of the watermovement. For example, when the water depth is changed at the downstream end of the flume, the flow upstream is quickly affected if Froude number is much less than unity a new quesi-steady state of the flow will be reached in a short time but it takes a long time before the bed profile reaches a new equilibrium state over the whole length of the flume. A small disturbance of the water surface is propagated at the velocity $u \pm \sqrt{gh}$. In the next paragraph the propagation velocity of a small disturbance of the bed profile and that of a disturbance in the composition of sand mixture will be discussed.

To arrive at solutions for the six dependent variables, viz. z, h, u, p_i, q_{st} and q_{si}, as functions of x and t, six equations are needed: eqs. (2-1)(or(2-2)), (2-4) (or(2-5)), (2-6), (2-3), (2-7) and (2-8). If we rewrite these equations, using $\tau_b/\rho_w R_b = W$ and $\alpha = 1/(1-\lambda)$, and if we assume that δ and α are constant, we get

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z}{\partial x} = -w \qquad (a)$$

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0 \qquad (b)$$

$$q_{st} = f_t \ (= f_m) \qquad (c)$$

$$q_{si} = p_i f_i \qquad (d)$$

$$\alpha\frac{\partial q_{st}}{\partial x} + \frac{\partial z}{\partial t} = 0 \qquad (e)$$

$$\alpha\frac{\partial q_{si}}{\partial x} + \delta\frac{\partial p_i}{\partial t} + p_i\frac{\partial z}{\partial t} = 0 \quad \text{(for aggradation)} \qquad (f)$$

$$\alpha\frac{\partial q_{si}}{\partial x} + \delta\frac{\partial p_i}{\partial t} + p_{io}\frac{\partial z}{\partial t} = 0 \quad \text{(for degradation)} \qquad (f')$$

when our interest is only in the flow of water, eqs. (c), (d), (e) and (f) may be omitted. In this case by using eqs. (a) and (b) we get the wellknown propagation velocity of small water surface disturbance C1, 2

$$C_{1,2} = (dx/dt)_{1,2} = u \pm \sqrt{gh} \qquad (2\text{-}9)$$

De Vries [1] derived the propagation velocity of a small disturbance of the bed for uniform sand C3, using eqs. (a), (b), (c) and (e).

$$C_3 = \frac{1}{1-\lambda}\frac{1}{1-F_r^2}\frac{1}{h}\frac{\partial f_t}{\partial u}\cdot u \qquad (2\text{-}10)$$

In case of a sand mixture, we will use the same analysis as done by de Vries for uniform sand. To avoid complexity we may assume $|C1,2| \gg$ C3 and C4 (C4 is the propagation velocity of a small disturbance of mixture composition) because the unsteady response of sediment is much slower than that of water, therefore the unsteady term $\partial u/\partial t$ and $\partial h/\partial t$ in eqs. (a) and (b) can be neglected. Putting eq. (e), eqs. (c), (d), into eqs. (e), (f) and using:

$$\frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx = du \tag{g}$$

$$\frac{\partial z}{\partial t}dt + \frac{\partial z}{\partial x}dx = dz \tag{h}$$

$$\frac{\partial p_i}{\partial t}dt + \frac{\partial p_i}{\partial x}dx = dp_i \tag{i}$$

we reduce these equations as follows

$$\begin{pmatrix} 0 & \beta & 0 & g & 0 & 0 \\ 0 & \alpha p_i(\partial f_t/\partial u) & p_{i(o)} & 0 & \delta & \alpha(\partial g_{si}/\partial p_i) \\ 0 & \alpha(\partial f_t/\partial u) & 1 & 0 & 0 & 0 \\ dt & dx & 0 & 0 & 0 & 0 \\ 0 & 0 & dt & dx & 0 & 0 \\ 0 & 0 & 0 & 0 & dt & dx \end{pmatrix} \begin{pmatrix} \partial u/\partial t \\ \partial u/\partial x \\ \partial z/\partial t \\ \partial z/\partial x \\ \partial p_i/\partial t \\ \partial p_i/\partial x \end{pmatrix} = \begin{pmatrix} -w \\ 0 \\ 0 \\ du \\ dz \\ dp_i \end{pmatrix} \tag{2-11}$$

$$(A) \qquad X = Y \tag{2-11'}$$

where $\beta = u(1 - 1/F_r^2)$

Mathematically, the characteristic velocities are obtained by putting the determinant of coefficient (A) in eq. (2-11) to zero, that is

$$|A| = dt\left\{-\beta\delta(dx)^2 + \alpha\beta\frac{\partial g_{si}}{\partial p_i}dx\,dt - g\delta\alpha\frac{\partial f_t}{\partial u}dt\,dx + \alpha^2 g\frac{\partial g_{si}}{\partial p_i}\frac{\partial f_t}{\partial u}(dt)^2\right\} = 0$$

This gives

$$dt = 0 \tag{2-12}$$

$$C_3 = \left(\frac{dx}{dt}\right)_z = -\frac{g\alpha}{\beta}\frac{\partial f_t}{\partial u} = \frac{1}{1-n}\,\frac{1}{1-F_r^2}\,\frac{1}{h}\frac{\partial f_t}{\partial u}\cdot u \tag{2-13}$$

$$C_{4i} = \left(\frac{dx}{dt}\right)_{p_i} = \frac{\alpha}{\delta}\frac{\partial g_{si}}{\partial p_i} = \frac{1}{1-n}\,\frac{1}{\delta}\left(f_i + p_i\frac{\partial f_i}{\partial p_i}\right) \tag{2-14}$$

Eq. (2-12) means C1,2 = $\pm\infty$ as compared with C3 or C4i, that is the assumption of this analysis. C3 and C4i describe the propagation velocity of a small disturbance of the bed and that of the frequency p_i of composition of sand mixture. Here we must take care about that these results are based on $|C1,2| \gg$ C3 or C4 i, therefore this analysis yields meaningful results only if Froude number is much smaller or much larger than unity. C3 and C4i can be calculated if the thickness of the moving layer and sediment transport function f_t or f_i are known. So, in the paragraph 2.5 and 2.6 f_i and δ are discussed.

2.4. Time variation of composition of bed sand

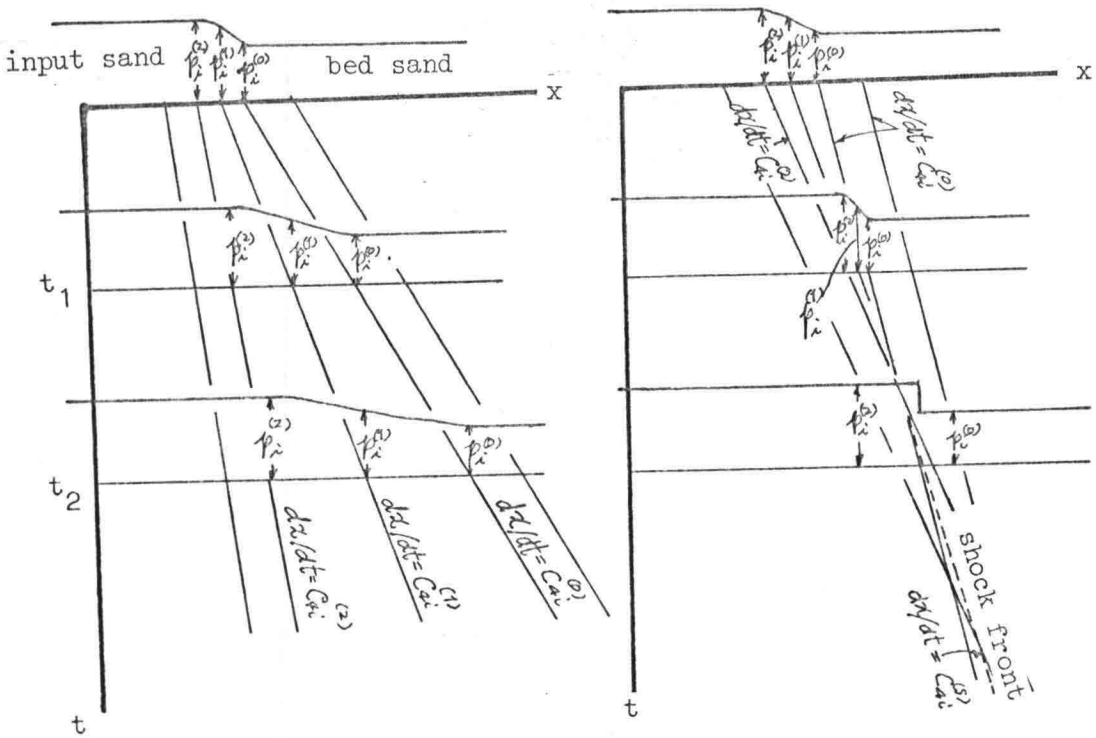
When sand with a different grain size composition is introduced into a river or flume, the composition of the bed will gradually change. The velocity C4i of this disturbance front of input sand has already been discussed in paragraph 2.3; it can be expressed by eq. (2-14).
If the characteristic velocity of di sand in bed $C4i^{(o)}$ $(= \alpha/\delta(f_\lambda^{(o)} + p_\lambda^{(o)}(\partial f_\lambda^{(o)}/\partial p_\lambda))$ where (o) means values pertaining to the bed)) is larger than that of input sand $C4i^{(1)}$ and $^{(2)}$ (where (1) or (2) means values concerning to input sand), the time change of the composition p_i can be obtained along the characteristic direction as figured in the x (distance along the channel) - t (time) diagram (Fig. 2-3 (a)). But in case of $C4i^{(o)} < C4i^{(1)}$, when the upstream discontinuity reaches to the downstream discontinuity, a shock front is formed and the time change of p_i can be figured as in Fig. 2-3 (b). Here final velocity of shock front $C4i^{(s)}$, which should be between $C4i^{(o)}$ and $C4i^{(2)}$, can be written as follows, using continuity relations: $C_{4i}^{(s)} \cong (p_\lambda^{(2)} C_{4i}^{(2)} - p_\lambda^{(o)} C_{4i}^{(o)}) / (p_\lambda^{(2)} - p_\lambda^{(o)})$

2.5. Sediment transport function f_i

We assumed that the amount g_{si} of transported sand di may be expressed as follows

$$g_{si} = p_\lambda f_\lambda \qquad (2\text{-}6)$$

and when $p_i = 1$ (uniform sand), then $g_{si} = f_i$. We may use one of modified transport formula of uniform sand for f_i.
Hirano[2] and Michiue[3] pointed out that if we use critical shear stress for individual grains of mixture, a transport formula of uniform sand can be applied to a sand mixture.
Here Mayer-Peter & Müllers' type of formula instead of Sato-Kikkawa-Ashida



(a) $C^{o}_{4i} > C^{1}_{4i}$ (b) $C^{o}_{4i} < C^{1}_{4i}$

Fig.2-3 Propagation of diaturbance of sand composition p_i

formula used by Michiue will be used. So, f_i can be written as follows,

$$f_i = C\sqrt{\Delta g d_i^3}\,(\tau_{*ei} - \tau_{*ci})^n \qquad (2\text{-}15)$$

where $\Delta(=\rho_s/\rho_w - 1)$ is the relative submerged density of sand, $\tau_{*ei}(=\mu R_{bi}/\Delta d_i)$ and τ_{*ci} are the nondimensional effective shear stress and critical shear stress, respectively and μ is a ripple factor, R_b is hydraulic radius, i is the energy slope and c and n are "constants". And

$$f_t = C\sqrt{\Delta g d_m^3}\,(\tau_{*em} - \tau_{*cm})^n \qquad (2\text{-}16)$$

where suffix m means values concerning to the mean diameter sand.

Non dimensional effective shear stress $\tau_{*ei}(=\mu R_{bi}/\Delta d_i)$

To calculate the effective shear stress in eq. (2-15) we must apply a wall connection for hydraulic radius at first and next estimate, a ripple factor μ. (Side wall correction for R).
Einsteins' method for correction of wall roughness can be used in calculating the hydraulic radius concerning only to the bed R_b[4], using Chézy coefficient C' instead of Manning coefficient n used by Einstein. That is

$$R_b = h\,(1 - 2R_w/B) \qquad (2\text{-}17)$$

where suffix b is concerning to the bed and w to the wall, h is waterdepth and B is the width of the channel.
Hydraulic radius concerning to the wall R_w may be calculated from

$$R_w = u^2/C_w i \qquad (2\text{-}18)$$

with $C_w = 18\log\frac{12R}{k_w + 0.3\delta'}$ by trial and error method, using known wall roughness k_w and $\delta' = 11.6\nu/u_* = 11.6\nu/\sqrt{gR_w i}$. After determining R_w, R_b can be calculated, using (eq. (2-17).
The wall roughness k_w of the flume used in our experiment is order of 10^{-5}m.
(ripple factor).
One may consider the total bed shear stress τ_b $(=\rho g R_b i)$ which is already corrected for side wall effects, to be the sum of two components, viz. a grain shear stress τ_g and a form drag τ_f;

$$\tau_b = \tau_f + \tau_g \quad \text{or} \quad (Ri)_b = (Ri)_f + (Ri)_g \qquad (2\text{-}19)$$

where suffix f is concerning to the form drag and g is pertaining to the grain drag. Mayer-Peter & Müller [5] divided energy slope i_b into i_f and i_g, keeping the hydraulic radius R unchanged and Einstein [6] divided R_b into R_f and R_g, keeping i unchanged.

Meyer-Peter & Muller assumed that only the grain drag contribute to the sand transport and related the grain shear stress to the total bed shear stress by means of a bed form factor ;

$$\tau_e = \tau_g = \mu \tau_b \qquad \tau_e : \text{effective shear stress}$$

And if we use the Chézy formula for the mean velocity u ,

$$u^2 = C_b^2 R i_b \ , \quad u^2 = C_g^2 R i_g$$

μ can be written as follows

$$\mu = (C_b / C_g)^2 \tag{2-20}$$

where C is the Chézy coefficient. A closer fit of the Meyer-Peter & Müller transport formula to the experimental data was obtained if the theoretical value of the exponent in eq. (2-20) was changed from 2 to 3/2. Prins [7] also modified μ in the range $0.4 < C_b/C_g < 0.8$,

$$\mu = 1.3 (C_b/C_g)^{3/2}$$

fitting his experimental data to the Meyer-Peter & Müllers' equation. It is unlogical, however, to change the exponent 2 because eq. (2-20) is derived using Chézy formula.

From experimental observation it appears that immediately downstream of the dune crest some particles are lifted up by the wake; therefore it seems reasonable to assume that not only the roughness drag but also some part of the form drag contributes to the sand transport, say,

$$\tau_e = \tau_g + \alpha \tau_f = \{ \alpha + (1-\alpha)(\tau_g/\tau_b) \} \tau_b$$

and so,

$$\mu = \alpha + (1-\alpha)(C_b/C_g)^2 \tag{2-21}$$

But α should be determined by experiment. C_b can be calculated by $C_b = u/\sqrt{R_b i}$, and C_g can be calculated by using logarithmic distribution of flow velocity

$$C_g = u/\sqrt{R_b i} = 18 \log \frac{12 R_b}{k_s + 0.3\delta'} \qquad (m^{1/2} sec^{-1})$$

where $k_s = d_{90}$, $\delta' = 11{,}6\,\nu/u_*$.

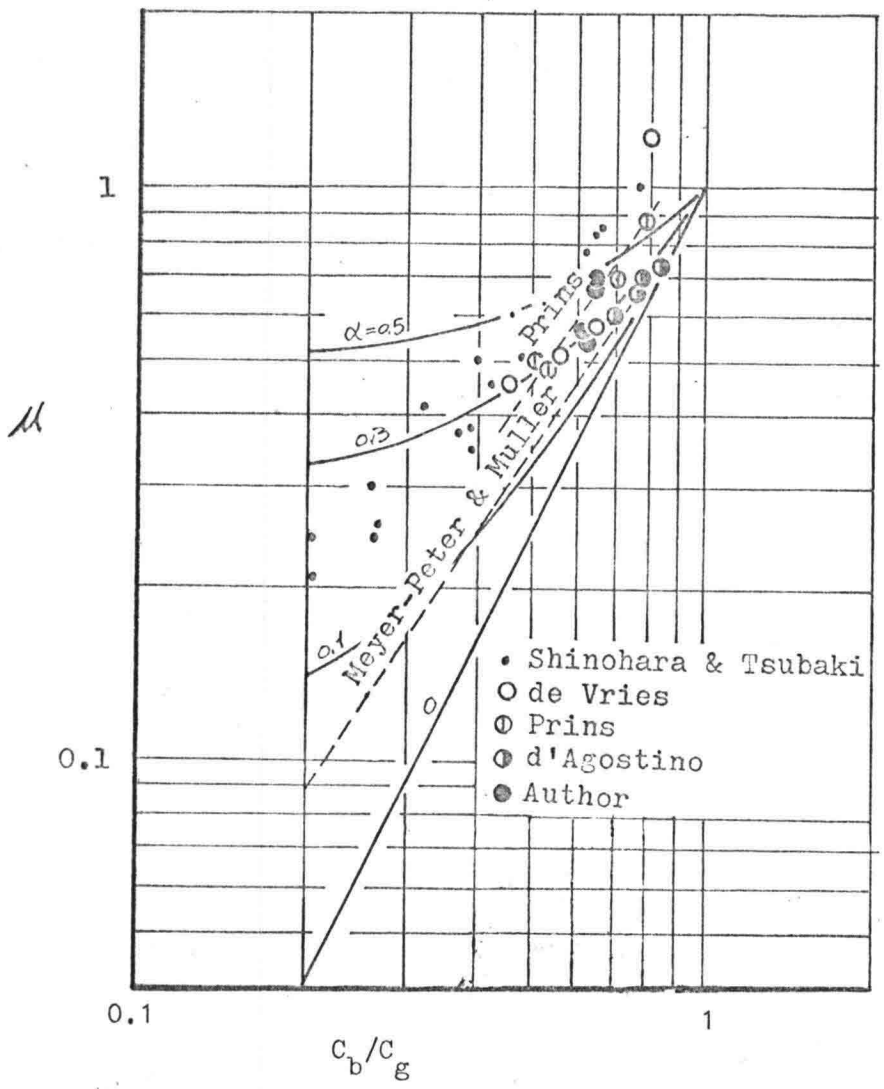


Fig.2-4 Relation between C_b/C_g and ripple factor

Fig. (2-4) shows the relation between μ and C_b/C_g, calculated by Meyer-Peter & Müllers' equation of sediment transport and experimental data. Nearly all data obtained at Delft are situated between $\mu=(C_b/C_g)^{3/2}$ (Meyer-Peter & Müller) and $\mu=1.3(C_b/C_g)^{3/2}$ and fit eq. (2-21) in case of $= 0.2 \sim 0.3$.

Non dimensional critical shear stress τ_{*ci}

Since the report by Shields, it is generally assumed that the non-dimensional critical shear stress τ_{*c} for uniform sand is constant in turbulent flow (say $u_*d/\nu > \sim 100$), where u_* is the shear velocity and ν is the kinetic viscosity); values for this constant range from almost 0.045 to about 0.065.
Shields ~ 0.06, Iwagaki ~ 0.05, Meyer-Peter & Müller $0.03 \sim 0.05$ [5].
Because of variation in exposure of the flow, the smaller grains of a sand mixture are less easily moved than grains of the same size in case of uniform sand; for the larger grains of a mixture the reverse holds.
Several formulae for the critical shear stress of sand mixture have been proposed (Kramer[8], Sakai[9], Tsuchiya[10], Gessler[11], Egiazanoff[12]).
Among them Egiazanoff's formula is simple and based on theoretical analysis considering drag and gravity forces on grains of uniform sand and velocity of the top of individual sand of mixture.
The non-dimensional critical shear stress of individual grain whose diameter is di in the mixture τ_{*ci} is written as follows

$$\tau_{*ci}\left(=\tau_c/(\rho_s-\rho_w)gd_i\right)=(2/3)\tau_{*cm}/C_{ox}(\log 19\, d_i/d_m)^2 \qquad (2\text{-}22)$$

where dm is mean diameter of sand mixture. C_{ox} is drag coefficient, which is equal to 0.4 for completely turbulent flow and τ_{*cm} is the non-dimensional critical shear stress for uniform sand whose diameter is dm. Eq. (2-22) means that if $d_i < d_m$, then $\tau_{*ci} > \tau_{*cm}$ and if $d_i > d_m$, $\tau_{*ci} < \tau_{*cm}$.
Ashida & Michiue[13] observed that in case of $d_i/d_m < 0.4$, τ_{ci} does not vary with di, then $\tau_{ci} = 0.85\tau_{cm}$.
Now, if we use Meyer-Peter &Müller's sediment transport equation we should use 0.047 for the value of τ_{*cm} and so we get

$$\tau_{*ci} = 0.078 / (\log 19 d_i/d_m)^2 \qquad (2\text{-}23)$$

Fig. 2-5 shows the relation eq. (2-15) (where $f_i=p_{*i}/p_i$), using eq. (2-23) for τ_{*ci} and C = 8 and n = 3/2 which are Meyer-Peter & Müller's constants.

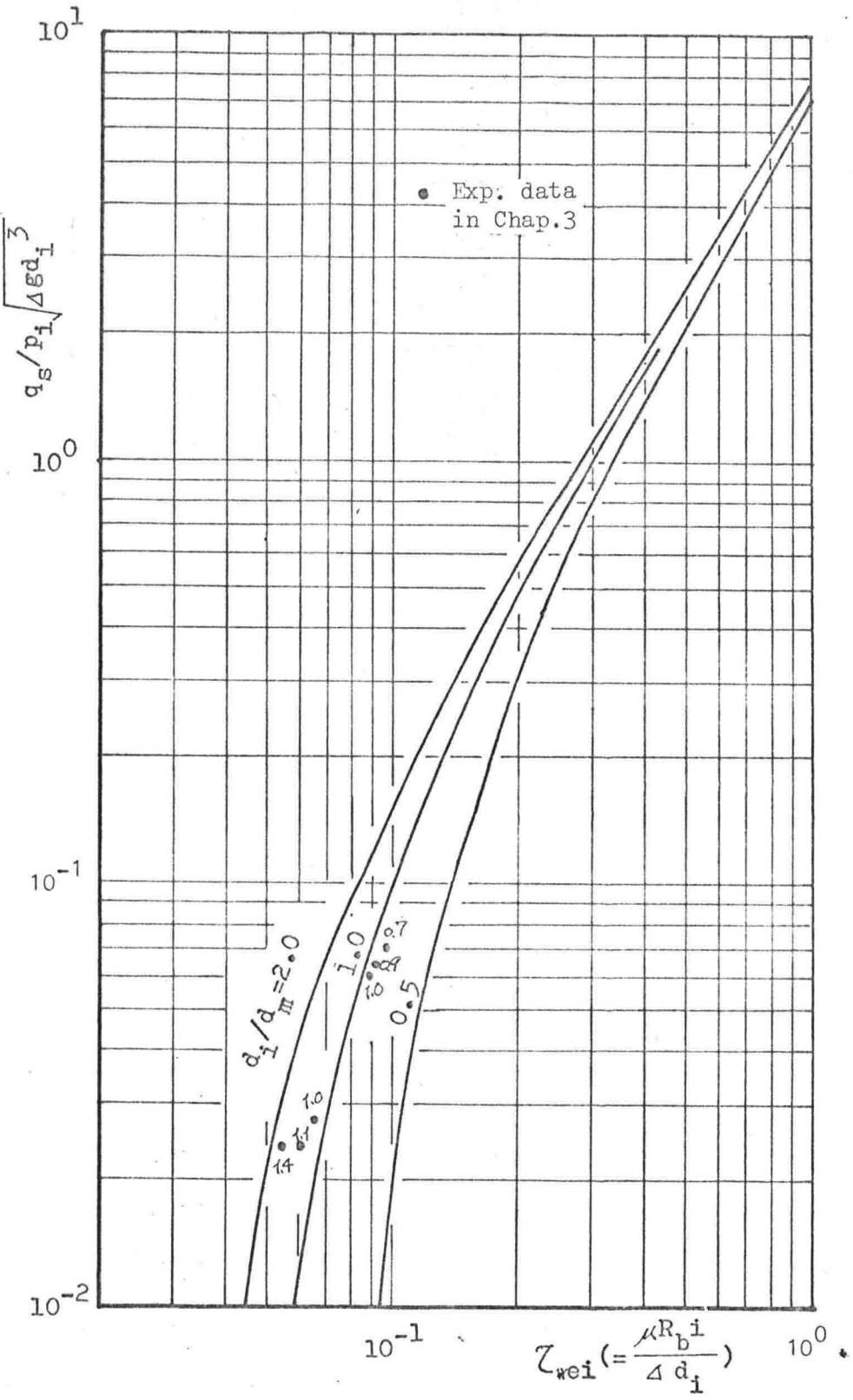


Fig.2-5 Transport function for sand mixture

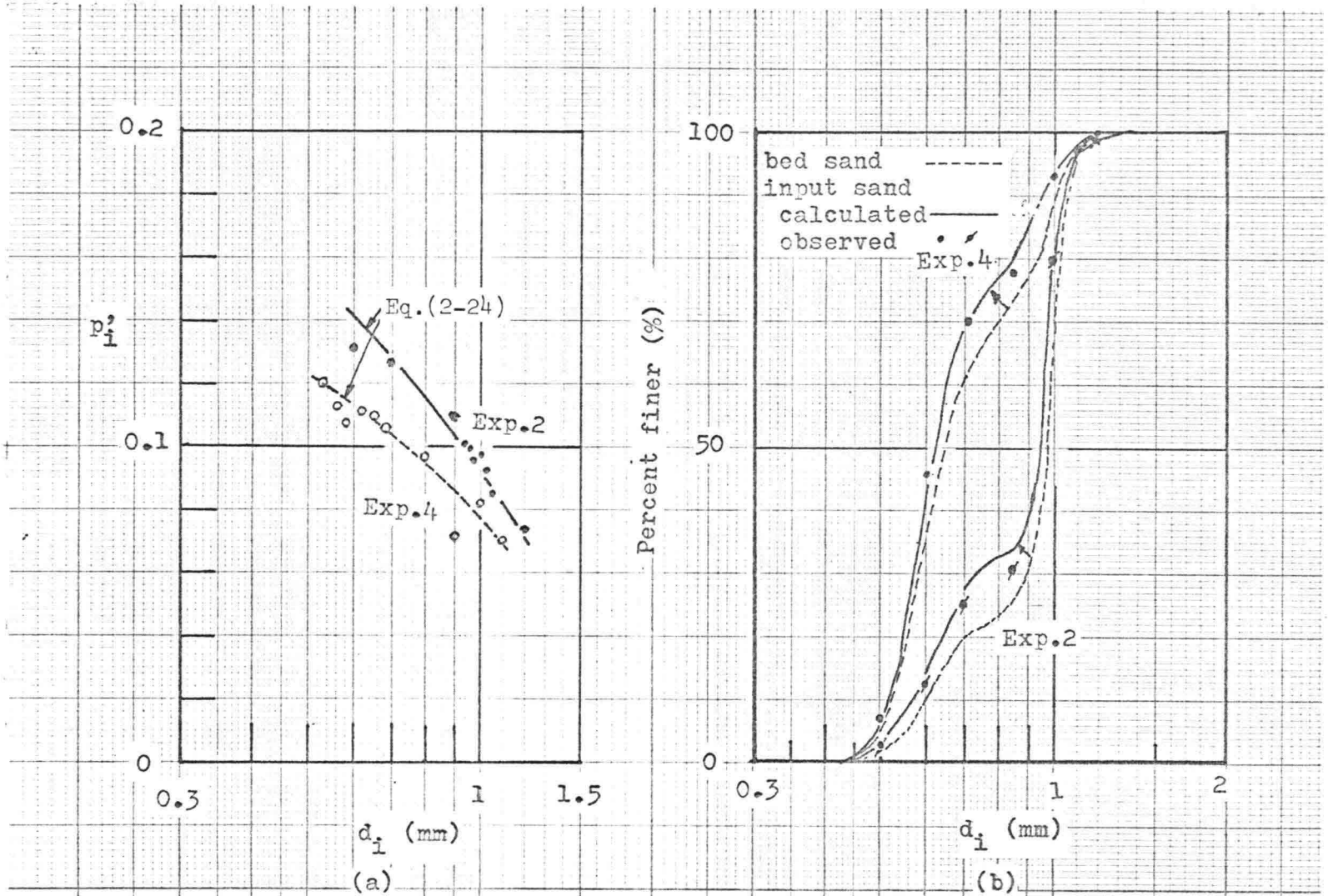


Fig.2-6 Relation between bed sand and input sand composition

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The value of $q_{si}/p_i\sqrt{sgd_i^3}$ becomes smaller as di/dm becomes smaller, and its difference is fairly large in the vicinity of the critical shear stress of τ_{*ei} but as τ_{*ei} becomes larger its difference becomes smaller.

In order to check the eq. (2-15), we can use the data which shows the relation between p_i' which is the frequency of output (or input) sand and p_i which is the frequency of bed sand.

$$p_i' = q_{si}/\sum_i q_{si} = p_i f_i/\sum_i p_i f_i \qquad (2\text{-}24)$$

By eq. (2-24) we can calculate p_i' from known bed frequency p_i using eq. (2-18) for f_i; in Fig. 2-6 (b) solid lines are calculated from dot lines. That is, at first we divided sand mixture into 10 groups (d1, d2.. ... d10) each of which occupies 10% in mixture (p1 = p2 = ... = p10 = 0.1) and next for each group we calculate p_i' using eq. (2-24) (Fig. 2-6(a)). From Fig. 2-6, the experimental data fit calculated values fairly well. Experimental conditions are listed in Table 3-2.

From these considerations we may use eq. (2-15) and assumption eq. (2-6) for the sediment transport equation for sand mixture.

2.6 Moving layer thickness δ

Mathematically C4i indicates the propagation velocity of a small disturbance of composition p_i and physically it seems to indicate the transport velocity of sand which has the diameter di.

So, in case of uniform sand C4 should be equal to C3, because in case of mixture the front of bed height disturbance should be one of the disturbance front of composition p_i and if we consider almost uniform sand di = dm every C4i is almost equal and its value is nearly C3.

From C4 = C3, we get moving layer thickness δ which is assumed a half of the dune height ($=\Delta H/2$). Using eqs. (2-13) and (2-14), we get

$$\frac{\delta}{h}\left(=\frac{\Delta H}{2}\right)=(1-F_r^2)\frac{f_m}{u\cdot(\partial f_m/\partial u)} \qquad (2\text{-}25)$$

where suffix ^{m} means values concerning to the mean value, (for dm).

We can calculate the dune height by eq. (2-25) using proper transport formula of sediment f_m. Of Meyer-Peter & Müllers' equation of sediment transport (eq. 2-15) is used, eq. (2-25) becomes

$$\frac{\Delta H}{h}=\frac{1}{n}\left(1-\frac{\tau_{*c}}{KF_r^2}\right)(1-F_r^2) \qquad (2\text{-}26)$$

in which $K = \frac{\mu}{\Delta}\frac{g}{C_b^2}\frac{R_b}{d}$

and if we can assume that hydraulic radius R_b is equal to water depth h, eq. (2-26) becomes,

$$\frac{\Delta H}{h} = \frac{1}{n}\left(1 - \frac{\tau_{*c}}{K_1}\right)\left(1 - F_r^2\right) \tag{2-27}$$

where, $K_1 = i\frac{\mu}{\Delta}\frac{h}{d}$.

Fig. 2-7 shows the eqs. (2-26) and (2-27). It is seen that $\Delta H/h$ is mainly determined by Froude number F_r and K or K_1, which contains the effects of the slope i and h/d.

P. Mardjikoen[14] also derived the relation between dune height and $(1 - F_r^2)$ by using the work of de Vries and experimental data of Shinohara -Tsubaki and Znamenskaya.

$$\frac{\Delta H}{h} = 0.88\left(\frac{1 - F_r^2}{b}\right) - 0.024 \tag{2-28}$$

where b is defined by $f = a.u^b$ (a = constant) and by using Meyer-Peter & Müllers' equation of sediment transport b is expressed as follows

$$b = 3/(1 - 0.4Y) \tag{2-29}$$

where $Y = g_s/\sqrt{\Delta g d^3}$

But in deriving eq. (2-29) he assumed b as independent on u although the results eq. (2-29) shows that b is dependent on u.

But if b is kept constant, letting a have dimensions, eq. (2-28) coinsides with eq. (2-26) (or(2-27)) in case of neglecting the critical shear stress.

Fig. 2-8 shows the relation between observed dune height and calculated dune height by eq. (2-26). In calculation, n = 3/2 and $\tau_{*c} = 0.047$, which are Meyer-Peter & Müllers' values, are used, μ is calculated by eq. (2-21) and C_b is calculated by $C_b^2 = u^2/gi$. Data are quoted from experiments done by De Vries[15], Shinohara-Tsubaki[16], Prins[4], d'Agostino[17] and from author's experiments described in the next chapter.

If we consider the accuracy of measuring the dune height, eq. (2-26) may be said to describe the dune height fairly well. Also Yalin[18] proposed an experimental equation, using a lot of data:

$$\frac{\Delta H}{h} = \frac{1}{6}\left(1 - \frac{h_{cr}}{h}\right)$$

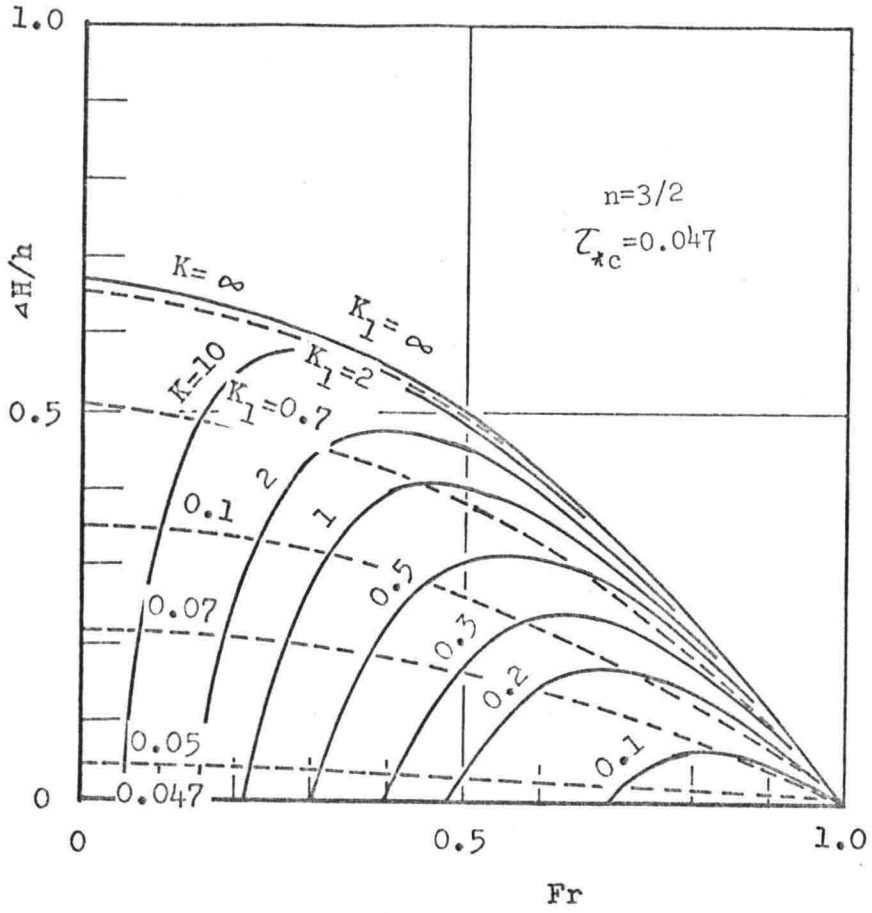


Fig.2-7 Relation between relative dun height ΔH/h and Froude number F_r

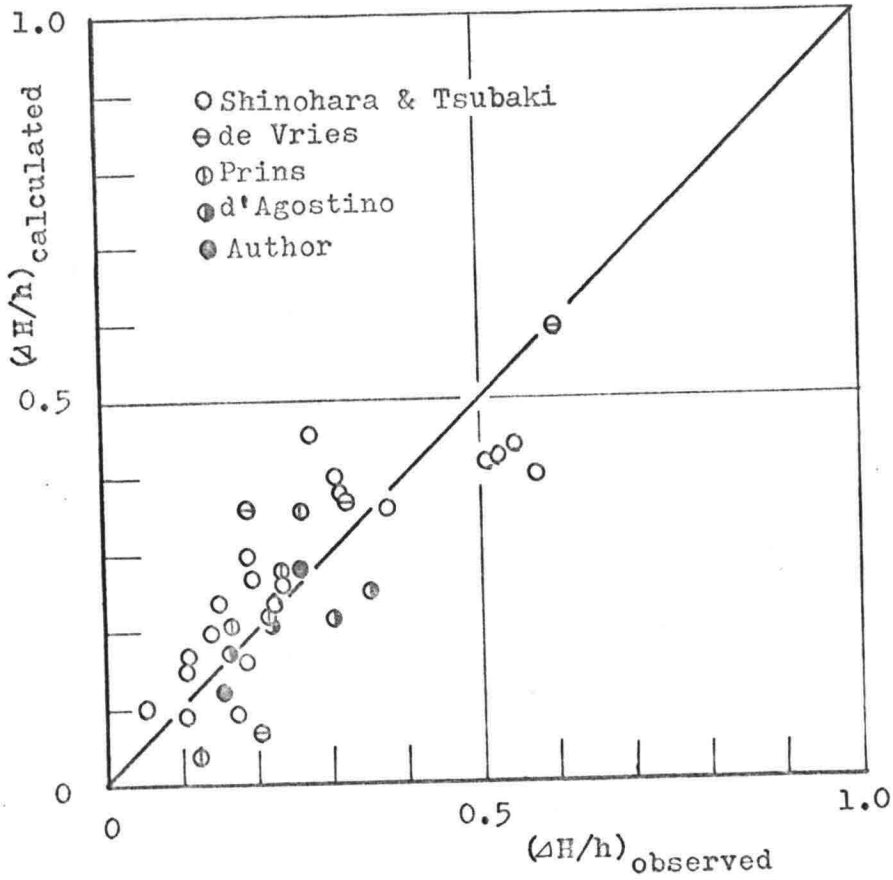


Fig.2-8 Relation between observed and calculated dune height

(h_{cr}: critical flow depth for the beginning of sediment move). which means $\Delta H/h$ can not exceed 1/6 and Nordin et al[19] found that 1/3 is better than 1/6. Fig. 2-8 shows that there are even values of $\Delta H/h$ which exceed 1/3. According to Shinohara-Tsubakis' work[16], $\Delta H/h$ is mainly a function of the non-dimensional effective shear stress τ_{*e} . Eq. (2-26) in which KF_r^2 is equal to τ_{*e} , shows that if the variation of F_r is not very large, $\Delta H/h$ is determined only by τ_{*e} .

3. Experimental Analysis.

3.1. General

Some experiments were done to check the equation of sediment transport (eq.(2-15)) and propagation velocity of small disturbance of the bed form and composition P_i (eq. (2-13) and eq.(2-14)) and the variation of the composition of sand mixture.

3.2. Experimental apparatus and procedure

The experiment has been done in a flume of the laboratory of Fluid Mechanics of the Delft University of Technology. The flume is outlined inf Fig. 3.1. This flume has a sand elevator which is raised automatically at a desired constant velocity which may range from o to about 10 cm/hour (this is corresponding to 0.0425 m^3/hour in bulk volume).
Water discharge is measured by an orifice plate and controlled by a valve; the water surface level is adjusted by the weir at the downstream end of the flume. The sand output which is trapped by a settling basin near the end of the flume is measured by weighing. The water surface profile and the bed profile are measured by a point gage and a bed profile indicator, respectively, mounted on a carriage which runs over the flume with a constant velocity. Bed profiles are registrated by paper recorder and by magnetic recorder.
Before the beginning of the experiment, an equilibrium state is established; for both experiments this took approximately 70 hours (Fig. 3-2).
The sampling of the sand of the bed is done after stopping the flow, keeping the water depth larger than the original flow depth. The surface grain layer on the whole area of upstream side of a dune is scraped off in order to achieve a sample which is representative for the moving layer.
The sample weight is about 100g; after drying, this sand is sieved using calibrated wire sieves.

3.3. Experimental conditions.

(sand)
The composition of the sand used for input P_i' (which is different from the composition of the bed sand P_i) is shown in Fig. 3-3. The porocity λ of this

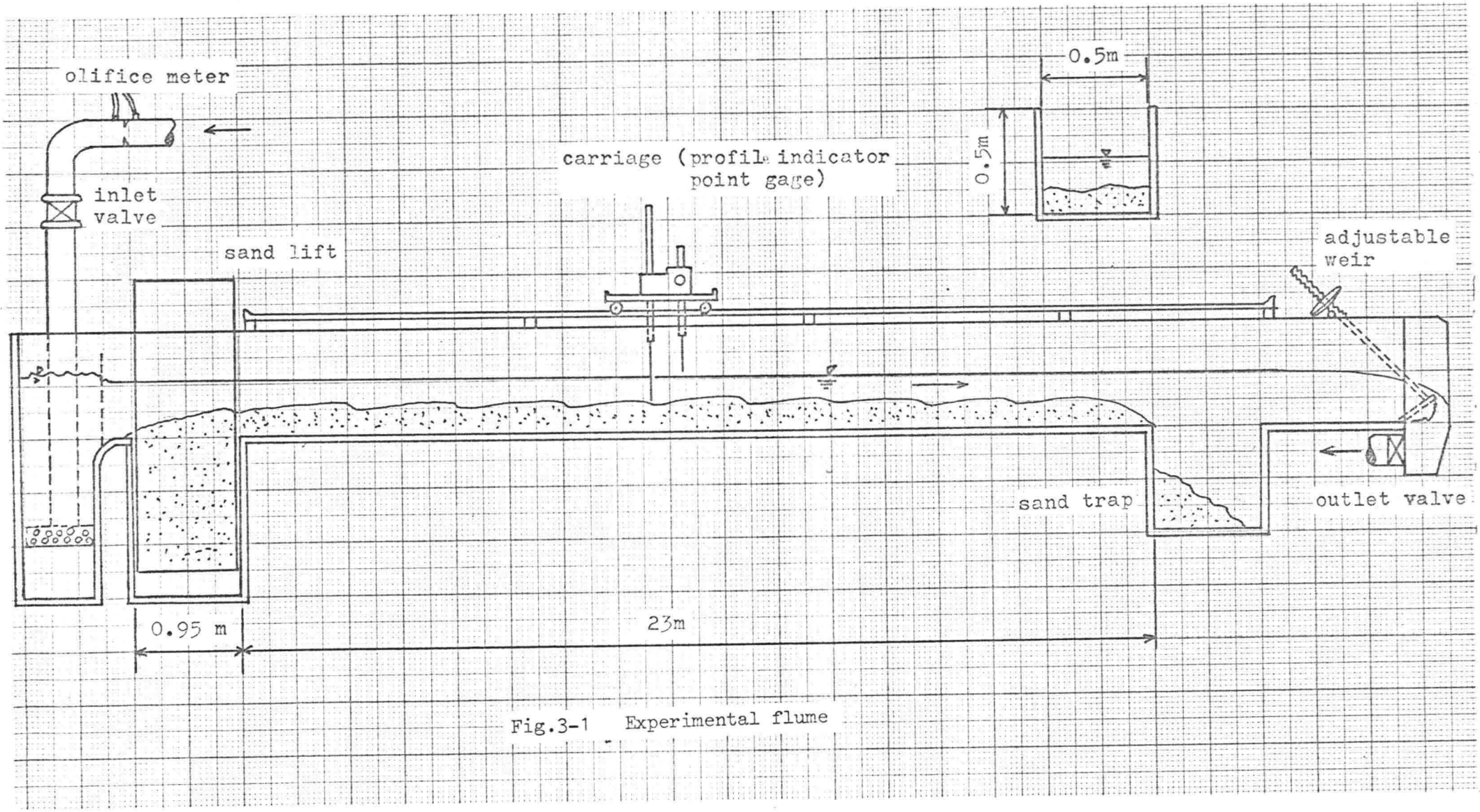


Fig.3-1 Experimental flume

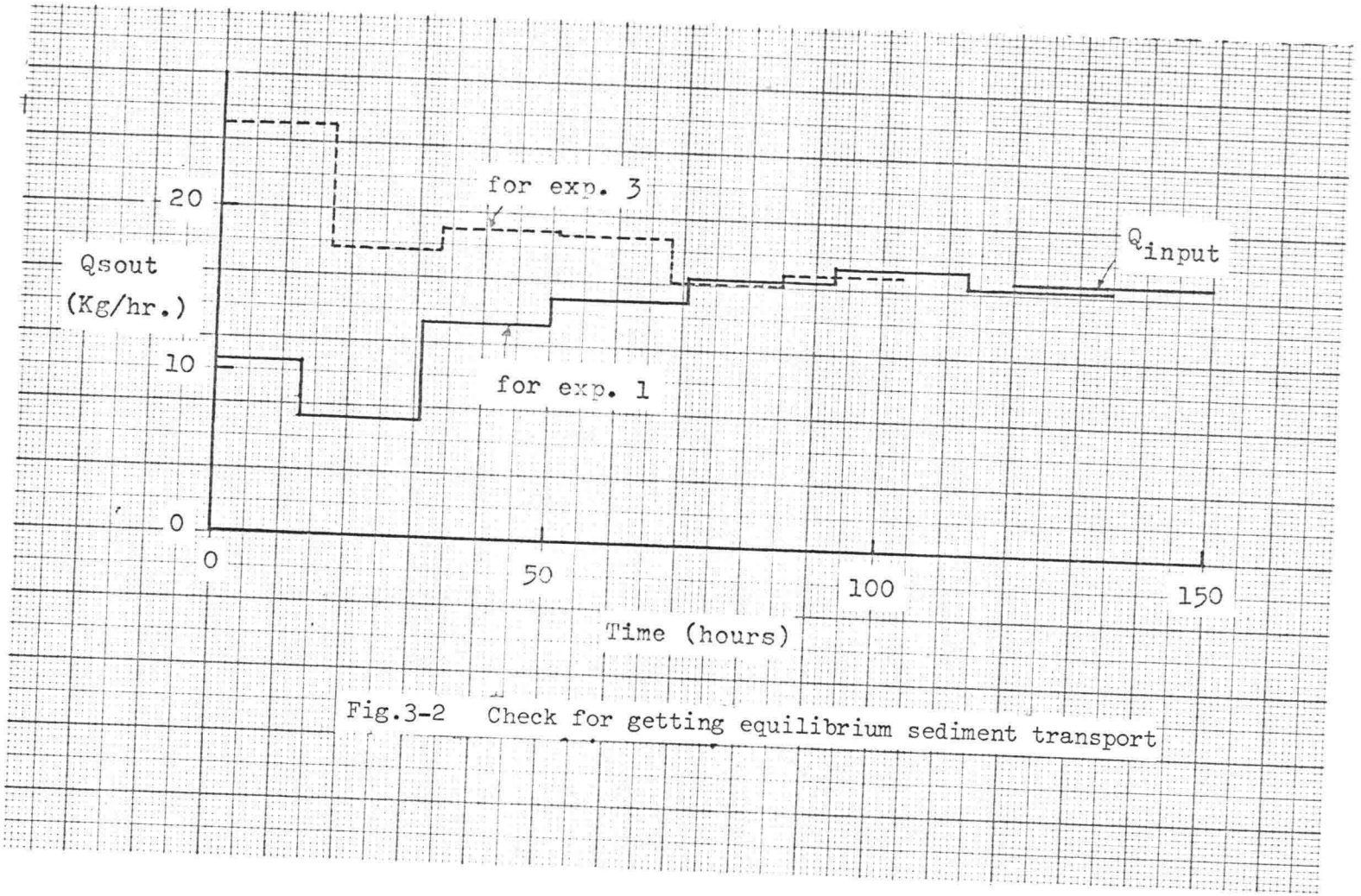


Fig.3-2 Check for getting equilibrium sediment transport

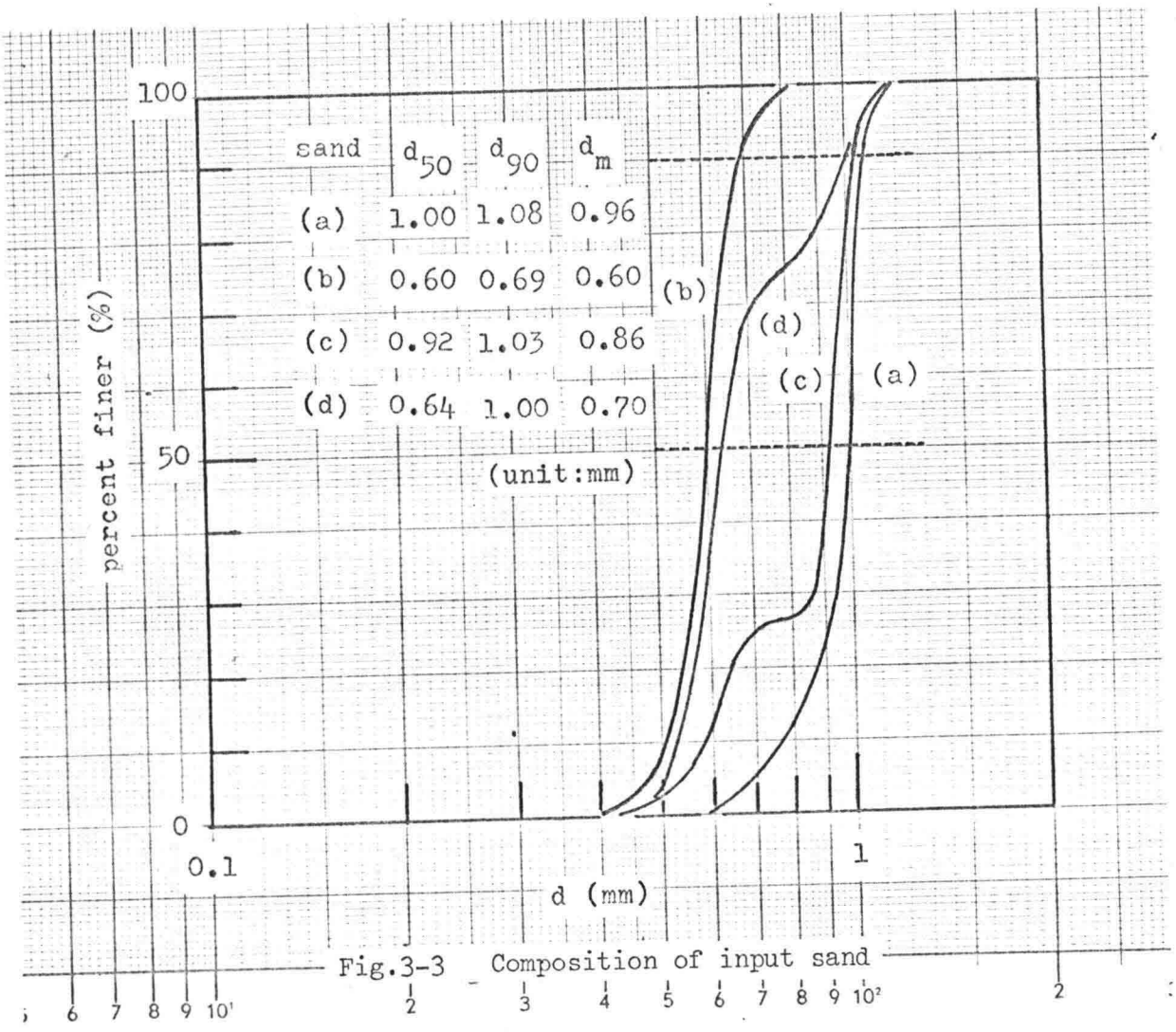


Fig.3-3 Composition of input sand

sand is almost constant, with a value of 0,40; the particle density ρ_s is 2650 kg/m^3. (conditions).

Experimental conditions are listed in Table 3-1.

In Exp. 1.1′.3 and 3′ uniform sand is used. Exp. 1′ and 3′ are done to examine the propagation velocity of a small disturbance of the bed form which is made by a sudden change of the water depth (Δh) at the end of the flume. In Exp. 2,2′,4 and 4′ a sand mixture is used in order to study the velocity of a small disturbance of the composition P_i.

Table 3-1 Experimental conditions

Exp. No	1	1′	2	2′	3	3′	4	4′
Water discharge q (cm^2/s)	900							
bed sand	(a)		(a)	(c)	(b)		(b)	(d)
input sand			(c)	(c)			(d)	(d)
non-steady conditions Δh=1 cm	-	Δh	ΔP_i	Δh	-	Δh	Δp_i	Δh

(summary of experiment)

A summary of the data of experiments 1,2,3 and 4 is listed in Table 3-2.

Table 3-2 Summary of experiment

Exp. no	1	2	3	4
shape is	1,10 x 10^{-3}	1,10 x 10^{-3}	1,06 x 10^{-3}	1,00 x 10^{-3}
water depth h(m)	0,185	0,190	0,185	0,200
flow velocity U(m/s)	0,49	0,47	0,49	0,45
Froude Number ($U/\sqrt{gh}$)	0,36	0,34	0,36	0,32
q_s(m^2/5 sand (a)	0.035	0,026	0	0,009
sand (b)	0	0,009	0,035	0,026
$q_s/P_i \Delta q d_i^3$ sand (a)	0,028	0,026	--	0,024
sand (b)	--	0,069	0,059	0,063
dune height ΔH (m)	0,03	0,04	0,05	0,055
composition of bed P_a	1	0,80	0	0,30
P_b	0	0,20	1	0,70

The shape of the water surface and bed is shown in Fig. 3-4 and 3-5, respectively. As the influence of up- and down-stream ends of the flume can be seen from Fig. 3-5, only the data of the bed between about 3 m to 15 m from upstream

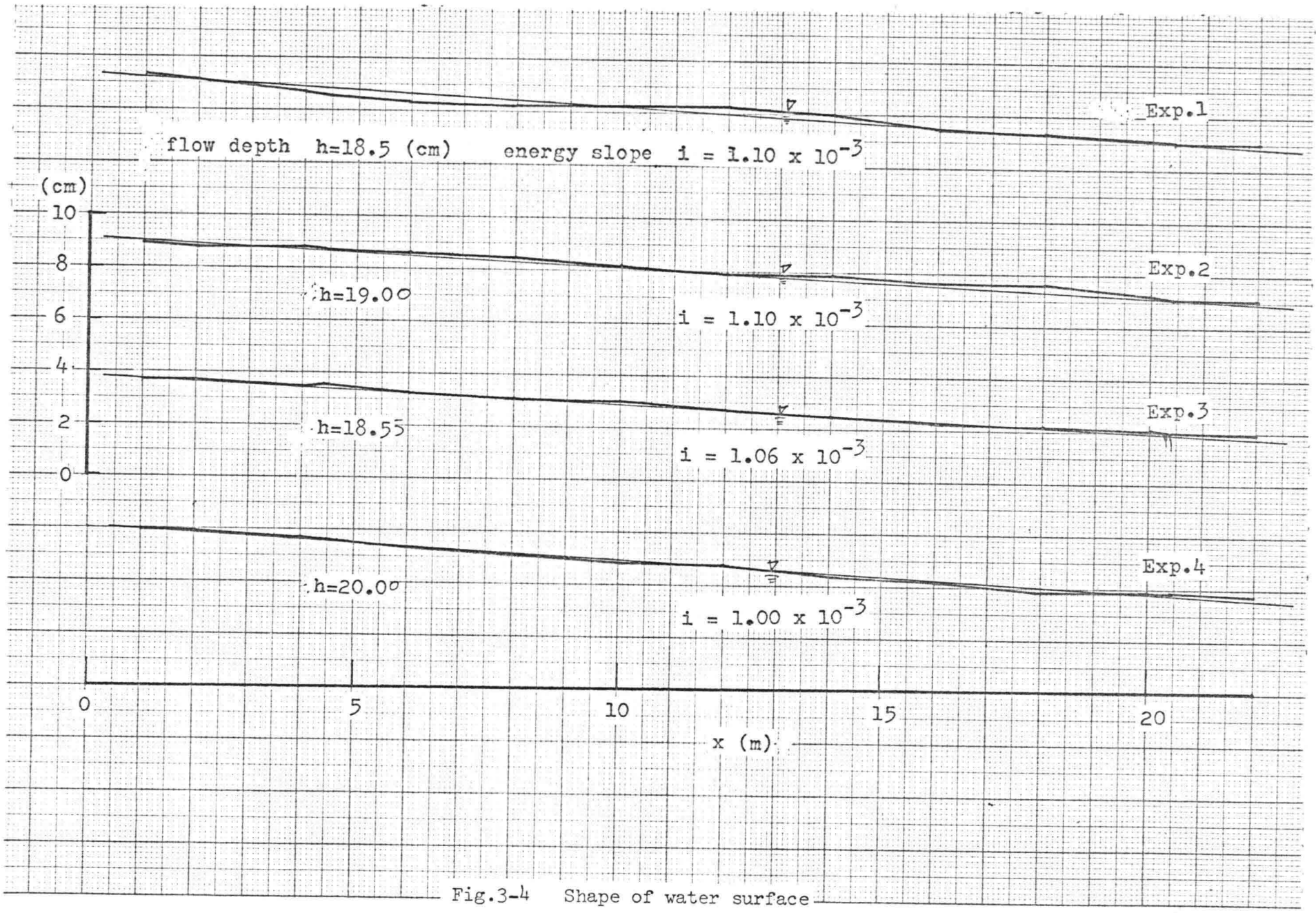


Fig.3-4 Shape of water surface

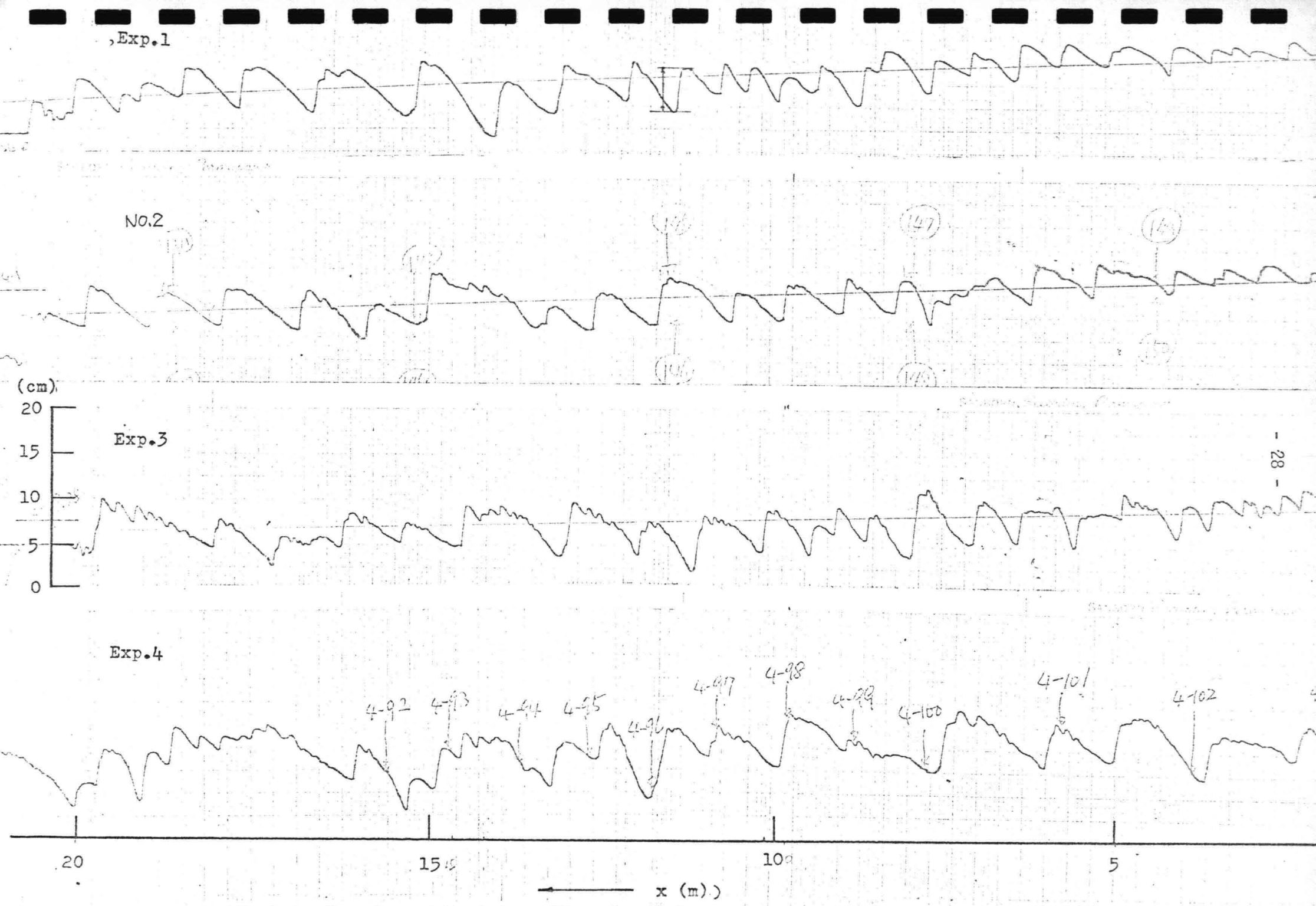


Fig.3-5 Examples of the bed profile

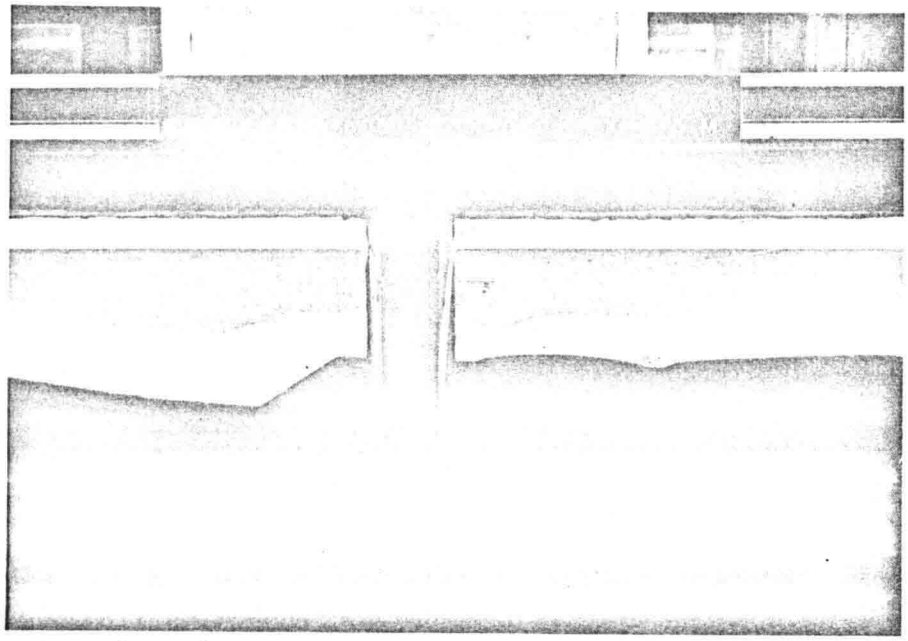


Photo-1 Side view of bed (Downstream direction is lefthand side)

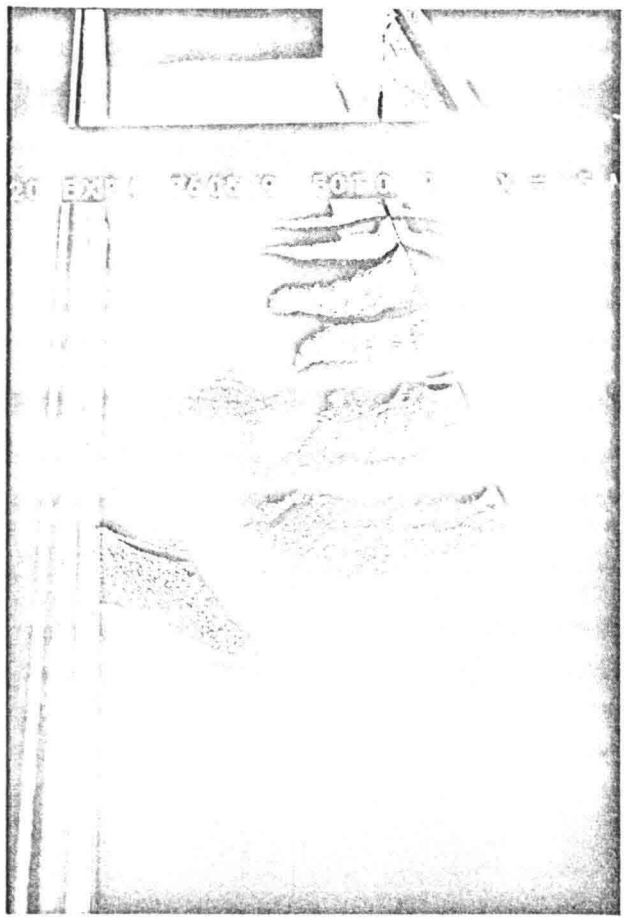


Photo-2 Bed surface, looked in upstream direction

end of the flume are reliable. The bed features can also be seen in Photos 1 and 2.

3.4. Analysis of experimental results.

3.4.1 Equation of sediment transport of sand mixture

The dots in Fig. 2-5 show experimental data. For the sand whose diameter is almost equal to the mean diameter (di/dm=1), we may say that eq. (2-15) coincides with the experimental data, and for both smaller (di/dm<1) and (di/dm >1) sand than the mean diameter sand, eq. (2-15) and data deviate from the line for di/dm=1 with same tendency. In case of di/dm>1 or di/dm<1, the number of experimental data is not enough to say that eq. (2-15) is correct or not. But Fig. 2-6 shows a good agreement of experimental values with calculated values based on eq. (2-15) and eq.(2-24). So it seems that eq. (2-15) can be used in case of transport of a sand mixture.

3.4.2. Propagation velocity of a small disturbance of the bed height.

Fig. 3-6 showns the propagation velocity of a small bed height disturbance. In this figure, straight lines are calculated by eq. (2-13). As to the experimental data, the slope (dx/dt) seems to be almost equal to that of calculated line for the larger time t. But for the samller t, the calculated lines do not coincide with the experimental data. If we take into consideration that this incoincidence is arisen by the influence of the nonuniformity of the bed at the beginning of the flume, we may say that the propagation velocity of the small disturbance of the bed height can be expressed by eq. (2-13) to some extent.

3.4.3.Propagation velocity of a small disturbance of the composition.

Fig. 3-7 (a),(b) show the time variation of the composition P_1(suffix 1 indicates the smaller sand (b) in Fig. 3-3) of Exp. 2 and P_2 (suffix 2 means larger sand (a) in Fig. 3-3) of Exp. 4 along the flume. In Exp. 2, at t=o the bed composed of only one kind of sand (d_1= 1,0mm, P_1=1) and input sand at χ=0 is composed of two kinds of sand (d_1= 1,0mm P_1'= 0,25 and d_2= 0,6 mm P_2'= 0,75) in order to examine how the disturbance of P_i propagates downstream, when we change only the compostion of input sand without changing flow and bed height conditions. In Exp.4, initial bed is composed only of sand d_1 (P_1=1) and the input sand consists of a mixture sand d_1(P_1'= 0,75) and sand d_2 (P_1'= 0,25). In Fig.3-7 the distribution curves of composition P_i can be calculated as described in paragraph 2-4, using the characteristic velocities C_{4i};

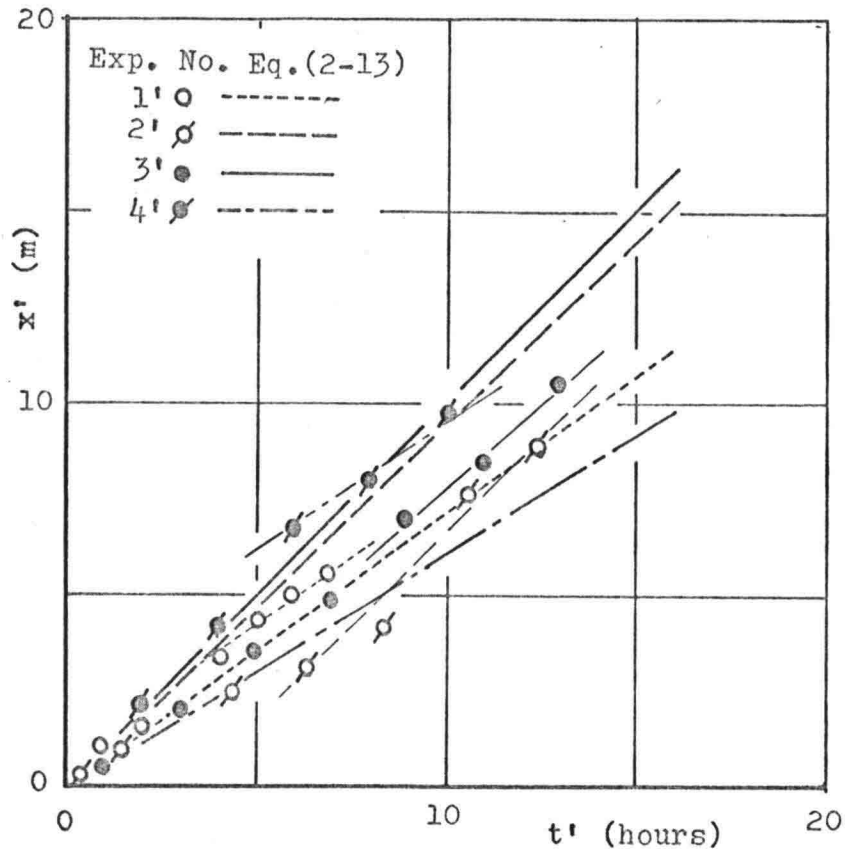


Fig.3-6 Propagation velocity of small disturbance of bed height

$$C_{4i} = \frac{\alpha f_i'}{\delta} + \frac{\alpha p_i}{\delta}\frac{\partial f_i}{\partial p_i} \qquad (2\text{-}14)$$

where

$$f_i = C\sqrt{\Delta g d_i^3}\,(\tau_{*ei} - \tau_{*ci})^n \qquad (2\text{-}15)$$

This C_{4i} should be calculated for each P_i. In these experiments, to calculate $\partial f_i/\partial p_i$ in eq. (2-14), we can put

$$\frac{\partial \tau_{*ei}}{\partial p_i} \cong 0$$

Because, even though Cg in μ in $\tau_{*ei}\,(=\mu R_i'/\Delta d_i)$ is expressed by $C_g \cong 18\log(12R_b/d_{90})$ and d_{90} depends on P_i, we can assume τ_{*ei} is independent of P_i as $d_{90} \cong d_{max}$ and changes only slightly with P_i. The nondimensional critical shear stress $\tau_{*ci}\,(=.0378/(\log 19\, d_i/d_m)^2)$, is varying with P_i because $d_m\,(=\sum p_i d_i)$ is depending on P_i. Therefore

$$\frac{\partial \tau_{*ci}}{\partial p_i} = \frac{d\tau_{*ci}}{d d_m}\frac{\partial d_m}{\partial p_i}$$

So, C_{4i} can be written as follows,

$$C_{4i} = \frac{\alpha}{\delta} C\sqrt{\Delta g d_i^3}\,(\tau_{*ei} - \tau_{*ci})^n + \frac{\alpha}{\delta} n C\sqrt{\Delta g d_i^3}\,(\tau_{*ei} - \tau_{*ci})^{n-1}\left(-\frac{d\tau_{*ei}}{d d_m}\right)\left(\frac{\partial d_m}{\partial p_i}\right) p_i \qquad (3\text{-}1)$$

where, $d\tau_{*ei}/d d_m = 0.156\,(\log 19\, d_i/d_m)^{-3} d_m^{-1}$

as to $\partial d_m/\partial p_i$, for our experiments the mean diameter d_m can be written as

$$d_m = d_1 p_1 + d_2 p_2 = p_1(d_1 - d_2) + d_2$$

and so

$$\frac{\partial d_m}{\partial p_1} = (d_1 - d_2)$$

Now we can calculate characteristic velocities for each P_i by using eq.(3-1). In calculation, we used eq. (2-25) for δ and c= 8, n = 3/2 and experimental conditions (Exp. 2 and 4) listed in Table 3-2.

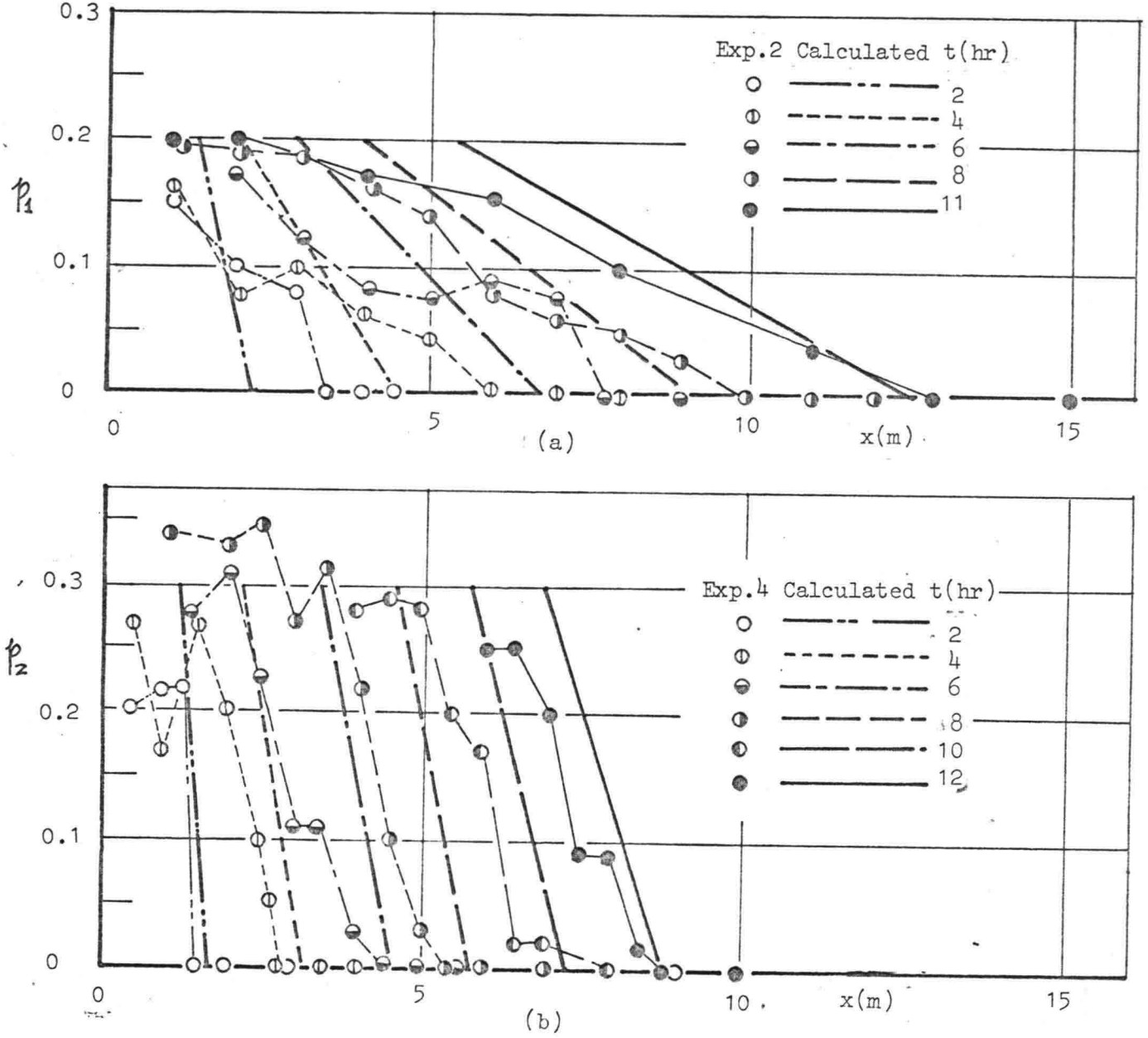


Fig.3-7 Time variation of bed sand composition

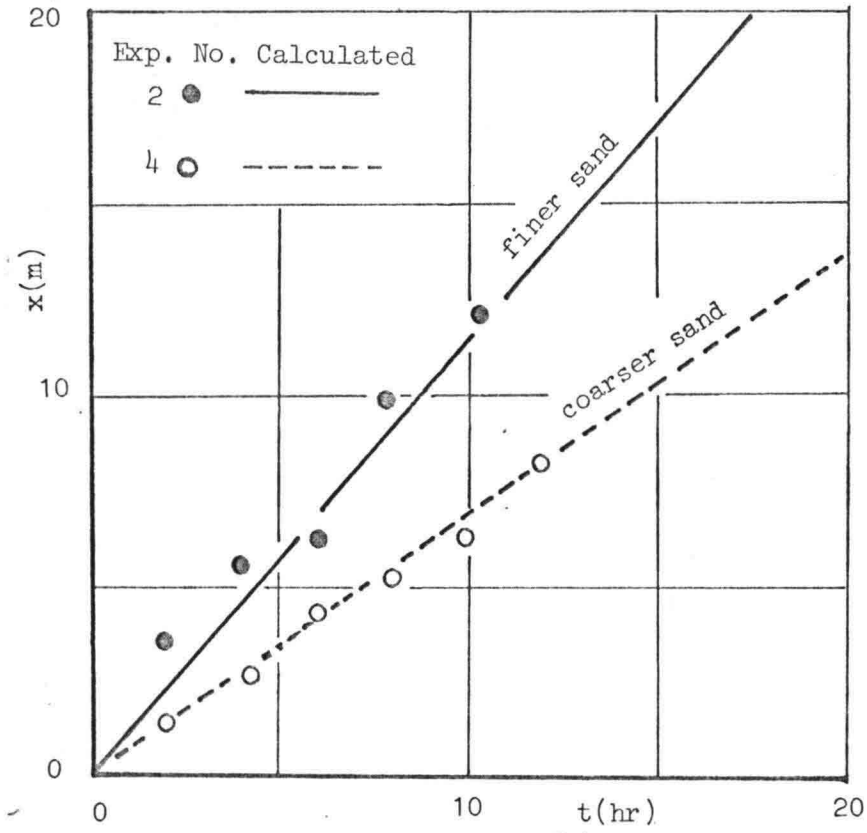


Fig.3-8 Propagation velocity of disturbance of composition (p_i=0)

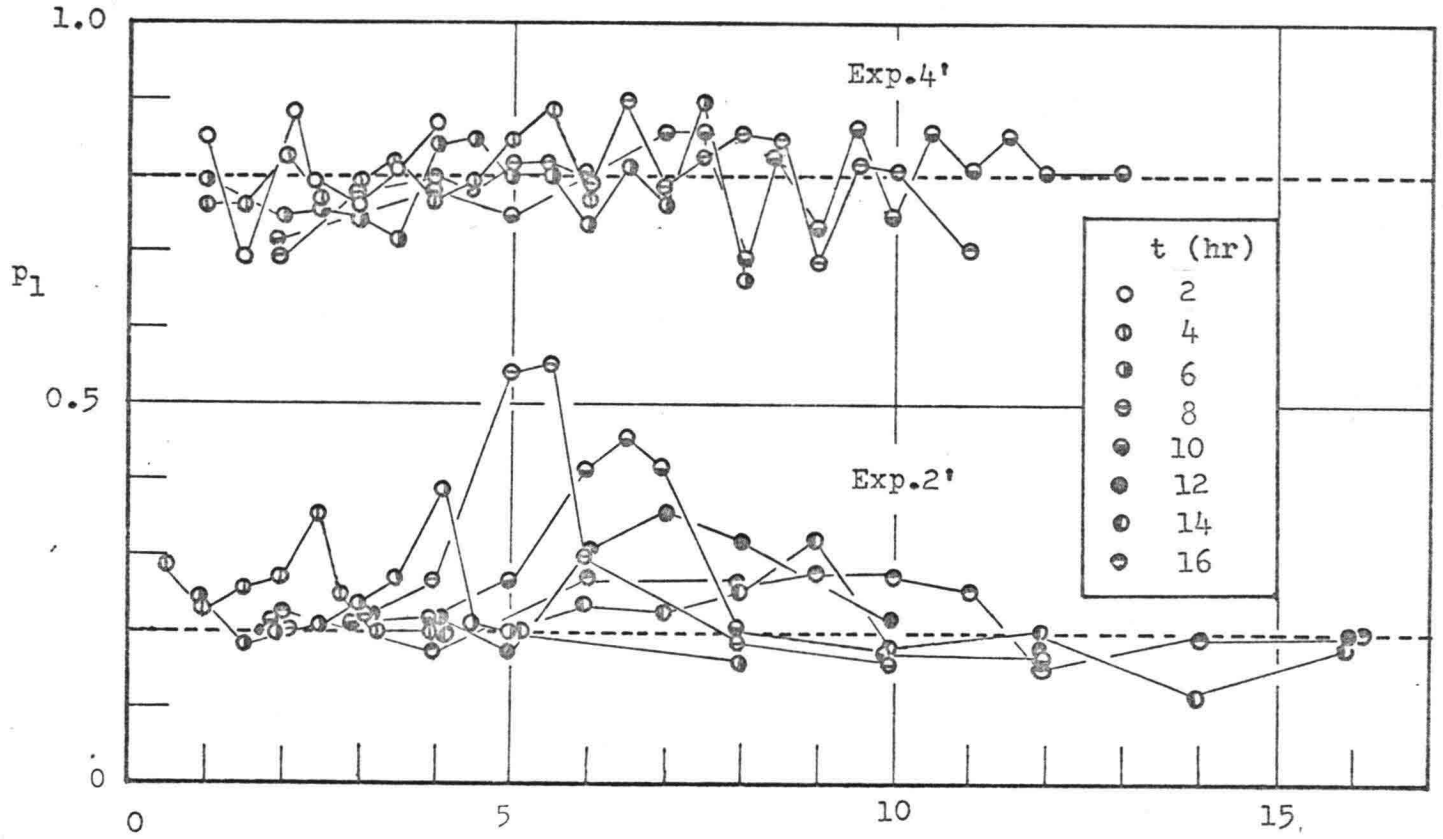


Fig.3-9 Time variation of bed sand composition caused by disturbance of bed form

Because of the difficulty of sampling sand correctly, the experimental data in Fig. 3-7 vary irregularly especially in Exp.2, but we can see the macro tendency of the propagation phenomena of P_i. And we may say that although many simplifying assumptions are made in theoretical analysis, the theoretical curves roughly coincide with the experimental data, taking into consideration the inaccuracy of this kind of experiment. But in case of Exp. 2, P_i distribution of data seems to be milder than the theoretical one. This is because in the theoretical analysis we neglected the bed lebel change. If the difference of P_i between bed and input sand becomes large, $\partial q_{sm}/\partial x$ can not be neglected. In case of Exp. 2, $\partial q_{sm}/\partial x < 0$ near the front , so $\partial z/\partial t > 0$ and some transported sand should be accumulated on the bed, which results in smaller advancing velocities. If we look at the characteristic velocity only at $P_i = 0$ ($\Delta P_i = 0$), we can see a good agreement between calculated and theoretical values (Fig. 3-8).

In Exp. $2'$ and $4'$, small steps of the bed height are introduced in the sand mixture bed, keeping the composition of input sand equal to that of initial bed sand.
Fig. 3-9 shows the time variation of P_i along the channel in case of Exp.$2'$ and $4'$. From the results of Exp $2'$ in this figure, we can see the increase of P_1 near the front of bed height step. This phenomena seems to be caused by the difference of velocities between finer sand and coarser sand.
This experimental fact suggests that even if the input sand and initial bed sand composition are the same some bed form disturbance caused in the past affects the local composition of river bed sand at present.

4. Conclusion

The main results of this paper may be summarized as follows;

1) The continuity equation of sand which has same diameter in sand mixture is pexpressed by eq. (2-1) for degradation of bed and eq. (2-2) for aggradation of bed. In deriving eq.(2-1) or eq.(2-2), we assumed a moving layer with thickness δ.
2) The propagation velocity of a small disturbance of composition P_i in sand mixture may be written by eq. (2-14) as a function of δ and of the transport function of sand mixture .
3) The moving layer thickness δ, which is approximately equal to half of the

dune height, may be expressed by eq. (2-25).

4) The transport function of sand mixture f_i may be expressed by eq. (2-15), in which the critical shear stress is expressed by eq. (2-23) and the ripple factor is modified by eq. (2-21).

Acknowledgement.

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References

1) de Vries, M.: Consideration about non-steady bed-load transport in open channels, I.A.H.R. Leningrad 1965.
2) Hirano, M. : Study on the degradation of river bed accompanied by armouring, Report of JSCE, vol 195.1971 pp 55-65.
3) Michiue, M.: Study on sediment transport and variation of bed height. Doctor Thesis of Kyot Univ. JAPAN 1972
4) Prins, A.: Dominant discharge, Research report S 78-III Delft Hydraulic Laboratory, 1969.
5) Meyer-Peter,E, and R. Müller: Formulas for bed-load transport, Proc. of 2nd Cong. of IAH.R., Stockholm 1947 pp 61-62.
6) Einstein, H.A.,: The bed load function for sediment transportation in open channel flow, U.S. Dept. of Agric., Soil Concerv. Sew. Techn. Bull. No 1026, 1950, pp 1-71
7) Prins, A.: 4)
8) Kramer, H.: Sand mixture and sand movement in fluvial models, Trans. ASCE, Vol 100, 1935, pp 798-878
9) Sakai, T.: About the critical shear stress for bed sand, JSCE, vol 31. no 12, 1946, pp.1-8
10) Yoshito.T.: Study on the citical shear stress of sand mixture, Anual Report of I.D.P.R. Kyoto Univ. no.6, 1963 pp 228-253
11) Gessler,J.: Self-stabilizing tendencies of alluvial channels, Proc. ASCE, WWHD, May 1970, pp.235-248
12) Egiazaroff, I.V.: Calculation of nonuniform sedement concentration, Proc. ASCE, HY4, 1965, pp225-246
13) Ashida, K. and M. Michiue, : Study on sediment transport and bed variation of sand mixture, Anual Report. of I.D.P.R., Kyoto Univ. no 14B A74, pp 259-273
14) Mardjikoen,R.: Some characteristic of sand waves in open channels with movable bed, Delft Hydraulic Laboratory Report R7, 1966 pp 1-14
15) de Vries, M.: Application of luminophores in sand- transport studies, Delft Hydraulic Laboratory Publication no 39.1966
16) Shinohara, K. and T.Tsubaki.: On the characteristics of sandwaves formed upon the beds of the open channels and rivers, Report of Research Institute for Applied Mechanics Kyushu Univ. JAPAN Vol VII no 25, 1959, pp 15-45

17) d'Agostino : Umpublished report (T.H. Delft)

18) Yalin, M.S.: Geometrical properties of sand waves, Proc. ASCE, vol 90 No HY5, 1964.

19) Nordin, C.F. and J.A. Algert : Geometrical properties of sand waves (Yalin.M.S.), A discussion, Proc. ASCE, vol 91 No HY 5, 1965

Main symbols

ΔH : dune height
δ : moving layer thickness ($\cong \Delta H/2$)
p_i : frequency of sand (di) in bed
λ : porocity of sand
q_{si} : transport rate of sand di per unit time per unit width
q_{st} : total transport of sand (= Σq_{si}) "
q_{sm} : transport rate of mean diameter sand "
z : bed level
x : distance along channel
t : time
f_i : transport function (= q_{si}/p_i)
u : flow velocity of x direction
h : water depth
τ_b : bed shear stress
ρ_w : density of water
R_b : hydraulic radius with respect to bed
g : acceleration of gravity
W : = $\tau_b/\rho_w R_b$
α : = $1/(1-\lambda)$
$C_{1,2}$: propagation velocity of water surface disturbance
C_3 : propagation velocity of channel bed disturbance
C_4 : propagation velocity of composition of sand di
Fr : Froude number = $u/\sqrt{gh}$
β : = $u(1-1/Fr^2)$
C : constant in transport function
n : constant in transport function
ρ_s : density of sand
Δ : relative submerged density of sand = $\rho_s/\rho_w - 1$
μ : ripple factor
i : bed slope
d_i : sand diameter
R_w : hydraulic radius with respect to side walls
B : channel width
C_w : Chézy coefficient with respect to side walls
δ' : viscous sublayer thickness

u_* : shear velocity

ν : kinetic viscosity

C_b : Chézy coefficient with respect to bed

C_g : Chézy coefficient with respect to sand grain

τ_e : effective shear stress

τ_{*ei} : nondimensional effective shear stress = $u_*^2/\Delta g d_i = \mu R_b i/\Delta d_i$

τ_{*ci} : nondimentsional critical shear stress = $u_{*ci}^2/\Delta g d_i$

d_m : mean diameter

p_i' : frequency of sand di of out(or in-)put sand

K : = $\mu g R_b/\Delta C_b^2 d$

K_1 : = $i\mu h/\Delta d$