

A wooden architectural model of a neighborhood. It features a central canal (gracht) with several small wooden buildings (houses) along its banks. The streets are labeled with Dutch names like 'Kade', 'gracht', and 'school'. The model is made of light-colored wood and is set against a green background representing grass.

Appendices

The Wednesday Afternoon Adventure Club

A toolkit that supports skilled 3rd and 4th grade mathematicians in developing their spatial ability with the help of design and inquiry-based learning

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Master thesis, TU Delft
MSc. Integrated Product Design

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A. Project brief

IDE Master Graduation Project

Project team, procedural checks and Personal Project Brief

In this document the agreements made between student and supervisory team about the student's IDE Master Graduation Project are set out. This document may also include involvement of an external client, however does not cover any legal matters student and client (might) agree upon. Next to that, this document facilitates the required procedural checks:

- Student defines the team, what the student is going to do/deliver and how that will come about
- Chair of the supervisory team signs, to formally approve the project's setup / Project brief
- SSC E&SA (Shared Service Centre, Education & Student Affairs) report on the student's registration and study progress
- IDE's Board of Examiners confirms the proposed supervisory team on their eligibility, and whether the student is allowed to start the Graduation Project

STUDENT DATA & MASTER PROGRAMME

Complete all fields and indicate which master(s) you are in

Family name	Lugthart	IDE master(s)	IPD ☒	Dfi ☐	SPD ☐
Initials	R.L.L.	2 nd non-IDE master			
Given name	Roosje	Individual programme (date of approval)			
Student number	4677293	Medisign	☐		
		HPM	☐		

SUPERVISORY TEAM

Fill in the required information of supervisory team members. If applicable, company mentor is added as 2nd mentor

Chair	Mathieu Gielen	dept./section	HCD - CDSC	! Ensure a heterogeneous team. In case you wish to include team members from the same section, explain why. ! Chair should request the IDE Board of Examiners for approval when a non-IDE mentor is proposed. Include CV and motivation letter. ! 2nd mentor only applies when a client is involved.
mentor	Mark Sypesteijn	dept./section	HCD - HICD	
2 nd mentor	Leonie Sonneveld			
client:	Wetenschapsknooppunt			
city:	Delft	country:	The Netherlands	
optional comments				

APPROVAL OF CHAIR on PROJECT PROPOSAL / PROJECT BRIEF -> to be filled in by the Chair of the supervisory team

Sign for approval (Chair)

Name Date Signature

CHECK ON STUDY PROGRESS

To be filled in by SSC E&SA (Shared Service Centre, Education & Student Affairs), after approval of the project brief by the chair. The study progress will be checked for a 2nd time just before the green light meeting.

Master electives no. of EC accumulated in total		EC
Of which, taking conditional requirements into account, can be part of the exam programme		EC

☐	YES	all 1 st year master courses passed
☐	NO	missing 1 st year courses

Comments:

Sign for approval (SSC E&SA)

Name Date Signature

APPROVAL OF BOARD OF EXAMINERS IDE on SUPERVISORY TEAM -> to be checked and filled in by IDE's Board of Examiners

Does the composition of the Supervisory Team comply with regulations?

YES	☐	Supervisory Team approved
NO	☐	Supervisory Team not approved

Comments:

Based on study progress, students is ...

☐	ALLOWED to start the graduation project
☐	NOT allowed to start the graduation project

Comments:

Sign for approval (BoEx)

Name Date Signature



DESIGN
FOR our
future

TU Delft

Personal Project Brief – IDE Master Graduation Project

Name student

Roosje Lugthart

Student number

4,677,293

PROJECT TITLE, INTRODUCTION, PROBLEM DEFINITION and ASSIGNMENT
Complete all fields, keep information clear, specific and concise

Project title

Creating an enjoyable and creative learning experience around spatial thinking for primary school children.

Please state the title of your graduation project (above). Keep the title compact and simple. Do not use abbreviations. The remainder of this document allows you to define and clarify your graduation project.

Introduction

Describe the context of your project here; What is the domain in which your project takes place? Who are the main stakeholders and what interests are at stake? Describe the opportunities (and limitations) in this domain to better serve the stakeholder interests. (max 250 words)

I will graduate for ‘het Wetenschapsknooppunt’, a company, part of TU Delft, to create more creative and explorative learning experiences for primary schools in the Netherlands.

TU Delft takes interest in improving primary school education (Delta, 2023) with the intention that tackling the understanding of mathematics early on will result in a higher number of children wanting to pursue a technical career and therefore, possibly choosing to study at TU Delft. In collaboration with Wetenschapsknooppunt, the university wishes to improve the primary school education around mathematics’ subject of spatial thinking because this subject seems to play an important role in overall technical understanding.

93% of primary school children in The Netherlands reached the fundamental level of understanding in mathematics in 2022-2023, only 42% reached the target level. These numbers are significantly lower than those for Dutch language and reading (DUO, 2023). With fewer children reaching a significant level in mathematics, logically the amount of children who pursue a technical career path decreases. Furthermore, some children perform badly because of ‘maths anxiety’, without even lacking skills in the subject (TEDEd, 2017).

Developing maths anxiety or other negative feelings towards mathematics can occur early at primary school. Because the content around spatial thinking increases difficulty during the 3rd and 4th grade, the project will focus on improving education for children in these grades.

Towards the end of this project, if its final form is more clear and fits their values: the Science Centre TU Delft and ‘de Makotheek’ could also be interested partners.

→ space available for images / figures on next page

introduction (continued): space for images

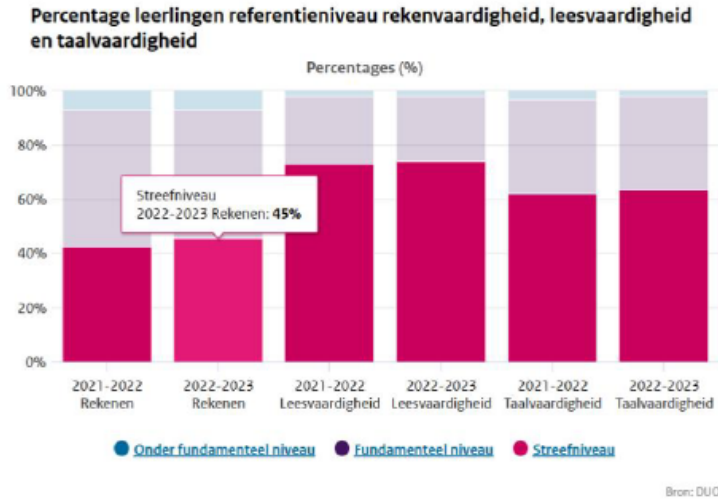


image / figure 1 Percentage of children who reach the target level of understanding in maths (DUO)

Referenties

- Bongers, S. (2023, December 20). Eerste generatiestudenten: hoe kan de TU hen bereiken? - Delta. Delta.
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<https://www.youtube.com/watch?v=7snnRaC4i15>

image / figure 2 Sources



TU Delft

Personal Project Brief – IDE Master Graduation Project

Problem Definition

What problem do you want to solve in the context described in the introduction, and within the available time frame of 100 working days? (= Master Graduation Project of 30 EC). What opportunities do you see to create added value for the described stakeholders? Substantiate your choice. (max 200 words)

Currently, classes around spatial thinking are mainly theoretical and not interactive. For example, it is difficult to understand the dimensions of a (complex) geometric shape without having the shape in front of you and only seeing 2D sketches or words describing it.

TED-Ed explains that interactive and creative tools are needed to improve childrens’ experience in mathematics classes and can encourage creative thinking and developping spatial skills. However, teachers are hesitant to implement creative thinking in their classes themselves due to the lack of supporting tools (Schoevers, 2020). Some teachers are even hesitant to help children who ask questions that differ slightly from the existing methods because they fear not knowing the answer themselves (Keijzer, 2024).

Following this abstract and theoretical way of teaching, there are many children who do not enjoy maths, find it too abstract and difficult, or can even experience ‘maths anxiety’ (it is expected that around 20% of the world population suffers from this (TEDEd, 2017)). Can designing an additional tool to the spatial thinking classes decrease the negative stigma around mathematics and help children to experience the subject in a more creative and fun way? And would this help teachers to implement creative learning into the current mathematics classes?

Assignment

This is the most important part of the project brief because it will give a clear direction of what you are heading for. Formulate an assignment to yourself regarding what you expect to deliver as result at the end of your project. (1 sentence) As you graduate as an industrial design engineer, your assignment will start with a verb (Design/Investigate/Validate/Create), and you may use the green text format:

Design a tool that makes use of design and inquiry based learning in mathematics classes to improve the development of children’s spatial skills for primary school children in grades 3 and 4 (groep 5 en 6) in the Netherlands.

Then explain your project approach to carrying out your graduation project and what research and design methods you plan to use to generate your design solution (max 150 words)

For this assignment I will start by conducting research on mathematics education in primary schools to understand what the exact subjects are in the 3rd and 4th grade and what part of these are most difficult to understand for children. Furthermore, I wish to learn about maths anxiety and understand how creative learning can help overcome this. This research will include desktop research, observing the current mathematics classes in primary schools, interviewing children and teachers about their experience and meeting up with experts on the fields of (mathematics) education and spatial thinking.

After the research stage, I would like to continue visiting primary schools to test and get feedback on ideas and prototypes, both from children and teachers. I would therefore like to start prototyping and visualising ideas as soon as possible to also test children’s reactions on visual appearance and understandability.

For me, an ideal outcome of this project would be to design and embody a product/tool that is a valuable addition to the current mathematics program, is both appealing and understandable by children, improves their experience around mathematics and is easy for teachers to implement in their classes.

Project planning and key moments

To make visible how you plan to spend your time, you must make a planning for the full project. You are advised to use a Gantt chart format to show the different phases of your project, deliverables you have in mind, meetings and in-between deadlines. Keep in mind that all activities should fit within the given run time of 100 working days. Your planning should include a kick-off meeting, mid-term evaluation meeting, green light meeting and graduation ceremony. Please indicate periods of part-time activities and/or periods of not spending time on your graduation project, if any (for instance because of holidays or parallel course activities).

Make sure to attach the full plan to this project brief. The four key moment dates must be filled in below

Kick off meeting

9 dec 2024

Mid-term evaluation

7 mrt 2024

Green light meeting

14 mei 2024

Graduation ceremony

13 juni 2024

In exceptional cases (part off) the Graduation Project may need to be scheduled part-time. Indicate here if such applies to your project

Part of project scheduled part-time	✓
For how many project weeks	25
Number of project days per week	4,0

Comments:
In the last six weeks of the project I will study fulltime (5 days) instead of parttime (4 days).

Motivation and personal ambitions

Explain why you wish to start this project, what competencies you want to prove or develop (e.g. competencies acquired in your MSc programme, electives, extra-curricular activities or other).

Optionally, describe whether you have some personal learning ambitions which you explicitly want to address in this project, on top of the learning objectives of the Graduation Project itself. You might think of e.g. acquiring in depth knowledge on a specific subject, broadening your competencies or experimenting with a specific tool or methodology. Personal learning ambitions are limited to a maximum number of five. (200 words max)

During my time at IDE, I experienced that I enjoy designing for children and that I find it important to always design in a sustainable way by adding value and to either improve existing products or to only create new ones that can make an impact.

Besides my interest in designing for children, I’m also passionate about improving their maths experience in school. Personally, I enjoyed the subject both in primary school and highschool, but I’m aware that there are many children who do not. Whilst I was a tutor in maths I also experienced this negative feeling some students can have about it. I would like to get rid of the negative stigma around maths classes and let other children, who might have a harder time understanding the subject at first, have a positive experience as well.

During this product I would like to:

- Learn to design for a group of children and how to implement their different needs and wishes
- Design a physical product that adds value to an already existing system
- Learn how to visualise this product in such a way that it is appealing and understandable by children but also valuable enough for teachers/schools to implement in their mathematics classes
- Learn about the difficulties that different children have around learning maths

B. Observations at school

Observation at primary school in Delft
(following the EDI-method)

4th grade, (17-12-2024)

observeren groep 6 de Horizon 17-12-24

voor de les:

- allemaal high-five met juf. als ze binnenkomen.
- op het bord: "je mag lezen (vandaag kleuren)" en "ik voel me goed, ik voel me de, ik voel me niet fijn"
- ze kunnen een van de drie aanklikken.
- kleuren een kleurplaat voor kerstdiner van morgen.
- leerdoelen vandaag: breuken & klokkijken
- les duurt 1 uur: beide halfuur (ze doen edith methode).
- methode pluspunt: klok 3: les 10.
- juf benoemt de leerdoelen + dat ze gaat uitleggen en helpen; vrijdag een toets.
- kinderen hebben een 'wisbord' en stift, op het bord staan allemaal breuken, kinderen schrijven en op. Juf neemt ze één voor één op.
- Als je bord leeg is, doe je hem omhoog. (stil).
- bijna iedereen lukt het, gaat best snel
- het is bijna stil, sommige kinderen zeggen wel 'yes!' 'bijna!' 'ahh!' → leuk
- iemand 'nee, dat mag niet...', als kind iets wegstreept wat er niet was. Juf 'ik zag het ook hoor'.

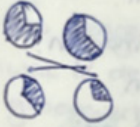
doel: "ik kan splitsen en delen tot hele breuken"

- juf "doel van vandaag is: "

→ klas herhaalt het doel hardop

→ eerst "ik doe het voor" op het bord. (5min +/-)

→ Juf legt uit wat een hele breuk is en doet een opdracht voor.



→ Daarna "we doen het samen" (5min +/-)

'welk deel is gekleurd'

- juf vraagt 'hoe pak je deze som aan?'

• vraag kind: ik dacht dat het eerste streepje er ook al één was

- later: kinderen noemen in één keer het antwoord.

• kind "Dit is echt makkelijk"

→ Daarna "jullie doen het samen" (10-15min)

- klassendienst deelt werkboeken uit.

- juf "Als je eerder klaar bent mag je de conditietraining doen".

(ze moeten in paren van twee sommen maken, één voor één noemen ze een breuk op.)

*tussendoor ook nog: 'wat is de noemer en wat de teller?'

Na de som samen. maken ze het werkblad alleen af. Eerst komen de antwoorden van de gezamenlijke som op het bord en kijken ze die samen na.

- timer van 5min op het bord en een rood stoplicht (→ niet praten). Lukt goed; het is bijna helemaal stil.

- als timer afgaat is iedereen klaar. Juf benadrukt hoe goed iedereen het heeft gedaan.

→ "jullie doen het alleen".

2^e leerdoel: klokkijken.

opdracht: "van vroeg naar laat"

leerdoel "ik kan uitrekenen hoeveel minuten en uren het later is".

kleine uitleg met voorbeeld van wanneer meisje op school het is, kan ik mijn wekker op tijd zetten.

- waarom leren we dat? "Als ik kan kijken hoe laat moet zijn."

→ 3uur 15min + 30min

"ik doe het voor"

10:55	13:55	14:00	14:30
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"Hoeveel later? Laat zien hoe je rekent".

→ 3uur en 35min later.

- Juf: "kan iemand mij hierbij helpen?"

→ "ben ik nu klaar?" → neemt ze goed mee in proces.

kinderen: "nee!" Je moet nog het antwoord schrijven.

"Wij doen het samen"

13:40	:	
-------	---	--

2 uur en 15 minuten.

- ze moeten nu zelf de lijn tekenen en het op hun wisbordjes laten zien.

→ Juf tussendoor "ik zie al heel veel goede getallenlijnen"

→ stimuleert steeds hoe goed het gaat.

"Jullie doen het samen" (3min timer).

- niet alle kinderen werken meteen samen. Sommigen werken individueel.

- Som kijkt kind 1 naar werk van kind 2.

- juf "Wie hadden als tweetal alles goed?"

- bijna iedereen een tweetal niet.

→ Juf "Wat vonden jullie moeilijk? Of waren het slordigheidsfouten?" + vonden jullie het moeilijk?

→ moeilijk: 'over het hele uur tellen' → getal verandert! opnieuw tellen.

juf: "Ja dat is ook moeilijk. Want hoeveel minuten heeft een uur?" → weer een vraag om te checken of het wordt begrepen.

→ zelfde tweetal: "60!"

→ niet: wat ging er mis? niet goed.

Methode werkt in blokken:

- les 1-4: één leerdoel per les. (en 6-9)
- les 5 en 10: twee leerdoelen
- de toets gaat over leerdoelen van het vorige blok(?)

↳ die herhalen ze wel in conditietraining, zodat ze er wel steeds nog even mee bezig zijn.

Alleen soms lukt dat niet helemaal en gaat het minder goed omdat ze het toch een beetje zijn vergeten.

"Julie doen het zelf" →

- eerst 5 min stoplicht op rood. → sommen uit de les
- 10 min stoplicht op oranje. → sommen uit werkblad, les 10 afmaken en verder met conditietraining.

(er is niet elke les tijd voor de conditietraining, soms is het moeilijk als er ook al meer doelen worden uitgelegd.)

- werkboek en conditietraining zit in één boek.

- toetsboek: meerdere toetsen in één boek.

- vroeger: laptop.

↳ daar kon je de berekening niet nakijken.

en je kon minder goed meekijken hoe ver iedereen was.

Observation at primary school in Delft (Wardorf school); 3rd and 4th grade (10-02-2025)

Meekijken [redacted] groep 5 en 6.

10-2-'25

Ruimtelijke figuren groep 5.

- Ze hebben al 20 figuren geleerd
- nu 3D.
- Ze moeten in groepjes voorwerpen in de klas zoeken:
 - ↳ cilinders, ballen, kubussen en balken.

"Wat zijn jullie ermee eens dat alles een balk is?"
 ↳ "nee, die etui deukt een beetje in dus dat is geen mooie balk". * creatieve rekenaar?

- Juf "als ik deze balk in de breedte doorsnijdt, wat zie je dan?"
- en dan "en in de lengte?" } teken op je wisbord.

- (kubus) "Wat valt je op aan deze vorm?"
- "het is drie dimensionaal"

"als je van de voorkant kijkt is het eigenlijk een vierkant"

- * Sommige kinderen zijn tussendoor 3D kubussen aan het tekenen op hun wisbord. * persona in klasoverzicht

"Wat zie je als je de cilinder snijdt?"

"en de bal?" → "Plat want hij is lekdus geen mooie cirkel"

"Wat zijn andere voorwerpen die men met een bal-vorm?"

- "Als je middenin de bal zou zijn, zijwaarts is de afstand dan steeds hetzelfde naar de rand of anders?"
- "Hetzelfde".

* Wissel 5 → 6.

- groep 5 pakt nu laptops en groep 6 wisbordjes.
- ze hebben geen schriften, wel koptelefoons voor concentr.
- * blok 6, week 2, les 8 → meten & meetkunde GROEP 6.

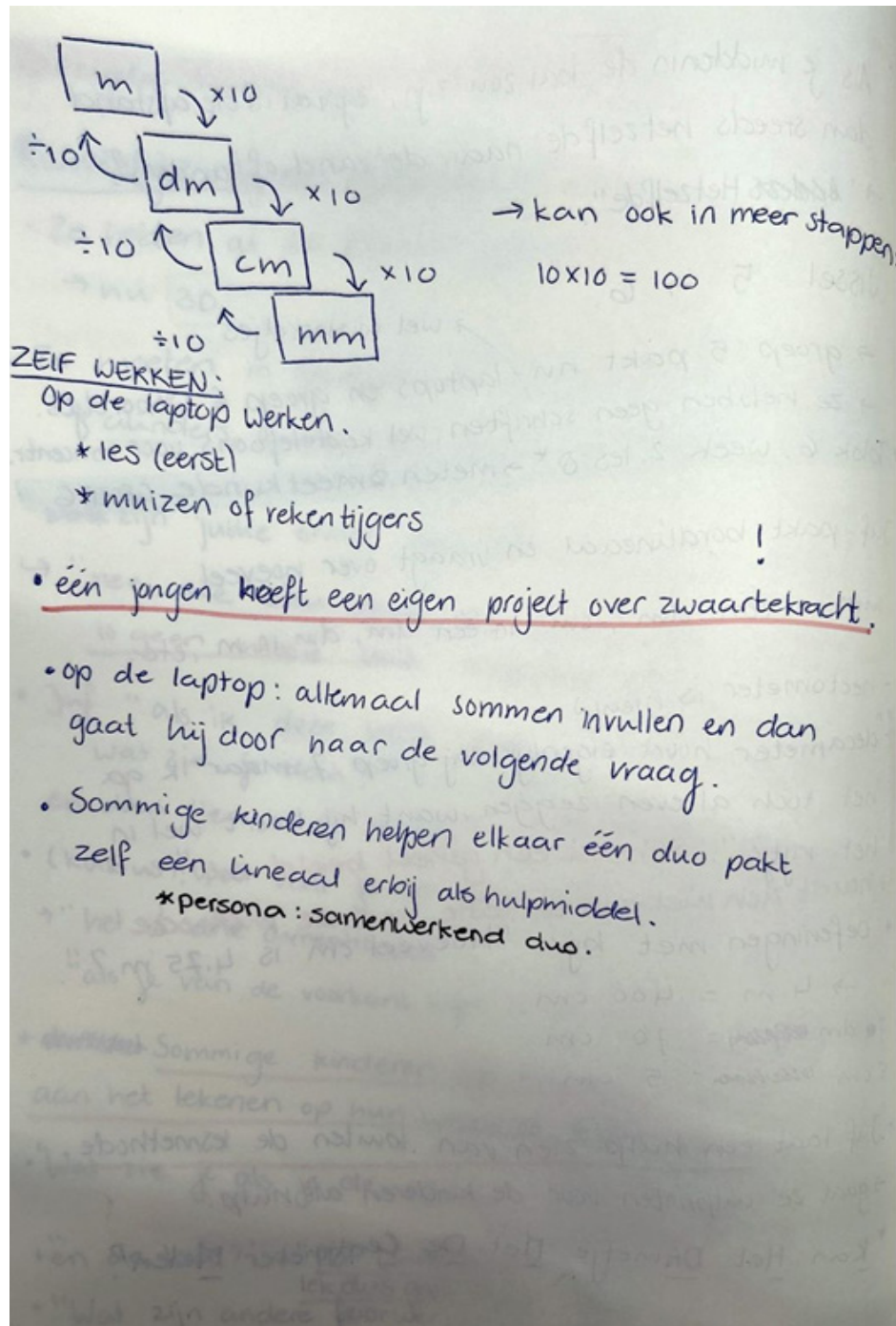
Juf pakt bordlineaal en vraagt over hoeveel mm in één dm, cm in één dm, dm in m, etc.

- hectometer is nieuw
- "decameter hoort eigenlijk bij groep 7, maar ik ga het toch al even zeggen, want hij hoort wel in het rijtje". Schetst een geheel beeld en:
- ↳ houdt zich niet aan limitatie van de methode

- oefeningen met bijv. "Hoeveel cm is 4,75 m?"
- 4 m = 400 cm
- 75 cm = 75 cm
- 5 cm = 5 cm

- Juf laat een hulp zien van buiten de lesmethode.
- gaat ze uitprinten voor de kinderen als hulp.

* Kan Het Dametje Het De Centimeter Meten?



Observation at primary school in Delft (Wardorf school); 3rd and 4th grade (10-02-2025):

skilled mathematicians

Observatie rekentijgers 12-2-'25

- 5 kinderen van groep 5.
- 4 kinderen van groep 6. → 2 hiervan deden ook nog 'XL rekenboek'.
- ~~je wordt gekozen als je~~
- Alle kinderen vonden dit leuker dan de normale sommen.
- Ze kunnen kiezen welke opdrachten ze doen, maar mogen niet te ver door; boek moet af aan eind van het blokjaar (?)
- Ze vinden het over het algemeen leuk om samen te werken en doen wel vaak dezelfde opdr. zodat ze kunnen overleggen.

FreinetLab?

- leuke opdrachten:
 - puzzels, sudoku's, tafels, hersenkrakers, tekenen, doolhof
 - bij Pasen gooien ze een ei dat niet mag breken
 - stomme opdr. (want te moeilijk)
 - getallen reeksen invullen als ze het niet snappen.
 - staat vaak niet veel uitleg.
 - ④ ⑤ ⑩ ② → dit was te moeilijk
 - 'maak 4'
 -  'zet het juiste getal in het lege vak' → ze snappen de connectie niet alleen bij:  → 5

- 'Je wordt rekentijger als je snel klaar bent'
- Ze waren het bijna allemaal al sinds groep 4.
- * alleen jongens.
- Ze vonden rekenen leuk (gr. 6)
- ze zouden wel meer met fysieke dingen willen spelen bij rekenen. (gr. 6)
- 'Ijsjes maken en tellen hoeveel ijsjes en bolletjes er zijn'
- ze vinden rekentijgers leuk omdat ze de normale les te makkelijk vinden en hierbij wel worden uitgedaagd
- * bij een opdracht moesten ze dingen met een passer tekenen (gr. 6)
- * bij één opdracht moesten ze zelf een dolthof tekenen (gr. 5)
- * XL boek voor gr. 6/7/8
- * rekentijgers is per ~~klas~~ groep

- groep 5 en 6 apart (om de beurt)

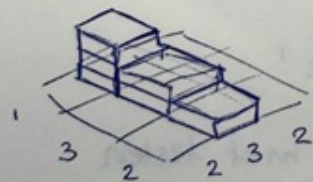
→ allebei deden dezelfde extra opdracht

: met duplo ~~torens~~ bouwen en in aanzicht plaatsen. er waren drie moeilijkheidsgraden: een veld van 3×3 , 4×4 en 5×5 blokken.

Gr. 6 had die van 3 al een keer gedaan, gr. 5 begon bij 3×3 . Ze zijn bijna allemaal toe gekomen aan 5×5 . (4×4 was voor gr 5 en 6 allebei goed te doen.).

Gr. 6 had 5×5 bijna af en goed maar moest stoppen. Gr. 5 kwam wat minder ver.

- iedereen vond het leuk → wel frustratie
ex. bij ^(iemand uit) groep 6 dat hij het net niet af mocht maken.



C. Interviews/meetings with experts

C1. R. Keijzer; professor mathematics (rekenen en wiskunde) at teacher training college ipabo

- meetkunde is waarloos in NL
- breuken: eerste moment met de rekenen aan dagelijks leven.
- meetkunde is meer te relateren.
wiskunde in het wild: meetkunde
↓
wiskundige oriëntatie
rekenangst = versmettelijk
→ Jansen
→ Pablo: 25% heeft r. angst
begint in groep 4 - benamingen
school: maak het leuk
leraren edv-aanpak, lezen gewoon op
meetkunde: in NL: blokken loppensessie
rekendag: drill & practice
maatschappelijk verantwoord.
g.v. dag: NLers wel goed in rek. maar niet leuk,
rekenen niet leuk: maatschappelijk.
meiden VO: niet stoer om wisk. leuk te vinden

(SLO) Stanya Oldengarm
s.oldengarm@slo.nl.
link: meisjes hebben rust nodig
jongens leken
gr 2 → 3: kleine grote overgang.
edv aanpak/
probleem: concreet materiaal dat leren
in de weg zit
- er moet meer reflectie / gesprek
- kinderen snappen het niet maar doen
per ongeluk goed
Susanne Spoerds
→ goede rekenaars → snelle
→ goede
→ creatieve
rekendag: brek.
- volgens Burjien
dem respons model → moeilijke schaal som vs
kind.
- pijl: probleem oplossen → (schaal)

C2. Dr. H. Kekkonen; researcher and assistant professor at TU Delft – Department of Applied Mathematics

meeting 07-01-2025
• tested in class: 12/13/14 y. old.
- f.e. 'what is the biggest shape you can make using only one type of surface?'
- tested with children who are good in maths, they already were following extra curriculums.
- understanding of 360° : less: something bends.
exactly 360° : flat shape.
- scaling up: $\triangle^\circ$ 
- relate it to real life objects.
- use foam in class: less breakable.
• no inspiration from current classes, hyperbolic shapes is not explained in 3D.
- start with a shape or object they understand.
- knitted hyperbolic shape to explain the shape.

C3. Mathematics teacher at teacher training college Thomas Moore Hogeschool

Meeting [redacted] Liefhebber pabo TMH 13-1-25

- meetkunde → beste focus voor ruimtelijk inzicht (meten zit dichtbij)
- het handlingsmodel (v. Groenestijn)
 - ↳ 4 niveaus om rekenen aan te bieden
 - steeds abstracter.
- ! **niveau 1 is visueel → kan met een 3D tool als inhouding gelaten voor een nieuw onderw.**
- nieuwe kerndoelen worden opgesteld dit jaar
 - meer creatief / nadenken / begrip / minder EDI
 - niet tussen leraren.
- verschil tussen scholen: alle lessen volgen of meer alleen 'de leerdoelen halen'.
 - ↳ en daar zelf indeling aangeven
 - + hier is ruimte voor extra materiaal
- buiten scholen, bewegings scholen.
- rekenangst:
 - vmbo-mbo pabo (kunnen bijv. breuken nooit gehad)
 - leren dingen op de pabo die ze zelf niet op school leerden
 - rekenboets uit bsa. → studenten soms wel 'bang' om die niet te halen.
 - referentietoetsen (bijv. cito) → elk halfjaar
 - (10 van lesmethoden).
 - daaruit onderscheidt sterke r., mindere r., midden.
 - ! pabo jaar 3 differentieren: wel aandacht voor 'zwakke r.' minder voor 'sterke r.'
- conclusie: focus op sterke rekenaars
 - als je product interessant is staan scholen er wel voor open. stel scholen 'de juiste vragen'

om mee te denken.

- Robert (TMH) is ook geïnteresseerd
- focus op meetkunde
- er is niet echt focus op de pabo hoe om te gaan met sterke rekenaars.
- het hangt per school af hoe ze met hun leerdoelen omgaan.
- je kan een tool beter door de hele les heen integreren dan alleen tijdens de laatste 10 (zelfst.)
- samenwerking TU Delft x scholen?
- directeur v.d. basisschool wordt 'beoordeeld' op de resultaten v.d. school & leraren worden beoordeeld op resultaten v.d. klas.
- + Robert had als directeur juist belang om sterke rekenaars te ondersteunen en begeleiden om zo het gemiddelde v.d. klas & school omhoog te halen.
- Streefniveau omlaag: minder mensen naar de TU.
 - kiezen voor techniek
- onderwerpen als breuken zijn nog niet heel moeilijk in groep 6, meer gr. 7/8.
- er is geen onderwerp dat over het algemeen als extra moeilijk wordt ervaren (binnen meetkunde)
- ~~toetsen andere test~~ bijles & begeleiding:
 - focus op goede antwoorden krijgen. Om zsm. een voldoende toets niveau te behalen, om de referentietoets te halen. → 'Geen' focus op begrip en berekeningen / stappen.

C4. S. Sjoers, researcher and writer on skilled mathematicians

Meeting [redacted] Spers 22-01-2025

- potentieel sterke rekenaars
- meertalig
- leerling gedrag. + prestaties + stimuleren van talent
- leraren: meer signaleren.
- gesprekken: leerlingen + ouders
- intuïtie
- praktijk: leraren zien het als één groep.
- TIMSS-score: groep 6.
- x **bron**: qua 'zwakke' r. → top 5% v.d. wereld
- wanneer: signaleren al in elke klas bijr.
- overlap: creatieve r. & hoogbegaafd (niet technisch ondersteund)
- rekenangst: creatieve r. vooral **bron**
 - door 'rare' vraagstelling → taal
 - bang hoe ze de vraag moeten
 - dressed up problems → versimpeld maar daar door juist raak
 - losgetrokken van de realiteit

waardvol: creativiteit ontwikkelen

- alles → afwisselen
- geen
- multipotentiaalt: goed ≠ leuk bron**
- lv. hoogbegaafd: veel talenten
- Waarom niet leuk - vooroordelen
 - jongens vs meisjes
- wel potentie: vooral meisjes (techniek)
- 15 hoogste.
- aan de gang met 2F (eind vmbo)
- te theoretisch / niet creatief uitdagend.
- veel aandacht aan de onderkant: extra niveaus
 - niet aan de bovenkant
- 2003: 5%
 - 2007: 4%
 - 2011: 4%
 - nu: 7%
- ↓
- there should be a higher / more challenging / creative level for sterke rekenaars to achieve / work towards.

handelingsmodel. iedereen

1. berekeningen formeel handelen
2. denkmodel voorstellen abstract
3. afbeeldingen v. - concreet
4. doen informeel handelen

sterke rekenaars

- 3 en 4 combineren **existing tools create confusion + not necessary**
2. niet nodig (bijv. hulpmiddel rekenen)
→ helpt om verband tussen 3 en 1 te doen

- geen product voor stap 3 en 4
- wel juist voor stap 1
- 'help met de stappen begrijpen'

- domeinen combineren: product.

! www.sterke-rekenaars.nl

verschilt heel erg: wat kinderen leuk vinden qua onderwerpen.

- motivatie kinderen > basisboekjes

tool:

gamma rekenmachine
(zeft: 'wat denk' je? geef een schatting')

- plusmateriaal is heel talig → maakt het vaag!
→ voorlezen: wordt makkelijker. onduidelijk / zorgt voor angst als context wordt vereenvoudigd.

leuk: meetkunde esher
opdrachten niet gedaan: materiaal is er niet.

- tool zou alle kinderen helpen → vooral
→ we mogen best focussen op alleen sterke rekenaars.
• vooral groot probleem bij scholen waar er
domein: combinatie van domeinen veel zijn.
↳ dat is leuk en mist nu.

ontwikkelingen

- meer aandacht: wordt gezien als probleem.
- contact: artikel schrijven
- terugkoppeling als ik iets heb antwoorden.

meeting

27-01-'25

- spatial ability → not better in brainstorming
→ better in prototyping
(out of correlation test spatial ability x creativity).
- rotation / orientation / visualisation.
↳ important aspects of spatial ability.
- 2D pictures of blocks are made better by boys.
- time pressure: works worse for girls.
↳ 3D products could work better for girls.
• boys have good spatial ability / understanding because of games (like minecraft).
verbal skills can be very important
→ correlation between spatial ability & creative output might be interrupted because of bad/good verbal skills
or because of bad verbal questioning?
- ↳ These 3 aspects are often tested to test the spatial ability of a child.
- 3D objects or videos showing the rotation of a block / shape can have the same effect.
↳ other times 3D objects are really useful: paper

D. Guide to signal stronger mathematicians

Signaleringsinstrument (potentieel) sterke rekenaars

Kenmerken van de leerling:	goed	snel	creatief
1. kan de vragen die voorkennis activeren beantwoorden	x	x	
2. oogt nieuwsgierig naar nieuwe rekenkennis	x	x	X
3. stelt verdiepende vragen tijdens de instructie over de inhoud van de stof			X
4. vraagt naar de toepassingsmogelijkheden van de instructiestof			X
5. koppelt uit zichzelf de lesstof aan andere lesstof		x	X

E. Examples of exercises children found enjoyable/fun

23 Getalraadsels

Welk getal moet in het lege hokje staan? Hoe weet je dat?

45	35	25	50
9	7	5	10

81	27	36	45
9	3	4	5

Nu moeilijker. Wat heb je gedaan?

25	30	45	50
7	11	12	

4	12	16	20
12	18	20	

18	24	27	30
12	18	20	

Eerst _____
daarna _____

Eerst _____
daarna _____

Eerst _____
daarna _____

Sudoku's

Zet in elke rij, elke kolom en elk blok de getallen 10, 20, 30, 40, 50 en 60.

10	50	20	60	40	30
30	30	60	10	50	20
60	40	30	20	50	10
50	20	30	40	60	10
20	60	10	50	30	40
30	40	50	20	10	60

Vul nu de tientallen 10 tot en met 90 in.

70	30	20	10	60	90	80	50	40
50	40	10	80	70	30	90	20	60
90	80	60	50	40	20	30	10	70
60	70	80	70	30	50	20	90	10
40	20	30	90	10	80	70	60	50
10	50	90	60	20	70	40	80	30
30	10	70	20	80	60	50	40	90
20	60	50	30	90	10	70	80	
80	90	40	20	50	10	60	30	20

Waar kun je het beste beginnen?

1 Puzzelvierkanten

Wat is de waarde van elk figuurtje?
Horizontaal, verticaal en diagonaal zie je hoeveel ze samen waard zijn.

+	✓	♥	+	○	22
♥	○	♥	○	♥	21
+	✓	♥	★	○	20
★	♥	♥	✓	+	22
♥	○	♥	★	★	21

⚡	■	☾	■	⊙	18
☾	⚡	⚡	☾	☾	15
⊙	☾	⚡	⊙	☾	11
⚡	⊙	⊙	⚡	⊙	9
⚡	☾	☾	⚡	⚡	17

Op de getallenlijn liggen 16 en 24, allebei 4 plaatsen van het 'mooie' getal 20. $16 = 20 - 4$ en $24 = 20 + 4$.
Als je 16×24 snel wilt uitrekenen, kun je dat doen met kwadraten. Een kwadraat van een getal is het getal vermenigvuldigd met zichzelf, bijvoorbeeld 20×20 . Kijk maar:

keersom	getal ertussen	kwadraat van dat getal	kwadraat verschil met de andere getallen	af trekken
16×24	20	$20 \times 20 = 400$	$4 \times 4 = 16$	$400 - 16 = 384$

Reken op die manier uit:

keersom	getal ertussen	kwadraat van dat getal	kwadraat verschil met de andere getallen	af trekken
6×14	10	$10 \times 10 = 100$	$4 \times 4 = 16$	$100 - 16 = 84$
19×21	20	$20 \times 20 = 400$	$1 \times 1 = 1$	$400 - 1 = 399$
15×25	20	$20 \times 20 = 400$	$5 \times 5 = 25$	$400 - 25 = 375$
12×28	20	$20 \times 20 = 400$	$8 \times 8 = 64$	$400 - 64 = 336$
23×37	30	$30 \times 30 = 900$	$4 \times 4 = 16$	$900 - 16 = 884$
32×28	30	$30 \times 30 = 900$	$8 \times 8 = 64$	$900 - 64 = 836$

Bedenk zelf nog 3 van deze (moeilijke) sommen en reken ze uit. (Er zijn heel veel mogelijkheden.)

keersom	getal ertussen	kwadraat van dat getal	kwadraat verschil met de andere getallen	af trekken
23×27	25	$25 \times 25 = 625$	$2 \times 2 = 4$	$625 - 4 = 621$
26×24	25	$25 \times 25 = 625$	$1 \times 1 = 1$	$625 - 1 = 624$
47×53	50	$50 \times 50 = 2500$	$3 \times 3 = 9$	$2500 - 9 = 2491$

Lukt het ook met een som als 7×9 ? **X ja**

Leg uit: $7 \times 9 = 63$ $1 \times 1 = 1$ $63 - 1 = 62$ $7 \times 9 = 63$

En met 7×57 ? **X ja** $6 \times 6 = 36$ $57 - 6 = 51$ $51 \times 51 = 2601$ $2601 - 36 = 2565$

14 Tafelsommen anders

Lees dit goed en jij kent die truc ook!

16 x 24 = 384. Reken ik zo uit m'n hoofd uit. Koud kunstje!

Op de getallenlijn liggen 16 en 24, allebei 4 plaatsen van het 'mooie' getal 20. $16 = 20 - 4$ en $24 = 20 + 4$.
Als je 16×24 snel wilt uitrekenen, kun je dat doen met kwadraten. Een kwadraat van een getal is het getal vermenigvuldigd met zichzelf, bijvoorbeeld 20×20 . Kijk maar:

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6 Toervervierkanten

Een toervervierkant is een vierkant met getallen erin.
Je kunt die getallen in alle richtingen bij elkaar optellen: de uitkomst is altijd hetzelfde.

Maak een toervervierkant met de getallen 1 tot en met 9.

4	3	8
9	5	1
2	7	6

Welk getal staat in het midden van je toervervierkant? **5**

Waarom? **Omdat 5 het midden van 9 en 1 is 5. Dat brengt de getallen in evenwicht.**

Maak nog een toervervierkant met de getallen 1 tot en met 9.

Hoe heb je het nieuwe toervervierkant gevonden?

door dat de andere drie getallen de eerste toervervierkant om te draaien.

2	7	6
9	5	1
4	3	8

Tel bij elk getal 4 op. Wat is nu de uitkomst van elke optelling? **12 x**

Waarom? **omdat vijfteen plus vier machtien is x**

Welke getallen gebruik nu je in dit vierkant? **zeventien x**

Blijft het ook een toervervierkant als je alle getallen x 2 doet? **ja x**

Maak een toervervierkant van 4 x 4 hokjes.

7	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

keurig

E

Welk dier hoort in welk hok?

Spelregels

- De hokken zijn van een arend (A), een beer (B), een cavia (C) of een das (D).
- In elke rij (van links naar rechts) mag een dier maar één keer voorkomen.
- In elke kolom (van boven naar beneden) mag een dier maar één keer voorkomen.
- In elk blok van dezelfde kleur zit een dier ook maar één keer.

		D	
D		B	C
		C	
B	C	A	D

A	B	C	
D		B	
C	D	A	
B			

C			D
B	D	A	C
A		D	B
	B	C	

C	B	D	A
	A	C	
B	C		D
	D	B	

Het kan ook met letters.

C	B	D	A
A	D	B	C
B	C	A	D
D	A	C	B

Kan jij zelf ook zo'n puzzel maken?

a	b	c	d
d	C	a	B
B	A	d	C
C	d	B	a



2 Noteer de kikkersprongen.
Kleur hoe het kikkerspel kan gaan met 2 blauwe en 2 groene kikkers.

start

zet 1				
zet 2				
zet 3				
zet 4				
zet 5				
zet 6				
zet 7				
zet 8				
zet 9				
zet 10				

3 Noteer alle kikkersprongen.
Kleur hoe het kikkerspel kan gaan met 1 blauwe en 1 groene kikker.

Schrijf erbij welke kikker is verplaatst.
Als de groene kikker beweegt, zet je er een G naast;
als de blauwe kikker beweegt, een B.

G begint:

G		
B		
G		

B begint:

F. Ideas to design directions

Type of domain

Numbers (getallen)

combining domains

Connections (verbanden)

Proportions (verhoudingen)

Geometry (meetkunde)

domain independent

Type of challenge

Discover; f.e. introduces future topics or revises old topics

Relation to real life situations

Combining domains

Deepening; harder level than current curriculum

extra

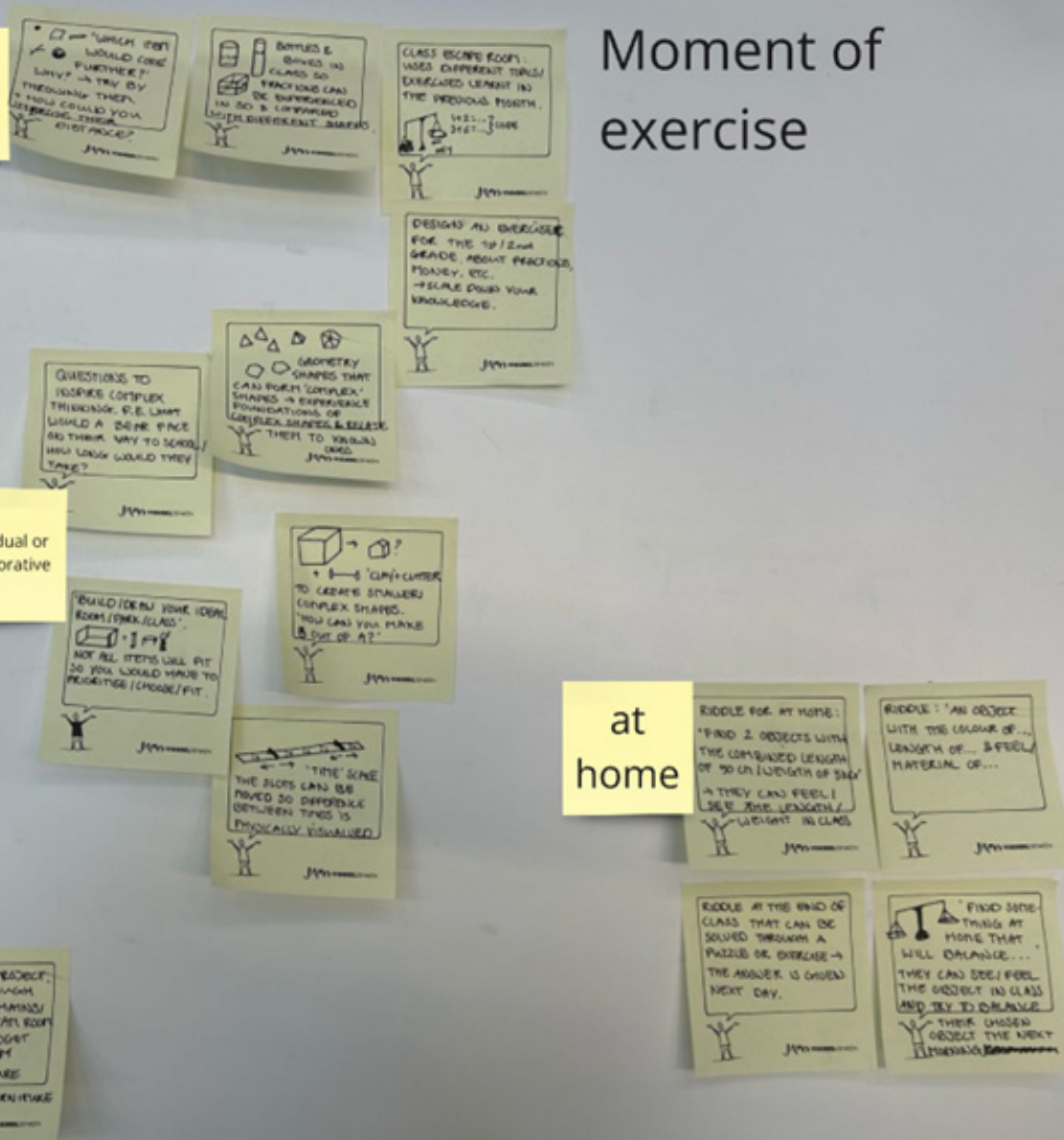
Collaborative
/ in-class
explanation

individual or
collaborative

individual

Moment of
exercise

at
home



G. Informed consent form for testing



[English]

Informed consent form research project mathematics classes in primary schools – test and evaluation

Dear parent or caregiver,
For my graduation project for the masters Integrated Product Design at the TU Delft, I am doing research on behalf of het Wetenschapsknooppunt regarding mathematics classes at primary schools. I will conduct tests with children of groep 5 and 6 to test my prototypes.

Aim of the research and test

I am graduating for ‘het Wetenschapsknooppunt’, a company working together with TU Delft to create more design and inquiry based learning experiences for primary schools in the Netherlands. I will focus on improving the mathematics classes based around spatial thinking.

I will ask your child to test a product I designed that is meant to improve the experience of learning spatial skills in mathematics classes. After testing, I will ask the children questions about their experience and attitude towards the product.
The prototype will always meet needed safety requirements, as agreed upon with the TU Delft. Before the testing takes place I will introduce myself, the test and what I’m doing the test for with the children. After this introduction, they can ask questions if they want to. During the testing, I will again explain clearly which actions they are asked to do.

Data and rights of participants

The data of your child will be saved anonymously, I will not mention names anywhere in my research findings. I will only save the age, grade (and gender) of your child (if the latter seems to have impact on the outcome of my research). Furthermore, audio and video data will not be used in my report or any other form of presentation, I will try to make photo’s or video’s without faces and if so, will always cut or blur any visible faces. Audio data will also not be shared in any form and is solely used as personal references for my research. Audio and video data will always be saved in a safe environment, not shared with other parties and will be deleted 6 months after the end of my project.

Participating children can always ask questions about the research, and may leave the interview at any given time if they do not wish to continue, without further consequences. They do not have to give any reasons to do so.

Contact information researcher

Roosje Lugthart (master student Integrated Product Design) –

Consent of the participation of your child

Via the questions below, you can indicate if your child can participate in the test and interview and if audio and video recordings can be made of this test and interview.

1. Do you give consent for the participation to the researcher to interview your child for this research?
- Yes, my child may participate in the test and be interviewed
- No, my child may not participate in the test and may not be interviewed

2. Do you give consent to the researcher to make audio and video recordings of the test and interview with your child, used on the conditions described above ?

Yes, audio and video recordings may be made of the test and interview, if used on the conditions described above
No, the researcher may not make audio and video recordings of the test and interview

3. Do you give consent to the researcher to possibly use video recordings of you child in their graduation report or graduation video (photo's or video's will always be without faces or faces will be blurred. Video's will never contain audio). This report and video can be uploaded on the TU Delft library and then read or seen by anyone who looks for it.

Yes, video recordings may be used in a graduation report and video, if used on the conditions described above
No, the researcher may not use video recordings in their graduation report or video

City and date:
.....

Name of child:
.....

Name of the parent or caregiver:
.....

Signature of the parent or caregiver:
.....

City and date:
.....

Name of the researcher:
.....

Signature of the researcher:
.....



H. Concept tests with children

H1. Test Sponge Questions that spark creative thinking (08-04-2025)

Introduction

Student Pauline Timmers conducted a test on repair with the 3rd and 4th grade. I helped in this test by being the ‘supervisor’ of one out of 5 stations of her test. Because she only needed exercises of her own on the other four of the stations, I could come with my own exercise and test something for my thesis.

Introduction/ Aim of the test

Through research in this report was found that skilled mathematicians are in need of challenging exercises. One way of doing so is by providing questions that require them to think outside of the box, or combine previously gathered information with personally developed insights. For this reason I wanted to test if questions like this, related to spatial ability in some way, but different from those currently provided in most mathematics classes were indeed challenging and fun to answer for skilled mathematicians.

Besides being challenging enough for the skilled mathematicians , the questions also need to be doable by the rest of the class. This is why I came up with so called ‘sponge questions’. These are open questions that students can answer differently regarding their personal level of knowledge on the subject. This means that both skilled mathematicians and the rest of the class can answer questions related to mathematics without it being too hard or easy for either group.

I came up with three questions related to the subject of the class (repair related to bikes) that also require spatial ability. Through this test I wanted to certify whether or not these questions were indeed doable and fun for both the skilled mathematicians and the rest of the class and whether there was a significant difference in the types of answers that skilled mathematicians gave compared to their fellow classmates. Lastly, I was curious what skilled mathematicians thought of the questions.

Test set up

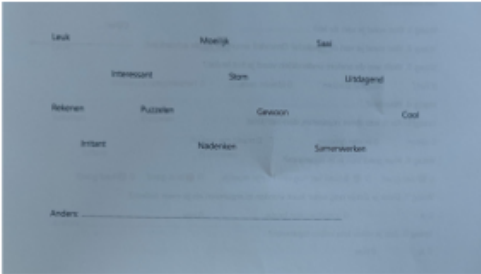
For this test, groups of 4-5 children came by my ‘station’ for 10 minutes. I had three questions, printed out. I shortly explained the different questions/exercises and then let the children choose one to do. Besides, they could choose if they wanted to make the exercises together or individually.

Vormen herkennen	Naam:.....
1a. Welke vormen kun je herkennen in de fiets? (Bijvoorbeeld, cirkel, bol, vierkant, kubus, cilinder, etc.) - Omcirkel en benoemde figuren in de foto hiernaast 	
1b. Welke onderdelen kun je niet omschrijven aan de hand van vormen die je kent? 	
Van stadsfiets naar racefiets	Naam:.....
2. Wat kun je toevoegen of veranderen aan deze fiets om hem sneller te maken? Schrijf de verbeteringen op of teken ze in de foto. 	
Op reis met de fiets	Naam:.....
3a. Stel je voor dat je met deze fiets naar Italië gaat fietsen. Wat voor soort wegen en obstakels kom je dan tegen? 	
3b. Hoe kan je de fiets zo aanpassen dat hij bestand is tegen die obstakels? 	

Results

22 children participated in the test day, however only 20 came to my station. All 20 of these students answered at least one question, most did two or all three. Some worked individually, but most collaborated with fellow students.

In the evaluation form that all students filled in at the end of the class, this was the question I provided for my test:



The words were circled as follows:

	Filled in by percentage of skilled mathematicians	Filled in by percentage rest of the rest of the class	Filled in (total amount)
Leuk	8/8 : 100%	9/12 : 75%	17/20: 85%
Moeilijk	0/8: 0%	1/12: 8,3%	1/20: 5%
Saai	0/8: 0%	0/12: 0%	0/20: 0%
Interessant	5/8 : 62,5%	5/12: 41,7%	10/20: 50%
Stom	0/8: 0%	0/12: 0%	0/20: 0%
Uitdagend	2/8: 25%	3/12: 25%	5/20: 25%
Rekenen	0/8: 0%	2/12: 16,7%	2/20: 10%
Puzzelen	5/8 : 60%	3/12 : 25%	8/20: 40%
Gewoon	1/8 : 12,5%	3/12 : 25%	4/20: 20%
Cool	4/8 : 50%	7/12: 58,3%	11/20: 55%
Irritant	0/8: 0%	0/8: 0%	0/20: 0%
Nadenken	3/8: 37,5%	5/12: 41,7%	8/20: 40%
Samenwerken	6/8: 75%	4/12: 33,%	10/20: 50%
Anders, ...			
- Grappig		1	
- Super		1	
- Super leuk	1		

Sterke rekenaars: 8 Rest van de klas: 12

Conclusion

The answers that were given most by skilled mathematicians were : ‘leuk’, ‘interessant’, ‘puzzelen’ and ‘samenwerken’. The biggest differences between skilled mathematicians and the rest of the class were that skilled mathematicians thought significantly more of puzzles and collaboration.

The most important conclusion is that all children liked the exercises and could easily fill up 10 minutes thinking about 1-3 questions. Besides, none of the skilled mathematicians found the questions boring or not fun to do. Therefore I will use similar questions in my final design, as addition in the exercise book.

H2. First validation test (14-05-2025)

Validation test – test set up (14-05-2025)

Introduction/aim of the test

After designing my provisionally design, I wanted to validate the product by testing it with children. The previous – concept – test helped to see if the concept had potential and if the measuring tool worked visually and ergonomically. For this test I have designed an exercise book complete with different spatial exercises that stimulate children to use the measuring tool and, redesigned map.

The two most important aspects I want to test are if the product is fun and challenging for the children. I will test this by observing how the children are interacting with the product and what their reactions are during the test. I will observe factors like: facial expressions, body language, reactions and interaction with each other. At the end of the test I will give the children an evaluation sheet in



Test set up

I will test the product with three groups of 3-4 children at a time, 15 minutes per group. The children are skilled mathematicians of a combined 3rd and 4th grade class. The teacher decided on which children are placed together. In total, I want to test all questions of the exercise booklet, however these are too many to test with all children in 15 minutes. Therefore, I will divide the questions between the three groups, as follows:

- Group 1: exercise 1, exercise 2 and exercise 3
- Group 2: exercise 4, exercise 5a and exercise 5b
- Group 3: exercise 6, exercise 7 and exercise 8

For each group, I will shortly explain the overall story of the booklet and for the second and third group I will give a recap of the exercises that the previous group has done. The children will each receive a booklet, with all questions and I will tell them which exercises they are asked to do. If the time runs out before all three questions per group are finished, I will let the next group start where the previous one ended. Questions 7 and 8 are design based questions and are less dependent on the measuring tool. Therefore it is less important if these two questions cannot be tested due to time running out.

After three times 15 minutes, I will ask all the children to fill in the evaluation form on the next page. Furthermore, the exercise booklet used for this test is shown on the pages afterwards. (However, a Dutch translated one is used).

Evaluatie van de test Naam:.....

Wat vond je van het rekenen met dit product?

.....

.....

Wat voor cijfer (1-10) zou je geven aan het rekenen met dit product?

Bij de volgende vragen is het de bedoeling dat je een cijfer omcirkelt tussen de 1 en 7.

Hoe leuk vond je het rekenen met dit product, vergeleken met de opdrachten uit de normale rekenlessen?

Veel minder leuk even leuk veel leuker

1 2 3 4 5 6 7

Hoe leuk vond je het rekenen met dit product, vergeleken met de opdrachten uit rekentijgers?

Veel minder leuk even leuk veel leuker

1 2 3 4 5 6 7

Hoe uitdagend vond je het rekenen met dit product, vergeleken met de opdrachten uit de normale rekenlessen?

Veel minder uitdagend even uitdagend veel uitdagender

1 2 3 4 5 6 7

Hoe uitdagend vond je het rekenen met dit product, vergeleken met de opdrachten uit rekentijgers?

Veel minder uitdagend even uitdagend veel uitdagender

1 2 3 4 5 6 7

Wat zou je ervan vinden om dit product vanaf nu te gebruiken in het rekentijger uur op woensdag?

.....

.....

Vind je dit product passen bij het rekentijger boek? Waarom wel of niet?

.....

.....

Most important observations of the test:

- the children did not immediately connect the different tiles. They knew that they could but they realised that they could also measure the distances by laying the tiles next to each other, without wires.
- Parts of the roads on the map were quite small or had turns that are too sharp to measure well.
- The connection between the tiles worked well with, children could easily connect and fasten them now.
- There were some variations in outcomes of the measure – questions, not everyone had the same answers
- The children also measured with the short sides of the tiles and assumed that these were 1,5 cm (for the 3cm ones).
- Converting centimetres to metres was ‘really’ easy (the scale is now 1cm=10m)
- The children seemed to enjoy the story around the exercises. They were involved, could follow were the crew was going on the map.
- The children were intrigued by the characters and asked who everyone was when they were mentioned in the story (I did not explain their names beforehand). When someone was mentioned, they t
- Took that character and placed it in front of them or held it. “So Rosa always knows where to go right, because she has the compass?”.

Results of the evaluation form (filled in 8 times):

Questions	Answers
What did you think of doing maths with this product*?	- Fun (<i>leuk</i>) ; 7 times - Educational (<i>leerzaam</i>) ; 2 times - Challenge (<i>uitdaging</i>) ; 1 time
What grade would you give for doing maths with this product from 1-10?	Average: 8,6/10
How fun did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 6,4/7
How fun did you think it was to do exercises with this product, compared to the exercises of rekentijgers** (1-7)?	Average: 5,7/7
How challenging did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 5,3/7
How challenging did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?	Average: 3,2/7
What would you think of using this product from now on during the ‘rekentijger’ hour on Wednesday?	- Very fun (<i>heel leuk</i>) ; 5 times - Really good idea (<i>heel goed idee</i>) ; 2 times - Fun, but rekentijgers is also fun (<i>Leuk, maar ik wil ook rekentijgers doen</i>) ; 2 times - Please (<i>heel graag</i>) ; 1 time
Do you think this product fits to the rekentijgers exercises?	- No ; 4 times - Yes ; 2 times - A little ; 2 times

Why?	No: - this is bigger, - rekentijgers is harder, - you need a board and other things, - it is not as challenging as rekentijgers, - it is big Yes: - it is both extra work - it has a good challenge
------	---

*I explained that with ‘product’ I meant the tiles, map and exercise booklet all together

**rekentijgers = the booklet they use at this school to provide extra exercises for skilled mathematicians

Conclusions of the test/design changes:

- The map should be larger, mostly in order to make the roads longer and to increase the distances (it can consist of two separate parts that fit together). Then, the 5cm tiles are of more value next to the 3cm ones.
- Create two parts that can fit together to make one larger map
- The roads should have less sharp turns in order to create more precise measurements
- The scale should increase. Multiplying by 10 is too easy, the scale can be ‘1cm = 12 (or 15) m’ for example. (Keep in mind that the measurements of the houses still make sense)
- The pictures in some exercises should be of low opacity in order to make drawing on them easier.
- Doors and house numbers should be laser cut on the map.
- Think of a way/instruction to increase stimulation of connecting multiple tiles instead of using them as separate tiles. Or change the design to separate blocks and skip the ‘connection ruler?’
- Idea: add an exercise that requires children to build something on the map (with the tiles)
 - o Add holes on non ruler sides that they can use to stack multiple tiles on top of each other
- Either give measurements to each side of the triangle or give instructions to only use one side of the tiles.
- Keep the symmetry, drawing and design exercise.
- Increase difficulty in the measuring exercises (can be done by only increasing the scale of the map).
- Keep the rotation exercise but increase difficulty.
- Keep the orientation exercise, but increase the amount of possibilities or decrease differences between the possibilities.

Reflection on test with teacher

On the 15th of May I met up with the teacher of the class I tested with to get her feedback on the product and its potential. The most important improvements that she suggested are listed below. Some of them are in line with what I had already concluded, which strengthens their importance.

- The map should be larger. You can create a folding line or two parts so that it still fits in the classroom and protects the inside of the map.
- You can use loose pieces of iron wires to measure the bended roads more easily.
- You can indicate which exercises are least and most difficult by adding one, two, or three stars. If you do so, it is okay to have ‘really difficult’ exercises since the children will feel less

- disappointed knowing that they weren’t able to complete those compared to the easier ones.
- Indicating a distance between 1-5 is not extremely challenging for some of these children
 - It is good to keep the speed calculation exercise; you can increase difficulty by letting them recalculate from seconds to minutes, or minutes to hours. Include a table which they can use to make these calculations.
 - Add lines on which children can write their calculations; helps them to reflect and helps teachers to check answers.
 - Adding a scale to the map is very relevant for these ages. You can indeed increase difficulty by changing 1 cm = 10 m to 1cm = 12 m.
 - You can create more exercises around measuring and calculating with scales.
 - Add short explanations to exercises that use specific terms like ‘symmetry lines’. These children just revised it but for some children it might have been some time.

Overall she was excited about the product and said that the children could definitely use it in class. She liked the physical aspect and mentioned how well children react on learning with physical objects and materials. The most practical remark that she had was if there were more booklets and exercises, since this booklet would be done in an hour. I explained that this booklet is one story I created but that the idea is to have multiple stories, in which she saw more potential. To invest in this product, she wanted at least a few booklets so that it was worth investing in.

Furthermore, she highlighted that the level of mathematical understanding in this class (and school) is not average because many parents in this neighbourhood have studied at the TU Delft. So questions that might not be extremely challenging for these children, could very well be way more challenging for children in other classes, schools, or cities.



H3. Second validation test (14-05-2025)

Aim of the test

This test is quite similar to the last one (done on 14-05-2025). Therefore, this test will mainly provide extra data and insights for the aspects that were tested before. Therefore, still, the two most important aspects I want to test are if the product is fun and challenging for the children. I will test this by observing how the children are interacting with the product and what their reactions are during the test. I will observe factors like: facial expressions, body language, reactions and interaction with each other. At the end of the test I will give the children an evaluation sheet in which they can write their experiences with the product.

The version that will be tested with now is an improved version compared to the one that was tested with last time. These improvements can be read in the conclusion from last test and seen in figure X. This test will therefore give a validation if those design changes are improvements compared to the last design. I will compare the answers that children gave to the questions during the previous test compared to this one and get an impression of, for example, if the questions are more challenging now and if measuring distances on the map is more smoothly now compared to during the previous test.

Test set up

I will again, test the product with three groups of 3-4 children at a time, 15 minutes per group. The children are skilled mathematicians of a combined 3rd and 4th grade class (different class than last time, but also from a Waldorf school). The teacher decided on which children are placed together. In total, I want to test all questions of the exercise booklet, however these are too many to test with all children in 15 minutes. Therefore, I will divide the questions between the three groups, as follows:

- Group 1: exercise 1, exercise 2, exercise 3 and exercise 4
- Group 2: exercise 5a, exercise 5b, exercise 5c, exercise 6 and exercise 7
- Group 3: exercise 8 and exercise 9

I will test more exercises per group, since during the last test the children were quicker than I thought answering each question and we could do more than three exercises in 15 minutes. Questions 8 and 9 are separated for one group since I want to give the students enough time to build something physically for this question.

For each group, I will shortly explain the overall story of the booklet and for the second and third group I will give a recap of the exercises that the previous group has done. The children will each receive a booklet, with all questions and I will tell them which exercises they are asked to do. If the time runs out before all questions per group are finished, I will still let the next group start with their planned questions. For this test it is important that question 9 is done since this (type of) question is not yet tested before.

The set up for this test is:



After three times 15 minutes, I will ask all the children to fill in the evaluation form shown on the next page.

Evaluatie van de test

Naam:.....

Wat vond je van het rekenen met dit product?

.....
.....

Wat voor cijfer (1-10) zou je geven aan het rekenen met dit product?

Bij de volgende vragen is het de bedoeling dat je een cijfer omcirkelt tussen de 1 en 7.

Hoe leuk vond je het rekenen met dit product, vergeleken met de opdrachten uit de normale rekenlessen?

Veel minder leuk			even leuk			veel leuker
1	2	3	4	5	6	7

Hoe leuk vond je het rekenen met dit product, vergeleken met de opdrachten uit rekentijgers?

Veel minder leuk			even leuk			veel leuker
1	2	3	4	5	6	7

Hoe uitdagend vond je het rekenen met dit product, vergeleken met de opdrachten uit de normale rekenlessen?

Veel minder uitdagend			even uitdagend			veel uitdagender
1	2	3	4	5	6	7

Hoe uitdagend vond je het rekenen met dit product, vergeleken met de opdrachten uit rekentijgers?

Veel minder uitdagend			even uitdagend			veel uitdagender
1	2	3	4	5	6	7

Wat zou je ervan vinden om dit product vanaf nu te gebruiken tijdens de rekenlessen?

.....
.....

Vind je dit product passen bij het rekentijger boek? Waarom wel of niet?

.....
.....

Observations during the test:

- Most children connected or at least asked if they could connect the tiles together to measure. When they heard they did not need to, they used the tiles separately, without connecting them. "That is easier".
- Both groups who did measuring exercises placed the tiles on the map, in a line. One group took them off after the path they had to measure was covered. The other group kept them laying in this path.
- Both groups only counted the distance of the measured path afterwards. "Okay so that is five times 5 cm and two times 3: 21 cm!. They did not add the different tiles while measuring.
- Changing the scale from 1 cm = 10 metre to 1 cm = 12 metre worked well. The students were all counting in their heads and making the equations to split it up '12 times 16, is 10 times 12 and 6 times 12'. Sometimes they made mistakes while counting.
- During exercise 4, the students measured different path options before deciding on the quickest one. This indicates that is was indeed less visible immediately which path is the shortest; an improvement from the last version. One of the children started to measure the paths in the booklet instead of in the map. Meaning that is could still be more clear that you should measure on the real map or change the scale of the picture in the booklet.
- The design exercise was received really well. When the materials were laid out their faces seemed to light up and they immediately stood up and started grabbing different materials. They also all started thinking out loud as to what they would make.
- For the drawing exercise, some students tried to draw in perspective but it was clear that this is still quite hard to do.
- For exercise 8, the students thought of many different ideas to cross the lake and it was interesting to hear how to interpreted the question differently and all started to think outside of the box in order to answer the question.
- Because the map is now bigger, both the 3cm and 5cm tiles could be used easily. It also clearly helped that some paths were less curved than before so that it was easier to measure for the students.
- When reading exercise 5c, someone said 'I don't' want to do this one' because it seemed like much effort. He continued however one of his peers did read it more carefully and started filling number in and realised it wasn't that hard. Then the first student also filled it in.
- Overall, most exercises that I changed seemed more difficult then during the previous test but they were all still doable with the right guidance from me or with the students helping each other.
- The symmetry exercise seemed harder for this class than for the previous one because they did struggle to see if the lake was only symmetrical in two ways or more.
- The addition of the comic story had a positive impact on the students, they liked to read the story.
- The students were also excited about the characters. During this test I first showed the first page of the booklet to let the students see who all five characters were. This seemed to help (next to seeing them in the comic story) because during the story when one of the characters was mentioned, they knew who it was and picked that character up sometimes. For some questions, they placed the characters on the map regarding their positions in the story.

Quotes and dialogues between the children (in Dutch)

Group 1

- “Even kijken”, jongen is klaar met naam opschrijven en bladert alvast door boekje
- Over de poppetjes: “Oehh mogen wij daar eentje van hebben?”
- “Heb je ook een liniaal? Of mogen we die elf pakken?” Op dat moment pakt Roosje de meetblokjes erbij
- “Ik vind het raar dat dit 3 is en dit. Jongen heeft de meetblokjes vast” -> Roosje: de lange zijde is 3
- Over de buigbare verbindingen tussen de meet blokjes”: Hooft het zo dat we het aan elkaar moeten maken?”
- “We hebben echt geluk dat we dit mogen doen. Veel leuker dan dictee”
- “Ahwww de poppetjes. Ahwww Rosa.” Jongen aait poppetje (zie foto)
- “Hoe maak je dit allemaal? Echt onmogelijk”
- “Is dit Ory? Ory is weggelopen in het meer.” Meisje verplaatst het poppetje over de kaart.
- Roosje: Ik denk helaas dat de tijd om is. “Neeeeee”

Group 2

- Ze lezen de tekst op de kaart: “Winkel winkel winkel winkel supermarkt”
- Opdracht 3: ze wijzen meteen op de kaart de plekken aan. “Symmetriemeer”
- Jongen wilde de meetblokjes in het boekje gebruiken. Maar je moet op de kaart meten.
- Jongen over opdracht 5c: “hier heb ik geen zin in”
- Daarna bladert hij verder naar opdracht 6, met tekenen van het symmetrie meer. Daarna weer terug naar 5c, en neemt een antwoord over van z'n klasgenoot en gaat daarna met die opdracht verder.

Group 3

- Ander meisje komt klas uit gelopen en kijkt naar de rekentijger meisjes: “moeilijk?”
- Meisje 1 schud nee
- Meisje 2 mwah gebaar

- Meisje 2 zei: “we kunnen een vlot maken.”
- Meisje 1: “maar een vlot is zo simpel. Dat is niet creatief”

- Meisje 2 opeens beetje dominant, eerst rustig.
- Meisje 2: “we gaan een vlot maken van klei.”
- Meisje 1: “Maar ik wil geen vlot maken.”
- Meisje 2: “Dan doen we het toch allebei”
- Even later, meisje 2: “maak jij het touw, dan maak ik het vlot”

- Jongen kleit iets wat lijkt op een M. “Een M van meer, oehh ik weet iets”

“[name] wil je dit voor mij tapen?” Vraagt een Jongen die 2 satéprikkers en elastiek aan elkaar wilt maken. Samenwerken aan de opdracht.

- Meisje 2: “we moeten onder de stokjes nog iets doen zodat het vast zit”
- Meisje 1: “kan ik de lijm gebruiken?”
- Meisje 2: “je moet tapen dat is veel beter”
- Meisje 1: “dan doen we lijm en tape”

Results of the evaluation form (filled in 6 times):

Questions	Answers
What did you think of doing maths with this product*?	- Fun (<i>leuk</i>) ; 6 + because you could build something (1 time)
What grade would you give for doing maths with this product from 1-10?	Average: 9/10
How fun did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 6,2/7
How fun did you think it was to do exercises with this product, compared to the exercises of rekentijgers** (1-7)?	Average: 3,8/7
How challenging did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 6,2/7
How challenging did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?	Average: 3,8/7
What would you think of using this product from now on during the ‘rekentijger’ hour on Wednesday?	- More fun (<i>leuker</i>) ; 3 times - (Really) fun (<i>super leuk</i>) ; 3 times
Do you think this product fits to the rekentijgers exercises?	- No ; 2 times - Yes ; 3 times - Both ; 2 times
Why?	No: - Because rekentijgers is equations (<i>sommen</i>) Yes: - ‘Just because’ - Because there is a lot of challenge in this Both: - Because this is more arts and crafts

*I explained that with ‘product’ I meant the tiles, map and exercise booklet all together

**rekentijgers = the booklet they use at this school to provide extra exercises for skilled mathematicians

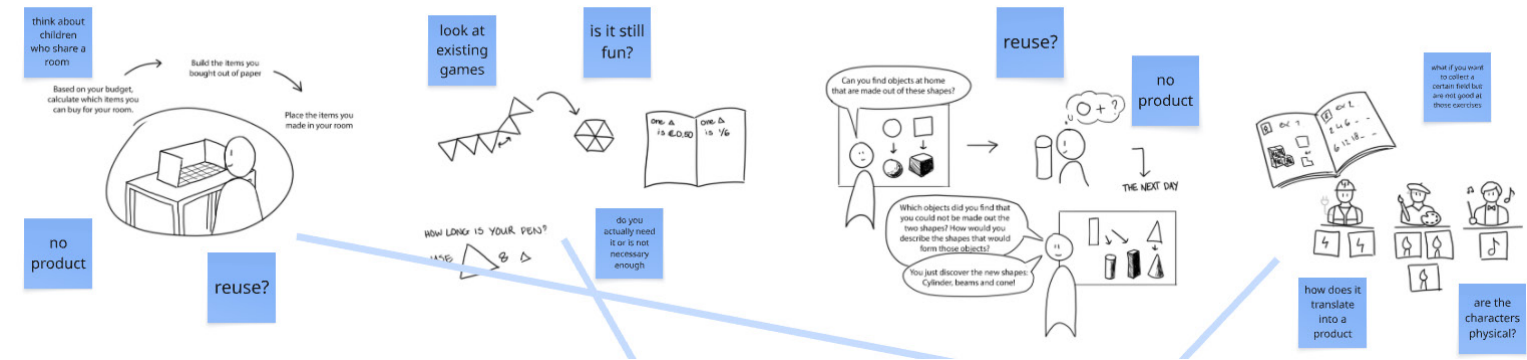
Conclusions/design changes

- In exercise 4 the picture should have the exact measurements as the physical map or clearly be way smaller so that it is immediately clear that they should measure on the physical map instead of in the booklet.
- Some questions could still be a bit more difficult but it is already a good improvement from last time.
- An extra orientation exercise could be added, since this really involved looking at the map. Maybe the different options could be in between paths instead of on them to increase difficulty.
- The symmetry, drawing and design exercises on the end should stay (the drawing exercise could have a hint in the explanation).
- The comic story is a positive addition to the booklet and helps children to follow the story themselves.
- The connection pieces for the tiles can stay since some students did like to connect the tiles in order to ‘create the entire path’. However students should not be obliged to use them since using the tiles separately also works well.
- Usage of the tiles does not seem necessary, however it does still seems like a fun addition because it adds the need to calculate the distance instead of just reading it in one go as would happen with using one ruler.
- A recommendation that stays is to add an exercise for which the children need to build something (a building for example) on the map with the blocks.
- The scale stays as it is (1 cm = 12 metres), this allows for challenging calculations but is doable enough for the students.

I. Prototyping and ideation

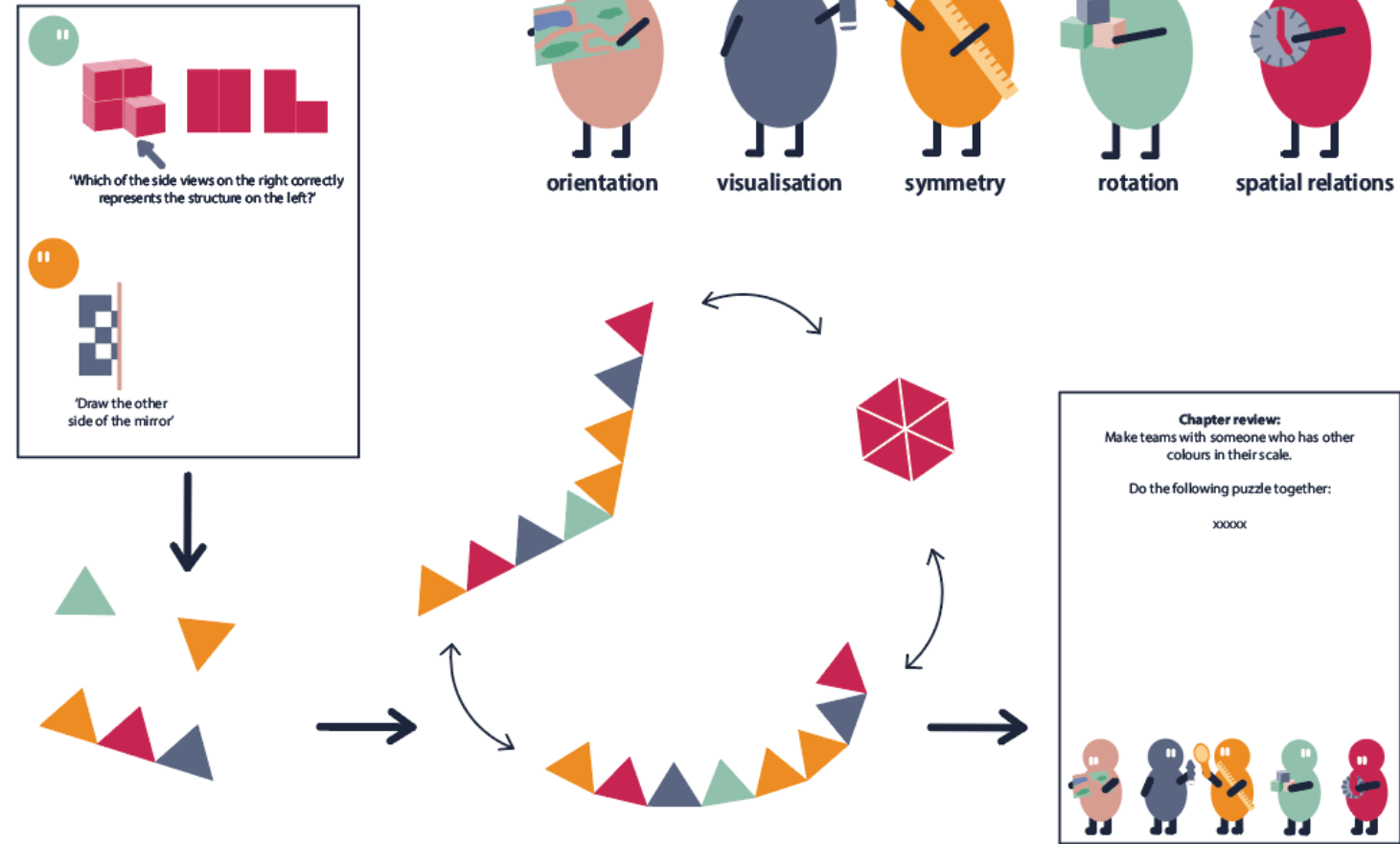
I1. How to improve each design direction

Design direction ideas + what is still missing

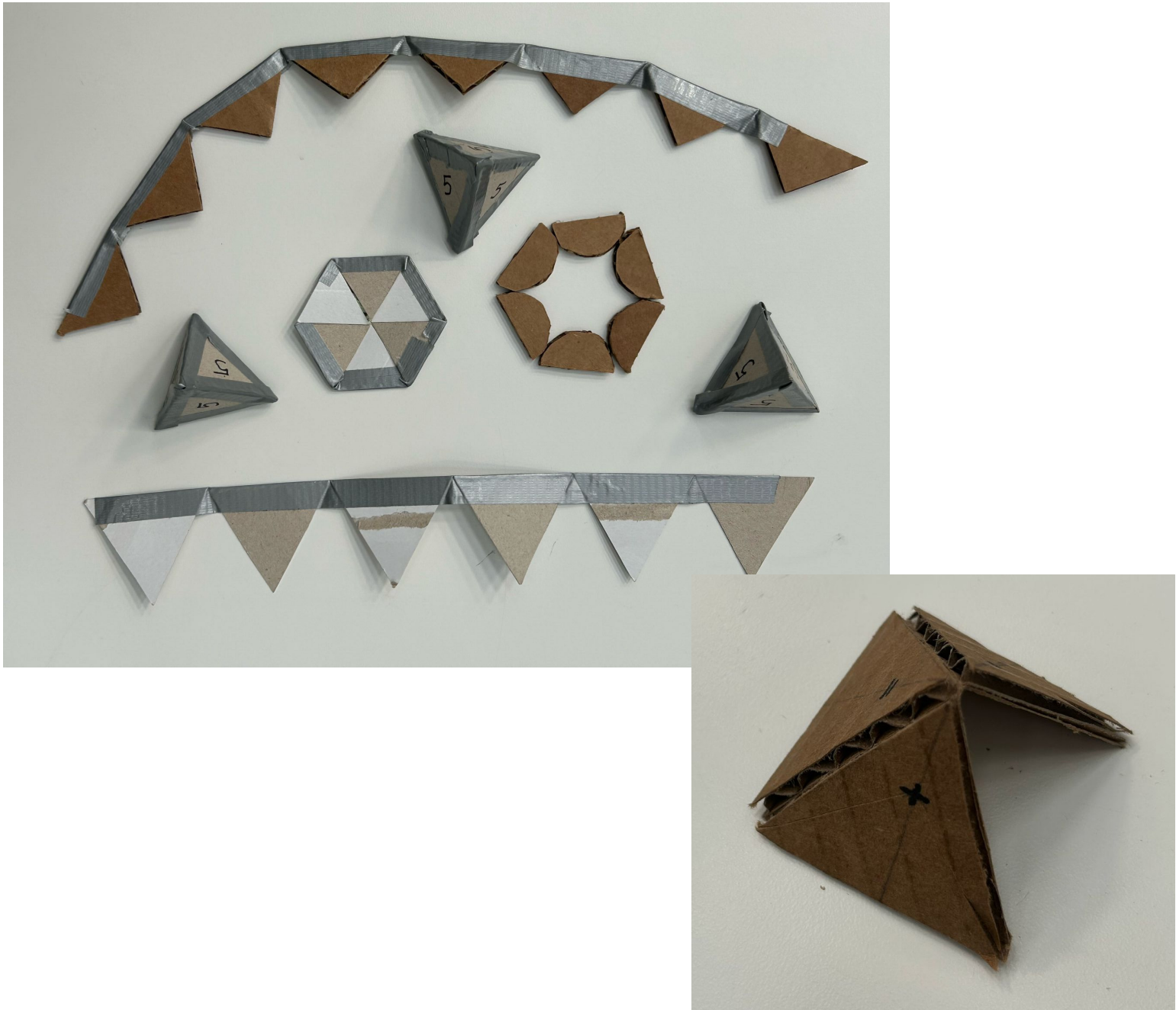


I2. Flexible measuring tool

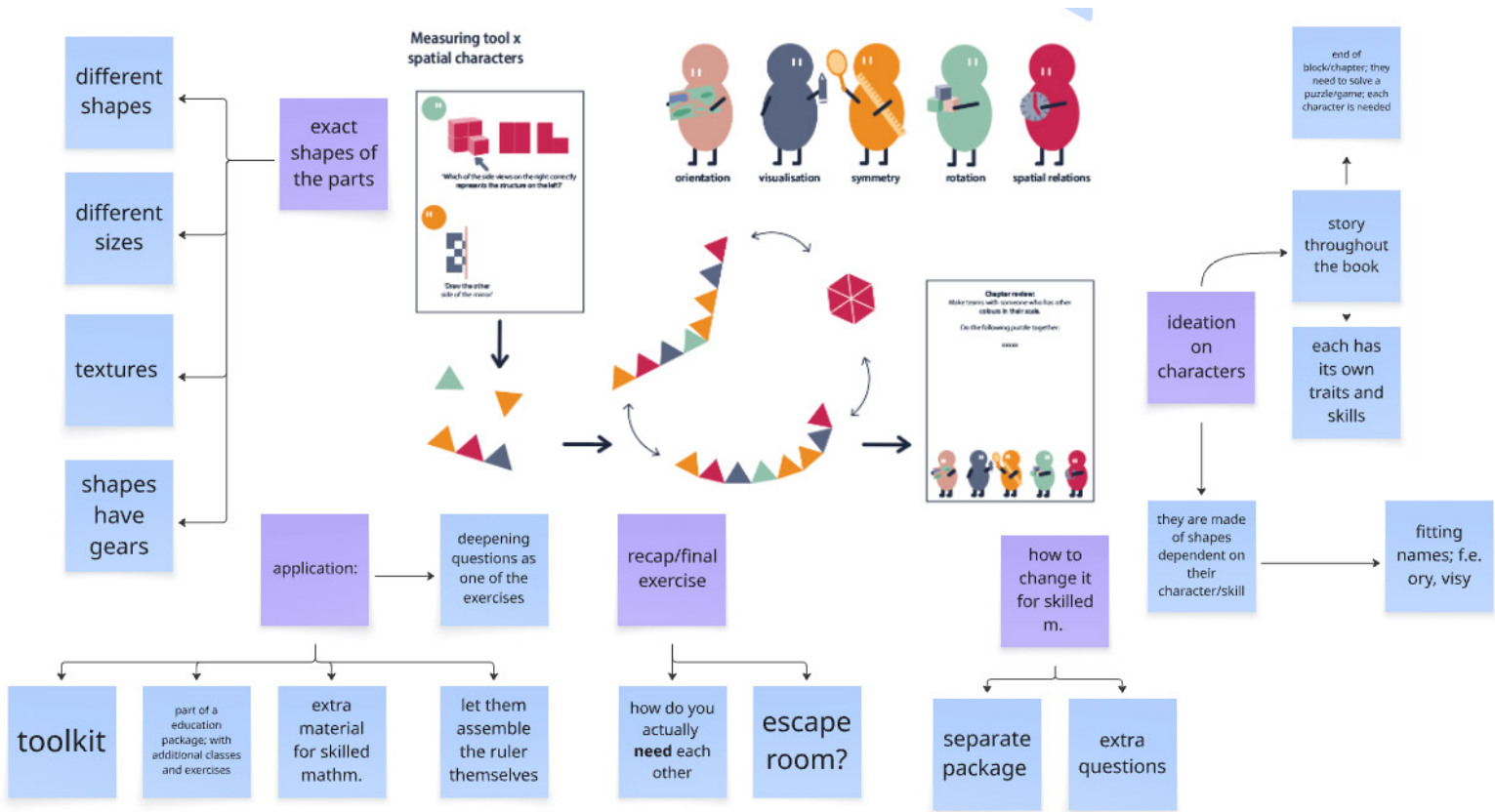
Measuring tool x spatial characters



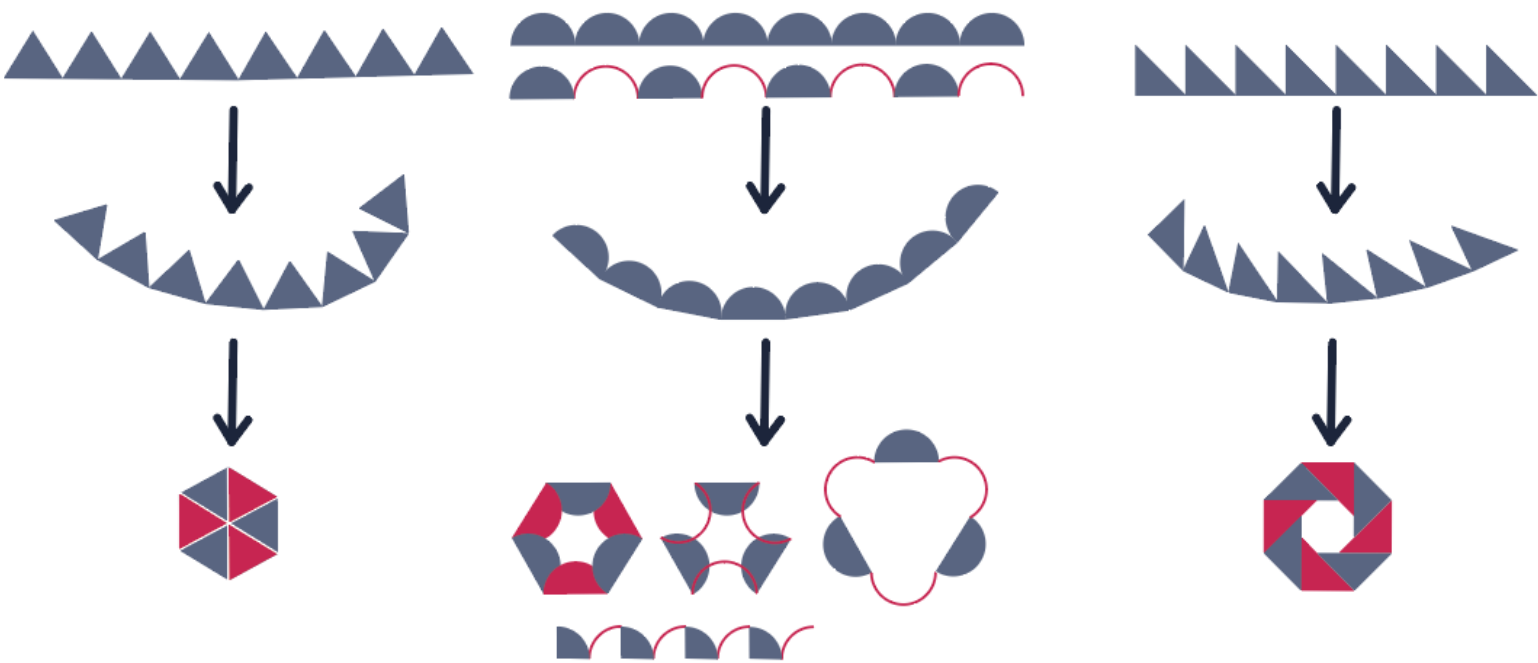
13. First prototypes; trying different shapes and sizes



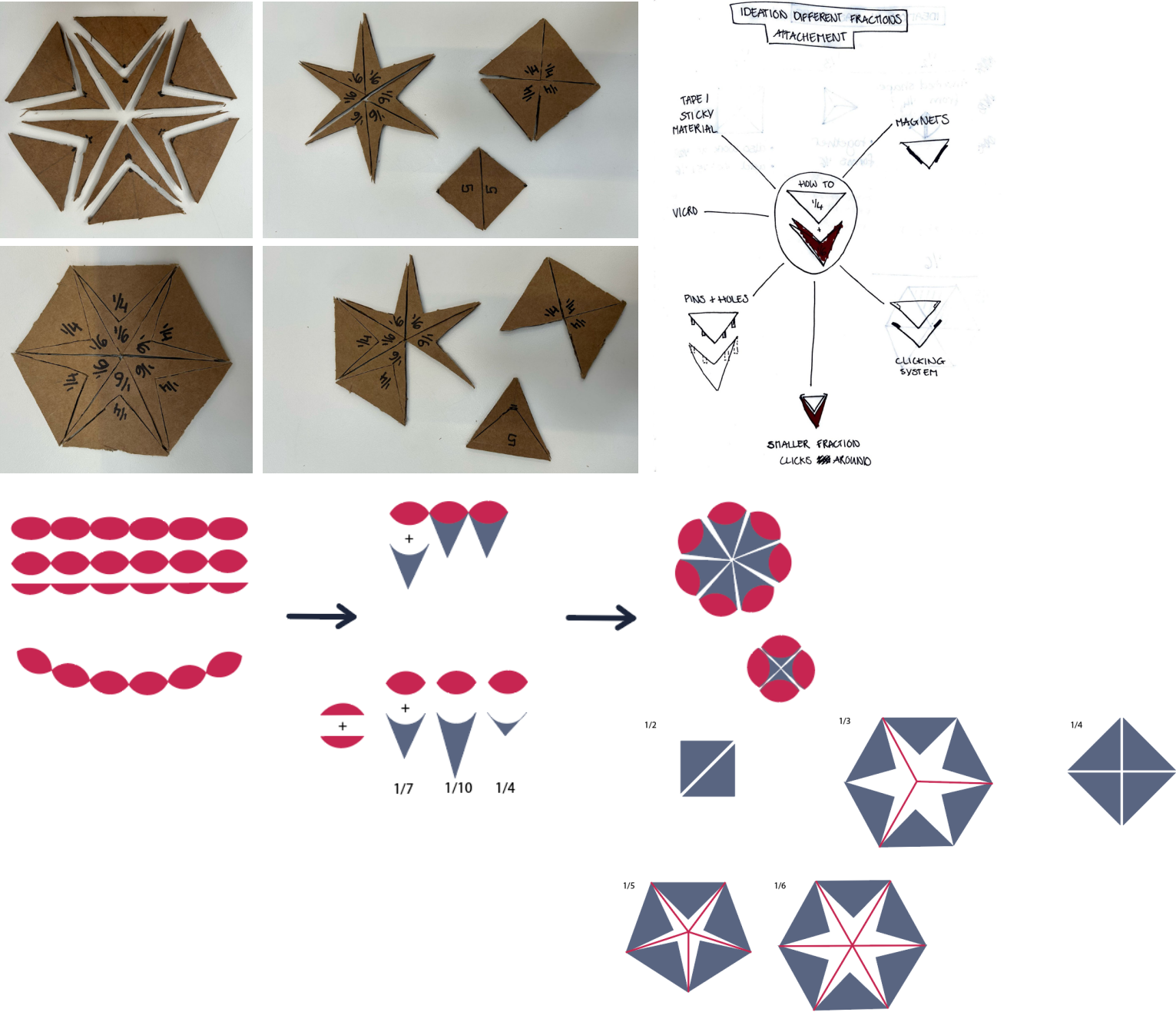
14. Improvements and opportunities of the flexible measuring tool



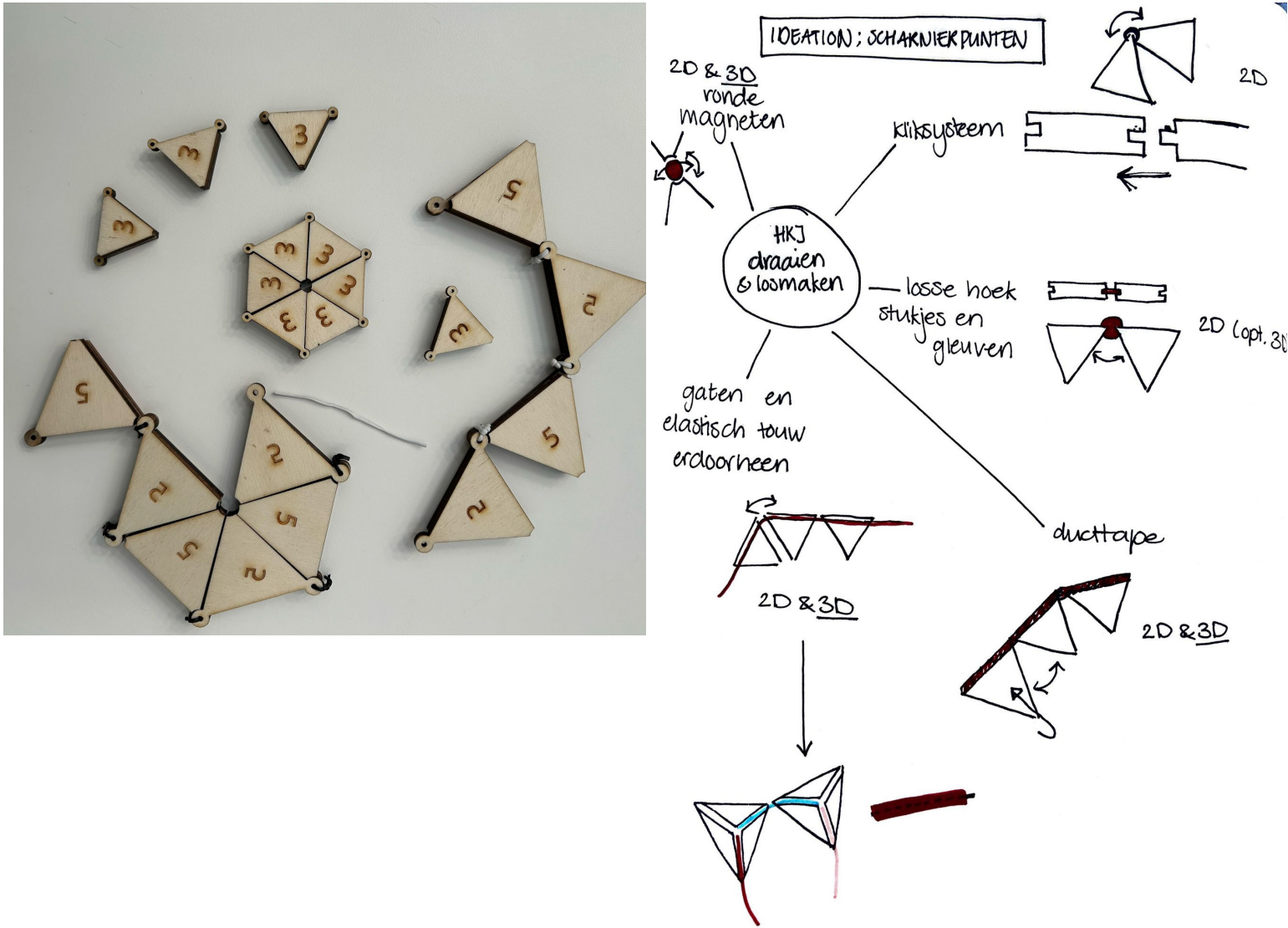
15. Ideating on shape: application of straight and bended ruler and fraction circle



16. ideating on fraction pieces and how to assemble them



17. First wooden prototypes and ideating on different connection methods



18. Second wooden iteration: prototype that was used to test with

