A CONSTITUTIVE MODEL FOR REINFORCED CONCRETE BASED ON STRESS DECOMPOSITION

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Abstract

A numerical model is proposed for reinforced concrete behavior which combines some commonly accepted ideas for modeling plain concrete, reinforcement and interaction behavior in a consistent manner. The basic idea is that the total stress that exists in a reinforced concrete element can be rigorously decomposed into individual contributions of the plain concrete, the reinforcement and the interaction between these constituents.

Introduction

In spite of more than two decades of research *numerical predictions* of the failure behavior of reinforced concrete structures still show a considerable scatter. Often the failure load and the ductility of reinforced concrete structures can only be computed accurately by tuning material or system parameters to the specific structure. The lack of reliability in predictions and the fact that the model parameters often have to be adapted to the structure in order to obtain a good agreement with experimental data, has motivated the search for improved models for reinforced concrete.

In this contribution we shall outline a phenomenological model for reinforced concrete behavior which unifies some established concepts for particulate phenomena like tension-softening and uses accepted guidelines, e.g., the CEB-FIP model code (1990). The fundamental assumption is that the total stress in a reinforced concrete element consists of the contributions of both constituents, concrete and reinforcement, and a so-called interaction stress contribution, which incorporates effects like dowel action and tension-stiffening. In the literature the tension-stiffening effect is usually referred to as the ability to gradually redistribute the load in a structure from concrete to steel under the formation of primary and secondary cracks. In our approach the *tension-stiffening effect* is conceived as the additional stiffness due to the interaction between concrete and reinforcement in the direction of the reinforcement, whereas the formation of primary and secondary cracks is modeled with the constitutive model of plain concrete in tension, the tension-softening model. In the presence of reinforcement the fracture-energy in this model is distributed over a tributary area by using the crack spacing as formulated in the CEB-FIP model code

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(1990), thus ensuring discretization-independent results.

General Framework

In this study it is assumed that the behavior of cracked, reinforced concrete can be obtained by superposition of the stiffness of plain concrete, a stiffness of the reinforcement and an additional stiffness due to interaction between concrete and reinforcement. This leads to the following summation of stress contributions

$$\sigma = \sigma_c + \sigma_s + \sigma_{ia} \tag{1}$$

with σ_c the stress contribution of the plain concrete, σ_s the contribution of the reinforcing steel, and σ_{ia} the interaction stress due to tension-stiffening, Figure 1.

The constitutive behavior of concrete will be modeled with a smeared model in which the damaged material is considered to be a continuum in which the notions of stress and strain apply. Consequently, the damage is also considered distributed and it is assumed that the damage can be represented by two internal parameters, κ_t in tension and κ_c in compression. These internal parameters are related to the released energy per unit damaged area by an equivalent length h. For tensile cracking the concept of the fracture energy G_f and an equivalent length, or crack band width, has extensively been used in finite element calculations. In this fashion the results which are obtained with a finite element analysis are objective with respect to mesh refinement, de Borst (1986). In this study, the concept of released energy and equivalent length is also proposed to model the compressive softening behavior by introducing a compressive fracture energy G_c and using the equivalent length h. However, it is recognized that the underlying failure mechanisms in compression may be more related to the volume of the elements than to a representative length of the elements.

The behavior in compression is modeled with a compression softening model as defined by a parabolic equivalent stress-equivalent strain diagram. In it, the Young's modulus of the concrete E_c has been assumed to be given by CEB-FIP model code regulations. The maximum equivalent strain κ_u has been related to the element size h and to the compressive fracture energy G_c, which is assumed to be a material parameter. In reinforced concrete, the compressive strength of the plain concrete is usually reduced because it is assumed that the compressive strength of plain concrete is affected by cracking in the lateral direction. Following Feenstra (1993) the compressive strength of reinforced concrete in biaxial tension-compression has been reduced by a constant factor of 20%.

In this study a linear softening diagram will be used for approximating the post-peak tensile behavior. This is sufficiently accurate for the envisaged class of applications (reinforced concrete panels and shear walls, plates and shells). The tensile strength of concrete $f_{ct,m}$ has been related to the compressive strength f_{cm}, in accordance with the CEB-FIP model code (1990). The fracture energy in tension, G_f, is assumed to be a material parameter and has been related to the compressive strength of the material f_{cm} and the maximum aggregate size $d_{\max}$, in accordance with the CEB-FIP model code (1990).

In reinforced concrete usually a number of cracks develop during the process of loading until the cracking process stabilizes and no further cracks develop in the structure. The crack spacing at stabilized cracking is determined mainly by the amount of reinforcement. It is assumed in this study that the material model for plain

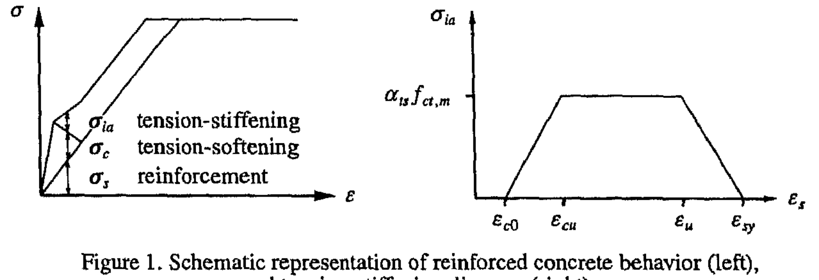

Figure 1. Schematic representation of reinforced concrete behavior (left), and tension-stiffening diagram (right).

concrete, based on fracture energy, can be applied to reinforced concrete with the total amount of fracture energy dissipated over the equivalent length. Because the fracture energy is assumed to be a material parameter, only the average crack spacing has to be determined.

The total amount of released energy at stabilized cracking is determined by the fracture energy of a single crack G_f and the average crack spacing l_s. In general, the dimensions of the finite elements in simulations of reinforced concrete structures, and thus the equivalent length h, are much larger than the average crack spacing, l_s, and it is assumed that the released energy can be determined by

$$G_f^{rc} = \min(G_f \ , \ G_f \frac{h}{l_s}) \tag{2}$$

with G_f the fracture energy of a single crack, h the equivalent length and l_s the average crack spacing. It is noted that if the equivalent length h is smaller than the average crack spacing l_s, the model will result in an overestimation of the released fracture energy and consequently, a too stiff behavior. The average crack spacing is a function of the bar diameter, the concrete cover and the reinforcement ratio according to the CEB-FIP model code (1990), which reads

$$l_s = \tfrac{2}{3}\, l_{s,max} = \tfrac{2}{3} [\, 2\, s_0 + \phi_s/(\gamma\, \rho_s) \,] \tag{3}$$

with s_0 the minimum bond length, ϕ_s the diameter of the reinforcement, a factor γ equal to four for deformed bars and equal to two for plain bars, and the reinforcement ratio $\rho_s = A_s/A_c$ with A_s the total area of reinforcement and A_c the cross area of the tensile member. The minimum bond length s_0 is usually taken equal to 25 mm in the absence of more precise data. For the treatment of plane two-dimensional structures like panels, reinforced with different layers of reinforcing grids in two orthogonal directions, the reader is referred to Feenstra (1993) or Feenstra and de Borst (1995).

After a stabilized crack pattern has developed, stresses are still transferred from reinforcement to concrete between the cracks due to the bond action which increases the total stiffness of the structure, Figure 1. The additional stress due to tension-stiffening is assumed to be given as a function of the strain in the direction of the reinforcement and will be active on the effective tension area. The interaction stress is assumed to be given by a trilinear function according to Cervenka, Pukl and Eligehausen (1990), Figure 1. The interaction model has been validated with experiments on reinforced concrete panels subjected to in-plane shear and normal loading (Feenstra 1993, Feenstra and de Borst 1995).

Application to Reinforced Concrete Shear Walls

The analysis of shear wall panels is a good example of the possible application of the developed models. The panel which will be presented in this study is panel S1, which has been tested at the E.T.H. Zürich by Maier and Thürlimann (1985). The panel is loaded initially by a vertical compressive force, and then loaded by a horizontal force until the experiment became unstable and the failure load had been reached.

Panel S1 is subjected to an initial vertical load of 433 $kN \equiv 2.5$ N/mm^2 which results in an initial horizontal displacement of 0.06 mm in the experiment. The calculated initial displacement is equal to $-80 \cdot 10^{-6}$ mm which indicates a possible eccentricity in the experimental set-up. After the initial vertical load, the horizontal load is applied with indirect displacement control (de Borst 1986). The load-displacement diagram of panel S1 is shown in Figure 2, which shows a reasonable agreement between the experimental and the calculated response.

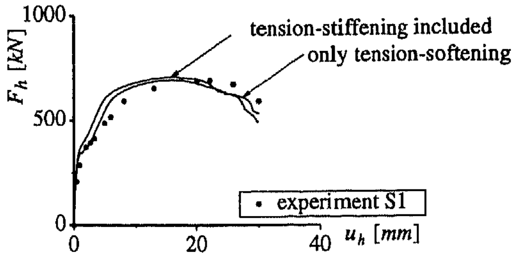

Figure 2. Load-displacement diagram for panel S1.

Appendix. References

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