

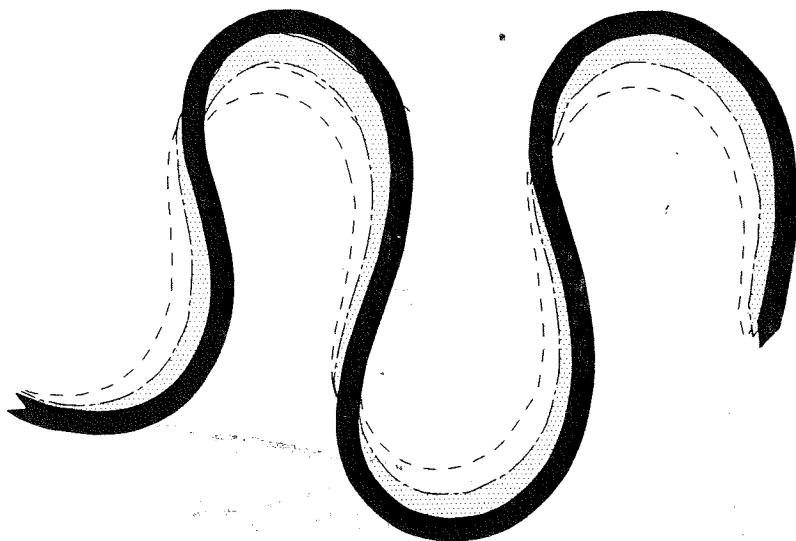
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Simulation of meandering river processes

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A. Crosato



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Delft
University of Technology

Faculty of Civil Engineering

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SIMULATION OF
MEANDERING RIVER PROCESSES

Technische Universiteit Delft
Faculteit CiTG
Bibliotheek Civiele Techniek
Stevinweg 1
2628 CN Delft

by

A. Crosato

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ABSTRACT

A simple mathematical model for the simulation of river meandering processes is presented and analysed. The model is schematized as follows: computation of steady-state flow field and riverbed topography; and computation of bank erosion rate as a function of the near-bank hydraulic and morphological properties. The model is linear for the prediction of flow field and river bed topography, while geometrical non-linearities arise from the bank erosion equation.

The flow and river bed model is derived by fully coupling flow field, bed topography and sediment transport and it (roughly) accounts for the secondary flow momentum convection.

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1. INTRODUCTION

1.1 General - Meanders are common in all kind of water courses from the smallest rills to the largest rivers. However, rivers with well-developed meanders are mainly found in alluvial plains.

Meandering rivers are characterised by mild slopes and large width-to-depth ratios (the latter are small if compared with those characteristic of braided rivers). They have a pool along the outer bank of bends and a point bar at the opposite side. The thalweg, or line of maximum depth, goes from pool to pool, so from one bank to the other. Cross-sections in bends tend to have a triangular shape, with the deepest point near the outer bank, whereas cross-sections in the straight reach which connects two opposite loops tend to be rectangular.

Flow in meander bends is typically three-dimensional. Secondary currents are produced by the interaction of the curvature and the vertical gradient of the primary flow velocity. (Direct measurements of secondary flows in meander bends were made among others by Thorne & Hey [1979]; Bathurst, Thorne & Hey [1977]). Both sectional shape and curvature vary throughout a bend, particularly at the entrance and exit, and the maximum velocity locus requires a finite distance from the bend entrance to cross from the inner bank to near the outer bank. Where the bend flow is stabilized the flow conditions are said to be "fully developed". Between two successive meander loops the secondary circulation due to the action of the upstream one decays and is replaced by the new circulation pattern of the one more downstream.

Meander bends are generally subject to a process of erosion and deposition: material is eroded along the outer banks whereas deposition takes place along the inner banks. Eroded material is transported towards the middle of the channel and deposited along the same bank but further downstream. The erosion-deposition process commonly yields a progressive development of the loop, which is the combination of translation and extension of the loop form ("meander migration"). The diversity of forms and sizes of migrating meanders along a river stretch is explained by the different circumstances in which meanders originate, by their story, and by the non-uniform erodibility of banks. Evidence of meander migration is provided by time sequential bank line diagrams, by which it is also possible to estimate its rate, see Figure 1.

Methods of measuring bank erosion include field measurements, comparison of time sequential maps and aerial photographs. A brief compilation of previous work was published by Hooke [1989] and a collection of measurements of bank erosion of 174 rivers was given by Brice [1982].

According to field observations (Hickin & Nanson [1984]) bank erosion rate tends to decrease with the increase in bend size. At the highest sinuosity the width of bends tends to stabilize (however, bank erosion proceeds in deforming more and more the loops). It is a common observation (Jefferson [1902], Bates [1939]) that a general relationship exists between the size of rivers and the size of bends. Friedkin [1945] from his experimental studies observed that for the same bank material and upstream boundary conditions (initiating angle of inflow) the size of bends becomes greater with the increase in discharge and/or slope.

In some cases meander growth proceeds to the point that one loop becomes overextended. At that time the loop can be shortcut by some discharge flowing across the neck (neck cut-off). Initially, only a fraction of flow crosses the neck but this fraction progressively increases and eventually the old bend way is abandoned by the flow. Meander cut-off have been recently studied by Klaassen & van Zanten [1989], and Biglari [1989] who have designed a simple criterion to estimate the possibility of occurrence of a natural short cut of the river course.

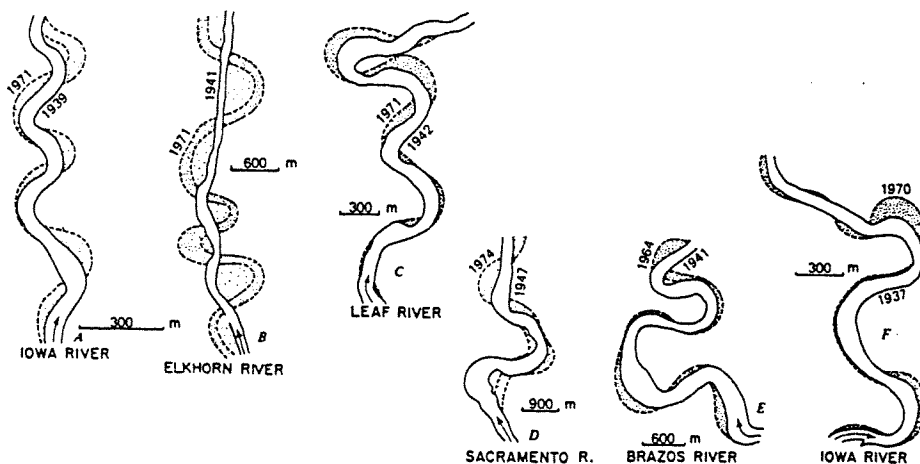


Figure 1 Time-sequential banklines, showing bank erosion and meander development (after Brice [1983]).

1.2 Review of existing theories on river meandering - The origin of meandering, and more in general the meander processes have been intriguing scientists for centuries. Several attempts have been made to understand the basic mechanisms of meander formation. Laboratory studies have been used intensively and have contributed considerably to the understanding of the phenomena involved (see among other Friedkin [1945], Leopold & Wolman [1957], Ackers & Charlton [1970], Martvall & Nillson [1982] and Fujita & Muramoto [1982]). On the theoretical side most of the works can be classified under one of the following categories:

- Regime Theory;
- Minimum Stream Power;
- Mathematical Modelling of Flow in Meander Bends;
- Stability Analysis;
- Steady Bed Deformation;
- Simulation of Meander migration.

The Regime Theory was mainly initiated by British scientists on the basis of an extensive collection of field observations from channels in India: Kennedy [1895], Lacey [1929], Lane [1937]. With the purpose to define the geometry of alluvial channels they derived several empirical relationships. In the original form, the regime-equations related the meander lengths only to the "formative" discharge. Later versions have modified this simple relations to include some dependences on sediment properties, Blench [1966].

The theory of Minimum Stream Power suggests that the regime geometry and slope of an alluvial river are dependent variables, because their formation in the long term depends on the water discharge and sediment load imposed upon the river by the drainage basin.

Minimum Stream Power states that the alluvial stream, width, depth and slope are so adjusted that the stream power per unit of channel length is a minimum, consistent with physical relations of flow continuity, resistance and sediment transport. This theory includes the statistical analysis known as the "Theory of Minimum Variance" exposed by Langbein & Leopold [1966]. More recently Yang [1976] stated that the time rate of energy expenditures explains the formation of meandering streams. Chang [1984] developed an analysis that establishes the maximum curvature for which a river does the least work in turning and explains meandering development by the river's tendency to seek minimum channel slope for the given conditions.

Mathematical Modelling of Flow in Meander Bends has been studied in detail first by Rozovskii [1957]. The problem is extremely complex and the Navier-Stokes equations must be simplified in order to obtain theoretical approximations. Many works published are based on the simplified hypothesis of small curvature and shallow water, while they do not consider the sidewall region of the cross section: Engelund [1974]; Kikkawa, Ikeda & Kitagawa [1976]; Odgaard [1981]; Falcon & Kennedy [1983]. De Vriend [1981] analyzed the flow in curved open channels including the entrance and exit regions of a bend and the sidewall region of the cross section. Struiksma [1983] with a simple model proved that the point bar at the initial part of circular channel bends is due to the lateral redistribution of water and sediment (non uniformity of the flow), which is originated by the flow disturbance caused by the bend entrance.

An approach to explain the origin of meandering, based on hydrodynamic equations, considers meandering as a stability problem. The general idea is to assume that small perturbations of channel bed, "Bar Theory", or of channel bends, "Bend Theory", may grow to a meandering pattern. This can be assessed by a Stability Analysis.

The so-called "Bar theory" is based on the assumption that the development of migrating alternate bars in a straight channel would eventually lead to the formation of meanders. The basic idea is that the flat alluvial bed of a straight channel, if under appropriate conditions, becomes unstable. Bed instability is assumed to be the cause of the alternate bar formation. Thus study into the cause of fluvial instability is carried out by theoretically examining submerged bed forms in a shallow channel with straight non-erodible banks. Meandering and braiding are treated as different degrees of the same instability phenomenon. This approach was among others supported by Hansen [1967]; Callander [1969]; Engelund [1970]; Parker [1976]; Fredsøe [1978].

More recently Ikeda, Parker & Sawai [1981] introduced the point of view of being a bend instability at the origin of meandering. They developed a stability analysis in terms of the growth rate of lateral bend amplitude in a mildly curved channel with erodible banks "Bend Theory". Bend instability does not occur in straight channels and differs from the alternate bar instability. Nevertheless, they found that the two mechanics operate at similar wave lengths. This led to the conclusion that migrating alternate bars gradually evolve into point bars.

Few years later Blondeaux & Seminara [1983], [1985], unified the Bar and Bend Theories. Deepening the analysis of Ikeda et al [1981], they demonstrated that bend growth and migrating alternate bars development are caused by two different phenomena: resonance, forced by the development of the channel sinuosity, and channelbed instability, respectively. The wave lengths of the approximately steady alternate bars selected by the bend resonance phenomenon were found to be about three times as large as those predicted by the bar instability. Furthermore, their order of magnitude was in agreement with the wave lengths of meanders at their initial stage.

Stating that the migration speed of alternate bars is too high to influence such a slow phenomenon as meander migration, Olesen [1983] and Struiksma et al [1985] based their works on another approach. They assumed that the development of a Steady Bed Deformation of the alternate bar type as a response to upstream flow disturbances (called "overshoot phenomenon"), rather than the presence of migrating alternate bars, is at the origin of river meandering. This assumption implies that meandering can be initiated by any flow disturbance, i.e. an obstacle, a change of curvature etc. They also found that the wave lengths of the steady bed oscillation were more or less in accordance with those of meanders at their initial stage.

Recently Seminara & Tubino [1988], [1989] have demonstrated the existence of interactions between migrating alternate bars and meander formation. Discussion on initiation of meandering is still open. Most probably both the overshooting phenomenon and migrating alternate bars are at the origin of meander formation. Bank erodibility and the relative intensity of the two phenomena will probably determine whether the overshoot phenomenon (steady bars) or the presence of migrating alternate bars will prevail in influencing the erosion pattern.

In the last years various models for the simulation of meander migration have been developed. The main ones are described in the next Section 1.3.

1.3 Review of existing theories on bank erosion processes - The progressive erosion of river banks is the cause of the lateral growth and downstream shift of river bends, phenomenon known as "meander migration". The rate of bank retreat is influenced by a number of different factors, such as: (i) near-bank hydraulic and morphological characteristics of the channel (velocity, shear stress, bed topography etc.); (ii) physical characteristics of the banks (bank elevation, cross-sectional shape, composition

of the soil, presence of vegetation, water content etc.); and (iii) quality of the eroding water (temperature, electrochemical qualities etc.). Banks subjected to ephemeral flow exhibit a variable resistance to erosion. Moreover, bank erosion products participate in the sediment transport process, (only that part which does not dissolve into wash load) therefore, they also influence the erosion process (Murphey Rohrer [1983], Neill [1987]).

Hickin [1978] suggested, on the basis of field observations, that the physical process causing bank erosion is the shear stress exerted on the bank by the main flow. With a different point of view Kitanidis & Kennedy [1984] suggested that the dominant mechanism responsible for bank erosion may be the secondary flow. However, being the intensity of the secondary flow much smaller than that of the main flow it is more widely accepted that bank erosion processes are governed by the main flow.

Ariathuri & Arulanandan [1978] developed a simple relation for the rate of erosion of cohesive banks. According to them the rate of bank retreat is directly proportional to the difference between the flow shear-stress on the bank and a critical one, below which no erosion occurs. Arulanandan et al [1980] defined some relations to assess the critical shear-stress and the coefficient of erodibility for cohesive soils.

The geometric evolution of particular bends was qualitatively explained among others by Daniel [1971], Hickin [1974] and Brice [1974].

More recently various models have been developed to simulate the meander migration processes of which the first ones are based on empirical relationships.

Hickin & Nanson [1975], from a work on the Beatton River in North Eastern British Columbia, suggested the existence of a relationship between bed migration rate and the ratio of half width to centre-line radius of curvature of river bends. Ferguson [1983] developed a simulation model of which the empirical support comes from the work of Hickin & Nanson [1975]. He demonstrated that asymmetric bends and secondary loops can develop from a regular waveform, such as the sine-generated curve, through the operation of a relationship between local channel curvature and migration rate. Since it involves only time and geometry, and not mass or force, Ferguson's meander migration model is a kinematic rather than a dynamic model.

Ikeda, Parker & Sawai [1981] assumed bank migration rate to be simply proportional to the difference between the near-bank tangential velocity excess and its reach-averaged value (the latter applies to the conditions of uniform flow in an infinite long straight channel). They coupled the erosion law with a steady-state flow and bed topography model in which the bed deformation applies for infinite long circular bends. Their meander migration model has successfully been applied to existing rivers, see Bridge [1983], Beck [1983], Johannesson & Parker [1985], and Parker & Andrews [1986].

Osman & Thorne [1988] developed a geomechanical river bank migration model adopting the relation designed by Ariathuri & Arulanaudan [1978].

Mosselman [1989] analysed the influence of bank erosion products on the bed development. His main conclusions are that in most cases (i) the computations of river bed topography and bank erosion can be decoupled, (ii) for most rivers the input of bank erosion products do not significantly affect the bank erosion process. He also developed a model to compute the retreat of cohesive banks in which width variation has also been taken into account, Mosselman [1990] (personal communication).

1.4 The proposed meander migration model - Simulation of meander migration requires a mathematical description of all the phenomena involved, i.e. the flow in channel bends and its effects on the bed development and on the erosion-deposition process. Furthermore, a meander migration model should also take into account the phenomena responsible for the occurrence of meander cut-offs. Cut-offs often occur during floods when also the flood plains discharge part of the water. For this reason the simulation of the occurrence of meander cut-offs implies the need to simulate the flow field in the flood plains of the river. However, the appearance of a meander cut-off invalidate measurements of bank erosion rate, in the area and downstream, which are necessary to calibrate the simulation models. Therefore for practical reasons meander cut-offs are not taken into account for the development of the proposed model. Besides, for the sake of simplicity, the applicability of the model is also restricted to those rivers which do not experience either aggradation or degradation and the back-water effects are not taken into account.

For the simulation of the planimetric changes of rivers a steady-state flow field and bed topography model is considered appropriate.

This is based on the assumption that neither the channel bed development is influenced by bank erosion (leading to the decoupling of the computation of flow and channel bed topography from the computation of channel migration) nor the latter is influenced by the presence of migrating alternate bars. This is true for those rivers characterized by relatively low bank erodibility. While, in addition, for rivers with a fully developed meandering pattern the second statement is generally true, as it has been demonstrated that alternate bar migration tends to stop when the river starts to meander, see Seminara & Tubino [1987].

The model is based on the assumption that under appropriate conditions the steady deformation of flow (and channelbed) originated by an upstream flow disturbance is of the alternate bar type. A deformation of this kind can be observed, for example, downstream of the entrance of a circular river bend where a steady bar is formed at the inner side (point bar) and a pool at the outer side, which are often followed by a typical steady oscillation of the bed, if the bend is long enough, Struiksmas et al [1985]. This bed configuration is called "overshoot phenomenon", because it can be regarded as an overshoot of the transverse bed slope with respect to that corresponding to the fully developed conditions (fully developed conditions apply for infinite long bends with a constant radius of curvature). The phenomenon is due to the redistribution of flow and sediment (non-uniformity of the flow) in the first part of bends which is mainly caused by the change of channel curvature at the bend entrance. This can be simulated mathematically only by fully taking into account the interactions between the movement of the water and that of the sediment. The overshoot of the transverse bed slope is found to have the same order of magnitude as the bed slope in fully developed bend flow conditions. For this reason it cannot be neglected when computing the bed topography in channel bends. The structure of the proposed model, called "MIANDRAS", is the same as the one adopted in Ikeda et al [1981], i.e. the model is splitted into two parts: computation of flow field and channel bed topography; and computation of bank erosion (river migration).

The model for the flow and channel bed prediction is based on the linear analysis of the 2-D model for riverbed topography developed by Koch & Flokstra [1980], Olesen [1983], Struiksmas et al [1985] (see Struiksmas & Crosato [1989]). The governing equations are obtained by fully coupling flow field, sediment transport and bed topography, and by taking into account the deviation of sediment transport direction from the direction of

the bed shear stress due to the transverse bed slope. The influence of the secondary flow momentum convection is also (roughly) taken into account by weighing the transverse bed friction (the importance of including it has been clearly demonstrated by Johanneson & Parker [1987]). The equations are simplified by linearization and by disregarding some smaller terms. The influence of the non-linear terms on the flow and bed prediction is a subject for further studies. The coupling of the model of the flow and channel bed prediction with a bank erosion equation makes the model non-linear on the whole (arising of geometrical non-linearities related to the planimetric changes). In principle the bank erosion model does not differ from that of Ikeda et al [1981]. The only difference is the insertion of an additional term to take into account the process of failure of vertical cohesive banks.

2. THE MATHEMATICAL MODEL

2.1 General - The mathematical model is schematized as follows: (i) computation of steady-state flow and channel bed deformation (flow and channel bed deformation model); and (ii) computation of time-dependent river migration (bank erosion model). The erosion scheme is depicted in Figure 2.

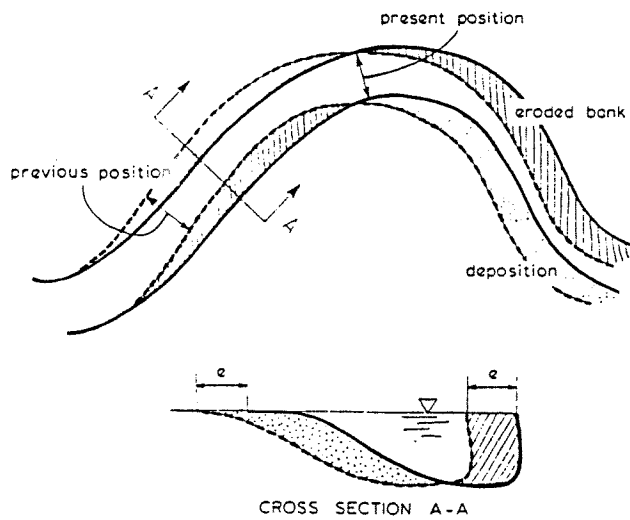


Figure 2 Erosion scheme (e = erosion rate)

This approach still allows for a varying discharge: the regime can be modelled by a step-wise approximation of the discharge hydrograph so that during one computational time-step the discharge is kept constant. However, the variation of discharge leads to a flow and bed deformation response which is lagging behind. Therefore, the model can be provided with a simple decoupled time-adaptation formulation which is governed by a characteristic time scale, see Section 2.2.5.

The flow and bed deformation model is derived from the linear version of the more complete non-linear model developed by Koch & Flokstra [1980] and

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Struiksmā et al [1985], which is also described and analysed in Struiksmā & Crosato [1989]. The linear analysis of the model shows that under the assumption of a mildly curved channel and for the flow conditions of most meandering rivers (damping range, see Struiksmā & Klaassen [1988]) the linearized set of equations still retains the main parameters influencing flow and bed deformation. Furthermore, when computing the steady-state conditions the equations can be further simplified. For this case the influence of the longitudinal bed slope on the sediment transport rate and of the secondary flow inertia are found not to influence significantly the results. The resulting simplified linear version is particularly appropriate for solving large-scale problems, while it still provides a fairly good prediction of the equilibrium configuration of flow and channel bed in meandering rivers. The prediction of flow and bed topography for finite amplitude meanders (strongly curved channel) has not yet theoretically been checked, while it has been proved acceptable when compared with experimental measurements, see Section 6.3.

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Similarly to Ikeda et al [1981] the flow-induced bank erosion rate is assumed simply proportional to the near-bank main velocity deformation, i.e. the deviation from the reach-averaged value. However, another term has been introduced to take into account the influence of bank elevation on the failure of vertical cohesive banks. Bank retreat due to bank failure has been assumed proportional to the near-bank water depth excess from the reach-averaged value, see Section 2.3. Consequently, the aim is to estimate the near-bank flow and water depth deformations to which bank erosion rate is related. Deviation from uniform flow conditions can be caused by: (i) local conditions, i.e. the flow deformation due to the curvature of the channel, etc.; (ii) upstream conditions, i.e. the deformation due to an obstacle, a change of channel curvature etc (overshooting phenomenon); and (iii) downstream conditions. The last are not taken into account here. The deviation from uniform flow conditions (infinite long straight channel) is given by the sum of two terms: a deformation due to the curvature of the channel axis (local conditions) and a deformation caused by the redistribution effects (upstream conditions).

The mathematical model is also based on the following assumptions:

- the channel width is constant in space and time;
 - the vertical pressure profile is hydrostatic;
 - shallow-flow approximation is appropriate;
 - mildly-curved channel approximation is appropriate;
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- the spatial variation of the hydraulic roughness (Chézy coefficient) can be neglected;
- the rate of sediment transport is assessable by local conditions (dominant bed load);
- the influence of grain sorting is insignificant (uniform bed material);
- the Froude number is small to moderate (rigid lid approximation);
- the bed slope is mild.

2.2 Flow and channel bed deformation model - The mathematical model for the computation of flow and channel bed deformations has been derived here with a quasi-steady approach, i.e. the interaction between steady water motion and time-dependent bed evolution is considered. This approach is widely accepted for computations of time-dependent river bed deformation in flows with small to moderate Froude number (Jansen [1979]). When neglecting the time-dependent terms the equations describe the time-averaged (equilibrium) configuration of flow field and bed topography. The steady-state equations are adopted for the computations of meander migration, while the time-dependent ones are used for the analysis of the time scale of the bed development process.

2.2.1 The basic equations - The flow deformation is assessed by solving the two-dimensional depth-averaged continuity and momentum equations using the co-ordinate system depicted in Figure 3:

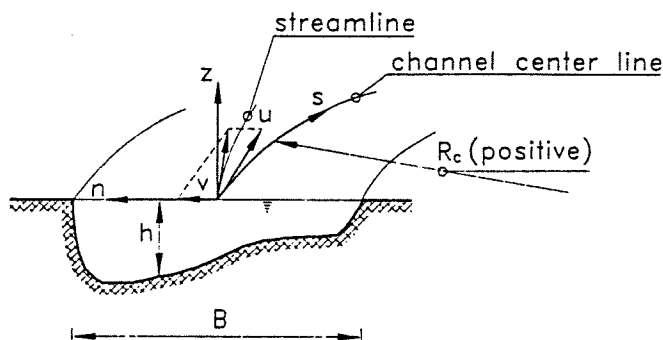


Figure 3 Co-ordinate system and velocity components for depth-averaged flow

$$\frac{\partial(hu)}{\partial s} + \frac{\partial(hv)}{\partial n} + \frac{hv}{(R_c + n)} = 0 \quad (2.1)$$

$$u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial n} + \frac{uv}{(R_c + n)} + g \frac{\partial z_w}{\partial s} + \frac{g}{C^2} \frac{u \sqrt{u^2 + v^2}}{h} = 0 \quad (2.2)$$

$$- \frac{u^2}{(R_c + n)} + u \frac{\partial v}{\partial s} + v \frac{\partial v}{\partial n} + g \frac{\partial z_w}{\partial n} + \frac{g}{C^2} \frac{v \sqrt{u^2 + v^2}}{h} = 0 \quad (2.3)$$

in which s = longitudinal co-ordinate; n = transverse co-ordinate; u = depth-averaged tangential velocity; v = depth-averaged transverse velocity; h = water depth; R_c = radius of curvature of the channel centre-line; z_w = water level; g = acceleration due to gravity; C = Chézy coefficient.

The bed shear-stress is related to the depth-averaged flow velocity according to the Chézy's equations. The stream-line curvature is approximated by the following expression (de Vriend, [1981]):

$$\frac{1}{R_s} = \frac{1}{(R_c + n)} + c \quad \text{where } c = - \frac{1}{u} \frac{\partial v}{\partial s} \quad (2.4)$$

in which R_s = stream-line radius of curvature; R_c = centre-line radius of curvature; c = curvature induced by the non uniformity of the flow.

To simulate the interaction between flow and sediment, the flow equations are combined with the time-dependent depth-averaged sediment balance equation:

$$\frac{\partial z_b}{\partial t} + \frac{\partial S_s}{\partial s} + \frac{\partial S_n}{\partial n} + \frac{S_n}{(R_c + n)} = 0 \quad (2.5)$$

in which z_b = bed level; t = time; S_s and S_n = components of the volumetric sediment transport, including pores, per unit of width in the s and n direction, respectively. These components can be expressed as:

$$S_s = S_t \cos \alpha \quad (2.6)$$

$$S_n = S_t \sin \alpha \quad (2.7)$$

in which S_t = total volumetric sediment transport including pores, per unit of width; and α = angle between s -direction and sediment transport direction. The transport S_t can be computed with various transport formulas,

as for example the Engelund-Hansen's [1967], and the Meyer-Peter and Müller's [1948].

Due to the gravity force acting on the grains moving along a sloping bed, the direction of the sediment transport does not coincide with the direction of the bed shear-stress. It can be derived from the bed shear-stress direction with a correction which takes into account the transverse bed slope influence (no longitudinal bed slope influence is considered here). The following formula is adopted, according to van Bendegom [1947]:

$$\tan \alpha = \frac{\sin \delta - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n}}{\cos \delta} \quad (2.8)$$

in which δ = angle between s-direction and bed shear stress direction; $f(\theta)$ = weighing function for the influence of the sloping bed; and θ = the Shields parameter, defined by:

$$\theta = \frac{u^2 + v^2}{C^2 \Delta D_{s0}} \quad (2.9)$$

in which Δ = relative submerged density of the sediment; and D_{s0} = the median grain size.

For a review on the weighing function $f(\theta)$ reference is made to Odgaard [1981]. In this paper the following formula is adopted (see also Zimmerman and Kennedy, [1978]):

$$f(\theta) = \frac{0.85}{E} \sqrt{\theta} \quad (2.10)$$

in which E = calibration coefficient (for laboratory channels $E = 0.5$ according to Van Mierlo [1986], for rivers $E = 1.0$).

For the direction, δ , of the bed shear-stress the following expression which includes the effect of the spiral motion (Koch and Flokstra, [1980]) is considered:

$$\delta = \arctan \left(\frac{v}{u} \right) - \arctan \left(A \frac{h}{R_*} \right) \quad (2.11)$$

in which A = coefficient which weighs the influence of the spiral motion, having the following expression: $A = 2\alpha_1 \kappa^{-2} (1 - \sqrt{g}/\kappa C)$ when the velocity profile is logarithmic (Jansen, [1979]), where κ = Von Karman constant and

α_1 = calibration coefficient (values between 0.4 and 1.2); R_* = the effective local radius of curvature of the stream-line. To compute R_* the inertia of the spiral motion should be taken into account, but it is neglected here. This means that R_* is approximated by the local radius of curvature of the stream-line R_s , which can be assessed from the flow field (Eq. [2.4]).

The expression for the deviation of sediment transport direction from s-direction, α (for small δ) becomes:

$$\tan \alpha = \frac{v}{u} - \frac{Ah}{R_s} - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} \quad (2.12)$$

2.2.2 Linearization of the equations - For the linearization of the equations every quantity is assumed to be given by the sum of two terms: the reach-averaged value (infinite long straight channel); and a perturbation term, e.g.:

$$h = h_0 + H'; \quad u = u_0 + U' \text{ etc.}$$

$$\text{being } \frac{H'}{h_0} = \epsilon; \quad \frac{U'}{u_0} = \epsilon, \text{ with } \epsilon \ll 1$$

The (depth-averaged) transverse velocity is assumed of first order:

$$v = V', \text{ being } \frac{V'}{u_0} = \epsilon.$$

The parameters describing sediment transport are assumed as follows:

$$S_s = S_0 + S'_s$$

$$\text{with } S'_s \ll S_0$$

Where:

$$S_0 \propto u_0^b$$

$$S'_s = S_0 \left(b \frac{U'}{u_0} \right)$$

The exponent b originates from the linearization of the sediment transport formulas and it is assumed to be a constant, viz.:

$$b = \frac{u_o}{S_o} \frac{\partial S_o}{\partial u_o} \quad (2.13)$$

It implies that the sediment transport formula is approximated by a power law in which b expresses the non-linearity of the sediment transport rate. Adopting the Engelund & Hansen transport formula b is constant and equal to 5, whereas adopting the Meyer-Peter & Müller transport formula b is variable and depends on the Shields parameter. If the value of the Shields parameter lies close to its critical value (incipient motion), the parameter b is large and strongly variable. This is the case for most gravel bed rivers.

The transverse sediment transport is assumed small with respect to the longitudinal one:

$$S_n = S'_n, \text{ being } \frac{S'_n}{S_s} = c$$

The angle between s -direction and sediment transport direction is also small, therefore:

$$\begin{aligned} \tan \alpha &= \alpha \\ \alpha &= \alpha' \\ S'_n &= S_o \alpha' \end{aligned}$$

Linearization of Eqs. [2.1]...[2.4] yields the following (see Appendix B):

- zero order set of equations (reach-averaged values) which applies to the conditions of uniform flow in an infinite long straight channel:

$$h_o u_o = \frac{Q}{B} \quad (\text{continuity equation}) \quad (2.14)$$

$$i_{os} = - \frac{u_o^2}{C^2 h_o} \quad (\text{Chézy equation}) \quad (2.15)$$

in which: Q = discharge; B = channel's width (it is assumed to be a constant all along the channel and with time); i_{os} = longitudinal water surface slope;

- first order equation for the flow deformation (in which the smaller terms have been disregarded):

$$\frac{\partial^2}{\partial s \partial n} (U') + \frac{1}{\lambda_w} \frac{\partial}{\partial n} (U') =$$

$$= \frac{u_o}{2\lambda_w h_o} \frac{\partial}{\partial n} (H') - u_o \frac{\partial}{\partial s} \left(\frac{1}{R_c} + c' \right) - \frac{u_o}{2\lambda_w} \left(\frac{1}{R_c} + c' \right) \quad (2.16)$$

$$\text{in which } \lambda_w = (C^2 h_o)/2g; \quad c' = -\frac{1}{u_o} \frac{\partial V'}{\partial s}.$$

(The equation for the flow deformation is obtained by eliminating the water level term, z_w , by cross differentiation of Eqs. [2.2] and [2.3]. The derivation is described in Appendix B.)

Combination and linearization of Eqs. [2.1], [2.5], [2.12], applying a rigid lid approximation, yield the following:

- first order equation for the channel bed deformation:

$$\frac{h_o}{S_o} \frac{\partial}{\partial t} (H') + \frac{\partial}{\partial s} (H') - \frac{h_o}{f(\theta_o)} \frac{\partial^2}{\partial n^2} (H') =$$

$$= (b-1) \frac{h_o}{u_o} \frac{\partial}{\partial s} (U') - Ah_o^2 \frac{\partial}{\partial n} \left(\frac{1}{R_c} + c' \right) \quad (2.17)$$

(The derivation of the equation for the bed deformation is also described in Appendix B.)

The equations for the flow and channel bed deformation are both linear and non-homogeneous, the non-homogeneity arising from the presence of terms which are a function of the channel centre-lines curvature only.

2.2.3 Near-bank flow velocity and water depth deformations - The aim is to estimate the near-bank flow velocity and water depth deformations to which the bank erosion rate is related (see Section 2.3). A transverse shape of the deformations (which is a solution of the equations) will be imposed to the system, so that the equations will become mono-dimensional (the variables will be a function of s only). Once the transverse shape of the deformations is known it is possible to derive their near (eroding) bank value.

The total deformation of flow velocity and water depth is given by the sum of two terms: deformation caused by the local channel curvature (a solution of the non-homogeneous equations); and deformation caused by the redistribution of flow and sediment, originated for example from upstream changes of the channel curvature, which is assumed to be of the alternate bar type (solution of the homogeneous equations).

Solution of the non-homogeneous equations

A solution of the non-homogeneous equations is the deformation caused by the local channel curvature in fully developed bend flow conditions which are present along infinite long channel bends with constant radius of curvature, as depicted in Figure 4. A characteristic of these conditions is that the derivatives $\partial/\partial s$ of h and u vanish, implying that the variables are a function of local conditions only; the deviation of sediment transport direction from the s -direction and the depth-averaged transverse velocity are both equal to zero: $\alpha' = 0$ and $V' = 0$.

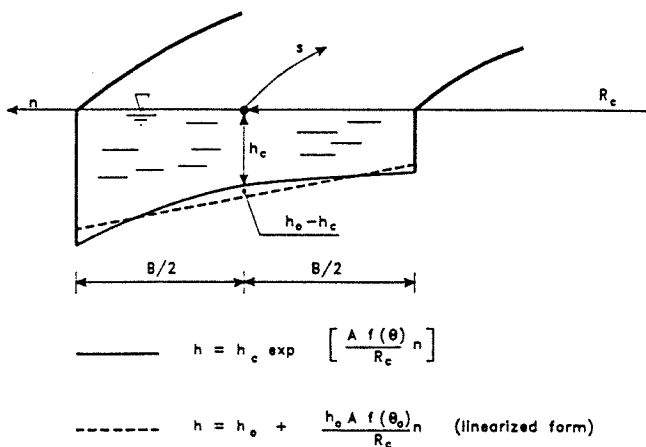


Figure 4 Schematization of bed topography in fully developed bend conditions

The equations describing the fully developed bend flow conditions are derived in Appendix A and read:

$$\frac{\partial h}{\partial n} = \frac{A f(\theta)}{(R_c + n)} h \quad (2.18)$$

$$\frac{\partial u}{\partial n} = \frac{u}{2h} \frac{\partial h}{\partial n} - \frac{u}{2(R_c + n)} \quad (2.19)$$

Eqs. [2.18] and [2.19] are then simplified by linearization, the result is:

$$\frac{\partial h'_o}{\partial n} = A \frac{f(\theta_o) h_o}{R_c} \quad (2.20)$$

$$\frac{\partial u'_o}{\partial n} = \frac{u_o}{2h_o} \frac{\partial h'_o}{\partial n} - \frac{u_o}{2R_c} \quad (2.21)$$

In which $h'_o = u'_o$ = water depth and velocity deformations in fully developed bend flow conditions, respectively.

The equation for the flow deformation, Eq. [2.16], taking into account Eq. [2.21], becomes:

$$\frac{\partial}{\partial s} \left(\frac{u_o}{R_c} + \frac{\partial u'_o}{\partial n} \right) = 0 \quad (2.22)$$

And the equation for the bed deformation, Eq. [2.17], taking into account Eqs [2.20] and [2.21], becomes:

$$\frac{\partial h'_o}{\partial s} = (b-1) \frac{h_o}{u_o} \frac{\partial u'_o}{\partial s} \quad (2.23)$$

For fully developed bend flow both Eqs. [2.22] and [2.23] are satisfied (being the derivatives $\partial/\partial s$ both equal to zero by definition). Therefore fully developed bend flow-conditions are a solution of the non-homogeneous equations.

The expressions for the near-bank value of the deformations corresponding to fully developed bend conditions $\hat{u}_o = (u'_o)_{n=B/2}$ and $\hat{h}_o = (h'_o)_{n=B/2}$, are obtainable from Eqs. [2.20] and [2.21]. By assuming that the centre-line values of velocity and water depth are equal to the respective reach-averaged values (see Appendix A), the near-bank velocity and water depth deformations are:

$$\hat{u}_0 = \left(\frac{1}{2} \frac{u_0}{h_0}\right) \hat{h}_0 - \frac{u_0}{2} \frac{1}{R_c} \left(\frac{B}{2}\right) \quad (2.24)$$

$$\hat{h}_0 = f(\theta_0) h_0 A \frac{1}{R_c} \left(\frac{B}{2}\right) \quad (2.25)$$

in which $\hat{h}_0$ and $\hat{u}_0$ = near bank-values of the deformation of water depth and main velocity, respectively.

The error induced by assuming the centre-line values of velocity and water depth as equal to the corresponding reach-averaged values is relatively small for the case of mildly curved channels. The difference between the reach-averaged water depth, h_0 , and the centre-line water depth, h_c , can be estimated by the following expression:

$$(h_0 - h_c) = \frac{1}{24} \left(\frac{Af(\theta_0)B}{R_c}\right)^2 h_0 \quad (2.26)$$

Solution of the homogeneous part of the equations

The homogeneous parts of Eqs. [2.16] and [2.17] are:

$$\frac{\partial^2 u'}{\partial s \partial n} + \frac{1}{\lambda_w} \frac{\partial u'}{\partial n} = \frac{u_0}{2\lambda_w h_0} \frac{\partial h'}{\partial h} - u_0 \frac{\partial c'}{\partial s} - \frac{u_0}{2\lambda_w} c' \quad (2.27)$$

and

$$\frac{h_0}{S_0} \frac{\partial h'}{\partial t} + \frac{\partial h'}{\partial s} - \frac{h_0}{f(\theta_0)} \frac{\partial^2 h'}{\partial n^2} = (b-1) \frac{h_0}{u_0} \frac{\partial u'}{\partial s} - A h_0^2 \frac{\partial c'}{\partial n} \quad (2.28)$$

respectively (when the channel is straight the equations are simply homogeneous):

in which u' and h' = velocity and water depth perturbations (straight channel).

The solution of the homogeneous part of the equations is a harmonic function. For meandering rivers the most suitable transverse shape is that of the alternate bar type.

The corresponding channel's perturbed bed is schematized in Figure 5.

Furthermore u' and h' must fulfill the following conditions:

$$\left. \frac{\partial h'}{\partial n} \right|_{n = \pm B/2} = 0 \quad \text{and} \quad \int_{-B/2}^{+B/2} h' \, dn = 0 \quad (\text{similarly for } u')$$

while the condition of impermeable side walls puts constraints on v' :

$$v' \Big|_{n = \pm B/2} = 0$$

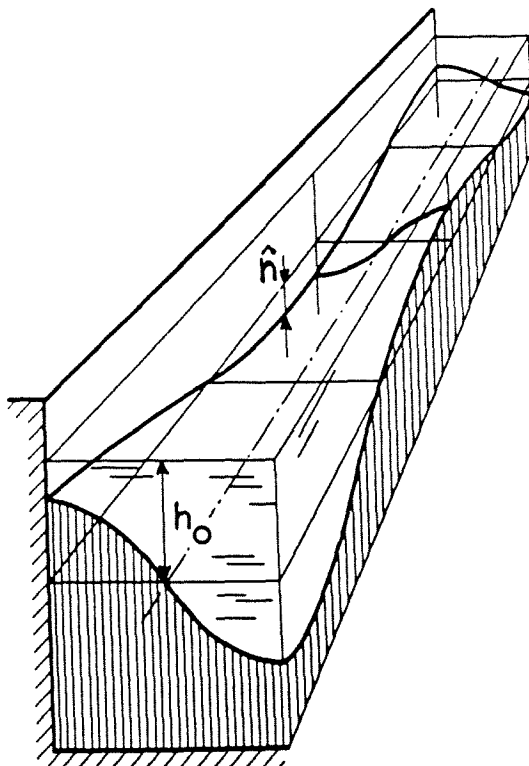


Figure 5 Schematization of the channel's perturbed bed

The following expressions describe the transverse shape of the perturbations:

$$h' = \hat{h} \sin(k_B n) \quad (2.29)$$

$$u' = \hat{u} \sin (k_B n) \quad (2.30)$$

$$v' = \hat{v} \cos (k_B n) \quad (2.31)$$

$$\text{while} \quad \hat{c} = - \frac{1}{u_o} \frac{\partial \hat{v}}{\partial s}$$

in which the amplitudes of h' , u' , v' etc. are a function of s and t . They correspond to the near-bank value of water depth and main velocity of the perturbations, respectively; k_B = wave number in the transverse direction. It can be written as follows:

$$k_B = \frac{m\pi}{B} \quad (2.32)$$

with $m = 1, 2, 3, \dots$ (mode that determines the transversal pattern).

$m = 1$ applies for the case of meandering rivers (bed and flow deformation of the alternate bar type). The higher harmonics are found to be negligible for the case of a mildly curved channel, Taal [1989].

Inserting the harmonic form of the perturbations in Eqs. [2.27] and [2.28] yields the following expressions for the variation of the near-bank value of the perturbations with s :

$$\frac{\partial \hat{u}}{\partial s} + \frac{\hat{u}}{\lambda_w} = \left[\frac{1}{2\lambda_w} \frac{u_o}{h_o} \right] \hat{h} - \frac{u_o}{k_B} \frac{\partial \hat{c}}{\partial s} - \frac{u_o}{2\lambda_w k_B} \hat{c} \quad (2.33)$$

$$\frac{h_o}{S_o} \frac{\partial \hat{h}}{\partial t} + \frac{\partial \hat{h}}{\partial s} + \frac{\hat{h}}{\lambda_s} = \frac{h_o}{u_o} (b-1) \frac{\partial \hat{u}}{\partial s} + A h_o^2 k_B \hat{c} \quad (2.34)$$

$$\text{with } \lambda_s = \frac{f(\theta_o)}{k_B^2 h_o} = \frac{1}{(m\pi)^2} h_o \left(\frac{B}{h_o} \right)^2 f(\theta_o)$$

Total solution of the equations

Being the equations linear and non-homogeneous, the total solution is equal to the sum of the solution of the homogeneous part of the equations and of one solution of the non-homogeneous equations. Therefore the total deformations of velocity and water depth are: $U' = u_o' + u'$, $H' = h_o' + h'$.

The sum of the near-bank perturbations leads to the final expressions describing the total near-bank deformation (the total bed deformation for a curved channel is schematized in Figure 6):

$$\frac{\partial U}{\partial s} + \frac{U}{\lambda_w} = H \left(\frac{1}{2\lambda_w} \frac{u_o}{h_o} \right) - \frac{u_o}{k_B} \frac{\partial}{\partial s} \left[\hat{c} + \frac{1}{R_c} \left(\frac{m\pi}{2} \right) \right] - \frac{(2-\sigma)}{2\lambda_w} \frac{u_o}{k_B} \left[\hat{c} + \frac{1}{R_c} \left(\frac{m\pi}{2} \right) \right] \quad (2.35)$$

$$\frac{h_o}{s_o} \frac{\partial H}{\partial t} + \frac{\partial H}{\partial s} + \frac{H}{\lambda_s} = \frac{h_o}{u_o} (b-1) \frac{\partial U}{\partial s} + A h_o^2 k_B \left[\hat{c} + \frac{1}{R_c} \left(\frac{m\pi}{2} \right) \right] \quad (2.36)$$

with $U = \hat{u}_o + \bar{u}$; and $H = \hat{h}_o + \bar{h}$

in which U, H = near-bank (total) deformation of main velocity and water depth, respectively.

The coefficient $(2-\sigma)$ is added to the transverse bed friction term in order to simulate the effects of the secondary flow momentum convection, not previously included in the model. The importance of including this has been demonstrated among others by Kalkwijk & de Vriend [1980]; de Vriend [1981]; Johannesson & Parker [1987]. With $\sigma = 2$ the transverse bed friction term becomes equal to zero, with $\sigma > 2$ the same term becomes negative. In the latter case the effect is a transverse shift of the maximum velocity locus towards the outer bend, effect comparable to that of the secondary flow momentum convection. The value of coefficient σ (to be calibrated) is expected to lie between 2 and 4.

A similar approach to simulate the effects of the secondary flow momentum convection was followed by Johannesson & Parker [1989]. Struiksma [1990a, personal communication] derived an expression for σ from their simple, but verified, theory:

$$\sigma = 1 + 90 \frac{C}{\sqrt{g}} \left(\frac{h_o}{B} \right)^2 \quad (2.37)$$

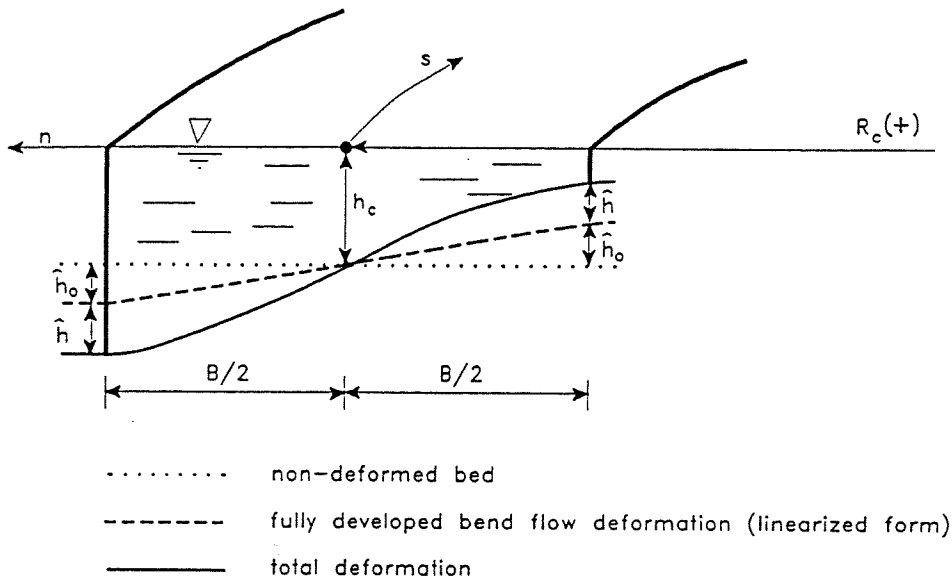


Figure 6 Total bed deformation in a curved channel

Equations [2.35] and [2.36] can be re-written as a system of two second-order differential equations, one for the water depth deformation only and one for the tangential velocity deformation only. The two equations are identical in their homogeneous part, whereas they are different in the source term, the latter being a function of the curvature terms and other local parameters. The equation for the water depth is given below:

$$\frac{\partial^2 H}{\partial s^2} + \frac{\partial H}{\partial s} \left[\frac{1}{\lambda_s} - \frac{(b-3)}{2\lambda_w} \right] + \frac{H}{\lambda_s \lambda_w} + \frac{h_o}{S_o} \frac{\partial^2 H}{\partial s \partial t} + \frac{h_o}{\lambda_w S_o} \frac{\partial H}{\partial t} = F(\text{curvature}) \quad (2.38)$$

in which $F(\text{curvature})$ = a function of the curvature terms and of local parameters, having the expression:

$$F(\text{curvature}) = -\frac{h_o}{k_B} (b-1) \frac{\partial^2 \Gamma}{\partial s^2} + \left[A h_o^2 k_B - \frac{(2-\sigma)(b-1)h_o}{2 \lambda_w k_B} \right] \frac{\partial \Gamma}{\partial s} + \frac{A h_o^2 k_B}{\lambda_w} \Gamma \quad (2.39)$$

$$\text{with } \Gamma = \hat{c} + \frac{1}{R_c} \left(\frac{m\pi}{2} \right)$$

The time-dependent bed deformation model, Eq. [2.38], is analyzed in Sub-section 2.2.5 to estimate the time-scale of the bed deformation development.

The steady-state model (obtained neglecting the time-dependent terms from Eq. [2.36]) is adopted for the simulation of the meandering process. It is analyzed in Sub-Section 2.2.4, and in the Chapters 3 and 4.

2.2.4 Steady-state model - The steady-state flow and bed deformation model adopted for the simulation of meander migration is obtained neglecting the time dependent terms from Eq. [2.36]. The equation describing the near-bank water depth deformation then becomes:

$$\frac{\partial H}{\partial s} + \frac{H}{\lambda_s} = \frac{h_o}{u_o} (b-1) \frac{\partial U}{\partial s} + A h_o^2 k_B \left[\hat{c} + \frac{1}{R_c} \left(\frac{m\pi}{2} \right) \right] \quad (2.40)$$

while the equation describing the near-bank tangential velocity deformation remains Eq. [2.35] (no time-dependent terms).

The form of Eqs. [2.35] and [2.40] shows that λ_w and λ_s are the characteristic adaptation lengths of tangential velocity and bed topography deformations, respectively. The ratio between the adaptation lengths, $\alpha = \lambda_s / \lambda_w$, can be regarded as the parameter characteristics of the interaction between flow and bed-deformation development (redistribution effects), and for this reason it is called: "interaction parameter". It is a characteristic of the channel and can be written as:

$$\alpha = \frac{2g}{(m\pi)^2} \left(\frac{B}{h_o} \right)^2 \frac{f(\theta_o)}{C^2} = \frac{\lambda_s}{\lambda_w} \quad (2.41)$$

As can be seen from Eq. [2.41], α is proportional to the square of the with-to-depth ratio and of $1/C$. This means that the shallower the water is, and, in a similar way, the higher the channel roughness is, the bigger α becomes. Consequently bigger values of α are expected for channels having a braided pattern which are generally shallower and rougher than the meandering ones. The interaction parameter is also dependent on the mode that determines the transverse deformation, see Eq. [2.32], as it is proportional to the square of the ration $1/m$.

For a meandering river $m=1$, while $m>1$ is expected for rivers λ/m ,

The behaviour of the steady-state model can be studied analysing the steady-state second-order differential equation which can be derived by neglecting the time-dependent terms from Eq. [2.38]:

$$\frac{\partial^2 H}{\partial s^2} + \frac{\partial H}{\partial s} \left[\frac{1}{\lambda_s} - \frac{(b-3)}{2\lambda_w} \right] + \frac{H}{\lambda_s \lambda_w} = F \text{ (curvature)} \quad (2.42)$$

(F(curvature) is defined in Eq. [2.39])

The curvature terms are composed by both the channel centre-line curvature, $1/R_c$, and the curvature of the stream-lines due to the non-uniformity of the flow field, $\hat{c}$. When the latter is taken into account, the equations of the system are of fourth order. This can be demonstrated by deriving an approximate expression for $\hat{c}$ (see Appendix C):

$$\hat{c} = -\frac{B}{\pi} \left(\frac{1}{h_0} \frac{\partial^2 H}{\partial s^2} + \frac{1}{u_0} \frac{\partial^2 U}{\partial s^2} \right) \left[1 + \left(\frac{B}{\pi R_c} \right) \right] \quad (2.43)$$

In this case the four solutions have generally this form: two are purely exponential and two are identical harmonics. However, the exponential functions are very strongly damping and only noticeable near the boundary, while the harmonic solutions are dominating, see Figure 7.

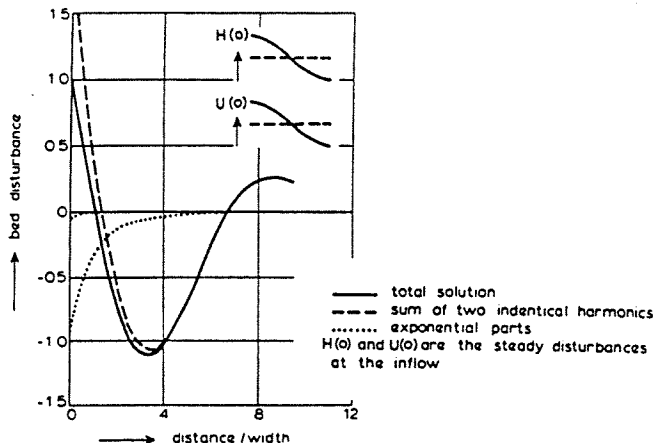


Figure 7 Longitudinal bed deformation caused by an upstream disturbance (fourth order system).

When the terms $\hat{c}$ are neglected, the two differential equations become of second order. The solutions in this case can be either harmonic or purely exponential. Only the harmonic range of solutions is considered to be of interest here, corresponding to a steady deformation of the alternate bar type, as depicted in Figure 5. It has been demonstrated that when computing the deformation field far enough from the boundary the solution of the fourth order system is very similar to the one of the second order system (Struiksma & Crosato [1989]). For this reason the terms $\hat{c}$ can be neglected from the equations.

The harmonic solution of Eq. [2.42] with $\hat{c}=0$ has the following form (the solution for the main velocity is given by a similar expression):

$$H(s) = H(0) \exp \left[-\frac{s}{L_D} \right] \sin \left[\frac{2\pi}{L_p} (s + s_p) \right] \quad (2.44)$$

where $H(0)$ and $H(s)$ = amplitude of water depth disturbance at Section 0 (upstream boundary) and at Section s (generic), respectively; s_p = spatial lag; $2\pi/L_p$ and $1/L_D$ = wave number and damping coefficient, respectively, having the following expressions:

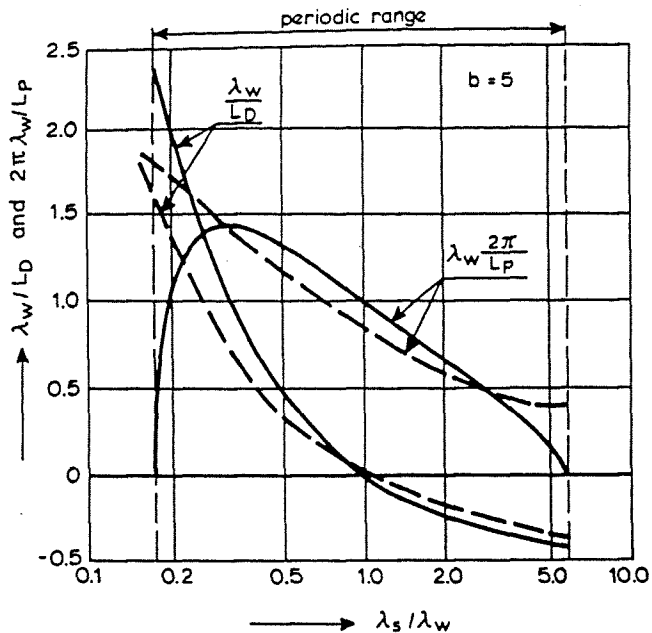
$$\frac{2\pi}{L_p} = \frac{1}{2\lambda_w} \left\{ (b-1) \left(\frac{\lambda_w}{\lambda_s} \right) - \left(\frac{\lambda_w}{\lambda_s} \right)^2 - \frac{(b-3)^2}{4} \right\}^{1/2} \quad (2.45)$$

$$\frac{1}{L_D} = \frac{1}{2\lambda_w} \left[\frac{\lambda_w}{\lambda_s} - \frac{(b-3)}{2} \right] \quad (2.46)$$

In some cases the roots are purely imaginary, which implies that the harmonic characters of the solution vanishes.

The complex roots mainly depend on the interaction parameter α , Eq. [2.41], and on the degree of non-linearity of sediment transport with main flow velocity, the exponent b , Eq. [2.13] (see Struiksma et al. [1985]; de Vriend & Struiksma [1983]).

In Figure 8 the wave number and the damping coefficient (made dimensionless) (Eqs. [2.45] and [2.50], respectively) are compared with the corresponding ones given by the more complete linear model analysed by Struiksma et al. [1985], and by Struiksma & Crosato [1989], in which $\hat{c}$ is retained, and the influence of the longitudinal bed slope on the sediment transport rate and of the spiral motion inertia are also accounted for, for the case $b = 5$ (Engelund & Hansen transport formula) and far enough from the boundary.



— presented linear model
 - - - Struiksma et al [1985]

Figure 8 Relative wave number and damping coefficient as a function of λ_s/λ_w . Comparison between the proposed linear model and the linear model development by Struiksma et al [1985].

The two linear models have a good agreement for λ_s/λ_w larger than about 0.25. For values of λ_s/λ_w less than 0.25 the two models still give comparable results: due to the high values of the damping coefficient any oscillation present upstream is damped out before becoming appreciable. For very small values of λ_s/λ_w (extremely high damping) the models become similar to the model developed by Ikeda et al. [1981]: the deformation caused by upstream disturbance is immediately damped out, so that the channel is soon characterized by fully developed bend flow conditions.

The wave lengths and the damping rate of the steady bed oscillations are quite sensitive to the changes of the exponent b derived from the selected transport formula, see Figure 9. As already mentioned, according to the

Engelund & Hansen transport formula b is constant and equal to 5. But, according to the Meyer-Peter & Müller formula b is a function of the Shields parameter and can be relatively large and strongly variable when the parameter is close to its critical value (incipient motion). For this reason the estimation of the value of b (degree of non linearity of the sediment transport) has to be done with great care.

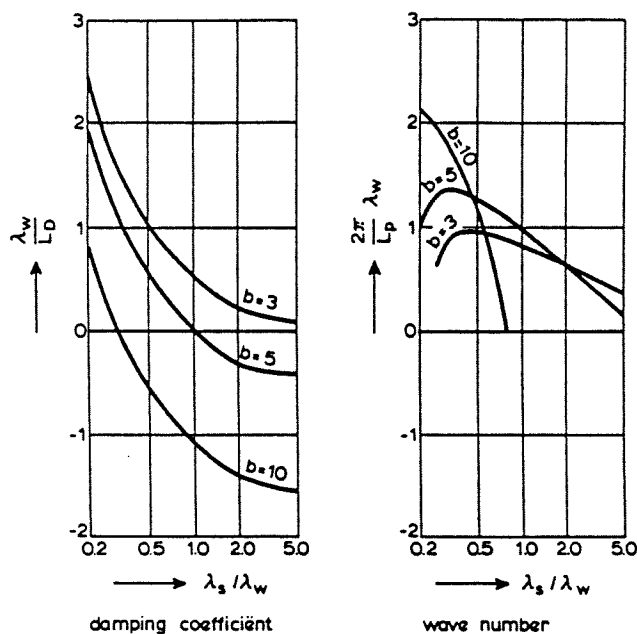


Figure 9 Variations of damping coefficient and wavenumber due to changes of the exponent b

Figure 10 shows the longitudinal profile of the bed level along the outer bank of a bend having a constant radius of curvature with a straight inflow section as predicted by the model when retaining $\hat{c}$. In this case those mathematical models, which disregard the redistribution effects, predict a constant (with s) transverse bed slope all along the bend, and correspondingly a horizontal and not harmonic bed level profile along the banks (dotted line: deformation corresponding to the fully developed bend flow conditions). The overshoot of bed slope due to the redistribution of flow and sediment movement in the first part of the bend, has the same order of

magnitude as the transverse bed slope corresponding to the fully developed bend flow conditions (Struiksma [1983b]), therefore it can not be neglected when computing the bed topography.

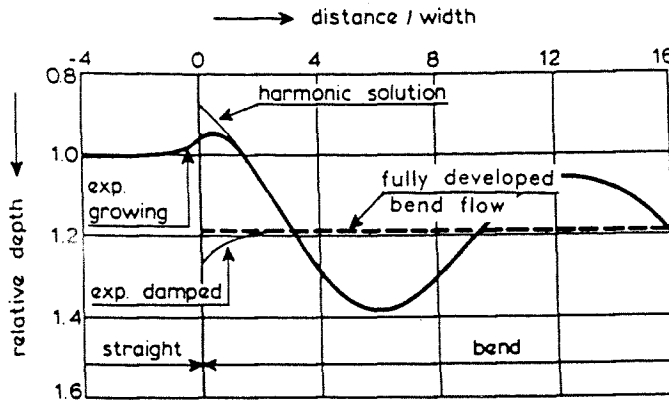


Figure 10 Longitudinal profile of bed level along the outer bank of the initial part of a circular bend

2.2.5 Time scale of the transverse channelbed deformation development - The analysis of the time-dependent equation for the bed development, Eq. [2.38], is necessary for the derivation of the time scale of the development of the channel bed deformation process Struiksma [1989, personal communication].

With the assumption of an infinite long meandering channel, as depicted in Figure 11, the curvature can be described by an equation like the following:

$$r = \bar{r} \sin(Ks) \quad (2.47)$$

in which $\hat{f}$ = maximum value of curvature (amplitude); K = meander wave number, which can be written as follows:

$$K = \frac{2\pi}{L} \quad (2.48)$$

in which L = meander wave length as measured along s .

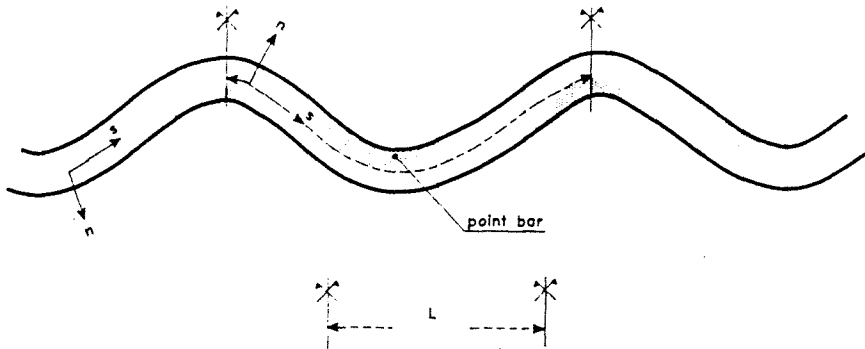


Figure 11 Schematization of an infinite long meandering channel

Eq. [2.38] becomes:

$$\begin{aligned} & \frac{\partial^2 H}{\partial s^2} + \frac{\partial H}{\partial s} \left[\frac{1}{\lambda_s} - \frac{(b-3)}{2\lambda_w} \right] + \frac{H}{\lambda_s \lambda_w} + \frac{h_o}{S_o} \frac{\partial^2 H}{\partial s \partial t} + \frac{h_o}{\lambda_w S_o} \frac{\partial H}{\partial t} = \\ & = - \frac{h_o}{k_B} (b-1) \frac{\partial^2 \Gamma}{\partial s^2} + \left[A h_o^2 k_B - \frac{(2-\sigma)(b-1)h_o}{2\lambda_w k_B} \right] \frac{\partial \Gamma}{\partial s} + \frac{A h_o k_B}{\lambda_w} \Gamma = \\ & = \hat{A} \hat{f} \sin(Ks) + \hat{B} \hat{f} \cos(Ks) \end{aligned} \quad (2.49)$$

with:

$$\hat{A} = \left[A \frac{k_B}{\lambda_w} + \frac{(b-1)}{k_B} K^2 \right] h_o$$

$$\hat{B} = \left[A h_o k_B - \frac{(2-\sigma)(b-1)}{2\lambda_w k_B} \right] h_o K$$

The behaviour of the system follows from the study of the homogeneous equation:

$$\frac{\partial^2 H}{\partial s^2} + \frac{\partial H}{\partial s} \left[\frac{1}{\lambda_s} - \frac{(b-3)}{2\lambda_w} \right] + \frac{H}{\lambda_s \lambda_w} + \frac{h_o}{S_o} \frac{\partial^2 H}{\partial s \partial t} + \frac{h_o}{\lambda_w S_o} \frac{\partial H}{\partial t} = 0 \quad (2.50)$$

Suppose the solution can be written as:

$$H = e^{-t/T_1} \hat{A}_1 \sin(Ks) + e^{-t/T_2} \hat{A}_2 \cos(Ks) \quad (2.51)$$

in which T_1 and T_2 = time scales of development of the two modes of bed deformation; $\hat{A}_1$ and $\hat{A}_2$ = integration constant.

Substituting H in Eq. [2.50] with the expression given in Eq. [2.51], the following time scales can be found:

$$T_1 = \frac{\lambda_s h_o}{S_o} \left[\frac{1 - (K \lambda_w)}{1 - (K \lambda_w) + \frac{b-3}{2} \frac{\lambda_s}{\lambda_w} (K \lambda_w) - \frac{\lambda_s}{\lambda_w} (K \lambda_w)^2} \right] \quad (2.52)$$

$$T_2 = \frac{\lambda_s h_o}{S_o} \left[\frac{1 + (K \lambda_w)}{1 + (K \lambda_w) - \frac{b-3}{2} \frac{\lambda_s}{\lambda_w} (K \lambda_w) - \frac{\lambda_s}{\lambda_w} (K \lambda_w)^2} \right] \quad (2.53)$$

The two modes of bed deformation have two different time scales, which means that there is a phase lag of the developing bed deformation which is time-dependent. This can be regarded as a progressive shift of the maximum value of the bed deformation toward a definite static position (the progressive shift is zero at $t = \infty$, see Eq. [2.51]).

The two time scales, T_1 and T_2 , are found to be a function of the meander wave number, $K = 2\pi/L$ (supposing one point bar per channel bend this is also the bed deformation wave number).

It is interesting to analyse the terms between square parenthesis in Eqs. [2.52] and [2.53] for the case of meandering rivers.

In a meandering channel the meander wave length as measured along s is much bigger than the channel width, while λ_w is comparable with the channel width. For this reason the ratio λ_w/L is small. Consequently the term $(K\lambda_w)$ is small when the ratio λ_w/L is smaller than $1/30$.

Under this hypothesis and considering that for meandering rivers the term $[(b-3)/2](\lambda_s/\lambda_w)$ can be assumed as approximately equal to 1, or even smaller (see at this purpose Chapter [4]), the terms between square parenthesis in Eqs. [2.52] and [2.53] can both be assumed as tending to the value 1. When this is true the bed development time scale can be assumed independent from the meander wave number:

$$T_1 = T_2 = T$$

$$T = \frac{\lambda_s h_o}{S_o} = \left(\frac{1}{mn}\right)^2 \frac{B^2 f(\theta_o)}{S_o} \quad (2.54)$$

This is also true when $\lambda_s \rightarrow 0$.

Approximately the same time scale (with a maximum deviation of 20%) was found computationally by Struiksma [1988] by using a fully non-linear 2-D model developed at DELFT HYDRAULICS (see Koch & Flokstra [1980]).

When the river discharge changes, the channel bed does not instantly adapt itself to the new conditions. The duration of the "transition period" is a function at the time scale of the bed development, T . This is very variable from river to river: for some rivers it has the order of magnitude of several days, for others of years! Only when the bed adaptation is relatively fast (small T) the assumption of an instantaneous adaptation of the bed to the new conditions, which is at the base of the adoption of a steady-state flow and bed development model, can be justified. Therefore, for computations of bed deformation with a varying discharge it can be convenient to take into account the time-adaptation of the bed development (even if roughly) also when using a steady-state model.

For this purpose the following expression to determine the transverse bed deformation at any time is suggested, Struiksma [1989, personal communication]:

$$H(t) = H(\infty) [1 - e^{-t/T}] \quad (2.55)$$

in which: $H(t)$ = value of the near bank water depth deformation at $t=t$;
 $H(\infty)$ = equilibrium near-bank water depth deformation as computed with the steady-state model, which is equal to the deformation at $t = \infty$.

The adoption of this (rough) time-adaptation formulation can be sometimes preferable to the adoption of a fully time-dependent model, for example when computer memory and/or computational time must be restricted to the minimum. This can be true when dealing with meander migration models, which are used to compute the planimetric changes of rivers for rather large periods of time (having the order of magnitude of several years) and on a relatively large scale.

2.3 Bank erosion model - Bank erosion is influenced by various factors such as: (i) near-bank flow (velocity, shear stress, etc); (ii) physical properties of the banks (bank elevation, composition of the soil, water content, presence of vegetation etc.); and (iii) quality of the eroding water (temperature, electrochemical properties, etc.).

Near-bank flow is the cause of bank erosion, while soil characteristics of the eroding banks have a significant effect upon erosion rates. The resistance to erosion is determined by the shear strength and cohesive strength of bank materials, which are influenced by several conditions, including: temperature of eroding water, antecedent water content, pore pressure etc. In addition the stability of cohesive materials is influenced by suspended sediment content and chemical quality of the eroding water. Banks subjected to ephemeral flow exhibit a variable resistance to erosion. Bank elevation influences bank stability, see Osman & Thorne [1988].

In order to simulate in a simple way all these factors, the physical characteristics of the banks can be roughly summed up into time-averaged erosion coefficients (to be calibrated with field data), while the erosion rate can be simply assumed proportional to the near-bank flow properties (which are influenced by the geometrical characteristics of the channel and by the upstream conditions).

Similar to Ikeda, Parker and Sawai [1981] the flow-induced rate of bank erosion is assumed simply proportional to the near-bank tangential velocity excess from the reach-averaged value. In other words, considering that the value of u near the outer bank of a channel bend exceeds the reach-averaged value, u_0 , by a certain amount U , it is assumed that whenever $U > 0$, the

outer bank will be eroded, otherwise it will be subjected to deposition. It follows that the reach-averaged value of the streamwise velocity, u_o , is the critical value for bank erosion: with uniform flow conditions the river banks are assumed stable. This can be written in the following way:

$$\frac{\partial n}{\partial t} = E_o U \quad (2.56)$$

where n = transverse co-ordinate; t = time; E_o = time-averaged erosion coefficient; U = near-bank tangential velocity deformation with respect to the uniform flow conditions (excess from the reach-averaged value).

Equation [2.56] can be derived through the simplification of the general law for bank erosion rate due to lateral fluvial entrainment of cohesive material, introduced by Ariathuri & Arulanadan [1978]:

$$\frac{\partial n}{\partial t} = \xi \left[\frac{\tau_B - \tau_c}{\tau_c} \right] \quad \text{for } \tau_B \geq \tau_c \quad (2.57)$$

$$\frac{\partial n}{\partial t} = 0 \quad \text{for } \tau_B < \tau_c$$

where ξ = erodibility coefficient; τ_B = flow shear stress on the bank; and τ_c = critical shear stress below which no erosion occurs.

Linearization of Eq. [2.57] together with the assumption that the critical conditions for bank erosion are coincident to the uniform flow conditions leads to Eq. [2.56], see Mosselman [1989].

If the bank is steep bank instability becomes important. Banks with a lower toe level are likely to be less stable and consequently more susceptible to erosion. For this reason an additional term is included in the erosion law when computing meander migration of a river with vertical cohesive banks. Surveys of the East Fork River (Andrews [1982]) showed that bank instability and to some extent the adjustment of the channel width is controlled by the process of scour and fill. Bank erosion caused by bank instability is assumed proportional to the near-bank water depth deformation. The erosion law becomes:

$$\frac{\partial n}{\partial t} = E_u U + E_h H \quad (2.58)$$

where E_u and E_h = time-averaged erosion coefficients (to be calibrated); H = near-bank water depth deformation with respect to the reach-averaged value.

The input of material caused by the erosion of the banks affects the bed development and therefore the flow etc. Bank erosion products can be easily incorporated in the model. However, in general the input of eroded material does not influence appreciably the characteristics (wave lengths and damping) of the bed development (Mosselman [1989]).

3. RESONANCE AND INITIATION OF MEANDERING

3.1 General - Many scientists tried to explain the origin of meandering with various approaches. The experimental observations of Leopold & Wolman [1957] have been followed by the development of rational models which gradually have been increasing the knowledge of the processes involved. The first ones were based on the assumption that the formation of submerged migrating alternate bars in a straight channel would eventually lead to the development of meanders. The basic idea is that the flat alluvial bed of a straight channel, if under appropriate conditions, becomes unstable. Bed instability was assumed to be the cause of the formation of migrating alternate bars. Callander [1966] was the first one who introduced the point of view of analysing bed instability in an unpublished note mentioned by Hansen [1967]. The latter, in a work similar to and contemporaneous with Adachi's [1967], developed the so-called "Bar Stability" analysis. In this theory a shallow channel with straight non-erodible banks is considered. An unsteady double-harmonic perturbation of the original plane bed is introduced and, using the small amplitude oscillation theorem, it is studied whether this perturbation amplifies or attenuates, corresponding to meandering or non-meandering, respectively. The theory yields an expression for the most probable wave length of meanders. Braiding is treated as a different degree of the same instability phenomenon. Callander [1969], Engelund [1970], Engelund & Skovgaard [1973], Parker [1976], and Fredsøe [1978] improved the analysis by introducing additional factors without changing the basic approach. Among other things the importance of taking into account the influence of the transverse bed slope when estimating the sediment transport direction was established. The weakest point of this approach is that the analyses were performed for the case of a straight channel with fixed side walls, while meandering is mainly dependent on bank erosion processes (erodible banks).

A basic change is due to Ikeda, Parker and Sawai [1981], who performed a stability analysis on a mildly curved channel with erodible banks. Their "Bend Stability" analysis is based on the growth rate of the lateral bend amplitude. It led to the assessment of critical meander wave lengths above which the loops tend to grow. This analysis differs from the "Bar Stability" analysis as it is based on a totally different approach. Nevertheless, they found that the two mechanisms operate at similar wave lengths. This led to the conclusion that there is a continuous evolution of alternate bars, which slowly migrate downvalley, into point bars.

Blondeaux and Seminara [1985] re-examined the linear analysis developed by Ikeda et al [1981] by adopting a more accurate model. They based their model on fully coupling flow field, bed topography and sediment transport introducing in this way the flow-redistribution effects. Their discovery was that alternate bars and bend amplification are controlled by two distinct mechanisms: bed instability and resonance (forced by the channel sinuosity), respectively. They found that resonant flow occurs when curvature forces a bed oscillation represented by an approximately steady perturbation of the alternate bar type. Bend instability does not select the most unstable wave length of bar instability but rather which is closest to resonance. The wave lengths selected by the resonance phenomenon were found to be about three times as large as those selected by the traditional "Bar Stability" theories. Furthermore, their order of magnitude was in accordance with the wave lengths of meanders at initial stage.

More or less contemporarily, at the so-called "Delft School" (after Parker & Johannesson [1989]) Olesen [1983]; Struiksma et al [1985] studied the phenomenon with the assumption that the celerity of migrating alternate bars is too high to influence such a slow phenomenon as bank erosion in existing meandering rivers. The formation of meanders was assumed to be caused by the development of a steady perturbation of flow (and channel bed) of the alternate bar type. This perturbation is originated by the flow and sediment redistribution which follows from those flow disturbances present more upstream. This assumption implies that meandering can be initiated by the presence of an obstacle to the flow, by changes of curvature (exit of bends), etc., and whenever the channel response is a downstream steady double-harmonic deformation of flow and channel bed. This is in accordance with the experimental experience of Friedkin [1945] and with what has been observed by Brice [1983]: "meanders can be initiated in a straight reach by grow of a bend at the reach entrance (Figure 1 (F), at top). Similarly, meander development in the straight (1941) reach of Figure 1 (B) is related to the growth and migration of a meander at its upstream end".

Similar to the steady bars selected by the resonance phenomenon discovered by Blondeaux & Seminara [1985], the steady deformation due to upstream disturbances was also found to have wave lengths two to three times larger than those selected by the "Bar Stability" theory.

Very recently Tubino & Seminara [1988], [1989], Seminara & Tubino [1989] have mathematically demonstrated a clear interaction between migrating alternate bars and meander formation. This interaction was also observed experimentally by Ackers & Charlton [1970] and Fujita & Muramoto [1982] in tests carried out on flume models, with banks constituted by (highly erodible) sandy material. Consequently, for those experimental conditions the erosion speed of banks could be assumed comparable to the bed development speed.

Based on this consideration it is still realistic to assume that for those rivers having a relatively low bank-erodibility the erosion pattern is influenced by the presence of a steady bed (and flow) deformation rather than by the presence of relatively fast migrating alternate bars. Field observations of Ikeda [1989] show that there is a class of meandering rivers which do not exhibit migrating alternate bar formation even when straightened, but which have clear point bars. For those rivers a steady bed deformation originated by an upstream channel disturbance seem to give a better explanation for the initiation of meandering. Anyway, the phenomenon of meander formation cannot be considered fully understood yet. It is likely that both migrating alternate bars and upstream-induced steady bars influence bank erosion. Both phenomena can be contemporary present in a channel, as observed in the Straight-Flume Experiment described in Section 6.2. Most probably bank erodibility and the relative intensity of the two phenomena will determine whether one of the two phenomena will prevail in influencing meander formation.

3.2 Resonance and initiation of meandering: the steady-state model - The approach adopted by the proposed meander migration model is based on the assumption of being a steady bed deformation of the alternate bar type, rather than the presence of migrating alternate bars, the major factor influencing the erosion pattern and therefore meandering. This is true for those rivers characterized by relatively low bank erodibility having the channel shift speed one order of magnitude higher than the alternate bar celerity. The basic assumption is also in agreement with the discovery of Blondeaux and Seminara [1985] that resonant flow and, according to their model, initiation of meandering is accompanied by the development of approximately steady alternate bars.

It is possible to demonstrate that initiation of meandering, as predicted by the proposed steady-state model, is coincident with a resonance of flow (and bed) which is forced by the development of the channel's sinuosity.

Resonance occurs when a flow disturbance is varying in a periodical way with s (like the curvature in a sinuous channel) and when appropriate conditions are satisfied. Resonance conditions are found coincident to initiation of meandering when adopting the erosion law defined by Eq. [2.56]. As demonstrated by Parker & Johanneson [1989] this resonance phenomenon is practically the same as the phenomenon discovered by Blondeaux & Seminara [1985].

Equation [2.42] is a second order differential equation describing the variation of the amplitude of water depth deformation along the longitudinal direction, s .

A similar one is obtained for the tangential velocity deformation. The source term is a function of the curvature and of local parameters and is defined in Eq. [2.39]:

$$a_1 \frac{\partial^2 H}{\partial s^2} + a_2 \frac{\partial H}{\partial s} + a_3 H = F \text{ (curvature)} \quad (3.1)$$

in which:

$$a_1 = 1;$$

$$a_2 = \left[\frac{1}{\lambda_s} - \frac{(b-3)}{2\lambda_w} \right];$$

$$a_3 = \frac{1}{\lambda_s \lambda_w}.$$

Equation [3.1] has a form similar to the general equation describing damped and forced mechanical vibrations, but instead of varying with time, H is varying with s . For the case of a sinuous river it is realistic to assume that the forcing function has a harmonic character.

When the damping term, a_2 , is equal to zero, corresponding to:

$$\frac{\lambda_w}{\lambda_s} = \frac{(b-3)}{2} \quad (3.2)$$

and when also expressing the forcing function in a trigonometric form, Eq. [3.1] becomes (see Eq. [2.49]):

$$a_1 \frac{\partial^2 H}{\partial s^2} + a_3 H = \hat{A} \hat{f} \sin(Ks) + \hat{B} \hat{f} \cos(Ks) \quad (3.3)$$

in which: K = meander wave number, as defined in Eq. [2.48]; $\hat{r}$ = amplitude (maximum value) of the stream line curvature; $\hat{A}$ and $\hat{B}$ = amplitudes, defined in Eq. [2.49].

Simply observing the form of Eq. [3.3] it can be concluded that, in analogy with forced mechanical vibrations, a resonance phenomenon might occur when the wave number of the forcing function, K , is the same as the wave number of the natural oscillation of the system, $k_0 = 2\pi/L_p$, defined by Eq. [2.45]. In this case:

$$k_{res} = \left(\frac{a_2}{a_1}\right)^{1/2} = \left(\frac{1}{\lambda_w \lambda_s}\right)^{1/2} \quad (3.4)$$

For the case of a non-damping system, the present model predicts a non-damped steady deformation of the alternate bar type as the channel's response to an upstream disturbance. Starting from a straight channel and adopting the erosion law defined by Eq. [2.56], the initially straight channel evolves to a sinuous one having, at initial stage, the same wave length as the natural oscillation.

The channel sinuosity becomes the forcing function yielding resonance of the system. The resonance phenomenon in this case is coincident with initiation of meandering.

When the damping of the system is not equal to zero the channel response to an oscillatory forcing function is always bounded. However, it is possible to derive the wave number of the forcing function yielding the maximum amplitude of the channel's response:

$$k_{max} = \frac{1}{\lambda_w} \left\{ \frac{\lambda_w}{\lambda_s} - \frac{1}{2} \left[\left(\frac{\lambda_w}{\lambda_s} \right)^2 + \left(\frac{b-3}{2} \right)^2 - (b-3) \frac{\lambda_w}{\lambda_s} \right] \right\}^{1/2} \quad (3.5)$$

In Figure 12 the wave number and the damping coefficient of the natural oscillation (Eqs. [2.45] and [2.46], respectively); the resonance wave number and that of the forcing function yielding the maximum channel response (Eqs. [3.4] and [3.5], respectively) are plotted together for two values of b . As can be seen, for a relatively large range of values of the interaction parameter, λ_s/λ_w , the wave numbers are very similar (the range becomes smaller with the increasing of b). It can be concluded that for values of λ_s/λ_w within this range initiation of meandering corresponds to resonance conditions.

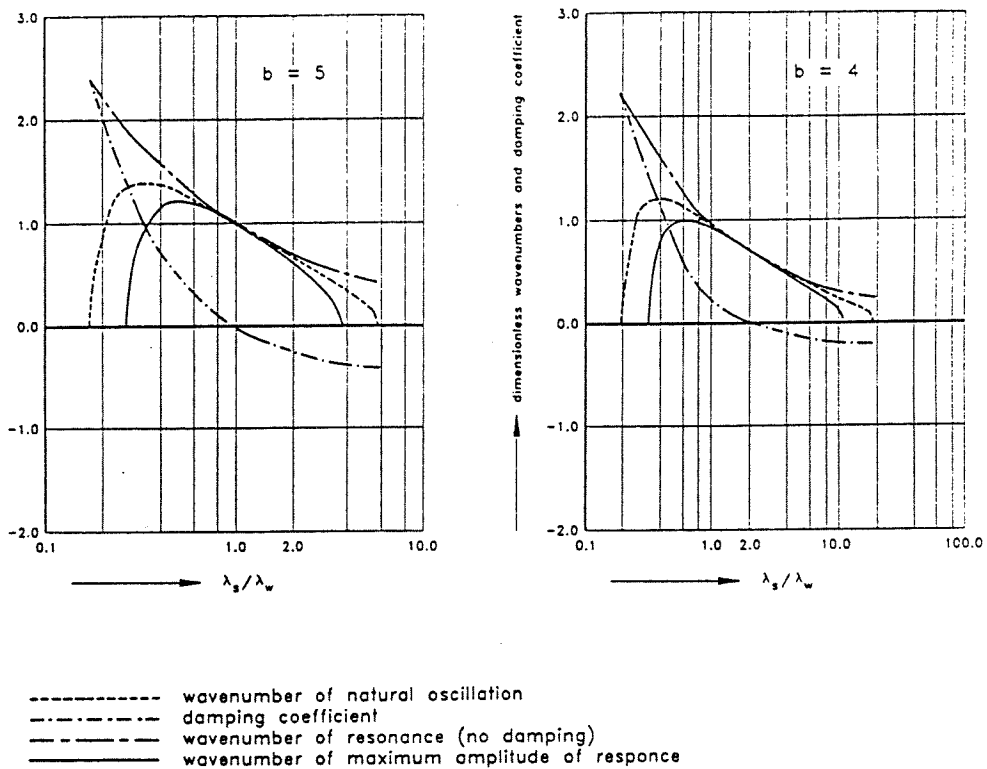


Figure 12 Resonance wave number; natural oscillation wave number; damping of the system; and wave number of the forcing function leading to the maximum amplification of deformation (damping system) as functions of λ_s/λ_w , for $b = 4$ and 5 .

4. THE DEVELOPMENT OF MEANDERS

4.1 General - Observations show that at low sinuosities meanders tend to have a relatively regular shape. Their migration proceed in shifting the loops towards the downstream direction and in enlarging the loop form. At high sinuosities bends that are actively migrating downstream tend to show a marked asymmetry in shape. This asymmetry is so general that it allows one to assess the flow direction from aerial photographs, the majority of loops being convex toward the downstream direction, Raisz [1955].

The present meander migration model predicts a similar development of the loop form. For very small values of the interaction parameter (strongly damping system) the development of finite amplitude meanders can be explained with a geometric analysis similar to the one performed by Parker et al [1983], Parker & Andrews [1986]. However, even if presenting the same geometrical characteristics (related to the erosion law) as Parker & Andrews' [1986] model, the present one shows an additional asymmetry arising from the bed topography prediction (point bars): the predicted bed topography in developing meanders shows, with the increase in bend sizes, a gradual shift of the point bar and, therefore, of the highest velocity locus (and erosion rate), which for low sinuosities lie downstream of the bend apex, towards the upstream part of the channel bends. This decreases the downstream migration speed of meanders and yields asymmetry of the loop form.

When by growth a meander has developed a path length which is long in relation to the spacing of pools and riffles in stream, the formation of new pools and riffles along its path initiate "lobing" or "double heading", Brice [1983], Hook [1983]. The present model can simulate this further channel "lobing" when the path length is long in relation to the spacing of pools and riffles which is given by their characteristic wave length (wave number: Eq. [2.45]).

In this chapter the mathematical model is analysed from the point of view of the prediction of the point bar position with the increase in bend size, Struiksma [1990, personal communication]. This analysis is very useful for the check of the computational results, to see if they are affected by appreciable numerical errors, which can be done by comparing the computed water depth isolines and meander-development with the analytical results.

4.2 Point bar shift - Equation [2.42] describes the variation of the amplitude of water depth deformation along the longitudinal direction, s . It is a second-order differential equation in which the source term is a function of the curvature and of local parameters, Eq. [2.39]. Equation [2.42] can be re-written in the following way:

$$a_1 \frac{\partial^2 H}{\partial s^2} + a_2 \frac{\partial H}{\partial s} + a_3 H = b_1 \frac{\partial^2 \Gamma}{\partial s^2} + b_2 \frac{\partial \Gamma}{\partial s} + b_3 \Gamma \quad (4.1)$$

in which (a_1 , a_2 , a_3 are defined in Eq. [3.1])

$$b_1 = -\frac{h_o}{k_B} (b-1)$$

$$b_2 = A h_o^2 k_B - \frac{(2-\sigma)(b-1) h_o}{2 \lambda_w k_B}$$

$$b_3 = \frac{A h_o^2 k_B}{\lambda_w}$$

with: $\Gamma = \frac{1}{R_c} \left(\frac{m\pi}{2} \right)$, channel's curvature term.

For a sinuous channel the curvature can be written as:

$$\Gamma = \hat{f} \exp (iKs) \text{ (excitation)} \quad (4.2)$$

in which: $\hat{f}$ = amplitude (maximum of curvature); K = meander wave number, Eq.[2.48].

Considering one point bar per channel bend, the water depth deformation can be written:

$$H = \hat{A} \exp [i K (s + s_p)] \text{ (response)} \quad (4.3)$$

in which: s_p = phase lag between the maximum bed deformation (point bar) and the maximum channel curvature (bend apex.).

Inserting the expression of Eqs. [4.2] and [4.3] in Eq. [4.1] the following equation is obtained:

$$\hat{A} \exp (iKs_p) = \hat{f} \frac{(-b_1 K^2 + b_2 iK + b_3)}{(-a_1 K^2 + a_2 iK + a_3)} \quad (4.4)$$

which can be written as:

$$\hat{A} [\cos (Ks_p) + i \sin (Ks_p)] = \hat{f} (R_1 + iR_2) \quad (4.5)$$

with:

$$R_1 = \frac{(-K^2 b_1 + b_2)(-K^2 a_1 + a_2) + K^2 b_2 a_2}{(-K^2 a_1 + a_2)^2 + K^2 a_2^2}$$

$$R_2 = \frac{K b_2 (-K^2 a_1 + a_2) - K a_2 (-K^2 b_1 + b_2)}{(-K^2 a_1 + a_2)^2 + K^2 a_2^2}$$

The phase lag, s_p , can be derived from Eq. [4.5]:

$$s_p = \frac{1}{K} \arctan (R_2/R_1) = \frac{1}{K} \arctan \left[\frac{K b_2 (-K^2 a_1 + a_2) - K a_2 (-K^2 b_1 + b_2)}{(-K^2 b_1 + b_2)(-K^2 a_1 + a_2) + K^2 b_2 a_2} \right] \quad (4.6)$$

A diagram can be made of the phase lag, s_p , against the wave number, K (made dimensionless with the natural oscillation of the system $k_0 = 2\pi/L_p$), see Figure 13. The diagram is obtained using the data of the Straight Flume Experiment, see Section 7.2.

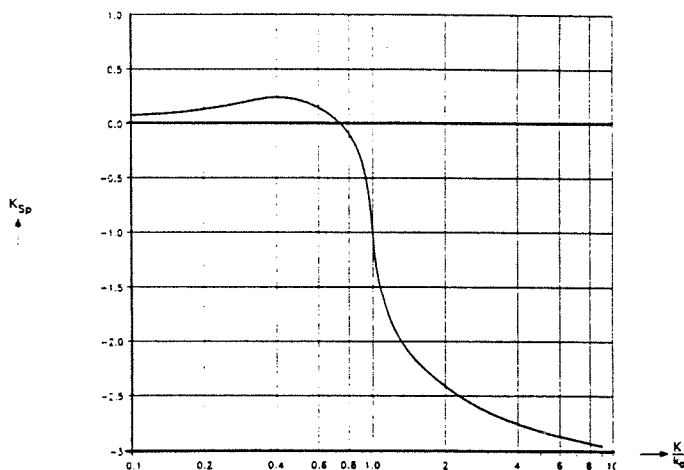


Figure 13 Dimensionless phase lag between the point bar position and the maximum curvature locus, Ks_p , against the ratio between the meander wave number and the natural oscillation wave number, K/k_0

The analysis of the graph in Figure 13 leads to the following conclusions: If meandering is initiated by a steady bed deformation induced by an upstream disturbance (see Chapter 3) the meander wave length is initially the same as that of the bed oscillation ($K/k_0 = 1$) with a negative phase lag between the point bar and the bend apex, s_p . This means that the point bar is initially downstream of the bend apex. The maximum velocity locus is always downstream of the maximum bed deformation (the phase lag between the velocity deformation and the bed deformation can be easily determined: it tends to λ_w for $K \rightarrow 0$). Consequently, at initial stage the maximum velocity locus and therefore the erosion rate is downstream of the bend apex which yields a characteristic prevalent downstream migration together with lateral growth (to a lower extent) of the channel bends.

With progression of bank erosion the bends grow, so does the meander wave length, while the meander wave number, $K = 2\pi/L$, decreases. Assuming that k_0 remains approximately constant, in Figure 13 it can be observed that for decreasing values of the ratio K/k_0 the phase lag between the point bar and the bend apex becomes positive. This means that for higher sinuosities the point bar preceeds the maximum curvature locus (bend apex). This configuration leads to a progressive distortion of the loop form favouring the transverse growth of bends.

Simulation of meander migration starting with the data adopted to draw the graph of Figure 13 are illustrated in Chapter 8. Computational results agree with this analysis.

5. THE COMPUTER PROGRAM MIANDRAS

5.1 General - The equations describing the steady-state near bank flow and channel bed deformations, Eqs. [2.35] and [2.42] respectively, have been incorporated in a computer program, called "MIANDRAS". The program computes flow field and bed topography at every time taking into account the time-scale of the bed development when necessary, according to Eq. [2.55]. Bank erosion is also computed at every time, as a function of the near-bank velocity and water depth deformations, according to Eq. [2.58]. MIANDRAS can be used to compute velocity and water depth profiles (and isolines) of an assigned channel, and when banks are erodible, also its planimetric changes. Bank erodibility can vary along the channel. Therefore, it is possible to take into account the variations of bank erodibility and the presence of bank protection works along the river course. The properties of the soil related to bank erodibility (erosion coefficients) are specified for regions in the river valley. The program establishes in which region the eroding bank is located and therefore which the values of the erosion coefficients are that must be applied. The prediction of velocity and bed topography has been tested with two experiments carried out at DELFT HYDRAULICS, see Chapter 6.

Some coefficients can be adjusted to calibrate the model: E ; α_1 ; σ ; E_u and E_h . They weigh the influence of the transverse bed slope on the sediment transport direction (Eq. [2.10]); of channel curvature on the bed shear-stress direction (Eq. [2.11]); of secondary flow convection (Eq. [2.35]) on flow field and bed topography; of flow-induced bank erosion and of bank retreat due to cohesive bank failure (Eq. [2.58]), respectively.

The program input-output flux is schematized in Figure 14.

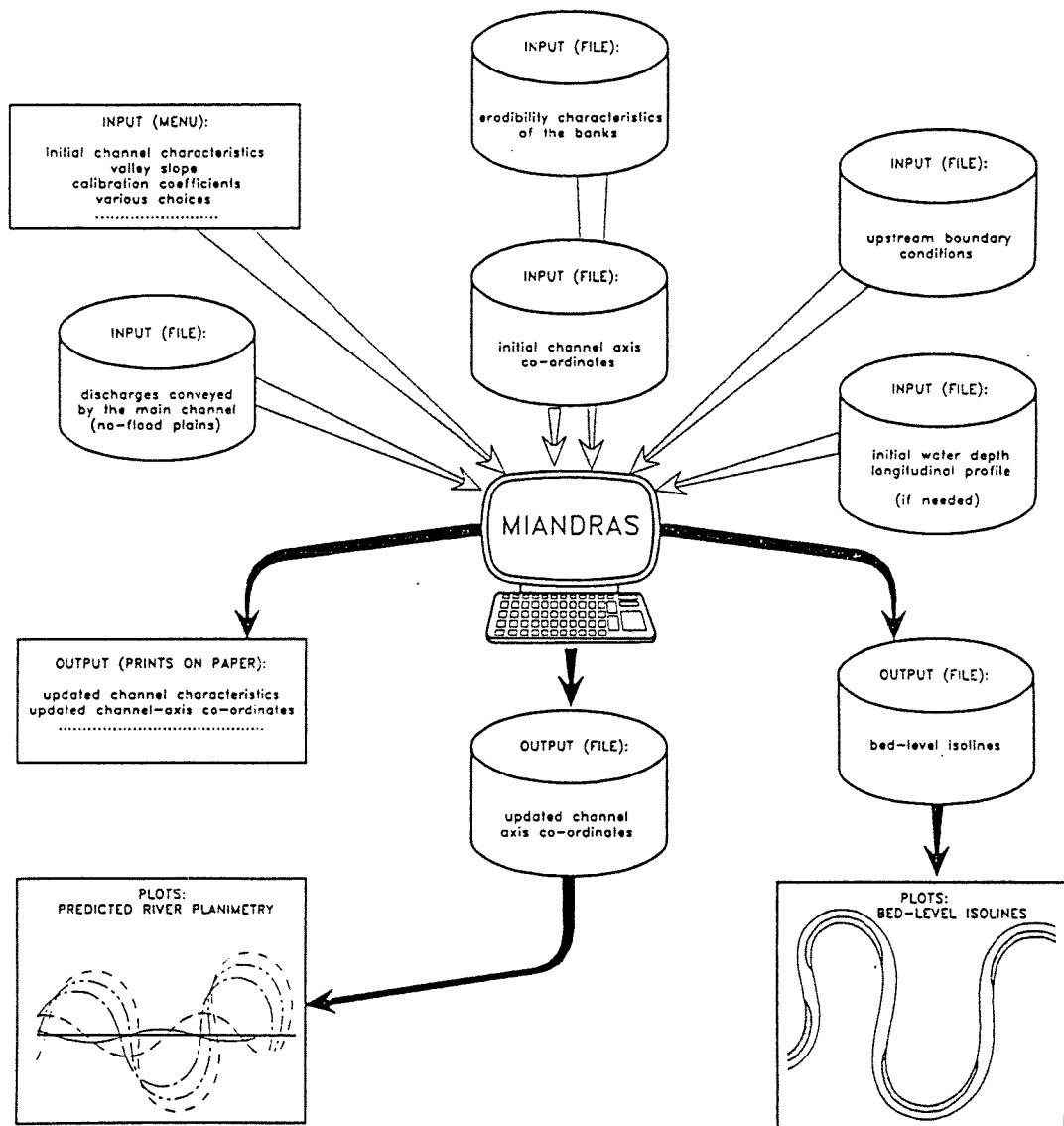


Figure 14 Input-output schematization, MIANDRAS

5.2 Numerical model - The differential equations, Eq. [2.42], Eq. [2.35] and Eq. [2.58] are solved separately. Computations proceed as follows:

- I Eq. [2.42], for the computation of the longitudinal near-bank water depth profile, is solved at a certain time-level with given boundary and, when needed, initial conditions (the latter are needed only if the [2.55]).
- II Eq. [2.35], for the computation of the longitudinal near-bank tangential velocity profile, can be solved when the near-bank longitudinal water depth profile is known and with a given boundary condition.
- III Eq. [2.58], for the computation of bank erosion relative to the duration of the last time-step, is solved using the computed tangential velocity and, when needed, water depth profiles (the latter is needed when computing the retreat of vertical cohesive banks).

At the new time level the cycle restarts.

The flow-chart of MIANDRAS is schematized in Figure 15.

The boundary conditions necessary to solve Eq. [2.42] to compute the water depth longitudinal profile, are the near-bank (for MIANDRAS, left bank) water depth excess (with sign) from its reach-averaged value and its gradient at the upstream section. The boundary condition necessary to solve Eq. [2.35] to compute the tangential velocity profile, is the near-bank velocity excess (with sign) from its reach-averaged value at the upstream section.

The reach-averaged values of velocity and water depth are computed with the Chézy equation.

Considering, for the bed deformation, Eq. [2.42] instead of Eq. [2.40] allows for the adoption of separate explicit numerical schemes for the computation of velocity and water depth deformations. However, this implies that the dependency of the water depth deformation on the velocity deformation, which is still present in Eq. [2.40], is lost. Nevertheless this can affect the results only if starting the computations from a straight channel while imposing non-equilibrium boundary conditions (i.e. upstream flow deformation without the corresponding bed deformation), which is nonsense

when adopting a steady-state model (Eq. [2.42]), even when taking into account the time-scale of the bed development according to Eq. [2.55].

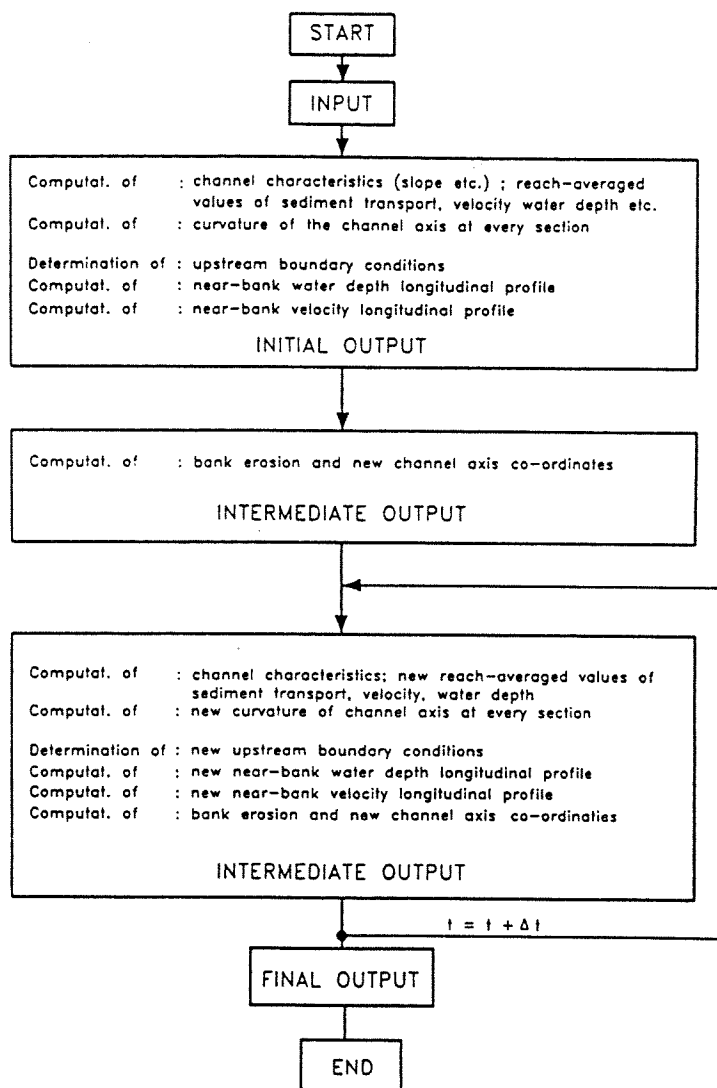


Figure 15 Flow-chart schematization, MIANDRAS

5.2.1 The numerical scheme of the solving equations - The equation for the near-bank water depth deformation, Eq. [2.42], and the equation for the tangential velocity deformation, Eq. [2.35], are both solved numerically with the Runge-Kutta method. This is a simple, explicit numerical method designed for first order differential equations, like Eq. [2.35], but extendable for higher order differential equations, like Eq. [2.42]. The adopted co-ordinate system is depicted in Figure 16.

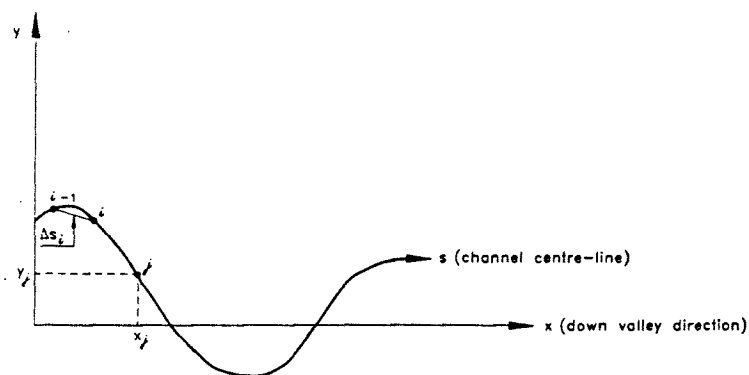


Figure 16 Computational co-ordinate system

The co-ordinate s of the generic section j is:

$$s_j = \int_0^j ds \quad (5.1)$$

s_j is determined computationally by:

$$s_j = \sum_{i=1}^j \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} = \sum_{i=1}^j \Delta s_i \quad (5.2)$$

in which Δs_i distance between the generic section i and section $i-1$.

When the sections are close enough to each other the error made in computing s_j with Eq. [5.2] instead of with Eq. [5.1] is small.

Bed equation

Eq. [2.42] can be written as

$$\frac{\partial^2 H}{\partial s^2} + A' \frac{\partial H}{\partial s} + B' H + (C' \frac{\partial^2 \Gamma}{\partial s^2} + D' \frac{\partial \Gamma}{\partial s} + E' \Gamma) = 0 \quad (5.3)$$

with:

$$\Gamma = \frac{1}{R_c} \left(\frac{m\pi}{2} \right)$$

$$A' = \frac{1}{\lambda_s} - \frac{(b-3)}{2\lambda_w}$$

$$B' = \frac{1}{\lambda_s \lambda_w}$$

$$C' = \frac{h_o}{k_B} (b-1)$$

$$D' = - \left[Ah_o^2 k_B - \frac{(2-\sigma)(b-1)h_o}{2\lambda_w k_B} \right]$$

$$E' = - \frac{Ah_o^2 k_B}{\lambda_w}$$

A' , B' , C' , D' , E' are known coefficients, they are constant with s , but variable with time. They are re-computed at every time-step.

The terms between parenthesis in Eq. [5.3] are a function of the channel centre-line curvature, because $\hat{c}$ (curvature due to the non uniformity of the flow) is neglected from the solving equations. The curvature of each section can be computed when the channel planimetry is known (see next Sub-Section 5.2.2) and is re-computed at every time-step.

For this reason:

$$C' \frac{\partial^2 \Gamma}{\partial s^2} + D' \frac{\partial \Gamma}{\partial s} + E' \Gamma = F(s)$$

$F(s)$ = the source term (function of s) of Eq. [5.3].

The boundary conditions, necessary to solve Eq. [5.3] are:

$$H(0) = H_{j=0}$$

$$\frac{\partial H(0)}{\partial s} = \left(\frac{H_{j=1} - H_{j=0}}{\Delta s_{j=1}} \right)$$

The numerical solution of Eq. [5.3] is obtained with the Runge-Kutta numerical method extended for second-order differential equations.

Equation [5.3] can be written in the following way:

$$\frac{\partial H}{\partial s} = f$$

$$\frac{\partial f}{\partial s} = - [A'f + B'H + F(s)]$$

According to the Runge-Kutta method, Eq. [5.3] can be solved as follows:

$$H_{j+1} = H_j + \frac{1}{6} (K_0 + 2K_1 + 2K_2 + K_3)$$

$$f_{j+1} = f_j + \frac{1}{6} (M_0 + 2M_1 + 2M_2 + M_3)$$

in which:

$$K_0 = \Delta s_{j+1} f_j$$

$$M_0 = - \Delta s_{j+1} [A'f_j + B'H_j + F(s_j)]$$

$$K_1 = K_0 + \left(\frac{\Delta s_{j+1}}{2} \right) M_0$$

$$M_1 = - \Delta s_{j+1} \left[A' \left(f_j + \frac{M_0}{2} \right) + B' \left(H_j + \frac{K_0}{2} \right) + F \left(s_j + \frac{\Delta s_{j+1}}{2} \right) \right]$$

$$K_2 = K_0 + \left(\frac{\Delta s_{j+1}}{2} \right) M_1$$

$$M_2 = - \Delta s_{j+1} \left[A' \left(f_j + \frac{M_1}{2} \right) + B' \left(H_j + \frac{K_1}{2} \right) + F \left(s_j + \frac{\Delta s_{j+1}}{2} \right) \right]$$

$$K_3 = K_0 + \left(\frac{\Delta s_{j+1}}{2} \right) M_2$$

$$M_3 = - \Delta s_{j+1} [A'(f_j + M_2) + B'(H_j + K_2) + F(s_{j+1})]$$

$$\text{with: } F(s_j + \frac{\Delta s_{j+1}}{2}) = \frac{F(s_j) + F(s_{j+1})}{2}$$

Flow equation

The numerical solution of Eq. [2.35] is obtained with the Runge-Kutta numerical method for first-order differential equations.

Equation [2.35] can be written as:

$$\frac{\partial U}{\partial s} = M'U + N'H + (P' \frac{\partial \Gamma}{\partial s} + Q'\Gamma) \quad (5.4)$$

with:

$$\Gamma = \frac{1}{R_c} \left(\frac{m\pi}{2} \right)$$

$$M' = - \frac{1}{\lambda_w}$$

$$N' = \frac{1}{2\lambda_w} \frac{u_o}{h_o}$$

$$P' = - \frac{u_o}{k_B}$$

$$Q' = - \frac{(2-\sigma)}{2\lambda_w} \frac{u_o}{k_B}$$

M' , N' , P' , Q' are known coefficients, they are constant with s , but variable with time. They are re-computed at every time-step.

The terms between parenthesis in Eq. [5.4] are a function of the channel centre-line curvature. For this reason:

$$P' \frac{\partial \Gamma}{\partial s} + Q'\Gamma = G(s)$$

$G(s)$ = the source term of Eq. [5.4].

The boundary condition necessary to solve Eq. [5.4] is:

$$U(0) = U_{i=0}$$

Eq. [5.4] can be solved numerically as follows (Runge-Kutta method):

$$U_{j+1} = U_j + \frac{1}{6} (R_0 + 4R_1 + R_2)$$

in which:

$$R_0 = \Delta s_{j+1} [M' U_j + N' H_j + G(s_j)]$$

$$R_1 = \Delta s_{j+1} \left[M' \left(U_j + \frac{R_0}{2} \right) + N' \left(\frac{H_j + H_{j+1}}{2} \right) + G \left(s_j + \frac{\Delta s_{j+1}}{2} \right) \right]$$

$$R_2 = \Delta s_{j+1} [M' (U_j + 2R_1 - R_0) + N' H_{j+1} + G(s_{j+1})]$$

$$\text{with: } G \left(s_j + \frac{\Delta s_{j+1}}{2} \right) = \frac{G(s_{j+1}) + G(s_j)}{2}$$

Bank erosion equation

One of the basic assumptions of the model is a constant channel width. Keeping the channel width constant (with s and t) is equivalent to the assumption that deposition will take place at the bank opposite to the eroding one in such a way that the depositing-bank shift towards the channel centre-line equals the bank erosion rate, see Figure 2. Furthermore, the assumption of a constant channel width together with the anti-symmetry of the transverse shape of velocity and water depth perturbations (to which bank erosion is related) allows for the computation of U and H near one bank only. In the computer program MIANDRAS bank erosion is related to the left bank, towards which the positive direction of n is conventionally pointing. The erosion rate, Δn , is computed at every time according to Eq. [2.58]. The channel centre-line is then shifted with the same step-length Δn (if positive towards the left bank, if negative towards the right bank). The numerical solution of Eq. [2.58] is the following:

$$\Delta n_j = [(E_u)_j U_j + (E_h)_j H_j] \Delta t \quad (5.5)$$

in which $(E_u)_j$, $(E_h)_j$ are the erosion coefficients of the eroding bank at section j ; Δt = time-step duration; U_j , H_j = near-bank velocity and water depth deformations at section j , respectively.

The accuracy of bank erosion computations is improved by computing Δn_j with a predictor-corrector method:

$$\Delta n_j(t)_c = [\Delta n_j(t) + \Delta n_j(t - \Delta t)]/2 \quad (5.6)$$

in which t = present time-level; $t - \Delta t$ = previous time-level; $\Delta n_j(t)$ and $\Delta n_j(t - \Delta t)$ = bank erosion rates at section j computed at time t and $t - \Delta t$, respectively; $\Delta n_j(t)_c$ = corrected bank erosion rate at time t .

5.2.2 Computation of the curvature - The curvature of the stream-lines can be approximated by the channel centre-line curvature (the error induced by this approximation was found acceptable, see Section 2.2.4). For this reason the term $\hat{c}$ (additional curvature of the stream-lines due to the non-uniformity of the flow) is neglected from the solving equations.

The channel centre-line curvature is computed for every section j as the radius of curvature of the circle passing through the three sections $j-1$, j , and $j+1$.

The sign of the curvature is computed according to the following conventional rules (see Figure 17): (a) the positive versus of n points towards the left bank; (b) if the radius of curvature has the same versus as n , the curvature is positive, otherwise it is negative.

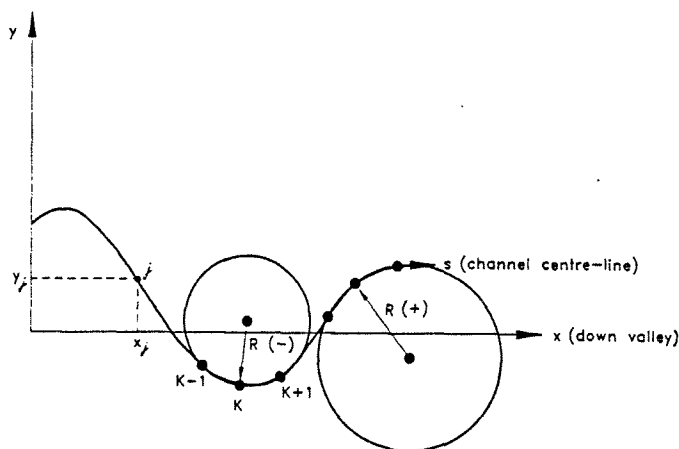


Figure 17 Computational channel centre-line curvature

Sometimes, due to the errors made in detecting the right position of the channel's centre-line planimetry (i.e. from satellite images or historical maps), it is necessary to smooth the sudden changes of the channel's curvature. This can be obtained by taking into account more than three sections for the computation of the channel's curvature ("smoothing of curvature"). Smoothing of curvature is done as follows:

$$C_j = \frac{C_{j-1} + 2C_j + C_{j+1}}{4} \quad (5.7)$$

in which: C_j , C_{j-1} , C_{j+1} = values of computational channel's curvature at section j , $j-1$ and $j+1$, respectively.

Curvature smoothing can be repeated for a number of times. With one time of curvature smoothing five sections are accounted for in the computation of the curvature (the section in which the curvature is computed plus two sections before and two after), with two times of curvature smoothing seven sections, and so on.

The computation of the curvature and therefore all the computational results can be greatly improved by writing a "smoothing routine" which es, calculates, in the plane x-y (river valley), the polinomial which best interpolates the channel's planimetry (with the minimum square distances method, for example). Once the polinomial is known, the curvature at every point is also known and not affected by sudden changes of the river's planimetry: no more need for something like "curvature smoothing".

6. COMPARISON WITH MEASUREMENTS: STEADY-STATE FLOW AND CHANNEL BED
PREDICTION

6.1 Straight Flume Experiment - The prediction of the equilibrium flow field and bed topography is compared with measurements carried out along a straight flume at DELFT HYDRAULICS.

The experimental conditions were a well defined and steady flow, while at the upstream boundary a disturbance was induced by restricting the inflow section with a plate, causing a steady deformation at the inflow section, see Figure 18.

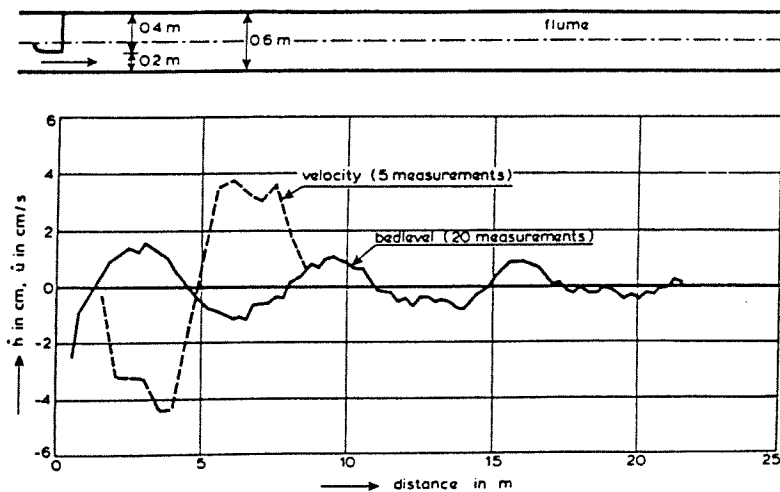


Figure 18 Longitudinal profile of time-averaged near-bank bed level and velocity profiles along the right side wall. Straight Flume Experiment

Table 1 gives the relevant conditions of the experiment, while flow and bed behaviour characteristics are summarized in Table 2.

flume width	discharge	water depth	flow velocity	water surface slope	Chézy coeff.	Froude number	Shields parameter	sediment transport
B	Q	h_o	u_o	i_s	C	F	θ_o	S**
(m)	(m^3/s)	(m)	(m/s)	(%)	($m^{1/2}/s$)	(-)	(-)	(m^3/h)
0.60	$6.85 \cdot 10^{-3}$	0.044	0.26	3.00	22.6	0.39	0.37	$2 \cdot 10^{-3}$

(** measurements of sediment transport were not satisfactory)

Table 1 Conditions of the Experiment

flow adaptation length λ_w (m)	bed adaptation length λ_s^{**} (m)	interaction parameter λ_s/λ_w (-)	Exp. b	
			Meyer-Peter & Müller tr. formula (-)	Engelund & Hansen tr. formula (-)
1.14	0.86	0.75	4.84	5.00

Table 2 Flow and Bed Development Characteristics (** E = 0.5).

Fine and almost uniform sediment was used, having the following characteristics: $D_{10} = 0.162$, $D_{50} = 0.216$, $D_{90} = 0.271$ mm.

It can be assumed that for the measurements of bed topography the duration of the experiment was sufficiently long to ensure the establishment of the equilibrium conditions. The "noise" of bed forms (ripples and alternate bars) was smoothed by averaging a large number (20) of independent soundings. The time-averaged longitudinal bed level profiles measured at 0.1 m from the side walls showed the development of a steady, mildly damping wave originated near the disturbance, all along the channel. The wave length of this steady oscillation was about two times larger than the one of the observed alternate bars which were migrating downstream, see Figure 19. The time-averaged velocity profile (only five independent sets of measurements available) had the same waving behaviour as the bed, but with a phase lag, see Figure 18.

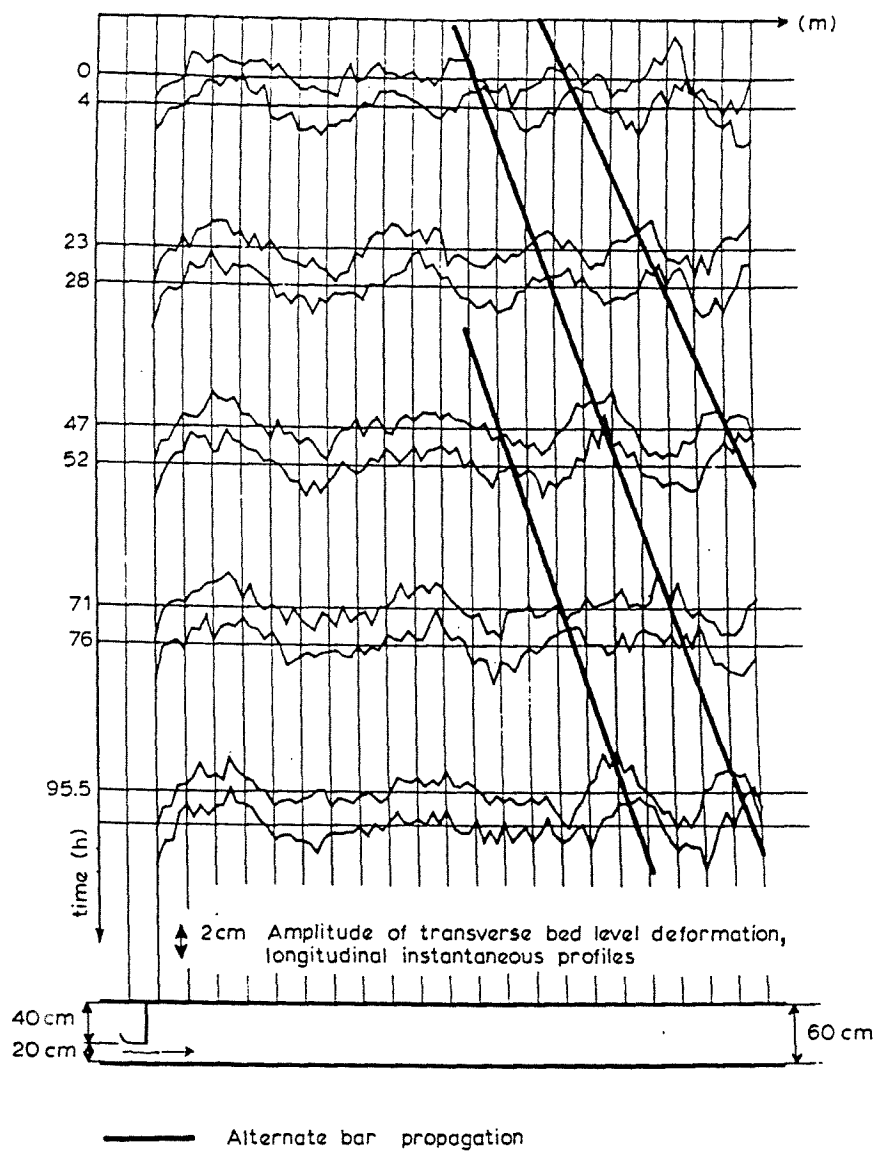


Figure 19 Observed alternate bar propagation during the Straight Flume Experiment

The computed and measured values of wave length and damping coefficient of the steady oscillation are summarized in Table 3. The computed values were obtained without calibration of coefficient E (influence of the transverse bed slope on the sediment transport direction, Eq. [2.10]), for which the value of 0.5 was chosen, according to van Mierlo [1986]. Most probably a calibration of E would even give better results. In Figure 20 the measured longitudinal near-bank profile of the equilibrium bed topography and the computed one (Meyer-Peter & Müller transport formula) are plotted.

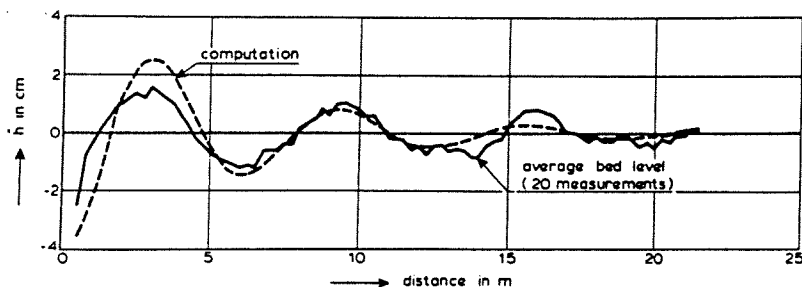


Figure 20 Measured and computed near-bank longitudinal bed level profile. Straight Flume Experiment.

When predicting the flow and bed deformation in the area close to the boundaries, $\hat{c}$ must be retained in the equations. As already mentioned in Section 2.2.4, the two additional, purely exponential solutions yielded by retaining $\hat{c}$ become appreciable here, while the harmonic solutions of the equations for the two cases: a) retaining $\hat{c}$, b) disregarding $\hat{c}$, are not so different. In the experiment they differ only 12 per cent, see Table 3.

$\hat{c}$ neglected**				$\hat{c}$ retained**				Measured	
Meyer-Peter & Müller tr. formula		Engelund & Hansen tr. formula		Meyer-Peter & Müller tr. formula		Engelund & Hansen tr. formula		Damping coeff. of deformation (m^{-1})	Wave length of deformation (m)
$1/L_d$ (m^{-1})	L_p (m)	$1/L_d$ (m^{-1})	L_p (m)	$1/L_d$ (m^{-1})	L_p (m)	$1/L_d$ (m^{-1})	L_p (m)		
0.18	6.3	0.14	6.3	0.11	7.2	0.09	7.2	0.09	6.6

(** $E = 0.5$, $\alpha_1 = 0.5$, $\sigma = 2.0$)

Table 3 Computed and measured wave length and damping coefficient of the observed deformation. Straight Flume Experiment

6.2 Curved Flume Experiment - The prediction of bed topography has also been studied by means of a curved flume experiment carried out at DELFT HYDRAULICS, see Struiksma [1983]. The equilibrium bed topography was measured in a curved laboratory flume with fixed banks. The planform of the bend (see Figure 21a or 21b), with a radius of curvature of 12 m, width of 1.5 m and a bend length of 29.32 m, does not fit into the characteristics of freely meandering rivers. For example, it is in contrast with the finding of Leopold et al [1964] that the meander length is about ten times the channel width. Table 4 gives the relevant experimental conditions.

flume width	discharge	water depth	flow velocity	water surface slope	Chézy coeff.	Froude number	Shields parameter	sediment transport
B	Q	h_o	u_o	i_s	C	F	θ_o	S
(m)	(m^3/s)	(m)	(m/s)	(%)	($m^{1/2}/s$)	(-)	(-)	(m^3/h)
1.5	0.047	0.080	0.39	2.36	28.4	0.44	0.26	$7.2 \cdot 10^{-6}$

Table 4 Experimental Conditions

The sediment was characterized by a median grain size $D_{50} = 0.45$ mm and a gradation parameter $\sigma_g = 1.19$. For the measurements of bed topography, the duration of this experiment is assumed to have been sufficiently long to ensure equilibrium conditions. The "noise" of bed forms was smoothed by averaging a large number (> 20) of independent soundings.

The measured and computed water depth isolines, are plotted in Figure 21a and 21b, respectively.

Computations: the curvature due to the non-uniformity of the flow is neglected ($\hat{C} = 0$); the adopted transport formula is that of Engelund & Hansen; the values of coefficients α_1 , E and σ are 0.5, 0.5 and 4, respectively.

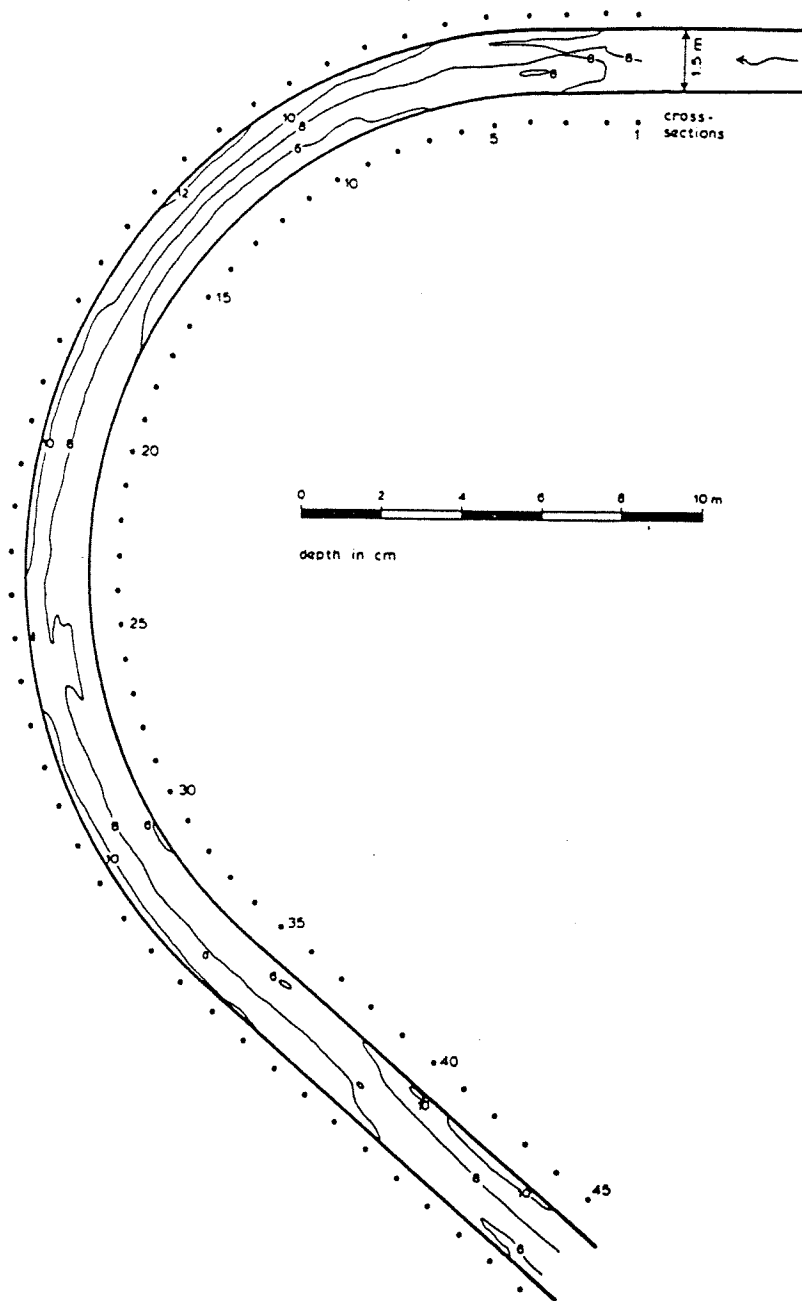


Figure 21a Measured water depth isolines. Curved Flume Experiment
(Struiksma [1983])

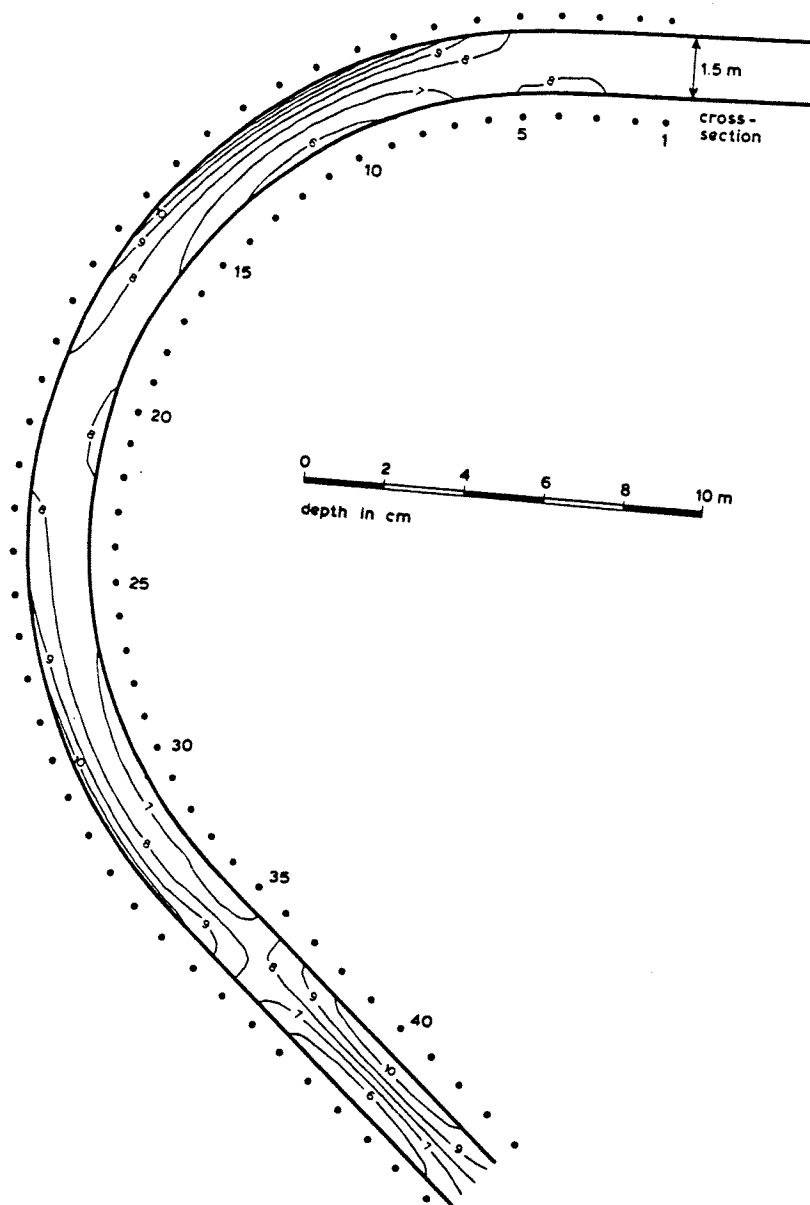


Figure 21b Computed water depth isolines. Curved Flume Experiment
(Struiksma [1983])

7. SIMULATION OF MEANDERING

7.1 General - MIANDRAS has been tested computationally for the prediction of initiation and further developments of meanders. These computational tests are only qualitative.

Initiation and further developments of meanders have been obtained from the initial conditions of the Straight Flume Experiment by assuming erodible non-cohesive banks (erosion law Eq. [2.56]). Appropriate erosion coefficient and time-step duration were chosen. The upstream boundary conditions (in the experiment: the measured near-bank velocity and water depth deformation, including water depth gradient, which were obtained by partially obstructing the flow with a transversal plate, see Figure 18, at a certain section) were kept constant with time in the first computational test; variable with time (but starting with the same values as in the first computation) in the second one. Channel planimetry and bed topography were plotted at regular intervals. The difference between the results of the two computations can give a qualitative idea about the influence of the upstream boundary conditions on the computational results of meander migration.

7.2 Constant upstream boundary conditions - Computations of meander migration have been carried out from the initial conditions of the Straight Flume Experiment (assuming erodible banks). The values of the initial parameters are summarized in Table 2. The values of the upstream boundary conditions as well as the value of the channel discharge were kept constant during the computations.

The results, i.e., the computed meander initiation and further meander developments are plotted in Figure 22 (non-distorted scale). At an initial stage meanders show a regular sinusoidal shape characterized by a gradual increase in amplitude and wave length and by downstream migration of the loops. The characteristic asymmetry of meanders evolving from the initially regular sinusoidal shape, as observed among others by Carson & Lapointe [1983] and Raisz [1955], is reproduced rather well by the model.

The difference between the proposed meander migration model and those which disregard the flow redistribution effects can be clarified by this computational result.

When computing the planimetric changes of an initially straight channel, having a non uniform inflow, those models based on the approach of Ikeda et al [1981] for the computation of flow and channel bed deformations would predict the gradual development of the uniform flow conditions downstream of the flow disturbance. According to those models, apart from a single curve close to the disturbance, no sinuous planimetry would develop. The proposed model, on the contrary, predicts the development of a sinuous planimetry whenever an upstream disturbance originates a deformation of the alternate bar type (within the harmonic range of solutions), either damped or non-damped non-growing.

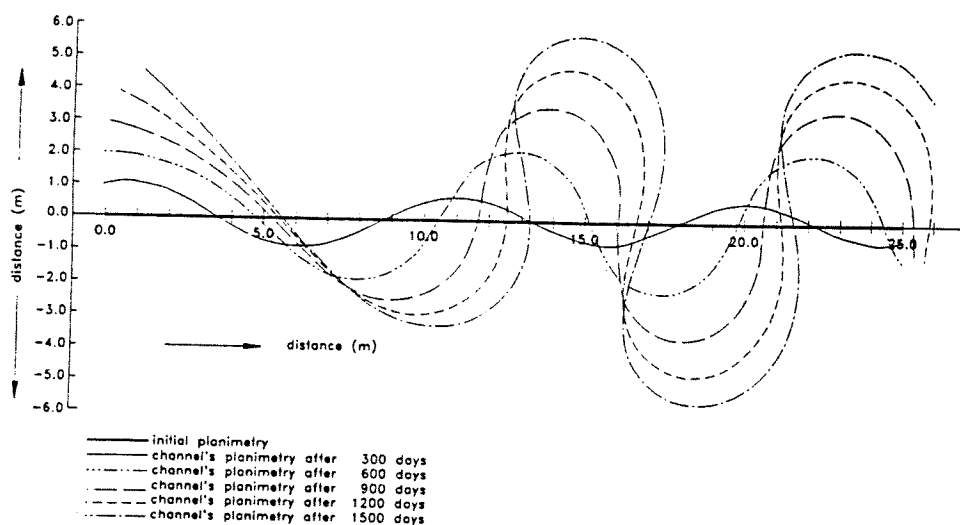


Figure 22 Simulation of the development and migration of meander loops from the initial conditions of the Straight Flume Experiment with constant boundary conditions.

7.3 Variable upstream boundary conditions - Another computational test has been performed from the initial conditions of the Straight Flume Experiment. The values of the initial parameters are summarized in Table 2. The discharge was kept constant during the computations, while the value of the upstream boundary conditions could vary. Their initial values were the same as in the previous computation.

The time variation of the boundary conditions was given by the following expressions:

$$H(0)_i = \left[\frac{H(1)_{i-1} + H(0)_{i-1}}{2} \right]$$

$$U(0)_i = \left[\frac{U(1)_{i-1} + U(0)_{i-1}}{2} \right]$$

$$\left[\frac{\partial H(0)}{\partial s} \right]_i = \left[\frac{H(1)_{i-1} - H(0)_{i-1}}{2} \right]$$

in which i = present time-level; $i-1$ = previous time-level; 0 = upstream boundary; 1 = first section downstream of the upstream boundary; s = longitudinal co-ordinate.

The result is a gradual decrease in values of the upstream deformations (and gradient) via a damped oscillation becoming soon equal to zero. The initial deformation was enough to induce meandering. The obtained initiation and further developments of meanders are plotted in Figure 23.

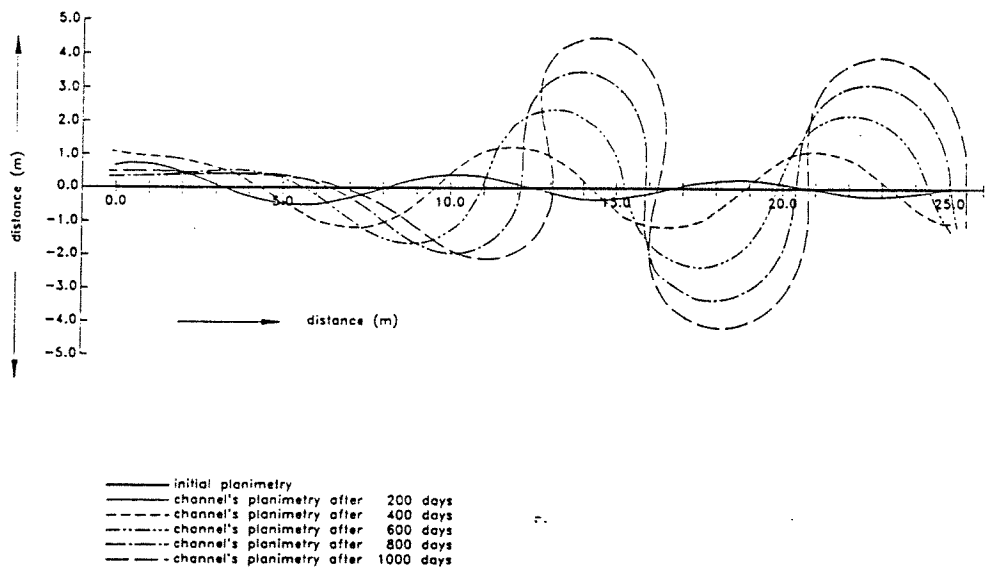


Figure 23 Simulation of the development and migration of meander loops from the initial conditions of the Straight Flume Experiment with variable boundary conditions

7.4 Observations - The difference between the planimetric configurations obtained with the two computational tests can be observed by comparing Figures 22 and 23 (centre-line alignments as function of time). The bed topography prediction can be detected from Figures 24 and 25 in which the point bars are also drawn (the two planimetries correspond to the same duration of computations and the contour lines of the point bars are comparable, being defined by the same value of the water depth).

The results show that the time variation of the upstream boundary conditions can strongly affect the prediction of planimetric changes and bed topography in that part of the channel close to the upstream boundary. Their influence is obviously stronger for longer term predictions. This leads to the conclusion that the upstream boundary should be chosen relatively far from the area of interest. In addition, the upstream boundary should be located in an area in which velocity, water depth and future planimetric changes are known, (for example a stabilized or straight reach). However it is better if the channel would have an approximately uniform flow. Furthermore, computations should not be carried out for excessive long periods.

To determine the applicability horizon of the model from the point of view of the duration of the prediction period, the influence of the boundary conditions should be a subject of further studies.

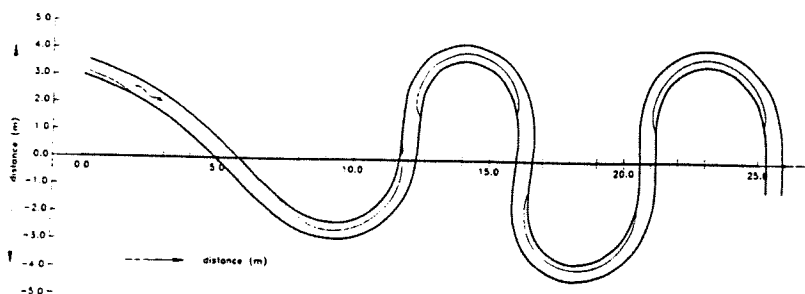


Figure 24 Planimetric configuration and point bar locations obtained from the initial conditions of the Straight Flume Experiment keeping constant boundary conditions.

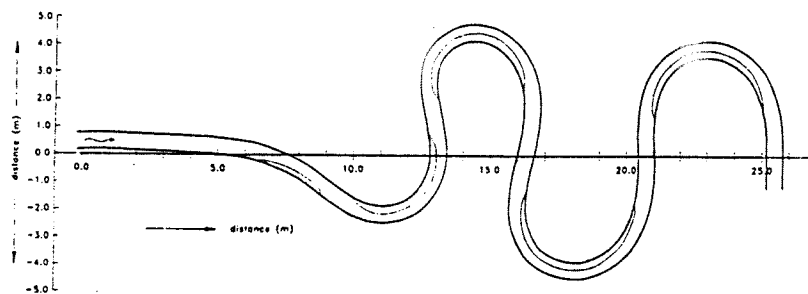


Figure 25 Planimetric configuration and point bar locations obtained from the initial conditions of the Straight Flume Experiment with variable boundary conditions.

8. ACCURACY OF COMPUTATIONS

8.1 General - Accuracy of computations is generally influenced by various factors: truncation errors, distance between sections, time-step etc. For computations of meander migration the accuracy of computations was found to be affected by (i) the maximum distance between sections; (ii) the time-step; (iii) the erosion rates; and (iiii) the number of sections accounted for in the computation of the channel centre-line curvature (via curvature smoothing). Furthermore, these parameters are related to each other.

8.2 Instability of the computations - Computations of meander migration showed that there is a relation which defines a threshold between stable and unstable planimetric computations. The computations are stable when:

$$\Delta t \leq \epsilon \frac{\Delta s}{\partial n / \partial t} \quad (8.1)$$

with $\epsilon \approx 0.2$

in which: Δs = maximum space step; Δt = time-step; $\partial n / \partial t$ = maximum bank erosion speed, defined by:

$$\frac{\partial n}{\partial t} = E_u u_o + E_h h_o$$

where u_o , h_o = reach-averaged values of velocity and water depth, respectively; E_u , E_h = erosion coefficients, see Section 2.3.

The form of Eq. [8.1] is similar to the relation designed by Courant, Isaacson and Rees [1952] for the stability of explicit numerical schemes for computations of riverbed deformation: the celerity of propagation of an infinitesimal bed perturbation is replaced by the term $\partial n / \partial t$, which represents the celerity of river migration.

The value of ϵ , defining the threshold between stable and unstable computations was estimated by performing a large number of computational tests. However, these were based on the data from the flume experiments only (Chapter 6). The value of ϵ should also be estimated with data coming from other sources.

8.3 Influence of time-step duration (stable computations) - Computations of meander migration, starting from the conditions of the Straight Flume Experiment (constant discharge and boundary conditions), performed with different time-steps show that with the increase of the value of the time-step accuracy problems can appear (computations that are stable but not accurate).

The results of some of these computational tests are plotted in Figure 26.

The definition of a criterion for the determination of the optimal time-step duration requires further studies. For the time being the advice is to perform test computations with different time-steps in order to estimate the optimal one.

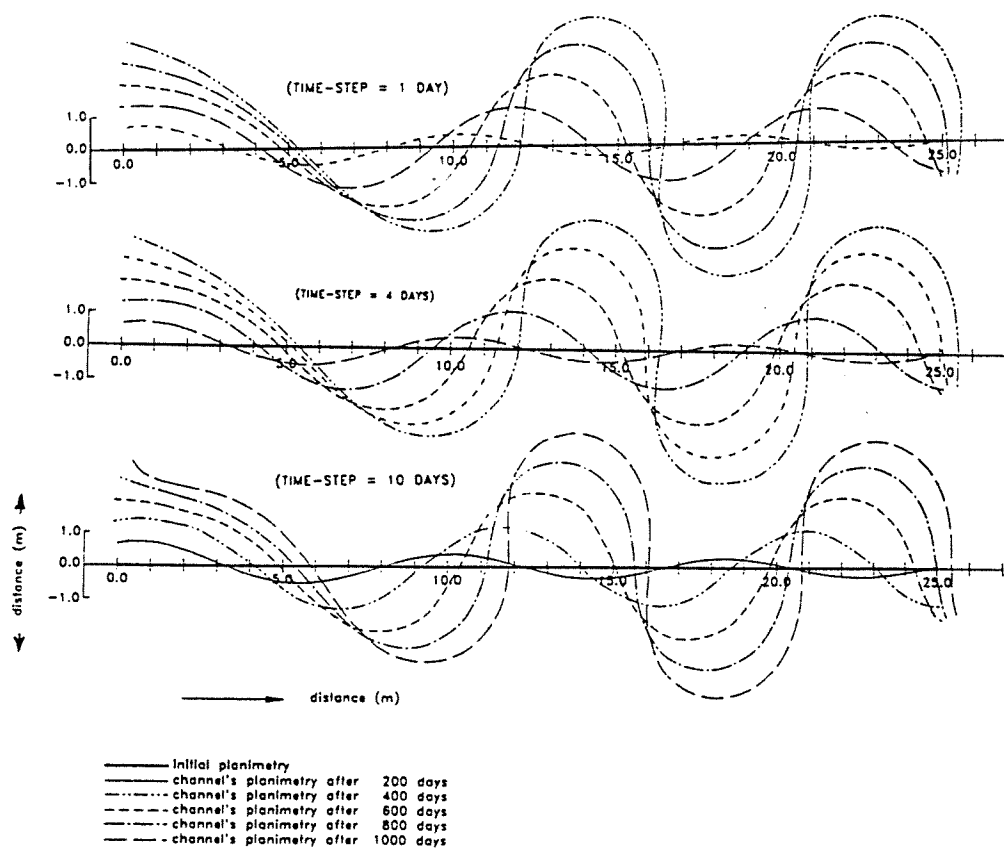


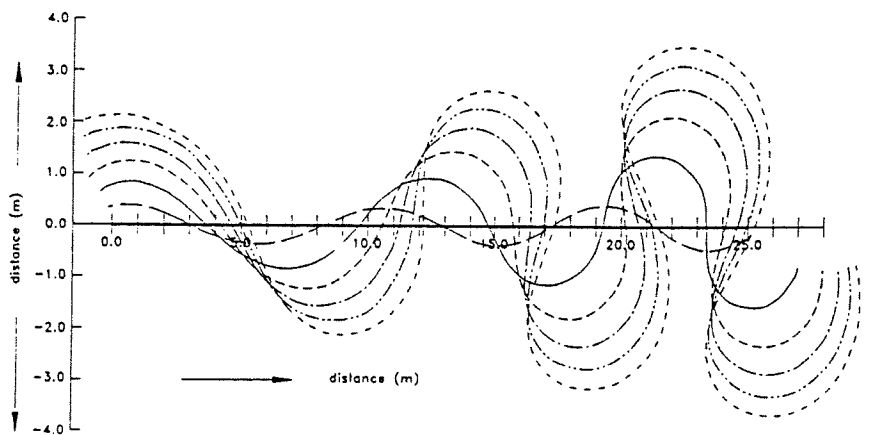
Figure 26 Meander migration predictions obtained with different time-steps

8.4 Influence of maximum space-step (stable computations) - Maximum space-step is the maximum distance between two successive sections, its value influences the accuracy of computations (computations that are stable but not accurate).

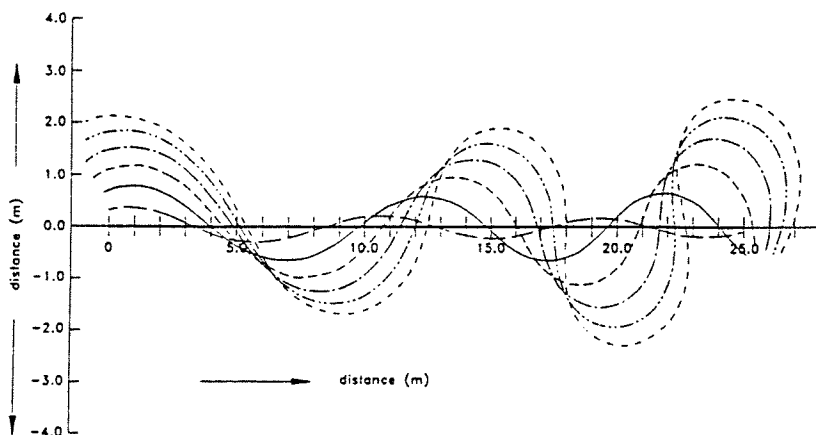
A reasonable criterion to determine the optimal value of $\Delta n_{\max}$ should take into account the values of the adaptation lengths of flow and water depth deformations, λ_w and λ_s , respectively. Maximum space-step can be assumed, for example, as 1/4 or 1/5 of the value of the smaller between λ_w and λ_s .

8.5 Influence of curvature smoothing - Curvature smoothing is one way to choose how many sections to consider when computing the channel curvature, see Eq. [5.8]. With "one-smoothing" five sections are accounted for (the section for which curvature is computed, plus the two sections before and the two after), with "two-smoothing" seven sections, and so on. Curvature smoothing improves the accuracy of computations when it is impossible to detect the initial channel centre-line co-ordinates with a relatively good accuracy. Curvature smoothing is sometimes the only way to avoid instability of the computations when this is related to sudden changes in the bank erodibility which often lead to the formation of (unrealistic) acute angles in the channel planimetry. However, one should consider that curvature smoothing decreases the derivatives of the curvature with s . This leads to a decrease in the computed bed and in flow deformations and therefore of the erosion rates. This can be observed from Eq. [2.39] which describes the source term of the bed equation, Eq. [2.42]. As a consequence the application of curvature smoothing in a second moment (after calibration) curvature smoothing leads to the need of re-calibrating the values of the erosion coefficients (see Figure 27).

The problems related to the uncorrect assessment of the initial channel planimetry which are especially related to satellite imagery, can be solved by writing a "smoothing routine", see Section 5.2.2., which finds an analytical form to describe the channel's planimetry in the co-ordinate system x - y (river valley). This can be done with the method of the minimum square distances, for example. Once the analytical expression for the planimetry is known, also its curvature at any point is known with no need for something as "curvature smoothing".



(no curvature smoothing) boundary conditions: constant



— initial planimetry
 - - - channel's planimetry after 200 days
 - - - channel's planimetry after 400 days
 - - - channel's planimetry after 600 days
 - - - channel's planimetry after 800 days
 - - - channel's planimetry after 1000 days
 - - - channel's planimetry after 1200 days

(with curvature smoothing) boundary conditions: constant

Figure 27 Meander migration predictions obtained with and without curvature smoothing

9. DISCUSSION AND CONCLUSIONS

In this report, a simple model for the simulation of meandering processes of alluvial rivers has been described. It is applicable to those channels which are characterized by: (i) neither aggradation nor degradation of the bed; (ii) an approximately constant width; (iii) shallow water; (iiii) relatively slow bank erosion processes. The last limitation is due to the assumption that neither the development of bed topography is influenced by bank erosion, nor the latter is influenced by the presence of alternate bars migrating downstream (migrating alternate bars are present only in channels with a relatively low sinuosity), which is the case with most of meandering alluvial rivers. The occurrence of meander cut-offs is not taken into account for the development of the model.

The mathematical model is split up into two parts: computation of steady-state flow (and bed) deformation, and computation of time-dependent river migration (flow-induced bank erosion is assumed to be a function of the near-bank velocity excess with respect to its reach-averaged value).

Although a similar model has already been developed by Ikeda et al [1981], the new feature of this model lies in the prediction of the steady-state flow field and bed topography prediction, which have been improved by taking into account the interaction between flow and sediment (redistribution effects). This follows from the full coupling of flow field, bed topography and sediment transport. Flow and sediment redistribution play an important role in the bed development, as it is responsible, for example, for the formation of point bars in channel bends. The secondary flow momentum convection is also taken into account.

The basic equations are simplified by linearization and by disregarding some smaller terms which are believed not to affect appreciably the results. The perturbations are assumed small when compared with the reach-averaged values. However, this assumption is often in contrast with observations. For example, in the Curved Flume Experiment the mean water depth is 8 cm, while the superimposed water depth deformation is 4 cm (maximum value).

The influence of the non-linear terms on the bed development and therefore on the prediction of meander migration should be subjected to further studies. It is likely that some non-linear terms, which are small for the conditions of a mildly curved channel, become non-negligible with stronger channel's curvatures. Furthermore, some non-linear terms will damp the

longitudinal deformations of flow and river bed which follow the upstream flow disturbances. According to the linear model this deformation can even grow in the longitudinal direction (non-damping system). However, this is not what happens in reality.

The prediction of the steady-state flow field and bed topography has been tested with experiments carried out at DELFT HYDRAULICS. The predicted equilibrium bed topography agrees rather well with the measured one. These satisfactory results support the idea that, in spite of the strong simplifications, the model takes into account the main parameters influencing the bed deformation.

The development of meander loops has been obtained computationally from the initial conditions of a straight channel with a distorted inflow (conditions present during the Straight Flume Experiment). Initiation of meandering is found to be coincident with the occurrence of a resonance phenomenon which is forced by the development of the channel sinuosity. The computed development of meanders, evolving from a regular sinusoidal shape to an asymmetric one, show downstream migration and lateral growth of the loops, in good accordance with field observations. However, the model does not predict the ceasing of the lateral bend growth when the bend is at its maximum size, a phenomenon which has been observed in nature. Indeed, the model simulates a decrease in lateral bend growth with the increase in bend size. This is due to: (i) the general decrease in velocities following the increase in channel sinuosity; and (ii) the decrease in value of local channel curvature at the bend apex, due to the progressive upstream expansion of the bend (leading to a decrease in near-bank velocity excess at the bend apex). Predictions of river migration have been found to be strongly dependent on the imposed upstream boundary conditions, especially for that part closer to the boundary. Furthermore, predictions of river migration become inaccurate, when performed for excessively long periods of time. Consequently, the upstream boundary should be chosen far enough from the stretch of interest, and the predictions performed for relatively long periods should be interpreted as merely qualitative. Further studies are needed to investigate more in detail the influence of the upstream boundary conditions on the prediction of meander migration.

With the proposed meander migration model the development of loops from a straight channel can be initiated whenever an upstream disturbance (an obstacle, a change of curvature etc) originates a steady harmonic deformation

of the flow, either damped or non-damped. This can be regarded as the main difference between the proposed model and those which disregard the redistribution effects. The latter, when computing bank erosion of an initially straight channel with non uniform inflow, predict bank retreat only in the region close to the upstream boundary without the development of a sinuous planimetry.

Recently a clear interaction between migrating alternate bars and meander formation has been found mathematically. This has also experimental support. It is beyond doubt that in some cases migrating alternate bars can be the cause of meandering, especially for those channels with relatively highly erodible banks (i.e. bank erosion speed comparable with the celerity of alternate bars propagation). However, it is more realistic to assume that both phenomena, the upstream-induced steady deformation and the presence of migrating alternate bars, influence the erosion pattern and therefore the meander formation. Most probably the erodibility of banks and the relative intensity of the two phenomena, when coexisting in the same channel, as for example in the Straight Flume Experiment, will determine which of the two will prevail in influencing bank erosion. It is believed that for those channels characterized by relatively low bank erodibility (i.e. bank erosion speed much lower than the celerity of alternate bars propagation) a steady bed deformation, rather than the relatively fast migrating alternate bars, will influence the erosion pattern.

NOTATIONS

A	weighing coefficient of spiral flow intensity
B	channel width
b	exponent of sediment transport law
C	Chézy coefficient
C	computational channel's curvature
D	grain size
E	calibration coefficient (for the influence of the transverse bed slope on the sediment transport direction)
E_o	time-averaged erosion coefficient
E_h	time-averaged erosion coefficient (failure of vertical cohesive banks)
E_u	time-averaged flow-induced-erosion coefficient
F	Froude-number
g	acceleration due to gravity
H	near-bank water depth deformation
h	water depth
i_s	water surface slope
K	meander wave number
k_{max}	wave number of forcing function yielding maximum response
k_{res}	resonance wave number
k_o	wave number of natural oscillation
k_B	transverse wave number
L	meander wave length as measured along s
L_D	damping length of flow and bed deformation (natural response)
L_p	wave length of flow and bed deformation (natural response)
m	mode that determines the transversal pattern of deformation (integer number)
n	transversal co-ordinate
Q	discharge
R_c	radius of curvature of the centre-line
R_s	radius of curvature of the stream-line
R_*	effective radius of curvature of the stream-line
S	sediment transport including pores per unit of width
S	general section
S_t	total volumetric sediment transport including pores per unit of width
s	stream-line coordinate
s_p	phase lag
t	time
U	near-bank tangential velocity deformation

NOTATIONS (continued)

u	flow velocity in s-direction
v	flow velocity in n-direction
z_b	bed level
z_w	water level
α	angle between sediment transport direction and s-coordinate
α	interaction parameter
α_1	calibration coefficient (for the influence of channel's curvature on bed-shear stress direction)
Γ	curvature term
Δ	submerged relative density of sediment
δ	angle between bed-shear stress direction and s-direction
c	number defining the threshold between stable and unstable computations
θ	Shields parameter
κ	Von Kármán constant
λ_s	adaptation length of bed deformation
λ_w	adaptation length of main flow
ξ	erodibility coefficient
ρ	density of fluid
σ	calibration coefficient (for the secondary flow convection)
'	superscript for perturbation
^	superscript for amplitude
o	subscript indicating zero-order
c	subscript indicating channel's centre-line
n	subscript indicating direction n
s	subscript indicating direction s

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APPENDIX A

FULLY DEVELOPED BEND FLOW

The conditions of a fully developed bend flow define a hypothetical situation characterized by non-variability of the flow parameters, such as velocity and water depth, along s . In reality these conditions can be observed in long channel bends having a constant radius of curvature.

Considering the two-dimensional, depth-averaged sediment balance equation for a curved channel:

$$\frac{\partial z_b}{\partial t} + \frac{\partial S_s}{\partial s} + \frac{\partial S_n}{\partial n} + \frac{S_n}{(R_c + n)} = 0 \quad (\text{A.1})$$

in which: s = longitudinal co-ordinate; n = transverse co-ordinate; S_s and S_n = components of volumetric sediment transport, including pores, per unit of channel width, in the s and n direction, respectively; z_b = bed level; R_c = radius of curvature of the channel centre-line; t = time.

When $\partial/\partial t = 0$ and $\partial/\partial s = 0$, together with the boundary condition (no sediment transport through the side walls):

$$S_n \Big|_{n = \pm B/2} = 0, \quad \text{with } B = \text{channel width}$$

the transverse sediment transport component is zero in the whole cross section. Consequently the sediment transport direction is coincident with the s -direction.

Similarly considering the two-dimensional, depth-averaged flow continuity equation for a curved channel:

$$\frac{\partial(h u)}{\partial s} + \frac{\partial(h v)}{\partial n} + \frac{h v}{(R_c + n)} = 0 \quad (\text{A.2})$$

in which u = tangential velocity; v = transverse velocity; h = water depth.

When $\partial/\partial s = 0$, together with the boundary condition of impermeable side walls:

$$v \Big|_{n = \pm B/2} = 0$$

the depth-averaged transverse velocity is zero in all the cross sections.

Considering now the expression for the sediment transport direction:

$$\tan \alpha = \frac{v}{u} - \frac{Ah}{R_s} - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} \quad (\text{A.3})$$

in which α = direction of sediment transport; $f(\theta)$ = weighing function for the influence of the sloping bed; and θ = the Shields parameter.

And applying Eq. [A.3] for the conditions of a fully developed flow ($\tan \alpha = 0$ and $v = 0$):

$$-\frac{Ah}{R_s} - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} = 0, \quad (\text{A.4})$$

with:

$$\frac{1}{R_s} = \frac{1}{(R_c + n)} - \frac{1}{u} \frac{\partial v}{\partial s} \quad (\text{de Vriend [1981]}) \quad (\text{A.5})$$

Equation [A.3] becomes:

$$-\frac{Ah}{(R_c + n)} - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} = 0 \quad (\text{A.6})$$

The basic assumptions of the model allow for a rigid lid approximation ($\partial z_b / \partial n = - \partial h / \partial n$):

$$\frac{\partial h}{\partial n} = \frac{A f(\theta) h}{(R_c + n)} \quad (\text{A.7})$$

Equation [A.7] is equivalent to the expression which represents the force balance on a grain on the sloping channel bed (equilibrium situation).

The expression giving the variation of the water depth along the direction n can be derived from Eq. [A.7] it is:

$$h = h_c \frac{(R_c + n)^{Af(\theta)}}{R_c^{Af(\theta)}} \quad (\text{A.8})$$

where h_c is the water depth at the channel centre-line ($n=0$) which is not coincident with h_0 .

When in Eq. [A.7] n can be disregarded with respect to R_c (mildly curved channel), the expression for the variation of h with n becomes:

$$h = h_c \exp \left[\frac{Af(\theta)}{R_c} n \right] \quad (A.9)$$

To obtain the expression for the variation of the main velocity along n in the fully developed bend flow conditions, it is necessary to consider the two-dimensional depth-averaged momentum equations with $v = 0$ and $\partial/\partial s = 0$ (except for z_w);

$$g \frac{\partial z_w}{\partial s} = - \frac{g}{C^2} \frac{u|u|}{h} \quad (A.10)$$

$$- \frac{u^2}{(R_c + n)} + g \frac{\partial z_w}{\partial n} = 0 \quad (A.11)$$

simplifying the water level, z_w , by cross differentiation of Eqs. [A.10] and [A.11], and applying the differentiation rule:

$$\frac{\partial z_w}{\partial n \partial s} = \frac{\partial}{\partial s} \frac{\partial z_w}{\partial n} - \frac{1}{(R_c + n)} \frac{\partial z_w}{\partial n}$$

The following equation is obtained:

$$\frac{\partial}{\partial s} \left[\frac{u^2}{(R_c + n)} \right] = - \frac{g}{C^2} \frac{\partial}{\partial n} \left[\frac{u^2}{h} \right] - \frac{g}{C^2} \frac{u|u|}{h} \frac{1}{(R_c + n)} \quad (A.12)$$

In which the first term can be neglected (see definition of fully developed bend flow). When u is positive the following equation is obtained:

$$\frac{\partial u}{\partial n} = \frac{u}{2h} \frac{\partial h}{\partial n} - \frac{u}{2(R_c + n)} \quad (A.13)$$

linearization of Eqs. [A.7] and [A.13] is made with the assumption:

$$u = u_o + u'_o \quad \text{with} \quad u'_o \ll u_o$$

$$h = h_o + h'_o \quad \text{with} \quad h'_o \ll h_o$$

in which u_o , h_o = reach-averaged values of velocity and water depth, respectively; u'_o , h'_o = perturbation terms caused by the local curvature of the channel.

Assuming $n \ll R_c$, the linearized equations are:

$$\frac{\partial h'_o}{\partial n} = A \frac{f(\theta_o) h_o}{R_c} \quad (A.14)$$

$$\frac{\partial u'_o}{\partial n} = \frac{u_o}{2h_o} \frac{\partial h'_o}{\partial n} - \frac{u_o}{2R_c} \quad (A.15)$$

The expression for the near-bank value of the perturbations, $\hat{u}_o = (u'_o)_{n=B/2}$ and $\hat{h}_o = (h'_o)_{n=B/2}$ can be obtained from Eqs. [A.14] and [A.15], with the assumption that the centre-line values of velocity and water depth are equal to the respective reach-averaged values. From:

$$h'_o = A \frac{f(\theta_o) h_o}{R_c} n \quad (A.16)$$

$$u'_o = \frac{u_o}{2h_o} h'_o - \frac{u_o}{2R_c} n \quad (A.17)$$

the following expressions are obtained for the near-bank perturbations:

$$\hat{h}_o = A \frac{f(\theta_o) h_o}{R_c} \cdot \frac{B}{2} \quad (A.18)$$

$$\hat{u}_o = \frac{u_o}{2h_o} \hat{h}_o - \frac{u_o}{2R_c} \cdot \frac{B}{2} \quad (A.19)$$

Inserting, $k_B = \frac{m\pi}{B}$, $\lambda_w = \frac{C^2 h_o}{2g}$, $\lambda_s = \frac{f(\theta_o)}{k_B^2 h_o}$, Eqs. [A.18] and [A.19] become:

$$\frac{\hat{h}_o}{\lambda_s} = k_B h_o^2 A \frac{1}{R_c} \left(\frac{m\pi}{2} \right) \quad (A.20)$$

$$\frac{\hat{u}_o}{\lambda_w} = \left(\frac{1}{2\lambda_w} \frac{u_o}{h_o} \right) \hat{h}_o - \frac{u_o}{2\lambda_w k_B} \frac{1}{R_c} \left(\frac{m\pi}{2} \right) \quad (A.21)$$

respectively.

APPENDIX B

DERIVATION OF FLOW AND BED EQUATIONS, LINEARIZATION

Flow equation

The flow equation is derived by combining the two-dimensional depth-averaged flow continuity and momentum equations for a steady shallow flow:

$$\frac{\partial(h u)}{\partial s} + \frac{\partial(h v)}{\partial n} + \frac{h v}{(R_c + n)} = 0 \quad (B.1)$$

$$u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial n} + \frac{h v}{(R_c + n)} + g \frac{\partial z_w}{\partial s} + \frac{g}{C^2} \frac{u \sqrt{u^2 + v^2}}{h} = 0 \quad (B.2)$$

$$- \frac{u^2}{(R_c + n)} + u \frac{\partial v}{\partial s} + v \frac{\partial v}{\partial n} + g \frac{\partial z_w}{\partial n} + \frac{g}{C^2} \frac{u \sqrt{u^2 + v^2}}{h} = 0 \quad (B.3)$$

in which: s = longitudinal co-ordinate; n = transverse co-ordinate; u = depth-averaged tangential velocity; v = depth-averaged transverse velocity; h = water depth; R_c = radius of curvature of the channel centre-line; z_w = water level; g = acceleration due to gravity; C = Chézy coefficient.

The water level, z_w , is eliminated by cross differentiation of Eqs. [B.2] and [B.3], applying the following differentiation rule (de Vriend [1971]):

$$\frac{\partial^2 z_w}{\partial n \partial s} + \frac{1}{(R_c + n)} \frac{\partial z_w}{\partial s} = \frac{\partial^2 z_w}{\partial n \partial s} \quad (B.4)$$

(This rule applies also for the other quantities; h , u and v).

The following equation is obtained:

$$\begin{aligned} & \frac{\partial}{\partial n} \left(u \frac{\partial u}{\partial s} \right) + \frac{\partial}{\partial n} \left(v \frac{\partial u}{\partial n} \right) + \frac{\partial}{\partial n} \left(\frac{u v}{R_c + n} \right) + \frac{\partial}{\partial s} \left(\frac{u^2}{R_c + n} \right) - \frac{\partial}{\partial s} \left(u \frac{\partial v}{\partial s} \right) + \\ & - \frac{\partial}{\partial s} \left(v \frac{\partial v}{\partial n} \right) + \frac{u}{(R_c + n)} \frac{\partial u}{\partial s} + \frac{v}{(R_c + n)} \frac{\partial u}{\partial n} + \frac{u v}{(R_c + n)^2} + \\ & + \frac{g}{C^2} \left[\frac{\partial}{\partial n} \left(\frac{u \sqrt{u^2 + v^2}}{h} \right) + \frac{\partial}{\partial s} \left(\frac{v \sqrt{u^2 + v^2}}{h} \right) + \frac{1}{(R_c + n)} \left(\frac{u \sqrt{u^2 + v^2}}{h} \right) \right] = 0 \end{aligned} \quad (B.5)$$

The combination of Eq. [B.5] with the flow continuity equation, Eq. [B.1], (taking into account the differentiation rule) yields:

$$\begin{aligned}
 & u \frac{\partial^2 u}{\partial s \partial n} + v \frac{\partial^2 u}{\partial n^2} - \frac{u}{h} \frac{\partial h}{\partial s} \frac{\partial u}{\partial h} - \frac{v}{h} \frac{\partial h}{\partial n} \frac{\partial u}{\partial n} + \frac{u}{(R_c + n)} \frac{\partial v}{\partial n} + \\
 & + \frac{2u}{(R_c + n)} \frac{\partial u}{\partial s} + u^2 \frac{\partial}{\partial s} \left(\frac{1}{R_c + n} + c \right) - \frac{\partial v}{\partial s} \frac{\partial v}{\partial n} - v \frac{\partial^2 v}{\partial s \partial n} + \frac{v}{(R_c + n)} \frac{\partial u}{\partial n} + \\
 & + \frac{g}{C^2} \left[u \left(\frac{\sqrt{u^2 + v^2}}{h} \right) \left(\frac{1}{R_c + n} + c \right) + \left(\frac{\partial u}{\partial n} - \frac{u}{h} \frac{\partial h}{\partial n} + \frac{v}{h} \frac{\partial h}{\partial s} \right) \left(\frac{\sqrt{u^2 + v^2}}{h} \right) + \right. \\
 & \left. + \frac{1}{\sqrt{u^2 + v^2}} \left(\frac{u^2}{h} \frac{\partial u}{\partial n} + \frac{uv}{h} \frac{\partial v}{\partial n} - \frac{uv}{h} \frac{\partial u}{\partial s} + \frac{v^2}{h} \frac{\partial v}{\partial n} \right) \right] = 0 \quad (B.6)
 \end{aligned}$$

$$\text{in which: } c = - \frac{1}{u} \frac{\partial v}{\partial s}$$

According to de Vriend [1981] the term $\left(\frac{1}{R_c + n} + c \right)$ is an approximation of the streamline curvature: $1/R_s$.

Eq. [B.6] is then simplified via linearization, assuming:

$$u = u_o + U',$$

$$h = h_o + H',$$

$$v = v'$$

with:

$$\frac{U'}{u_o} = \epsilon, \quad \frac{H'}{h_o} = \epsilon, \quad \frac{v'}{u_o} = \epsilon,$$

In which u_o , h_o = reach-averaged values of tangential velocity and water depth, respectively (they are constant with s , with n , and with time); U' and H' perturbation of velocity and water depth, respectively.

The assumption of shallow water yields the condition:

$$h_o/B = \epsilon$$

in which B = channel width

The assumption of mildly curved channel yields the conditions:

$$B/R_c \approx \epsilon + n \ll R_c; \quad \frac{1}{R_c} \approx c$$

The flow adaptation length is assumed comparable with the channel width:

$$\lambda_w = \frac{h_o C^2}{2g} \approx B$$

u , h , v are substituted in Eq. [B.6] with their expressions.

The non-linear terms are disregarded. The following equation is obtained:

$$\frac{\partial^2 U'}{\partial s \partial n} + u_o \frac{\partial}{\partial s} \left(\frac{1}{R_c} + c' \right) + \frac{1}{\lambda_w} \left\{ \frac{u_o}{2} \left(\frac{1}{R_c} + c' \right) + \frac{\partial U'}{\partial n} + \frac{u_o}{2h_o} \frac{\partial H'}{\partial n} \right\} = 0 \quad (B.7)$$

$$\text{with } c' = - \frac{1}{u_o} \frac{\partial v'}{\partial s}$$

Bed equation

The bed equation is derived by combining the two dimensional depth-averaged flow continuity equation, Eq. [B.1], with the sediment balance equation taking into account the expression for the sediment transport direction. The sediment balance equation:

$$\frac{\partial z_b}{\partial t} + \frac{\partial S_s}{\partial s} + \frac{\partial S_n}{\partial n} + \frac{S_n}{(R_c + n)} = 0 \quad (B.9)$$

In which S_s and S_n = components of volumetric sediment transport, including pores, per unit of channel width, in the s and n direction, respectively ;
 z_b = bed level; t = time.

The expression for the sediment transport direction:

$$\tan \alpha = \frac{v}{u} - \frac{Ah}{R_s} - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} \quad (B.10)$$

with:

$$S_n = S_s \tan \alpha, \text{ and } \frac{1}{R_s} = \frac{1}{R_c} - \frac{1}{u} \frac{\partial v}{\partial s} \text{ (de Vriend [1981])}.$$

in which α = direction of sediment transport; $f(\theta)$ = weighing function for the influence of the sloping bed; and θ = the Shields parameters.

Substituting in Eq. [B.9] S_n with $S_s \tan \alpha$, the following equation is obtained:

$$\begin{aligned} & \frac{\partial z_b}{\partial t} + \frac{\partial S_s}{\partial s} + \frac{\partial S_s}{\partial n} \left[\frac{v}{u} - Ah \left(\frac{1}{R_c + n} + c \right) - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} \right] + \\ & + S_s \left\{ \frac{\partial}{\partial n} \left(\frac{v}{u} \right) - \frac{\partial}{\partial n} \left[Ah \left(\frac{1}{R_c + n} + c \right) \right] - \frac{\partial}{\partial n} \left(\frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} \right) + \right. \\ & \left. + \frac{v}{u(R_c + n)} - \frac{Ah}{(R_c + n)} \left(\frac{1}{R_c + n} + c \right) - \frac{1}{f(\theta)(R_c + n)} \frac{\partial z_b}{\partial n} \right\} = 0 \quad (B.11) \end{aligned}$$

Combining Eq. [B.11] with the flow continuity equation, Eq. [B.1], the following expression is obtained:

$$\begin{aligned} & \frac{\partial z_b}{\partial t} + \frac{\partial S_s}{\partial s} + \frac{\partial S_s}{\partial n} \left[\frac{v}{u} - Ah \left(\frac{1}{R_c + n} + c \right) - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} \right] + \\ & + S_s \left\{ - \frac{1}{h} \frac{\partial h}{\partial s} - \frac{1}{u} \frac{\partial u}{\partial s} - \frac{v}{hu} \frac{\partial h}{\partial n} - \frac{v}{u^2} \frac{\partial u}{\partial n} - A \frac{\partial h}{\partial n} \left(\frac{1}{R_c + n} + c \right) + \right. \\ & - Ah \frac{\partial}{\partial n} \left(\frac{1}{R_c + n} + c \right) - \frac{\partial}{\partial n} \left[\frac{1}{f(\theta)} \right] \frac{\partial z_b}{\partial n} - \frac{1}{f(\theta)} \frac{\partial^2 z_b}{\partial n^2} + \\ & \left. - \frac{Ah}{(R_c + n)} \left(\frac{1}{R_c + n} + c \right) - \frac{1}{f(\theta)(R_c + n)} \frac{\partial z_b}{\partial n} \right\} = 0 \quad (B.12) \end{aligned}$$

Similarly to the flow equation (Eq. [B.6]) also Eq. [B.12] is simplified via linearization, but with the following additional assumptions:

$$S_s = S_o + S'_s, \text{ with } \frac{S'_s}{S_o} = \epsilon$$

where:

$$S_o :: u_o^b$$

$$S'_s = S_o \left(b \frac{U'}{u_o} \right)$$

consequently $S_s = S_o \left(1 + \frac{b}{u_o} U' \right)$.

The assumption of a small to moderate Froude number allows for a rigid lid approximation:

$$\frac{\partial z_b}{\partial n} = - \frac{\partial h}{\partial n};$$

$$\frac{\partial z_b}{\partial t} = - \frac{\partial h}{\partial t}$$

Linearization of Eq. [B.12] yields:

$$\begin{aligned} & - \frac{u_o}{S_o} \frac{\partial H'}{\partial t} + (b-1) \frac{\partial U'}{\partial s} - \frac{u_o}{h_o} \frac{\partial H'}{\partial s} + \\ & + A u_o h_o \frac{\partial}{\partial n} \left(\frac{1}{R_c} + c' \right) + \frac{u_o}{f(\theta_o)} \frac{\partial^2 H'}{\partial n^2} = 0 \end{aligned} \quad (B.13)$$

Linearization of the flow continuity and momentum equations

Each of the two-dimensional depth-averaged flow continuity and momentum equations (Eqs. [B.1], [B.2] and [B.3], respectively) can be linearized with the same previously described assumptions.

Linearization and dimensional analysis of Eqs. [B.1], [B.2] and [B.3] yield the following:

- zero order set of equations (which applies for an infinite long straight channel):

$$h_o u_o = \text{constant} = \frac{Q}{B} \quad (\text{continuity equation}) \quad (\text{B.15})$$

$$\frac{\partial z_{wo}}{\partial s} = - \frac{u_o^2}{C^2 h_o} \quad (\text{Chézy equation}) \quad (\text{B.16})$$

- first order set of equations:

$$u_o \frac{\partial H'}{\partial s} + h_o \frac{\partial U'}{\partial s} + h_o \frac{\partial V'}{\partial n} + \frac{h_o V'}{R_c} = 0 \quad (\text{B.17})$$

$$u_o \frac{\partial U'}{\partial s} + \frac{V' u_o}{R_c} + g \frac{\partial z'_w}{\partial s} + \frac{g}{C^2} \left\{ - \frac{u_o^2}{h_o^2} H' + 2 \frac{u_o}{h_o} U' \right\} = 0 \quad (\text{B.18})$$

$$- \frac{u_o^2}{R_c} - \frac{2u_o}{R_c} U' + \frac{u_o^2}{R_c} \left(\frac{n}{R_c} \right) + u_o \frac{\partial V'}{\partial s} + \frac{g}{C^2} \frac{\partial z'_w}{\partial n} + \frac{g}{C^2} \frac{V' u_o}{R_c} = 0 \quad (\text{B.19})$$

The flow equation, Eq. [B.8], can be also derived by combining Eqs. [B.17], [B.18] and [B.19] already linearized in which the following term has been disregarded from Eq. [B.19]:

$$\frac{u_o^2}{R_c} \left(\frac{n}{R_c} \right)$$

This term is small and it is believed that it does not appreciably affect the computations.

APPENDIX C

STREAM-LINE CURVATURE CAUSED BY THE NON-UNIFORMITY OF FLOW

According to de Vriend [1981] the stream-line curvature can be approximated by:

$$\frac{1}{R_s} = \frac{1}{(R_c + n)} + c, \text{ where } c = -\frac{1}{u} \frac{\partial v}{\partial s} \quad (C.1)$$

in which: s = longitudinal co-ordinate; n = transverse co-ordinate; v = depth-averaged transverse velocity; R_s = stream-line radius of curvature; R_c = centre-line radius of curvature; c = curvature induced by the non-uniformity of the flow.

The stream-line curvature induced by the non-uniformity of the flow (in fully developed bend conditions $c = 0$) can approximately be derived in the following way, via linearization:

Assuming:

$$h = h_0 + h'; \quad u = u_0 + u'; \quad v = v'; \quad \text{with } h' \ll h_0, \quad u' \ll u_0, \quad \text{and } v' \ll u_0$$

in which u_0 , h_0 = reach-averaged values of tangential velocity and water depth, respectively; u' , h' , v' = perturbation terms of tangential velocity, water depth and transverse velocity (the non-perturbed depth-averaged transverse velocity is zero), respectively.

In linearized form c becomes:

$$c' = -\frac{1}{u_0} \frac{\partial v'}{\partial s} \quad (C.2)$$

In linearized form the two-dimensional depth-averaged flow continuity equation for a curved channel becomes:

$$\frac{\partial v'}{\partial n} + \frac{1}{R_c} v' = -\frac{u_0}{h_0} \frac{\partial h'}{\partial s} - \frac{\partial u'}{\partial s} \quad (C.3)$$

Imposing a transverse sinusoidal shape to u' and h' :

$$u' = \hat{u} \sin(k_B n) \quad \text{and} \quad h' = \hat{h} \sin(k_B n)$$

in which: $\hat{Q}$ and $\hat{h}$ = amplitudes of the transverse velocity and water depth profiles, respectively (they are constant with respect to n , while they can vary along s); k_B = transverse wave number, $k_B = \frac{m\pi}{B}$ ($m = 1$ applies to meandering rivers).

The following linear, non-homogenous differential equation with constant coefficients (with respect to n) is obtained:

$$\frac{\partial v'}{\partial n} + \frac{1}{R_c} v' = - \left(\frac{\partial \hat{h}}{\partial s} \frac{u_o}{h_o} + \frac{\partial \hat{Q}}{\partial s} \right) \sin (k_B n) \quad (C.4)$$

Eq. [C.4] can be solved applying the Bernoulli's method.

This is done by assuming:

$$K = - \left(\frac{\partial \hat{h}}{\partial s} \frac{u_o}{h_o} + \frac{\partial \hat{Q}}{\partial s} \right)$$

(K is constant with respect to n , but it is variable along s)

and

$$v' = gt$$

(g and t are both a function of n)

Equation [C.4] becomes:

$$g \left(\frac{\partial t}{\partial n} + \frac{t}{R_c} \right) + t \frac{\partial g}{\partial n} = K \sin (k_B n) \quad (C.5)$$

Equation [C.5] is equivalent to the system:

$$\frac{\partial t}{\partial n} + \frac{t}{R_c} = 0$$

$$t \frac{\partial g}{\partial n} = K \sin (k_B n)$$

$$v' = gt$$

The system can be solved as follows:

$$\frac{\partial t}{\partial n} = -\frac{1}{R_c} t \rightarrow t = \exp\left(-\frac{n}{R_c}\right)$$

$$\frac{\partial g}{\partial n} = \exp\left(\frac{n}{R_c}\right) K \sin(k_B n) \rightarrow g = K \int \left[\exp\left(\frac{n}{R_c}\right) \sin(k_B n)\right] dn$$

The integral can be solved with the method of Integration by Parts, which leads to:

$$\int \left[\exp\left(\frac{n}{R_c}\right) \sin(k_B n)\right] dn = \left(\frac{R_c^2}{1 + R_c^2 k_B^2}\right) \exp\left(\frac{n}{R_c}\right) \left[\frac{1}{R_c} \sin(k_B n) - k_B \cos(k_B n)\right] \quad (C.6)$$

The expression for v' ($=tg$) becomes:

$$v' = K \left(\frac{R_c^2}{1 + R_c^2 k_B^2}\right) \left[\frac{1}{R_c} \sin(k_B n) - k_B \cos(k_B n)\right] + C_1 \quad (C.7)$$

in which C_1 = constant of integration which can be derived from the boundary conditions:

$$v' \Big|_{n=B/2} = 0 \quad (\text{impermeable side walls})$$

$$C_1 = -\frac{K}{R_c} \left(\frac{R_c^2}{1 + R_c^2 k_B^2}\right) \quad (C.8)$$

consequently Eq. [C.7] becomes:

$$v' = K \left(\frac{R_c^2}{1 + R_c^2 k_B^2}\right) \left[\frac{1}{R_c} \sin(k_B n) - k_B \cos(k_B n) - \frac{1}{R_c}\right] \quad (C.9)$$

Being $c' = -\frac{1}{u_0} \frac{\partial v'}{\partial s}$ the linearized form of the stream-line curvature induced by the non-uniformity of the flow is:

$$c' = -\frac{1}{u_0} \frac{\partial K}{\partial s} \left(\frac{R_c^2}{1 + R_c^2 k_B^2}\right) \left[\frac{1}{R_c} \sin(k_B n) - k_B \cos(k_B n) - \frac{1}{R_c}\right] \quad (C.10)$$

at $n = 0$, $c = \hat{c}$ (maximum value):

$$\hat{c} = \frac{1}{u_o} \frac{\partial K}{\partial s} \left(\frac{R_c^2}{1 + R_c^2 k_B^2} \right) \left(k_B + \frac{1}{R_c} \right) \quad (C.11)$$

with $k_B = m \frac{\pi}{B}$ and $k = - \left(\frac{\partial \hat{h}}{\partial s} \frac{u_o}{h_o} + \frac{\partial \hat{u}}{\partial s} \right)$, Eq. [C.11] becomes:

$$\hat{c} = - \frac{B}{m\pi} \left(\frac{1}{h_o} \frac{\partial^2 \hat{h}}{\partial s^2} + \frac{1}{u_o} \frac{\partial^2 \hat{h}}{\partial s^2} \right) \left\{ \frac{1 + \left(\frac{B}{m\pi R_c} \right)}{1 + \left(\frac{B}{m\pi R_c} \right)^2} \right\} \quad (C.12)$$

The term $\left(\frac{B}{m\pi R_c} \right)^2$ can be disregarded with respect to 1 even when $R_c \approx B$ (strongly curved channel).

The term $\left(\frac{B}{m\pi R_c} \right)$ can only be disregarded with respect to 1 with the assumption of a mildly curved channel: $B \ll R_c$.