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APPLICATIONS OF DOMAIN DECOMPOSITION TECHNIQUES FOR THE MULTISCALE MODELING OF SOFTENING MATERIALS

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Summary. In this contribution we describe a methodology for the study of softening brittle materials at different scales of observation. The goal is to account for a higher resolution at those areas that undergo the non-linear processes. We apply the FETI (Finite Element Tearing and Interconnecting) technique to glue different domain resolutions during the non-linear analysis. The present framework is suitable for multiscale problems in which the scale separation principle does not hold and, consequently, the use of classical homogenization techniques is no longer possible.

1 INTRODUCTION

The present contribution focuses on the applications of the FETI method to tackle multiscale problems. A general scheme is sketched in Fig. 1. Computations start at a coarse level where the mesh size is fine enough to provide an accurate linear solution. The mesh is partitioned into several domains and a predictor is used to anticipate the linear/non-linear character of the following calculations at each domain. Before entering the non-linear regime the resolution of the domain is improved and the mesh is refined in a suitable manner. This process is continued through all non-linear areas. In our case the non-linearities are caused by the growth of damage in the specimen.

2 THE FETI METHOD

Domain Decomposition techniques are often used to partition the computation of large systems. They combine direct solvers for the local domains and iterative solvers for the interface problem.

The FETI approach [1] can be seen as a particular Domain Decomposition method in which we introduce Lagrange forces to fullfill the compatibility conditions between different domains $\Omega_{(s)}$ using the signed Boolean matrices $\boldsymbol{B}$:

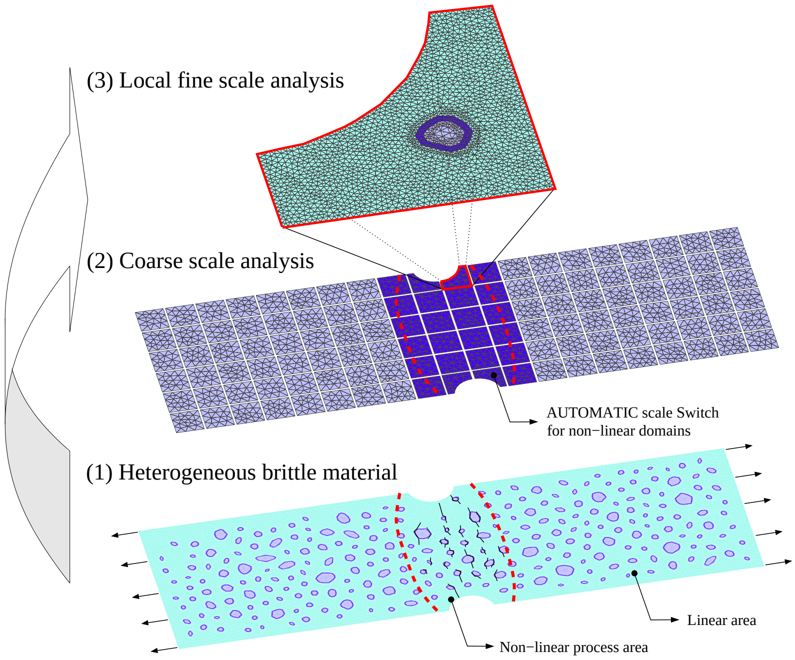

Figure 1: Scheme of the multiscale analysis

$$\sum_{s=1}^{N_s} \boldsymbol{B}^{(s)}\boldsymbol{u}^{(s)} = \boldsymbol{0}. \tag{1}$$

The global structural system $\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$ is split into a set of subsystems which are connected using Lagrange multipliers.

In this contribution we introduce a simple methodology to glue different domain resolutions. At the interface between coarse and fine domains some nodes are matching (independent nodes). In order to restrain the d.o.f. (degrees of freedom) of a non-conforming interface we apply LMPC (Linear Multi Point Constraints) at the nodes of the fine mesh which do not have a corresponding pair at the adjacent coarse domain (dependent nodes). The collocation of the dependent nodes on to the interface is illustrated in Fig. 2.

The set of homogeneous LMPC can be cast in a matrix form as $\boldsymbol{C}\boldsymbol{u} = \boldsymbol{0}$ and they are implemented by adding extra equations via the use of Lagrange Multipliers. These extra equations can be contained in the modified boolean matrices $\bar{\boldsymbol{B}}^{(s)}$ by simply concatenating row-wise the constraint matrices $\boldsymbol{C}^{(s)}$ and the original boolean matrices $\boldsymbol{B}^{(s)}$. The $\bar{\boldsymbol{B}}^{(s)}$ matrices are needed to construct an adequate preconditioner for the interface problem [4]. The extended field of Lagrange Multipliers contains the forces arising from the multipoint constraints $\boldsymbol{\mu}$ and the ones that glue the adjacent domains $\boldsymbol{\lambda}$.

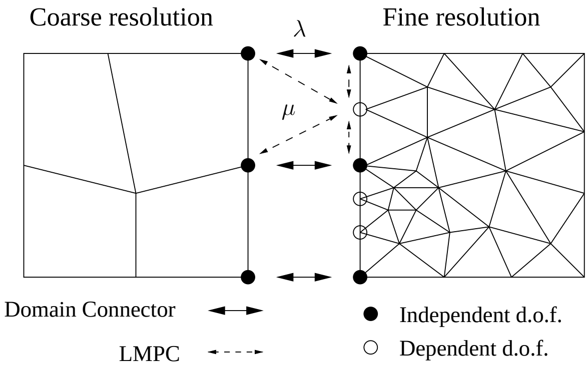

Figure 2: LMPC at non-conforming adjacent domains.

3 EXAMPLE: LOCALIZATION TEST ON A L-SHAPED STRUCTURE

In this section we present an illustrative example to show the performance of the LMPC gluing different domain resolutions. An L-shaped structure composed by an homogeneous material is loaded according to the test depicted in Fig. 3 (Left). The test is chosen in order to localize the strains at the reentrant corner of the plate. We use a Gradient Enhanced Damage model [2] to simulate failure phenomena. In this model, regularization is achieved by introducing a non-local state variable (equivalent strain) which represents an invariant of the strain tensor. It should be mentioned that this model gives rise to a non-symmetric system and, therefore, a solver like GMRES or Bi-CGSTAB [3] is needed for the interface problem. We combine the coarse mesh with the fine mesh as depicted in Fig. 3 (Right). The fine domains are only used at the areas that will undergo the non-linear processes.

In Fig. 4 we observe the contours of non-local equivalent strain and damage at the last loading state. The fine domains cover the non-linear process zone while the coarse domains remain elastic. There is full displacement compatibility between all domains.

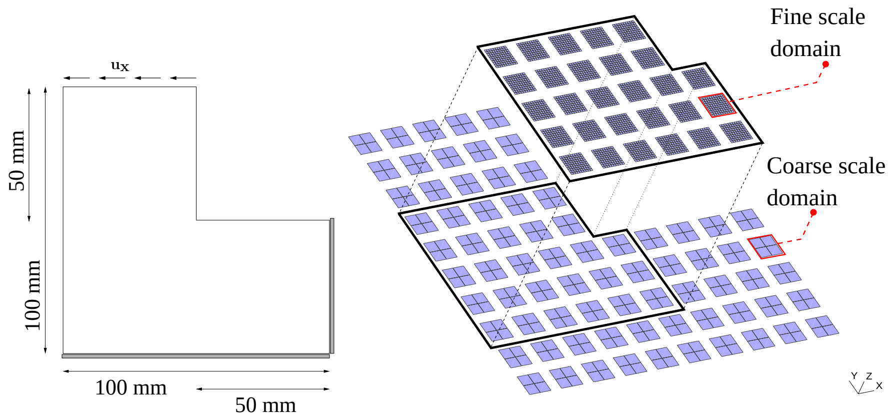

Figure 3: Boundary conditions (Left) and domain decomposition (Right).

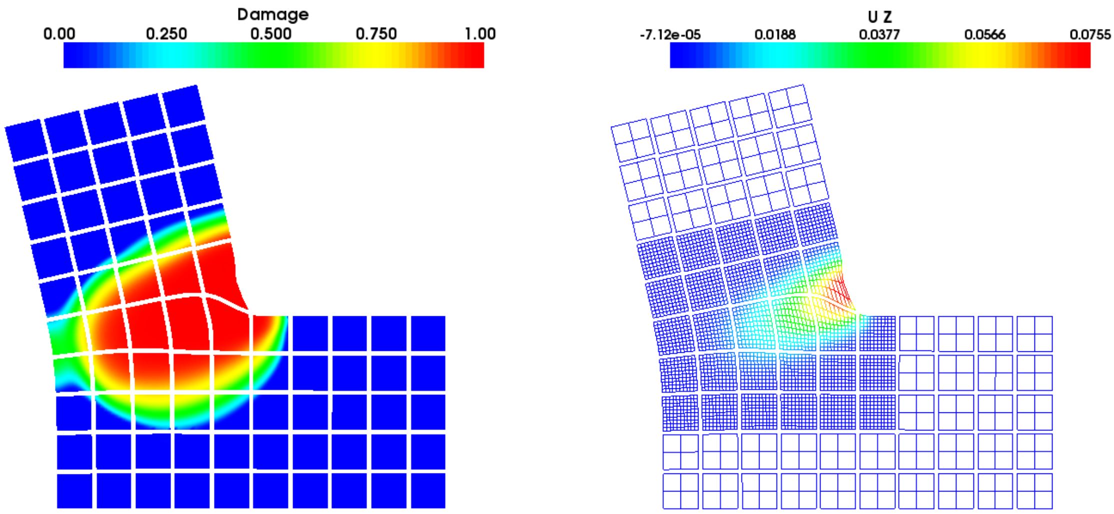

Figure 4: Damage contours (Left) and non-local equivalent strain contours (Right) (Magnification factor 15).

4 CONCLUSIONS

The FETI technique is a suitable tool for the multiscale analysis of softening materials. The decomposition into several domains with possible different resolutions and the use of a linear/non-linear criteria to switch between them are the ingredients for a complete multiscale framework. When the critical non-linear areas are Switched On, the correct physics of the problem is captured and the computations turn to be simpler and faster than a complete monoscale analysis. It is advised to decompose the structure into a set of domains which are smaller than the size of the expected non-linear process zone. In this way, the surface of the Switched On area will be small and the size of the whole problem will be minimized.

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