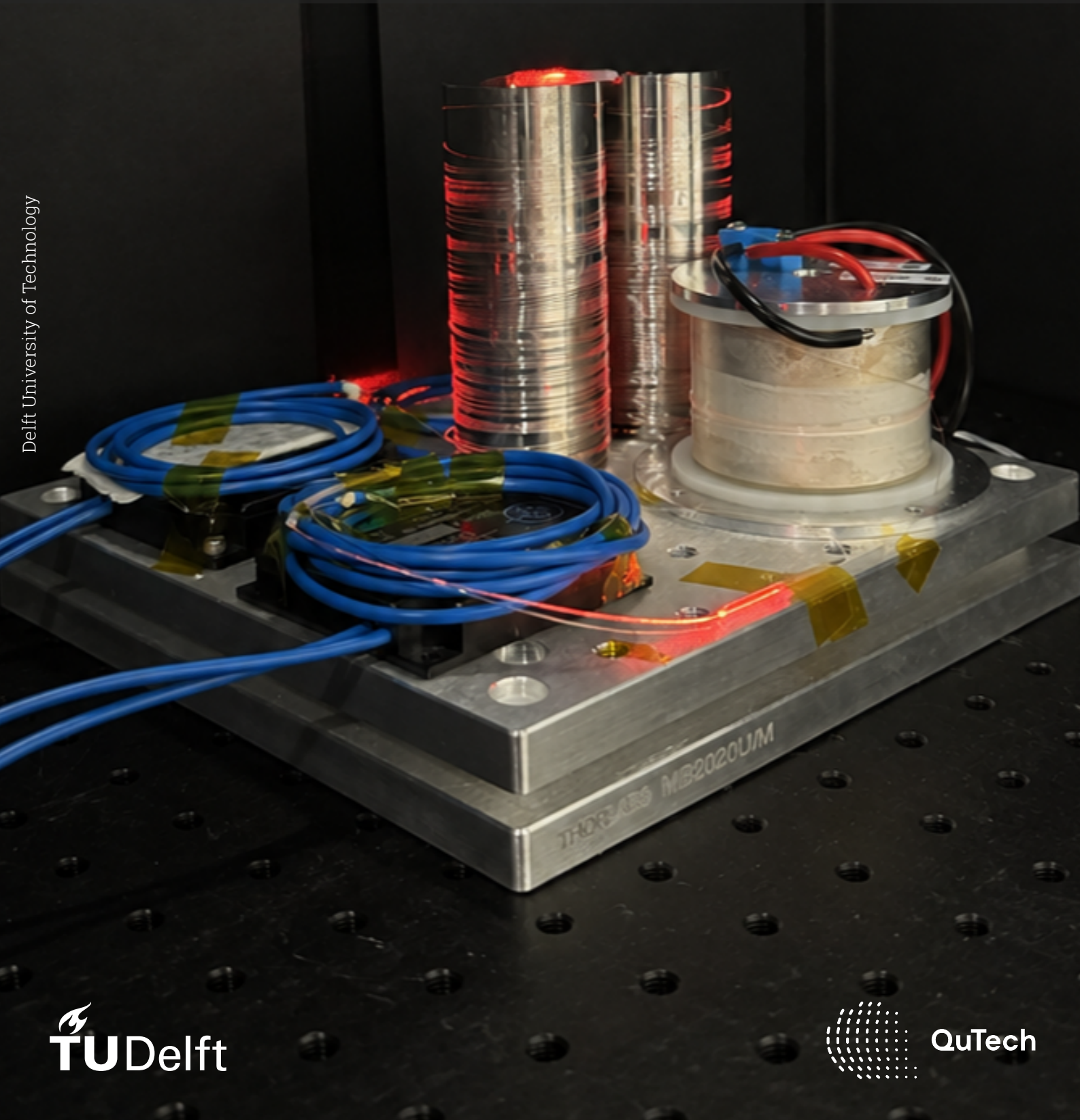


A Time-Delay Interferometer for Photonic Quantum State Measurements

Mark Tabaksblat
Hanson Lab

Delft University of Technology



by

Mark Tabaksblat

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Responsible supervisor:	Prof. Dr. Ronald Hanson
Daily supervisor:	Tim Turan, MSc
Thesis committee:	Prof. Dr. Ronald Hanson Prof. Dr. Miriam Blaauboer Prof. Dr. Toeno van der Sar
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Abstract

Quantum entanglement between spatially separated nodes is a key ingredient for future technologies, such as secure communication, distributed quantum computing, and quantum-enhanced sensing. Photon-mediated entanglement motivates development of quantum nodes with an optical interface, and the tin-vacancy center is a compelling building block. It is a spin-photon interface with coherent and indistinguishable emission, compatible with integration in nanostructures. Additionally, it has shown long spin coherence times at temperatures above 1 K. For photonic qubits, time-bin encoding is robust against photon loss and phase noise in fibers, making it especially suitable for long-range entanglement mediation.

An important challenge is thus, to demonstrate entanglement between the spin of the tin-vacancy center and a photonic time bin qubit. Quantum state tomography of the entangled state requires measurements of the photonic qubit in various bases. To this end, we have constructed a Time Delay Interferometer (TDI), in the form of an Unbalanced Mach-Zehnder Interferometer. A probe signal facilitates feedback to set ('lock') the phase of the TDI to any angle, allowing controlled quantum state measurements in a continuum of bases. We can lock the phase successfully in only 30 ms as shown by a subsequent probability distribution with a Root Mean Square of 0.046π , centered around the target phase. During locking, no quantum state measurements on single photons are possible, which necessitates passive phase stability. Laser frequency noise limited verification of passive phase stability on time scales below 200 ms. During the time between 200 ms and 8 s, the RMS increased with only 0.05π . This promises quantum state tomography measurements with little overhead.

Next, we measured the single photon purity of emission when attempting excitation of a tin-vacancy center. A second order autocorrelation $g^{(2)}(0) = 0.07(7)$ clearly demonstrates antibunching. In addition, we probed the indistinguishability of two-photon interference in the TDI. We measured a raw Hong-Ou-Mandel visibility $V_{\text{raw}} = 0.85(14)$ and a corrected indistinguishability of $0.87(14)$ (upper bound at 1). Together with measured imperfections in the interferometer, this data determines an upper bound on the fidelity of single shot quantum state measurements of $0.72(4)$.

This work lays the foundation to demonstrate spin-photon entanglement between a tin-vacancy spin qubit and an emitted time bin photonic qubit. This is an important step in realizing long-range quantum entanglement via emission based protocols.

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Table of contents

Abstract	iii
Acknowledgments	v
1 Quantum Networks	1
1.1 Long range quantum entanglement	1
1.2 Quantum node	1
1.2.1 Group IV Vacancy Centers	2
1.2.2 Tin-vacancy Center	3
1.2.3 Optical Transitions	3
1.2.4 Linewidth Broadening	5
1.3 Photonic qubit encoding	5
1.4 Summary	6
2 A Time Delay Interferometer	7
2.1 Quantum State Tomography	7
2.2 Unbalanced Mach-Zehnder Interferometer	7
2.2.1 Quantum State Measurements	8
2.2.2 Measurements with an Imperfect TDI	9
2.2.3 Measurements with Spectral Diffusion	9
2.3 Summary	9
3 Realization of the Interferometer	11
3.1 Phase Locking stage	11
3.2 Measurement Stage	12
3.3 Requirements of the TDI	14
3.4 Design of the TDI	14
3.5 Summary	15
4 Characterization of the Interferometer	17
4.1 Phase Stability	17
4.2 Delay and Losses	19
4.3 Interleaved Execution of Stages	20
4.4 Summary	21
5 Single photon generation	23
5.1 Single Photon Collection and Filtering	23
5.2 Optical Transition Selection	23
5.3 Charge-Resonance Check	24
5.4 Optical Pulse Generation	24
5.5 Tuning	25
5.6 Summary	26
6 Characterization of Single Photons	27
6.1 Second-order Autocorrelation Measurement	27
6.2 Hong-Ou-Mandel Interference	29
6.3 Single Shot Readout Fidelity prediction	30
6.4 Summary	31
7 Conclusion and Outlook	33
7.1 Conclusion	33

7.2 Outlook	33
References	35
A Complete Optical setup	41
B Theoretical model of an imperfect TDI.....	43
B.1 The Model	43
B.2 Compensating for Photon Quantum State Tomography	44
B.3 Compensating for Spin-Photon Tomography	46
B.4 Single-shot Readout Fidelity	46
C Possible Improvements of the TDI	49
C.1 Other Designs	49
C.2 Different Locking Mechanisms	49
C.3 Detection Improvements	50

- Chapter 1 -

Quantum Networks

1.1. Long range quantum entanglement

Quantum entanglement has been used to study fundamental aspects of the theory of quantum mechanics [1], as it predicts correlations between systems beyond a bound set by classical theories [2, 3]. This bound has been violated by measurements on spatially separated entangled systems, such as entangled vacancy centers in diamond [4], cold atoms [5], superconducting circuits [6], quantum dots [7] and photons [8].

Quantum entanglement on spatially separated nodes has gained additional interest, fuelled by visions of quantum technologies utilizing long range entanglement. In quantum sensing long range entanglement can be employed to enhance sensitivity [9] for example, of atomic clocks [10], and optical interferometry in telescope arrays [11]. In quantum communication, entanglement is a prerequisite for many functions of a quantum internet, such as quantum state teleportation [12], quantum repeaters to bridge long distances [13] and device independent quantum key distribution [14] - already with impressive demonstrations over large distances [15]. Finally, a full-blown quantum internet would include quantum computers in this quantum network, opening the door to distributed quantum computing. [14, 16]. As a result, entanglement distribution between spatially separated nodes has remained the subject of intensive work.

To generate entanglement between spatially separated nodes, one can transmit quantum information via a ‘flying qubit’ [17], and through interactions with the stationary qubits, entanglement can be generated between the nodes [18, 19]. A promising realization for this flying qubit is a photon, because it can be transported via existing glass-fiber internet infrastructure, or free-space, at room temperatures. Impressive demonstrations of entanglement have been done via satellite links [20], and over metropolitan fiber-networks [21, 22]. However, challenges still remain, such as photon loss resulting in loss of quantum information, scaling exponentially with distance [23]. To circumvent this, proposals of quantum repeaters exist, which utilize entanglement between stationary nodes and photons, or cluster states of entangled photons, to swap entanglement onto the stationary nodes. [13]

1.2. Quantum node

The stationary qubits require an optical interface, to enable quantum state dependent interactions with an optical field [24]. A promising form of stationary nodes is the spin-photon interface, and many schemes already exist to create photon-mediated entanglement between spins [19]. These schemes rely on photon-photon or spin-photon entanglement, making demonstrations of this an important stepping stone in developing long range entanglement.

Diamond color centers form a particularly promising spin-photon interface to perform quantum networking tasks. Color centers are defects in a crystal lattice, with localized electronic states in the bandgap of the host material, which can give rise to a qubit. The implantation of these qubits in a solid-state host offers promises of integration of qubits in compact and scalable devices. However, it also introduces a noisy environment which can disturb the qubit’s quantum state and the energy levels [25]. Diamond is a promising host-material, because it mitigates some of the disadvantages compared to vacuum spin-photon interfaces. Diamond is optically transparent, and has a large bandgap of 5.5 eV, which can host discrete electronic states separated by energies in the optical domain [26]. Quantum state decoherence is mitigated, because diamond is magnetically, electrically and mechanically quiet, thus emulating a vacuum. Its high Debye temperature results in low phononic occupation numbers [27]. Also, it can be grown using Chemical Vapor Deposition, in a purified form without Carbon-13 atoms to reduce magnetic noise [28] though Carbon-13 atoms can also be used as memory qubits, providing coherence times up to 1 minute [29], or a second at room temperature. [30].

Among the many color centers in diamond, group IV color centers have recently gained attention.

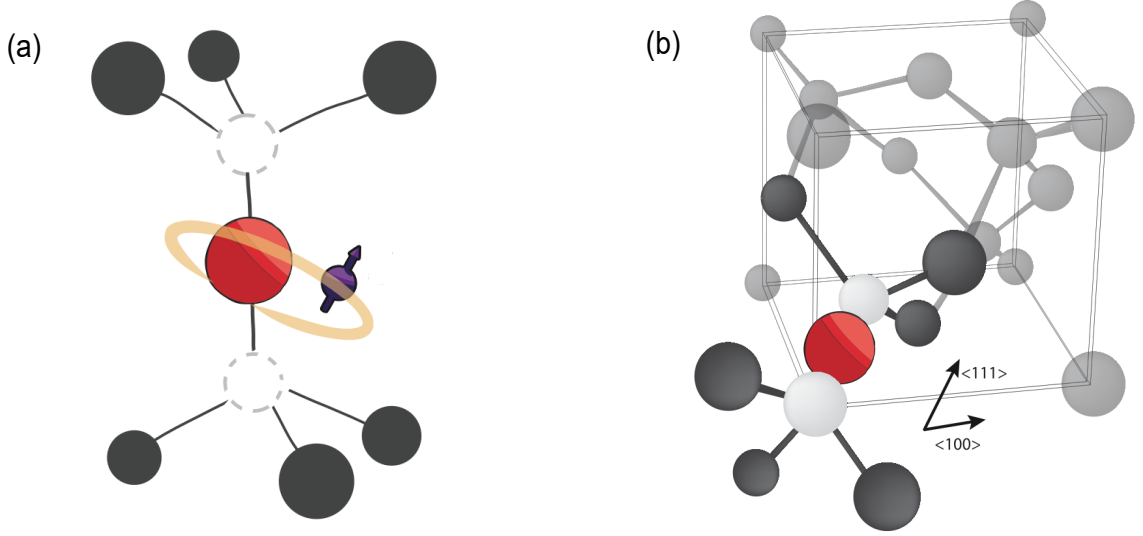


Figure 1.1: The crystallographic structure of a group IV color center. Taken from [36]. (a) Isolated group IV color center. Six carbon atoms are denoted by black circles and two vacancies are denoted by white circles. The group IV atom is given by the red circle. A hole is denoted by the blue circle orbiting the group IV atom. (b) Group IV color center integrated in the diamond lattice, where additional carbon atoms are denoted by semi-transparent black circles. The symmetry axis of the color center is the crystallographic axis $\langle 111 \rangle$ and is drawn in as well.

Their energy levels are first order insensitive to electric fields [31]. This allows for low spectral diffusion and pure dephasing, even when incorporated in crystal nanostructures containing electrical noise [32]. This is especially important since entanglement protocols rely on high coherent coupling of the spin to the right optical modes. It has been shown that this can be increased with modulating the density of states via the Purcell effect [33, 34]. Doing so by nanostructures offers promises of scalability, also opening the door to integrated photonic circuits [35]. We will now explain the structure of a group IV vacancy center.

1.2.1. Group IV Vacancy Centers

In group IV vacancy centers a non-Carbon group IV atom (Pb, Sn, Si, Ge) is situated in the middle of two vacancies in the diamond lattice along the $\langle 111 \rangle$ axis (see Fig. 1.1). This gives it a point-symmetric D_{3d} structure around the group IV atom, explaining its first order insensitivity to electric fields. The structure of a group IV color center is shown in Fig. 1.1

The group IV atom carries 4 valence electrons, and the six surrounding carbon atoms each contain a dangling bond from the two vacancies. The outer orbitals of the tin atom hybridize with the dangling bonds. The resulting electronic eigenstates of this structure are given by 12 different states where we have already accounted for spin degeneracy. The negatively charged group IV center has 11 localized valence electrons; 6 from the carbon atoms' dangling bonds, 4 from the group IV atom, and 1 captured from a nearby impurity. This means that only one of the 12 states is not filled; we can thus describe the electronic structure as a single hole which has spin 1/2. Therefore, the spin projection of the spin along any axis can have two eigenvalues, $\pm 1/2$ that can emerge from a measurement of the projection operator of the spin in a certain basis. Conventionally, one measures the spin in three orthogonal bases denoted by X-, Y- and Z-basis. For example, a measurement in the Z-basis, will yield one of the eigenvalues $+1/2, -1/2$ of the operator $\hat{S}_z$ whose corresponding eigenstates are popularly denoted as $|\uparrow\rangle$ and $|\downarrow\rangle$. Taking spin into account, the 8 lowest energy hole eigenstates that are, ordered in ascending energy: $\{|e_{g-} \uparrow\rangle, |e_{g+} \downarrow\rangle\}, \{|e_{g+} \uparrow\rangle, |e_{g-} \downarrow\rangle\}, \{|e_{u-} \uparrow\rangle, |e_{u+} \downarrow\rangle\}, \{|e_{u+} \uparrow\rangle, |e_{u-} \downarrow\rangle\}$. Each pair in brackets has equal energy, in the absence of an external magnetic field. The energy level of the center is defined by which of these states is occupied by the hole. An occupation of a $|e_g\rangle$ state means that the center is in the ground state, and an occupation of a $|e_u\rangle$ state means that the center is in the excited state. Both the ground and the excited state are split into an upper and a lower branch, depending on in which pair of states the hole resides [37].

While the qualitative structure of the group IV energy levels is the same, there are quantitative differences with important implications. Of the group IV vacancy centers, SiV has been widely studied, with promising results of entanglement of quantum nodes with coherence times on the order of seconds, utilizing control over the optical interface, electron spin, and nuclear spin in diamond crystal cavities [22]. Recently, tin-vacancy centers (SnV) have attracted attention. We will continue to explain the structure of group IV vacancy centers, highlighting the properties specific the SnV and its merits. Unless stated otherwise, we always mean the SnV^- (charge -e), when speaking of the tin-vacancy center or SnV.

1.2.2. Tin-vacancy Center

A graphical representation of the structure of the energy levels of the SnV is given in Fig. 1.2. The energy splitting between the lower branch and the upper branch of the ground (excited) states are 830 GHz (3030 GHz). This is a favorable property, because for temperatures up to 1.8 K phononic transitions from the lower branch to the upper branch do not dominate qubit decoherence [38]. This allows for coherent spin control below 1.7 K [39], and even up to 4 K with applied strain, which can further increase the energy splitting between upper and lower branch [40]. Such temperatures eliminate the need for a dilution refrigerator, or even allows the use of a simple bath cryostat [41].

In contrast, the timescale of decay from the upper branch to the lower branch of the ground state has been measured to be on the order of 30 ps [38]. If an external magnetic field is applied, the degeneracy of these branches breaks due to the spin Zeeman effect and the orbital Zeeman effect. This is denoted in the right panel of Fig. 1.2. This energy splitting lets us define our spin qubit as by the states $\{|e_{g+} \uparrow\rangle, |e_{g-} \downarrow\rangle\}$ of the lower branch of the ground state.¹ Readout and initialization can occur via transitions to the lower branch of the excited state, which has energies in the optical domain. We will now describe these optical transitions.

1.2.3. Optical Transitions

In Fig. 1.2, four optical transitions between the lower and upper branches of the ground and excited states are denoted with blue arrows. The splitting between the lower and upper branches is partially due to the spin-orbit coupling, which has a different magnitude in the ground- and excited manifold. This makes all transitions distinct in frequency. We are mainly interested in decay from the excited state lower branch to the ground state lower branch, as single photons from this decay will be used as photonic qubits in our entanglement protocol. Excitation of the SnV can be done resonantly with photons with wavelength of about 619 nm (2.00 eV). 80 % of the time, such an excitation will be followed by relaxation by emission of a photon [41]. This number is called the quantum efficiency (QE). Photonic emission can be accompanied by emission of a phonon; we call this emission in the Phonon Side Band (PSB). Due to energy lost to the phonon, the photon is red shifted compared to emission of only a photon. The latter is called emission in the Zero Phonon Line (ZPL). The ratio of decays happening in the ZPL is given by the Debye-Waller factor (DW), which for the SnV is 0.57(1) [43]. In the ZPL, the emitted photon will be linearly polarized along the electric dipole moment associated with the coupling of the dipole operator between the specific ground- and the excited states [31].

There are four different decay channels, denoted by the blue arrows in Fig. 1.2. Due to the described phononic decays from upper branches to lower branch, we will utilize the transition between the two lower branches (C transition) for single photon emission. Phononically assisted decays cannot be used for creating entanglement, due to the entanglement with phonons lost to the environment. Tracing out the phonon will result in a decohered quantum state, yielding this transition unfavorable for spin-photon entanglement. However, when exciting to the lower branch of the excited state, there is also decay from the D transition that emits a photon with slightly longer wavelength. The fraction of ZPL emission into the C transition is given by the Branching ratio, with a value estimated to be 0.8 [44].

Concludingly, there is an upper limit of $\zeta = QE \times DW \times BR = 36 \pm 3\%$ efficiency for single photon excitation that can yield high quality entanglement. It is estimated that for SnV this quantity is the highest of all group IV vacancy centers, making it especially suitable to achieve high emission based entangling rates [45].

A magnetic field lifts the degeneracies of the upper and lower branch. Experimentally, the magnetic field is never perfectly aligned with the energy quantization axis that split the upper- and lower branches.

¹To be precise, strain alters these states. This is also needed to form a functional qubit; microwave driving between the two states would not be possible without the modifications to the two states made by strain orthogonal to the symmetry axis. Microwave driving, and thus these modifications are not addressed in this work.

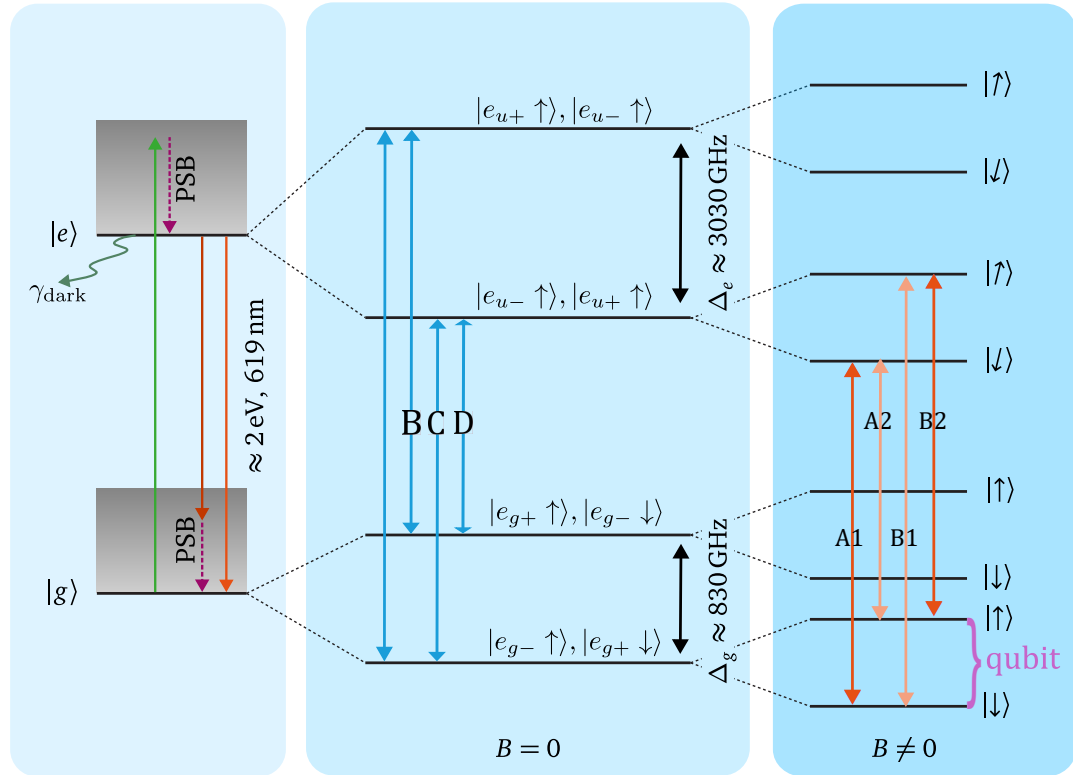


Figure 1.2: 'The optical transitions and electronic level structure of the SnV center. The states in the left region depict the unperturbed ground and excited states, which are split by an optical transition at 619 nm. The middle region shows the splitting of the ground and excited state branches due to SO coupling and the JT effect. Due to these interactions, the ZPL splits into four transitions, two of which are named 'C' and 'D'. The right region shows the additional splitting of these states in the presence of a magnetic field. The SnV qubit is then defined as the spin-up and spin-down states of the lower branch of the ground state, which is depicted in purple.' Adapted from [42].

However, due to the different energy splittings of the ground and excited state, the resulting energy quantization axis under magnetic field will be slightly misaligned. This is shown in Fig. 1.2 by arrows that are slightly misaligned. It is also shown that under a magnetic field, the C transition has split into four frequency resolved transitions denoted by A1, A2, B1, and B2. Because of the strong overlap $\langle \uparrow | \downarrow \rangle$ and $\langle \downarrow | \uparrow \rangle$, the A1 and B2 transitions are largely spin conserving. As a result, driving the A1 and B2 transition will often result in decay back to the original state. However, since $\langle \uparrow | \downarrow \rangle \neq 0$, there is a finite probability of decay to the opposing spin ground state. The expected amount of times an emitter can be excited before a spin flip prevents resonant excitation, is called the cyclicity. This number will be high for the transitions A1 and B2 under a well-aligned magnetic field. Thus, we can use these two transitions for readout of the spin state. When resonantly exciting transition A1 (B2), we will see significant fluorescence only if the SnV was in qubit state $|\downarrow\rangle$ ($|\uparrow\rangle$). Because of their finite cyclicity, we can also use these transitions for initializing the spin; if we excite them for long enough, the state will eventually decay to the opposite spin ground state. This process is called ‘spin pumping’ [46].

Lastly, it is also possible that the SnV goes into a dark state, where it emits no photons. This has been linked to previous optical excitation pulse (‘and thus called dark state pumping’), where the rate in units of previous excitations scales linearly with optical power. [47]. It is theorized that this is because the tin vacancy center has a charge state $-2e$ (SnV⁻²), which does not emit photons [43]. When driving the SnV⁻ to the excited state, an electronic state 2.00 eV above the valence band will be unoccupied. Thus, the same optical pulse can drive another electron from the valence band into this state, explaining the ionization. It can be undone by applying a green laser pulse [43, 44, 47]. This can excite an electron from the valence band into a nearby defect, creating a hole in the valence band, which diffuses to the SnV. These available states in the valence band will stimulate the relaxation of an electron from the tin vacancy to the valence band, deionizing to SnV⁻ [48]. This is called ‘repumping’.

1.2.4. Linewidth Broadening

A decay from the excited state with relative energy $\hbar\omega$ and lifetime τ yields a photonic wavepacket where the electric field oscillates at frequency ω and is contained in an envelope of exponential decay: $E(t) = H(t)e^{-t/(2\tau)}e^{-i\omega t}$, where $H(t)$ is the Heavyside step-function. The Fourier transform of this product yields a Lorentzian frequency distribution. The power spectral density in angular frequency coordinate ν is given by [49]

$$S_{\omega}(\nu) = \frac{1}{\pi} \frac{\Gamma_2}{(\nu - \omega)^2 + \Gamma_2^2}, \quad (1.1)$$

where $\Gamma_2 = (2\tau)^{-1}$ determines the linewidth; the Full Width Half Maximum of this distribution is $2\Gamma_2$. Photons with this temporal shape are referred to as ‘transform limited’ photons; their linewidth is minimized, given the finite excited state lifetime τ .

However, the energy of the transition can fluctuate. This destroys indistinguishability of single photons, and has been called a main challenge in building quantum networks with solid state emitters [25]. One of the reasons is the charge environment of the SnV. Even though the energy levels and thus optical transition frequencies of the SnV are first order insensitive to electric fields, higher orders persist [50]. When energy fluctuations happen during the decay, this induces pure dephasing. Pure dephasing is loss of coherence of the single photon, and it broadens the linewidth of the photon as $\Gamma_2 = (2\tau)^{-1} + \Gamma_{\phi}$ [51], where Γ_{ϕ} is the pure dephasing rate. When the energy of the transition fluctuates between emissions, this causes frequency differences between emitted photons. We describe this by a Gaussian distribution of frequencies, so $f \sim \mathcal{N}(f_0, \sigma)$, and call it spectral diffusion. Spectral diffusion not only changes the frequency of the emitted photons, but can also cause ineffective excitation because the emitter has drifted out of resonance. [43] In this work, we mitigate this using Charge-Resonance checks (see Chap. 3), which is a probabilistic way of finding resonance. However, in other platforms, such as quantum dots [52] and NV centers [53], deterministic ways of remaining on resonance have been applied.

1.3. Photonic qubit encoding

For a photon, there are multiple ways to encode the quantum information of a qubit into the physical system. Examples of bases for encoding are in the Fock, polarization, orbital angular momentum, spatial mode, frequency bins, and time bins [54]. A popular way is to encode the information in a two-dimensional time-bin basis, such that it forms a qubit. For these photons, one can define an early

$(|E\rangle)$ and late $(|L\rangle)$ temporal mode, which span the 2-dimensional Hilbert subspace in which the quantum state is encoded. One of the merits of this encoding is that it is robust against mechanical deformations or stresses in optical fibers used for long-range communication [55], it is robust to photon loss (any loss of photons can be noticed immediately and heralded away as an experimental failure) and generalizing it to higher dimensions does not add much complexity in optical setups [54].

1.4. Summary

In short, we have introduced the SnV as a promising spin-photon interface for optically distributed entanglement. We have focussed on the optical decay channels and identified the C-transition in the ZPL as the emission that can be used to create entanglement. We have also shortly touched upon tools for initialization readout of the spin qubit. Finally, we have introduced the qubit encoding of the photon that will be entangled

- Chapter 2 -

A Time Delay Interferometer

In this chapter, we will first describe quantum state tomography on an entangled spin time-bin photon pair. Then, we will describe how a Time Delay Interferometer enables the necessary measurements on the photonic qubit.

2.1. Quantum State Tomography

In order to reconstruct the density matrix of a single qubit, one can do measurements of the quantum state in three orthogonal bases. It is a well known result that

$$\rho = \frac{1}{2} \sum_{i=0}^3 \langle \sigma_i \rangle \sigma_i, \quad (2.1)$$

where σ_i denotes the Pauli operators I, X, Y, Z and $\langle \cdot \rangle$ denotes the expectational value. This decomposition naturally generalizes to a multi-qubit system. For example, any two-qubit density matrix can be expanded in the tensor-product Pauli basis as [55]

$$\rho = \frac{1}{4} \sum_{i,j=0}^3 \langle \sigma_i \otimes \sigma_j \rangle \sigma_i \otimes \sigma_j. \quad (2.2)$$

A long term goal of the project is to measure the fidelity of a spin-photon Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|E\rangle + |\downarrow\rangle|L\rangle)$. In the ordered basis $\{|\uparrow\rangle, |\downarrow\rangle\} \otimes \{|E\rangle, |L\rangle\}$, one can verify that the density matrix of the Bell state ρ_{Φ^+} , is given by

$$\rho_{\Phi^+} = \frac{1}{4}(I \otimes I + Z \otimes Z + X \otimes X - Y \otimes Y) \quad (2.3)$$

Thus, a calculation of the fidelity of our produced state with density matrix ρ gives

$$\mathcal{F} = \text{Tr}(\rho_{\Phi^+} \rho) = \frac{1}{4}(1 + \langle Z \otimes Z \rangle + \langle X \otimes X \rangle - \langle Y \otimes Y \rangle). \quad (2.4)$$

Experimentally, one determines $\langle \sigma_i \otimes \sigma_j \rangle$ by repeating N measurements of σ_i for the first qubit and σ_j for the second qubit, assuming no state evolution happens between the measurements on the different qubits. If the outcomes are given by s_i^k and s_j^k , then the correlator can be estimated as $\sum_{k=1}^N s_i^k \times s_j^k / N$. Measurements on the Pauli matrices (take, for example, X) are equivalent to a projective measurement on that basis $\{|X^+\rangle, |X^-\rangle\}$ with assigned values 1 and -1.

2.2. Unbalanced Mach-Zehnder Interferometer

To perform X, Y, Z basis measurements on the photonic quantum state, we will use an in-fiber Unbalanced Mach-Zehnder Interferometer. A Mach-Zehnder type interferometer consists of two subsequent beamsplitters. It is unbalanced because one of the arms between the beamsplitters has a longer optical path length, which can be conveniently realized by a spool of fiber. Fig. 2.1 shows the interferometer schematically. To allow for equatorial plane measurements of our time bin qubit on the Bloch sphere, the path length difference should create a time delay T equal to the time delay between the two timebins of the basis of the photonic qubit: $|E\rangle$ and $|L\rangle$. Then the part of the early wavepacket that travelled through the lower arm of the interferometer will mix on the second beamsplitter with the part of the late wavepacket that travelled through the short path. This is also depicted by the wavepackets

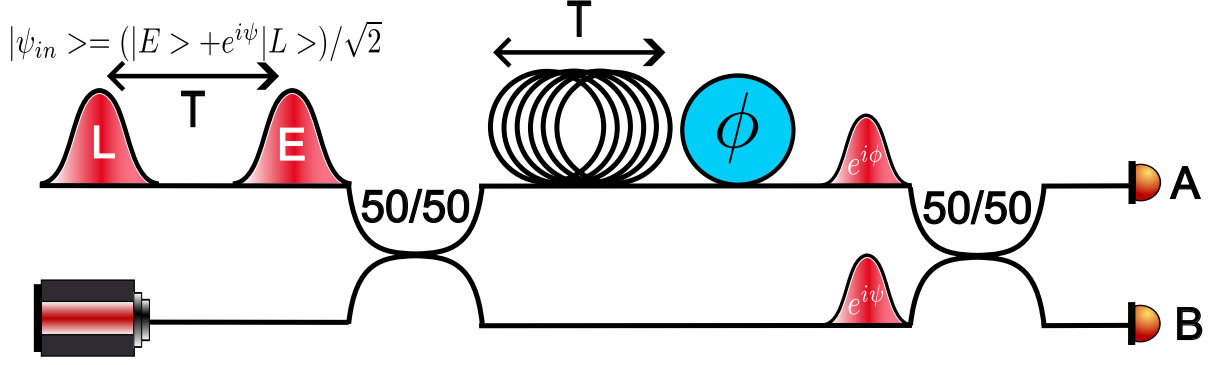


Figure 2.1: Mach-Zehnder type in-fiber time delay interferometer. Incoming photon in port 1 is denoted by $|\psi_{in}\rangle$ and the two peaks denote the superposition of the late and the early time bin, with time T in between. A beamsplitter splits the fiber into two arms. The upper arm's delay line of time T is shown by a coil of optical fiber. The symbol ϕ denotes a relative phase difference of ϕ between the optical arms. A and B denote the two photodiodes at the end of each arm of the second beamsplitter. In the lower arm, a continuous wave laser provides a probe signal to lock ϕ . The difference signal of the two photodiodes is the output signal that can be used to apply feedback to ϕ .

at the beginning of the second beamsplitter in Fig. 2.1. A detection in one of the output ports of the interferometer at this time, will be called a detection in the Middle timebin. We will call the arrival time of the state $|E\rangle$ after the travel time through the short arm the Early time bin, and the arrival time of state $|L\rangle$ after the travel time through the long arm the Late timebin. The three time bins are separated by time T .

The wavepackets that has travelled through the longer arm will acquire a phase ϕ compared to photons travelling through the shorter arm, which depends on the optical path length difference between the two arms, and the free-space wavelength λ , as

$$\phi = \frac{2\pi nL}{\lambda} = 2\pi T f, \quad (2.5)$$

where $f = c/\lambda$ is the carrier frequency of the single photon.

In the previous part, we have neglected the phase shifts given on the photons by the beamsplitters. Indeed, for lossless beamsplitters, we can absorb this into the phase difference ϕ of the two arms, as long as we only use one input port, and disregard global phases at the output ports. Adopting this convention, we see that two wavepackets entering from input port 1 and exiting via port A will acquire a phase difference ϕ . In contrast, they will acquire a phase difference $\phi + \pi$ if they exit through port B. This is a manifestation of con- and destructive interference at the two ports. If the other input port is used, this effective phase difference will differ by π . These properties are derived mathematically in Appendix B.

2.2.1. Quantum State Measurements

Here we assume the splitting ratio 50/50 and no losses. Let $|m_A\rangle$ and $|m_B\rangle$ be the states of a photon in the middle timebin in output arms A and B. Let $|\psi_{out}\rangle$ be the output state corresponding to the transformation of the TDI of input state $|\psi_{in}\rangle = (|E\rangle + e^{i\psi}|L\rangle)/\sqrt{2}$. Then we can calculate that

$$|\langle\psi_{out}|m_A\rangle|^2 = \frac{1 + \cos(\psi - \phi)}{2} \quad \text{and} \quad |\langle\psi_{out}|m_B\rangle|^2 = \frac{1 - \cos(\psi - \phi)}{2} \quad (2.6)$$

We call a measurement of a photon a click. With Eq. (2.6), we can verify that for $\phi = 0$ a click in the middle time bin in port A corresponds to a projective measurement of the incoming photon onto $|X^+\rangle$. Similarly, a click in the Middle time bin in port B corresponds with a projection onto $|X^-\rangle$. If $\phi = \pi/2$ the same holds for $|Y^+\rangle$ and $|Y^-\rangle$ respectively. Lastly, a click in the Early (Late) timebin will constitute a measurement in the Z-basis; a projection of the input state onto the states $|E\rangle$ ($|L\rangle$) [56]. This is summarized in Tab 2.1.

The TDI therefore provides the necessary measurements in different bases for quantum state tomography. Combined with similar measurements on the spin, we can now estimate the correlators of Sec. 2.1, needed for density matrix and fidelity estimation of our intended spin-photon Bell state.

Click in	Early timebin	Late timebin	Middle timebin, arm A	Middle timebin, arm B
Projection onto	$ E\rangle$	$ L\rangle$	$(E\rangle + e^{i\phi} L\rangle)/\sqrt{2}$	$(E\rangle - e^{i\phi} L\rangle)/\sqrt{2}$

Table 2.1: Top row: possible states in which a photon can be registered at the output ports of the TDI, with distinction made in terms of timebin and output arm where necessary. Bottom row: corresponding state on which a measurement denoted in the top row projects the input state.

2.2.2. Measurements with an Imperfect TDI

We can verify that the properties of table 2.1 still hold for balanced losses. If this is not the case, however, the tomography of our photonic quantum state will contain an error. In Appendix B, we will show that we can correct for this, and by extension, for the tomography of a joint quantum state.

However, there are cases when single-shot readout fidelity of a photon is important. For example, when the result of the readout determines the next step in a process [57], or cases in which a click in one of the timebins heralds a certain quantum state [22]. In those cases, imbalanced losses can negatively affect the fidelities. For example, in [22], a TDI is used to herald the Bell states $|\Phi^+\rangle$ (by click in detector A) or $|\Phi^-\rangle$ (by click in detector B) between two spins that have interacted with a photon. In the case of [22], they correct for this by creating an imbalanced photonic quantum state $|\Psi\rangle = \alpha|E\rangle + \beta|L\rangle$, with $|\alpha|^2 \neq |\beta|^2$. The ratio $|\alpha|^2/|\beta|^2$ should then be equal to the transmission through the long arm divided by the transmission through the short arm.

2.2.3. Measurements with Spectral Diffusion

We have seen that for a state $|+\phi\rangle = (|E\rangle + e^{i\psi}|L\rangle)/\sqrt{2}$ the probability of a click in detector A is unity, if the phase of the TDI equals ϕ . We define the measurement fidelity of the TDI as

$$F = \frac{\mathbb{P}(A|+\phi) + \mathbb{P}(B|-\phi)}{2}, \quad (2.7)$$

where $\mathbb{P}(A|+\phi)$ is the probability of a click in detector A for state $|+\phi\rangle$. This yields that the fidelity of our measurement, if we are detuned by $\cos(\Delta\phi)$, equals

$$F' = \frac{1}{2} + \frac{1}{2} \cos(\Delta\phi) \quad (2.8)$$

This detuning can be due to laser frequency noise, or due spectral diffusion (see Sec. 1.2.4). We assume that the laser frequency and photon emission frequency are normally distributed as $f_l \sim \mathcal{N}(0, \sigma_l^2)$ and $f_p \sim \mathcal{N}(\mu, \sigma_p^2)$. Now, we can derive via normal distribution scalings [58] that the variable $\Delta\phi = 2\pi(f_p - f_l)T$ (see Eq. 2.5) is distributed as

$$\Delta\phi \sim \mathcal{N}(2\pi T\mu, (2\pi T)^2(\sigma_p^2 + \sigma_l^2)) \quad (2.9)$$

Let us define the variance of this distribution as $\tilde{\sigma}^2 = (2\pi T)^2(\sigma_p^2 + \sigma_l^2)$. We can now calculate the expected fidelity of a measurement as

$$F = \int_{-\infty}^{+\infty} \left(\frac{1}{2} + \frac{\cos(\Delta\phi)}{2} \right) f(\Delta\phi) d\Delta\phi = \frac{1}{2} + \frac{\cos(2\pi T\mu)}{2} e^{-\tilde{\sigma}^2/2} \quad (2.10)$$

where $f(\Delta\phi)$ is the probability density function of the distribution of $\Delta\phi$.

To investigate the influences of spectral diffusion, we have plotted this in Fig. 2.2, for a time delay $T = 193$ ns, which we will show to be delay of the TDI. From this plot, we can conclude that with our current delay time, we need control over the center frequency of the emission far below the linewidth of the emitter.

2.3. Summary

In this section, we have introduced the quantum state tomography tools essential to showing spin-photon entanglement. Afterwards, we introduced the Time Delay Interferometer that will be constructed for measurements of the photonic quantum state. We briefly discussed the effect of imperfections in the interferometer, and the challenge that spectral diffusion poses.

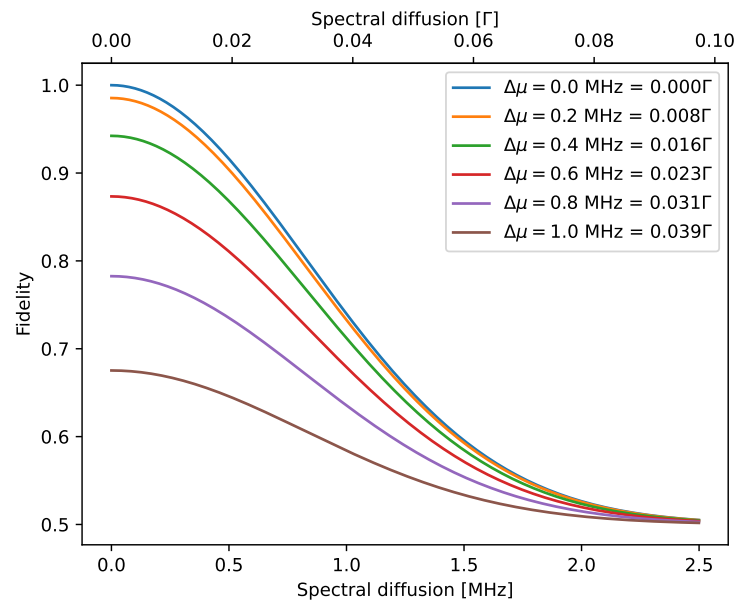


Figure 2.2: Influence of spectral diffusion and laser detuning $\Delta\mu$ on the fidelity of the TDI. Fidelity on the vertical axis is plotted against the standard deviation of the spectral diffusion on the horizontal axis. Lower horizontal axis shows the absolute value, upper horizontal axis is in units of the lifetime limited linewidth Γ for a lifetime of 6.2 ns.

- Chapter 3 -

Realization of the Interferometer

The previous chapter established the relation between the phase of the interferometer and the bases of the photonic quantum state measurements. To allow quantum state measurements at deterministic bases, we divide the experiment in two stages: the ‘phase locking stage’ and the ‘measurement stage’. We also identify the key parameters that influence the performance. Next, we show the design of the TDI that seeks to optimize it.

3.1. Phase Locking stage

In the phase locking stage, we will apply feedback to fix the phase to a setpoint ϕ_{set} , which we call ‘locking’ the phase to ϕ_{set} . We will do so by applying feedback to an error signal which is zero for phase $\phi = \phi_{\text{set}}$, and has a sufficiently large derivative in $\phi = \phi_{\text{set}}$. We will create this error signal using a probe signal; a coherent state entering the bottom arm of the interferometer. The power measured in the two arms of the interferometer will be proportional to $1 + x \cos(\phi)$ and $1 - x \cos(\phi)$ [59].¹ Here x is a measure of the visibility, depending on how complete the interference is, which is 1 for a perfect interferometer. More generally, subtracting the output of the photodetectors in the two arms yields a signal $V_{\text{diff}}(\phi) = \alpha + \beta \cos(\phi)$, where α and β take into account any imbalances in the interferometer. We will show that our methodology for error signal creation is not hindered by these imperfections.

The basis of this error signal generation relies on generating the in-phase component $I \propto \cos \phi$ and the quadrature component $Q \propto \sin \phi$ of V_{diff} . It is inspired by the Pound-Drever-Hall method [60] and has been executed (with small modifications) before in [61]. In this method, the phase in the optical arms is modulated by an amount $\phi_{\text{mod}}(t) = K \sin(\Omega t + \phi_0)$, such that the total phase of the interferometer is $\phi(t) = \phi_{\text{env}}(t) + \phi_{\text{mod}}(t)$. ϕ_0 is due to any delay in the response to the modulation, for example due to a finite electric capacity of the actuator. We will take Ω to be much larger than the natural fluctuations. Now, we will take our input signal V_{in} to be $V_{\text{diff}} = \alpha + \beta \cos \phi$ and write it as

$$V_{\text{in}} = \alpha + \beta \{\cos(\phi + K \sin(\Omega t + \phi_0))\} \quad (3.1)$$

$$= \alpha + \beta \{\cos \phi \cos(K \sin(\Omega t + \phi_0)) - \sin \phi \sin(K \sin(\Omega t + \phi_0))\} \quad (3.2)$$

$$= \alpha + \beta \left\{ \cos \phi \left[J_0(K) + 2 \sum_{n=1}^{\infty} J_{2n}(K) \cos(2n(\Omega t + \phi_0)) \right] \right. \quad (3.3)$$

$$\left. - \sin \phi \left[2 \sum_{n=1}^{\infty} J_{2n-1}(K) \sin((2n-1)(\Omega t + \phi_0)) \right] \right\}, \quad (3.4)$$

where J_i denotes the i ’th order Bessel function of the first kind, following from the Jacobi-Anger expansion. Subsequently, we multiply by $\sin(\Omega t + \Delta\phi)$ ² (‘demodulation’) and then low pass filter the signal with a cutoff frequency far below Ω . The product of two sinusoids can be written as two sinusoids oscillating with the difference and the sum of the two frequencies. As a consequence, there is only one term $-J_1(K) \cos(\phi_0 + \Delta\phi) \sin \phi$ that does not oscillate at a frequency of Ω or higher. Thus after low-pass filtering, only the quadrature component remains. Similarly, if we multiply the signal by $\sin(2\Omega t - \Delta\phi)$ (demodulate with double the modulation frequency) the only signal not filtered out by the low-pass filters is $-J_2(K) \sin(2\phi_0 - \Delta\phi) \cos \phi$, yielding the in-phase component of our signal.³

¹In our case, the laser will enter via the other arm of the interferometer than the single photons, see Sec. 3.4 This yields an effective phase difference of π

²One might wonder about the origin of $\Delta\phi$ in the coming calculation. This is a parameter that can be set in the IQ modulation. Our calculation will show that the magnitude of our resulting signals will depend on the value ϕ_0 set by the experiment. $\Delta\phi$ can be tuned in IQ modulation to maximize signal strength.

³Sometimes the signal V_{in} is used as the in-phase component. However, this demands the ability to compensate for an offset due to finite visibility.

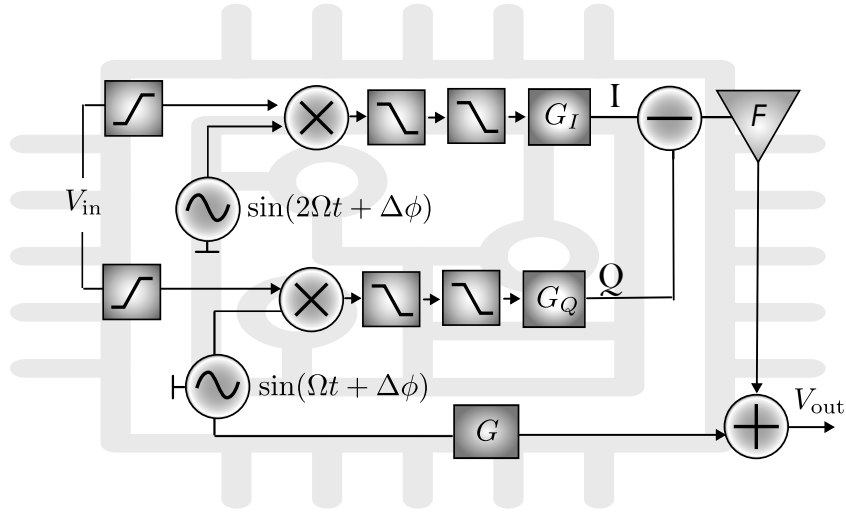


Figure 3.1: Schematics of the electrical circuit for applying feedback to the phase of the interferometer. Two paths take the same input V_{diff} , to which a phase modulation and feedback has been applied. The signal is high-pass filtered to suppress noise - before demodulation the signal should only contain frequencies on the order of Ω . The upper path generates the in phase signal by demodulation at frequency 2Ω and while the lower path generates the quadrature signal by demodulation at frequency Ω . Both arms apply low pass filtering to isolate these signals, and after appropriate scaling by gains G_I and G_Q and subtraction these form the error signal. Based on this error signal, feedback is applied. The feedback is superposed with the modulation signal (scaled with a factor G , bottom arm) to form the output signal V_{out} , driving an actuator of the phase.

This process is shown in Fig. 3.1, where the upper arm generates the in-phase (I) component, and the arm below that generates the quadrature (Q) component. The processes in the two arms only differ in demodulation frequency and the scaling factors.

We generate our error signal ϵ by choosing these scalings appropriately, and subtracting the two components. As demonstrated in Eq. (3.7) we can create a signal with a deterministic arbitrary phase-offset with respect to our input signal V_{in} . Now, if proportional, integral, or derivative feedback is applied to ϵ , this can lock ϕ to setpoint ϕ_{set} , owing to the sufficiently negative derivative in $\phi = \phi_{set}$. Feedback is denoted in Fig. 3.1, by the symbol ' F ', and is superposed by the modulation generated in the lowest arm. They form signal V_{out} , that drives an actuator of the phase.

$$I = \cos \phi \propto \beta - J_2(K) \sin(2\phi_0 - \Delta\phi) \cos \phi \quad (3.5)$$

$$Q = \sin \phi \propto \beta - J_1(K) \cos(\phi_0 + \Delta\phi) \sin \phi \quad (3.6)$$

$$\epsilon = \sin(\phi_{set}) \cos \phi - \cos(\phi_{set}) \sin \phi = -\sin(\phi - \phi_{set}) \quad (3.7)$$

3.2. Measurement Stage

To distinguish single photons from the probe signal, we use time-filtering. The single photon measurements will happen after the locking stage, and we will call this the measuring stage. During the measurement stage the probe signal is blocked, rendering feedback to the phase impossible. The key functionalities during the measuring and locking stage are displayed in table 3.1.

The first planned use case of the interferometer, and the original goal of this project, is to measure the fidelity of a created Bell-state between the SnV spin qubit and the time-bin encoded photonic qubit. To better estimate the needed duty cycle, it is important to have rough estimate of our measurement rate. Therefore, we will now describe the proposed entanglement sequence, its practical implementation and its duration. A scheme to create these spin-photon entangled states with time bin photonic state can be found in [62]. The spin is prepared in the state $|\uparrow\rangle$ by means of spin pumping (see Chap. 2). Then a $\pi/2$ microwave pulse is applied to the spin to create a superposition state, yielding a combined quantum state

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_s + |\downarrow\rangle_s) |vac\rangle_p, \quad (3.8)$$

Where the subscript s stands for the spin and p stands for the photonic mode. After this an optical excitation pulse is sent to the spin qubit, which will only excite the bright spin state $|\uparrow\rangle$. The SnV will

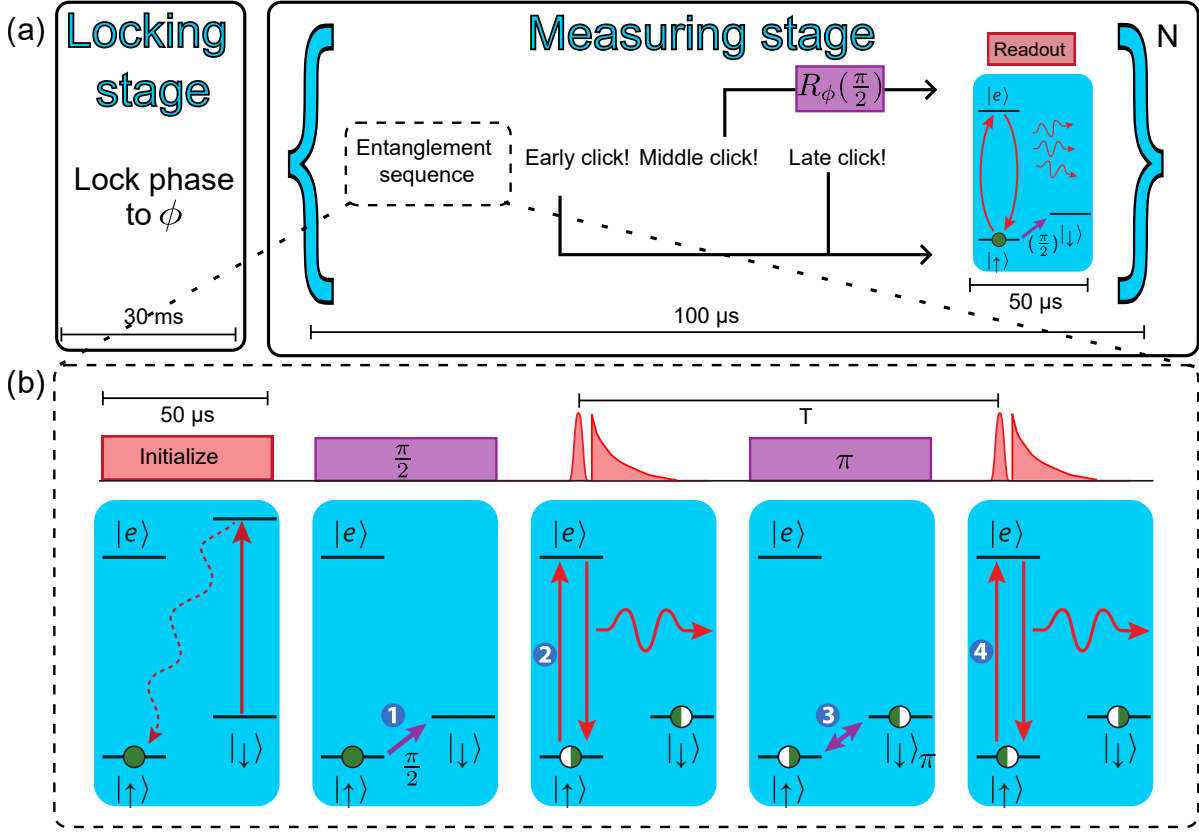


Figure 3.2: Operational sequence of the envisioned spin-photon entanglement experiment. Dominant time durations have been drawn in. Time axes are not to scale. **(a)** Operation during the two stages of the Time Delay Interferometer. During the locking stage, the phase ϕ of the interferometer is being locked. At the current state of the interferometer this process takes about 20 ms. During the measuring stage of the interferometer, the entanglement sequence is executed. If an APD clicks in the late time-bin, a $\pi/2$ rotation is applied to the spin. Afterwards, the spin is read out in the Z-basis. **(b)**, adapted from Bernien [62]. A depiction of the entanglement sequence, consisting of microwave pulses (purple) and optical pulses (red). First the spin is prepared in a superposition (1). Then, an optical excitation pulse of approximately 3 ns is applied (2) followed by flipping the spins (3) and another optical excitation pulse (4). More details are in the main text. Initialization and readout times have been estimated based on the work by Waas [46]

decay to the bright ground state, emitting a photon which is collected in a certain optical mode. Let us set the probability of this process to 1; it will not matter as long as it is constant over the entanglement sequence. We will denote the photonic quantum state by $|E\rangle$, where E stands for early. The resulting quantum state will be

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_s |E\rangle_p + |\downarrow\rangle_s |vac\rangle_p). \quad (3.9)$$

Then a π pulse is applied to the spin, which yields

$$\frac{1}{\sqrt{2}}(|\downarrow\rangle_s |E\rangle_p + |\uparrow\rangle_s |vac\rangle_p), \quad (3.10)$$

and with another excitation pulse, our resulting quantum state will be

$$|\psi\rangle_{sp} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_s |E\rangle_p + |\uparrow\rangle_s |L\rangle_p), \quad (3.11)$$

The scheme is shown in Fig 3.2(b). It has however, not yet been executed. In this thesis, single photons have only been generated in well-defined time bins.

Immediately after measuring a single photon, a measurement on the spin of the SnV will follow. As described in Chap. 2, a measurement in the early time bin, or the late time bin, constitute a Z-basis measurement and as such the spin must also be read out in the Z-basis. A click in the late timebin,

constitutes a measurement in the basis on the equator with azimuthal angle given by the phase of the interferometer ϕ . A measurement in equal basis must be applied to the spin, which is realized by firstly a $\pi/2$ rotation $R_\phi(\frac{\pi}{2})$ around the vector $(-\sin \phi, \cos \phi, 0)$, and a subsequent Z-basis measurement on the spin. This entanglement sequence and the subsequent conditional measurement on the spin, are repeated a certain amount of times, limited by the time during which the interferometer is phase-stable. It is shown in the right panel of Fig. 3.2(a).

	Locking stage	Measuring stage
Entanglement sequence	Active	Inactive
Phase modulation and feedback	Active	Inactive
Locking laser light	Incident	Blocked
Single photon measurements	Active	Inactive

Table 3.1: Various functionalities of the TDI (denoted in the left column), during the two stages of the experiment (denoted in the upper row). Note that during the implementation of the two stages described in Sec. 4.3, the entanglement sequence was not actually played, and a weak coherent state replaced single photons

3.3. Requirements of the TDI

Gathering sufficient data from the experiment shown in Fig. 3.2 can be time-consuming. Therefore, it is important that we optimize the duty cycle of the TDI, which we define as the fraction of time that the TDI can spend in the measurement stage. To this end, it is important that the phase converges to the setpoint in a short time when applying feedback, and does remains close to the setpoint for a long time without feedback. We will call the latter ‘passive stability’. In addition, it must be engineered to have low optical loss to achieve high photon detection rates. We will now describe the design of the interferometer, taking into account the two stages and these requirements.

3.4. Design of the TDI

At its core, the interferometer will follow the form of Fig. 2.1. A laser set to wavelength of 619 nm (Toptica, DLC TA-SHG pro) provides the probe signal, which is routed through three Acousto-Optic Modulators (AOMs) as shown in the bottom arm in Fig. 3.3. They receive a 200MHz electrical signal from drivers (Zepto RF-Driver) which can be gated to provide fast switching between transmitting and blocking the probe signal. The optical fields in the output arms of the interferometer are split by 90/10 beamsplitters (Thorlabs PN635R2A2), where the 90 ports are connected to Avalanche PhotoDiodes (APD, Laser Components, COUNT-T). The APDs can detect single photons during the measuring stage, and are gated during the locking stage to prevent damage from the probe signal. The 10 ports of the beamsplitters are connected to a differential photodiode (Thorlabs, PDB435A) providing signal V_{in} . This signal contains a modulation at high frequency ($\Omega = 10$ kHz), which we cannot accurately distinguish with the APDs, explaining the need for the differential photodiode. On V_{in} , the signal processing steps shown in Fig. 3.1 are performed digitally by a Field Programmable Gate Array, shown in the top right of Fig. 3.3 (Red Pitaya Stemplab 125-14, programmed with a Python API called PyRPL [63]). Via signal V_{out} , it will drive a piezo-electric fiber stretcher⁴ (custom, PiezoDrive, 630HP optical fiber stretcher), which has low optical loss.

The stages and the functionalities during these stages are coordinated by a control system (Qblox quantum control system, Cluster). It synchronizes various modules of the type Quantum Timetagger module, which timestamps the counts registered by the APDs, and the Quantum Control Module which sends Transistor-to-Transistor Logic signals to control the equipment.

To increase the passive stability of the interferometer, we have made the following design choices of the interferometer. Firstly, the delay line is wrapped around invar cylinders. Invar has a very low thermal expansion coefficient of $1 \times 10^{-6} \text{ K}^{-1}$ to $2 \times 10^{-6} \text{ K}^{-1}$ [64]. Therefore, any change in temperatures of the cylinders will apply minimal strain and thus minimal change of optical path length to the delay line. In addition, the breadboard containing the optical components is temperature stabilized, via a Thermal Electric Cooler element (TEC, Thorlabs, TECF1S). The TEC is clamped between two optical

⁴A piezo-electric fiber stretcher is a cylinder-shaped piezo which can expand in the radial direction. Five windings of the optical fiber are wrapped around it. By applying a voltage to the piezo, the fiber can be stretched, changing the optical path length in the upper arm which determines the phase.

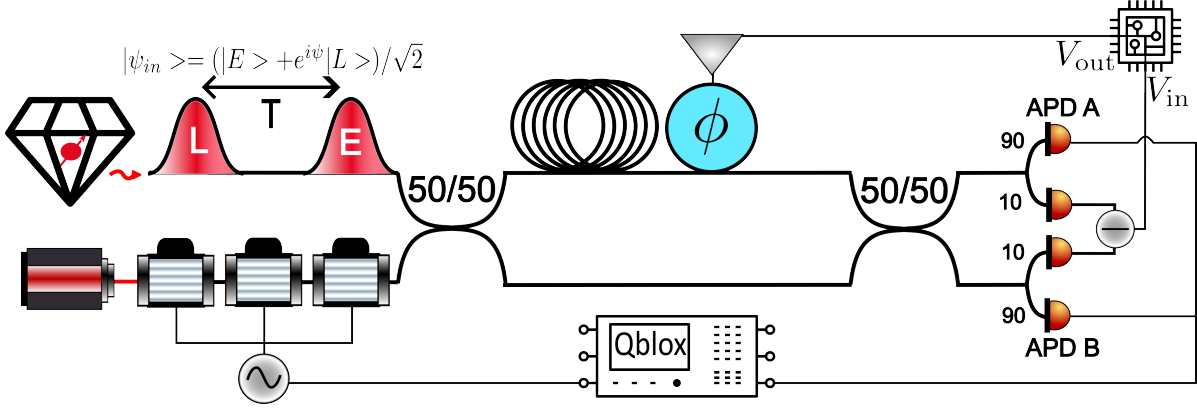


Figure 3.3: Mach-Zehnder type in-fiber time delay interferometer. Incoming photon is denoted by $|\psi_{in}\rangle$ and the two peaks denote the superposition of the late and the early time bin, with time T in between. Through the other arm, a laser is incident, which can be blocked by two AOMs connected to a driver. A beam splitter splits the fiber into two arms. In the upper arm a delay line of time T is shown by a coil of optical fiber. The yellow symbol with text ϕ denotes a relative phase difference of ϕ between the optical arms, and is in practice created by a fiber stretcher. The inner photodiodes form the differential photodiode (indicated by the - sign) and the outer photodiodes are APDs. Qblox controls the FPGA state, the drivers, and the gating of the APDs via TTL signal.

breadboards, connected by thermally insulating screws. The TEC is driven by a temperature controller (Thorlabs, TC300B) applying feedback based on readings from a Negative Temperature Coefficient thermistor (NTC, Thorlabs TH10K). This NTC is placed on the upper optical breadboard, close to the cylinders containing the delay line, as this is the most sensitive part of the setup. These temperature stabilization components can be seen in Fig. 3.4(a). Lastly, an enclosure made of plastic and styrofoam, surrounds the phase-sensitive part of the interferometer to increase passive stability. This is visible in Fig. 3.4(b).

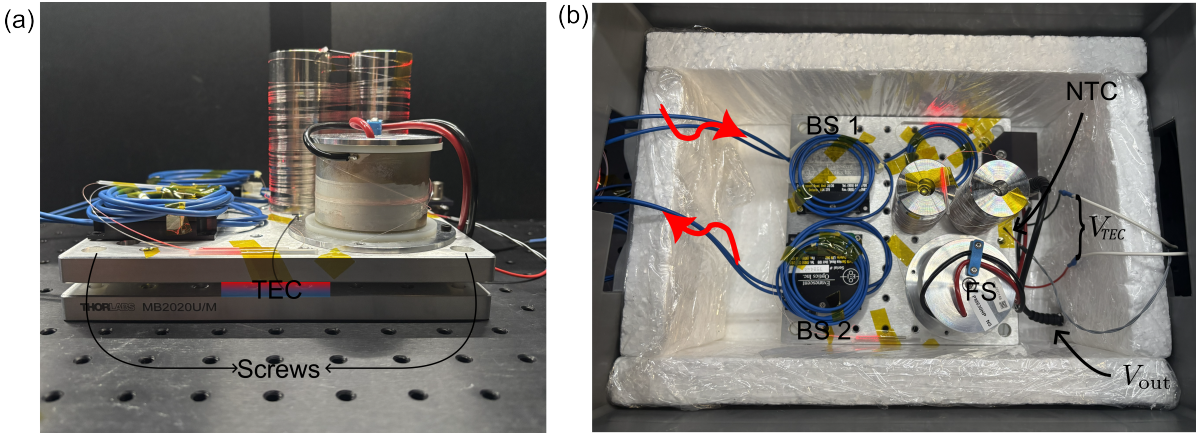


Figure 3.4: Physical realization of the part of the Unbalanced Mach-Zehnder Interferometer requiring phase stability. For this configuration, inspiration was drawn from [65, 66] (a) Side view of the optical breadboard containing the interferometer. Consists of two optical breadboards connected via thermally insulating screws, clamping the TEC element in the middle. The TEC element is drawn in; in reality it is below the cylinders and not visible in this image. (b) Top view of the interferometer. Delay line is wrapped around the two invar cylinders, and connected to the fiber stretcher, denoted by FS. Via a coaxial cable a voltage V_{out} can be supplied to the fiber stretcher. The two beamsplitters are denoted with BS1 and BS2. Photons indicate the direction of the light. An NTC measures the temperature of the breadboard close to the delay line. Feedback to the temperature is applied via the voltage denoted by V_{TEC} over the TEC element drawn in (a). The optical breadboards are contained in an enclosure made from a plastic box (dark gray) with padding made of styrofoam (white) and can be closed with a lid similar to the sides shown.

3.5. Summary

In this chapter we have showed the experiment that can be executed with the TDI to show spin-photon entanglement in the future. We have divided the experiment in two stages of the experiment, and described their practical implementation, aiming for a high duty cycle and low optical loss.

- Chapter 4 -

Characterization of the Interferometer

In this chapter, we will characterize the performance of the TDI. Firstly, we will attempt to lock the interferometer to a deterministic phase, and observe phase drifts, and the timescales on which they occur. Subsequently, we will extract the time-delay and imbalances of the interferometer. Lastly, we will take on the technical challenge of interleaving the locking and measuring stage of the experiment (see Chap. 3). During the locking stage, we will lock the phase, and during the measuring phase we will measure phase-sensitive effects with our APDs on a weak coherent state. We will compare these results to the measured phase stability during the measuring stage, and the measured imbalances.

4.1. Phase Stability

To characterize the phase stability of the interferometer, we have locked the phase to a deterministic value $\phi_{set} = \pi/4$, stopped locking at time $t = 0$, and monitored the signal of the differential photodiode V_{out} . We extract the phase drift from the in-phase and quadrature components, using Eq. 3.7. We repeated this 400 times, yielding a distribution of 400 phases as a function of time. The experiment has been done with temperature stabilization as described in Sec. 3.4, and without, to characterize the effects. The phase distributions are shown in Fig. 4.1, for timestamps $t \in \{0\text{ s}, 0.1\text{ s}, 0.2\text{ s}, 1\text{ s}, 4\text{ s}, 7\text{ s}\}$. We choose these timestamps because we found that in the first 100 ms the reconstructed phase drifted rapidly, as can be seen by comparing the upper left and middle plots in Fig. 4.1. After 100 ms, the distribution of phases evolved much less rapidly, as can be seen by comparing the upper middle and upper right plots.

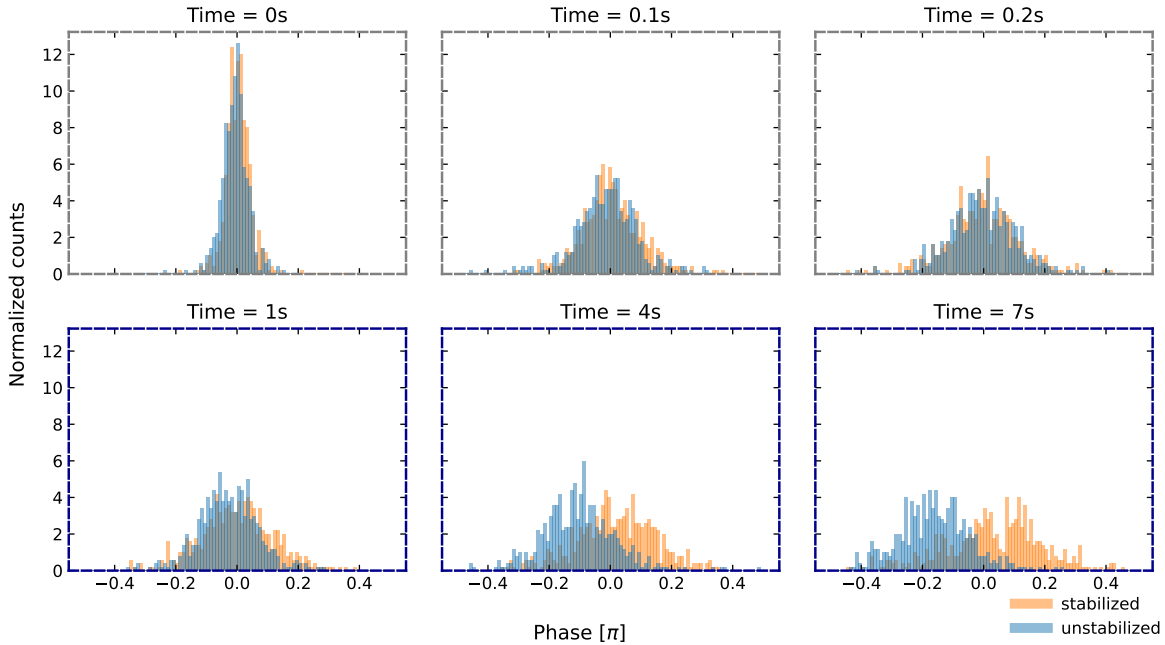


Figure 4.1: Normalized distribution of phase offset from lock setpoint ϕ_{set} after releasing lock at $t = 0$. The distribution is shown for a temperature stabilized and a non-stabilized case. The phase drift has been measured by releasing lock 400 times and tracking the phase of the interferometer over time, using Eq. 3.7. Each plot shows the phase distribution of phases at the labelled timestamp.

However, looking at longer timescales, we see that slow evolution of the phase distribution persists.

The lower histograms in Fig. 4.1 show a slow broadening of the phase distribution. Note the different time spacings compared to the upper histograms. For the non-temperature stabilized interferometer a systematic drift towards negative phase differences is visible. Given the fact that this does not happen to the temperature stabilized interferometer, we attribute this drift to temperature drifts of the interferometer. From this we conclude that most likely the temperature has been continuously decreasing during the duration of the experiment (approximately 1.5 h). However, this has not been monitored during so it remains speculation.

To quantify the phase drift, we have calculated the Root Mean Square (RMS) of the distribution for the complete time spectrum. The result is shown in Fig. 4.2(a). The RMS values at zero time are 0.045π and 0.046π for the stabilized resp. unstabilized case. After the initial rapid phase drift after releasing lock, RMS for the unstabilized case shows a dip, while the RMS for the temperature stabilized case continues to increase. For this difference, we have found no conclusive explanation. However, on longer timescales the RMS of the stabilized case increases less than the RMS of the unstabilized case. Between 0.2 s and 8 s, the RMS only increased by 0.05π for the stabilized case. As a result, after 3 s the RMS for the temperature stabilized case drops below the RMS of the non-temperature stabilized case. This is in correspondence with the systematic drift of the unstabilized interferometer towards negative phases, which increases the RMS of the phase drift.

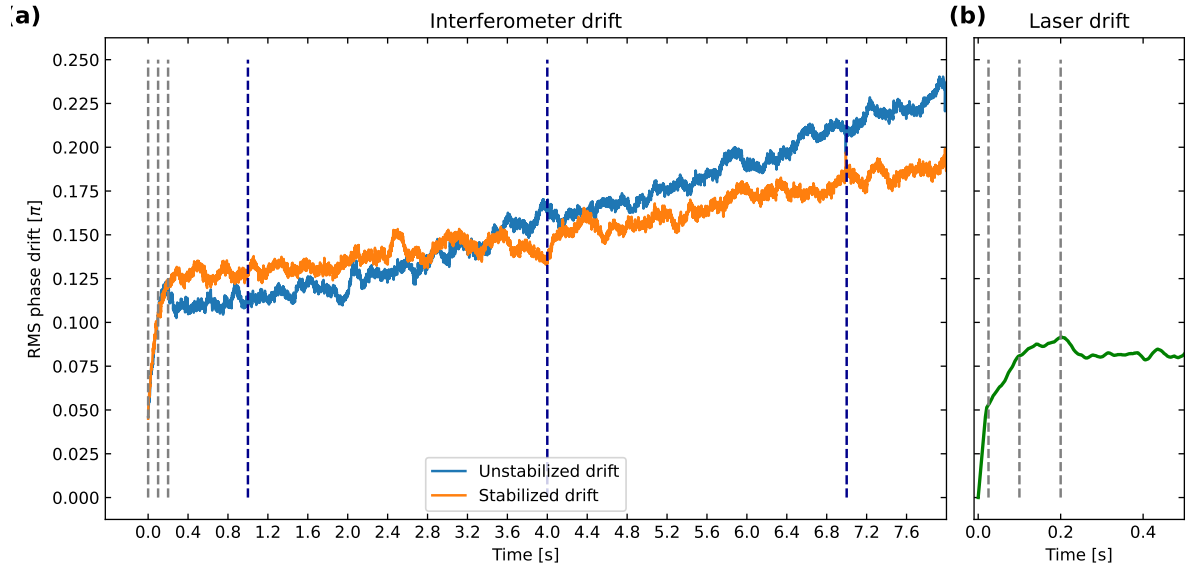


Figure 4.2: RMS of phase distribution as a function of time (a) Corresponding to the distribution measured by the temperature stabilized interferometer, and the non-temperature stabilized interferometer. The vertical gray dashed lines signify the timestamps of the upper three histograms in Fig. 4.1, and vertical dark-blue dashed lines correspond to the timestamps of the lower three histograms. (b) Corresponding to the effective phase distribution calculated as a result of the wavelength of the laser. Vertical gray dashed lines correspond to the timestamps of the three histograms in Fig. 4.3.

Now we will investigate the reason behind this sudden increase in reconstructed phase drift. As explained in Sec. 2.2, changes in phase can be attributed to changes in optical path length (inherent to the interferometer) and changes in laser wavelength. We hypothesized that frequency noise in the laser causes the rapid phase drift on short timescales. To test hypothesis, we have taken a long timetrace of the wavelength-meter (HighFinesse Wavemeter WS8-2) connected to the laser, yielding a set of 3000 independent sub-traces of the wavelength over a time of 0.6 seconds. From these sub-traces, we have reconstructed the effective phase drift resulting from wavelength (see Eq. 2.5), with respect to the zero-time of the subtrace. We plot the phase distribution of the phase drift due to laser noise in a fashion similar to Fig. 4.1.¹ The result is shown in Fig. 4.3. Qualitatively, the shape of these histograms at times 100 ms and 200 ms matches well the shape of the histograms in Fig. 4.1.

To better quantify the influence of laser noise as a function of time, we have plotted RMS of the phase distributions due to laser noise as shown in Fig. 4.3, on a continuous time spectrum. The result is shown

¹The histogram at 25 ms is chosen somewhat arbitrary as an ‘early time’, for at time exactly 0 we would only see a single bin at phase difference 0.

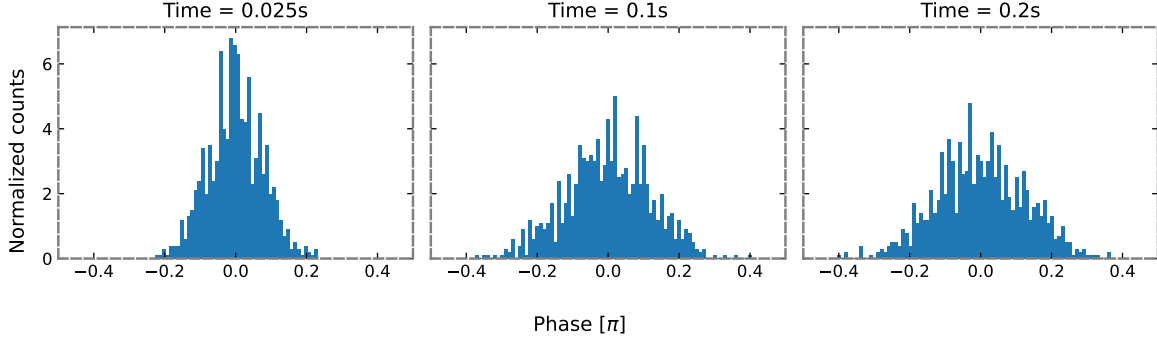


Figure 4.3: Normalized histograms of the effective phase distribution caused by laser wavelength fluctuations over time. A timetrace wavelengths over time has been split into 3000 independent traces of 0.6 seconds, yielding a distribution of 3000 phases as a function of time. Three histograms correspond to the distribution at three different timestamps.

in Fig. 4.2(b). One can see that both the magnitude of the phase drift and the timescale match with the initial rise in Fig 4.2(a). One can see that after an initial rise during the first 0.2 seconds, the RMS does not change systematically anymore, as can be expected for a stabilized laser. Thus, the later, slow drift in reconstructed interferometer phase in Fig. 4.2 over the timescale of seconds cannot be attributed to laser noise. Like the unstabilized case in Fig. 4.2(a) the RMS due to laser noise in 4.2(b) shows a dip after 0.2 seconds. This dip could be due to the dynamics of the feedback loop controlling the wavelength of the laser, in combination with the input from the wavemeter. However, this dip is not visible for the temperature stabilized case. No explanation has been found for this discrepancy between the two cases.

4.2. Delay and Losses

The first important metric of the interferometer is the time delay T between the two arms, as this sets the delay between the two time bins. Due to uncertainty in, the time needed for the microwave π pulse (see Sec. 3.2), this length is not yet certain. We have chosen it to be around 40 m, corresponding to a time delay on the order of what has previously been shown to be achievable [39, 67]. Making it longer would not be beneficial, mainly due to frequency noise, and photon loss, whose effects scale linearly resp. exponentially.

To measure the time delay more precisely, we generate an optical pulse using the optical path that is normally used to excite the SnV. See Sec. 5.4 for more details. This pulse is heavily attenuated and incident in the interferometer, after which an APD at one output of the interferometer registers the pulse. We repeated this experiment 750×10^3 times and the counts of the APD are timetagged and histogrammed. With this, the count-rate of the incident pulse is calculated as a function of time, and the result is shown in Fig. 4.4.

From Fig 4.4 we can extract the time delay of 192.85(8) ns, by measuring the time delay between the two peaks. A similar curve exists for the second APD. From integrating the counts in the two peaks (in the interval between the dashed gray lines in Fig. 4.4) for the two APDs, we can extract information on the imbalances in the interferometer. The integrated counts for both peaks (early and late) and both APDs (A and B) are shown in Fig. 4.4.

We will denote the reflection and transmission of beamsplitter i as T_i and R_i and the transmission in the long and the short arm as $\mathcal{T}_l$ and $\mathcal{T}_s$. We also incorporate finite transmissions in the output arms A and B of the interferometer, as $\mathcal{T}_A$ and $\mathcal{T}_B$. In this experiment, this models mainly different detection efficiencies of the APDs. It can also incorporate loss due to the 90/10 beamsplitters are placed in the output arms (see Chap. 3). However, for this experiment, and the experiments in Chap. 6, these were taken out. A schematic of the optical paths and the transformations of the optical field are given in Fig. B.1. Using this schematic, one can verify the formulas shown in Fig. 4.4.

From Fig. 4.4 we can extract information on the imbalance between the short and long arm, the imbalance of the second beamsplitter and the imbalance in detection efficiencies $\mathcal{T}_A/\mathcal{T}_B$. We assume lossless beamsplitters. Straightforward manipulation of the formulas gives the values shown in Tab.4.1. Errors are calculated using standard error propagation and shot noise errors. Using Tab.4.1, we see that $\mathcal{T}_A = 1.02\mathcal{T}_B$, which means that the APDs are approximately equally sensitive, as expected. Note that in

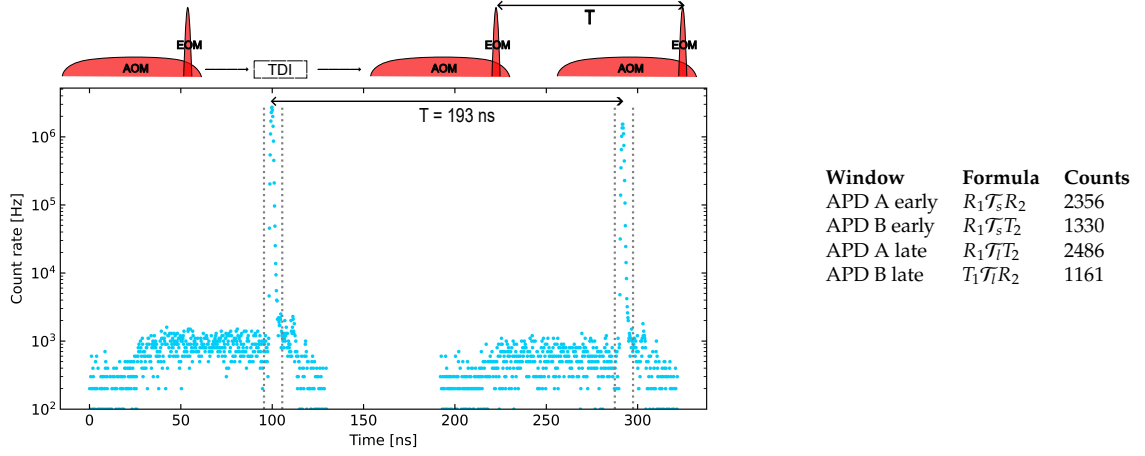


Figure 4.4: Figure: Count rate measured by an APD at an output arm of the TDI, after sending an optical pulse via the fast pulse path (see Sec. 5.4). Two peaks are visible, spaced in arrival time by $T = 192.95(9)$ ns, and we will denote them as an early peak and a late peak. Only one APD is shown, for clarity. Table: Integrated counts of the early and late peaks surrounded by the dashed gray line in the figure, both for APD A and APD B. Formula denotes the expected number of counts (up to arbitrary scaling), with symbols described in the main text, and drawn schematically in Figure B.1 in Appendix B.

Quantity	Formula	Extracted value
Detection imbalance	$r = \mathcal{T}_A / \mathcal{T}_B$	1.04(3)
Arms imbalance	$q = (R_1 \mathcal{T}_s) / (T_1 \mathcal{T}_l)$	1.95(5)
Beamsplitter 2 transmission	T_2	0.55(1)
Beamsplitter 2 reflection	R_2	0.45(1)

Table 4.1: Values concerning imbalances in the interferometer. Extracted from the data as shown in Fig. 4.4.

any measurement using only one input port of the TDI, $T_1 \mathcal{T}_l$ and $R_1 \mathcal{T}_s$ will always appear as a product (if at all). With only this measurement, we are not able to distinguish between imbalances in the first beamsplitter and imbalances in the losses in the two arms.

Using the formulas in the Tab.in Fig. 4.4, one can easily derive that the visibilities during incidence with continuous wave laser light in the output ports of the interferometer are given by

$$V_A = \frac{2}{\sqrt{\frac{R_1 R_2 \mathcal{T}_s}{T_1 T_2 \mathcal{T}_l}} + \sqrt{\frac{R_1 R_2 \mathcal{T}_s}{T_1 T_2 \mathcal{T}_l}}^{-1}} \quad V_B = \frac{2}{\sqrt{\frac{R_1 T_2 \mathcal{T}_s}{T_1 R_2 \mathcal{T}_l}} + \sqrt{\frac{R_1 T_2 \mathcal{T}_s}{T_1 R_2 \mathcal{T}_l}}^{-1}}, \quad (4.1)$$

yielding $V_A = 0.97(1)$ and $V_B = 0.91(1)$. In these visibilities, corrections for polarization mismatch have not been taken into account. Nevertheless, they are close to the visibilities measured with continuous wave light, being: $V_A = 0.95(2)$ and $V_B = 0.92(2)$.

4.3. Interleaved Execution of Stages

Now we will test our sequence of alternating locking- and measurement stages. However, since we do not yet have access to single photons in time-bin superposition with a well-defined phase, we replace the single photons by a coherent state. It is a heavily attenuated laser signal, such that APD B measures a count rate of 28(3) kHz in case of constructive interference. APD A was turned off during the entire sequence.

In this experiment, we swept the phase to which we locked the interferometer during the locking stage. In the subsequent measuring stage we measured the attenuated laser light for 45 ms. This was repeated 30 times for each phase, to decrease shot noise. For each phase, the counts registered from the APD were integrated, and converted to a count-rate. The result is shown in Fig 4.5. It shows a sinusoidal profile as expected by theory. The sinusoidal is offset in phase due to a constant offset in frequency between the signal during the measurement- and locking stage; a consequence of the AOMs.

The fit in Fig. 4.5 shows a visibility of 0.85(3), where we have not yet corrected for inaccuracies in the phase lock. Let us also assume a normal distribution of phase during the measurement stage (due

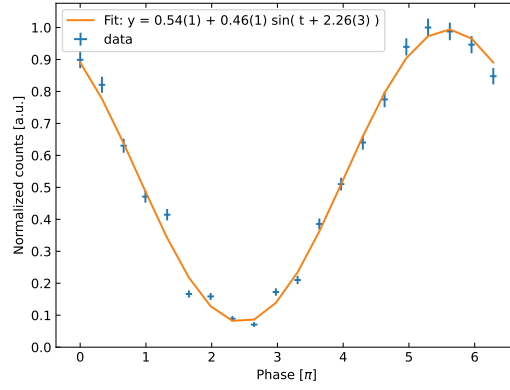


Figure 4.5: Normalized counts measured with an APD in the measuring stage (45 ms), when incident with a weak probe laser giving rise to a maximal count rate of 28(3) kHz. Phase setpoint is swept during the locking stage (100 ms). Vertical errorbars are given by shot-noise errors. Horizontal errorbars are given by the standard deviation of the expected phase distribution averaged over measurement time.

to phase drift). We can calculate the average standard deviation with the data in Fig. 4.2 during the measurement sequence to be 0.049. This yields a corrected visibility of 0.86(3). This is lower than the previously predicted value for the visibility by Eq. (4.1), due to the longer than usual delay line of 263 ns. If we assume a balanced first beamsplitter, we can correct $\mathcal{T}_l/\mathcal{T}_s$ with a factor 0.78, yielding a corrected visibility of 0.86. Thus, we do not conclude that there is a discrepancy between the two experiments.

Still, the data in Fig. 4.5 still show some deviations from the fit. The dark count rate was 100 kHz, so this cannot explain it. However, power fluctuations during the experiment might explain it. These later also been observed behind the attenuator, but have not been quantified.

4.4. Summary

Concludingly, we have shown that we are able to lock the interferometer to a deterministic phase. After locking for 30 ms we observed a phase distribution centered around the setpoint with RMS of 0.046π . This is likely limited by laser noise. On the timescales below 200 ms laser noise is the dominant source of the observed phase drift. Therefore, no reliable conclusions can be drawn with respect to the phase stability of the interferometer on short timescales. The non-temperature-stabilized case shows a smaller RMS phase spread between 0.2 s and 3 s after releasing phase lock, and this appears to be influenced by the unexpected RMS dip around 0.2 s. This dip does not show in the temperature stabilized case. Since a similar dip also appears in the effective phase derived from laser wavelength noise, it may be a laser artifact, but it is unclear why it does not show in the temperature stabilized case. Measurements with decreased laser noise are needed to quantify how phase stable the interferometer is on the short timescales. On timescales above 3 s the temperature stabilised case showed a lower RMS than the non-temperature stabilized case. The RMS increased with only 0.049π between 200 ms and 8 s.

Using short pulses and detection with APDs, we characterized the imbalances and the delay of the TDI. This predicted visibilities which matched well with measurements.

Lastly, we successfully interleaved the locking and measuring stage. This was shown by a clearly distinguishable oscillation in the measurement stage, due to phase locking. This shows that the TDI can in principle perform measurements in deterministic equatorial basis on timebin photonic qubit.s

- Chapter 5 -

Single photon generation

Having finished the interferometer, we now devote our efforts to generating single photons to measure. In this work, we only generate single photons in one timebin, not superpositions. Firstly, we describe the collection of single photons, as this is an important part of the subsequent experiments. Then, we aim to find a frequency at which we can resonantly excite the C-transition of an SnV. To optimize efficient resonant excitation, we implement a 'Charge-Resonance Check' protocol. Then, we need to make a short optical pulse to excite the SnV. To maximize population transfer to the excited state, we calibrate the optical power of this pulse. Lastly, we need to make sure that the photons are collected in a well-controlled optical mode, and any remaining excitation light is filtered out.

5.1. Single Photon Collection and Filtering

The emission following the optical pulse is captured by the same optical fiber as was used to deliver the pulse. This fiber has been tapered in 40% hydrofluoric acid, to allow for in-contact coupling. In this configuration, previous work [68] has shown coupling efficiencies above 90% at visible wavelengths. However, in our work it was used in lensed configuration [69], which decreases collection efficiency.

The APDs that should register the single photons have a dead time of 45 ns. Thus, it is of great importance that the laser light from the excitation pulse is filtered out sufficiently, for otherwise no single photons would be detected. Also, high optical powers can damage the APD. We filter out the laser light by exploiting the emitter dipole polarization by rotating the excitation light away from that dipole orientation. The ZPL emission of the SnV will be fully aligned with this dipole (see Sec. 1.2.2). After being collected by the fiber, it couples into free space and a half-waveplate and quarter-waveplate are used to rotate the polarization of the light. The waveplates are tuned to rotate the laser light such that it is completely reflected by the subsequent polarizing beamsplitter. The single photons have a finite probability of being transmitted, due to the misalignment with their polarization. After transmission, a dichroic beamsplitter is used to separate the PSB and the ZPL. Subsequently, a band pass filter is used to filter emission from the D transition out, and a quarter- and half-waveplate are used to align the polarization of the optical mode with the slow axis of the optical fiber. This is routed to the TDI for single photon measurements.

Because the filtering of the laser light is not perfect, we additionally use time-filtering to prevent counting laser light as single photons. Thus, we define a window shortly after the leaked laser pulse, in which single photons will be registered by the APDs. Also, to minimize dark counts, this window is limited to a 29 ns (a few lifetimes). As an example, multiple single photon decays in the ZPL, as well as the windows of registering counts, are visible in Fig. 6.1.

5.2. Optical Transition Selection

Because our excitation light is coupled via fiber into the waveguide, many different emitters in the waveguide can be excited, depending on the frequency of the light. To select an emitter, a Photo-Luminescent Excitation (PLE) scan is performed. Firstly, we apply a repump pulse for 100 μ s (see Sec. 1.2.2). Then, we apply a red pulse for 300 μ s. This is repeated a thousand times. We repeated this sequence 1000 times across different frequencies of red light (2.5 GHz range, 0.02 GHz steps). If the laser light is on resonance with an emitter's C-transition, it will optically excite the emitter, which will decay to the ground state. As a result we will see a peak in counts from PSB emission, which is filtered following a method described in Sec. 5.1. A peak indicates a resonance with this frequency of the laser light, and they are visible in Fig. 5.1(a). We chose the excitation shaded in Fig. 5.1 for the experiments to come.

However, an optical pulse at the frequency of the 129.4 GHz peak will often not yield resonant excitation. We show this in Fig. 5.1(b). In this experiment, a repump pulse was only applied at the

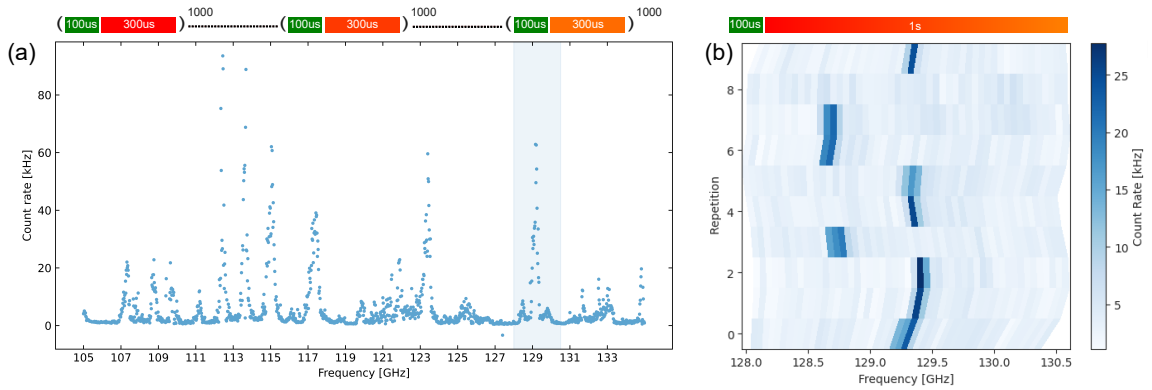


Figure 5.1: PhotoLuminescent Excitation scan. Emitters are excited, and PSB emission is collected. **(a)** Counts plotted for a single PLE. We apply a repump pulse and a thousand subsequent red pulses. The frequency is swept over a range of 30 GHz in steps of 0.02 GHz. **(b)** Zoom-in on the frequency spectrum shaded in (a). Multiple frequency sweeps are performed, and plotted as various repetitions on the vertical axis. Horizontal axis is again frequency, and counts are given by the blue color bar. Each repetition takes a second to complete. The axes of **(a)** and **(b)** are not an absolute frequency, but points have only relative meaning.

beginning of a repetition. Repetitions are plotted on the vertical axis. On the horizontal axis, we sweep the frequency of the laser over the range shaded in Fig. 5.1(a). The counts are plotted on a color scale, and one can see that between repetitions, the resonance frequency of the emitter shifts, in correspondence with the broad linewidth in Fig. 5.1(a). This can partly be due to the repump pulse (see Sec. 1.2.2). We conclude that we need some mechanism to ensure that the emitter is in the right charge state, and resonant with our laser frequency.

5.3. Charge-Resonance Check

To this end, we employ a Charge-Resonance check (CR-check). Firstly, we apply a repump pulse. Secondly, a resonant ‘probe’ light is applied, and PSB is collected to check if the emitter is on resonance. Let C be the average amount of counts collected in the PSB during an excitation. If C is smaller than some threshold C_{repump} , we conclude that we are not exciting the emitter, and likely the SnV is not in the $-e$ charge state. Thus, another repump pulse is applied. If C is greater than threshold C_{pass} we are sufficiently on resonance, and we continue with the experiment. If C is smaller than the hard threshold, but greater than some soft threshold, we continue the probe excitation, hoping to eventually surpass the hard threshold.

The utility of the CR-check is two-fold. Firstly, it increases the count-rate of the experiments that will be executed, as excitations will likely be resonant and thus effective. Secondly, by setting a stringent (high) pass threshold, one makes sure that the experiment only happens if the resonance frequency is very close to the intended frequency, thus limiting spectral diffusion [50].

In Fig. 5.2 we show the protocol (inset), and the results. The horizontal axis denotes the counts that we register in PSB during the probe pulse, relative to the thresholds used in the protocol. The vertical axis shows how often these values occurred. A steep decay at the low counts, is expected and would be due to noise counts. This peak would correspond to the emitter being in the dark state, and receiving a poissonian distribution of counts, due to a low and constant probability of receiving a count. Ideally, at higher counts, we would also see a peak, due to the emitter being in the right charge state and emitting photons, as shown in [45]. This is not well visible. Apparently we are not exciting the emitter very efficiently; the integral behind C_{pass} is small compared to the rest of the curve. A reason could be that the resonance frequency of the emitter has quite a large variance (see Fig. 5.1(b)), often yielding off-resonant driving.

5.4. Optical Pulse Generation

To prevent spontaneous decays from the excited state manifold to the ground state manifold during the excitation pulse, it is important that we drive a full π rotation in timescale shorter than the excited state lifetime [47]. This is realized by combining two AOMs (Gooch & Housego Fiber-Q 633 nm) with an amplitude electro-opto modulator (aEOM Jenoptik AM635) in series. We call this the ‘fast-pulse

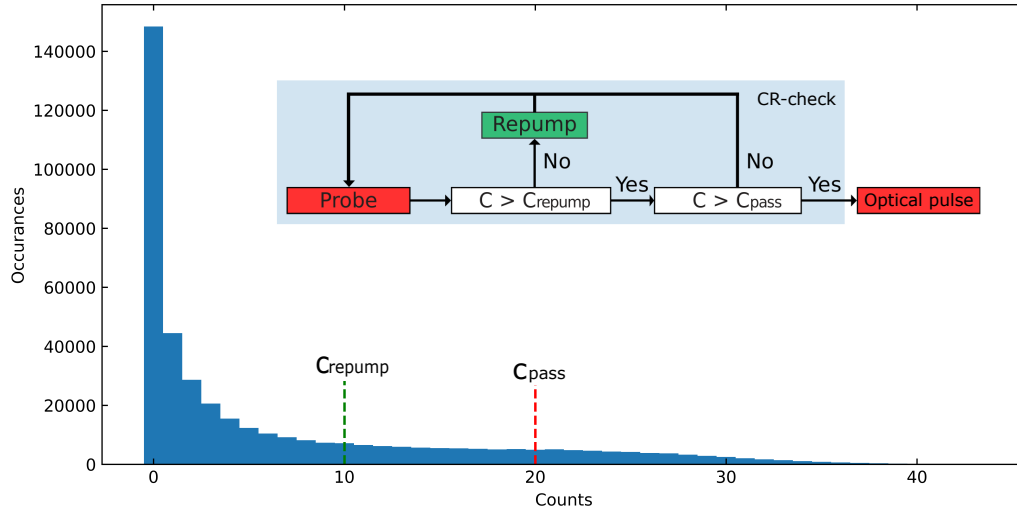


Figure 5.2: CR-check. The inset shows the protocol, as described in the main text. Inset taken from Brevoord [45]. The outer figure shows a histogram of the PSB counts collected during the probe pulse. The thresholds of the protocol are denoted by dashed lines in the figure.

path'. The two AOMs have a rise- and fall time of approximately 30 ns and are driven by an AOM driver (Zepto RF-Driver) at a frequency of 200 MHz. When these AOMs are 'off' (gated, and not driven), they have a combined suppression on the order of 100dB. The aEOM is a Mach-Zehnder interferometer that can be tuned to destructive interference with a suppression of about 30dB. The aEOM is driven out of and back into destructive interference by a custom 'pulse box' creating an electrical pulse of about 1 ns. This creates a window of transmission with a rise and fall time on the order of 200 ps. To create the optical excitation pulse, we open the AOMs for a window of 100 ns. We want to ensure low laser leakage in the window after the aEOM pulse, where we expect to see single photons. Therefore, we combine the opening of the aEOM with the falling shoulder of the peak created by the AOMs. As such we leverage the full suppression of the aEOM and the AOMs in the window of single photon collection. The shape of the pulse has already been shown (two-fold) in Fig. 4.4, where the fast-pulse path was directly connected to the TDI. In Fig. 4.4 the long window of count rate around 1 kHz is due to the AOMs opening, and the sharp peak up till count rate around 1 MHz is due to simultaneous opening of the EOM.

5.5. Tuning

To tune the optical pulse, we play a repump pulse, excite the emitter via the fast-pulse path and collect the resulting PSB fluorescence. Repeating the experiment a million times and histogramming the counts allows us to calculate the count rate of the emission as a function of time, which should show an exponential decay. The results are shown in Fig. 5.3(a). We also show fit of the exponential decay yielding an excited state lifetime of 6.2(3) ns, longer than the value reported for bulk diamond. [43]. The lifetime depends on several factors such as implantation depth, and the prolonged lifetime could be due to proximity to the interface with air [47]. Previous experiments using waveguides have shown similar lifetimes [70]

The rotation angle between the ground- and excited state poles on the Bloch Sphere, depends on the excitation efficiency, optical power, and pulse duration. The excitation efficiency depends on the frequency detuning, the orientation of the polarization of the optical field with respect to the electric dipole of the emitter, and the emitters position relative to the optical field [59]. Thus, after choosing an emitter and frequency of excitation, we must tune the optical power such that it matches a full π rotation during the selected time window. We repeated the previous experiment, but varied the AOM drive amplitude to produce excitation pulses with optical powers from 0.5 nW to 3.0 nW. The result is shown in Fig. 5.3(b). A full π rotation will correspond to the highest amount of photons collected in the

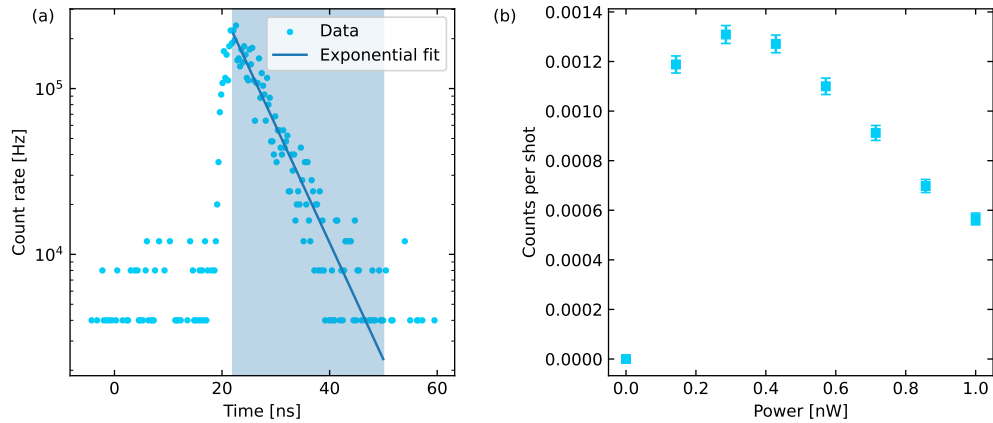


Figure 5.3: Collection of PSB fluorescence (described in Sec. 5.1), after excitation of the SnV via the fast-pulse path (see Sec. 5.4) (a) Excitation and subsequent collection was repeated 1×10^6 times. Counts histogrammed and the count rate was calculated as a function of time (scattered points). An exponential decay was fitted in the shaded window, from which the lifetime is extracted. (b) Experiment was repeated for different optical powers, which constitute the vertical axis. Average amount of counts collected in the shaded window in (a) are plotted, errorbars are calculated with shotnoise.

PSB due to spontaneous decay. At first glance, it seems that 2.5 nW is close to the optical driving power for the fixed EOM pulse duration. However, this experiment was not preceded by a CR-check. Since a CR-check improves resonance, during the experiment we might be driving with too much power. This has not been verified.

5.6. Summary

In this chapter, we have demonstrated the techniques that will be used for optical excitation of the SnV, and subsequent collection of the emission. We have shown the exponential decay of the excited state, and fitted the lifetime via emission. In a PLE-scan, we have shown the optical transition of the SnV that we target during the following experiments. We have shown that the resonance frequencies shows large variation between PLE scans. Additionally, we have introduced the CR-check as a tool to increase the probability of resonant excitation and limit spectral diffusion. The CR check shows that the excitation efficiency is low, which might be due to the strong spectral diffusion of the emitter.

- Chapter 6 -

Characterization of Single Photons

In this chapter, we use the TDI to analyze single photons produced using the methods outlined in the previous chapter. We quantify their single photon purity, and their indistinguishability. Note that we do not perform these experiments on photons in superpositions of time-bins, but only well-defined in a single time-bin. Therefore, phase stability of the interferometer is not needed in this chapter.

To this end, we supply two optical π pulses to the SnV using the fast-pulse path, spaced by time delay $T = 193$ ns. For a certain window after the pulse, photons are collected, filtered and the ZPL photons are incident on the interferometer. This experiment is repeated $N = 5 \times 10^5$ times, where we denote the repetition number by $1 \leq k \leq N$. The APDs in the output arms of the interferometer register the counts at three time bins spaced by 193 ns. The ‘early time bin’ is when the emission from the first π pulse arrives through the short arm of the TDI. The ‘late time bin’ is when the emission from the second optical excitation arrives through the long arm of the TDI. The middle time bin is when both the emission from the first optical excitation pulse arrives through the long arm, and emission from the second optical excitation arrives through the short arm. This bin will thus contain twice as many counts. The counts are timestamped and histogrammed, and the count rate is calculated as a function of time. The result is shown in Fig. 6.1.

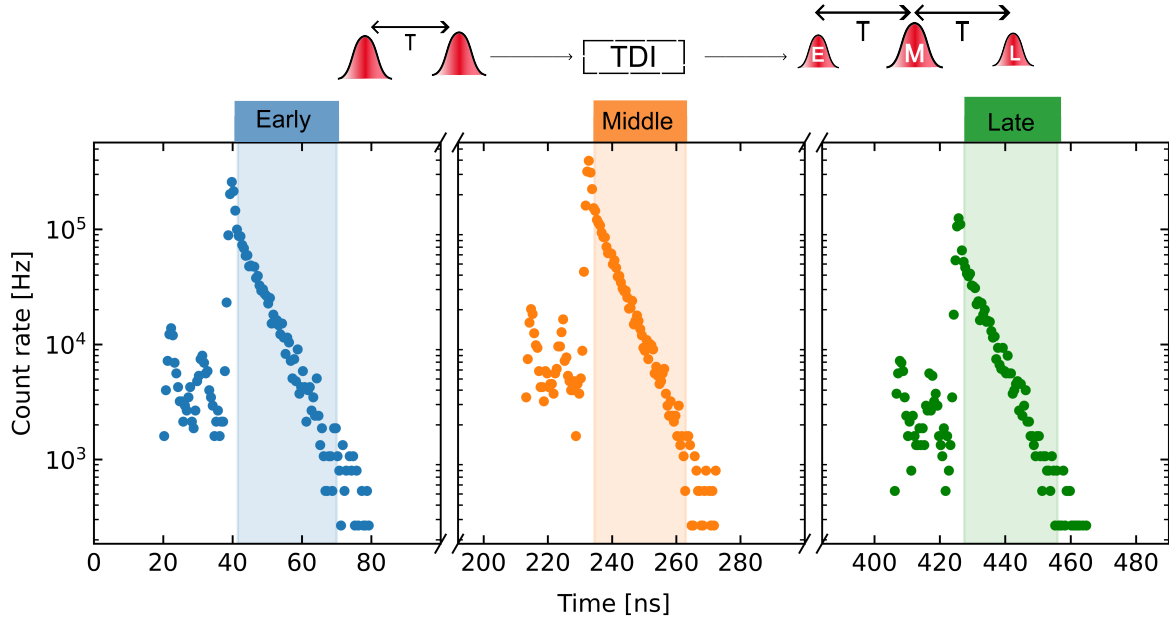


Figure 6.1: Counts (summed over the two APDs) of the interferometer, when two optical π pulses are played on the SnV. Optical π pulses are realized by two AOMs and an EOM, as depicted in the top of the figure. See Sec. 5.4 for more details. After these pulses, a window is defined (plotted in transparent filling) in which the counts are registered.

In each of the time-bins, we define a window which is denoted by a coloured fill in Fig. 6.1. We place these windows shortly after the excitation pulse, which is visible as a sharp peak. In these windows, we expect to find almost solely single photon emission from our targeted optical excitation.

6.1. Second-order Autocorrelation Measurement

To verify this, we perform a second-order autocorrelation measurement. In our case this measurement quantifies correlations in the intensity of an optical field at between different times spaced by τ . We will

denote these correlations by $g^{(2)}(\tau)$, and it is given by [59]

$$g^{(2)}(\tau) = \frac{\langle \hat{E}^{(-)}(t) \hat{E}^{(-)}(t + \tau) \hat{E}^{(+)}(t + \tau) \hat{E}^{(+)}(t) \rangle}{\langle \hat{E}^{(-)}(t) \hat{E}^{(+)}(t) \rangle \langle \hat{E}^{(-)}(t + \tau) \hat{E}^{(+)}(t + \tau) \rangle}. \quad (6.1)$$

We will now write $E^{(-)}(t)$ and $E^{(+)}(t)$ in terms of $\hat{a}^\dagger(t)$ and $\hat{a}(t)$. For $\tau = 0$, a special case occurs. A truly single photon source will create an optical mode $|\psi\rangle$ such that $\hat{a}(t)\hat{a}(t)|\psi\rangle = 0$, so $g^{(2)}(t) = 0$. But for $\tau \neq 0$, the operators $\hat{a}^{(\pm)}(t)$ commute, and we can write $g^{(2)}(\tau)$ in terms operator for number of photons $\hat{n}(t) = \hat{a}^\dagger(t)\hat{a}(t)$:

$$g^{(2)}(\tau) = \frac{\langle \hat{n}(t)\hat{n}(t + \tau) \rangle}{\langle \hat{n}(t) \rangle \langle \hat{n}(t + \tau) \rangle}, \quad (6.2)$$

which we can calculate as

$$g^{(2)}(\tau) = \frac{\int n(t)n(t + \tau)dt}{\int n(t)dt \int n(t + \tau)dt}, \quad (6.3)$$

where $n(t)$ is the number of photons registered at time t . Experimentally, measuring simultaneous counts is not straightforward due to dead time of the single photon detectors. This can be circumvented by using a beam splitter that the photons to two different detectors, to enable coincidences in two different output arms of the beamsplitter. The TDI already contains a beam splitter with two APDs at its output ports, which will be used to measure the optical field. We discretize the time coordinate by the repetition number k of the experiment. Let $n_D(k)$ denote the number of counts of detector $D \in \{A, B\}$ in the single photon detection window in repetition k of the experiment. Note that this quantity exists for all the three different time bins. Then we can calculate, for each of the time bins, $g^{(2)}$ as [46]

$$g^{(2)}(m) = \frac{N \sum_k n_A(k)n_B(m + k)}{\sum_k n_A(k) \sum_k^{N-m} n_B(m + k)}, \quad (6.4)$$

where N is the amount of values we sum over. In the Early (Late) time bin, photons only arrive from the first (second) excitation, and only if they travelled through the short (long) arm. If our light source is purely a single photon emitter, only APD A or APD B will click in those windows, and $g^{(2)}(0) = 0$. We assume no correlations between the different intervals of the experiment. Then in theory, $g^{(2)}(m) = 1$ will be constant for all $m \neq 0$ [58]. Thus, we calculate $g^{(2)}(m)$ by calculating $\sum_k \langle \hat{n}_A(k)\hat{n}_B(m + k) \rangle$ for $m \in \{-10, 9, \dots, 9, 10\}$ and normalizing this expression by the average for $m \neq 0$.

We have done this analysis for counts in the early and in the late time-bin, and taken the average over the two time-bins. Fig. 6.2 shows the result. A clear dip can be seen at $m = 0$, with a value of $g^{(2)}(0) = 0.07(7)$. This error has been calculated assuming shot noise.

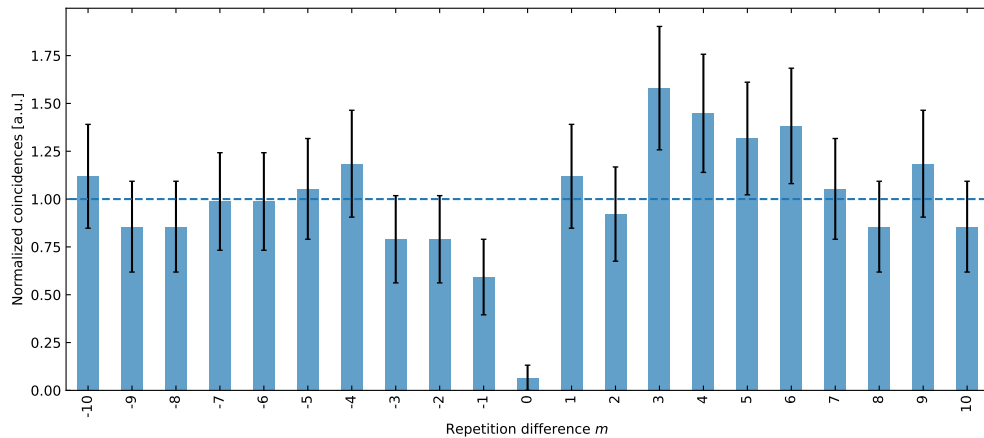


Figure 6.2: $g^{(2)}(m)$ measurement of the optical field registered by the APDs during the early time bin, and during the late time-bin, averaged. Histogram shows the calculated values for $g^{(2)}(m)$. Normalized to the average values for non-zero m , shown by the dashed line. Errorbars are calculated assuming shot noise.

This anti-bunching is a clear sign of a single photon emitter. If there were n single photon emitters present, it was derived that [71]

$$g^{(2)}(0) = \frac{\sum_{i=1}^n \sum_{j \neq i}^n I_i I_j}{(\sum_{i=1}^n I_i)^2}, \quad (6.5)$$

where I_i is the intensity of a single emitter. From our low value of $g^{(2)}(0)$ we can conclude that there is no other emitter present at a similar signal strength.

In this measurement, only once was a simultaneous click registered in the two APDs. It happened in the early time bin, which also experiences the highest count rate. This coincidence could be due to a weakly excited emitter, as argued above. Another factor explaining the offset from zero might be laser leakage, due to insufficient polarization filtering, and fiber fluorescence. Other sources have stated that laser leakage was the main source of offset in the value of $g^{(2)}(0)$ [46]. Finally, it is not likely that the coincidence is due to a dark count in a detector. The average count rate during the time bins is a factor 2.2×10^2 higher than the average count rate when no excitation pulses are being played. This would make a coincidence between a photon and a dark count approximately 2.2×10^2 times less frequent than a coincidence between a coincidence two photons in different repetitions of the experiment, which does not match the found value of $g^{(2)}(0)$.

6.2. Hong-Ou-Mandel Interference

To achieve high measurement fidelities with the TDI, we do not only require a high purity of our single photon source. It is also important that two interfering wavepackets arriving from the short arm and the long arm have high overlap. The relation of imperfect overlap and the measurement fidelities is derived in Appendix B.

We can characterize the overlap with a Hong-Ou-Mandel (HOM) interference experiment [72]. In the simplest experimental setup to show HOM interference, two photons are incident on ports 1 and 2 of a beamsplitter. Let us denote them by $|\psi\rangle_1$ and $|\phi\rangle_2$. Assume a lossless beamsplitter, with transmission T and reflection R . Then, using the beamsplitter transformations [59], we can derive that the probability of a click both in output port A and in output port B is given by $\mathbb{P}$ as

$$\mathbb{P} = T_2^2 + R_2^2 - 2T_2R_2|\langle\phi|\psi\rangle|^2 = 1 - 2T_2R_2 - 2T_2R_2|\langle\phi|\psi\rangle|^2 \quad (6.6)$$

The overlap $\eta = |\langle\phi|\psi\rangle|^2$ is called the indistinguishability of the two photons. For a balanced beamsplitter and for perfect indistinguishability, we see that the probability of a simultaneous click is zero.

Now, we extend the simplest case of HOM interference to the case of the TDI, in the experiment described above and shown in Fig. 6.1. Consider coincidences in the middle time bin, during the same repetition of the experiment. A photon from the first (second) excitation can arrive if it travelled through the long (short) path. Thus, two photons can arrive at the second beamsplitter simultaneously, giving rise to two-photon interference. This has equal probability of occurring as one photon arriving in the early time bin and one photon arriving in the late time bin, if we consider the two excitations to be identical. Because for both scenarios, one photon has to travel the long path, and another has to travel the short path.

The important distinction is what happens at the second beamsplitter. In the case of an early and late click, the photons were completely distinguishable (called the ‘distinguishable case’). This is opposed to the case of a coincidence in the middle time bin. There, in the perfect scenario, the two photons arriving should be completely indistinguishable (thus called the ‘indistinguishable case’) Let us define the raw visibility as

$$V_{\text{raw}} = 1 - \frac{\mathbb{P}(\text{coincidence, indistinguishable case})}{\mathbb{P}(\text{coincidence, distinguishable case})} \quad (6.7)$$

and define the visibility V as

$$\mathbb{P}(\text{coincidence, indistinguishable case}) = 1 - 2T_2R_2 - 2T_2R_2V, \quad (6.8)$$

such that in an ideal case, V should equal η . Then, simple algebra allows us to write

$$V = \frac{V_{\text{raw}}(1 - 2R_2T_2)}{2R_2T_2} \quad (6.9)$$

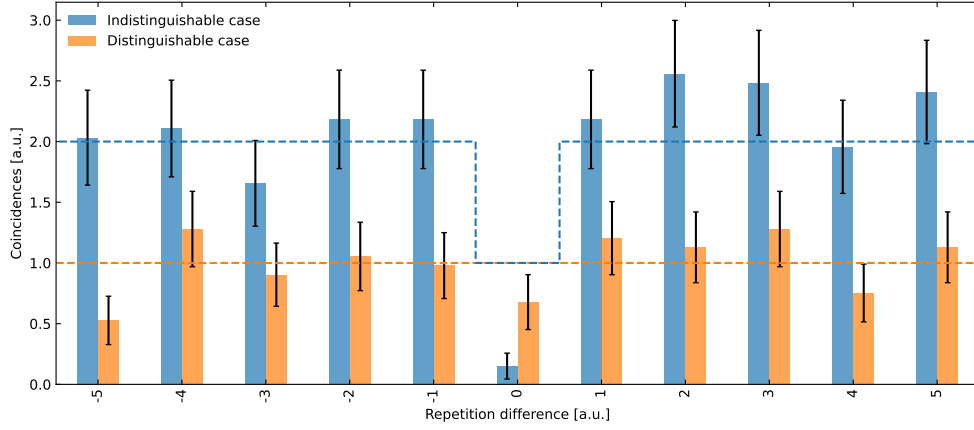


Figure 6.3: Hong-Ou-Mandel experiment. Vertical axis shows coincidences between clicks in APD A and APD B. Shown both for the indistinguishable case (coincidences in the middle time bin) and the distinguishable case (coincidences in the early and late time bins). Error bars assume shot noise. Horizontal axis shows the difference in repetition number of the sequence. Dashed lines show expected number of counts if not interference took place.

Previously, we have discussed coincidences during the same repetition of the experiment. To show the HOM dip, we will now also correlate the coincidences across various repetitions of the experiment. We will denote a coincidence between two bins with repetition difference m as the occurrence of a click in the first bin in repetition k , and a click in the second bin in repetition $k + m$. We have plotted it for both the indistinguishable case and for the distinguishable case. The non-zero repetition differences show approximately two times more counts for the indistinguishable case than for the distinguishable case, due to combinatorics. For non-zero repetition differences m , there can be a coincidence between two emissions from the first excitation, and also from the second excitation.

This is not so for the $m = 0$ and for the distinguishable case. In these cases only a combination of emission from the first excitation pulse and emission from the second excitation pulse can result in a coincidence. The expected number of coincidences, assuming no interference, has been plotted by dashed lines in Fig. 6.3.

We calculate the raw visibility by normalizing the coincidences for the indistinguishable case in the zero-repetition bin, by the mean (over the shown repetition differences) of coincidences for the indistinguishable case $V_{\text{raw}} = 0.85(14)$. After applying a correction for the second beamsplitter via Eq (6.9), we find $V = 0.87(14)$. This result is calculated using propagated uncertainty of the shot noise and known errors in the second beamsplitter. Note that values above 1 are unphysical. Note that this value of η has not yet been corrected for the finite value of $g^{(2)}(0)$. However, the fact that the value of $g^{(2)}(0)$ is only formed by 1 count means that the shot noise is equal to the value itself. Therefore, we have omitted a correction

This indistinguishability is the overlap between the wavepackets at the second beamsplitter, and thus contains effects of pure dephasing, spectral diffusion, and mismatch in temporal envelope. Our found value for the indistinguishability exceeds the value found by Arjona et al. [47], which has also measured two-photon interference from a single SnV with a TDI. Arjona et al. also show that shorter collection windows increase the indistinguishability of the registered photons, by mitigating spectral diffusion effects. Our found value for the corrected visibility exceeds the value that Arjona et al. have reported, even the shortest collection window of 11 ns, which is short compared to our window of 29 ns. Note however, that our excitation pulse is significantly shorter. Other works using a similar excitation pulse [46], have shown visibilities more in agreement with ours, even when interfering photons from distinct SnVs.

6.3. Single Shot Readout Fidelity prediction

In Appendix B, we give a derivation for the single shot readout fidelity of a single photon in an equatorial basis on the Bloch sphere (using the definition of fidelity used in Chap. 2). In this, we assume that

the overlap between two wavepackets of a photon generated in superposition does not differ from two wavepackets of distinct single photons (correcting for normalization of course). This derivation results in Eq. (B.40), takes into account the indistinguishability of two wavepackets at the second beamsplitter, which we conveniently quantified in the previous section. Other parameters are the imbalances in the TDI extracted in Sec. 4.2. We insert the found imbalances in Tab. 4.1 and the value for the indistinguishability $\eta = 0.87(14)$. Using Eq. (B.40), we predict a single shot readout $F = 0.72(4)$.

6.4. Summary

In this chapter, we connected our previous work on the TDI with the work on single photon generation. To confirm single photon emission we calculated a second-order autocorrelation of $g^{(2)}(0) = 0.07(7)$ (upper bound at 1), demonstrating strong antibunching. The photon indistinguishability was characterized using Hong-Ou-Mandel interference, resulting in a corrected indistinguishability of $\eta = 0.87(14)$ for two wavepackets arriving at the second beamsplitter. Furthermore, we have estimated single shot readout fidelity in timebin equatorial basis on the Bloch sphere to be $0.72(4)$.

- Chapter 7 -

Conclusion and Outlook

7.1. Conclusion

We have designed and constructed a Time Delay Interferometer of Mach-Zehnder type for measurements in time-bin superpositions on single photons emitted by an SnV. By controlling the phase of the TDI, we can select the azimuthal angle of the measurement basis on the equator of the Bloch sphere. A modulation-demodulation scheme enables phase stabilization at an arbitrary setpoint relative to the probe-signal frequency. After a locking stage of 30 ms, we found that the phase after locking followed a distribution centered around the setpoint with a Root Mean Square value of 0.046π .

We have identified that frequency noise of the probe signal is the dominant source of observed phase fluctuations up to time scales of 200 ms. When releasing lock, any phase drifts due to the changes in optical path length of the interferometer were overshadowed by the fluctuations due to laser noise. Because the passive stability influences the duty cycle of the interferometer, considerable efforts have been made to increase the passive stability of the interferometer. Two measurements were compared: with and without temperature stabilization. Over long timescales, the phase distribution of a non-stabilized configuration showed a collective drift toward negative phase values, consistent with a slowly decreasing optical path length. Remarkable is that this behavior has persisted over the entire experiment which took 1 hour. Temperature fluctuations in the laboratory on this timescale have been observed in the past, and form the current hypothesis for this behavior. This is supported by the fact that the temperature stabilized measurement does not show this. Regardless, we can conclude that temperature stabilization improves long-term passive stability on long timescales (> 3 s). During the time between 200 ms and 8 s, the RMS for the temperature controlled interferometer increased with only 0.05π . Contrary, on shorter timescales, temperature stability seemed to decrease phase stability. However, measurements with decreased laser noise are needed to conclusively prove this.

Next, the interferometer has been used for measurements of single photons. As a first step we have shown that we can excite a single SnV, and detect the emission. We have shown that it is a very pure single photon source, quantified by a second order autocorrelation of $g^{(2)}(\tau = 0) = 0.07(7)$. Subsequently, we have demonstrated Hong-Ou-Mandel interference between two photons in the two distinct time-bins separated by the time delay of the interferometer. The raw visibility of the Hong-Ou-Mandel interference was measured to be $0.85(14)$. This was higher than expected based on a previous work. After correction for imbalances in the setup by normalization, we found an indistinguishability of $0.87(14)$ (upper bound at 1) between the two photon wavepackets.

Using the value for the indistinguishability of the two wavepackets, and the characterization of imbalances in the interferometer will allow us to correct for them when doing quantum state tomography, as shown in Appendix B. However, fidelity of a single shot measurement is impacted by imbalances. Currently, we estimate that a measurement of an equatorial states can be done with fidelity of $0.72(4)$, if these imbalances are not compensated. This can become important when using the interferometer for heralding of quantum states. We have also discussed strategies to mitigate this.

7.2. Outlook

The TDI is operational, although the phase stability on time scales below 200 ms needs to be verified. This requires higher frequency stability of the probe signal. Currently, the frequency of the probe signal is controlled by a feedback loop driven by a computer. Possible improvements would be to apply feedback with an FPGA to react faster to noise. Other work on the TDI is not deemed urgent, though some suggestions are given in Appendix C.

Another vital next step is to verify that we can strongly suppress spectral diffusion utilizing Charge-Resonance checks. Successful locking requires that frequency detuning between the probe and the single photon emission is small compared to the reciprocal of the delay time. We have seen that we

need spectral diffusion far less than the lifetime limited linewidth of our SnV, requiring stringent Charge-Resonance checks. This can also decrease the rate at which single photons are generated, possibly resulting in low experimental rates, partly depending on the stability of the emitter.

Another necessary step is to demonstrate microwave control over the spin qubit of the SnV, as this is essential to executing the described entanglement protocol. Then, the interferometer can be used for its first purpose of demonstrating spin-photon entanglement between the SnV spin and a time-bin superposition photon. If Rabi driving takes too long, such that the excited state lifetime and Rabi driving exceed 193 ns, a longer delay line will have to be put in, requiring even higher frequency stabilities.

We conclude that challenges remain to show entanglement between the SnV and a time-bin photonic qubit. Nevertheless, this is a fundamental stepping stone in utilizing the SnV as a node in a quantum network.

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- Appendix A -

Complete Optical setup

In Fig. A.1 the full optical setup of the Time Delay Interferometer is shown. The specific models of the components are given in Chap. 3.

At the bottom left, denoted by 'Laser input', the continuous wave laser light enters the setup and passes through three AOMs, as shown in the bottom left arm in Fig. 3.3. The AOMs are driven by two drivers. The last two AOMs are powered by the same driver via a power splitter; the first AOM uses a separate driver, which is attenuated to prevent over-driving. In Fig. 3.3 only one driver and no power splitter are drawn to reduce complexity. The laser light enters the phase-stabilized box via the 'Laser arm'. Single photons from the SnV enter via the 'Single photon arm'. The phase-stabilized box is shown in more detail in Fig. 3.4.

After the phase-stabilized box, the single photons exit into arm A and arm B of the interferometer, which are both connected to a beamsplitter with a 90/10 splitting ratio. The 90 port is connected to APDs, and the 10 port is connected to the differential photodiode (see Fig. 3.3). At the bottom right one can see the connection between the PID controller and the NTC and TEC elements for temperature control, as shown in Chap. 3.

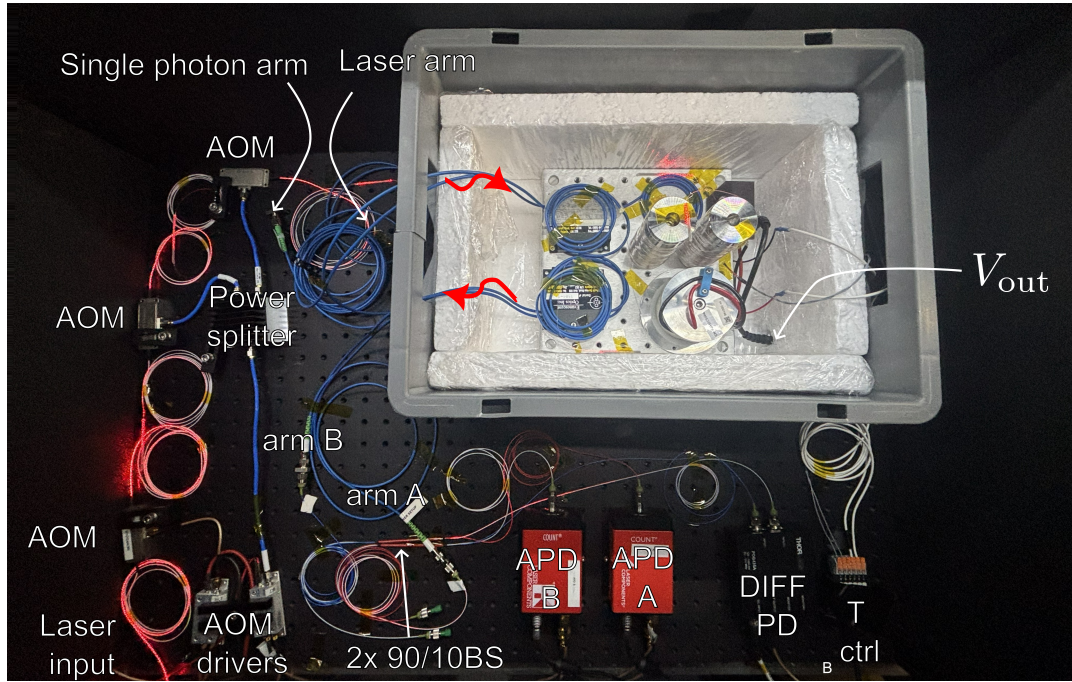


Figure A.1: The full optical setup of the TDI. Unconventional abbreviations denote the following. DIFF PD: Differential Photodiode. T ctrl: Temperature control. 90/10BS: beamsplitter with splitting ratio 90/10. V_{out} : voltage supplied to the fiber stretcher, see Sec. 3.1.

- Appendix B -

Theoretical model of an imperfect TDI

In this chapter we show how one can correct for any imperfections in the TDI due to optical losses and imbalances.

B.1. The Model

We assume a pure input state, $|\psi_{\text{in}}\rangle = \alpha|E\rangle + \beta|L\rangle$, where the overlap between the two wavepackets is not assumed to be perfect. We describe the TDI as a concatenation of a lossless beamsplitter, two arms with a certain loss and effective phase difference, another lossless beamsplitter, and two lossy collection arms. We work out the case where we register a click in the middle time bin. We can extract information from which APD clicked. Thus, we aim to construct operators Π_A and Π_B in the basis $|E\rangle, |L\rangle$ whose expectation values are the probability of a middle timebin click in APD A and APD B. In our model, we will use the notation shown in Fig. B.1.

We use the transformation of a general lossless beamsplitter [59]:

$$\text{BS}_i = \begin{pmatrix} \sqrt{T_i} & \sqrt{R_i}e^{-i\phi_i} \\ -\sqrt{R_i}e^{i\phi_i} & \sqrt{T_i} \end{pmatrix}, \quad (\text{B.1})$$

from the creation operators of a photonic wavepacket in input ports 1 and 2 to the creation operators at the two output ports 3 and 4 $a_1^\dagger, a_2^\dagger \rightarrow a_3^\dagger, a_4^\dagger$.

However, we are interested in the transformation of a state in early / late timebin basis. Thus, we want to describe BS1 as a transformation from creation operators of wavepackets in the early and late timebin to the creation operators in the long and short arm. We assume a click in the middle timebin, thus it is heralded that the early wavepacket took the long path and the late photon took the short path. We can thus drop time-dependence in the output basis and write the transformation of BS1 as $a_E^\dagger, a_L^\dagger \rightarrow a_l^\dagger, a_s^\dagger$ (E is for early, L is for late, l is for long, s is for short):

$$\text{BS}_1 = \begin{pmatrix} \sqrt{T_1} & 0 \\ 0 & e^{-i\phi_1}\sqrt{R_1} \end{pmatrix} \quad (\text{B.2})$$

To model loss, one ordinarily defines a beamsplitter with two outputs, among which a loss channel, and traces out the state at this output. However, for a pure state, we can derive that this is equivalent to the following transformation in the basis of creation operators of wavepackets in the long and short arm

$$\begin{pmatrix} e^{i\tilde{\phi}}\sqrt{\mathcal{T}_l} & 0 \\ 0 & \sqrt{\mathcal{T}_s} \end{pmatrix}, \quad (\text{B.3})$$

where $\mathcal{T}_l$ and $\mathcal{T}_s$ are the optical power transmissions of the short and the long arm. We gave the long arm an effective phase difference $\tilde{\phi}$ with respect to the short arm.

Finally, we can write the transformation of our input state in basis $|E\rangle, |L\rangle$ to the basis of $|m_A\rangle, |m_B\rangle$ as the transformation

$$\begin{pmatrix} \sqrt{T_2} & \sqrt{R_2}e^{-i\phi_2} \\ -\sqrt{R_2}e^{i\phi_2} & \sqrt{T_2} \end{pmatrix} \times \begin{pmatrix} e^{i\tilde{\phi}}\sqrt{\mathcal{T}_l} & 0 \\ 0 & \sqrt{\mathcal{T}_s} \end{pmatrix} \times \begin{pmatrix} \sqrt{T_1} & 0 \\ 0 & e^{-i\phi_1}\sqrt{R_1} \end{pmatrix} = \\ \begin{pmatrix} \sqrt{T_1T_2\mathcal{T}_l}e^{i\tilde{\phi}} & \sqrt{R_1R_2\mathcal{T}_s}e^{-i\phi_1-i\phi_2} \\ \sqrt{T_1R_2\mathcal{T}_l}e^{i\phi_2+i\tilde{\phi}+i\pi} & \sqrt{R_1T_2\mathcal{T}_s}e^{-i\phi_1} \end{pmatrix} \quad (\text{B.4})$$

Neglecting phase differences between the upper and the lower row (which do not matter for probabilities), and correcting for limited detection efficiencies $\mathcal{T}_A, \mathcal{T}_B$ in arm A and B, we can write this as

$$\text{TDI}_{|E\rangle, |L\rangle \rightarrow |m_A\rangle, |m_B\rangle} = \begin{pmatrix} \sqrt{\mathcal{T}_A T_1 T_2 \mathcal{T}_l} e^{i\tilde{\phi}} & \sqrt{\mathcal{T}_A R_1 R_2 \mathcal{T}_s} \\ \sqrt{\mathcal{T}_B T_1 R_2 \mathcal{T}_l} e^{i\phi_2+i\tilde{\phi}+i\pi} & \sqrt{\mathcal{T}_B R_1 T_2 \mathcal{T}_s} \end{pmatrix}, \quad (\text{B.5})$$

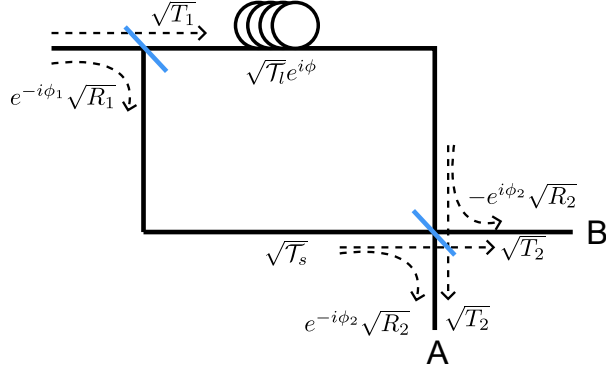


Figure B.1: A schematic of the transformations applied by the TDI. Blue slanted bars are beamsplitters; upper left is BS1 and lower right is BS2. Upper arm is the long arm, denoted by the delay line. Two output ports are denoted by A and B.

where ϕ is a linear combination of $\tilde{\phi}, \phi_1, \phi_2$.

B.2. Compensating for Photon Quantum State Tomography

Now we define operators $\hat{\Pi}_A, \hat{\Pi}_B$ whose expectation value for a pure state $|\psi\rangle$ gives the probability of a click in arm A and arm B. We can do this as

$$\hat{\Pi}_A = \mathcal{T}_A \begin{pmatrix} T_1 T_2 \mathcal{T}_I & \sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_I \mathcal{T}_s} e^{-i\phi} \\ \sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_I \mathcal{T}_s} e^{i\phi} & R_1 R_2 \mathcal{T}_s \end{pmatrix} \quad (\text{B.6})$$

$$\hat{\Pi}_B = \mathcal{T}_B \begin{pmatrix} T_1 R_2 \mathcal{T}_I & -\sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_I \mathcal{T}_s} e^{-i\phi} \\ -\sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_I \mathcal{T}_s} e^{i\phi} & R_1 T_2 \mathcal{T}_s \end{pmatrix}. \quad (\text{B.7})$$

Subtracting these two operators, we can define the operator for the expected amount of clicks in A minus the expected amount of clicks in B as

$$\hat{\Pi}_A - \hat{\Pi}_B = \begin{pmatrix} T_1 \mathcal{T}_I (T_2 \mathcal{T}_A - R_2 \mathcal{T}_B) & (\mathcal{T}_A + \mathcal{T}_B) \sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_I \mathcal{T}_s} e^{-i\phi} \\ (\mathcal{T}_A + \mathcal{T}_B) \sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_I \mathcal{T}_s} e^{i\phi} & R_1 \mathcal{T}_s (R_2 \mathcal{T}_A - T_2 \mathcal{T}_B) \end{pmatrix} \quad (\text{B.8})$$

We can easily verify that if the wavepacket arriving at beamsplitter 2 through the short and the long arm have non-unitary overlap $\sqrt{\eta} = \langle \psi_s | \psi_l \rangle$ (any phase effects are taken into account in ϕ), this yields an additional factor $\sqrt{\eta}$ on the diagonals. Then we can write $\hat{\Pi}_A - \hat{\Pi}_B$ as

$$\hat{\Pi}_A - \hat{\Pi}_B = \gamma \cos \phi \hat{X} + \gamma \sin \phi \hat{Y} + c_I \hat{I} + c_Z \hat{Z}, \quad (\text{B.9})$$

where we define

$$\gamma = (\mathcal{T}_A + \mathcal{T}_B) \sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_s \mathcal{T}_I} \sqrt{\eta}, \quad (\text{B.10})$$

$$c_I = \frac{T_1 \mathcal{T}_I (T_2 \mathcal{T}_A - R_2 \mathcal{T}_B) + R_1 \mathcal{T}_s (R_2 \mathcal{T}_A - T_2 \mathcal{T}_B)}{2}, \quad (\text{B.11})$$

$$c_Z = \frac{T_1 \mathcal{T}_I (T_2 \mathcal{T}_A - R_2 \mathcal{T}_B) - R_1 \mathcal{T}_s (R_2 \mathcal{T}_A - T_2 \mathcal{T}_B)}{2}. \quad (\text{B.12})$$

Of course, in case of no losses and perfectly balanced beamsplitters, the diagonals vanish, and we are left with $\cos \phi \hat{X} + \sin \phi \hat{Y}$.

We can calculate $\langle X \rangle$ and $\langle Y \rangle$ of a photonic quantum state by sweeping ϕ . This will yield an oscillation of the measured quantity $\langle \hat{\Pi}_A - \hat{\Pi}_B \rangle$. This oscillation will be offset by a factor $M = c_I \langle \hat{I} \rangle + c_Z \langle \hat{Z} \rangle$. Thus, we can fit this oscillation, and find the expectational values for X and Y as

$$\langle \hat{X} \rangle = \frac{\langle \hat{\Pi}_A - \hat{\Pi}_B \rangle_{\phi=0} - M}{\gamma} \quad \langle \hat{Y} \rangle = \frac{\langle \hat{\Pi}_A - \hat{\Pi}_B \rangle_{\phi=\pi/2} - M}{\gamma} \quad (\text{B.13})$$

However, calculating this from experimental data is involved; the reason is that the absolute values of some of the coefficients are not known due to losses. However, we will show now that we can calculate

it from the amount of clicks from both detectors in various time bins, and the ratios extracted in Chap. 4. Let us write the ratio of detection efficiencies $r = \mathcal{T}_A/\mathcal{T}_B$, and $q = R_1\mathcal{T}_s/(T_1\mathcal{T}_I)$, and remember that we also know the transmission T_2 and reflection R_2 of the second beamsplitter. Using $\mathcal{T}_A + \mathcal{T}_B = \mathcal{T}_B(1+r)$; $R_1\mathcal{T}_s\mathcal{T}_B = T_1\mathcal{T}_I\mathcal{T}_Bq$ and $R_1\mathcal{T}_s\mathcal{T}_A = T_1\mathcal{T}_I\mathcal{T}_B r q$, we can write Eq. (B.10), (B.11), (B.12) as

$$\gamma = T_1\mathcal{T}_I\mathcal{T}_B(1+r)\sqrt{qT_2R_2} \quad \eta \equiv T_1\mathcal{T}_I\mathcal{T}_B\gamma', \quad (\text{B.14})$$

$$c_I = \frac{T_1\mathcal{T}_I\mathcal{T}_B}{2} [(T_2r - R_2) + q(R_2r - T_2)] \equiv T_1\mathcal{T}_I\mathcal{T}_B \equiv T_1\mathcal{T}_I\mathcal{T}_B c'_I, \quad (\text{B.15})$$

$$c_Z = \frac{T_1\mathcal{T}_I\mathcal{T}_B}{2} [(T_2r - R_2) - q(R_2r - T_2)] \equiv T_1\mathcal{T}_I\mathcal{T}_B c'_Z, \quad (\text{B.16})$$

where still the coefficients, are not known, from Chap. 4, but at least up to a global prefactor $T_1\mathcal{T}_I\mathcal{T}_B$.

However, we can calibrate this out by proper normalization of measurement quantities. An experiment does not give access to the probability $\langle \hat{\Pi}_A - \hat{\Pi}_B \rangle$ directly, but to the number of clicks registered in each output arm. Let N be the number of photons arriving at the TDI over the course of a measurement. Then the measurable quantity closely related to Eq. B.9 is

$$D(\phi) = N \langle \hat{\Pi}_A - \hat{\Pi}_B \rangle = N \left(\gamma \cos \phi \langle \hat{X} \rangle + \gamma \sin \phi \langle \hat{Y} \rangle + c_I + c_Z \langle \hat{Z} \rangle \right) \quad (\text{B.17})$$

$$= NT_1\mathcal{T}_I\mathcal{T}_B \left(\gamma' \cos \phi \langle \hat{X} \rangle + \gamma' \sin \phi \langle \hat{Y} \rangle + c'_I + c'_Z \langle \hat{Z} \rangle \right), \quad (\text{B.18})$$

where we used $\langle \hat{I} \rangle = 1$ for a normalized input state. Sweeping ϕ produces a sinusoidal oscillation, with an offset that allows us to extract the unknown $NT_1\mathcal{T}_I\mathcal{T}_B$ as

$$O = NT_1\mathcal{T}_I\mathcal{T}_B (c'_I + c'_Z \langle \hat{Z} \rangle) \Rightarrow NT_1\mathcal{T}_I\mathcal{T}_B = \frac{O}{(c'_I + c'_Z \langle \hat{Z} \rangle)} \quad (\text{B.19})$$

Now suppose that we can extract $\langle \hat{Z} \rangle$, we will later show that this is the case. Then we can write

$$\begin{aligned} \cos \phi \langle \hat{X} \rangle + \sin \phi \langle \hat{Y} \rangle &= \frac{D(\phi) - O}{\gamma' NT_1\mathcal{T}_I\mathcal{T}_B} \\ &= \frac{(D(\phi) - O)(c'_I + c'_Z \langle \hat{Z} \rangle)}{\gamma' O} \end{aligned} \quad (\text{B.20})$$

In this formula, all symbols on the right-hand side are known except for $\langle \hat{Z} \rangle$; for $D(\phi)$ and O have been measured and fitted, and all symbols with an accent can be calculated with the data extracted in Chap. 4.

It remains to obtain the population term $\langle \hat{Z} \rangle$, which we read off the two outer time bins. A photon detected in the early bin has travelled through the short arm and one detected in the late bin has travelled through the long arm. Thus, in the early and late bin, we expect n_E and n_L counts as

$$n_E = N|\alpha|^2 R_1\mathcal{T}_s (R_2\mathcal{T}_A + T_2\mathcal{T}_B) = |\alpha|^2 NT_1\mathcal{T}_I\mathcal{T}_B q (R_2r + T_2) \quad (\text{B.21})$$

$$n_L = N|\beta|^2 T_1\mathcal{T}_I (T_2\mathcal{T}_A + R_2\mathcal{T}_B) = NT_1\mathcal{T}_I\mathcal{T}_B (T_2r + R_2). \quad (\text{B.22})$$

Now, we can easily verify that

$$\langle \hat{Z} \rangle = \frac{\frac{n_E}{q(R_2r+T_2)} - \frac{n_L}{q(T_2r+R_2)}}{\frac{n_E}{q(R_2r+T_2)} + \frac{n_L}{q(T_2r+R_2)}}, \quad (\text{B.23})$$

finishing our description of quantum state tomography with an imperfect TDI. Of course, $\langle \hat{Z} \rangle$ can also be determined without the TDI, which makes the analysis easier by removing imperfections. However, it is inevitable that approximately half of the clicks will happen in the early and late timebin, such that this way does not require taking extra data.

With $\langle \hat{Z} \rangle$ determined in this way, Eq. (B.20) fixes $\langle \hat{X} \rangle$ and $\langle \hat{Y} \rangle$.

B.3. Compensating for Spin-Photon Tomography

When compensating for spin-photon tomography, a slight adjustment needs to be made. We now aim to measure the operator $\hat{S} \otimes (\hat{\Pi}_A - \Pi_B)$, with $S \in \{\hat{X}, \hat{Y}\}$. Then we can define $G(\phi)$ similar to D before as

$$G(\phi) = N \langle \hat{\Pi}_A - \hat{\Pi}_B \rangle = NT_1 \mathcal{T}_I \mathcal{T}_B \left(\gamma' \cos \phi \langle \hat{S} \hat{X} \rangle + \gamma' \sin \phi \langle \hat{S} \hat{Y} \rangle + c'_I + c'_Z \langle \hat{S} \hat{Z} \rangle \right). \quad (\text{B.24})$$

$$= NT_1 \mathcal{T}_I \mathcal{T}_B \left(\gamma' \cos \phi \langle \hat{S} \hat{X} \rangle + \gamma' \sin \phi \langle \hat{S} \hat{Y} \rangle + c'_I + c'_Z \langle \hat{S} \hat{Z} \rangle \right), \quad (\text{B.25})$$

We again measure the offset $\mathcal{U}$ when sweeping ϕ . From our previous discussion we know that $NT_1 \mathcal{T}_I \mathcal{T}_B = \mathcal{O}/(c'_I + c'_Z \langle \hat{Z} \rangle)$, where $\mathcal{O}$ is still defined as the offset in the clicks in arm A minus the clicks in arm B when sweeping the phase. Thus, we can now calculate

$$\langle \hat{S} \hat{X} \rangle \cos \phi + \langle \hat{S} \hat{Y} \rangle \sin \phi = \frac{(G(\phi) - \mathcal{U})(c'_I + c'_Z \langle \hat{Z} \rangle)}{\gamma' \mathcal{O}}, \quad (\text{B.26})$$

allowing us to extract the correlators.

B.4. Single-shot Readout Fidelity

Now let us concern ourselves with the single shot readout fidelity. The Z-basis readout fidelity is not affected by imbalances, only the rate. However, the readout in bases on the equatorial plane is more involved. Because of imperfect interference, now for both detectors the state $|\pm_\phi\rangle = (|E\rangle + e^{i\phi}|L\rangle)/\sqrt{2}$ has a finite probability for both detectors of yielding a click. For a perfect interferometer, the $|+\phi\rangle$ should only give a click in arm A and the $|-\phi\rangle$ only in arm B. Following Chap. 2 will now define the fidelity for a measurement in the basis $\{|\pm_\phi\rangle\}$ as

$$F = \frac{\mathbb{P}(A|+\phi) + (B|-\phi)}{2}, \quad (\text{B.27})$$

where $\mathbb{P}(A|+\phi)$ as a conditional probability. It is the probability that detector A clicks when we send in a $\{|\pm_\phi\rangle\}$ state *if any detector clicks at all*. We can thus calculate it as

$$\mathbb{P}(A|+\phi) = \frac{\langle +_\phi | \hat{\Pi}_A | +_\phi \rangle}{\langle +_\phi | (\hat{\Pi}_A + \hat{\Pi}_B) | +_\phi \rangle}. \quad (\text{B.28})$$

Very similar to before, we can now calculate

$$\langle \hat{\Pi}_A + \hat{\Pi}_B \rangle = d_I \langle \hat{I} \rangle + d_Z \langle \hat{Z} \rangle + \delta(\cos \phi \hat{X} + \sin \phi \hat{Y}), \quad (\text{B.29})$$

with

$$d_I = \frac{1}{2} [T_1 \mathcal{T}_I (T_2 \mathcal{T}_A + R_2 \mathcal{T}_B) + R_1 \mathcal{T}_S (R_2 \mathcal{T}_A + T_2 \mathcal{T}_B)] \equiv T_1 \mathcal{T}_I \mathcal{T}_B d'_I, \quad (\text{B.30})$$

$$d_Z = \frac{1}{2} [T_1 \mathcal{T}_I (T_2 \mathcal{T}_A + R_2 \mathcal{T}_B) - R_1 \mathcal{T}_S (R_2 \mathcal{T}_A + T_2 \mathcal{T}_B)] \equiv T_1 \mathcal{T}_I \mathcal{T}_B d'_Z, \quad (\text{B.31})$$

$$\delta = (\mathcal{T}_A - \mathcal{T}_B) \sqrt{T_1 R_1 T_2 R_2 \mathcal{T}_I \mathcal{T}_S} \sqrt{\eta} \equiv T_1 \mathcal{T}_I \mathcal{T}_B \delta', \quad (\text{B.32})$$

where

$$d'_I = \frac{1}{2} [(T_2 r + R_2) + q(R_2 r + T_2)] \quad (\text{B.33})$$

$$d'_Z = \frac{1}{2} [(T_2 r + R_2) - q(R_2 r + T_2)]. \quad (\text{B.34})$$

$$\delta = (r - 1) \sqrt{q T_2 R_2} \quad (\text{B.35})$$

Now, for an arbitrary input state $|\psi\rangle$, we can calculate the conditional probability as

$$\mathbb{P}(A|\psi) = \frac{\langle \hat{\Pi}_A \rangle}{\langle \hat{\Pi}_A + \hat{\Pi}_B \rangle} = \frac{\langle \hat{\Pi}_A + \Pi_B \rangle + \langle \hat{\Pi}_A - \Pi_B \rangle}{2 \langle \hat{\Pi}_A + \hat{\Pi}_B \rangle} = \frac{1}{2} + \frac{1}{2} \frac{\langle \hat{\Pi}_A - \Pi_B \rangle}{\langle \hat{\Pi}_A + \hat{\Pi}_B \rangle}. \quad (\text{B.36})$$

$$= \frac{1}{2} + \frac{1}{2} \frac{c'_I \langle \hat{I} \rangle + c'_Z \langle \hat{Z} \rangle + \gamma'(\cos \phi \hat{X} + \sin \phi \hat{Y})}{d'_I \langle \hat{I} \rangle + d'_Z \langle \hat{Z} \rangle + \delta'(\cos \phi \hat{X} + \sin \phi \hat{Y})}, \quad (\text{B.37})$$

To calculate the fidelity of an equatorial basis measurement on the Bloch sphere, now define $|\pm_\psi\rangle = \frac{1}{\sqrt{2}}(|E\rangle \pm e^{i\psi}|L\rangle)$. We calculate the fidelity of the measurement as

$$F = \frac{1}{2} [\mathbb{P}(A|+\phi) + \mathbb{P}(B|-\phi)] = \frac{1}{2} [\mathbb{P}(A|+\phi) + 1 - \mathbb{P}(A|-\phi)] \quad (\text{B.38})$$

$$= \frac{1}{2} \left(1 + \frac{1}{4} \frac{c'_I + \gamma'(\cos \phi \langle \hat{X} \rangle + \sin \phi \langle \hat{Y} \rangle)}{d'_I + \delta'(\cos \phi \langle \hat{X} \rangle + \sin \phi \langle \hat{Y} \rangle)} - \frac{1}{4} \frac{c'_I - \gamma'(\cos \phi \langle \hat{X} \rangle - \sin \phi \langle \hat{Y} \rangle)}{d'_I - \delta'(\cos \phi \langle \hat{X} \rangle + \sin \phi \langle \hat{Y} \rangle)} \right), \quad (\text{B.39})$$

where we for ease of notation now define $\langle \hat{\Theta} \rangle = \langle +_\psi | \hat{\Theta} | +_\psi \rangle$ for $\Theta \in \{X, Y\}$. We further used that $\langle \hat{Z} \rangle = 0$, $\langle \hat{I} \rangle = 1$, and that for the basis states $\hat{X}, \hat{Y}$ operators have opposite sign. Knowing that $\cos \phi \langle \hat{X} \rangle + \sin \phi \langle \hat{Y} \rangle = \cos \phi \cos \psi + \sin \phi \sin \psi = \cos(\phi - \psi)$, we can even write this more succinctly as

$$F = \frac{1}{2} \left(1 + \frac{1}{4} \frac{c'_I + \gamma' \cos(\phi - \psi)}{d'_I + \delta' \cos(\phi - \psi)} - \frac{1}{4} \frac{c'_I - \gamma' \cos(\phi - \psi)}{d'_I - \delta' \cos(\phi - \psi)} \right). \quad (\text{B.40})$$

This equation can be used to calculate the fidelity of single-shot measurement in an equatorial basis, for a phase offset $\phi - \psi$ of the interferometer. For a perfect interferometer ($r = q = 1$ and $T_2 = R_2 = 1/2$), Eq. (B.40) becomes $F = 1/2 + \cos(\phi - \psi)/2$.

- Appendix C -

Possible Improvements of the TDI

A Time Delay Interferometer does not have to take the form of a Mach-Zehnder interferometer. In this discussion, we restrict ourselves to interferometers with a time delay established in fiber; we do not see a practical solution for a significant time delay. Since coupling into fiber is estimated to have a large loss, this option is disregarded. Thus, we can restrict the selection of possible TDI's to interferometers which are at least in-fiber until the delay line.

C.1. Other Designs

Two other designs have been found, which were deemed feasible, taking into account low loss of photons. The first is a Michelson-type interferometer, with a delay line in one of the arms. [73] This design exists for in-fiber interferometers, with Faraday mirrors at the end of both arms. No examples have been found with a significant time delay, though this does not seem to pose an additional problem. A merit of this design has been reported that it is feasible to construct the interferometer with Single Mode fiber, for the reverse direction of propagation compensates for birefringence effects. However, this comes at the cost of the complexity and loss of Faraday mirrors. Another issue with such an interferometer is that one of the output ports is also an input port. To retrieve single photons in both output ports of the interferometer, one would need to add a circulator in this arm.

The second design deemed feasible [65], also took the design of a Mach-Zehnder interferometer, and also employed two circulators. An advantage is that the continuous-wave probe laser light travel in the other direction than the single photons to be measured. Thus, this design does not need any beamsplitters in the output ports of the interferometer. Two disadvantages have been considered. Firstly, it was reported that one of the single photon detectors could not be used due to leakage from the circulator. Secondly, the insertion loss of the circulators would be higher than the loss induced by the 90/10 beamsplitters, or at most on the same order for single mode fibers. This would add the complexity of polarization controllers, as done in the work of Uysal et al. [56]

These are the main reasons why we do not have another design to recommend over the current. However, there are ways to relieve the need for the 90/10 beamsplitters, as will now be explained.

C.2. Different Locking Mechanisms

In the current form of the interferometer, 90/10 beamsplitters are only needed to track the phase via a probe signal. However, if one can do this using the same detectors as for detecting the single photons, they would no longer be needed. In the current locking mechanism, the maximum count rate of the APDs result limit the frequency of a small dithering signal that can be resolved due to shot noise. Combined with the very shallow roll-off of the low pass filters of the FPGA, this would limit the modulation to very low frequencies to be accurately resolved. Currently, it is unsure if the natural phase fluctuations would disturb this modulation.

However, locking schemes without a modulation can allow for registering the probe signal with the single photon detectors, due to the uncertainty in high-frequency phase noise. One approach is to change the frequency of the continuous wave light used for locking [73] with respect to the frequency of single photons by Δf . As the phase depends on the frequency as in Eq. (2.5), this allows to lock V_{diff} to zero, while the phase for the single photons is determined by the offset $2\pi\Delta f T$, tunable by tuning Δf .

Another approach is to not lock the phase at all, but let it drift freely and reconstruct it. This has been done [56], by utilizing the two different refractive indices of a slightly strained single mode fiber, to achieve two signals, which are dependent on the phase of the interferometer, but offset by a certain value depending on the optical axis of the fiber. This phase difference was virtually constant, and this allowed for phase reconstruction. However, due to the decreased beat-length in PM optical fibers, it is questionable if this can work for a PM-fiber based interferometer. Additionally, the single mode fiber

requires the use of polarization controllers.

C.3. Detection Improvements

Currently, during the locking stage, only integral feedback is applied. This yielded a phase distribution with RMS of 0.046π after locking for 30 ms. This might be improved if also proportional and derivative feedback are applied. However, when releasing lock the FPGA is currently only holds the output of the integral feedback constant; the contributions of proportional, and integral feedback vanish and the phase jumps as a result. By reprogramming the FPGA, this can be altered, to apply feedback more effectively.

During the measurement stage, we currently wait 2 ms before ungating the APDs. We have observed high spikes in the count rate of the APDs when ungating them shortly after the locking stage. The height of the spike had a positive correlation with the optical input power during the locking stage and a negative correlation with the wait time between the two stages. To not exceed the maximum count rate of the APDs, we implemented the 2 ms wait time. If we decrease this wait time, this would allow additional time in the measurement stage where the phase is close to the setpoint. The high spike is hypothesized to be due to remaining electron-hole pairs that have been generated during the locking stage. To mitigate this, other detectors could be considered, such as gated SNSPDs. Working with SNSPDs also increases detection efficiencies compared to APDs.