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DOI
10.1016/j.powtec.2018.01.079

Publication date
2018

Document Version
Accepted author manuscript

Published in
Powder Technology

Citation (APA)
Gao, P., Zhang, T. S., Wei, J. X., & Yu, Q. J. (2018). Evaluation of RRSB distribution and lognormal distribution for describing the particle size distribution of graded cementitious materials. *Powder Technology*, *331*, 137-145. https://doi.org/10.1016/j.powtec.2018.01.079

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Evaluation of RRSB distribution and lognormal distribution for describing the particle size distribution of graded cementitious materials

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Abstract: Graded blended cement made of graded Portland cement (PC), blast furnace slag (BFS) and fly ash (FA) is attractive for cement production. For manufacturing graded blended cement, a suitable mathematical expression should be introduced to describe the particle size distribution (PSD) of its components and control the quality of graded blended cement. This study aims to evaluate Rosin-Rammler-Sperling-Bennet (RRSB) distribution and lognormal distribution for describing the PSD of the components of graded blended cement. RRSB distribution and lognormal distribution are used to fit the PSD of ungraded and graded PC, BFS and FA. It is found that lognormal distribution exhibits smaller fitting errors for describing the PSDs of graded PC, BFS, FA and ungraded FA. What is more, lognormal distribution exhibits good simplicity and popularity. Hence, it is recommended to use lognormal distribution to control the PSD of graded blended cement in manufacturing process.

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Key words: Lognormal distribution; RRSB distribution; Blast furnace slag; Fly ash; Portland cement; Graded cement.

1. Introduction

Cementitious materials, such as Portland cement (PC), consist of polydisperse particles with particle size from nanoscale to microscale. The particle size plays an important role in affecting the performance of cementitious materials, such as setting, heat release and strength [1-8]. Hence, controlling the particle size distribution (PSD) of cementitious materials is an important issue in cement production.

The incorporation of supplementary cementitious materials (SCM), like blast furnace slag (BFS) and fly ash (FA), in PC is an efficient method to reduce CO_2 emissions and energy consumption in cement production. Controlling the PSD of SCM is also important to improve the strength of cement blended with SCM [9-12]. In recent years, Zhang et al. [10-12] proposed a method to prepare graded blended cement by mixing fine BFS (e.g. <8 μm), medium PC (e.g. 8-32 μm) and coarse FA (e.g. >32 μm) based on a close packing theory. The obtained graded blended cement contains only a small amount of PC (approximate 25 wt.%) and shows similar strength to pure PC. For this reason this method is attractive for cement production. A commercial patent was also published recently to promote the application of this method in cement industry [13].

However before applying this method for manufacturing graded blended cement, some aspects should be carefully taken into account. One aspect is how to control the PSD of the components of the graded blended cement in the manufacturing process. Mathematical expressions are good tools to describe the PSD of cements. By using mathematical expressions the PSD of cements can be described with only a few coefficients. These coefficients are helpful for cement quality control [14], because it will be easy to know whether the PSD of cements meet the standard or not by comparing the values of these coefficients with the standard values. For manufacturing graded blended cement, a suitable mathematical expression can also be used to control the PSD of graded cementitious materials, such as fine BFS, medium PC and coarse FA.

Rosin-Rammler-Sperling-Bennet (RRSB) distribution is a possible option because it has been widely used in cement industry. As shown in Fig. 1, RRSB distribution (Fig. 1a) exhibits a similar curve to the PSD of an ungraded PC (Fig. 1b). However, as shown in Fig. 1b the graded PC presents a sharp and narrow PSD which is significantly different form the ungraded PC. Hence, RRSB distribution might be inappropriate to describe the PSD of graded cementitious materials.

Another possible option is lognormal distribution. Lognormal distribution has been widely applied in natural and social sciences. It can be used to describe the PSD of graded particles [15], aerosol particles [16], ultrafine metal particles [17] and powders for ceramic sintering [18], etc. As shown in Fig. 1a, lognormal distribution shows a bell-shape in logarithmic to linear scale, which is similar to the PSD shape of the graded PC.

It seems that lognormal distribution is more suitable to describe the PSD of graded cementitious materials in comparison with RRSB distribution. However, only a few studies have systematically evaluated RRSB and lognormal distributions for describing the PSD of graded cementitious materials. This study aims to evaluate these two mathematical expressions for describing the PSD of graded cementitious materials. The evaluation will take into account the accuracy of mathematical expressions for describing the PSD of graded cementitious materials, the simplicity (the number of coefficients) of mathematical expressions and the popularity of mathematical expressions.

2. Introduction of RRSB distribution and lognormal distribution

2.1 RRSB distribution

RRSB distribution is a powered exponential distribution. It originated in 1933 when P. Rosin and E. Rammler used a mathematical expression to describe the PSD of materials prepared by grinding [19,20]. The initial RRSB distribution was:

$$R(x) = 100exp\,(-bx^n) \quad (1)$$

where $R(x)$ is the cumulative weight of particles larger than x (μm); b and n are coefficients.

The coefficients b and n were difficult to be solved until B.B Bennet introduced a characteristic particle size x_e:

$$b = (x_e)^{-n} \quad (2)$$

By combining Eq. (1) and Eq. (2), RRSB distribution was rewritten as:

$$R(x) = 100exp(-(x/x_e)^n) \quad (3)$$

And further rewritten as:

$$lnln\big(100/R(x)\big) = nln(x/x_e) \quad (4)$$

According to Eq. (4), $nln\big(100/R(x)\big)$ is proportional to $ln(x/x_e)$. Hence $(x/x_e,\ 100/R(x))$ can be plotted as a straight line in the "$lnln$ to ln" axis. The coefficient n, which is the slope of this straight line, can be determined with a protractor in the "$lnln$ to ln" axis. After that the coefficients b and x_e can be calculated with Eq. (1) and Eq. (2). This method was used to calculate the coefficients of RRSB distribution before computers were prevalent. Currently, these coefficients can be easily obtained based on linear regression method with the help of computers.

2.2 Lognormal distribution

Lognormal distribution is also called Galton's distribution. It was derived from normal distribution. Normal distribution, also called Gaussian distribution, is a well-known mathematical expression to describe the random variation that occurs in natural and social phenomena [21,22].

The function of normal distribution can be written as:

$$f(x) = \frac{1}{\delta\sqrt{2\pi}} exp\left(1 - \frac{(x - x_0)^2}{2\delta^2}\right) \tag{5}$$

where $f(x)$ is the probability density; x_0 is the arithmetic mean of x; δ is the standard deviation of x.

Although normal distribution is successfully used in many fields, it is not suitable to describe the distributions with skewed curves (also called skew distribution), which are quite common for the data with low mean values, large variances and no negative values [21,22]. In 1879, Galton proposed lognormal distribution to describe skew distributions [23]. As shown in Fig. 2, the normal distribution presents a bell-shaped curve, while the lognormal distribution exhibits a skewed-shaped curve. It should be emphasized that the lognormal distribution will also present a bell-shaped curve in logarithmic to linear scale.

Lognormal distribution can be written as:

$$f(x) = \frac{1}{x\delta\sqrt{2\pi}} exp\left(-\frac{(lnx - lnx_0)^2}{2\delta^2}\right) \tag{6}$$

where $f(x)$ is the probability density; lnx_0 is the arithmetic mean of lnx; δ is the standard deviation of lnx.

Two other parameters (offset y_0 and curve area A) are often involved to extend the applicability of lognormal distribution. Then lognormal distribution is rewritten as:

$$f(x) = y_0 + \frac{A}{x\delta\sqrt{2\pi}} exp\left(-\frac{(lnx - lnx_0)^2}{2\delta^2}\right) \tag{7}$$

Limpert et al. [22] used two models to illustrate the mechanism of normal and lognormal distributions (Fig. 3). As shown in Fig. 3a, a board with triangular barriers is used to illustrate the mechanism of normal distribution. The balls are fed in the middle-top of the board and dropped through the triangular barriers until arriving at the bottom receptacles. When the balls hit on a barrier, the probability of turning right or left is identical. If the number of the barrier rows is N, the number of the bottom receptacles will be $N+1$. For a large number of rows and balls, the distribution of the balls in the receptacles will present a bell shape.

For lognormal distribution, a board with scalene triangles is used (Fig. 3b). The balls are also fed in the left-top of the board. When the balls hit on the barriers, the probability of going right or left is identical. If the number of the barrier rows is N, the number of the bottom receptacles will be $N+1$. If the number of the rows and balls are large enough, the distribution of the balls in the receptacles will be bell-shaped because the space of the receptacles increases from left to right.

These two models reveal that both normal and lognormal distributions are caused by random variation.

3. Raw materials

In this evaluation the PSD of both ungraded and graded PC, BFS and FA will be used. The PSD data of ungraded and graded PC, BFS and FA are from Zhang et al. [11], in which the graded PC, BFS and FA were prepared by dividing the ungraded PC, BFS and FA with an air classifier, and the PSD of these materials were determined with a laser diffraction particle size analyser. As shown in Fig. 4b, 4c and 4d, the graded PC (labelled as C1 to C8), BFS (labelled as B1 to B7) and FA (labelled as F1 to F8) exhibit a bell-shaped PSD in the logarithmic to linear scale, respectively. As shown in Fig. 4a, FA also presents a bell-shaped PSD curve although this “bell” is wider than that of the graded PC, BFS and FA. The ungraded PC, BFS and FA are labelled as C0, B0 and F0, respectively.

4. Evaluation methods

4.1 RRSB distribution fitting

The coefficients of RRSB distribution n and x_e in Eq. (3) are calculated with linear regression method. The coefficients of RRSB distribution b is calculated with Eq. (2). The frequency weight of the RRSB distribution is calculated as:

$$\Delta G_{RRSB}(d_i) = (100 - R(d_i)) - (100 - R(d_{i-1})) \tag{8}$$

where d_i is the diameter of particles; $\Delta G_{RRSB}(d_i)$ is the frequency weight of the particles between d_{i-1} and d_i; $R(d_i)$ is the cumulative weight of the particles with size up to d_i.

4.2 Lognormal distribution fitting

By setting the offset (y_0) as zero in Eq. (7), the function of lognormal distribution is rewritten as Eq. (9). The coefficients of lognormal distribution d_0, d_i and A in Eq. (9) are calculated with linear regression method.

$$\Delta G_{Log}(d_i) = \frac{A}{d_i \delta \sqrt{2\pi}} exp\left(-\frac{(lnd_i - lnd_0)^2}{2\delta^2}\right) \tag{9}$$

where $\Delta G_{Log}(d_i)$ is the frequency weight of the particles between d_{i-1} and d_i; A represents the curve area coefficient; δ is the curve width; d_0 is the mean diameter.

4.3 Fitting errors

The fitting errors are represented as absolute error and relative error. The absolute error means the PSD difference between the fitting and the experimental data, and the relative error means the ratio between the absolute error and the experimental data.

The absolute error for the particles between d_{i-1} and d_i is calculated as:

$$AE(d_i) = \left|\Delta G_{\text{exp}}(d_i) - \Delta G(d_i)\right| \tag{10}$$

where $AE(d_i)$ is the absolute error; d_i is the diameter of particles; $\Delta G_{\text{exp}}(d_i)$ is the frequency weight of the particles between d_{i-1} and d_i given by experiment; $\Delta G(d_i)$ is the fit frequency weight of the particles between d_{i-1} and d_i.

If there are N fractions, the average absolute error (AAE) for the particles between minimum particle diameter (d_{min}) to maximum diameter (d_{max}) is calculated as :

$$AAE = \left(\sum_{d_{min}}^{d_{max}} AE(d_i)\right)/N \tag{11}$$

where N is the number of particle size intervals given by the laser diffraction particle size analyser.

The relative error of the particles between d_{i-1} and d_i is calculated as:

$$RE(d_i) = \frac{\left|\Delta G_{exp}(d_i) - \Delta G(d_i)\right|}{\Delta G_{exp}(d_i)} \times 100\% \tag{12}$$

The average relative error (ARE) for the particles between d_{min} and d_{max} is calculated as:

$$ARE = \left(\sum_{d_{min}}^{d_{max}} RE(d_i)\right)/N \tag{13}$$

In linear regression method, the fitting accuracy can be represented with the adjusted coefficient of determination ($Adj.R^2$). If $Adj.R^2$ is closer to 1, the regression accuracy is more acceptable.

$Adj.R^2$ is calculated from the coefficient of determination (R^2). R^2 is calculated as:

$$R^2 = 1 - \frac{\sum_{d_{min}}^{d_{max}} \left(\Delta G_{exp}(d_i) - \Delta G(d_i)\right)^2}{\sum_{d_{min}}^{d_{max}} \left[\Delta G_{exp}(d_i) - \left(\sum_{d_{min}}^{d_{max}} \Delta G_{exp}(d_i)\right)/N\right]^2} \tag{14}$$

Then $Adj.R^2$ is calculated as:

$$Adj.R^2 = 1 - \frac{(1 - R^2)(N - 1)}{(N - p - 1)} \tag{15}$$

where p is the number of coefficients. For RRSB distribution p = 2, because it contains coefficients x_e and n. For lognormal distribution p = 3, because it contains coefficients A, d_0 and δ.

5. Evaluation results

5.1 Coefficients of RRSB and lognormal distribution fitting

The obtained coefficients of RRSB and lognormal distributions are listed in Table 1, Table 2 and Table 3. The coefficients b and δ represent the "width" of RRSB and lognormal distributions, respectively. For the RRSB distribution fitting, the value of coefficient b is smaller for the graded materials with larger characteristic particle size x_e. For the lognormal distribution fitting, the value of coefficient δ is smaller for the graded materials with larger mean particle size d_0. Both values of coefficients b and δ show that the graded materials with smaller particle size have wider distributions. This is consistent with the PSD curve of graded materials (Fig. 4).

5.2 Fitting errors

As listed in Table 4 RRSB distribution shows smaller errors for fitting the PSD of the ungraded PC (C0) and BFS (B0), and exhibits larger errors for fitting the PSD of the ungraded FA (F0). For fitting the PSD of the graded PC (C1 to C8), BFS (B1 to B7) and FA (F1 to F8), lognormal distribution exhibits smaller errors (as listed in Table 5 to Table 7).

In order to more directly illustrate the fitting errors of RRSB and lognormal distributions, three representative fractions (fine, medium and coarse) of the graded PC, BFS and FA are selected, and their PSD are plotted in Fig. 5 together with the fitting results of RRSB and lognormal distributions. The PSD of the ungraded FA is also plotted in Fig. 5. As shown in Fig. 5a, Fig. 5b and Fig. 5c the lognormal distribution fitting is closer to the PSD of the graded PC, BFS and FA. Furthermore, lognormal distribution fitting is also closer to the PSD of the ungraded FA (Fig. 5d).

5.3 Adjusted coefficient of determination

Fig. 6 shows the $Adj.R^2$ of the fitting of RRSB and lognormal distributions. The lognormal distribution fitting shows smaller $Adj.R^2$ for describing the PSD of the ungraded PC (C0) and BFS (B0), and larger $Adj.R^2$ for describing the PSD of the ungraded FA (F0) (Fig. 6a). For all graded materials, C1 to C8, B1 to B7 and F1 to F8, the lognormal distribution fitting shows higher $Adj.R^2$ (Fig. 6b, Fig. 6c and Fig. 6d). The closer $Adj.R^2$ is to 1, the more accurate is the linear regression. The above results illustrate that the accuracy of lognormal distribution fitting is better for the graded PC, BFS and FA, and the ungraded FA, which is consistent with the results of fitting errors.

6. Discussion

According to the aforementioned results lognormal distribution is more accurate to describe the PSD of graded PC, BFS and FA. However, the applicability of a mathematical expression depends not only on its accuracy, but also on its simplicity and popularity [24]. If a mathematical expression comprise too many coefficients, it will be inconvenient to be applied. Moreover, if this mathematical expression is not well-known, it will be difficult to be widely accepted. In the following paragraphs the applicability of RRSB and lognormal distributions for describing the ungraded and graded PC, BFS and FA will be discussed in view of accuracy, simplicity and popularity.

6.1 Mathematical expression for describing the PSD of ungraded cementitious materials

On average, RRSB distribution is suitable for describing the PSD of ungraded PC and BFS. First, its accuracy is adequate according to the results shown in Table 4, and Fig. 6a. In addition, the simplicity of RRSB distribution is acceptable since it only contains two coefficients: n and b. Further, the popularity of RRSB distribution is good because it has been widely applied in cement industry.

However the accuracy of RRSB distribution is relatively low for describing the PSD of ungraded FA (as shown in Table 4, and Fig. 6a). For the ungraded FA from other reports [25-28], the RRSB distribution fitting also show larger errors (Table 8). This is probably because RRSB distribution was initially proposed to describe the PSD of materials made by grinding. However FA particles are normally formed in the cooling process of fused materials in the air without grinding.

6.2 Mathematical expression for describing the PSD of graded cementitious materials

According to the results in Table 5, Table 6, Table 7 and Fig. 6b, Fig. 6c, Fig. 6d, lognormal distribution presents high accuracy for describing the PSD of graded PC, BFS and FA. Hence, the accuracy of lognormal distribution is acceptable. Furthermore, the simplicity of lognormal distribution is adequate, because it contains three coefficients: the curve width δ; the curve area A; and the mean particle size d_0. It will be easy to evaluate the PSD from different resources. What is more, the popularity of lognormal distribution is ensured because it is a well-known distribution that has been applied in many fields.

In this evaluation, the graded PC, BFS and FA were obtained by using an air classifier to divide the ungraded PC, BFS and FA into several fractions. As schematically shown in Fig. 7a, this air classifier comprises three main zones: feed zone, gravitational-counterflow zone, and centrifugal-counterflow zone. The feed of the air classifier contains fine, medium and coarse particles. As illustrated in Fig. 7b, the particles in the gravitational-counterflow zone are affected by the combination of airflow force (F_A) and gravity force (F_G) [29]. A cut size is calculated according to the balance $F_G = F_A$. The particles with diameter smaller than the cut size will rise along with the airflow. However, due to stochastic factors, some fine particles will fall to the downside and some coarse particles will rise up [29]. When the particles pass through the gravitational-counterflow zone, the coarse particles will be separated. In the centrifugal-counterflow zone, the fine and medium particles are rotated by the airflow in the centrifuge (Fig. 7c). The movement of these particles is affected by the combination of three forces: airflow force, centrifugal force and drag force (Fig. 7d). The drag force F_D is defined as the force induced by the air movement [29]. Driven by these forces, the medium particles will flow towards the walls and then fall to the bottom, while the fine particles will rise up to outside along with the airflow. At the same time some medium particles will arrive at the upside, and some fine particles will come to the bottom as a result of stochastic factors. When the particles pass through centrifugal-counterflow zone, the medium and fine particles will be separated.

Using the above classifying process each ungraded material was classified into three fractions. Next, each fraction was fed in the air classifier again, and new fractions were obtained. After several passes the ungraded materials were graded into several fractions with desired PSD. Due to stochastic factors, each fraction unavoidably contained the fine, medium and coarse particles. To some extent, the graded cementitious materials were produced by random variation, which is similar to lognormal distribution (Fig. 3b). This is the possible reason why lognormal distribution is suitable for describing the PSD of graded cementitious materials.

7. Conclusions

Graded blended cement made of graded PC, BFS and FA is attractive for cement production. For manufacturing the graded blended cement, a suitable mathematical expression can be used to describe the PSD of its components and control the quality of the graded blended cement. RRSB distribution and lognormal distribution are two options to describe the PSD of the components of the graded. This study evaluated RRSB distribution and lognormal distribution for describing the PSD of graded cementitious materials by taking into account the accuracy of mathematical expressions for describing the PSD of graded cementitious materials, the simplicity (the number of coefficients) of mathematical expressions and the popularity of mathematical expressions. Based on the results of the evaluation, the following conclusions can be drawn:

(1) Lognormal distribution shows smaller fitting errors when used to describe the PSD of graded PC, BFS and FA. The reason is that the graded PC, BFS and FA were produced by random variation, which is similar to lognormal distribution. RRSB distribution shows smaller fitting errors for describing the PSD of ungraded PC and BFS. The reason is that RRSB distribution is only suitable for describing the PSD of materials made with grinding.

(2) The accuracy of RRSB distribution for describing the PSD of ungraded FA is relatively low. This is because there is no grinding process in the production of FA.

(3) The simplicity of lognormal distribution is adequate, because it contains three coefficients: the curve width δ; the curve area A; and the mean particle size d_0. It will be easy to evaluate the PSD from different resources. Further, the simplicity and popularity of lognormal distribution are also acceptable. It is recommended to use

lognormal distribution to control the PSD of graded blended cement in manufacturing process.

Acknowledgements

The authors would like to thank Mr. Hua Dong, Dr. Guang Ye, and Dr. Jie Hu for their help in the paper writing. This work was funded by the National key research and development program (2016YFB0303502), China Scholarship Council (CSC), Research Centre of TU Delft in Urban System and Environment (No. C36103), SCUT Doctoral Student Short-Term Overseas Visiting Study Funding Project, National Natural Science Foundation of China (No. 51672084), Guangdong special support for Youth Science and Technology innovation talents (2015TQ01C312), and Pearl River Science and Technology Nova Program of Guangzhou (201610010098). Their financial supports are gratefully acknowledged.

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Figure captions:

Fig. 1 PSD of the ungraded and graded PC, and the shapes of RRSB and lognormal distributions

Fig. 1 (a) Lognormal distribution and RRSB distribution, Fig. 1 (b) PSD of the ungraded and graded PCs

Fig. 2 Shapes of normal distribution and lognormal distribution

Fig. 2 (a) Bell-shaped curve of normal distribution, Fig.2 (b) Skewed curve of lognormal distribution

Fig. 3 Schematic representation of the models for normal distribution and log-normal distribution (After [22])

Fig. 3 (a) Normal distribution, Fig. 3 (b) Lognormal distribution

Fig. 4 PSD of graded and ungraded PC, BFS and FA measured with a laser diffraction particle size analyser (After [11])

Fig. 4 (a) PSD of ungraded PC, BFS and FA, Fig. 4 (b) PSD of graded PC, Fig. 4 (c) PSD of graded BFS, Fig. 4 (d) PSD of graded FA.

Fig. 5 The PSD determined by experiments *versus* the PSD fit by RRSB and lognormal distributions

Fig. 5 (a) Comparison for C2, C5 and C8, Fig. 5 (b) Comparison for B1, B4 and B7, Fig. 5 (c) Comparison for F1, F5 and F8, Fig. 5 (d) Comparison for F0

Fig. 6 $Adj.R^2$ of the RRSB distribution fit and lognormal distribution fit

Fig. 7 Schematic representation of the air classifier to prepare graded cementitious materials

(a) Main zones of the air classifier: left is feed zone, middle is gravitational-counterflow zone, right is centrifugal-counterflow zone; (b) Two forces in gravitational-counterflow zone; (c) Vertical view of centrifugal-counterflow zone; (d) Three forces in centrifugal-counterflow zone

Fig1a
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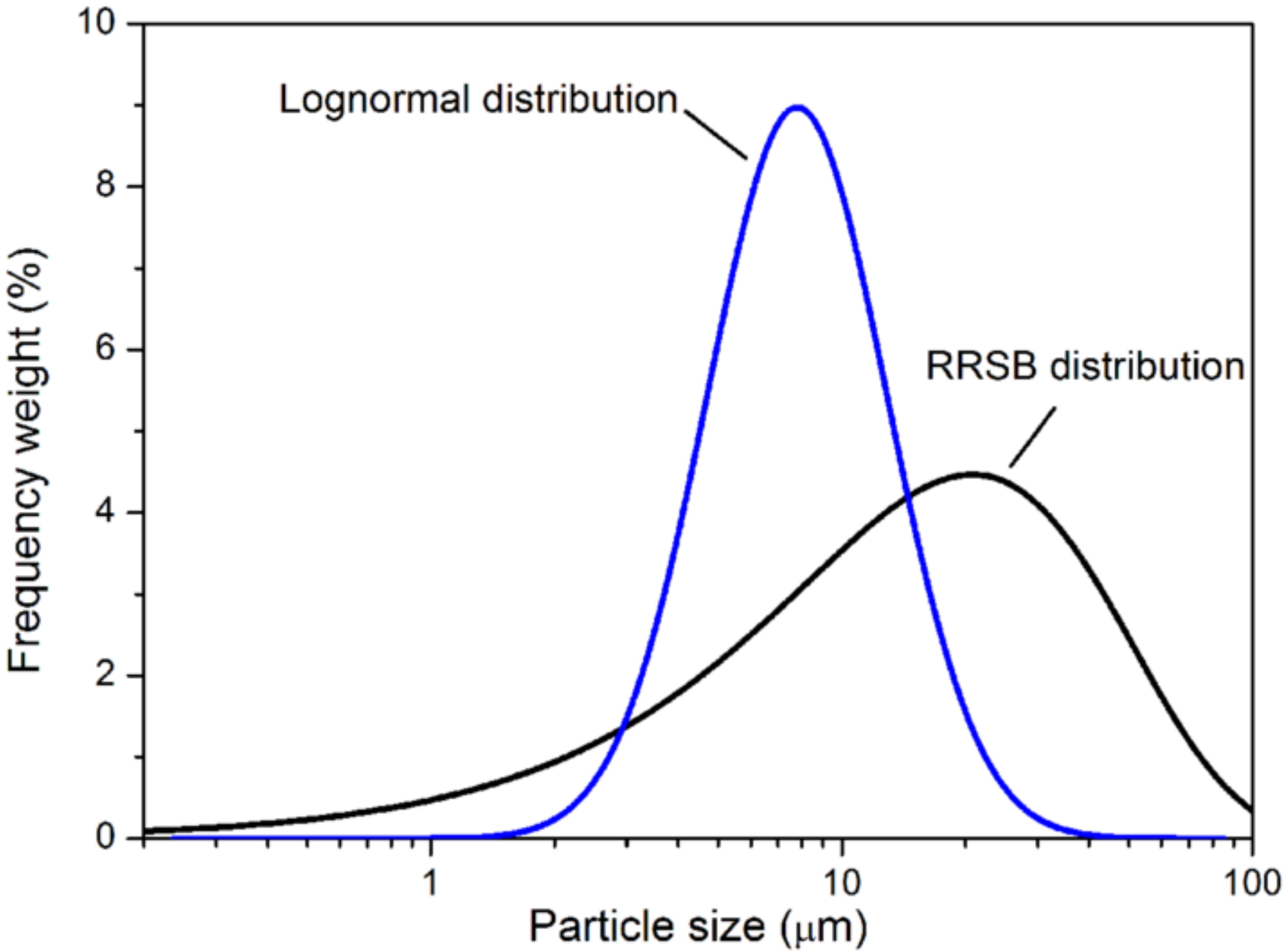

Fig1b
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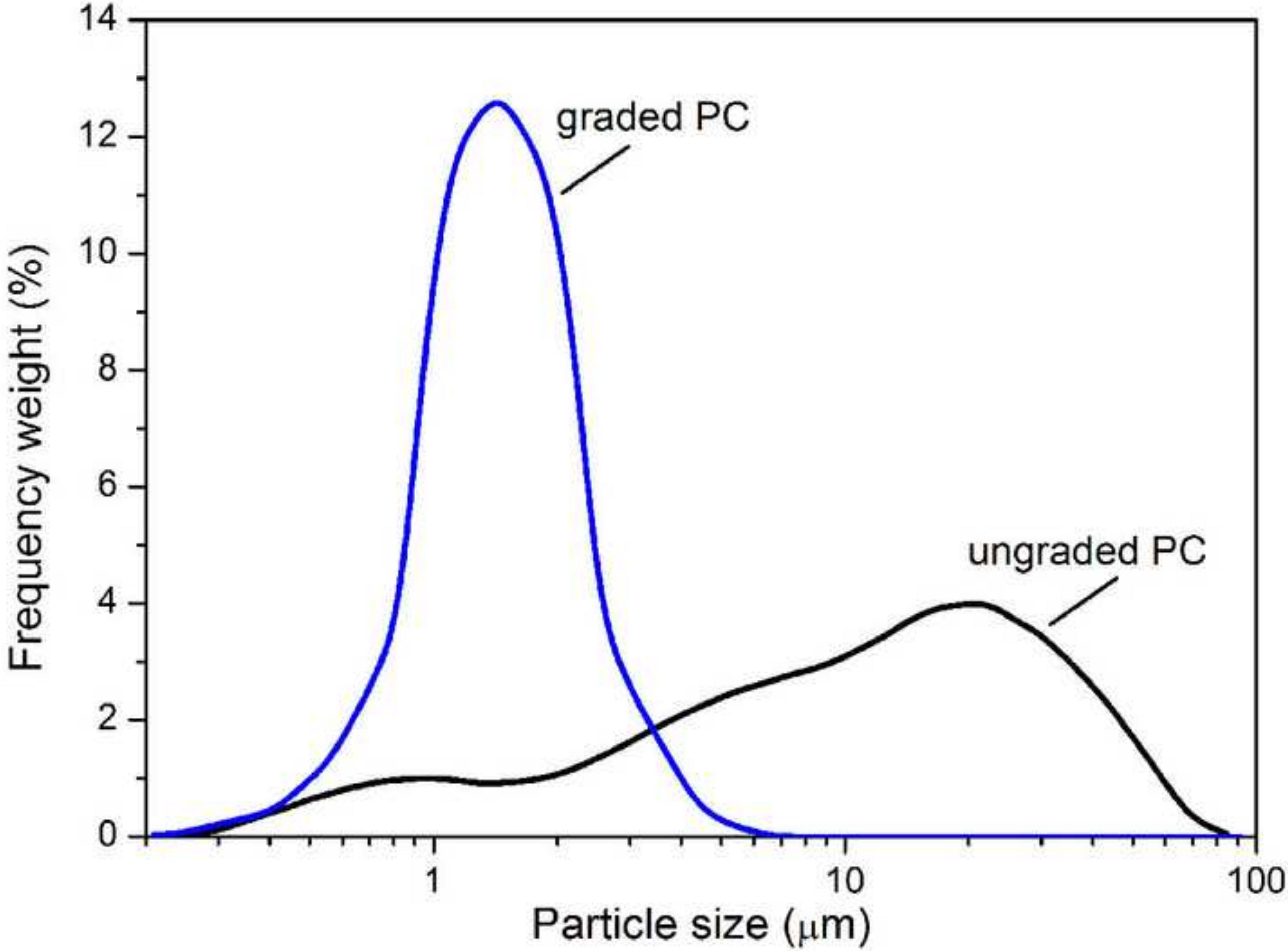

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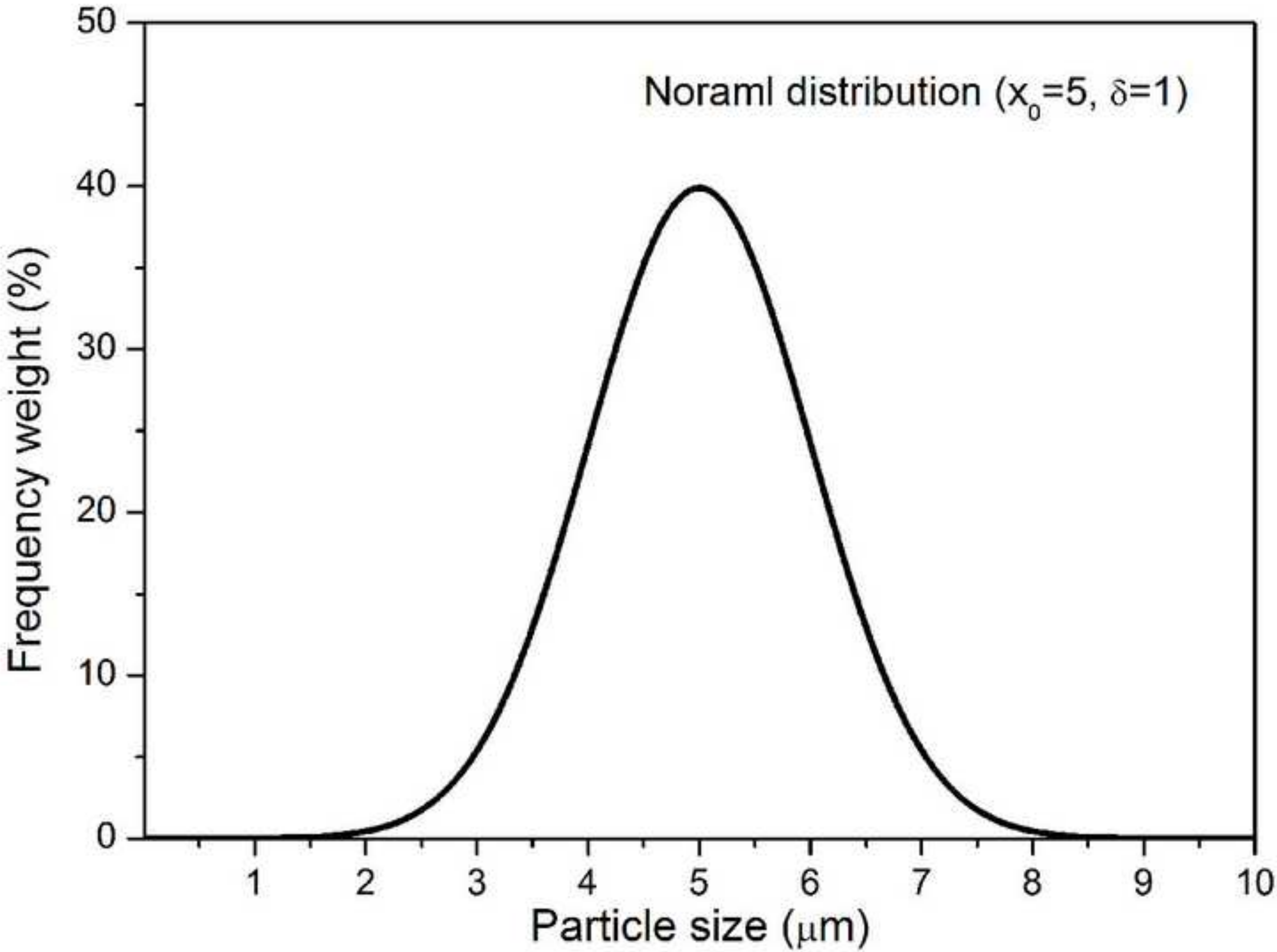

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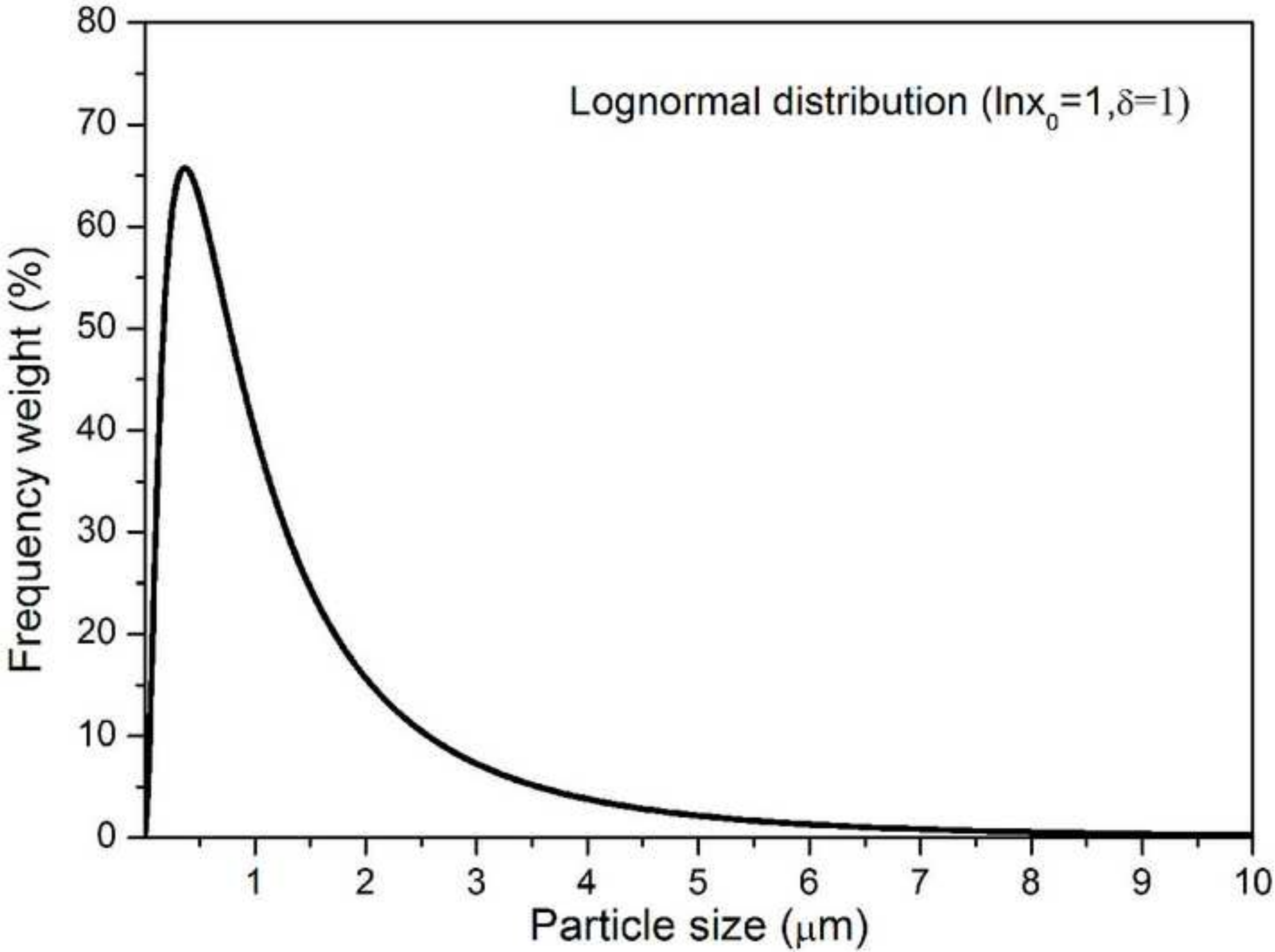

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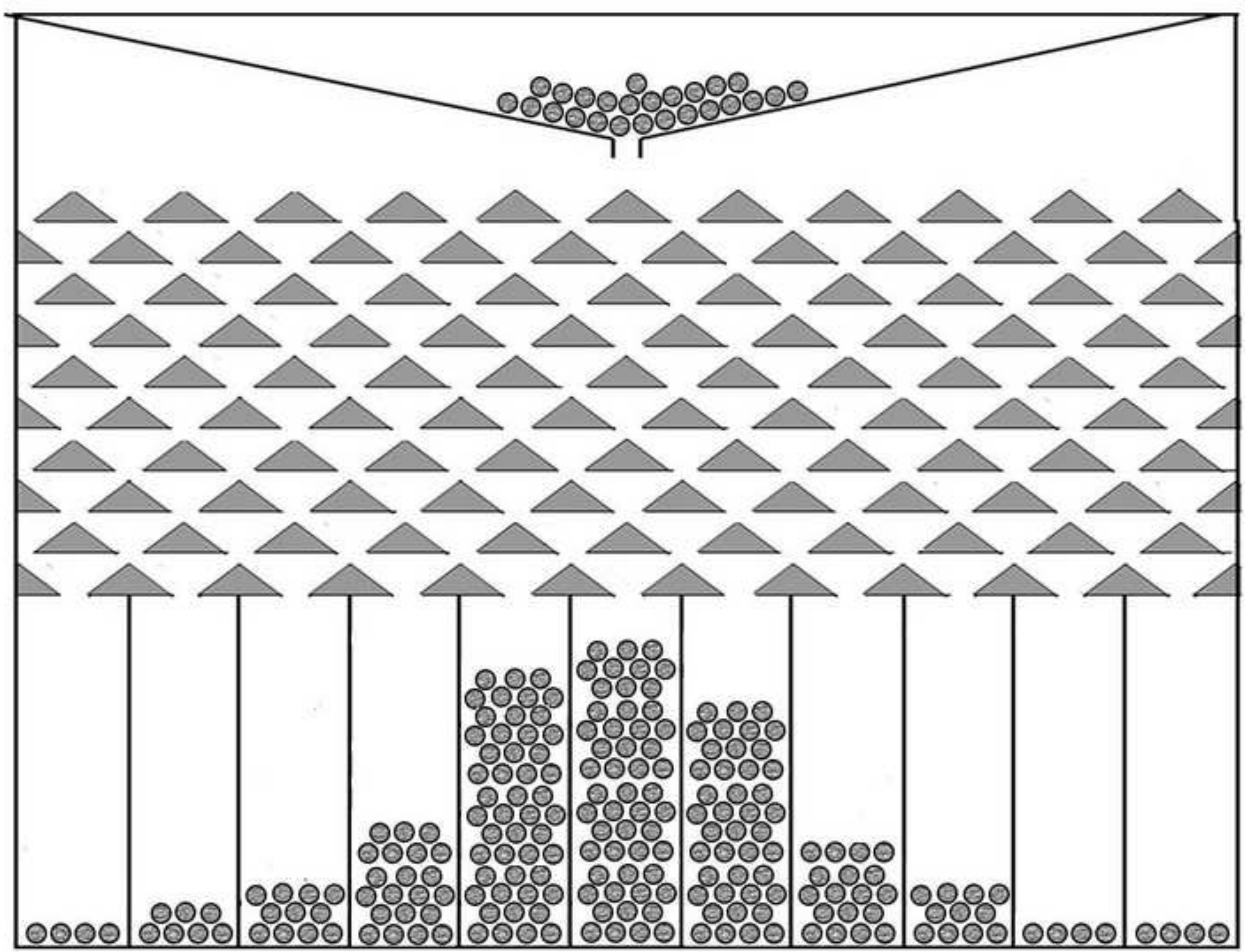

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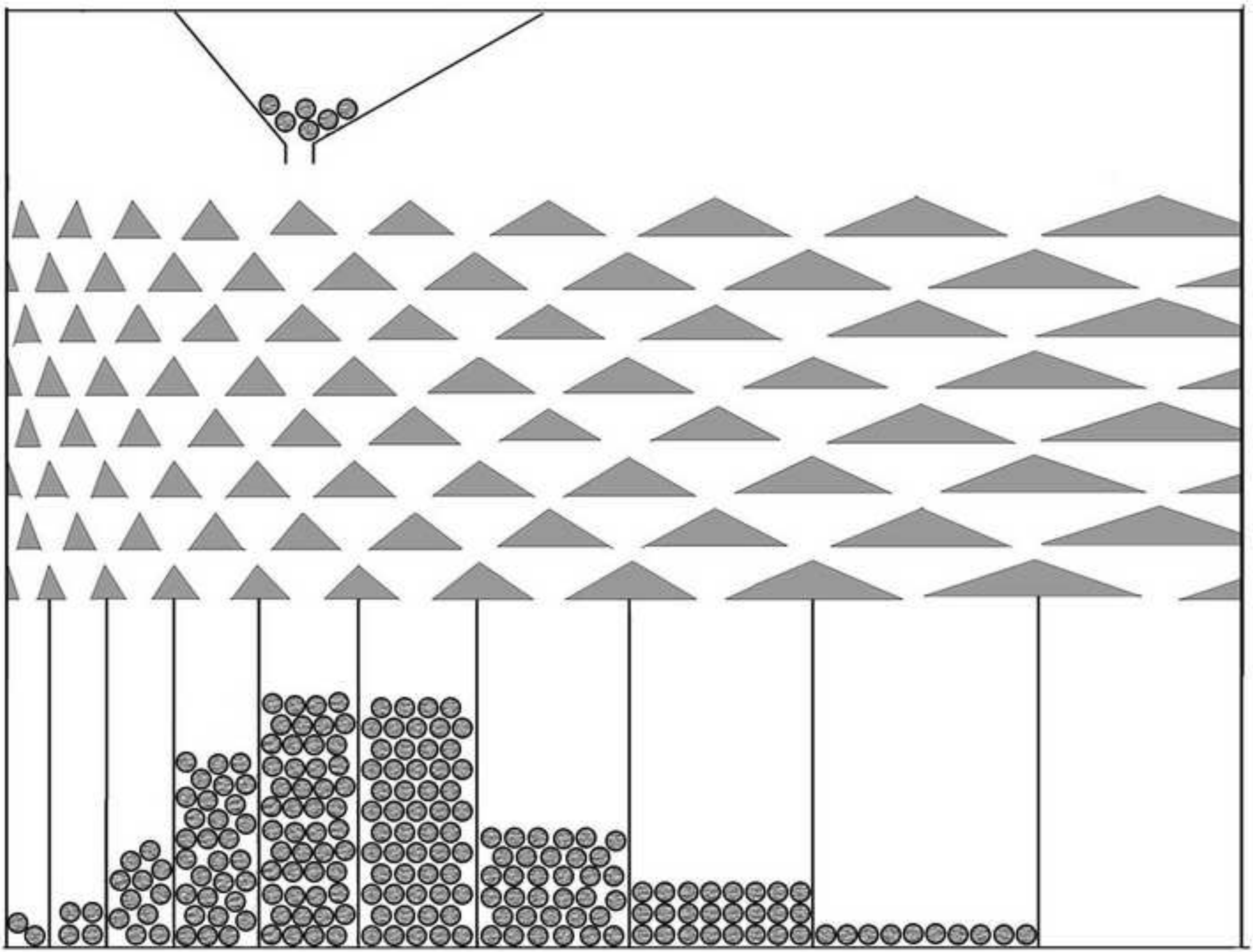

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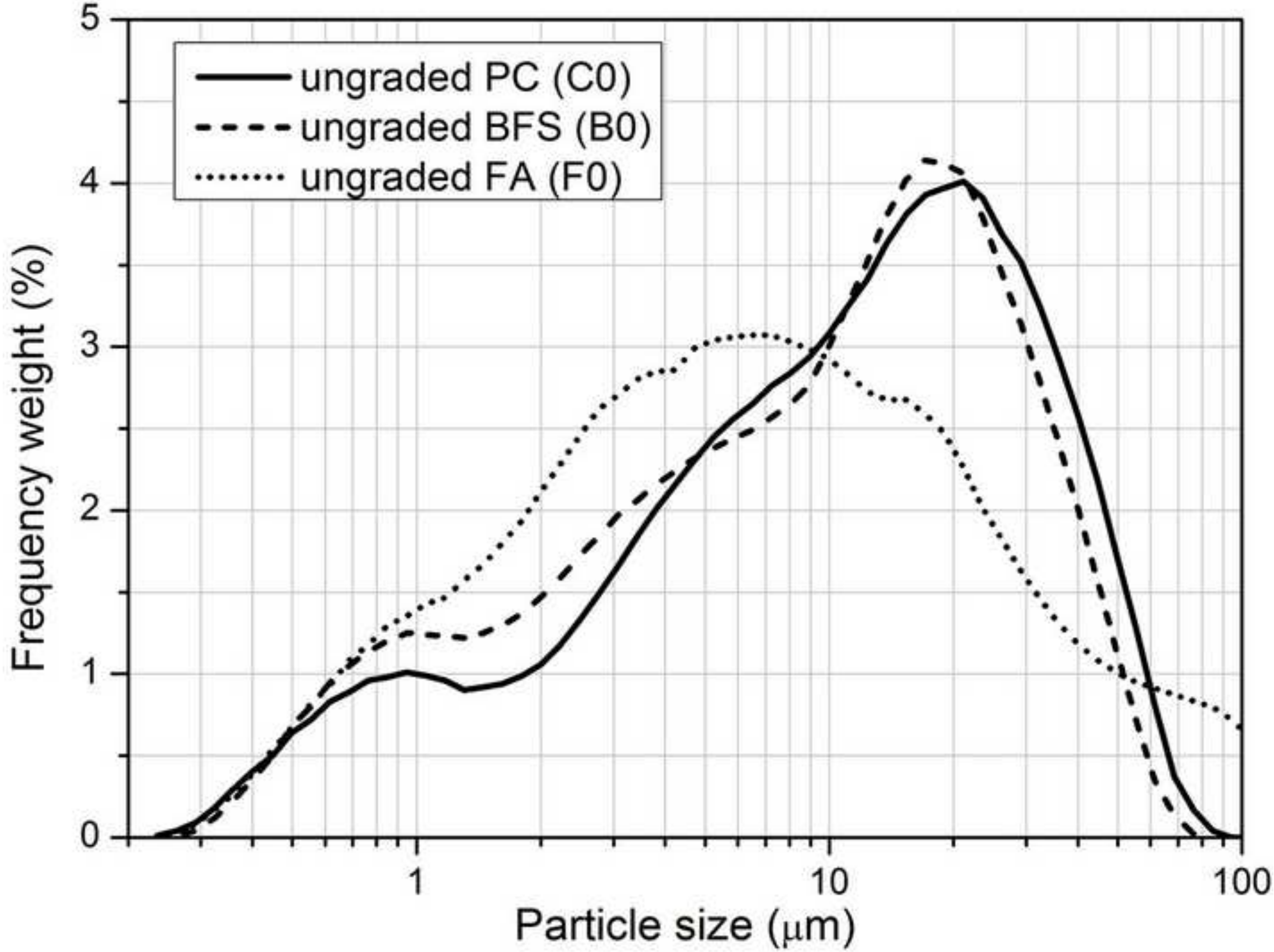

Fig4b

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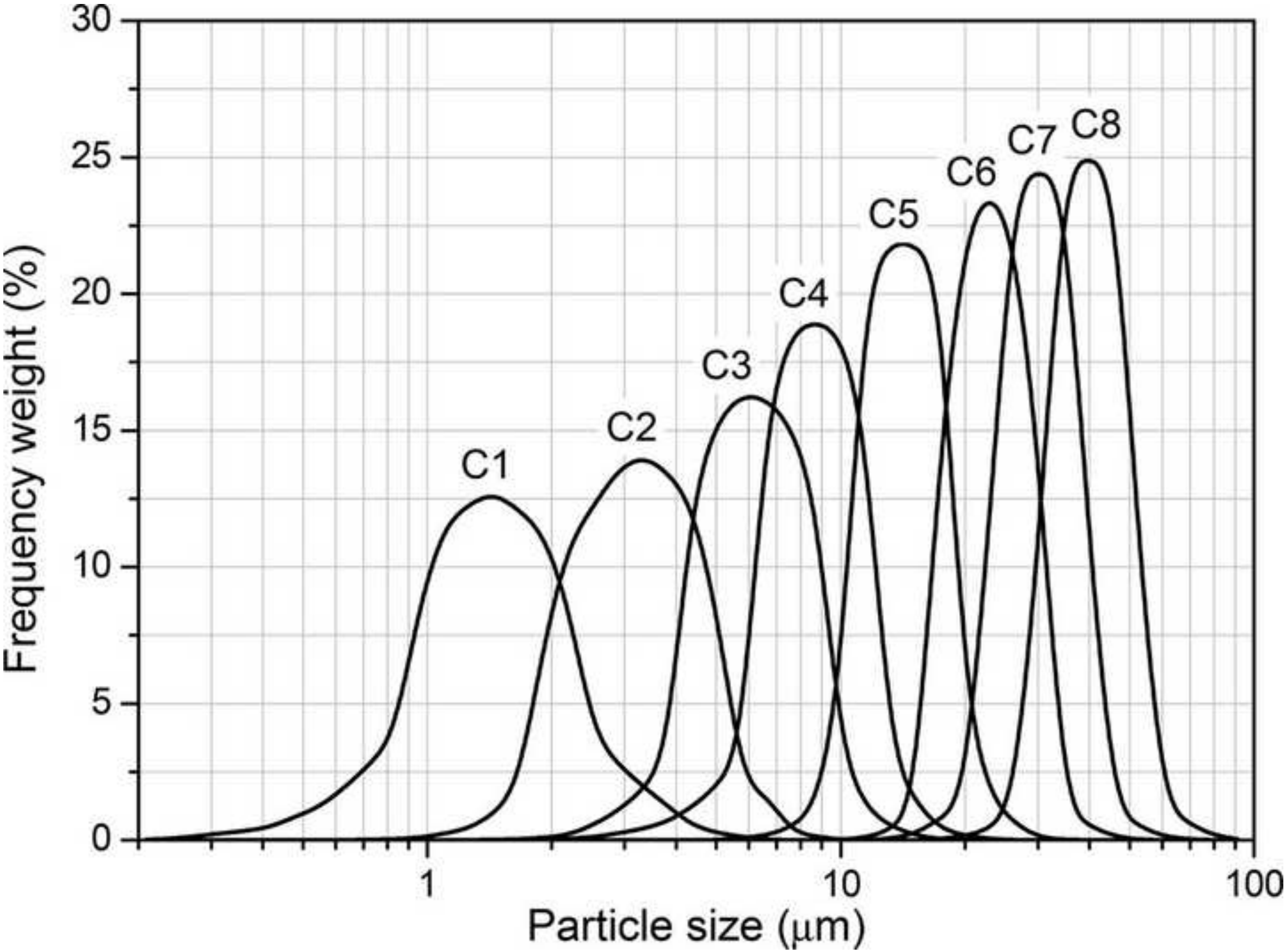

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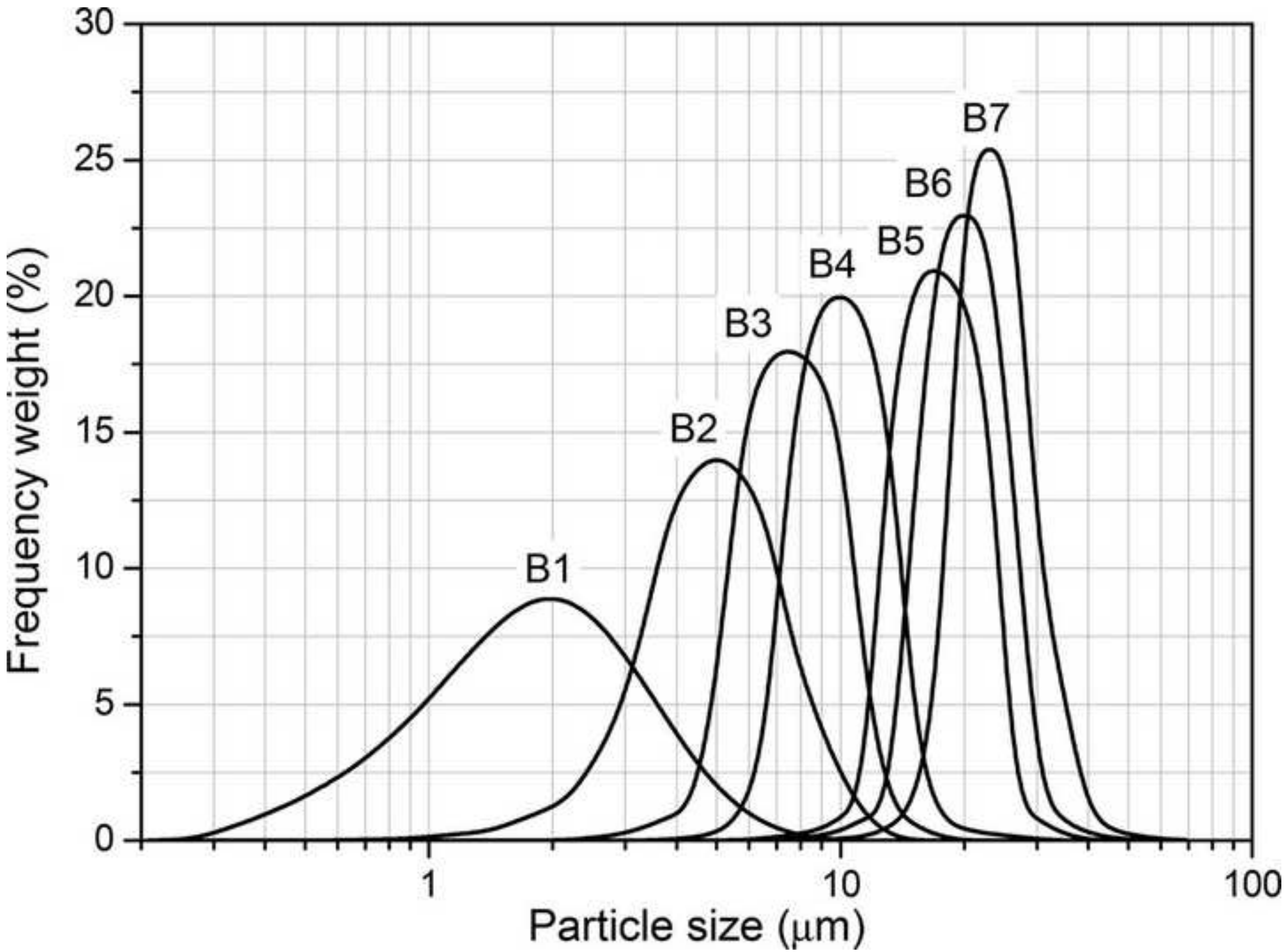

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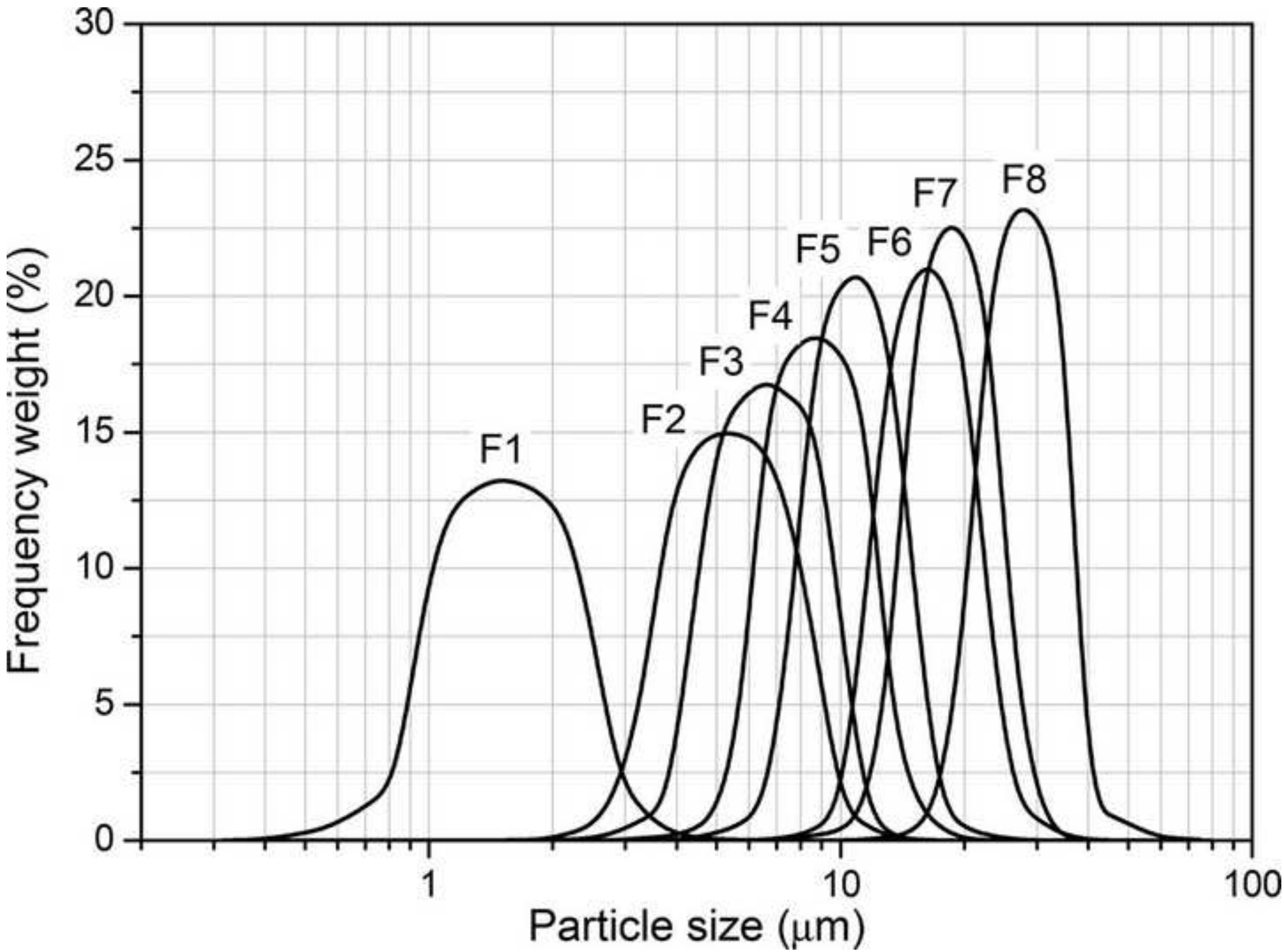

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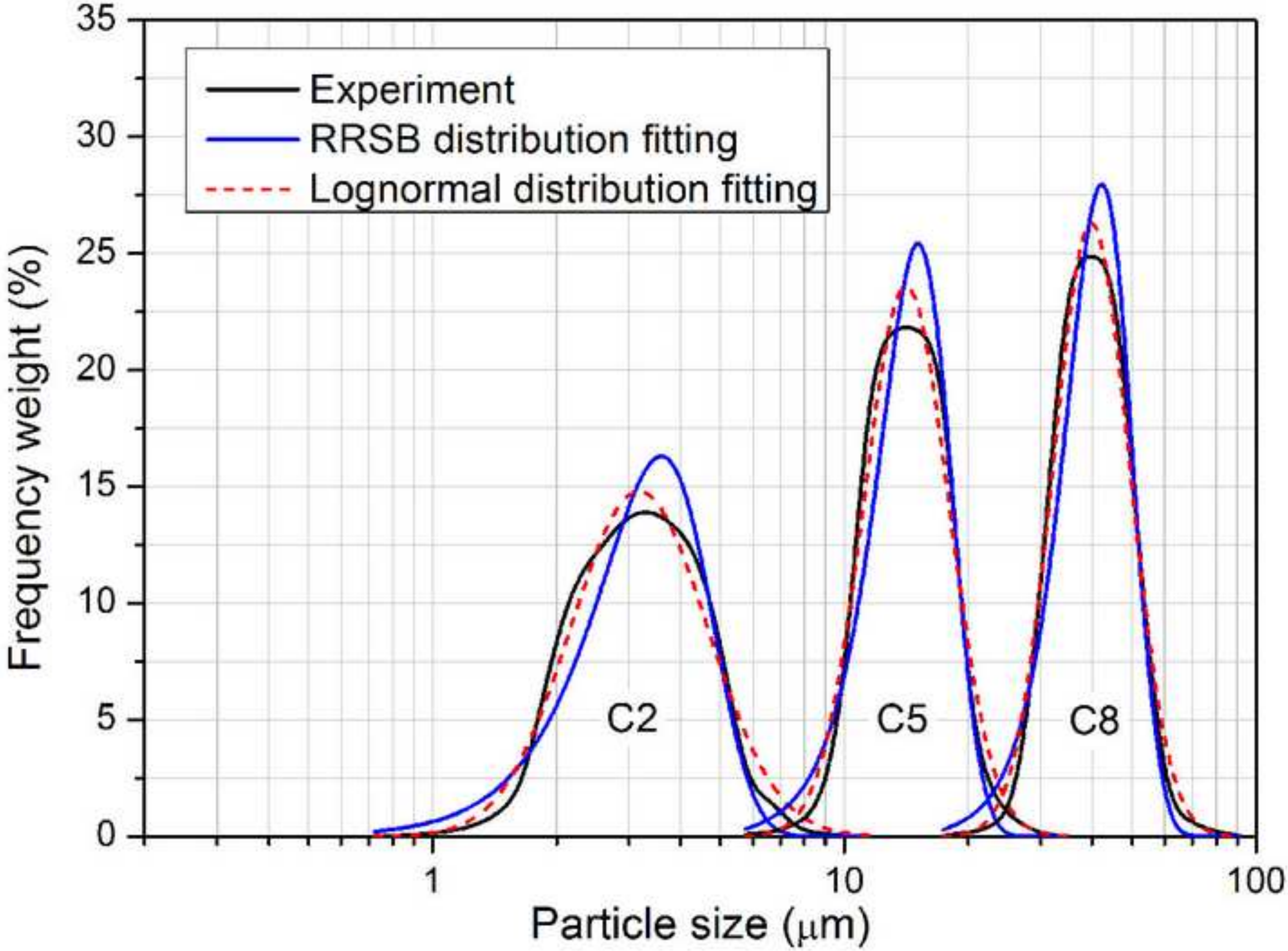

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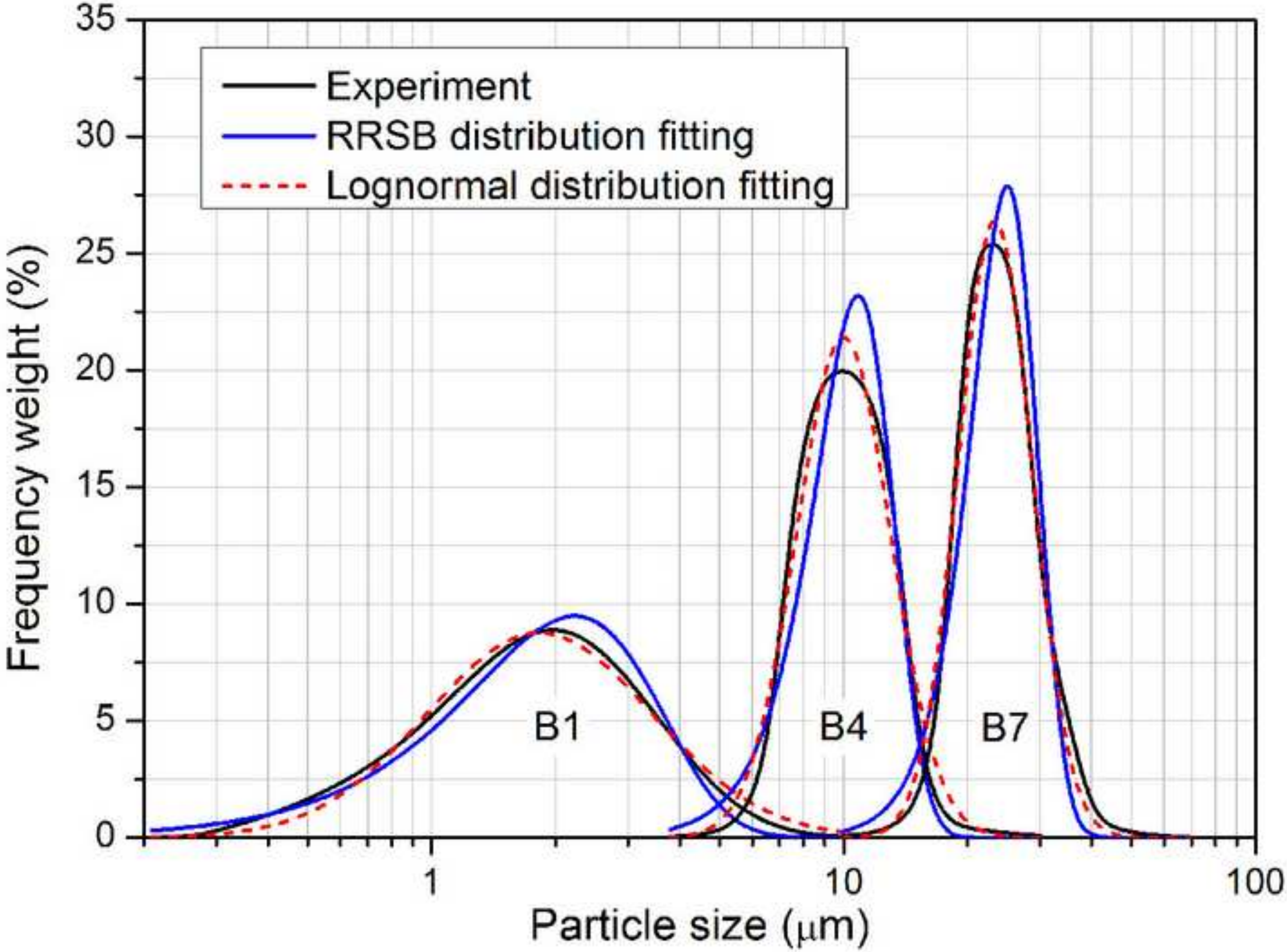

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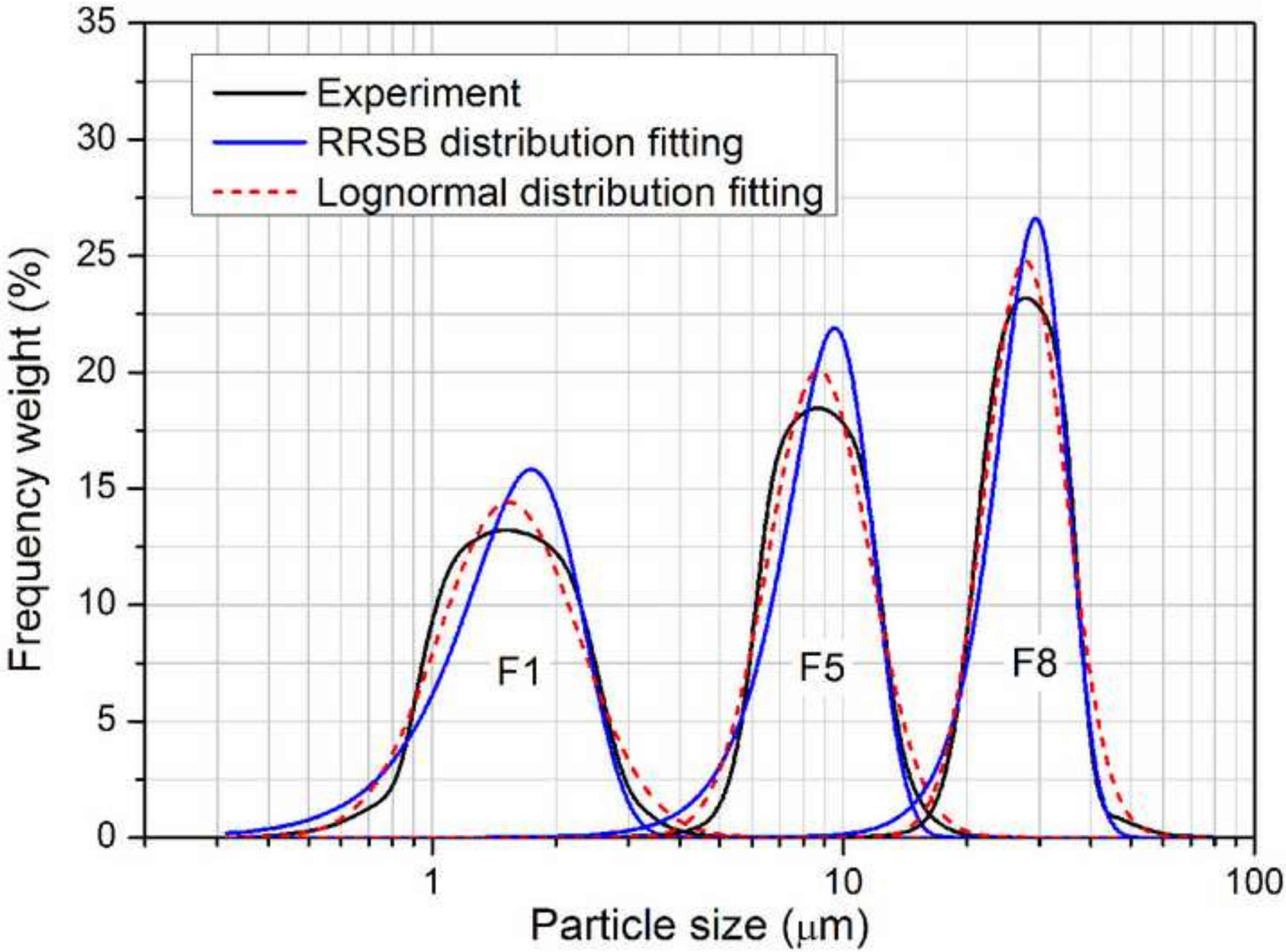

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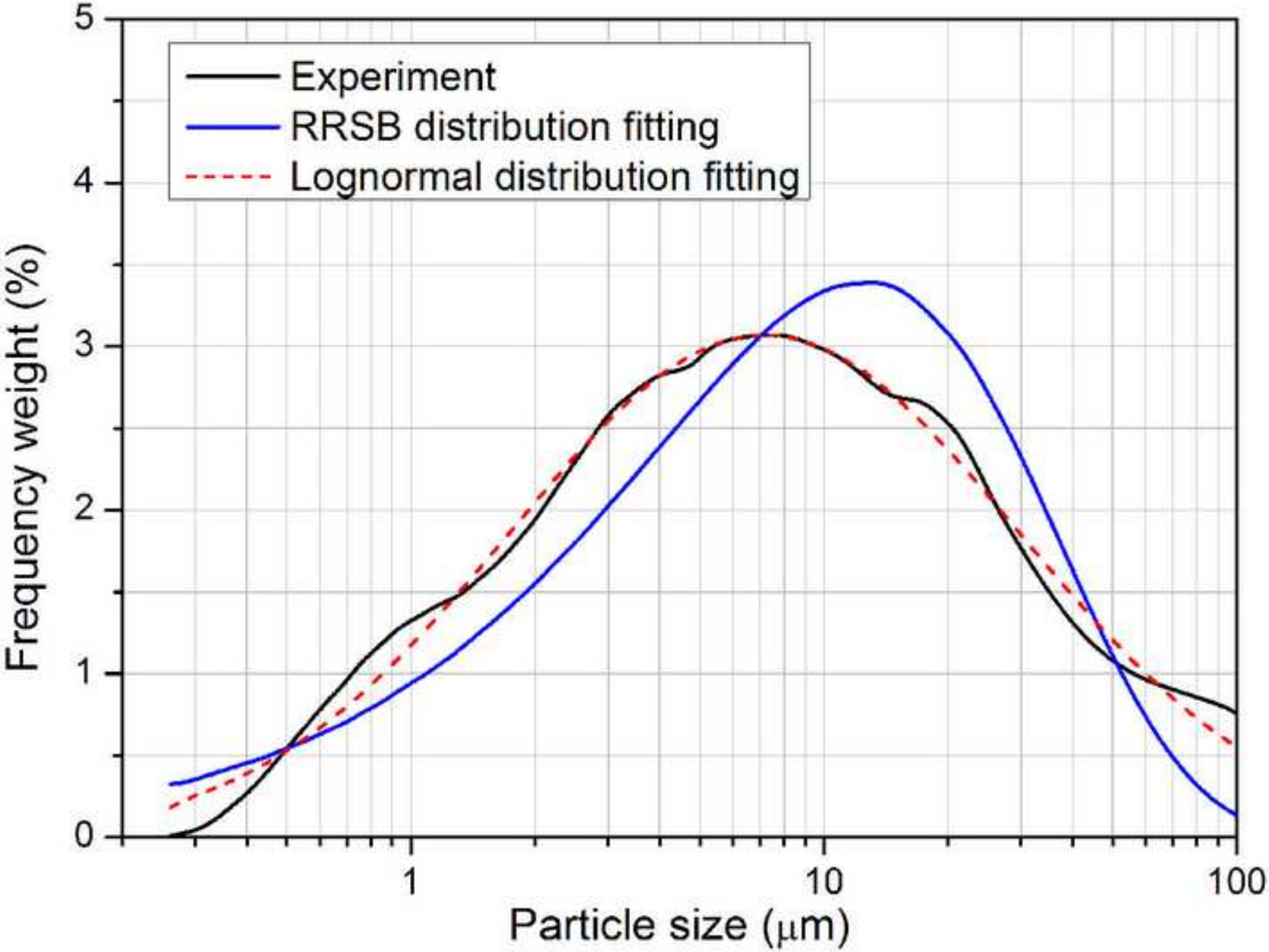

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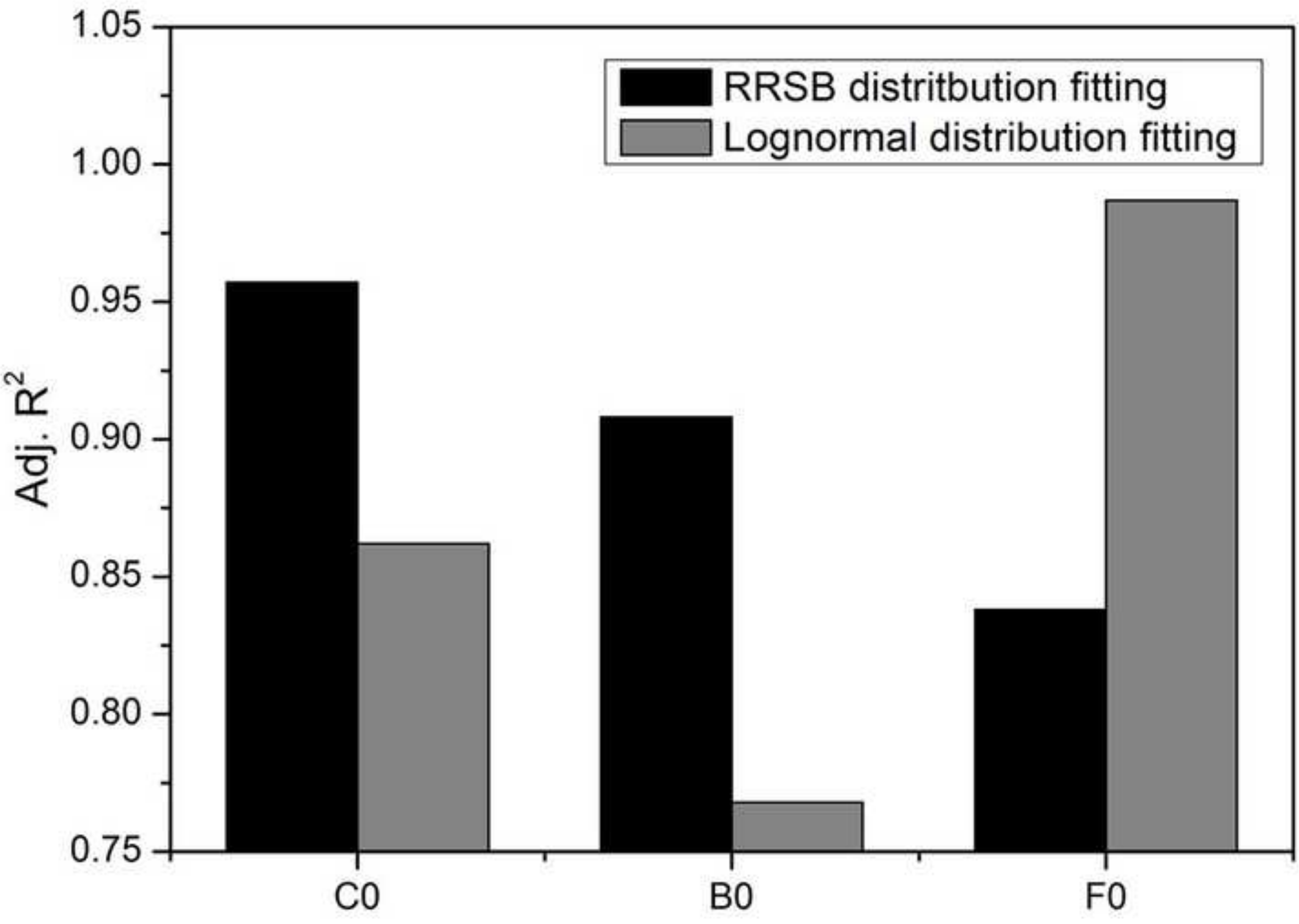

Fig6b

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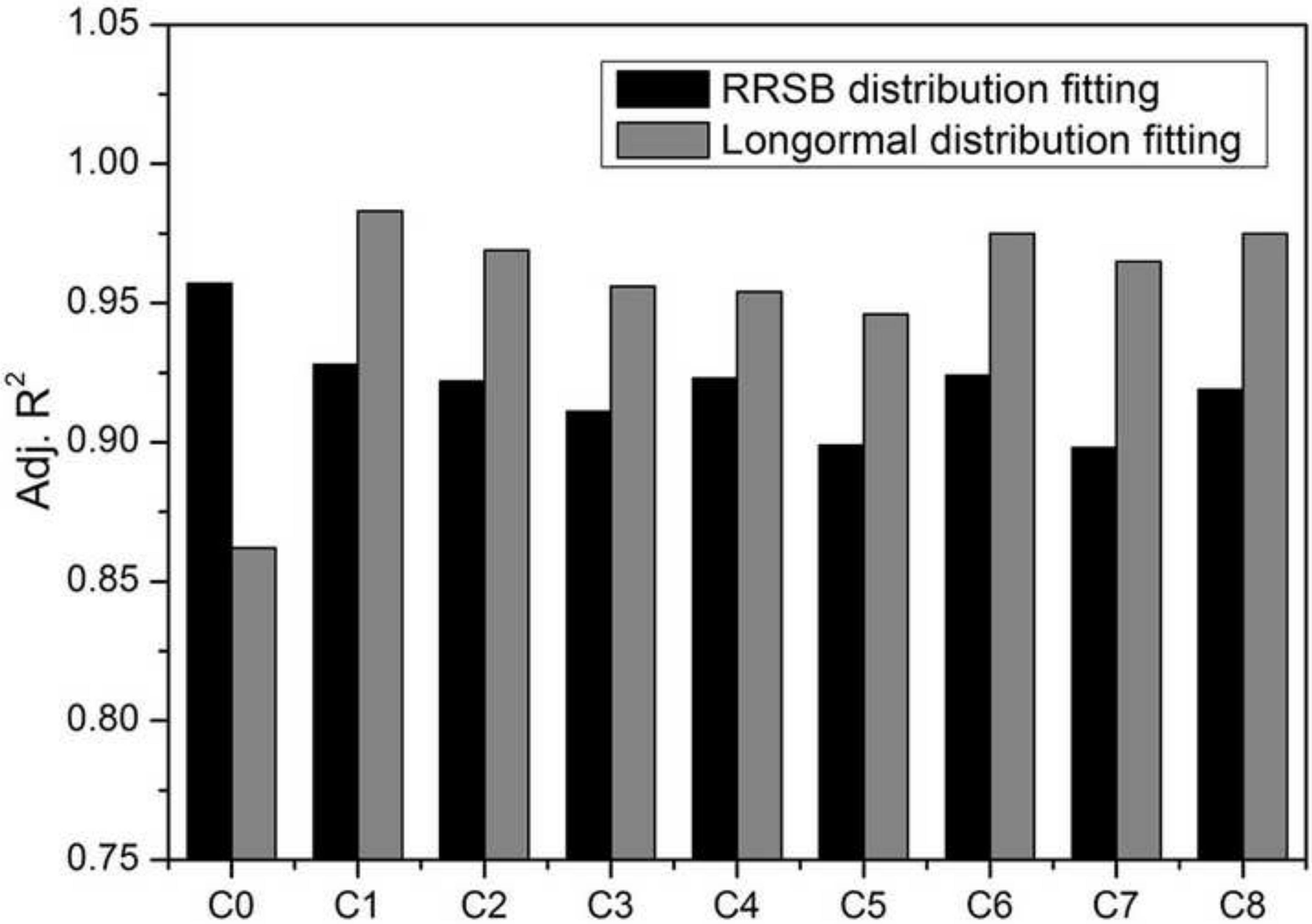

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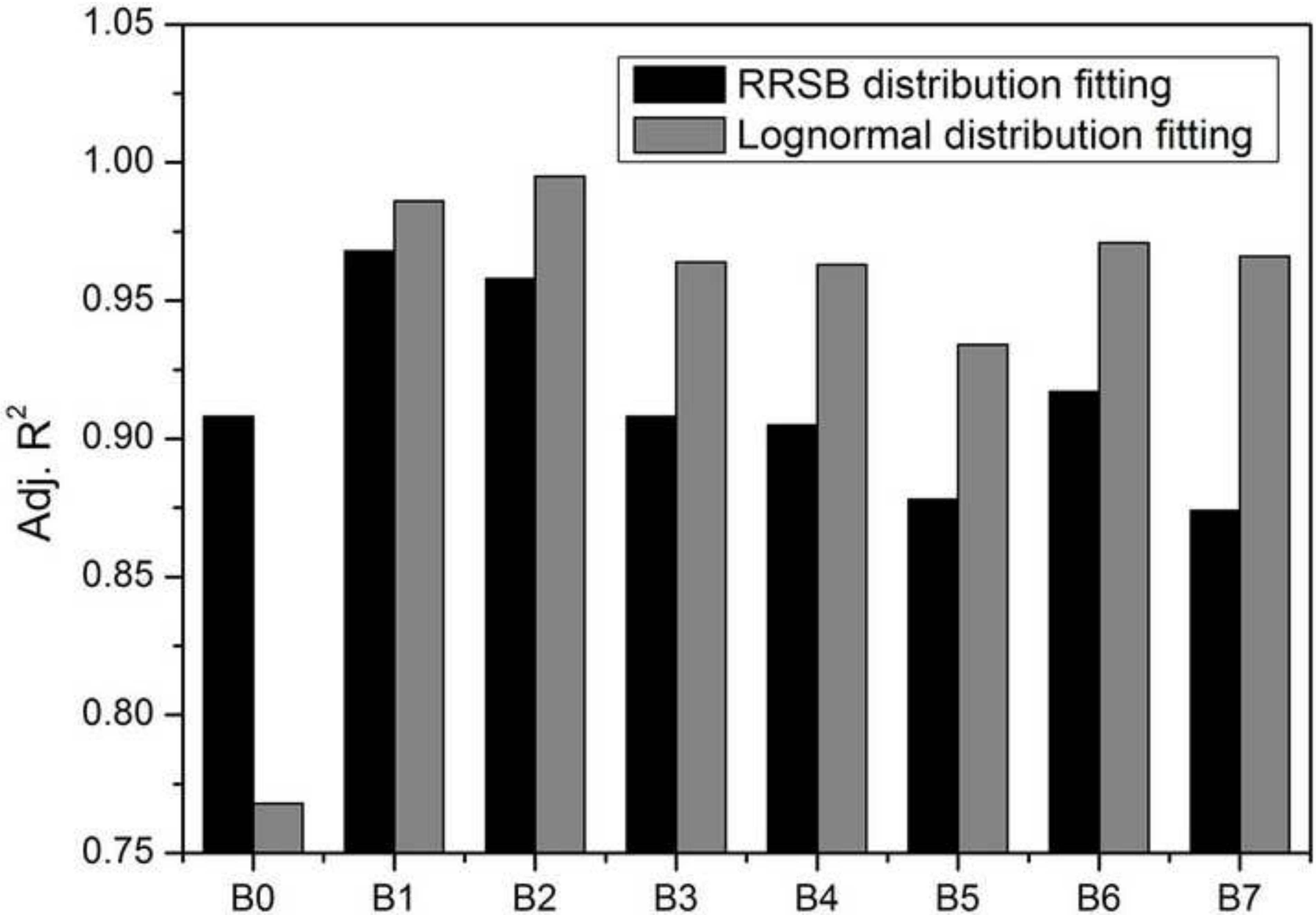

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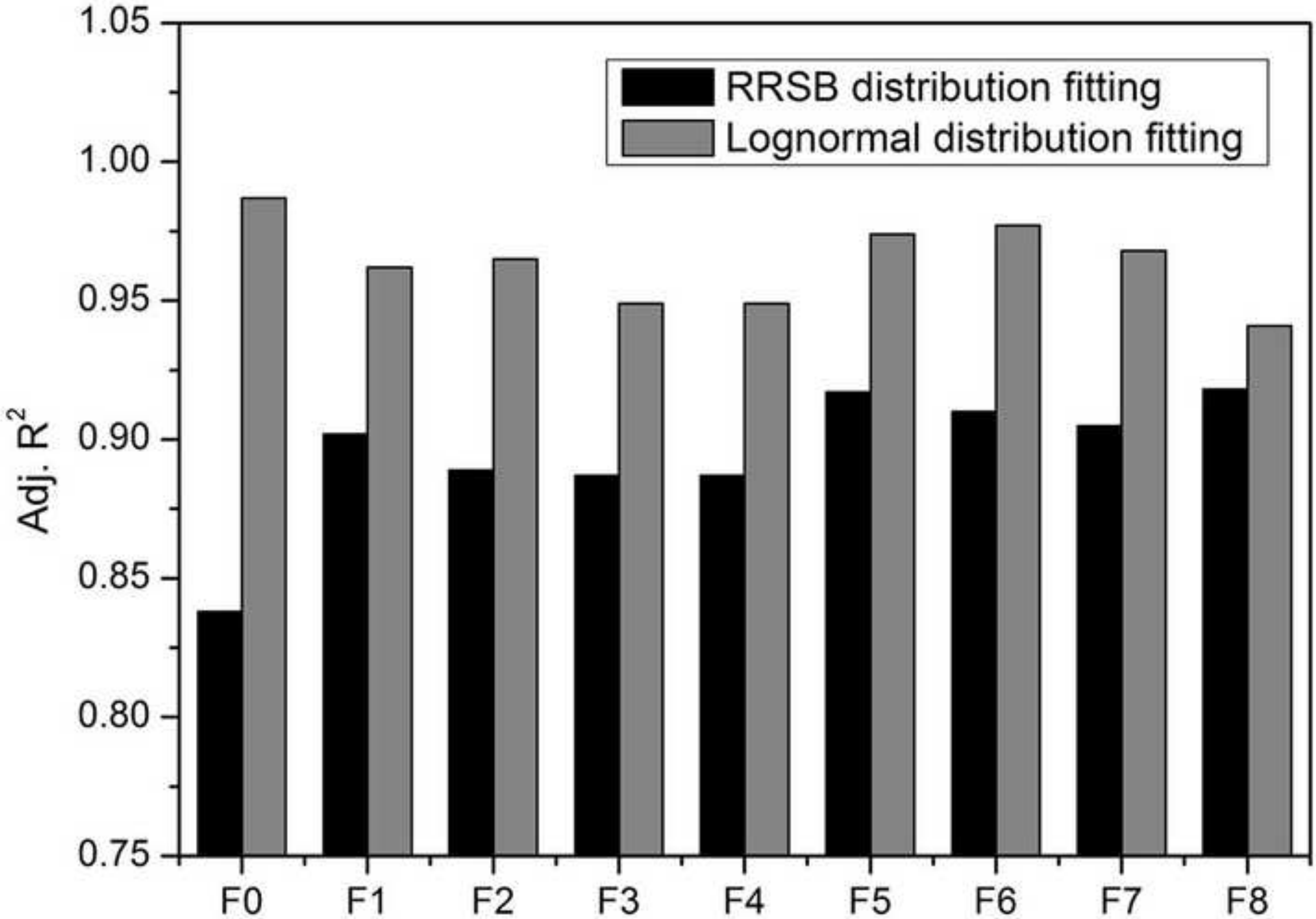

Fig7

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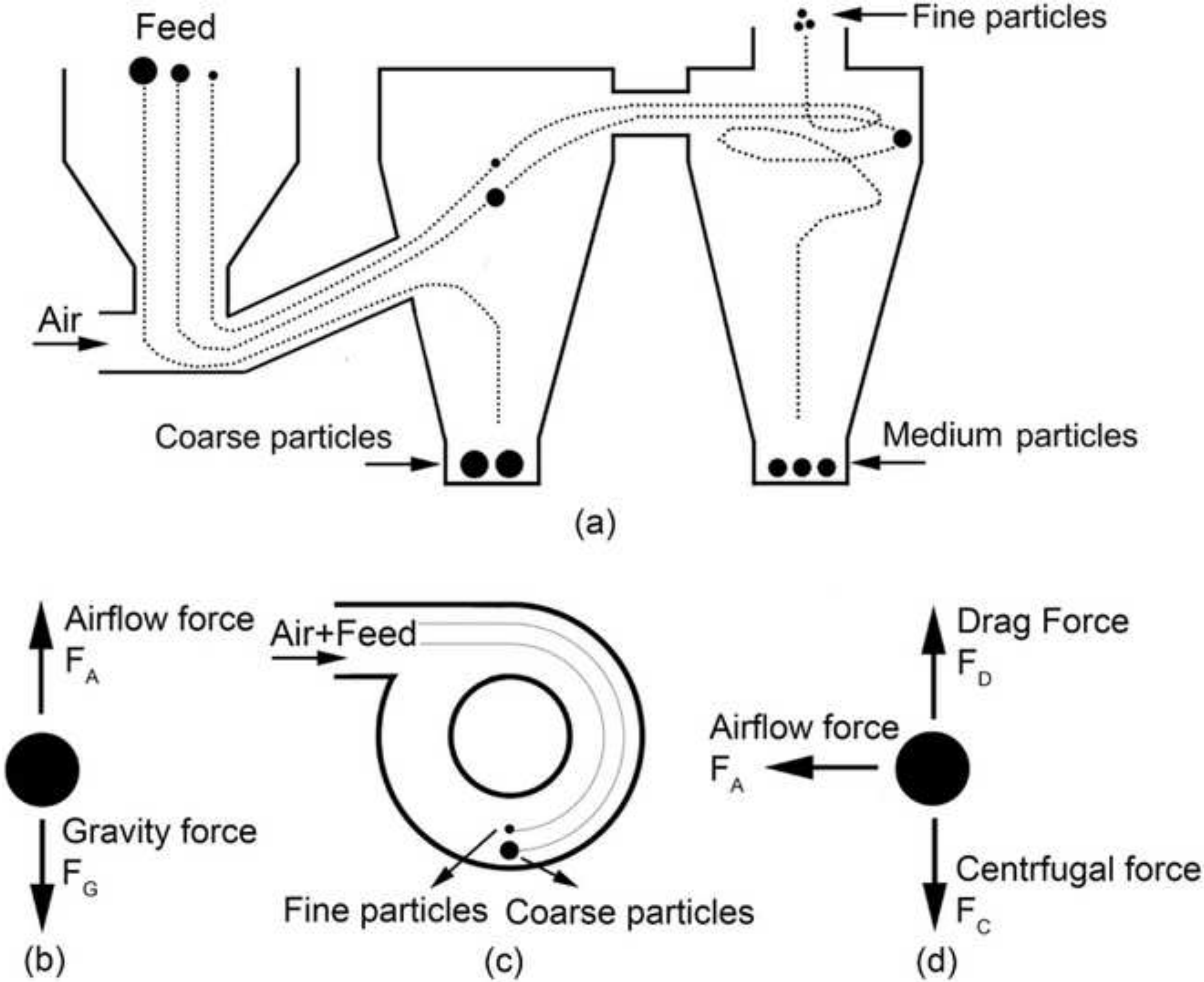

Table(s)

Table 1 Fitting coefficients of RRSB and lognormal distributions for ungraded and graded PC

Sample No.	RRSB distribution			Lognormal distribution		
	x_e	n	b	d_0	δ	A
C0	15.321	0.958	7.313E-02	43.979	1.104	245.231
C1	1.580	2.806	2.771E-01	1.701	0.404	21.129
C2	3.395	3.348	1.671E-02	3.651	0.374	48.247
C3	6.430	3.962	6.281E-04	6.772	0.310	90.676
C4	8.883	4.595	4.379E-05	9.277	0.260	124.267
C5	14.375	5.585	3.429E-07	14.873	0.225	204.842
C6	23.205	5.885	9.204E-09	24.01	0.218	335.194
C7	30.364	6.167	7.217E-10	31.437	0.203	434.289
C8	40.106	6.334	7.009E-11	41.428	0.199	573.082

Table 2 Fitting coefficients of RRSB and lognormal distributions for ungraded and graded BFS

Sample No.	RRSB distribution			Lognormal distribution		
	x_e	n	b	d_0	δ	A
B0	11.870	0.932	9.957E-02	56.680	1.318	265.486
B1	2.092	1.890	2.478E-01	2.696	0.623	30.598
B2	5.295	3.095	5.750E-03	5.717	0.371	72.692
B3	7.848	4.436	1.074E-04	8.196	0.279	111.173
B4	10.284	4.998	8.732E-06	10.652	0.252	146.129
B5	17.694	5.319	2.308E-07	18.356	0.239	254.718
B6	20.258	5.769	2.899E-08	20.949	0.217	288.593
B7	23.738	6.287	2.255E-09	24.188	0.191	325.372

Table 3 Fitting coefficients of RRSB and lognormal distributions for ungraded and graded FA

Sample No.	RRSB distribution			Lognormal distribution		
	x_e	n	b	d_0	δ	A
F0	11.791	0.863	7.313E-02	54.019	1.421	215.276
F1	1.651	3.240	1.969E-01	1.777	0.383	23.365
F2	5.720	3.702	1.570E-03	6.060	0.341	81.565
F3	6.886	4.198	3.038E-04	7.251	0.303	99.204
F4	9.023	4.667	3.477E-05	9.383	0.272	128.841
F5	10.992	5.143	4.426E-06	11.410	0.244	156.690
F6	16.615	5.213	4.336E-07	17.186	0.242	236.089
F7	18.926	5.652	6.047E-08	19.623	0.223	271.426
F8	27.918	5.910	2.853E-09	29.014	0.213	402.037

Table 4 Errors of RRSB and lognormal distributions for fitting the PSD of the ungraded PC, BFS and FA

Sample No.	Average absolute error		Average relative error (%)	
	RRSB	Lognormal	RRSB	Lognormal
C0	0.21	0.41	73.1	84.7
B0	0.30	0.46	80.0	165.0
F0	0.35	0.09	107.1	90.4

Note: RRSB represents RRSB distribution and Lognormal represents lognormal distribution

Table 5 Errors of RRSB and lognormal distributions for fitting the PSD of the ungraded and graded PC

Sample No.	Average absolute error		Average relative error (%)	
	RRSB	Lognormal	RRSB	Lognormal
C0	0.21	0.41	73.1	84.7
C1	0.84	0.45	68.1	40.2
C2	1.07	0.68	212.4	86.7
C3	1.21	0.92	140.9	49.0
C4	1.21	0.93	59.4	54.5
C5	1.50	1.37	106.6	35.0
C6	2.13	1.22	457.3	82.1
C7	1.81	0.99	120.2	50.5
C8	1.86	0.95	151.9	51.2

Table 6 Errors of RRSB and lognormal distributions for fitting the PSD of the ungraded and graded BFS

Sample No.	Average absolute error		Average relative error (%)	
	RRSB	Lognormal	RRSB	Lognormal
B0	0.30	0.46	80.0	165.0
B1	0.45	0.31	171.4	66.4
B2	0.71	0.29	56	45.6
B3	1.37	0.9	161.2	50.3
B4	1.58	0.99	179.3	54.2
B5	2.03	1.39	138.9	73.7
B6	1.7	1.02	104.6	64.2
B7	2.01	1.11	110.9	55.7

Table 7 Errors of RRSB and lognormal distributions for fitting the PSD of the ungraded and graded FA

Sample No.	Average absolute error		Average relative error (%)	
	RRSB	Lognormal	RRSB	Lognormal
F0	0.35	0.09	107.1	90.4
F1	1.14	0.73	199.6	60.6
F2	1.39	0.90	426.3	132.3
F3	1.62	1.17	155.4	235.9
F4	1.60	1.12	259.7	68.1
F5	1.51	0.88	160.0	49.1
F6	1.73	0.84	209.8	42.2
F7	1.77	1.25	107.9	123.8
F8	1.55	1.30	168.0	72.7

Table 8 $Adj.R^2$ of RRSB distribution fitting and lognormal distribution fitting for the ungraded FA in [25-28]

Sample number	$Adj.R^2$	
	RRSB distribution fitting	Lognormal distribution fitting
1[24]	0.733	0.958
2[25]	0.805	0.896
3[26]	0.820	0.952
4[27]	0.946	0.971

Note: In [26], there were four FA. The FA which was called HCA 2 is used in the present study