

Evaluation of Empirical Methods for Estimating Breaching Parameters of Dikes

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Abstract. Dike breaching parameters are important to support dike flood risk management, and can be estimated rapidly using empirical models. Due to the lack of dike specific models, sometimes models for man-made dams were used without validation. This paper compares relevant factors of dike, man-made dam and landslide dam breaches. With the aid of two dike breaching cases, the prediction capabilities of different dike, man-made dam and landslide dam models are studied. It is clearly shown that the models for man-made dams and landslide dams should not be used for dike breaching analysis.

Keywords. Dike breaching, breaching parameters, empirical models, flood risk

1. Introduction

Dike breaching parameters can be estimated with the aid of physical process based mathematical models or empirical equations to support risk management decisions (Zhang et al., 2013). Physical process based models are sophisticated (Visser, 1998; Zhu, 2006; Morris, 2011) and considered as more reliable methods, but their outcomes are closely related to required inputs such as the erodibility coefficient, critical shear stress against erosion, etc. However, such site-specific soil physical properties may not be easily available. As the rate of erosion varies in a wide range, from 0.001 to $1000 \text{ cm}^3\text{s}^{-1}\text{N}$, its estimate may be associated with large errors, which may vastly affect prediction results.

Empirical equations for assessing the breaching parameters of dikes are rather limited in the literature. As a result, empirical models for dams are sometimes used to estimate dike breaching parameters without validation.

This paper aims to evaluate empirical models for estimating breaching parameters of dikes. The differences in the breaching mechanisms of dikes, man-made dams, and landslide dams are compared first. Then the capabilities of some dike, man-made dam and landslide dam models are evaluated numerically with the aid of two case studies to estimate dike breaching parameters.

2. Comparison of Dikes, Man-Made Dams, Landslide Dams and Their Breaches

A comprehensive database of past dike breaches, with over 1000 cases, has been established at The Hong Kong University of Science and Technology. With the aid of this database, dikes and their breaching mechanisms can be compared with those of man-made dams (Xu and Zhang, 2009) and landslide dams (Peng and Zhang, 2012). The most relevant factors and their effects are detailed hereby:

- *Hydraulic conditions of the breaching process.* Both man-made and landslide dams store limited amount of water, which sets a limit to the progression of the breach. In the case of dikes, the amount of incoming water depends on meteorological and hydrological conditions. Long lasting floods may result in an extremely long development time and breach length.
- *Scouring.* Most dikes are founded on erodible soils, while man-made dams are often built on solid bedrock.
- *Construction materials.* Man-made dams and most dikes (except natural dikes such as dunes) are built of selected materials, under quality control and compacted, while materials of landslide dams are formed by natural deposition and are

highly variable.

- *Flood control measures.* Man-made dams are built with flood control measures (spillways, etc.); dike systems may include elements for such purposes; landslide dams are usually lack of any flood control measures or on-site flood defence.
- *Breach geometry.* Dikes are distributed systems, hence their breaches may be longer than those of dams.
- *Structural geometry.* Man-made dams are usually higher and have steeper slopes than dikes. The crests of landslide dams are usually wider and their slopes are gentler than those of dikes.

Based on the comparison, though similarities can be found, there are serious differences between dikes and dams from the perspective of the physical process. Therefore, the application of models for man-made dams or landslide dams to estimate dike breaching parameters is questionable.

3. Empirical Models for Estimating Breaching Parameters

Based on a comprehensive database, Danka and Zhang (2015) derived new multivariate equations to estimate three dike breaching parameters: breaching length along the dike, breaching depth and peak discharge (Table 1). Unfortunately, the breaching time was not studied due to limited data. Nagy (2006) proposed an empirical equation to approximate the breaching length as a function of the dike height (Table 1). In Danka and Zhang (2015), the two models were compared based on case studies and the mean bias factor (i.e., mean value of the ratio of observed value and predicted value). The multivariate models are shown to perform better.

Verheij (2002) proposed two equations (Table 1) to estimate the breaching length of dikes built of fine and coarse grained material, respectively as a function of the breaching time. Using the breaching time as control variable is closely related to the physical sense of the breaching process, but its *prior* estimate may include large uncertainties, which likely

increases prediction uncertainties.

Several existing models for estimating the breaching parameters of man-made dams and landslide dams are summarised in Table 1. The following points can be noted:

- The man-made dam models of the US Bureau of Reclamation (1982, 1988) are simple and easy to use.
- The multivariate man-made dam models of Xu and Zhang (2009) are complex; their prediction capabilities are better than that of Froehlich's multivariate models in terms of the model biases (Xu and Zhang, 2009). The use of reservoir volume as a control variable may make the models difficulty to apply to dikes.
- Peng and Zhang (2012) developed equations for landslide dams. Both reservoir volume and dam volume are used as control variables, which make these models hardly applicable to dikes.

4. Case Studies

4.1. Methodology for Comparing Different Empirical Methods

The studied empirical models are listed in Table 1. The symbols are defined in the footnotes. Sophisticated man-made dam and landslide dam models use reservoir volume (v) and dam volume (d) as control variables, which can be hardly defined for dikes. In the case of a dike, the reservoir volume can be taken approximately as the amount of total breach water (i.e. $Q_{total} = v$), which is available for back analysis. In risk assessment, the range of its value might be estimated based on a study of regional dike breaching cases and the related inundation. The dam volume may be assumed as the volume of a 200 m stretch of dike not including the foundation.

The applicability of complex man-made dam or landslide dam models is questionable from the very first moment as control variables are difficult to be estimated. For risk management purposes their use is challenging.

Table 1. Empirical equations for estimating dike, man-made dam and landslide dam breaching parameters studied in this paper.

	References	Model ^{a, b}	Values of indicator variables
Dikes	Danka and Zhang (2015)	$L = 0.08 \frac{W^{3.06} e^m}{h^{2.11} e^f}$	$m: 0.38 (c), 0.42 (f), 0.35 (org.);$ $t: 0.94 (comp.), 0 (earth.)$
		$D = 0.91h^{1.02} e^f$	$f: 0.74 (ext.), 1.21 (int.), 0.81 (slope),$ $1.15 (others)$
		$Q_{peak} = 2.55 \frac{h^{1.14} W^{0.58} e^m}{e^f}$	$m: 2.84 (c), 2.60 (f), 1.47 (org.);$ $t: 0.93 (comp.), 0 (earth.)$
		$Q_{peak} = 4.15 \frac{h^{1.67} e^m}{e^f}$	$m: 3.47 (c), 3.20 (f), 2.01 (org.);$ $t: 1.0 (comp.), 0 (earth.)$
Verheij (2002)		$L = 67 \log \left(\frac{T}{522} \right)$ (coarse grained – ‘sandy’)	
		$L = 20 \log \left(\frac{T}{288} \right)$ (fine grained – ‘clayey’)	
Nagy (2006)		$L = 5.1699e^{0.7498h}$	
Man-made dams	U.S. Bur. Recl. (1982, 1988)	$L = 3h$	
		$Q_{peak} = 19.1h^{1.85}$	
	Xu and Zhang (2009) ($H_{r1} = 15 \text{ m}, T_{r1} = 1 \text{ hour}$)	$\frac{L_t}{D} = 1.062 \left(\frac{h}{H_{r1}} \right)^{0.092} \left(\frac{v^{1/3}}{h_w} \right)^{0.508} e^{t+f+m}$, and $H_r = 15 \text{ m}$	$t: 0.061 (core.), 0.088 (c.f.), -0.089 (earth.);$ $f: 0.299 (ext.), -0.239 (int.);$ $m: 0.411 (h), -0.062 (m), -0.289 (l)$
		$\frac{L_a}{D} = 0.787 \left(\frac{h}{H_{r1}} \right)^{0.133} \left(\frac{v^{1/3}}{h_w} \right)^{0.652} e^{t+f+m}$	$t: -0.041 (core.), 0.026 (c.f.), -0.226 (earth.);$ $f: 0.149 (ext.), -0.389 (int.);$ $m: 0.291 (h), -0.140 (m), -0.391 (l)$
		$\frac{D}{H_{r1}} = 0.453 - 0.025 \left(\frac{h}{H_{r1}} \right) + (t + f + m)$	$t: 0.145 (core.), 0.176 (c.f.), 0.132 (earth.);$ $f: 0.218 (ext.), 0.236 (int.);$ $m: 0.254 (h), 0.168 (m), 0.031 (l)$
		$\frac{Q_p}{\sqrt{g}v^{5/3}} = 0.175 \left(\frac{h}{H_{r1}} \right)^{0.199} \left(\frac{v^{1/3}}{h_w} \right)^{-1.274} e^{t+f+m}$	$t: -0.503 (core.), -0.591 (c.f.), -0.649 (earth.);$ $f: -0.705 (ext.), -1.039 (int.);$ $m: -0.007 (h), -0.375 (m), -1.362 (l)$
		$\frac{T}{T_{r1}} = 0.304 \left(\frac{h}{H_{r1}} \right)^{0.707} \left(\frac{v^{1/3}}{h_w} \right)^{1.228} e^{t+f+m}$	$t: -0.327 (core.), -0.674 (c.f.), -0.189 (earth.);$ $f: -0.579 (ext.), -0.611 (int.);$ $m: -1.205 (h), -0.564 (m), 0.579 (l)$
	Landslide dams	Peng and Zhang (2012) ($H_{r2} = 1 \text{ m}, T_{r2} = 1 \text{ hour}$)	$\frac{L_t}{H_{r2}} = \left(\frac{h}{H_{r2}} \right)^{0.752} \left(\frac{h}{w} \right)^{0.315} \left(\frac{d^{1/3}}{h} \right)^{-0.243} \left(\frac{v^{1/3}}{h} \right)^{0.682} e^m$
$\frac{L_b}{h} = 0.004 \left(\frac{h}{H_{r2}} \right) + 0.050 \left(\frac{h}{w} \right) - 0.044 \left(\frac{d^{1/3}}{h} \right) + 0.088 \left(\frac{v^{1/3}}{h} \right) + m$			$m: 0.775 (h), 0.532 (m)$
$\frac{D}{H_{r2}} = \left(\frac{h}{H_{r2}} \right)^{0.882} \left(\frac{h}{w} \right)^{-0.041} \left(\frac{d^{1/3}}{h} \right)^{-0.099} \left(\frac{v^{1/3}}{h} \right)^{0.139} e^m$			$m: -0.316 (h), -0.520 (m)$
$\frac{Q_{peak}}{g^{1/2}h^{5/2}} = \left(\frac{h}{H_{r2}} \right)^{-1.417} \left(\frac{h}{w} \right)^{-0.265} \left(\frac{d^{1/3}}{h} \right)^{-0.471} \left(\frac{v^{1/3}}{h} \right)^{1.569} e^m$			$m: 1.276 (h), -0.336 (m), -1.532 (l)$
	$\frac{T}{T_{r2}} = \left(\frac{h}{H_{r2}} \right)^{0.262} \left(\frac{h}{w} \right)^{-0.024} \left(\frac{d^{1/3}}{h} \right)^{-0.103} \left(\frac{v^{1/3}}{h} \right)^{0.705} e^m$	$m: -0.635 (h), -0.518 (m)$	

^a Breaching parameters: L – breaching length (rectangular channel), L_t – breaching length at the top of a trapezoid breaching channel, L_a – averaged breaching length of a trapezoid breaching channel, L_b – breaching length at the bottom of a trapezoid breaching channel, D – breaching depth, Q_{peak} – peak discharge, T – breaching time.

^b Control variables: h – dike height, h_w – height of water behind structure ($\approx h$), w – dike width, m – material/erodibility (c – coarse soil, f – fine soil, $org.$ – organic soil, h, m, l – high, medium, low erodibility), f – failure mechanisms ($ext.$ – external erosion, $int.$ – internal erosion, $slope$ – slope instability, $others$ – other failure mechanisms), t – type of dike/dam ($comp.$ – composite dike, $earth.$ – earthen homogeneous or zoned dike, $core$ – dam with a core-wall, $c.f.$ – concrete faced dam), v – reservoir volume in 10^6 m^3 , d – volume of landslide dam 10^6 m^3 .

Table 2. Comparison of prediction capability of empirical models: input parameters, observed breaching parameters and prediction results

		Tarpa - Section 55+650 bank of Tisza	17 th Street Canal Dike	
Input data				
Height of dike, h [m]		3.0 (2.9-3.2)	3.5 (2.5-3.5) (~5.0-6.0 with I-wall)	
Width of dike, w [m]		17.5	25.0	
Dike material		Clay (fine)	Clay with organic subsoil (org.)	
Volume of dike, d [10^6 m ³]		0.00645 (assuming 32.25 m ³ /m and the length of 200 m)	0.00872 (assuming 43.61 m ³ /m; length of 200 m)	
Observation				
Breaching length, L [m]		60-70 (after 4-5 hours) 110-140 (at closure hours)	61 (after sliding) 137 (final)	
Breaching depth, D [m]		3.0-3.5	8-9	
Peak discharge, Q_{peak} [m ³ /s]		400-450	510-820	
Breaching time, T [hours]		4-5 / 72	~4	
Total breach water, Q_{total} [10^6 m ³]		over 60	over 60 (8hrs + 20 hrs) over 110 (8hrs + 40 hrs)	
Prediction				
Dikes	Danka and Zhang	$L = 73.3$ m	$L = 59.8$ m (composite); $L = 153.0$ m (earthen)	
		$D = 5.9$ m	$D = 10.3$ m	
		$Q_{peak}(h,w,m,t) = 803$ m ³ /s	$Q_{peak}(h,w,m,t) = 110$ m ³ /s (composite) $Q_{peak}(h,w,m,t) = 299$ m ³ /s (earthen)	
		$Q_{peak}(h,m,t) = 803$ m ³ /s	$Q_{peak}(h,w,m,t) = 92$ m ³ /s (composite) $Q_{peak}(h,w,m,t) = 251$ m ³ /s (earthen)	
Dikes	Verheij (2002)	$L = 34.0$ m (fine, $T = 4$ hrs)		
		$L = 35.9$ m (fine, $T = 5$ hrs) $L = 59.1$ m (fine, $T = 72$ hrs)		
Dikes	Nagy (2006)	$L = 49.0$ m	$L = 71.3$ m	
Man-made dams	US Bureau of Reclamation (1982, 1988)	$L = 9$ m	$L = 10.5$ m	
		$Q_{peak} = 146$ m ³ /s	$Q_{peak} = 194$ m ³ /s	
		$T_{calc} = 0.7$ h ($L = 60$ m) $T_{calc} = 1.5$ h ($L = 140$ m)	$T_{calc} = 1.5$ h	
	Man-made dams	Xu and Zhang (2009)	$L_t = 30.2$ m	$L_t = 75.4 / 151.9$ m ($m = low/high$, $v = 60 \cdot 10^6$ m ³) $L_t = 83.6 / 168.2$ m ($m = ll/h$, $v = 110 \cdot 10^6$ m ³)
			$L_a = 28.6$ m	$L_a = 70.4 / 139.2$ m ($m = low/high$, $v = 60 \cdot 10^6$ m ³) $L_a = 80.3 / 158.8$ m ($m = ll/h$, $v = 110 \cdot 10^6$ m ³)
			$D = 12.4$ m	$D = 12.4 / 15.8$ m ($m = low/high$)
Landslide dams	Peng and Zhang (2012)	$Q_{peak} = 161$ m ³ /s	$Q_{peak} = 202 / 783$ m ³ /s ($m = low/high$, $v = 60 \cdot 10^6$ m ³) $Q_{peak} = 259 / 1003$ m ³ /s ($m = ll/h$, $v = 110 \cdot 10^6$ m ³)	
		$T = 32.0$ h	$T = 29.5 / 5.0$ m ($m = low/high$, $v = 60 \cdot 10^6$ m ³) $T = 37.8 / 6.3$ m ($m = ll/h$, $v = 110 \cdot 10^6$ m ³)	
		$L_t = 77.5$ m	$L_t = 74.5 / 120.6$ m ($m = med./high$, $v = 60 \cdot 10^6$ m ³) $L_t = 85.5 / 138.4$ m ($m = m/h$, $v = 110 \cdot 10^6$ m ³)	
		$L_b = 35.3$ m	$L_b = 35.5 / 36.3$ m ($m = med./high$, $v = 60 \cdot 10^6$ m ³) $L_b = 43.2 / 44.0$ m ($m = m/h$, $v = 110 \cdot 10^6$ m ³)	
		$D = 2.8$ m	$D = 3.1 / 3.9$ m ($m = med./high$, $v = 60 \cdot 10^6$ m ³) $D = 3.2 / 4.0$ m ($m = m/h$, $v = 110 \cdot 10^6$ m ³)	
		$Q_{peak} = 3135$ m ³ /s	$Q_{peak} = 3147 / 52170$ m ³ /s ($m = low/high$, $v = 60 \cdot 10^6$ m ³)	
	$T = 21.3$ h	$T = 20.1 / 17.9$ h ($m = med./high$, $v = 60 \cdot 10^6$ m ³) $T = 23.2 / 20.6$ h ($m = m/h$, $v = 110 \cdot 10^6$ m ³)		

4.2. Tisza Dike Breach in Tivadar, Hungary at Section 55+650 on 6 March 2001

After a relatively dry period, cyclones passed by Central and Western Europe on 3 and 4 March 2001 followed by heavy rainfall. Warnings were released in advance, but the Hungarian Meteorological Service underestimated the precipitation for the upcoming 3 days: 50 mm rainfall was forecasted but around 135 mm was observed.

The water level of the Tisza rose quickly after the rain hit the region. Overtopping of dikes in the area seemed likely; hence the construction of sandbag wall was started immediately. The first slope failures were noticed between 5-8 am on 6 March next to the town of Tivadar. After 11 am, 16 other slope failures were recorded in 3.5 hours.

The dike system breached first at 55+650 on 6 March around 1 pm. The dike consisted of two zones of clay with different rates of compaction, and had a height of 2.9-3.2 m, slope inclinations of 1:2.5 on the water side and 1:2 on the defended site, and a crest width of about 4 m. The dike breaching process is as follows:

- The defence could not keep up with the rate of water level rising, and the 40-50 cm tall sandbag wall was overtopped.
- At 1 pm, the defended slope failed by 5-6 m along the river.
- Shortly after, micro-instability triggered failure moved the top layer of the crest, and opened a gap on the sandbag wall.
- The dike was severely overtopped and, in about 30 minutes, the dike breached. The breach length at this time was 5-6 m.
- Between 5-6 pm (in 4-5 hours), the breach length reached 60-70 m along the river. The final breach length at the time of closure was 110-140 m.

Overall more than 60 million m³ of water passed through the breach; the peak discharge was estimated between 4-450 m³/s (Table 2).

4.3. 17th Street Canal Dike Breach, New Orleans, August 2005.

17th Street Canal Dike, New Orleans was a composite dike (3-3.5 m earthen dike) with an embedded I-wall 1.5-2.0 m in height above the

dike, founded on a layer of highly erodible marsh. The approximate dike width was 25 m. Since dike breaching cases related to Katrina Hurricane were extensively studied in the literature (for instance Rogers et al. 2008; Steedman and Sharp, 2011), the breaching process is only reported briefly here:

- The water level rose behind the I-wall causing landward rotation of the wall.
- Due to the displacements of the I-wall, a gap formed between the soil and the wall, which was filled with water immediately, resulting in decreased flow path.
- The passive earth pressure was mobilized behind the wall. As the shear strength of the underlying marsh was not sufficient, the failure progressed backwards, causing further rotation of the I-wall.
- Finally, the dike with the embedded I-wall slid 15 m horizontally, opening a gap (61 m), inundating homes and destroying buildings in the Lakeview area.
- The I-wall on both sides of the breaching channel continued to fail piecemeal as its foundation material was being washed away. The I-wall could not slow down the breach growth, and the dike section behaved as an earthen dike due to the highly erodible underlying marsh. The final breaching length reached 137 m.

IPET (2006) presented breach discharge plots for the first 8 hours. The peak discharge is estimated between 510-820 m³/s. Including data of the first 8 hours and extrapolating for the upcoming 20 or 40 hours lead to over 60 to 110 million m³ of discharge (Table 2).

4.4. Comparison of Prediction Results

The input control variables, observed values of dike breaching parameters, and prediction results are all presented in Table 2. Among the 3 dike specific models, Danka and Zhang (2015) performed the best in the prediction of breaching length in both cases. The final breach length of the Hungarian case was not captured properly (73.3 m vs. 110-140 m); the prediction is close to what was observed after 4-5 hours (60-70 m). In Table 2, two conditions are evaluated related to the breach of the 17th Street canal dike: considering a composite dike ($L = 59.8$ m) and

assuming an earthen dike ($L = 153$ m). The predictions are close to those after the sliding (61 m) and the final stage (137 m). The breaching depth is slightly overestimated for the Hungarian case (3.5 m vs. 5.9 m), but properly estimated for the 17th Street Canal dike (8-9 m vs. 10.9 m). The peak discharge estimates appear to be the least reliable.

The man-made dam models of the US Bureau of Reclamation (1982, 1988) underestimate vastly the dike breaching parameters. The man-made dam equations of Xu and Zhang (2009) are unable to predict the dike breaching parameters of dikes of low erodibility. In the case of organic or highly erodible dikes, the predictions are more reasonable. However the use of the multivariate model is challenging as the reservoir volume must be assumed.

Key differences in the breaching processes of dikes and landslide dams are reflected in the prediction results as well. Using landslide dam equations, the predicted breaching channels are trapezoidal, rather than rectangular, and the estimated peak discharges are extremely large. Empirical equations for landslide dams are not recommended for dikes.

5. Conclusions

Comparison of dikes, man-made dams and landslide dams, and a study of dike breaching parameters estimated with various empirical models lead to the following conclusions:

- Key differences among the three structures can be found when their breaches are studied: the hydraulic conditions of dam and dike breaches are different; scouring plays a more important role in dike breaching development.
- Among the models for estimating dike breaching parameters, the Danka and Zhang equations perform the best. Nagy (2006) delivers reasonable breaching length estimates.
- Easy-to-use man-made dam models vastly underestimate dike breaching parameters. Multivariate man-made dam or landslide dam models are not practical for dikes. Some control variables are generally not available and these equations are not recommended for dikes.

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