Nonexistence of pure *S*- and *P*-polarized surface waves at the interface between a perfect dielectric and a real metal

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(Received 2 December 2013; published 19 February 2014)

It is known that, at optical frequencies, a simple interface between a perfect dielectric and a real metal can sustain the propagation of surface plasmon polaritons only for *P*-polarized electromagnetic waves, being *S*-polarized surface plasmons are prohibited. In this work, we formally show that, strictly speaking, both polarization states are in fact prohibited and that only *P*-polarized pseudosurface waves are allowed, which is what is encountered in the applications. The existence of such pseudosurface modes allows one to reconcile theory and experimental evidence, but also sets limits for them to be considered as modes bound to the interface.

DOI: 10.1103/PhysRevA.89.023834 PACS number(s): 42.25.−p, 73.20.Mf

I. INTRODUCTION

Surface plasmon polaritons (SPPs) are collective oscillations of electrons in metals which are coupled to electromagnetic waves and typically appear at metal-dielectric interfaces. At optical frequencies, the importance of such oscillations mostly resides in a resulting strong-field enhancement on a nanometer scale which has found plenty of important applications in different fields such as nanosensing, light harvesting, lighting, and super-resolution near-field imaging, and has given birth to the very active current research field called plasmonics [1–10]. Typically, the origin of such collective modes is proven by looking for possible solutions of Maxwell equations in the presence of an interface between a dielectric and a metal, when no sources for the field are present in the region of interest and there are no incident fields. This leads to the well-known dispersion curve for surface plasmons reported by several authors already (see, among others, [11,12]). From this approach also follows the known property that SPPs can only be excited by means of *P*-polarized light, with *S*-polarized surface plasmon polaritons being prohibited. In the present work, we take a deeper look at the existence conditions of SPPs and at the formal derivation of the dispersion curve of SPPs. More specifically, we will focus on the common case of an interface between a perfect dielectric and a real metal and we will show that, under the assumption usually made to derive the dispersion curve of SPPs, none of the two polarization states, *S* or *P*, can in fact give rise to a perfect surface wave. The origin of such nonexistence is, however, different for the two polarization states which allows one to reconcile theoretical findings and experiments by resorting to the approximations typically found in the literature. As will be clearer later in the paper, the main goal of our work is just to point out that, on a formal point of view, attention should be paid when introducing any simplification in the underlying physical model, since this can often lead to a solution which is no longer admissible. This is even more important if one aims to improve the agreement, not only qualitative but also quantitative, between theory, experiments, and numerical simulations, as pointed out by Barnes in a recent review on this subject [13]. Also, a more careful look at the foundations can lead to new interesting physical predictions as well, as recently reported by Norrman *et al*. [14] in a work where the inadequacy of approximate solutions usually found in the literature along with a new type of backward-propagating surface waves are discussed.

The paper is organized as follows. In Sec. II, we define the reference framework, the general properties, and the geometry for the materials involved. Additionally, we recall the conditions, coming directly from Maxwell equations and the jump conditions at the interface for the electric and magnetic fields, which a surface wave must satisfy to exist. In Sec. III, the case of a perfect surface wave is considered, i.e., a wave that propagates only parallel to the interface, and we prove that such a wave cannot exist, whatever its polarization state. In Sec. IV, the case of a pseudosurface is considered. Finally, in Sec. V, the results of our work are summarized.

II. SURFACE WAVE AT THE INTERFACE BETWEEN A PERFECT DIELECTRIC AND A REAL METAL

Let us suppose that we want to find conditions under which a monochromatic surface wave can exist at the interface between a dielectric, characterized by the relative electric permittivity $\varepsilon_1 = \varepsilon_{1r} + i\varepsilon_{1i}$, complex magnetic permeability $\mu_1 = \mu_{1r} + i\mu_{1i}$, and electric conductivity σ_1, and a metal endowed with relative electric permittivity $\varepsilon_2 = \varepsilon_{2r} + i\varepsilon_{2i}$, magnetic permeability $\mu_2 = \mu_{2r} + i\mu_{2i}$, and electric conductivity σ_2. All of these quantities are, in general, functions of the angular frequency ω, but we will not indicate this dependency explicitly throughout the paper. Additionally, we will consider the dielectric to be ideal (i.e., absorption free), which is equivalent to saying that $\varepsilon_{1i} = \sigma_1 = 0$. A simple sketch of the interface geometry is shown in Fig. 1. In any of the two homogeneous regions, the field should, of course, be a proper solution of Maxwell equations [for a monochromatic wave of angular frequency ω, the time dependence is assumed to be given by the factor $\exp(-i\omega t)$, $\omega > 0$, which is omitted throughout this paper],

$$\nabla \times \mathbf{E}(\mathbf{r}) = i\omega\mu_0\mu_l\mathbf{H}(\mathbf{r}), \quad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = -i\omega\varepsilon_0\varepsilon_l\mathbf{E}(\mathbf{r}), \quad (2)$$

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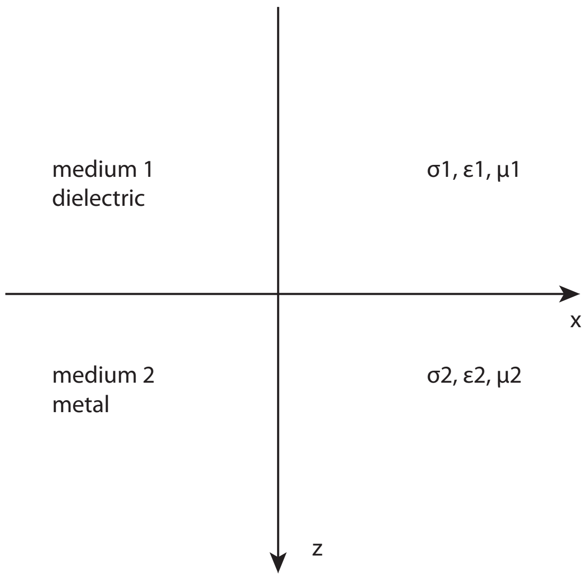

FIG. 1. Simple sketch of an interface between two media. In the case where medium 1 is a perfect dielectric, $\sigma_1 = 0$ and ε_1 is purely real.

where $l = 1,2$ denotes one of the two media. These equations must be complemented by the boundary conditions at the interface between mediums 1 and 2. We are interested in a unique solution that is evanescent in both domains, i.e., in the dielectric and the metal. In order to do so, we need to discuss separately the two orthogonal polarizations states, S and P.

A. Conditions for the existence of *S*-polarized surface waves

With reference to Fig. 1, an S-polarization configuration involves only $E_y(x,z), H_x(x,z)$, and $H_z(x,z)$ field components, with $E_y(x,z)$ essentially playing the role of the potential. We can write the expression of E_y in both media as

$$E_y^{(1)}(x,z) = A_E^{(1)} \exp(i\boldsymbol{k}_1 \cdot \boldsymbol{r}), \tag{3}$$

$$E_y^{(2)}(x,z) = A_E^{(2)} \exp(i\boldsymbol{k}_2 \cdot \boldsymbol{r}), \tag{4}$$

with $\boldsymbol{k}_1 = (k_{1x},0,k_{1z})$ and $\boldsymbol{k}_2 = (k_{2x},0,k_{2z})$ (generally complex) wave vectors and $\boldsymbol{r} = (x,y,z)$. $A_E^{(1)}$ and $A_E^{(2)}$ are the complex amplitudes for the two fields. The indexes 1 and 2 refer to the first and second mediums, respectively. The x components of the magnetic field in both media are given by Eq. (1),

$$H_x^{(1)}(x,z) = -A_E^{(1)} \frac{k_{1z}}{\omega\mu_0\mu_1} \exp(i\boldsymbol{k}_1 \cdot \boldsymbol{r}), \tag{5}$$

$$H_x^{(2)}(x,z) = -A_E^{(2)} \frac{k_{2z}}{\omega\mu_0\mu_2} \exp(i\boldsymbol{k}_2 \cdot \boldsymbol{r}). \tag{6}$$

First of all, at the boundary $(x,z=0)$, the tangential components of the magnetic and electric fields should be continuous. For S-polarized waves, this means that E_y and H_x are continuous. The continuity for the electric fields implies

$$A_E^{(1)} = A_E^{(2)}, \tag{7}$$

while that for the magnetic fields leads to the condition

$$\frac{k_{1z}}{\mu_1} = \frac{k_{2z}}{\mu_2}. \tag{8}$$

Since in this work we will only be dealing with natural materials, we can further simplify the problem by setting, from now on, $\mu_{1r} = \mu_{2r} = 1$ and $\mu_{1i} = \mu_{2i} = 0$ at optical wavelengths. However, we would like to recall that at longer wavelengths, or in the presence of properly designed metamaterials, such simplification might not apply and the analysis for S and P polarization becomes somehow specular. Under these assumptions, Eq. (8) leads to $k_{1z} = k_{2z}$. Additionally, the tangential components of the wave vector k_x should be preserved as well, namely, $k_{1x} = k_{2x}$. However, using the conditions $k_z^2 + k_x^2 = \omega^2\mu_0\varepsilon_0\varepsilon_l$, with again $l = 1,2$, it is easy to check that the conservation of both k_x and k_z is possible only in the case where the two media are actually the same, which is a possibility already excluded from the beginning. This forces one to exclude the existence of a surface wave associated to a S-polarization state, which is a well-known result. In the next section, we address the P-polarization case, which requires somehow a more careful analysis.

B. Conditions for the existence of *P*-polarized surface waves

In the P-polarization case, the magnetic field has only one component different from zero, which is H_y in our case. In both domains, we can write the solution as a plane wave,

$$H_y^{(1)}(x,z) = A_H^{(1)} \exp(i\boldsymbol{k}_1 \cdot \boldsymbol{r}), \tag{9}$$

$$H_y^{(2)}(x,z) = A_H^{(2)} \exp(i\boldsymbol{k}_2 \cdot \boldsymbol{r}), \tag{10}$$

where now $A_H^{(1)}$ and $A_H^{(2)}$ are the complex amplitudes for the two magnetic field components. As done before, it is better to list all of the conditions that the solutions (9) and (10) have to satisfy.

First of all, at the boundary $(x,z=0)$, the tangential components of the magnetic and electric fields should be continuous. For P-polarized waves, this means that H_y and E_x are preserved. E_x, in both domains, can be derived from H_y by means of the relations

$$E_x^{(1)}(x,z) = -A_H^{(1)} \frac{k_{1z}}{\omega\varepsilon_0\varepsilon_1} \exp(i\boldsymbol{k}_1 \cdot \boldsymbol{r}), \tag{11}$$

$$E_x^{(2)}(x,z) = -A_H^{(2)} \frac{k_{2z}}{\omega\varepsilon_0\varepsilon_2} \exp(i\boldsymbol{k}_2 \cdot \boldsymbol{r}). \tag{12}$$

Continuity of the H_y components at $z = 0$ implies that

$$A_H^{(1)} = A_H^{(2)}, \tag{13}$$

while the continuity of the E_x components leads to the condition

$$\frac{k_{1z}}{\varepsilon_1} = \frac{k_{2z}}{\varepsilon_2}. \tag{14}$$

Also, from any of the two boundary conditions for the fields, we have that the x component of the wave vectors must be preserved, that is,

$$k_{1x} = k_{2x}. \tag{15}$$

Generally speaking, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are both complex vectors. This means that it is possible to write them as

$$\boldsymbol{k}_1 = k_{1x}\boldsymbol{x}_0 + k_{1y}\boldsymbol{y}_0 + k_{1z}\boldsymbol{z}_0 = \boldsymbol{\beta}_1 + i\boldsymbol{\alpha}_1, \tag{16a}$$

$$\boldsymbol{k}_2 = k_{2x}\boldsymbol{x}_0 + k_{2y}\boldsymbol{y}_0 + k_{2z}\boldsymbol{z}_0 = \boldsymbol{\beta}_2 + i\boldsymbol{\alpha}_2, \tag{16b}$$

with $\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2 \in \mathrm{Re}^3$,

$$\boldsymbol{\beta}_1 = \beta_{1x}\boldsymbol{x}_0 + \beta_{1y}\boldsymbol{y}_0 + \beta_{1z}\boldsymbol{z}_0, \tag{17a}$$

$$\boldsymbol{\beta}_2 = \beta_{2x}\boldsymbol{x}_0 + \beta_{2y}\boldsymbol{y}_0 + \beta_{2z}\boldsymbol{z}_0, \tag{17b}$$

and

$$\boldsymbol{\alpha}_1 = \alpha_{1x}\boldsymbol{x}_0 + \alpha_{1y}\boldsymbol{y}_0 + \alpha_{1z}\boldsymbol{z}_0, \tag{18a}$$

$$\boldsymbol{\alpha}_2 = \alpha_{2x}\boldsymbol{x}_0 + \alpha_{2y}\boldsymbol{y}_0 + \alpha_{2z}\boldsymbol{z}_0, \tag{18b}$$

where $\boldsymbol{\beta}$ represents the propagation vector of the generic wave, while $\boldsymbol{\alpha}$ denotes the decaying vector. From Eqs. (16)–(18) and the assumption that only the H_y component is present, we have that

$$k_{1x} = \beta_{1x} + i\alpha_{1x}, \tag{19a}$$

$$k_{1y} = 0, \tag{19b}$$

$$k_{1z} = \beta_{1z} + i\alpha_{1z}, \tag{19c}$$

and

$$k_{2x} = \beta_{2x} + i\alpha_{2x}, \tag{20a}$$

$$k_{2y} = 0, \tag{20b}$$

$$k_{2z} = \beta_{2z} + i\alpha_{2z}. \tag{20c}$$

Since the fields in Eqs. (9) and (10) have to be solutions of the Helmholtz equation, the following conditions follow:

$$\boldsymbol{k}_1 \cdot \boldsymbol{k}_1 = \omega^2\mu_0\varepsilon_0\varepsilon_1, \tag{21a}$$

$$\boldsymbol{k}_2 \cdot \boldsymbol{k}_2 = \omega^2\mu_0\varepsilon_0\varepsilon_2. \tag{21b}$$

For the medium 1 (dielectric), this leads to

$$\boldsymbol{\beta}_1{}^2 - \boldsymbol{\alpha}_1{}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{1r}, \tag{22a}$$

$$\boldsymbol{\beta}_1 \cdot \boldsymbol{\alpha}_1 = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{1i}}{2} = 0. \tag{22b}$$

It follows that, in the dielectric, either there is no decaying ($\boldsymbol{\alpha}_1 = 0$) or the propagation and the decaying vectors are perpendicular to each other. Since we can always choose a Cartesian reference framework where $k_{1y} = k_{2y} = 0$ (as indeed we have done in this section), Eqs. (22) can be written in the equivalent way as

$$\beta_{1x}^2 + \beta_{1z}^2 - \alpha_{1x}^2 - \alpha_{1z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{1r}, \tag{23a}$$

$$\beta_{1x}\alpha_{1x} + \beta_{1z}\alpha_{1z} = 0. \tag{23b}$$

For the metal, we can write, in the same way,

$$\boldsymbol{\beta}_2{}^2 - \boldsymbol{\alpha}_2{}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{2r}, \tag{24a}$$

$$\boldsymbol{\beta}_2 \cdot \boldsymbol{\alpha}_2 = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{2i}}{2}, \tag{24b}$$

and, in terms of Cartesian components,

$$\beta_{2x}^2 + \beta_{2z}^2 - \alpha_{2x}^2 - \alpha_{2z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{2r}, \tag{25a}$$

$$\beta_{2x}\alpha_{2x} + \beta_{2z}\alpha_{2z} = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{2i}}{2}. \tag{25b}$$

Since, in the metal, $\varepsilon_{2i} \neq 0$, it follows that $\boldsymbol{\beta}_2$ and $\boldsymbol{\alpha}_2$ cannot be perpendicular to each other. This implies that, in a metal, the wave is propagating along $\boldsymbol{\beta}_2$ while it is attenuating along the direction of $\boldsymbol{\alpha}_2$. Finally, before concluding the section, we need to specify some facts on the signs for the real and the imaginary parts of the wave vectors in both half spaces. The solutions we are looking for must decay when moving away from the interface. This means that it must be $\alpha_{1z} \geqslant 0$ and $\alpha_{2z} \leqslant 0$ in order to have physically realizable fields. In the same way, a wave propagating along the z direction in both half spaces should have $\beta_{1z} \geqslant 0$ and also $\beta_{2z} \geqslant 0$, respectively.

III. P-POLARIZED PURE SURFACE WAVES: NO PROPAGATION ALONG THE z DIRECTION ($\beta_{1z} = 0$)

To be a true surface wave, the solution given in Eqs. (9) and (10) should not propagate in the z direction. This means that the real part of k_{1z} must vanish, namely, $\beta_{1z} = 0$. However, it is possible to prove that this solution cannot exist. In fact, Eq. (A.8b) would become

$$\beta_{1x}\alpha_{1x} = 0. \tag{26}$$

Since β_{1x} has to be different from zero (otherwise no wave would exist), we must conclude that $\alpha_{1x} = 0$ as well. Since the boundary condition in Eq. (15) can be separated into a real and imaginary part,

$$\beta_{1x} = \beta_{2x}, \tag{27a}$$

$$\alpha_{1x} = \alpha_{2x}, \tag{27b}$$

we get the condition $\alpha_{2x} = 0$, too. Hence, to summarize, the request $\beta_{1z} = 0$ leads to

$$\boldsymbol{\beta}_1 = (\beta_{1x},0,0), \tag{28a}$$

$$\boldsymbol{\alpha}_1 = (0,0,\alpha_{1z}). \tag{28b}$$

For the wave into the metal, we can have

$$\boldsymbol{\beta}_2 = (\beta_{2x},0,\beta_{2z}), \tag{29a}$$

$$\boldsymbol{\alpha}_2 = (0,0,\alpha_{2z}). \tag{29b}$$

Additionally, Eqs. (A.8) and (A.10) become

$$\beta_{1x}^2 - \alpha_{1z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{1r}, \tag{30a}$$

$$\beta_{1x}\alpha_{1x} = 0, \tag{30b}$$

and

$$\beta_{2x}^2 + \beta_{2z}^2 - \alpha_{2z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{2r}, \tag{31a}$$

$$\beta_{2z}\alpha_{2z} = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{2i}}{2}. \tag{31b}$$

There is still a further condition, derived from Eq. (14), that has not been discussed yet. That condition leads us to the following complex equation:

$$\frac{i\alpha_{1z}}{\varepsilon_{1r}} = \frac{\beta_{2z} + i\alpha_{2z}}{\varepsilon_{2r} + i\varepsilon_{2i}}, \tag{32}$$

which can be separated into a couple of equations,

$$\frac{\alpha_{1z}}{\varepsilon_{1r}} = \frac{\alpha_{2z}\varepsilon_{2r} - \beta_{2z}\varepsilon_{2i}}{\varepsilon_{2r}^2 + \varepsilon_{2i}^2}, \tag{33a}$$

$$\beta_{2z}\varepsilon_{2r} = -\alpha_{2z}\varepsilon_{2i}. \tag{33b}$$

Equations (30), (31), and (33) are five equations for only four unknowns, $\beta_{1x}, \beta_{2z}, \alpha_{1z}, \alpha_{2z}$. By combining Eqs. (31b) and (33b), one easily gets

$$\beta_{2z} = \sqrt{\frac{-\omega^2\varepsilon_{2i}^2}{2c^2\varepsilon_{2r}}} = k_0\sqrt{\frac{-\varepsilon_{2i}^2}{2\varepsilon_{2r}}}, \tag{34}$$

where c is the speed of light in vacuum and $k_0 = \omega/c$. It is evident that in order to have a solution (i.e., β_{2z} real), we must have $\varepsilon_{2r} < 0$, a condition attainable by using metals working below their plasma frequency. From the same equations, we get α_{2z}, which reads

$$\alpha_{2z} = -k_0\varepsilon_{2r}\sqrt{-\frac{1}{2\varepsilon_{2r}}}. \tag{35}$$

We see that, under the same requirement, $\varepsilon_{2r} < 0$, α_{2z} is positive as well, as it should be to not have a diverging solution in the metal (which is defined in the half space $z > 0$).

Once β_{2z} and α_{2z} are known, we can get α_{1z} from Eq. (33a),

$$\alpha_{1z} = -k_0\varepsilon_{1r}\sqrt{-\frac{1}{2\varepsilon_{2r}}}. \tag{36}$$

Again, we see that α_{1z} is negative, as it should be to not have a diverging wave into the dielectric (domain $z < 0$). Finally, we can compute the dispersion curve for this wave, which is the remaining unknown β_{1x}. We are, however, left with two equations to determine β_x: Eqs. (30a) and (31a). From the first of the two, we get

$$\beta_{1x}^2 = -k_0^2\varepsilon_{1r}\frac{\varepsilon_{1r} - \varepsilon_{2r}}{2\varepsilon_{2r}}, \tag{37}$$

while Eq. (31a) leads to

$$\beta_{1x}^2 = k_0^2\frac{\varepsilon_{2r}^2 + \varepsilon_{2i}^2}{2\varepsilon_{2r}}. \tag{38}$$

In order to find a valid value for $\beta_x = \beta_{1x} = \beta_{2x}$, both Eqs. (37) and (38) should lead to the same solution, which is clearly not possible, considering that Eq. (38) has no real solution for β_{1x} since $\varepsilon_{2r} < 0$. We can say that this is a direct consequence of having only four unknowns to be determined by solving a system of five independent equations. If we combine the results of Sec. II A with what has been obtained in the present section, we can conclude that *it is not possible to have a pure surface wave (i.e., endowed with a* $\beta_{1z} = 0$*) at the interface between a real metal and a perfect dielectric, no matter which polarization one is dealing with.*

This result represents the main message of our work. We would, however, like the reader to notice how this conclusion is a consequence of two distinct facts for the two polarization states. For *S*-polarized waves, it comes from three incompatible conditions (conservation of k_x and k_z at the interface, along with the conservation of the complex amplitudes A_E) and is essentially a consequence of the lack of contrast for the magnetic permeabilities of natural materials at optical wavelengths. On the other hand, for *P*-polarized waves, it is the lack of solutions for the dispersion relations in Eqs. (37) and (38), which does not allow the pure surface mode to exist. This conclusion appears to be in conflict with the myriad of studies performed so far on this subject, both theoretical and experimental, which have started from the assumption that a *P*-polarization-induced SPP does exist. In fact, phenomena induced by SPP excitation have been predicted and observed. One way to try to solve this apparent inconsistency can be based on including some absorption for the dielectric, i.e., an imaginary part of the electric permittivity $\varepsilon_{1i} \neq 0$ should be considered from the very beginning of the analysis. However, this would not justify the outcomes of experiments performed in air or vacuum. More importantly, we will show in the appendix A that even a $\varepsilon_{1i} \neq 0$ does not lead to an ideal surface wave. A reconciliation between theory and experiments is possible by surrendering the concept of pure surface wave without resorting to the presence of a lossy dielectric. In this approach, one considers a wave characterized by some propagation along the normal to the interface between the two media, but whose dynamics is still mostly dominated by the propagation along the surface. This second approach results in the usual derivation for SPPs.

In the next section, we briefly recall such derivation in order to comment on similarities and differences with respect to the ideal solutions presented in this section.

IV. *P*-POLARIZED PSEUDOSURFACE WAVES ($\beta_{1z} \neq 0$)

In this section, we will drop the condition $\beta_{1z} = 0$. This means that the wave in the dielectric side cannot be a pure surface wave since some propagation along the normal to the interface in the dielectric must be present as well. When the z component of the complex wave vector also has a real part (β_{1z}) in the dielectric, the calculations become much more complex and it is convenient to proceed without separating the wave vectors in a real and imaginary part. Since this derivation is presented in several references (see, for instance, [12]), we will skip all of the details in this case. The conditions (22) and (24) can be written as

$$k_{1x}^2 + k_{1z}^2 = k_0^2\varepsilon_1 \tag{39}$$

and

$$k_{2x}^2 + k_{2z}^2 = k_0^2\varepsilon_2. \tag{40}$$

These two relations, together with Eq. (14), lead to the following solutions, for $k_x = k_{1x} = k_{2x}$, k_{1z}, and k_{2z}:

$$k_{1z}^2 = k_0^2\frac{\varepsilon_1^2}{\varepsilon_1 + \varepsilon_2}, \tag{41a}$$

$$k_{2z}^2 = k_0^2 \frac{\varepsilon_2^2}{\varepsilon_1 + \varepsilon_2}, \tag{41b}$$

$$k_x^2 = k_0^2 \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}, \tag{41c}$$

which lead to [15]

$$k_{1z} = -k_0 \frac{\varepsilon_{1r}}{\left[(\varepsilon_{1r} + \varepsilon_{2r})^2 + \varepsilon_{2i}^2\right]^{1/4}} \times \exp\left[-i\frac{1}{2}\left(\arctan\frac{\varepsilon_{2i}}{\varepsilon_{1r} + \varepsilon_{2r}} + \pi\right)\right], \tag{42a}$$

$$\beta_{1z} = k_0 \frac{\varepsilon_{1r}}{\left[(\varepsilon_{1r} + \varepsilon_{2r})^2 + \varepsilon_{2i}^2\right]^{1/4}} \sin\left[\frac{1}{2}\arctan\frac{\varepsilon_{2i}}{\varepsilon_{1r} + \varepsilon_{2r}}\right], \tag{42b}$$

$$\alpha_{1z} = -k_0 \frac{\varepsilon_{1r}}{\left[(\varepsilon_{1r} + \varepsilon_{2r})^2 + \varepsilon_{2i}^2\right]^{1/4}} \cos\left[\frac{1}{2}\arctan\frac{\varepsilon_{2i}}{\varepsilon_{1r} + \varepsilon_{2r}}\right]. \tag{42c}$$

Equations (42) have been derived under the assumption that $\varepsilon_{2r} < 0$ (metal) and $\varepsilon_{1r} + \varepsilon_{2r} < 0$. Also, the choice of the sign for β_{1z} and α_{1z} has been made in order to have a wave decaying away from the interface and an energy flow going from the interface towards the dielectric [see also the expression of the Poynting vector, given by Eq. (49)]. Equation (41c) is usually called the dispersion curve for plasmons.

At this point, it is useful to look at one specific example to have an idea of the order of magnitude of these quantities. In the case of silver, at $\lambda = 633$ nm, we have $\varepsilon_2 = -18.28 + 0.48i$ [16]. If the dielectric is vacuum ($\varepsilon_1 = 1$), we get $\beta_{1z} = -0.0033366k_0$ and $\alpha_{1z} = -0.24042k_0$. This leads to a penetration distance, in the dielectric (vacuum), equal to

$$d_{1z} = \frac{1}{|\alpha_{1z}|} \simeq 419 \text{ nm} \simeq 0.66\lambda, \tag{43}$$

with an equivalent wavelength, for the wave propagating along z in medium 1, equal to

$$\lambda_z = \frac{2\pi}{|\beta_{1z}|} = 189\ \mu\text{m} \simeq 299\lambda. \tag{44}$$

Using the dispersion curve for k_x (see also, for instance, Ref. [12], p. 386), one gets that the corresponding component of the real part of the wave vector along x is

$$\beta_x = 1.0285k_0 \tag{45}$$

(as is well known, since in $\beta_x > k_0$, this solution cannot be excited by a free-space propagating wave). The equivalent λ_x reads

$$\lambda_x = \frac{2\pi}{\beta_x} \simeq 0.615\ \mu\text{m} = 0.972\lambda. \tag{46}$$

Also, we have

$$\alpha_x = 0.000781k_0, \tag{47}$$

with a penetration distance along x equal to

$$d_x = \frac{1}{\alpha_x} \simeq 129\ \mu\text{m} \simeq 204\lambda. \tag{48}$$

Hence, although the wave is not an ideal surface wave as those described in the previous section, its behavior does not depart too much from it. In fact, the propagation along the z axis exists, but it is characterized by an equivalent wavelength λ_z in the dielectric much longer than its penetration depth in the same medium (d_z), leading to an almost negligible propagation into the dielectric itself. On the other hand, the propagation along the interface is characterized by a penetration depth (d_x) much longer than the wavelength along the x axis. It follows that such pseudosurface wave is strongly confined along the z direction and mostly propagates along the surface, which agrees well with the picture of a surface mode. However, although small, the propagation along the z direction can never be neglected. By doing so, one would end up with an approximated solution that, from what we have seen in the previous section, does not exist. It is important to point out another main difference between an ideal surface and this pseudosurface wave. While a perfect surface wave would preserve its nature, the nature of the pseudosurface wave strongly depends on the values taken by the electric permittivities of the two media involved. In order to clarify this aspect, let us look at the propagation of the energy along the z and x directions, in medium 1.

The Poynting vector **S** is defined as

$$\boldsymbol{S} = \frac{1}{2}\text{Re}(\boldsymbol{E} \times \boldsymbol{H}^\star). \tag{49}$$

For a P-polarized pseudosurface wave, this leads to (in the dielectric side)

$$\boldsymbol{S} = \frac{1}{2}|A_H|^2 \exp(-2|\alpha_{1x}|x) \exp(2|\alpha_{1z}|z)(\beta_{1x}\boldsymbol{x}_0 + \beta_{1z}\boldsymbol{z}_0), \tag{50}$$

where $\boldsymbol{x}_0$ and $\boldsymbol{z}_0$ represent the unit vectors. We see that there is an energy flux flowing along the z and x axes. The field propagates almost parallel to the interface (i.e., $|\beta_{1x}| \gg |\beta_{1z}|$) and the ratio between the components of the Poynting vector, along z and along x, is

$$\rho = \frac{|Sz|}{|Sx|} = \frac{|\beta_{1z}|}{|\beta_{1x}|} \tag{51}$$

and depends only on the ratio between the real parts of the wave vectors along the z and x direction, respectively. In the case of the interface between vacuum and silver considered before, at $\lambda = 633$ nm, this leads to a ratio $\rho \simeq 3.2 \times 10^{-3}$.

However, we should bear in mind, on one hand, that the only conditions we required for such a pseudosurface wave to exist were

$$\varepsilon_{2r} < 0, \tag{52a}$$

$$\varepsilon_{1r} + \varepsilon_{2r} < 0, \tag{52b}$$

and that, on the other hand, the dependency of ρ on the material properties is not linear. We saw that there is always some energy flowing along the normal to the surface of separation between the perfect dielectric and the metal, with the component of the Poynting vector along z usually small compared to that along the surface. However, this strongly depends on the electric permittivity of the dielectric compared to that of the metal. In order to clarify this, in Fig. 2(a), we show

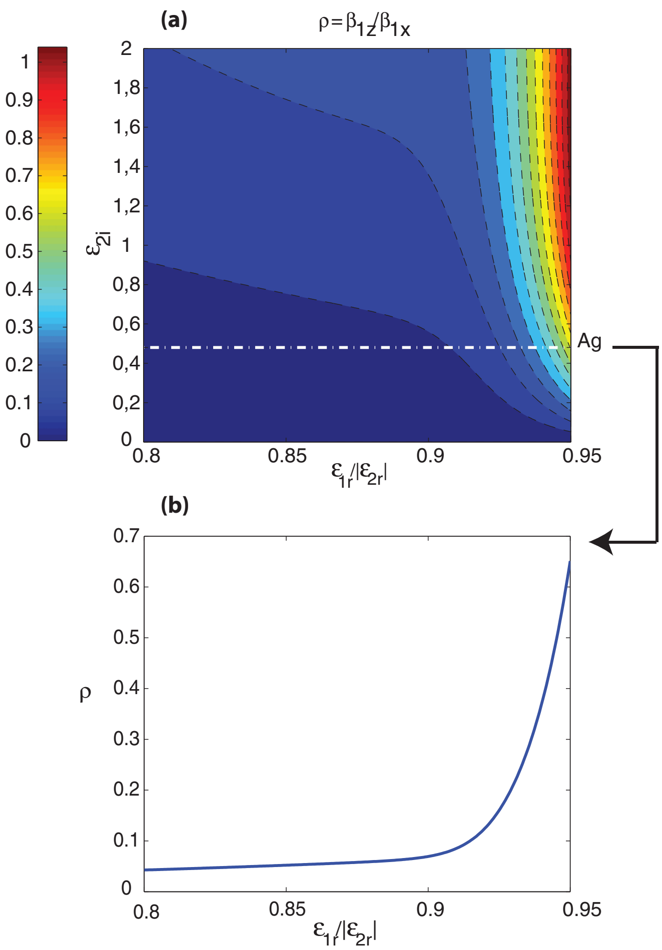

FIG. 2. (Color online) Ratio $\rho = S_z/S_x$ between the z component (normal to the interface) and x component (parallel to the interface) of the Poynting vector in the dielectric side as a function of variable $\varepsilon_{2i}, \varepsilon_{1r}/|\varepsilon_{2r}|$. $\lambda = 633$ nm. ε_{2r} is set to the value -20 and ε_{1r} is varied in a range of values such that the condition $\varepsilon_{1r} + \varepsilon_{2r} < 0$ is always fulfilled. (b) A slice of the 2D map, cut at the value for $\varepsilon_{2i} = 0.49$ corresponding to the example of silver discussed in the text.

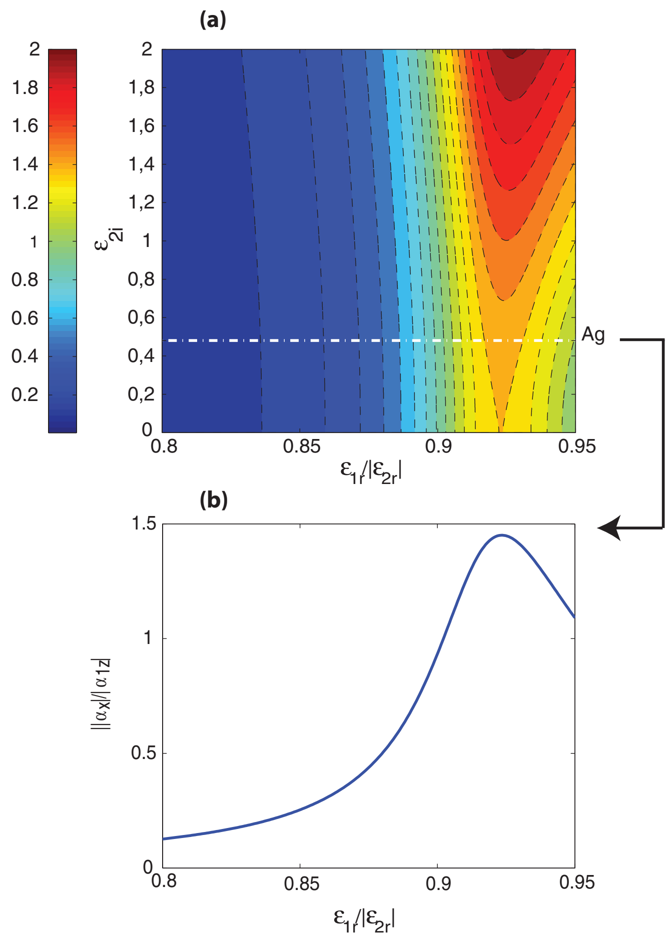

FIG. 3. (Color online) Ratio α_x/α_{1z} between the x component (parallel to the interface) and the z component (normal to the interface) of the decaying vector in the dielectric side as a function of variable $\varepsilon_{2i}, \varepsilon_{1r}/|\varepsilon_{2r}|$. $\lambda = 633$ nm. ε_{2r} is set to the value -20 and ε_{1r} is varied in a range of values such that the condition $\varepsilon_{1r} + \varepsilon_{2r} < 0$ is always fulfilled. Interestingly, while for less dense dielectrics the decaying along the normal is stronger than that along the surface, for a set of values of the ratio $\varepsilon_{1r}/|\varepsilon_{2r}|$, the wave is more confined along the x than along the z direction. Additionally, there is an intermediate region where the field is strongly localized in both directions. (b) A slice of the 2D map, cut at the value for $\varepsilon_{2i} = 0.49$ corresponding to the example of silver discussed in the text.

a plot of ρ as a function of $\varepsilon_{1r}/|\varepsilon_{2r}|$ and ε_{2i}, for an interface between a dielectric medium of varying real permittivity $\varepsilon_{1r} \in (1,19)$ and metal with $\varepsilon_{2r} = -20$ and $\varepsilon_{2i} \in (0,2)$. The values of ε_{1r} and ε_{2r} are such that it always holds $\varepsilon_{1r} + \varepsilon_{2r} < 0$. The wavelength is again $\lambda = 633$ nm. From the figure, it is evident that the ratio between β_{1z} and β_{1x} does not remain unaffected by a change of ε_{1r} and/or ε_{2i}. In fact, it is easy to see that as the ratio $\varepsilon_{1r}/|\varepsilon_{2r}|$ increases, the flux along z becomes more and more relevant and the wave tends to lose its surface-wave nature. Similar effects are observable when the metal becomes less and less ideal. We have found that ρ can increase by about one order of magnitude by properly changing the material properties. For the reader's convenience, in Fig. 2(b), we show a slice of the two-dimensional (2D) map, cut at the value for $\varepsilon_{2i} = 0.49$ corresponding to the example of silver discussed above, where the increase of ρ as a function of the ratio of the permittivities $\varepsilon_{1r}/|\varepsilon_{2r}|$ can be better appreciated. In Fig. 3, we plot the ratio between the x and z components of the decaying vector $\boldsymbol{\alpha}$ in the dielectric, again as a function of $\varepsilon_{1r}/|\varepsilon_{2r}|$ and ε_{2i}. Interestingly, we notice that while for low values of the permittivity of the dielectric the confinement along the z direction is much more marked than that along the x direction, it is possible to find a proper combination of values for ε_{1r} and ε_{2r} such that the wave results are confined along both directions. This is the case, for instance, of the interface between gallium phosphide and gold at $\lambda = 633$ nm, where one finds $\varepsilon_{1r} = \varepsilon_{\mathrm{GaP}} = 11.0113$ and $\varepsilon_2 = \varepsilon_{\mathrm{Au}} = -11.7532 + i1.2595$ (one should notice that the condition $\varepsilon_{1r} + \varepsilon_{2r} < 0$ still applies) [16]. With these values, one obtains $\alpha_x/\alpha_{1z} \simeq 0.5$, which results in a penetration distance $d_x = 0.037\lambda$ and $d_z = 0.02\lambda$. In other words, the mode becomes a sort of strongly spatially localized hot spot, bound on the surface between the two media.

V. CONCLUSIONS

To summarize, we have explicitly shown that no perfect surface waves can exist at the interface between a real metal and a perfect dielectric at optical wavelengths, a case which covers all situations where the generation of SPPs is invoked. This holds for S- and for P-polarized waves. It is possible to recover the existence of only P-polarized pseudosurface waves, under specific conditions for the materials involved and by including some propagation along the normal to the interface. For many practical cases of interest, such pseudosurface waves effectively mimic the ideal case. However, the denser the dielectric and the less ideal the metal becomes, the more a pseudosurface wave departs from the ideal SPP.

ACKNOWLEDGMENTS

This work was partly funded through the European Metrology Programme (EMRP) Project IND07 Thin Films. The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union.

APPENDIX: P-POLARIZED SURFACE WAVES AT THE INTERFACE BETWEEN A LOSSY DIELECTRIC AND A REAL METAL

In this Appendix, we would like to show that even the inclusion of a lossy dielectric in medium 1 does not lead to the existence of an ideal surface-wave solution in the same medium. For the P-polarization case, we will have the magnetic components, in both domains,

$$H_y^{(1)}(x,z) = A_H^{(1)} \exp(i\boldsymbol{k}_1 \cdot \boldsymbol{r}), \tag{A1}$$

$$H_y^{(2)}(x,z) = A_H^{(2)} \exp(i\boldsymbol{k}_2 \cdot \boldsymbol{r}), \tag{A2}$$

while for the electric components, we have

$$E_x^{(1)}(x,z) = -A_H^{(1)} \frac{k_{1z}}{\omega\varepsilon_0\varepsilon_1} \exp(i\boldsymbol{k}_1 \cdot \boldsymbol{r}), \tag{A3}$$

$$E_x^{(2)}(x,z) = -A_H^{(2)} \frac{k_{2z}}{\omega\varepsilon_0\varepsilon_2} \exp(i\boldsymbol{k}_2 \cdot \boldsymbol{r}). \tag{A4}$$

Continuity of the H_y and E_x leads to the conditions

$$A_H^{(1)} = A_H^{(2)} \tag{A5}$$

and

$$\frac{k_{1z}}{\varepsilon_1} = \frac{k_{2z}}{\varepsilon_2}, \tag{A6}$$

respectively. In the dielectric, we have

$$\boldsymbol{\beta}_1{}^2 - \boldsymbol{\alpha}_1{}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{1r}, \tag{A7a}$$

$$\boldsymbol{\beta}_1 \cdot \boldsymbol{\alpha}_1 = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{1i}}{2}. \tag{A7b}$$

The presence of $\varepsilon_{1i} \neq 0$ implies now that in medium 1 also, the propagation and decaying vectors can no longer be perpendicular to each other. Equations (A7) can be expanded as

$$\beta_{1x}^2 + \beta_{1z}^2 - \alpha_{1x}^2 - \alpha_{1z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{1r}, \tag{A8a}$$

$$\beta_{1x}\alpha_{1x} + \beta_{1z}\alpha_{1z} = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{2r}}{2}. \tag{A8b}$$

For the metal, nothing changes and we will still have

$$\boldsymbol{\beta}_2{}^2 - \boldsymbol{\alpha}_2{}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{2r}, \tag{A9a}$$

$$\boldsymbol{\beta}_2 \cdot \boldsymbol{\alpha}_2 = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{2i}}{2}, \tag{A9b}$$

and, in terms of Cartesian components,

$$\beta_{2x}^2 + \beta_{2z}^2 - \alpha_{2x}^2 - \alpha_{2z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{2r}, \tag{A10a}$$

$$\beta_{2x}\alpha_{2x} + \beta_{2z}\alpha_{2z} = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{2i}}{2}. \tag{A10b}$$

If we now impose the condition $\beta_{1z} = 0$, we get the following sets of equations to be satisfied at the same time,

$$\frac{i\alpha_{1z}}{\varepsilon_{1r} + i\varepsilon_{1i}} = \frac{\beta_{2z} + i\alpha_{2z}}{\varepsilon_{2r} + i\varepsilon_{2i}}, \tag{A11a}$$

$$\beta_{1x}^2 - \alpha_{1x}^2 - \alpha_{1z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{1r}, \tag{A11b}$$

$$\beta_{1x}\alpha_{1x} = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{1i}}{2}, \tag{A11c}$$

$$\beta_{1x}^2 + \beta_{2z}^2 - \alpha_{1x}^2 - \alpha_{2z}^2 = \omega^2\mu_0\varepsilon_0\varepsilon_{2r}, \tag{A11d}$$

$$\beta_{1x}\alpha_{1x} + \beta_{2z}\alpha_{2z} = \frac{\omega^2\mu_0\varepsilon_0\varepsilon_{2i}}{2}, \tag{A11e}$$

where we have also used the fact that $\beta_{1x} = \beta_{2x}$ and $\alpha_{1x} = \alpha_{2x}$. Hence, we are again left with a system of five independent scalar equations in the four unknowns, $\beta_{1x},\alpha_{1x},\beta_{2z},\alpha_{2z}$, analogously to what we obtained in Sec. III for the case of an ideal dielectric. In order to find a solution for any choice of the permittivities, one needs to include another unknown, which is, in fact, represented by $\beta_{1z} \neq 0$.

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