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Integrator Delay Zero Model for Design of Upstream Water-Level Controllers

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Abstract: A variety of methods are in use for the design of controllers for adjusting canal gate positions to maintain a constant water level immediately upstream from check gates. These methods generally rely on a series of tests on the water level's response to changes in canal gate position or flow, either by simulation or on the canal itself. This paper presents a method for tuning these controllers based on wave celerity through use of the integrator delay zero (IDZ) model. These equations can be used to determine the resonance peak height and resonance frequency. Unsteady-flow canal simulation models are used to show the response of controller design using these theoretical equations with a test case for ASCE Test Canal 1. A novel method is presented for avoiding disturbance amplification by considering the delay times in all canal pools downstream. **DOI:** 10.1061/(ASCE)IR.1943-4774.0000997.

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Introduction

The control of water levels upstream from canal gates is the most common method of canal automation in practice. If the correct flow (i.e., sum of downstream demand) enters the canal at the canal head gate, this method will distribute the flow correctly to all gates downstream. Errors in canal inflow will result in errors in the flow available within the last pool, either resulting in canal spills or providing insufficient flow to farm turnouts there. Operators are then expected to alter the canal inflow to correct such flow errors. In most cases, this control is done manually, although automatic control is becoming more common. With automatic control, if controllers on individual gates are not properly tuned, it is possible to get disturbance amplification, where the gates' positions and water levels oscillate with increasing amplitude in the downstream direction. This problem can be avoided if the controllers for all canal pools are tuned simultaneously. For example, Overloop et al. (2005) suggest that centralized control will avoid this disturbance amplification. Optimization procedures require a response model of the canal.

The usual practice for implementing automatic upstream control is to develop a simulation model of the canal, determine the response of the canal from simulation tests, use optimization to develop control parameters, and then to test the suitability of the controller through simulation. When adapted to the real canal, the parameters are further tuned through testing. This can be a time-consuming and thus expensive process (Overloop et al. 2005).

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The intent of this paper is to present a simple method for tuning proportional-integral (PI) controllers for the automatic control of water levels upstream from canal gates. For many cases, this can be done with the equation presented here based on canal geometry, rather than extensive simulation testing. In other cases, some response testing on the canal may be required.

Basic Theory

Response Model

Schuurmans et al. (1999b) proposed a simple model for canal pool response; the integrator delay (ID) model. This model relates changes in the water level at the downstream end of the pool to flow changes through gates at the upstream and downstream end of the pool. The model has two parameters: a delay time, τ, and a backwater surface area, A_s. The upstream part of the pool is considered to be at normal depth, while the downstream part of the pool is considered to be a reservoir. Pools that are entirely under backwater have no delay time. This model has been successfully used to design canal controllers that have subsequently been field tested (Litrico et al. 2007; Clemmens and Strand 2010). The ID model is

$$\Delta h(k) = \frac{T_s}{A_s}[Q_u(k - \tau_R) - Q_d(k)] \quad (1)$$

where $\Delta h(k)$ = change in water level; k = integer time step number; T_s = duration of the time step; Q_u = upstream flow rate; Q_d = downstream flow rate; and τ_R = closest integer representing the delay time = τ/T_s. The only difference between the response of the upstream and downstream gates results from the delay time, which does not apply to the downstream gate since the water level is immediately next to the gate. For canal pools under backwater, the backwater surface area can be approximated by the top width, B, times the pool length, L

$$A_s = BL \quad (2)$$

The top width can vary with length, and culverts and other obstructions can alter the effective area. But this provides a reasonable approximation. For pools that are not under backwater for their entire length, the backwater area can be much smaller. In this case, Eq. (2) should not be used and the canal pool response should be evaluated to determine A_s.

Litrico and Fromion (2004) show that the ID model is an oversimplification of a real canal's response. It does not consider oscillations that occur in the canal pool as a result of the flow changes. They propose to add a zero to the ID model (IDZ) to account for the water level response at frequencies that are sufficiently high that significant oscillations occur. If the controller is designed to consider the lowest frequency at which oscillations occur, higher-frequency oscillations should be sufficiently damped. A change in flow through the gate at the upstream end of a pool will cause a wave to travel the length of the pool. The zero in the IDZ model represents the height of this wave when it arrives at the downstream end. For long canal pools under normal depth, this wave is damped and essentially insignificant. For pools under backwater, the wave height can be substantial. A flow change in the gate at the downstream end of a pool will create a step change in water level immediately upstream from the gate, regardless of whether or not the water level is at normal depth. This suggests that the IDZ model could be used for the design of upstream water-level controllers (i.e., gate controller based on the water level immediately upstream from gate).

Resonance Peak Height

Litrico and Fromion (2004) developed a series of relationships to determine the resonance peak height based on the IDZ model. The equations determine the average magnitude of all resonances (water-level change for unit flow rate change at selected frequency). For upstream control, this study is interested only in the step change in water level at the downstream end of the pool for a step flow change at the downstream end of the pool. In the notation of Litrico and Fromion (2004) this response function is denoted p_{22}, which simply means the response of the water level at the downstream end of the pool to the flow change at the downstream end of the pool. Modifying their equation for p_{22} for a pool of infinite length gives the average resonance peak height

$$R_p = \frac{1}{B(c - v)} \tag{3}$$

where v = average flow velocity; and c = wave celerity = $\sqrt{gD}$, where g is the acceleration of gravity and D is hydraulic depth (cross-sectional area divided by top width). If the water depth at the downstream end of the canal pool is at (or close to) normal depth, Eq. (2) is a conservative estimate of the maximum resonance peak height. However, many canal pools are under backwater such that the depth is well above normal depth, particularly at low discharges relative to canal capacity. The resonance peak at low discharges can be significantly higher than that predicted by Eq. (3). Detailed equations for the maximum resonance peak height are provided in the Appendix. These account for reflection waves for pools entirely under backwater.

Filtered Proportional-Integral Controller (PIF)

Schuurmans (1997, 1999a) developed equations for proportional-integral (PI) controller coefficients based on the integrator delay model, such that robust stability would be assured (based on 45° phase margin criteria). These equations are applicable to either upstream or downstream water-level control, with the main difference related to the delay time. Equations were developed for both PI and filtered PI (PIF) controllers. Here, only the PIF controllers for upstream water-level control are considered because the wave created by gate movements can cause the controller to be unstable if not filtered. Further explanations of canal control theory can be found in Wahlin and Zimbelman (2015). The PIF controller equation is

$$\Delta Q(k) = K_P \Delta e_f(k) + K_I e_f(k-1) \tag{4}$$

where K_p and K_I = PIF controller coefficients; e_f is the filtered water level error; k = time step number; and ΔQ = control action (change in flow from $k-1$ to k); in this case a change in flow rate. A linear filter is used, namely

$$e_f(k) = F_c e_f(k-1) + (1 - F_c)e(k) \tag{5}$$

where

$$F_c = e^{-(T_s/T_f)} \tag{6}$$

and where $e(k)$ = water level error; F_c = filter constant where $0 < F_c < 1$; and T_f = period of waves above which damping is required. Use of the filter causes a delay in the ID model since, for example, for a gradually increasing water level, the filtered level lags behind since it considerers past (lower) water levels. The added delay, on average, is

$$t_{\text{delay}} = \frac{F_c}{1 - F_c} T_s \tag{7}$$

Since the ID model relates flow changes to water level changes, the flow changes have to be converted to gate position changes. This is essentially the universal factor proposed by Burt et al. (1996). This conversion can be made by inverting the gate discharge equation or can be approximated by determining the slope of the relationship between gate position change and discharge change (dW/dQ), where W is the gate opening. So for practical application to local gate controllers, Eq. (4) can be modified to

$$\Delta W(k) = \frac{dW}{dQ} [K_P \Delta e_f(k) + K_I e_f(k-1)] \tag{8}$$

The PIF controller coefficients from Schuurmans (1997) are

$$K_P = \frac{A_s}{2T_f} \tag{9}$$

$$K_I = K_P \frac{T_S}{T_I} \tag{10}$$

$$T_I = 6T_f \tag{11}$$

$$T_f = \sqrt{\frac{A_s R_p}{\omega_r}} \tag{12}$$

$$\omega_c = \frac{1}{2T_f} \tag{13}$$

Provided that

$$T_s \leq 0.15/\omega_c \tag{14}$$

where K_p and K_I = proportional and integral control constants; A_s = average backwater area (over different flow rates, assuming backwater area is a function of flow rate); T_f = filter time constant; T_I = integration time; T_s = control time step; R_p = resonance peak height; ω_r = resonance frequency; and ω_c = cross-over frequency. These equations ensure that the resonance frequency will be greater than the cross-over frequency, which is a requirement for control stability.

Solution of these equations for K_p and K_I requires selection of a control time step, T_s; an estimate for the backwater surface area, A_s; an estimate for the resonance peak height, R_p; and a suitable value of the resonance frequency, ω_r, such that Eq. (14) is still satisfied. For distant downstream control, the general approach is to determine the resonance frequency based on the travel time of a celerity wave

$$\omega_r = \frac{2\pi}{T_{cycle}} = \frac{2\pi}{L(\frac{1}{c+v} + \frac{1}{c-v})} \tag{15}$$

where T_{cycle} = travel time for a wave to travel from one end of the canal to the other and back. For upstream control, this resonance frequency can be taken as a conservative estimate, although the cycle of two control time steps, $\omega_r = (2\pi)/(2T_s)$, has more influence on fluctuations in water levels and gate positions.

Combining Eqs. (13) and (14) yields

$$\omega_c = \frac{1}{2T_f} \leq \frac{0.15}{T_s} \tag{16}$$

This results in

$$T_f \geq \frac{T_s}{0.3} \tag{17}$$

If taken as an equality, this leads to $F_c = 0.741$ from Eq. (6). This value of the filter constant is often used to overcome oscillations caused by a fixed sampling interval, so called antialiasing. For example, if the input signal is a sign wave around a constant value, sampling at an interval that is different from the frequency of the sine wave can result in water levels that trend in one direction for an interval and then trend in the opposite direction for an interval, when the signal is essentially constant. Wahlin and Zimbelman (2015, p 174) suggest a value of F_c greater than 0.667 to avoid antialiasing. Here, a value greater than $11/16 \approx 0.688$ is used since this is a common way filters are used in control software.

Alternative Proportional-Integral Controller

Litrico and Fromion (2006) developed alternative expressions for T_I and K_p based on both gain and phase margins, with K_I determined from Eq. (4)

$$T_I = \frac{T_d}{\omega_c} \tan\left(\frac{\pi}{180}\Delta\Phi + \omega_c\right) \tag{18}$$

$$K_p = A_s \frac{\omega_c}{T_d} \sin\left(\frac{\pi}{180}\Delta\Phi + \omega_c\right) \tag{19}$$

$$\omega_c = \frac{\pi}{2} 10^{-\Delta G/20} \tag{20}$$

$$\Delta\Phi_{\max} = 90(1\text{–}10^{-\Delta G/20}) \tag{21}$$

where ΔG = desired gain margin; $T_d = (T_f + T_s)/2$ = delay time; and $\Delta\Phi$ = phase margin. They suggest that stable performance can be obtained for $\Delta G = 10$, which gives a cross-over frequency $\omega_c \approx 0.5$ from Eq. (20), and with $\Delta\Phi = 0.7\Delta\Phi_{\max}$, which will allow stable control (for downstream control) if the actual time delay is up to 150% of the expected delay. By considering both the phase and gain margins, it is expected that this control will be slightly more aggressive than those of Schuurmans (1997) while still remaining stable. This simply gives an alternative method for determining controller gains for individual gates. The resonance frequency is determined from Eq. (15).

Multiple Pools

When upstream controllers are tuned individually, there is concern that the overall control will exhibit disturbance amplification because of interactions among pools. Overloop et al. (2005) and Clemmens and Schuurmans (2004) suggest the use of optimization to tune PI controller constants. The procedures used in these studies considered the ID model and did not consider wave action (zero in IDZ). Thus, in order to use these models, constraints need to be placed on values of K_p and K_I, as suggested by Eq. (14). The authors' experience with this optimization (Overloop 2005) suggests it is sufficient to constrain the value of K_p as computed previously [a value less than that computed from Eq. (9) or Eq. (19)], while the value of K_I has to be progressively decreased in the upstream direction to account for additional pools downstream.

Based on these observations, it is hypothesized that K_p from Eq. (9) can be used directly and K_I can be adjusted by considering the additional downstream resonance. The integral constant is a function of the resonance frequency. This relationship can be developed from Eq. (10) by substituting Eq. (9) for K_p and Eq. (11) for K_I, then substituting Eq. (13) for T_f, which gives

$$K_{I\_i} = \frac{A_{S\_i}}{2T_{f\_i}} \frac{T_s}{6T_{f\_i}} = \frac{A_{S\_i}T_s}{12} \frac{\omega_{r\_i}}{A_S R_{P\_i}} = \frac{T_s}{12} \frac{\omega_{r\_i}}{R_{P\_i}} \tag{22}$$

where subscript_i = pool being considered.

To avoid disturbance amplification downstream, it is assumed that the ω_r value for any pool includes the travel time for all downstream pools. Then for any pool i, the resonance frequency can be estimated from

$$\omega_{r\_i} = \frac{2\pi}{\sum_{j=i}^{N} T_{\text{cycle}\_j}} = \frac{2\pi}{\sum_{j=i}^{N} \frac{2\pi}{\omega_{r\_j}}} \tag{23}$$

where N = number of pools in the canal.

Eq. (23) is heuristic in nature, while the prior equations are all based on physics and well-established control-theory principals. An example application is presented in this paper where a simulation model of a real canal is used to test the various equations presented in preceding sections for developing upstream control coefficients. The range of conditions under which they are useful would require a more-thorough investigation.

Materials and Methods

A test canal was examined to test the various methods for upstream control described previously through unsteady-flow simulation. Simulation tests were run with *Sobek* version 2.12 (Deltares 2015). Water-level responses to determine pool properties were developed using step changes in gate discharge and observing water-level responses. Upstream controller parameters (i.e., proportional and integral constants) were tuned with the previously described methods. These controllers were tested using the custom-built control routines in *Sobek* version 2.12.

The ASCE Task Committee on Canal Automation Algorithms developed a series of test cases for evaluating the response of downstream water-level controllers (Clemmens et al. 1998).

Table 1. Test Canal 1 Physical Properties

Pool	Pool length (m)	Bottom width (m)	Canal depth (m)	Gate width (m)	Gate height (m)	Target level (m)	Test 1-1 initial flows (m^3/s)
(Headgate)	—	—	—	—	—	—	0.8
1	100	1.0	1.1	1.5	1.0	0.9	0.7
2	1,200	1.0	1.1	1.5	1.0	0.9	0.6
3	400	1.0	1.0	1.5	0.9	0.8	0.5
4	800	0.8	1.1	1.2	1.0	0.9	0.4
5	2,000	0.8	1.1	1.2	1.0	0.9	0.3
6	1,700	0.8	1.0	1.2	0.9	0.8	0.2
7	1,600	0.6	1.0	1.0	0.9	0.8	0.1
8	1,700	0.6	1.0	—	—	0.8	0.0

Table 2. Tuned Parameters for Test Canal 1 (Taken from Clemmens and Wahlin 2004)

Pool	Inflow rate during tuning (m^3/s)	Delay time (min)	Backwater surface area (m^2)
(1)	(2)	(3)	(4)
1	0.8	0.0	443
2	0.7	7.9	964
3	0.6	2.0	634
4	0.5	4.9	877
5	0.4	17.2	948
6	0.3	15.2	785
7	0.2	16.4	711
8	0.1	20.2	724

Test Canal 1 was used here for evaluating upstream controllers. It is relatively long and steep, roughly 9.5 km long with an elevation drop of 27 m. The canal has a bottom slope of 0.002 m/m and 1.0-m drops after each check gate. Canal side slopes are 1.5–1.0, horizontal to vertical. Additional details of Test Canal 1 are given in Table 1. There is no check gate at the end of the canal. Backwater from a check gate generally does not progress upstream to the next upstream gate, except for the first pool, since it is very short. Pools in this canal do not experience oscillations, except in the first pool. Check gates are all under free flow.

Response Example

The *Sobek* model of Test Canal 1 was used to demonstrate the IDZ response. The initial flow was set to 1.0 m^3/s, or 50% of capacity. The gate at the upstream end of Pool 1 was opened to increase the flow by 0.2 m^3/s. The initial water depth for this test was 0.9 m, with a top width of 3.7 m, speed of celerity of 2.37 m/s, and a backwater area of 443 m^2. This canal pool has a bottom width of 1.0 m, side slopes of 1.5:1 (Horizontal to vertical), and a length of 100 m.

Control Example: No Reflection Waves

For Test Canal 1, parameters for the ID model were tuned with step tests as described in Clemmens and Wahlin (2004). Those tuned parameters are shown Table 2. The resonance-peak heights were determined at maximum discharge (capacity) from Eq. (3) and from Eq. (31), as shown in Table 3. Computing resonance peak at maximum discharge generally results in the largest value of resonance peak height, and is thus conservative. The values from the two approaches are identical to three significant digits, except Pool 1 (pool 1 is short and does experience oscillations). Thus the resonance peak height can be estimated sufficiently accurately from Eq. (3) for pools that are not under backwater for their entire length and do not experience oscillations from reflection waves (i.e., water flows at normal depth over a portion of the pool length).

Different sets of proportional-integral filtered (PIF) controllers for Test Canal 1 were tuned with the procedures presented here (Schuurmans 1997). Initial conditions for the simulations were the starting conditions for Test Case 1-1, which is shown in column 2 of Table 2. The different tuning methods include

1. PIF controller (Schuurmans' Method) for each pool determined independently with the resonance peak height and the resonance frequency based on the canal pool celerity [Eqs. (3), (6), and (8)–(15)];

Table 3. Resonance Peak Heights for Test Canal 1 Based on Flow at Capacity

Pool	Inflow rate during tuning (capacity) (m^3/s)	Resonance peak height [Eq. (31)] (s/m^2)	Resonance peak height [Eq. (A8)] (m^2)
(1)	(2)	(3)	(4)
1	2.0	0.190	0.224
2	2.0	0.190	0.190
3	2.0	0.264	0.264
4	1.6	0.190	0.190
5	1.6	0.190	0.190
6	1.6	0.257	0.257
7	1.3	0.263	0.263
8	1.1	0.237	0.237

2. PIF controller (Schuurmans' Method) with the resonance peak height and the resonance frequency based on the canal pool celerity, but integral constants adjusted based on downstream resonance [Eqs. (3), (6), (8), (9), (11)–(15), (22), and (23)];
3. PIF controller (Litrico and Fromion's Method) for each pool determined independently with the resonance peak height determined from the canal pool celerity and the resonance frequency determined based on the maximum cross-over frequency [Eqs. (3), (6), and (18)–(21)]; and
4. PIF controller (Litrico and Fromion's Method) with the resonance peak height determined from the canal pool celerity and the resonance frequency determined based on the maximum cross-over frequency, but integral constants adjusted based on downstream resonance [Eqs. (3), (6), (10), and (18)–(23)].

Proportional and integral gains for these controllers are shown in Table 4. The Sobek unsteady-flow simulation model (Deltares 2015) was used to test the effectiveness of these controllers. A step change of 0.2 m^3/s was initiated at 4 h at the canal headgate. In separate tests, each of these controllers was used to bring the water levels back to their set points. Controller time step was 10 min, and simulation time step was 1 min. The length of simulation was 24 h. The size of calculation grid is 25 m.

Results

Response Example

Fig. 1 shows the response in water level in Pool 1 for Test Canal 1 for a step increase in discharge at the gate immediately downstream. These clearly show the step change (z-term in IDZ model) at the time of the change, and gradual change (I-term in

Table 4. Control Constants for Test Canal 1

Pool	K_p (m²/s) Methods 1 and 2	K_p (m²/s) Methods 3 and 4	Fc (–) Methods 1–4	K_I (m²/s) Method 1	K_I (m²/s) Method 2	K_I (m²/s) Method 3	K_I (m²/s) Method 4
(1)	(2)	(3)	(4)	(6)	(7)	(8)	(9)
1	6.032	4.32	0.688	1.6429	0.0165	0.888	0.046
2	2.569	3.67	0.688	0.1369	0.0166	0.295	0.050
3	2.816	3.46	0.688	0.2502	0.0136	0.399	0.040
4	3.033	4.04	0.688	0.2098	0.0200	0.393	0.059
5	1.994	3.00	0.713	0.0839	0.0221	0.201	0.065
6	1.575	2.39	0.702	0.0632	0.0222	0.154	0.064
7	1.542	2.31	0.688	0.0669	0.0334	0.158	0.089
8	1.637	2.43	0.688	0.0740	0.0740	0.172	0.172

IDZ model). The step drop shown in Fig. 1(a) is plotted at the resonance peak height given by Eq. (3) ($\Delta h = Rp\Delta t\Delta Q$). The step drop in Fig. 1(b) is plotted at the resonance peak height given by Eq. (31) in the Appendix. The sloped part is plotted at a rate $\Delta h/\Delta t = \Delta Q/A_s$, which represents the integrator in the IDZ model. The resonance in this pool is clearly seen by the water-level drop at roughly 22 min in Fig. 1(b). The step change from Eq. (3) should match the initial water surface's drop. The step change from Eq. (31) is intended to contain the reflection waves. Here Eq. (31)'s step is a bit larger than the initial water-level drop, which is on the conservative side. The water level drops below the IDZ line by a small amount. This difference could result from the linear assumption in the IDZ model. In numerous tests (not shown), this model has been shown to be a reasonable representation of canal response to a change in gate flow.

Control Example: No Reflection Waves

Fig. 2 shows the simulation results for Method 1 (PIF controller with resonance frequency based on celerity). The changes in flow rate are shown in Fig. 2(a). All flow rates changed by 0.2 m^3/s. Since the sudden flow change was not anticipated by the upstream controllers, the water levels deviated by up to 5 cm. The flow rates reacted strongly to remove the excess water that accumulated behind the gates. This is actually how the controller should respond. After this initial response, the controllers returned to nearly the final steady-state flow, with only a small amount of overshoot (overcorrection). The flows rates are overcorrected more and more by each pool successively downstream. This is shown more clearly in Fig. 2(b), which shows the resulting water levels. This is typical of what is known as disturbance amplification. In this case, the amplification is not great, so this control might be considered acceptable. But errors in pool property estimates (which are minimized in simulation studies) or changes over time might result in poor performance in actual operations.

Fig. 3 shows the results for Method 2, where downstream response is included in the calculations for the integral constant. The integral constants in Column 7 are much smaller than the integral constants in Column 6, except for the last pool downstream. This greatly reduces the disturbance amplification, as shown by the water level response in Fig. 3(b). The flow rates also do not overcorrect as much, as shown in Fig. 3(a). Only the last pool in Fig. 3(b) shows some water-level oscillation. The rest of the pools show little to none. So considering downstream response reduces disturbance amplification, but makes the controller respond more slowly, as shown by the greater deviations in water level for upstream pools in Fig. 3(b) compared to those in Fig. 2(b).

Figs. 4 and 5 (Methods 3 and 4) show the same results as Figs. 2 and 3 (Methods 1 and 2), except Methods 3 and 4 design the controller based on the alternative method of Litrico and Fromion (2006). Smaller proportional gains result in greater water level deviations, as shown by comparing the maximum water level deviations for pools in Figs. 2 and 4 compared to the proportional gains shown in Table 4 columns 2 and 3. This is because the controller is

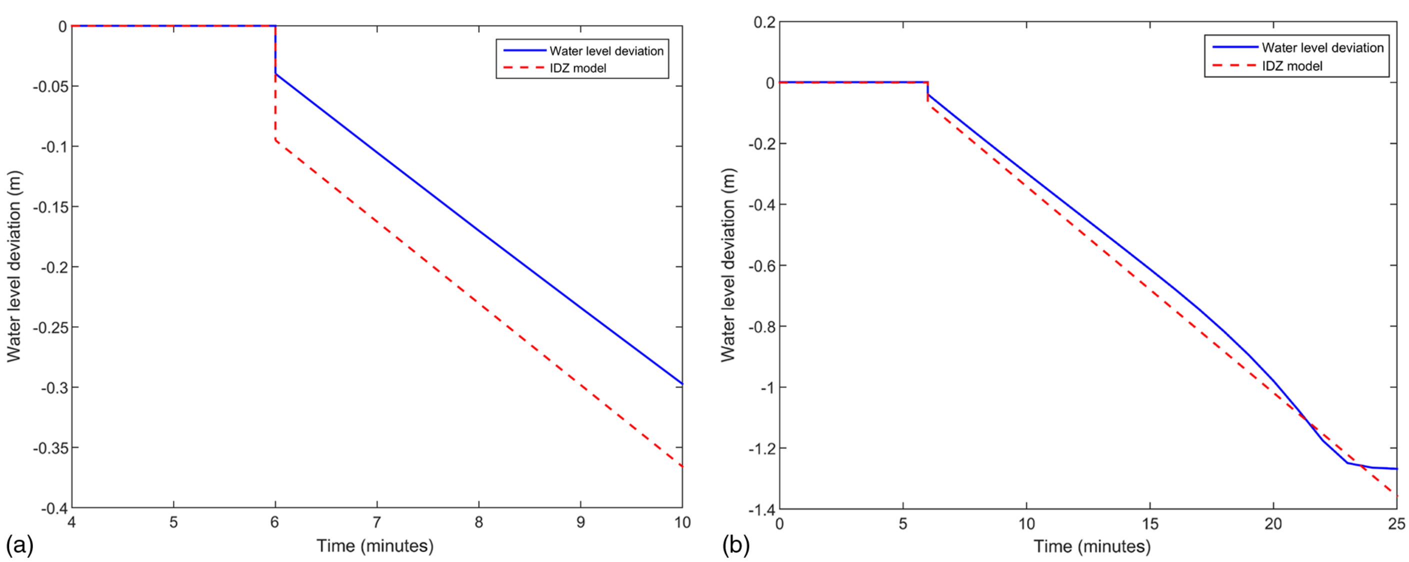

Fig. 1. Response of water level upstream from test canal Gate 1 to a gate flow change of 0.5 m^3/s

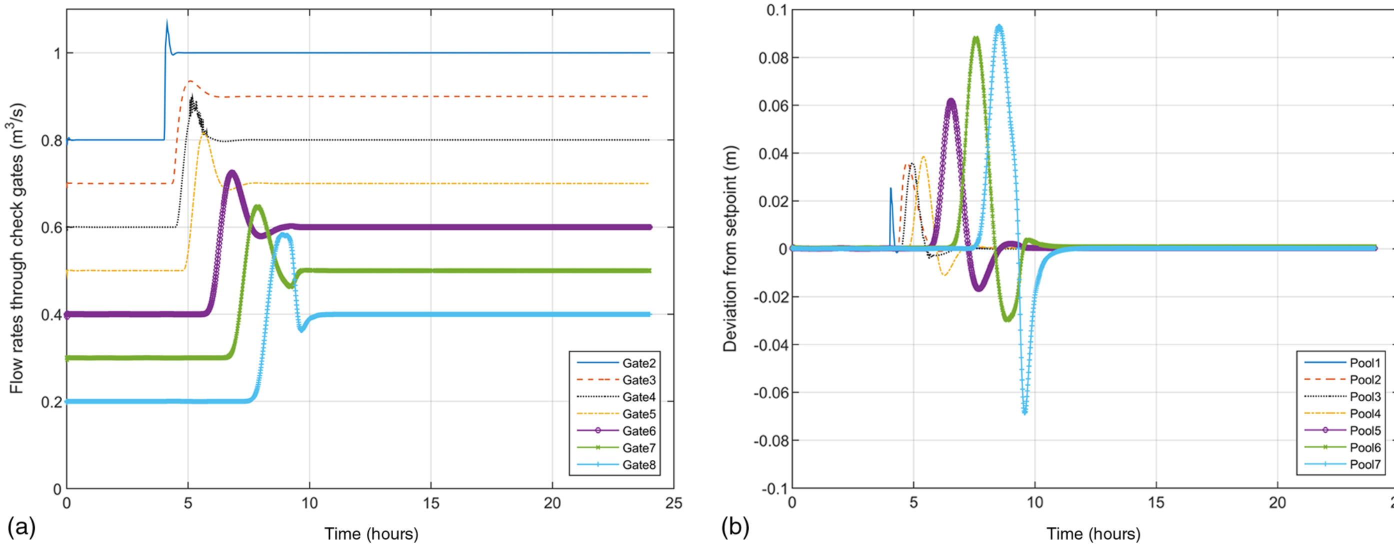

Fig. 2. Simulation results for step change of 0.2 m^3/s in Test Canal 1; Method 1

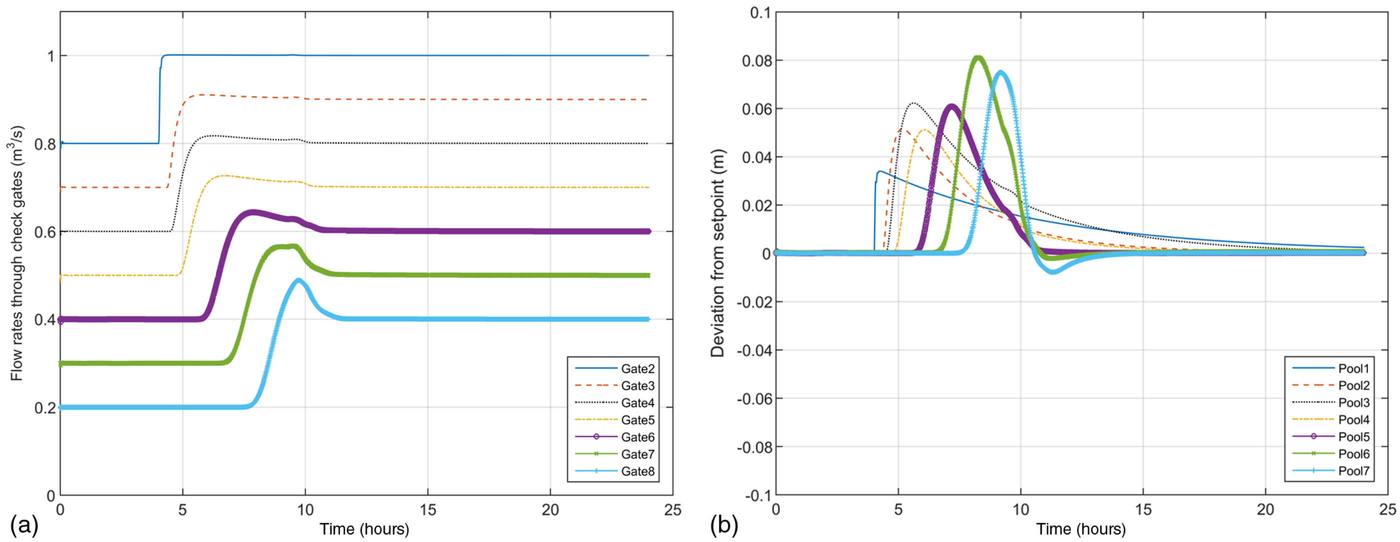

Fig. 3. Simulation results for step change of 0.2 m^3/s in Test Canal 1; Method 2

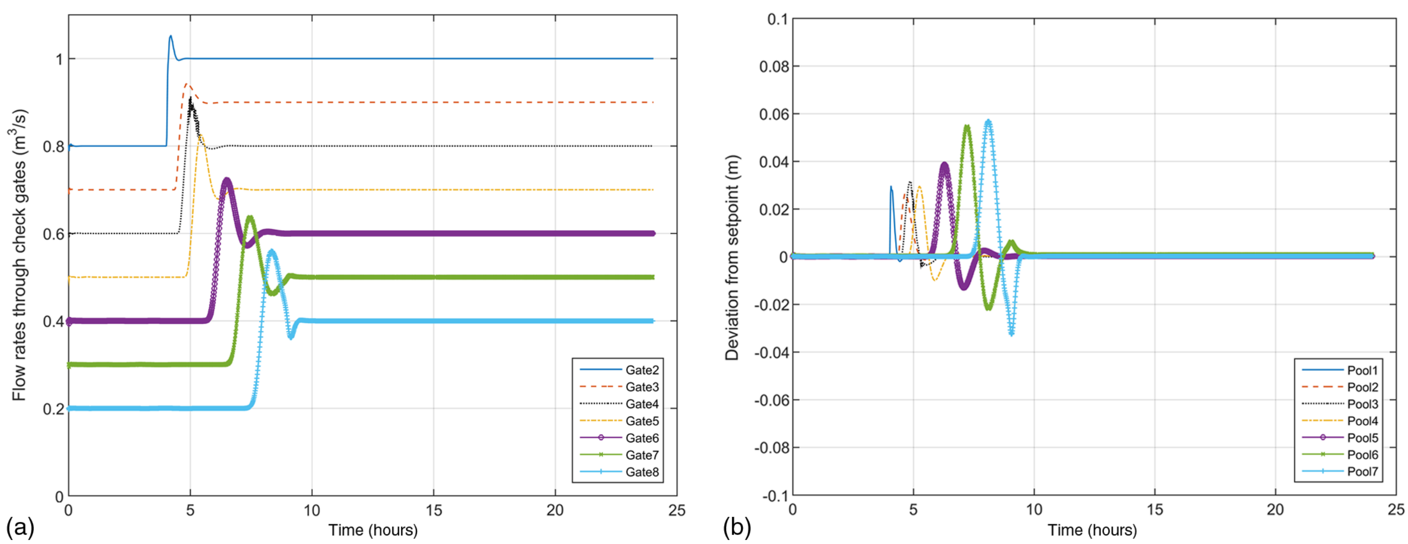

Fig. 4. Simulation results for step change of 0.2 m^3/s in Test Canal 1; Method 3

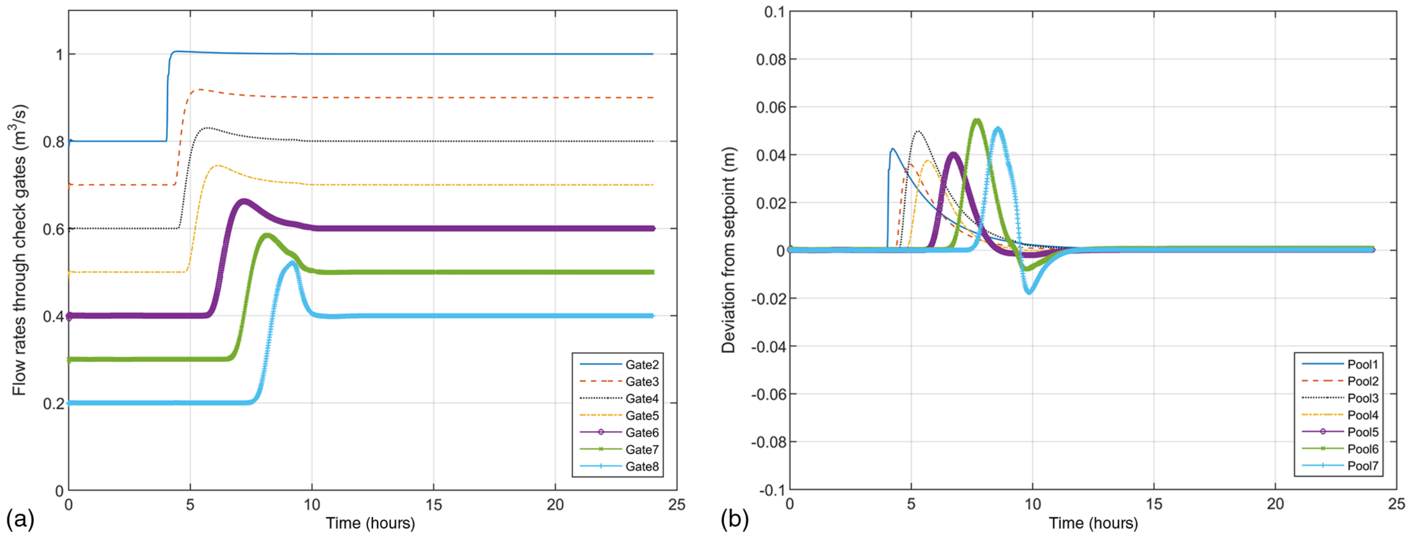

Fig. 5. Simulation results for step change of 0.2 m^3/s in Test Canal 1; Method 4

not reacting strongly enough. Comparing Figs. 4 and 5, Method 4, which considers the downstream delay times, removes most of the disturbance amplification. Method 4 was also run with the resonance peak for the first pool computed based on Eq. (31), rather than Eq. (3). Results were essentially the same as for Method 4, so are not shown here.

Discussion

First, it is clear that the IDZ model is an effective method for determining the resonance peak height needed for designing upstream controllers. The resonance peak-height values for the canal that did not exhibit oscillations (Test Canal 1) are shown in Table 3. Since there is little or no resonance from reflection waves (oscillations), the values from Eqs. (3) and (31) are nearly the same. Eq. (31) essentially considers the influence of these reflection waves on the resonance peak height. Thus the influence of reflection waves on resonance does not need to be considered for this canal.

Results from the alternative method (Litrico and Fromion 2006) for determining PIF controller constants provided better overall results that the original method (Schuurmans 1997) and should be the recommended approach. These tests did not evaluate canal pools with significant oscillations.

Conclusion

This research has shown that the IDZ model can be used directly to determine the resonance peak height for canal pools where upstream control is planned. For canal pools that do not exhibit oscillations from reflection waves, a controller design based on wave celerity [i.e., Eq. (3)] provides adequate upstream control response.

The use of controllers tuned for individual canal pools will result in disturbance amplification, where fluctuations in water levels get progressively worse in the downstream direction. This can be avoided by reducing the integral gain. A method was developed to reduce the integral gain by including the response time of all downstream pools in the controller design. This research demonstrates that for pools with limited backwater, the IDZ model parameters, and thus upstream controller parameters, can be determined with only knowledge of the pool geometry.

Appendix. Calculation of Resonance Peak Height

Uniform Flow

Litrico and Fromion (2009) developed equations for the resonance of canal pools. For uniform flow conditions, they developed the average resonance magnitude for water levels at either end of the pool, based on flow changes at either end of the pool. For upstream control, this study is only concerned with the water-level response at the downstream end of the pool based on a flow change through the downstream gate. The resulting equation is

$$R_p \approx \frac{1}{B(c-v)}\sqrt{\frac{1+\frac{\beta^2}{\alpha^2}e^{-2rL}}{1+e^{-2rL}}} \tag{24}$$

where B = top width; c = speed of celerity; v = average velocity; L = pool length; and $\alpha = c+v$, $\beta = c-v$, and $r = r_1 + r_2$, where

$$r_1 = \frac{\alpha\delta - \gamma}{\alpha(\alpha+\beta)} \tag{25}$$

$$r_2 = \frac{\beta\delta + \gamma}{\beta(\alpha+\beta)} \tag{26}$$

$$\gamma = g(1+k)S_b \tag{27}$$

$$\delta = \frac{2\,gS_b}{v} \tag{28}$$

$$k = \frac{7}{3} - \frac{4A}{3BP}\frac{dP}{dy} \tag{29}$$

and g = acceleration of gravity; S_b = bottom slope; A = cross-sectional area; P = wetted perimeter; and y = flow depth. When L approaches infinity, the term under the radical in Eq. (24) approaches 1, and thus Eq. (3) is the limit of Eq. (24) when the pool length approaches infinity.

The response described by Eq. (24) represents the average resonance. The actual response is cyclic about this average. Eq. (24) was developed from solution of the frequency response of the Saint Venant equations, expressed as transfer functions. The more-complete form of Eq. (24) from these relationships is

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$$R_p \approx \frac{1}{B(c-v)}\sqrt{\frac{1+\frac{\beta^2}{\alpha^2}e^{-2rL}+\frac{2\beta}{\alpha}e^{-rL}\cos\omega\tau}{1+e^{-2rL}-2e^{-rL}\cos\omega\tau}} \quad (30)$$

where ω = frequency; and τ = wave travel time [Eq. (15)]. The resonance determined from Eq. (30) is a maximum when $\cos(\omega\tau) = 1$, and a minimum when $\cos(\omega\tau) = -1$. Thus the maximum resonance peak height is

$$R_{p-\max} \approx \frac{1}{B(c-v)}\sqrt{\frac{1+\frac{\beta^2}{\alpha^2}e^{-2rL}+\frac{2\beta}{\alpha}e^{-rL}}{1+e^{-2rL}-2e^{-rL}}} \quad (31)$$

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