# Probabilistic design of relief wells systems as piping mitigation measure.

**Carlos Miranda**[1,2*], **Ana Teixeira**[2+], **Maximilian Huber**[2++], **Timo Schweckendiek**[1,2**]

[1] Delft University of Technology, Delft, The Netherlands
[2] Deltares, Delft, The Netherlands
*Carlos.Miranda@deltares.nl
+ Ana.Teixeira@deltares.nl
++ Maximilian.Huber@deltares.nl
** Timo.Schweckendiek@deltares.nl

**Abstract** Nowadays there is an on-going discussion about piping safety for dikes in the Netherlands. Relief wells represent an attractive solution as mitigation measure against piping, saving hinterland space. Nevertheless, they have been disregarded due to the uncertainties in its performance over its life cycle. The aim of this contribution is to demonstrate a probabilistic design of relief wells systems using fully and approximated probabilistic methods. We compare the results with the reliability target for piping as set in the Netherlands. For this purpose, statistical parameters of the influencing variables were studied, using collected data from existing projects or field observations in the Netherlands. Within this, we used the the design approach for relief wells, as proposed by U.S. Army Corps of Engineers.

## 1 Introduction

Netherlands is historically known for its continuous battle against flooding. The Netherlands has 3,600 km of dikes and dunes. These primary flood defences given in figure 1 are evaluated every five years. According to the last assessment of primary flood defences in 2013 [3], 680 km of dikes do not fulfil these given safety requirements.

Piping is a type of regressive erosion underneath a dike. This erosion process starts downstream and progresses upstream until it reaches the water source (e.g. river), creating pipes underneath the structure, which could lead to its collapse.

Up to now, design methods have been based on the use of semi-probabilistic safety factors for load and resistance parameters. These different safety factors are based on expert knowledge or probabilistic analyses on an acceptable low probability of failure. Latest developments in reliability (probabilistic) based design and the possibility to perform numerous computations allow introducing uncertainties from all the involved variables into the performance functions. This allows determining the probability of failure of the system, which will lead towards a more "rational" design, without the need of safety factors, which sometimes are not specified on design codes.

This contribution focuses on the probabilistic based design of relief wells. This includes

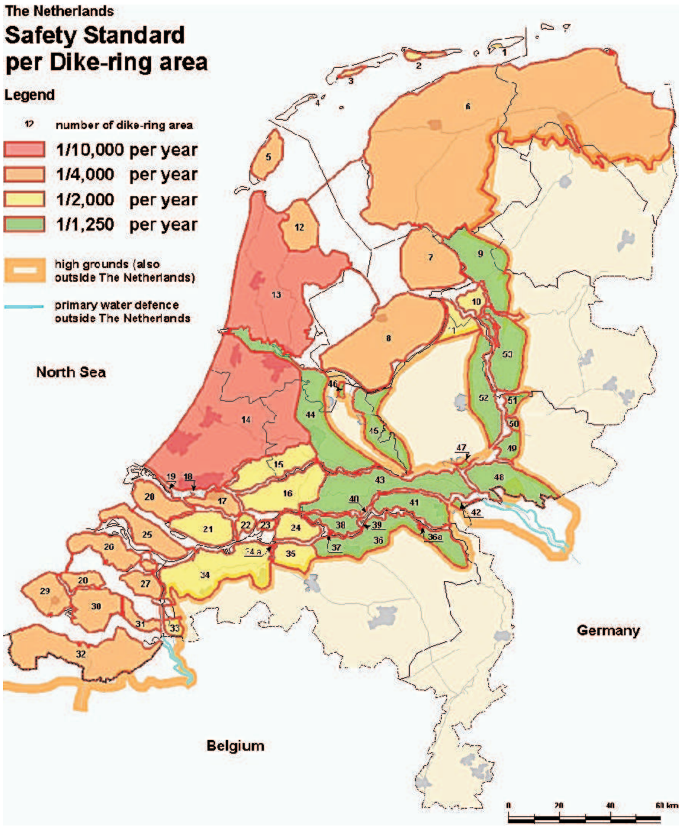


Figure 1: Safety standards per dike ring area in the Netherlands, [9].

the basics for the design of relief wells, the discussion of the limitations and strength of the proposed approach and a case study at the end of this contribution.

# 2 Relief wells

Relief wells are drainage systems in confined aquifers as shown in figure 2; relief wells are one of the possible mitigation measure against failure due to the piping mechanism. They consist of a riser pipe drilled in the soil through the impervious strata until the previous strata, allowing the underwater to reach the free surface, relieving the pore water pressure. Screens and filters are needed in order to avoid loss of coarse fine material and prevent clogging, which can lead to a decrease in wells' efficiency. A system of partially or fully penetrated wells is needed in order to obtain a reduced ground-water level and to ensure an allowable level. The goal of this design is to find the position of such wells in order to acquire the design requirements.

## 2.1 Design approach for relief wells

In general, one can distinguish between two system of relief wells: the fully penetrated wells in figure 3 (a) and the partially penetrated wells in figure 3 (b). We assume that

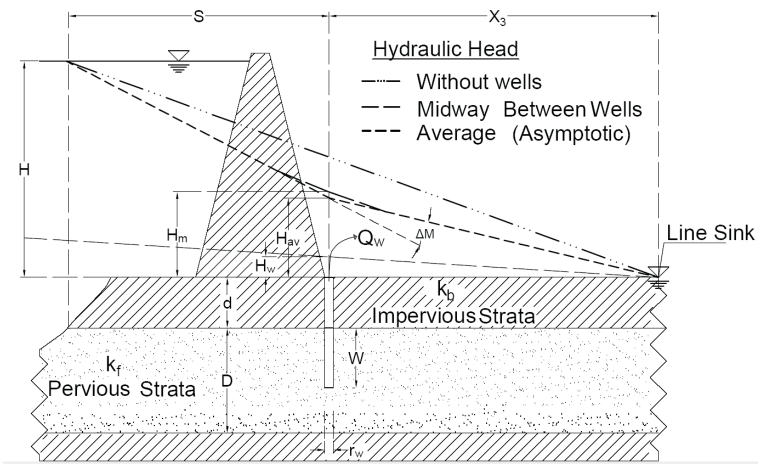


Figure 2: Nomenclature for relief wells system.

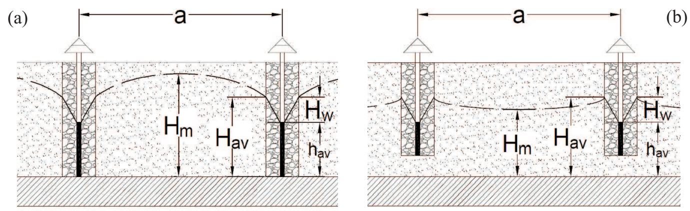


Figure 3: Hydraulic head in relief wells system for fully penetrated wells (a) and partially penetrated wells (b).

the drawdown of the hydraulic head in relief wells is in accordance to Dupuit Forchheimer assumption [1]. In the case of fully penetrated wells in figure 3 (a) the maximum hinterland head will always occur midway between wells. For partially penetrated wells in figure 3 (b) the efficiency is reduced through a smaller available flow discharge. Additionally, partial penetration induces a vertical flow and increases the velocity in the vicinity of the well; this increases the head losses. This effect decreases while moving away from the well and leads the maximum head to be on wells' plane.

We use the semi-empirical method, which is proposed by USACE [11], to evaluate the potential at the exit point in a multiple well systems. The procedure and formulas to apply are described in detail at [5]. We consider the head at well's plane $H_{av}$ and the head between wells $H_m$ given in equation 1 and 2, which are used to calculate net seepage slope $\Delta M$. Figures 2 and 3 shows the head at well's plane $H_{av}$ and the head between wells $H_m$ for fully and partially penetrated wells.

$$H_{av} = a \cdot \Delta M \cdot \theta_a + H_w \tag{1}$$

$$H_m = a \cdot \Delta M \cdot \theta_m + H_w \tag{2}$$

$$\Delta M = f(H, H_{av}, S, X_3) \tag{3}$$

In equation 1 and 2 $a$ refers to the well spacing given in figure 3. The net seepage slope

($\Delta M$) is defined as the difference between the slope formed in front of the well; from $\Delta H$ and $H_{av}$ , and the hinterland slope in figure 2. $\theta_a$ and $\theta_m$ are the so called well factors which are function of: $D/a$, $W/D$ and $a/r_w$. The hydraulic heads ($H_{av}$ and $H_m$) are corrected by adding the well losses $H_w$. The rest of the involved variables are given in the appendix.

## 2.2 Limitations of USACE method

Using the proposed USACE method for the design of relief wells, one has to keep the limitations of this approach in mind. The main limitation of the USACE method is the assumption of laminar flow. According to [1], we also assume 1 as a maximum Reynolds number value, which is a safe limit taking into account that the Reynolds number is highly sensitive to sand characteristic diameter. The method shows for practical application a limitation due to well factors which were determined for a given range of the ratio between well spacing ($a$) and aquifer's thickness ($0.25 < D/a < 4$); this limits to find the solution in between those limits. These losses can be estimated from experimental data, given in [10].

# 3 Probabilistic design of relief wells system

In order to reduce the complexity of the given system of relief wells, we consider only heave and uplift in the sequel case studies. The probability of failure of the system $P_f$ is, therefore, a parallel system of the probability of failure for uplift $P_{f,u}$ and for heave $P_{f,h}$ given in equation 4. We do not consider piping because there is not a method to consider this mechanism when drainage systems are applied.

$$P_f = P_{fu} \cap P_{fh} \tag{4}$$

We use the FORM and the Monte-Carlo approach for the case of piping mechanism under a dike.

## 3.1 Limit state functions

The limit state function defines the ultimate state of a mechanism, which is the boundary between desirable and undesirable performance of the mechanism considered. High water pressure in the sand layer under the impervious strata (blanket) can cause uplifting and even cracking of this layer. The limit state function for uplift is defined by the difference of resistance, which is the vertical effective stress at the bottom of the cover layer, and the load, which is the average head of the well $H_{av}$ and the average head between wells $H_m$ (equation 5). Heave can only occur, if the vertical gradient at the exit point exceeds the critical value for heave $i_c$. The limit state equation compares the critical gradient for heave and the existing vertical gradient on the blanket as given in equation 6.

$$Z_u = \frac{d \cdot (\gamma_s - \gamma_{water})}{\gamma_{water}} - \max\{H_{av}, H_m\} \tag{5}$$

$$Z_h = i_c - \frac{\max\{H_{av}, H_m\}}{d} \tag{6}$$

We use equation 7 within the Monte-Carlo approach for the simulation of the combination of uplift and heave.

$$Z_{u+h} = \max\{Z_u, Z_h\} > 0 \tag{7}$$

Additionally, we use the established Hohenbichler Rackwitz approach [2] to combine the uplift and heave mechanism within FORM.

## 3.2 Random variables

The basis for random variables are given in [4] and [6, 7]. We assume the gravity acceleration $g$, the well radius $r_w$, the well thickness $t_p$ and the specific weight of the water $\gamma_w$ as deterministic variables. We summarize the random variables with type of distribution, mean value and standard deviation in table 1.

## 3.3 Target reliability for piping

In order to obtain a probabilistic design we set our target based on the reliability of the system. Reliability is a measure of the probability that our system does not fail. The reliability index is defined as:

$$\beta \quad = \quad -\Phi^{-1}(P_f) \tag{8}$$

Herein,$\beta$ is the reliability index, $\Phi^{-1}$ is the inverse of the standard normal cumulative distribution function, and $P_f$ is the probability of failure of our system.

Table 1: Distribution type of the random variables, mean values $\mu$, standard deviations $\sigma$ used for the design of the relief wells system in case study A and case study B.

| | pdf | Case study A | | Case study B | |
|---|---|---|---|---|---|
| | | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ |
| $\gamma_{cover}$ [kN/m$^3$] | normal | 16.00 | 1.60 | 17.10 | 1.70 |
| $\gamma_w$ [kN/m$^3$] | determ. | 10.00 | - | 10.00 | - |
| $d$ [m] | lognormal | 3.00 | 0.15 | 4.11 | 1.23 |
| $D$ [m] | normal | 26.30 | 5.05 | 9.30 | 3.00 |
| $k_f$ [m/s] | lognormal | $1.74 \cdot 10^{-4}$ | $3.29 \cdot 10^{-4}$ | $5.79 \cdot 10^{-4}$ | $7.56 \cdot 10^{-4}$ |
| $k_b$ [m/s] | lognormal | $1.16 \cdot 10^{-6}$ | $1.16 \cdot 10^{-6}$ | $1.16 \cdot 10^{-6}$ | $1.16 \cdot 10^{-6}$ |
| $h_r$ [m] | Gumbel | -3.79 | 0.30 | -8.45 | 0.30 |
| $h_p$ [m] | normal | 4.30 | 0.25 | 8.70 | 0.10 |
| $H_e$ [m] | lognormal | 0.05 | 0.05 | 0.05 | 0.05 |
| $S$ [m] | normal | 28.50 | 3.42 | 22.86 | 2.29 |
| $C$ [-] | normal | 125 | 10 | 125 | 10 |
| $r_w$ [m] | normal | 0.15 | 0.00 | 0.15 | 0.00 |
| $i_c$ [-] | lognormal | 0.70 | 0.10 | 0.70 | 0.10 |

Dikes are usually long structures, which are influenced by longitudinal spatial variations. This spatial variation is considered via the length effect. The length effect is defined as the increase of the failure probability with the length of the dike due to imperfect correlations and/or independence between different cross sections and/or elements [8]. This indicates that decrease of the system reliability with the increase of its length. In order to be able to perform a probabilistic design, the reliability target has to be defined. Different researches [8] developed the formulation presented on equation 9 for translating dike ring requirement 1 into a (local) cross section safety requirement for piping and uplift.

$$P_{adm,loc} = \frac{0.1 \cdot P_{adm,ring}}{1 + \frac{\alpha}{l_{eq}} * L_{dr,s}} \tag{9}$$

Herein, $P_{adm,loc}$ stands for the local admissible failure probability, $P_{adm,ring}$ for the admissible failure probability for the dike ring requirements, $\frac{\alpha}{l_{eq}}$ is the length effect factor, and $L_{dr,s}$ is the length of the dike that is sensitive to the considered mechanism. Among others, [8] reports the ratio is $\alpha/l_{eq} = 0.0028$ for piping and $\alpha/l_{eq} = 0.0045$ for uplift.

# 4 Case studies

In this section we investigate the design of relief wells within a probabilistic based design framework in two case studies. We selected a cross section located in dike ring 36 for case study A and and a cross section in dike ring 52 for case study B, given in figure 1. The data for these two locations are given in table 1. One can derive from figure 1 that both locations have a required probability of failure of $1/1,250$ years. For both, 10 km of the dike stretches, which are sensitive to piping are considered and one can calculate the target reliability $\beta_{adm,loc} = 4.5$ using equation 9. We investigate several combinations of well spacing and well penetration in order to cover all possible combinations. We show the results of case study A and B in figure 4. The reliability index $\beta$ is plotted as contour lines among the possible combinations of well spacing $a$ and well penetration $W/D$, delimiting zones with equal reliability target. From the results we can derive that large ratios $W/D$ and small well distances $a$ show high reliability indices. For the investigated cases one can see that in case of a reliability index $\beta \geq 4.5$ fully penetration is needed (figure 4).

The FORM-sensitivity factors $\alpha^2$ for the investigated cases are given in figure 5. We present them grouped on basis of $W/D$ and show them for two different ratios of $D/a$. It can be observed that there is a significant scatter among the sensitivity factors. From figure 5 (a) and (d) we can observe that for partial well penetration the blanket permeability is the driven variable. For fully penetration, entrance losses are the driven variable. On the other hand, in 5 (d), larger well spacing, blanket and aquifer permeability's are the driven variables despite well penetration. In case of having small well spacing one can see in 5 (b) that the influence of the specific weight of the blanket is lower than when having larger well spacing.

## 5 Conclusions

This contribution presents the application of a probabilistic design of relief wells for piping mitigation solution. Within this, we use the USACE relief wells' design procedure in the framework of a probabilistic based design. Herein, the length effect due to the soil spatial variability is considered, as well as the combination of the sub-failure modes heave and uplift as a parallel system to achieve piping failure. Two case studies were investigated, and each uncertainty studied to identify which are the most important and influential in the evaluation of the performance of relief wells. FORM-sensitivity coefficients show that, using USACE method, the blanket and the aquifer permeability, as well as the hydraulic losses, are the dominant variables (from the 'load' side). However, a high discrepancy between these sensitivity coefficients was found for 'strength' side in partially and fully penetrated wells. As result of the reliability analysis, graphs like 4 shows the appropriate combinations of the design variables which fulfil our safety requirements. The optimum alternative should then be chosen after a cost analysis optimization, the core of such analysis can be found in [5].

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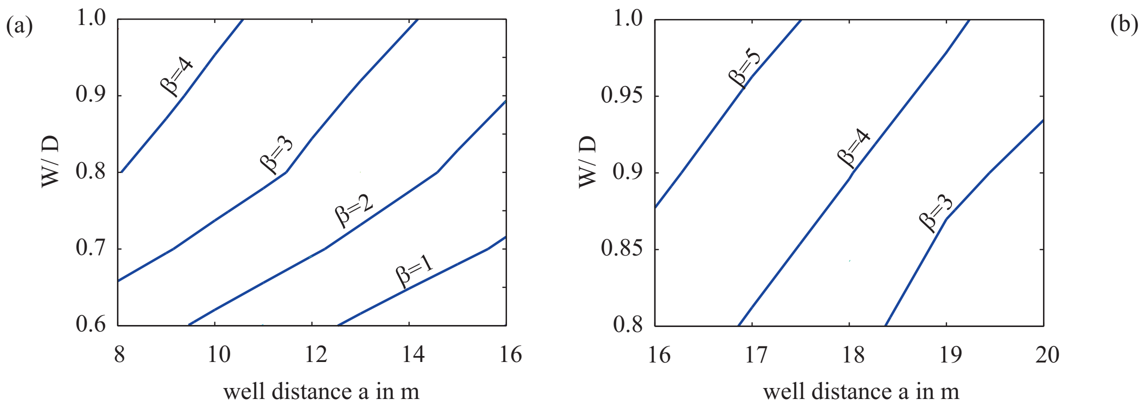


Figure 4: Ratio of well penetration $W$ and thickness of the aquifer $D$ in relation to the well distances $a$ for different reliability indices $\beta$ for case study A (a) and case study B (b).

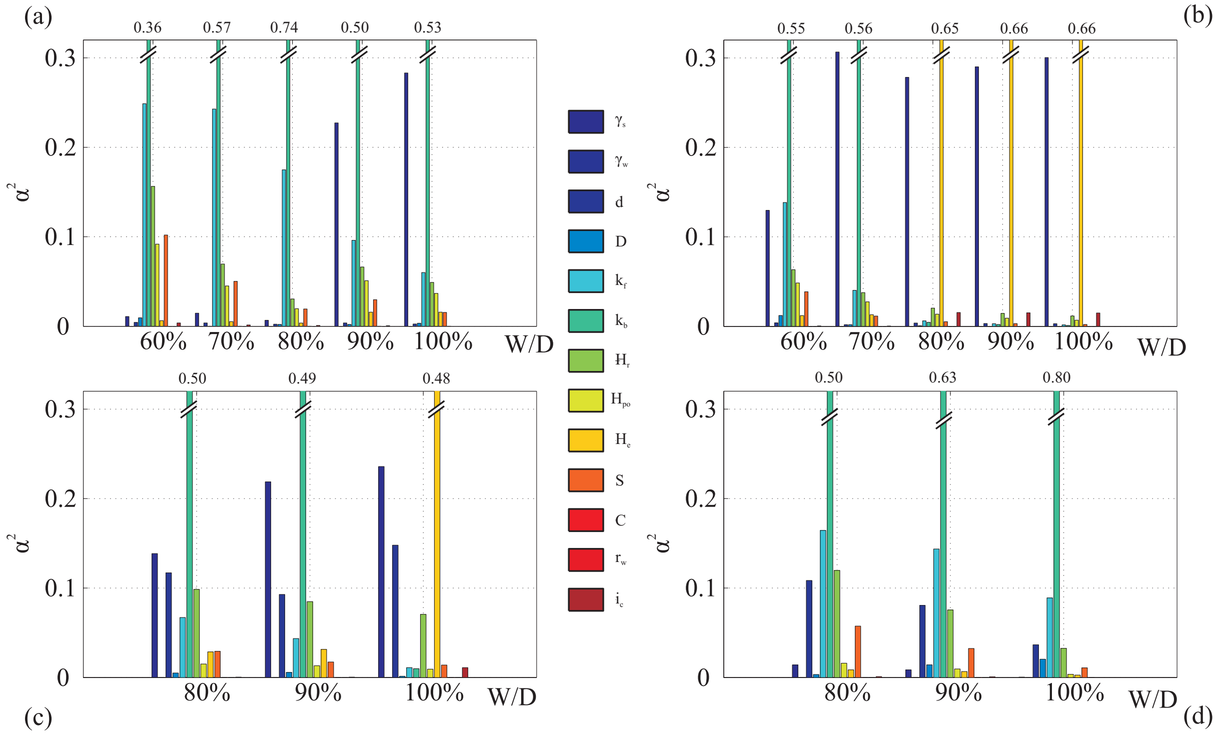


Figure 5: Sensitivity factors for cases for different well penetration $D/a$ =1.6 in case study A (a), $D/a$ =2.6 in case study A (b), $D/a$ =0.6 in case study B (c) and $D/a$ =0.46 in case study B (d).

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