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Smart abstraction of stochastic systems using memory

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1 Introduction

Autonomous systems in uncertain environments are increasingly common, with applications like autonomous driving, rescue robots, and smart grids [2]. Ensuring their safe deployment requires mathematical methods for formal verification, often using *stochastic models* to account for uncertainty. These systems have the form

$$x_0 \sim \lambda_0, \quad x_{k+1} \sim \tau(\cdot|x_k),$$

where $x_k \in X$ belongs to a continuous state space, and where λ_0 and τ are respectively the initial probability measure of the state and the transition kernel of the system. The states x_k are then realizations of a continuous Markov process denoted by $(X_k)_{k\in\mathbb{N}}$.

For safety-critical analysis of such systems, methods that are based on discrete approximations of stochastic systems, called *abstractions*, have recently surged [1]. Using these methods, one defines a *partition* the state space in *cells* $A_1, \dots, A_n$, and studies the discrete process defined by

$$Y_k = i \iff X_k \in A_i,$$

which consists of observing the successive cells in which the state lies without knowing the exact state. A fundamental observation about $(Y_k)_{k\geq 0}$ is that, although it consists of information about the state Markov process $(X_k)_{k\in\mathbb{N}}$, the process (Y_k) **is not a Markov chain**, as, in general,

$$\mathbb{P}[Y_k = i_k | Y_{k-1} = i_{k-1}, \dots, Y_0 = i_0] \neq \mathbb{P}[Y_k = i_k | Y_{k-1} = i_{k-1}].$$

2 Memory-dependent abstractions

The most classical abstraction-based approach consists in studying the simple Markov chain $(\tilde{Y})_{k\in\mathbb{N}}$, whose transition matrix is defined by 1-step observations of the cells process, that is

$$\mathbb{P}[\tilde{Y}_1 = i_1 | \tilde{Y}_0 = i_0] := \mathbb{P}[Y_1 = i_1 | Y_0 = i_0],$$

leading to an abstraction of size n^2 — the number of transitions between cells. But because of the non-Markovianity of the process $(Y_k)_{k\geq 0}$, this method introduces a non-negligible approximation error. In order to circumvent this, we propose **increasing the length of the observations**, and construct a ℓ-memory Markov chain $(\tilde{Y}_k^\ell)_{k\in\mathbb{N}}$ defined as

$$\begin{aligned}\mathbb{P}[\tilde{Y}_\ell^\ell = i_\ell | \tilde{Y}_{\ell-1}^\ell = i_{\ell-1}, \dots, \tilde{Y}_0^\ell = i_0] \\ := \mathbb{P}[Y_\ell = i_\ell | Y_{\ell-1} = i_{\ell-1}, \dots, Y_0 = i_0],\end{aligned}$$

leading to an abstraction of size $n^{\ell+1}$ — the number of transitions between sequences of cells.

3 Results

In this work, we show that, for **abstractions of the same size, increasing memory may lead to smaller approximation errors**, enabling a smart abstraction method, as shown in Figure 1.

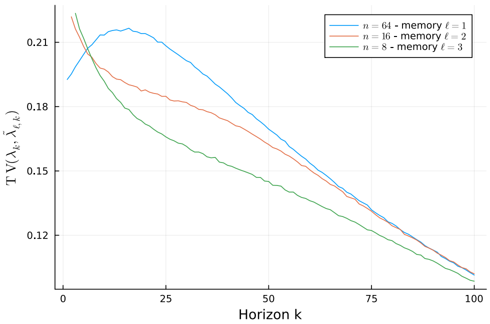

Figure 1: Approximation errors for different (uniform) partitions and memory. For the same size ($n^{\ell+1} = 4096$), abstractions with a larger memory lead to smaller errors.

More specifically, we combine concepts from transfer operator theory and symbolic control to demonstrate that the constructed memory Markov processes consists of a lifted Galerkin approximation of the transfer operator. We also establish guarantees on the total variation distance between the true state distribution at a given time and the distribution defined by our memory-dependent abstractions.

References

[1] A. Lavaei, S. Soudjani, A. Abate, and M. Zamani. Automated verification and synthesis of stochastic hybrid systems: A survey. *Automatica*, 146:110617, Dec. 2022.

[2] E. A. Lee and S. A. Seshia. *Introduction to Embedded Systems A Cyber-Physical Systems Approach*. MIT Press, 2016.

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