Handling Qualities Optimization in Aircraft Conceptual Design

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Challenge the future

Handling Qualities Optimization in Aircraft Conceptual Design

by

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Preface

"The risk I took was calculated but man, am I bad at math."

Anonymous

Summary

The handling qualities of an aircraft have always been a crucial field of study, being chiefly concerned with the safety and comfort of flight. Historically, the design-by-discipline approach has been used, thus relegating stability and control considerations to later stages of the design process. In recent years a new view emerged, which advocates the benefits of including handling qualities at earlier stages, such as conceptual and preliminary.

The objective of the research is to develop a handling qualities optimization module to be fitted in the Initiator, a conceptual design tool developed at TU Delft. The module shall handle both unaugmented designs (bare airframe), as well as augmented design, in which a suitable stability augmentation system (SAS) is included. To this end, a common stability paradigm, based on a modified Routh-Hurwitz criterion, has been implemented as a set of nonlinear constraints on the design space. In essence, the criterion has been transformed from a test to a design procedure, taking the form of a general polynomial-based regional pole placement method. The methodology is concerned with conventional aircraft configurations, specifically by sizing the horizontal tailplane and positioning of the wing. The module is focused on designs that possess optimal short period damping ratio and Control Anticipation Parameter (CAP), while the objective functions to be minimized are tailplane induced drag, zero-lift drag, and weight. Static stability and controllability are ensured in the relevant flight regimes using Torenbeek's X-plots, implemented in the form of additional constraints on the design space. Lastly, the stability augmentation system consists of a pitch damper and an angle of attack feedback. Two different methods have been investigated for the computation of the feedback gains: the first one entails the inclusion of such gains in the design vector. The second one makes use of an optimal control technique: the Linear Quadratic Regulator (LQR), augmenting the design vector with three required weighting factors. Hence, the module accepts three user inputs which specify the methodology: unaugmented design, augmented with method 1 (gains in the design vector) and augmented with method 2 (LQR).

The optimization has been run for the three different user-specified methods on an Airbus A320-200. The results show that the procedure is successful, increasing the CAP by 100 % for the unaugmented and augmented (method 1) designs. By using the LQR, the CAP showed an increase in 150 %. Moreover the unaugmented design achieved the required short period damping of 0.55, or a 34 % increase with respect to the baseline value, while the augmented designs were capable of achieving a damping ratio of 0.76, which corresponds to an increase of 85 %.

A convergence study has been done within the Initiator, to assess the performance of the optimized configurations concerning aerodynamic efficiency (L/D_{max}) and maximum takeoff mass (MTOM). The investigation led to the definition of *breakeven points*, which are used to indicate the values of short period damping ratio for which, at a given CAP, the optimized designs have the same performance as the baseline aircraft. Two general design guidelines have been extrapolated: for unaugmented designs, the breakeven points move closer to the baseline values as the CAP increases. Furthermore, at the baseline short period damping ratio and CAP, the optimized configuration is more efficient. For augmented designs, it was revealed that the performance benefits achievable are substantially higher and independent of the required handling qualities. Hence, no breakeven points can be defined. Optimal values of L/D_{max} and MTOM are achieved right from the baseline values of damping and CAP, while the gains are progressively increased to cope with the handling qualities demands. Due to the qualitative nature of these conclusions, it is deemed possible to extend them to other conventional configurations, thus providing general design guidelines.

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At the conclusion of my life as a student, looking back at the past two years feels like standing on top of a hill, and a considerably steep one at that. However, all along the way I had people supporting me in this effort. Most of them were there also through the final push: the master thesis. This page is therefore devoted to addressing them, and expressing my appreciation.

First and foremost, I'd like to thank my family. As cheesy as it sounds, I would not have been here if it wasn't for them, for a million different reasons. The most important one, however, is quite simple: they allowed to pursue my interests and dreams, always. Not once have I felt withheld, and for that, I will be forever grateful. A special thought goes to my sister, who I'm enormously proud of.

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Now, people who know me know that I have a knack for drama, therefore I will conclude the same way I began this experience, with a quote: *"man was not made to fly, but he can definitely work on it"*.

D. Cosenza Delft, November 2016

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Nomenclature

Latin symbols	Description	Unit	
A	state matrix	[-]	
Ac	augmented state matrix	[-]	
AR _{ht}	horizontal tail aspect ratio	[-]	
b_{ht}	horizontail tail span	[m]	
В	input matrix	[-]	
С	output matrix	[-]	
CAP	control anticipation parameter	[-]	
\overline{c}	mean aerodynamic chord	[m]	
c _{ineq}	optimization inequality constraint	[-]	
C_{D_f}	skin friction drag coefficient	[-]	
C_{D_i}	induced drag coefficient	[-]	
C_f	skin friction coefficient	[-]	
$C_{L_{lpha}}$	aircraft lift curve slope	[rad ⁻¹]	
$C_{L_{\alpha_{\mathrm{ht}}}}$	horizontal tail lift curve slope	$[rad^{-1}]$	
$C_{L_{\alpha_{ub}}}$	wing-fuselage lift curve slope	$[rad^{-1}]$	
$C_{L_{\dot{\alpha}}}$	derivative of lift w.r.t angle of attack rate	$[rad^{-1}]$	
C_{L_a}	derivative of lift w.r.t pitch rate	$[rad^{-1}]$	
$C_{L_{\delta_{\alpha}}}$	derivative of lift w.r.t elevator deflection	$[rad^{-1}]$	
C_{mac}	aerodynamic center pitching moment	[-]	
$C_{m_{\alpha}}$	lift coefficient	$[rad^{-1}]$	
$C_{m_{\dot{\alpha}}}$	derivative of pitching moment w.r.t angle of attack rate	$[rad^{-1}]$	
C_{m_a}	derivative of pitching moment w.r.t angle of pitch rate	$[rad^{-1}]$	
$C_{m_{\delta_{\alpha}}}$	derivative of pitching moment w.r.t elevator deflection	$[rad^{-1}]$	
D	feedthrough matrix	[-]	
D_c	non-dimensional derivative operator	[-]	
g	gravitational acceleration	$[m/s^2]$	
J	objective function	[-]	
K_h	composite correction factor	[-]	
k_{lpha}	angle of attack feedback gain	[rad/rad]	
k_q	pitch rate feedback gain	[rad/rad/s]	
K_Y^2	non-dimensional inertia factor	[-]	
lb	lower bound	[-]	
L	thickness ratio factor	[-]	
L_h	horizontal tail arm	[m]	
MTOM	maximum takeoff mass	[kg]	
q_1^{\star}	first element of state weighting matrix	[-]	
q_2^{\star}	second element of state weighting matrix	[-]	
$\frac{q_h}{q}$	horizontail tail dynamic pressure ratio	[-]	
Q	state weighting matrix	[-]	
R	control weighting matrix	[-]	
R_{l_s}	lifting surface correction factor	[-]	

$R_{ m array, \alpha}$	modified Routh array - α	[-]
R_{array,x_1}	modified Routh array - x_1	[-]
R _{array.modified}	modified Routh array - second condition	[-]
S_{ht}	horizontail tail planform area	[m ²]
$S_{ht_{wet}}$	horizontail tail wetted area	[m ²]
Sw	wing planform area	[m ²]
$\left(\frac{t}{c}\right)$	thickness over chord ratio	[-]
T_{θ_2}	incidence lag constant	[-]
ub	upper bound	[-]
u	input vector	[-]
V_0	initial speed	[m/s]
V_D	maximum dive speed	[m/s]
W_{ht}	horizontail tail weight	[kg]
x	design variable	[-]
x	state vector	[-]
ż	derivative of state vector	[-]
<i>x_{ac}</i>	aircraft aerodynamic center position	[m]
x_{cg}	aircraft center of gravity position	[m]
x_{pos_w}	longitudinal wing apex position	[m]
$\overline{x}_{cg_{aft,max}}$	maximum aft center of gravity position	[-]
$\overline{x}_{cg_{\text{fwd,max}}}$	maximum forward center of gravity position	[-]
$\overline{x}_{cg_{\text{NPaft}}}$	aft center of gravity position for neutral stability	[-]
$\overline{x}_{cg_{\text{TO}}}$	forward center of gravity position for takeoff rotation	[-]
$\overline{x}_{cg_{stall}}$	forward center of gravity position for stall	[-]
y	output vector	[-]
Greek symbols	Description	Unit
α	angle of attack	[rad]
α^{\star}	amplitude of cone in imaginary plane	[deg]
δ_e	elevator deflection	[rad]
$\frac{d\epsilon}{d\alpha}$	downwash gradient	[rad]
ζ	damping ratio	[-]
ζ_{sp}	short period damping ratio	[-]
ζ_{ph}	phugoid damping ratio	[-]
λ_{ht}	horizontail tail taper ratio	[-]
$\Lambda_{ht_{ ext{LE}}}$	horizontail tail sweep - leading edge	[deg]
$\Lambda_{ht_{0.25}}$	horizontail tail sweep - quarter chord	[deg]
$\Lambda_{ht_{0.5}}$	horizontail tail sweep - half chord	[deg]
μ_c	non-dimensional mass	[-]
$ ho^{\star}$	control weighting factor	[-]
ω_n	undamped natural frequency	[rad/s]
$\omega_{n_{ m sp}}$	short period undamped natural frequency	[rad/s]

Chapter 1

Introduction

1.1. Background

Since the beginning of controlled flight, with the Wright Brothers, aircraft designers have always been concerned with the flight characteristics of aircrafts. Historically during the conceptual and preliminary design phase handling qualities have never been treated with a substantial level of detail. The most common approach consisted in nothing more than simple static stability and controllability considerations, mainly relying on statistical and historical data [1, 2]. This entailed treating the various disciplines that come into play during the conceptual design phase in a sequential way, with stability and control treated as an analysis discipline, and not a design one. However this approach has flaws since shortcomings in handling qualities might come up in later stages, when making changes to the aircraft configuration becomes extremely difficult and, more than anything, quite costly. In this respect, Soban states that "*Although a relatively small fraction of life cycle costs are spent during the preliminary design phase, mistakes and misjudgments during this phase prove costly, and sometimes financially disastrous, to fix at later dates"* [3]. The merit of including handling qualities at the conceptual and preliminary design stage is, therefore, apparent, both in terms of cost effectiveness as well as safety of new designs.

In recent years, this field has expanded in scope and complexity, encompassing both airframe and control system design, aiming at achieving the highest level of handling qualities and at the same time obtaining performance benefits. The advent of Multidisciplinary Design Optimization (MDO) paved the way for the inclusion of stability and control as a parallel design discipline, applied to the investigation of relaxed static stability and unconventional configuration for performance increase, but still, the potential benefits of such approach were not exploited to the fullest. For this to come about, researchers focused on applying optimal control methodologies to increase the capabilities of MDO and allowing for the inclusion of complex control systems architectures during the conceptual and preliminary design phases. Hence, with the development of Automatic Flight Control Systems (AFCS) it became clear that it would have been possible to redesign an aircraft exploiting this new technology, achieving smaller stabilizing surfaces and therefore improvements in terms of drag, weight and fuel consumption.

1.2. Handling Qualities or Flying Qualities?

In practical applications in the aerospace industry, the terms handling qualities and flying qualities are often used interchangeably. Generally, they are used to refer to the flight characteristics of an aircraft or, to put in a different way, to indicate how "well" an aircraft flies. However, this definition is quite vague. Especially with the widespread diffusion of fly-by-wire, the complexity of this discipline has exponentially increased over the past years. It is no surprise then that finding a common way to refer to the flight characteristics of an aircraft has proved quite a challenge and has often spawned misunderstandings.

As an example, in [4], Philips gives the following definition of flying qualities: "the stability and control

characteristics that have an important bearing on the safety of flight and on the pilot's impressions of the ease of flying an airplane in steady flight and in maneuvers". His definition of flying qualities, therefore, encompasses an objective criteria, the safety of flight, and a subjective criteria, as he refers to the pilot's own judgment. Another definition can be found in the landmark article from Cooper and Harper [5]. In their work, in which they pioneered the improvement of the idea of a unified framework for the evaluation of handling qualities based on pilot rating scales, they defined them as "those qualities or characteristics of an aircraft that govern the ease and the precision with which a pilot is able to perform the tasks required in support of an aircraft role". The focus, therefore, was definitely switched onto the pilot subjective opinion, with respect to a certain task, or mission, that the aircraft has to accomplish. Furthermore, they also stated that "The generally accepted meaning of Flying Qualities is similar to this definition of Handling Qualities". Regardless of the discrepancies, it can be noted that a common feature emerges: there are definitely two sides to the same medal, one being the judgment of the pilot and the other being the capacity of the aircraft to accomplish a task while ensuring adequate safety.

To ensure consistency, for the remainder of this document the definition found in [6] is used: the term handling qualities "*is used to describe those parameters that characterize the stability, control and response of an aircraft and so govern the ease and precision with which the pilot is able to fly an aircraft"*. On the other hand, the term flying qualities "*relate to the pilot assessment of how well he is able to fly an aircraft to complete the range of tasks required and are wholly subjective in character*".



Figure 1.1: Handling Qualities vs Flying Qualities [7]

1.3. Handling Qualities Evaluation

Having given a univocal qualitative definition of handling qualities, it is now appropriate to introduce how said handling qualities have been, and still are, evaluated and consequently how an aircraft is characterized in this respect. Historically the first and most important way to determine flying characteristics has always been through flight testing. The pilot would, therefore, be the judge of the aircraft's handling qualities, marking the distinction between excellent, acceptable and unacceptable. A considerable amount of effort was then put into translating this pilot opinions into objective and general metrics, which could be applied to every (similar) aircraft. Finally, these metrics were then formalized and the recommended values in terms of classical aircraft response analysis collected in documents, issued by military organisms and regulating agencies, which specify the required values for damping ratios and frequencies for different categories of aircraft and different flight phases: these are referred to as *flying qualities requirements*. The aforementioned documents are used by airplane designers as guideline to achieve optimal handling qualities [6].

Section 1.3 is structured in the following way: firstly an overview of the Cooper-Harper rating scale is given, since it is deemed to be historically crucial to the evaluation of handling qualities. Secondly, the various metrics derived from the pilot's experiences are described, concluding with the presentation of the widely used military specifications, collected in [8]. An example of general the approach used to include handling

qualities considerations in the design process is shown in Figure 1.2.



Figure 1.2: Evaluation and use of handling qualities in the design process

1.3.1. Cooper-Harper Rating Scale

The work of Cooper and Harper [5] has already been introduced in the previous sections, but given its importance in the field of handling qualities and how extensively it has been used over the years during flight test campaigns [9], it was deemed fundamental to give a brief presentation of their work.



Figure 1.3: Cooper-Harper rating scale [5]

As it can be seen in Figure 1.3, the Cooper-Harper Rating scale ranges from 1 to 10, with one being the best handling qualities and 10 the worst.Furthermore, it is formulated in such a way that both keywords and numerical values are used. This is an important feature, and the specific keywords were selected so that the distinction between each level of rating would be univocal and unambiguous; however, this approach was also recognized as risky. Given the fact that this scale is formulated with linearity in mind, engineers might be tempted to treat the qualitative definitions provided by the keywords with mathematical tools

which are strictly valid in the linear domain [5]. It is, however, clear that this scale does not lend itself to such mathematical methods.

There are several factors that are taken into account, and that contribute to the definition a specified level of handling qualities:

- Performance and workload
- Compensation
- Failure or emergency operations
- · Operating margin and safety
- Pilots skill

The two terms appearing in the first factor, performance, and workload, refer to two distinct aspects of the human-machine interaction: the former describes "*the precision of aircraft control attained by the pilot, that is, the pilot-vehicle, performance*" [5]; the latter is intended to convey "*the amount of effort and at-tention, both physical and mental, that the pilot must provide to attain a given level of performance*" [5]. The term workload is therefore closely tied to the following factor, compensation: this term is intended to describe the required increase in pilot workload to achieve the desired performance. It can be immediately inferred that a decrease in performance would then entail a decisive increase in both workload and compensation, putting an increasing amount of mental and physical strain on the pilot. This eventuality is definitely not acceptable nor satisfactory, since the pilot, as a human being, can only cope with stress (both physical and mental) to a certain extent, after which it is no longer possible to complete a required task with adequate operational and safety margins. Indeed these considerations are taken into account, as the fourth factor in the list, and are inextricably tied to performance and workload, and more in general, to handling qualities. The connection between good handling qualities and safety has always been apparent, since the early days of flight [4], and is still a key requirement.

The last factor that appears in the Cooper-Harper rating scale is pilot skill: but how does it differ from the already mentioned pilot workload? As can be seen in Figure 1.3, pilot skill comes into play in the lowest levels of the rating scale, and indicates a situation in which the excessive workload condition has been surpassed, and the limiting factor has now become the capability of the pilot to actually continue performing the task, or the presence of extreme difficulty in controlling the aircraft. In this sense, the individual skill and level of training of a pilot is taken into account in the rating scale.

In conclusion, *"the test pilot is the final judge of the aircraft's handling qualities"* [6]. The effort then has been focused on the translation of these opinions into an objective assessment, which could then be used by engineers in the design phase to obtain the intended level of handling qualities. As an example, a few parameters encompassing stability, control and response that have been used in both flight testing and design are [6]:

- Stick displacement per g.
- Initial pitch acceleration per g.
- Time at peak pitch rate.
- Overshoot in normal acceleration response.
- Time to half/double amplitude

These parameters are only referred to longitudinal qualities, but similar ones can be found for lateraldirection handling qualities as well in [6], [10] and [11].

1.3.2. Handling Qualities Criteria

In the previous section, an introduction to the Cooper-Harper rating scale, and the transition between pilot rating and objective handling qualities evaluation metrics has been presented. This section is devoted to elaborating on this concept, illustrating how these opinions were "mapped" and how they could be used in the design process.

The procedure is a rather simple one: based on reports from various pilots it is possible to investigate which kind of characteristics, in terms of stability parameters, is common to the aircrafts that attained level 1, level 2 and level 3 handling qualities. From this point, charts can be drawn, which depict the conditions, as confirmed by the pilot's opinion, that need to be achieved for a specific level. These can then be translated in terms of classical modal response parameters, such as damping ratio and undamped natural frequency, and therefore investigated during the aircraft design process. As an example, the short period is going to be presented; it defined as an oscillating motion, in which essentially the incidence and airspeed of the aircraft remain constant while the angle-of-attack and pitch rate change quickly [7, 12].

In this respect, pilot ratings have been charted in terms of short period damping ratio ζ_{sp} and natural frequency $\omega_{n_{sp}}$, in order to point out the regions on these graphs that correspond to the three levels of handling qualities shown in Figure 1.3. One of the first examples of this approach is found in the work of Shomber and Gertsen [13]. An instance of such chart, which can be found in [6], for transport aircrafts in class III category (see Section 1.3.3) is presented in Figure 1.4.



Figure 1.4: Short Period "Thumb print" for class III transport aircraft [6]

It is possible to see that the best combination of ζ_{sp} and $\omega_{n_{sp}}$, or in other words the one that corresponds to level 1 handling qualities, is around 0.6 for the first and 3.0 [rad/s] for the second. These charts, of course, depend on the category of aircraft. and different charts are associated with different categories, even though their basis still lie in pilot ratings. Another interesting feature to note is that the areas which are significantly away from the optimum are characterized in words, so that is easy to infer that a low value of short period damping ratio corresponds to an extremely susceptible aircraft, while excessively high values determine a sluggish response. The same applies to the undamped natural frequency.

This kind of charts are valuable during the design process, because they provide general guidelines for engineers, in the matter of what they should try to achieve in terms of modal response parameters. Of course, these methods are quite rudimentary, and start becoming inadequate in the presence of significant stability augmentation systems (SAS), with additional dynamics introduced by actuators, filters, and a complex sensor apparatus.

More refined criteria for handling qualities evaluation exist, capable of investigating different aspect of the dynamic behavior of an aircraft as well as increasingly complex dynamics. These criteria are discussed in

the following paragraphs.

Control Anticipation Parameter Bihrle, in [14], refined the methods illustrated in the previous section, by developing a metric which is to be compared against the damping ratio ζ_{sp} of the aircraft, instead of the classical $\omega_{n_{sp}}$: the Control Anticipation Parameter (CAP). It is expressed as follows:

$$CAP = \frac{\omega_{n_{sp}}^2}{n_{\alpha}} \simeq \frac{\omega_{n_{sp}}^2}{C_{L_{trim}}/\alpha}$$
(1.1)

In words, it can be interpreted as "*the ratio of an aircraft's initial pitching acceleration to its change in steady state normal acceleration*" [15]. The core of this approach resides in the term n_{α} , which is constant with airspeed. The magnitude of the CAP determines the abruptness and intensity of pitching acceleration of the aircraft following an elevator input, with respect to the trimmed state normal acceleration. It can also be condensed in the statement "*does the nose follow the stick*?". The importance lies in the reaction of the pilot: a low CAP (sluggish response) will mean that lower pitching acceleration which might then lead to an overshoot on the intended flight path. On the other hand, high value of CAP means that the pilot would try to compensate for the excessive initial acceleration, thus ending up undershooting the intended flight path. An example of CAP chart mapping is presented in Figure 1.5.



Figure 1.5: CAP criterion for Landing Phase category III Aircraft [15]

On the ordinate axis the short period damping ratio ζ_{sp} is present. Similar charts can be found for different flight phases and aircraft's category, due of course to different requirements in terms pitch response between takeoff, cruise, and landing.

Bandwidth Criterion The bandwidth criterion bridges the gap between control theory and aircraft flight dynamics, in fact, it defines "*bandwidth frequency in a flying qualities sense*" [15]. Explicitly, the bandwidth of the aircraft is the "*highest open-loop crossover frequency attainable with good closed-loop dynamics*" [15]. These parameters can be used to determine how well the aircraft responds to a series of rapid inputs, and to what extent the pilot-in-the-loop influences its stability characteristics. This criterion best applied to aircraft with significant augmentation systems, for which the classical handling qualities metrics fall short. For a more in-depth discussion about the bandwidth and its description in terms of control theory, the reader is referred to [16].

Dropback Criterion The Dropback criterion was introduced by Gibson in [17] to properly characterize the aircraft response in the time domain, complementing the bandwidth criterion. It is defined as follows: "*a measure of the mid-frequency response to attitude changes...Excessive dropback results in pilot complaints of abruptness and lack of precision in pitch control - complaints common also to aircraft with excessive values of pitch attitude bandwidth"* [15].

The criterion works in the following way: an elevator step input is applied to the aircraft, and once a steady state pitch rate is reached the input is taken out. The interesting parameters are the maximum pitch rate and the maximum pitch angle occurred during the maneuver. The difference between the steady state pitch angle (before the input) and maximum attained value of the pitch angle is defined as the Dropback (Drb). The correlation with the bandwidth criterion lies in the fact that, if excessive dropback is found, then it is necessary to degrade the aircraft handling qualities, as indicated by the bandwidth criterion, by one level. It can be therefore seen how these two criteria mostly investigate the same aircraft characteristic: the abruptness in the response when an elevator input is applied. Figure 1.6 show a typical Drb chart, with the values normalized with respect to the steady state pitch rate. It is immediate to notice the limit between acceptable and excessive dropback.



Figure 1.6: Dropback criterion boundaries [15]

1.3.3. Military Specifications - MIL-F-8785C

As it was mentioned in Section 1.3, the results of test pilots experience, and the consequent data gathered, were collected in military specifications, providing the flying qualities requirements. It must be noted that, in this context, the term flying qualities also indicates handling qualities, as defined in Section 1.2. In these documents aircraft are categorized according to their characteristics, and the required values for modal response parameters are established with respect to different flight phases. Specifically, aircrafts are divided in four categories, as shown in Table 1.1 [8].

Class I	Small, light aircrafts. Maximum mass $\simeq 5700$ [Kg]
Class II	Medium weight, low-to-medium manoeuverability. Mass between 5700 and 30000 [Kg]
Class III	Large, heavy, low-to-medium manoeuverability. Mass > 30000 [Kg]
Class IV	High manoeuverability aircraft.

Table 1.1: Aircraft class definition - MIL-F-8785C [8]

Furthermore, the various flight phases are defined in Table 1.2.

Flight Phase A	Non-terminal flight phase that requires rapid manoeuvering, precision tracking or
	precise flight path control.
Flight Phase B	Non-terminal flight phase that requires gradual manoeuvers, without precision tra-
	-cking. Accurate flight path control might be required.
Flight Phase C	Terminal flight phase, which require gradual manoeuvers and usually precise
	path control.

Table 1.2: Aircraft flight phases definition - MIL-F-8785C [8]

Concerning the longitudinal motions, an example of the different requirements for the various levels of handling qualities, for class III aircrafts, are shown in Table 1.3 for the phugoid and Table 1.4 for the short period.

Level	1	2	3
Characteristics	$\zeta_{ph} \ge 0.04$	$\zeta_{ph} \ge 0$	Undamped with $T_2 \ge 55 [s]$

Table 1.3: Phugoid damping ratio requirements [8]

Elight Phase	Level 1		Lev	rel 2	Level 3	
	Min	Max	Min	Max	Min	Max
А	0.35	1.30	0.25	2.00	0.1	-
В	0.30	2.00	0.20	2.00	0.1	-
С	0.50	1.30	0.35	2.00	0.25	-

 Table 1.4: Short Period damping ratio requirements [8]

It can be seen that in general, the damping ratio requirements for the short period are much stricter than the ones for the phugoid. This is due to the fact the the phugoid usually has a very long period and presents small oscillations, thus the pilot has enough time to compensate without incurring in excessive workload. In fact level 2 handling qualities for the phugoid entail $\zeta = 0$, which means that the motion is undamped, while level 3 is still attainable with divergent motion, provided that the time to double the amplitude stays within certain boundaries. Furthermore, flight phase C, which corresponds to approach and landing, shows the highest requirements in terms of short period damping ratio, as it is expected.

In conclusion, the military specifications give the designer valuable guidelines, and the explicit values of the modal response parameters are commonly used in order to tune a particular design, either in terms of airframe geometry or, as it will be shown later, stability augmentation systems, in order to achieve satisfactory handling qualities.

1.4. State of The Art

In this section, the current status of the research field concerned handling qualities optimization is presented and discussed, to put the author's work into context. In this respect two well-defined sub-fields have been identified: the first one deals with the optimization of the aircraft configuration, intended as bare airframe. In this view, the optimization of the configuration in itself will provide satisfactory handling qualities, even though sometimes suboptimal. Furthermore, it will pave the way for a swifter implementation of suitable flight controllers, such as stability augmentation systems, at later stages of the design process, if need be. The second sub-field on the other hand focuses more on the optimization of handling qualities through concurrent design of (parts of) the airframe and flight control systems very earlier in the design process. Both of these approaches will be discussed and evaluated.

Regarding the former, in [18] Farag developed a set of equations which related few tailplane geometrical variables to the modal response parameters of the system, namely the short period damping ratio ζ_{sp} and undamped natural frequency $\omega_{n_{sp}}$. Even though rudimentary, this method provides powerful insight about the design procedure required to improve the handling qualities of conventional aircrafts. A shortcoming, however, is found in the amount of variables that these relations can take into account, which indeed is rather limited. A more in-depth approach was proposed by Bazile [19]: he explored the concept of the aerodynamic invariant, which is a parameter consisting of a combination of several design variables which does not vary with the location of the aircraft center of gravity location. This has profound implications on the capability of investigating the handling qualities of unconventional configurations, for which the mass distribution is difficult to estimate at early stages. However, it must be noted that these approaches do not entail a systematic search for the optimum, but rather a trial and error procedure which requires a good deal of user input. Teofilatto developed a modified version of the well-known Routh-Hurwitz criterion, which directly related a small number of geometrical variables to the various levels of handling gualities [20, 21]. It was therefore possibly to immediate and intuitively evaluate which parameters needed to be changed in order to satisfy the requirements, thus avoiding the costly trial and error procedure. However, the limited amount of variables which could be taken into account narrowed the scope of his methodology. The first attempt at optimizing the handling qualities of an aircraft through geometrical design in an automatic fashion was proposed by Soban, Biezad and Gelhausen[3]. They successfully implemented a multidisciplinary optimization routine which consistently achieved an optimal level of handling qualities for different aircraft configurations, by minimizing the difference between the desired modal response characteristics and the ones computed at every iteration. A preliminary evaluation of the performance of the optimized design was also carried out, through the calculation of the resultant parasitic drag. This tool, however, required the linearization and solution of a non-linear aircraft model at every iteration, thus requiring a rather high computational time. Furthermore, the lack of emphasis on the performance of the aircraft potentially entails the generation of less efficient designs.

Regarding the second research sub-field, which is concerned with the concurrent design of (parts of) the aircraft and a suitable automatic flight controller, one of the most interesting approaches was proposed by Kaminer [22]. The controller was designed, in terms of feedback gains, using Linear Matrix Inequalities (LMI), which provide a robust optimization tool and possess the capability of placing the poles of the closed-loop system in user-specified region. This a very powerful regional pole-placement method, which however works only for aircraft with stability augmentation systems. It was found that the optimizer tended to focus more on the design of the controller than on geometric changes on the aircraft. This limitation was encountered as well by Morris in reference [23]. Furthermore, even though the use of LMI to optimize the handling qualities of the configuration proved successful in both the cases mentioned above, however the computational time became non-negligible. This was due to the fact that, in order to place the poles of the system in a prescribed region of the complex plane, an optimization procedure has to be performed. Clearly, if such a method is to be included into a broader optimization framework, the increase in computational time could render this method less attractive, regardless of its efficacy. Another optimal control technique, the Linear Quadratic Regulator (LQR), was investigated by Welstead in reference [24]. The results proved that the LQR is indeed capable of stabilizing even configurations which would be inherently very unstable, by designing a robust controller. In fact, he applied this optimization methodology to a Cessna 180, while increasingly reducing the initial tail area and obtained in every case level 1 handling qualities with respect to the short period motion. Unfortunately, the LQR does not possess the capabilities of regional pole placement that the LMI approach has, so a set of constraints on the design space in terms of modal response parameters was required. However, it must be noted that the computation of feedback gains using the Linear Quadratic Regulator proved rather efficient, paving the way for its inclusion in complex optimization routines.

From the Literature review two opposite view emerged. One aims at optimizing handling qualities early in the design phase by suitable geometrical modifications to the aircraft, while it delegates the design of a flight control system to later stages. The second one instead focuses on an integrated aircraft-controller design approach, with the two being carried out simultaneously. No reference was found regarding an approach which unifies this two opposite views. Clearly, having a certain flexibility at disposal at the conceptual design phase is a desirable contingency. The author's research, therefore, aims at filling this gap, by developing a general handling qualities optimization tool, such that both augmented and unaugmented designs could be optimized depending on the designer's choice. In order to achieve that, a common stability criterion for handling qualities is required, which should have the characteristic of being independent of the aircraft configuration. Furthermore, it should also be uninfluenced by the presence of a stability augmentation system (or lack of it). It is deemed beneficial that such method possesses the capabilities of placing the poles of the system in a specified region, rather than at a single point, thus allowing for the exploration of a larger design space during the optimization. Several stability criteria, which show both the regional pole placement capability as well as the applicability to both augmented and unaugmented designs, have been found in open literature [25–27]. All these methods are concerned with the open and closed loop characteristic polynomials coefficients space, thus being independent of the presence of a stability augmentation system and at the same time being computationally efficient, due to the simple algebraic relations involved.

It is believed that another such method has the potentiality to be developed, starting from Teofilatto's modified Routh-Hurwitz criterion [20, 21]. This approach is also concerned with the characteristic polynomials coefficient space. Although never used as a regional pole placement method in the context of an optimization framework, it is deemed to be quite suitable to be included at the conceptual design stage, also in terms of computational time. To this end, the test nature of this methodology will have to be translated into an efficient design procedure (see Sections 2.6 and 3.2.5). Having zeroed in on a suitable stability criterion, the author's research objective and questions can now be presented.

1.5. Research Objective and Questions

As previously discussed, the main goal of this thesis is the creation of a general tool for the optimization of aircraft handling qualities (HQ) in conceptual design, capable of coping with both augmented and unaugmented designs. This is highly relevant because it allows for the investigation of safer and more fuelefficient configurations earlier in the design process, as well as lessens the chances of costly redesigns at later stages. The literature review, which has been performed by the author prior to this project, led to the identification of a gap in the body of knowledge, and the identification of a methodology which will form the core of the optimization tool. Hence, the research objective is as follows:

To create a tool for handling qualities optimization by using a modified Routh-Hurwitz criterion, capable of coping with both augmented and unaugmented designs.

The investigations that led to the present formulation of the research objective, and consequently the choice of the modified Routh-Hurwitz criterion as common stability paradigm are treated in detail in Sections 2.6 and 2.7. The main challenge and innovation in this respect consists in turning the modified Routh-Hurwitz criterion from a test into a design procedure: specifically, this is done by implementing the algebraic relations that stem from this methodology as a set of constraints on the design space for the optimization procedure. In other words, the modified Routh-Hurwitz criterion takes the form a regional pole placement method (see Section 2.7), thus focusing the analysis on the configurations that satisfy the desired handling qualities. Specifically, only the short period motion will be investigated. This can be justified by considering that the damping requirement on the phugoid is quite relaxed, as shown in Table 1.3, and is therefore deemed of secondary importance for the purpose of this research. To be more specific, the criteria that will be used, among the ones presented in Section 1.3.2, are the short period damping ratio dictated by the military specifications MIL-F-8785C (Table 1.4) and the Control Anticipation Parameter (Figure 1.5). The remainder of the metrics discussed in Section 1.3.2 will, therefore, be neglected. A further clarification is necessary: up until now, continuos reference has been made to the general term handling *qualities.* Henceforth this term will be only used to indicate the combination of CAP and ζ_{sp} . The optimized designs will, therefore, have to comply with certain minimum required short period damping ratio and Control Anticipation Parameter simultaneously.

Considering the aircraft design aspect of the thesis objective, the research is focused on conventional aircraft configurations, or tube and wing. In this respect, the chosen approach entails the modification of the horizontal stabilizer planform, as well as the determination of an optimal wing position, in order to achieve suitable longitudinal handling qualities. Lastly, in terms of objective functions, the tool will aim at minimizing tailplane zero-lift drag, induced drag and weight. All this leads to the following research questions:

Can an optimization methodology based on the modified Routh-Hurwitz criterion consistently achieve the desired handling qualities, for both unaugmented and augmented designs?

What are the effects of handling qualities optimization on the performance of the aircraft, in terms of aerodynamic efficiency and maximum takeoff mass, both in case of unaugmented and augmented designs?

A clarification must be made regarding the first research question: the requirement of consistency entail that the tool must indeed successfully achieve the desired handling qualities, independently of the configuration, be it augmented or unaugmented. The second research question is rather self-explanatory. It is hypothesized that for a given level of handling qualities, in case of unaugmented aircrafts, the tool will be able to find a suitable combination of design parameters which allows for a reduction in both weight and drag compared to the baseline configuration. Furthermore, by including stability augmentation system in the design optimization, this improvement is believed to be even greater due to the possibility of reducing the size of the stabilizing surfaces. This is directly related to the concept of more fuel-efficient configurations which was stated at the beginning of the section. For all the previous considerations, it is believed this tool will enhance the aircraft conceptual design capabilities of the department of Flight Performance and Propulsion at TU Delft.

1.6. Thesis Scope

Given the limited time frame available for the completion of the this project, the thesis scope has been defined at the beginning of the research. The proposed tool is to be fitted as a module within the Initiator, a conceptual and preliminary design tool developed at the Department of Flight Performance and Propulsion. The initial philosophy that forms the basis of this tool is found in the *Design and Engineering Engine* (DEE), devised by Gianfranco la Rocca [28], which was later on followed by the development of the *Initiator*, under the supervision of R. Vos. In fact in [29] Elmendorp, Vos and la Rocca indeed proposed a design method, and its implementation into a physics-based software, which allows for the investigation of conventional and unconventional configurations. Starting from user-specified top level requirements and chosen configuration, the tool can generate a first estimation of the aircraft geometry and conduct an evaluation of the associated performance parameters. A schematic flowchart representing the process diagram of the Initiator is illustrated in Figure 1.7.



Figure 1.7: Paradigm of the Initiator [29]

The disciplines involved in the analysis and optimization of the configuration include Weights, Aerodynamics, Performance, and Cost. All the analyses are carried out using lower fidelity tools, suitable at the conceptual design level: as an example, the aerodynamic analysis is carried out using the Athena Vortex Lattice software, designed by Drela [30], which allows for rapid evaluation of induced drag and loads estimation. The choice of the tools used in the Initiator is dictated mainly by a trade-off between acceptable accuracy and reasonable computational time. It is therefore of paramount importance to implement a fast optimization procedure. To have a better understanding of the structure of the Initiator, and consequently place the author's work into context, the N^2 diagram representing the design convergence is presented in Figure 1.8.

Class 1 Weight Estimation	мтом	мтом	MTOM, FF					J	
	Wing Thrust Loading	W/S, T/W			T/W				
		Geometry Modules	Geometry	Geometry			Wing geometry	Fuselage geometry	
OEM			Class 2 Weight Estimation	MTOM, FM, OEM	мтом	Range			
L/D, CD _{min} CL _{min}	Polar	Polar		Aerodynamic modules		Polar	Loading	Loading	
SFC					Engine Model	SFC			
	мтом		MTOM, FF			Mission Analysis			
			Wing weight (OEM)				EMWET		
t			Fuselage weight (OEM)					Fuselage Weight Estimation	

Figure 1.8: Initiator N2 diagram

It is possible to see that there are three main convergence loops: class 1, class 2 and class 2.5, which differ insofar as the analysis methods go. At present no module dedicated to the optimization of handling qualities exist. Some rudimentary considerations on longitudinal handling qualities are present in the Horizontal Stability Estimation module [31]. The author's module will, therefore, expand on these previous results. The determination of the position of the handling qualities optimization module within the framework of the Initiator is an integral part of the research, and is discussed in Section 3.2.1.

1.7. Thesis Structure

This report consists of four main parts. Chapter 2 treats the basic concepts of tailplane design as well as the general theory of static and dynamic stability. Furthermore, in Section 2.6 and 2.7, a detailed description of the classical and modified Routh-Hurwitz criterion is given. Chapter 3 treats the methodology used to develop the module. In Section 3.1 an overview of the optimization framework is presented. This includes a description of the objective functions and a sensitivity study, which has been performed preliminarily to assess the most suitable design variables. Section 3.2 contains the implementation of the main theories and approaches that have been used, along with the core and novelty of this research: the definition of constraints based on the modified Routh-Hurwitz criterion. Chapter 4 treats the verification and validation of two aspects of the methodology: the aerodynamic derivative estimation and the flight dynamics model. In Chapter 5, the capabilities of the module are presented through a test case. The optimization is tested for both augmented and unaugmented design, to ensure its consistency. Section 5.2 deals with two different topics: firstly, the effect of including actuators in the optimized design is investigated, to assess whether their presence entails a significant deviation from the nominal performance in terms of handling qualities. Secondly, it treats a convergence study done within the Initiator, in order to assess the benefits achievable through the proposed methodology, in terms of aerodynamic efficiency and maximum takeoff mass, with the aim of answering the second research question posed in Section 1.5 Lastly, the conclusion and recommendations are presented in Chapter 6.

Chapter 2

Theory

In the following Section a broad overview of the theoretical background of this project is given. In Section 2.1 the design guidelines for the sizing of a horizontail tailplane are discussed. Sections 2.2 and 2.3 treat static and dynamic stability of aircrafts, respectively. Consequently, a qualitative introduction to stability augmentation systems is given in Section 2.5. Finally, the classical Routh-Hurwitz criterion is discussed, follwed by an in-depth presentation of the modified Routh-Hurwitz criterion, which forms the core of this research project (Sections 2.6 and 2.7).

2.1. Horizontal Tailplane Design

In Section 1.6 it was stated that this research if focused on the sizing of the horizontal tailplane and wing positioning for conventional aircraft configurations. Generally, the two go hand in hand since the optimal tail size follows from a suitable wing location. This mutual interaction is treated more in detail in Section 3.2.6. The objective of this section is to give a qualitative overview of the functions and requirements concerning horizontal tailplanes.

Firstly, a tailplane needs to fulfill three main functions [32]:

- · Provide static and dynamic stability
- · Provide pitch control in all flight conditions
- Ensure that a condition of equilibrium (trim) is achievable in each flight condition

These functions apply to aircrafts with tube and wing configurations, for which the lack of a horizontal stabilizer would entail a condition of instability, as well as almost no controllability on the longitudinal axis. However, several concepts devoid of a horizontal tailplane have been proposed over the years. One of the most notable examples are flying wings. By careful positioning of the center of gravity, it is possible to achieve static stability for these configurations as well, even though in most of the cases they require the presence of a stability augmentation system to ensure adequate flight characteristics [33–35].

Having defined the function pertaining to horizontal tail surfaces, it is now appropriate to introduce the requirements that need to be met when designing one. These can be summarized as follows [2, 32]:

- It shall provide a sufficiently large contribution to longitudinal static and dynamic stability
- · It shall provide sufficient control capability
- · Control shall be possible with acceptable control forces
- · It shall be able to handle high angles of attack
- · It shall be able to counterbalance tail-off forces and moments

· It shall be able to handle high Mach numbers without flow separation

Clearly, many factors need to be taken into account, which makes the design of a the horizontal tail quite complex. Many of the requirements are conflicting and are satisfied by considerably different tail geometries. As an example, to fulfill the first requirement a high aspect ratio is beneficial; however, this is in conflict with the fourth requirement. Clearly increasing the aspect ratio yields a steeper lift curve slope, which in turns reduces the stall angle of attack. Moreover, to satisfy the sixth requirement, a high sweep angle is desirable. It is trivial though to see that the higher the sweep angle, the smaller the slope of the lift curve, which in turn reduces the contribution of the tail to static and dynamic stability.

Finding an acceptable tradeoff is therefore of paramount importance. Section 3.1.4 treats precisely the problematics that arise by these conflicting requirements, and how they have been overcome in this project.

2.2. Static Stability

As discussed in the previous section, the first function that must be provided by a horizontal tailplane is static stability. Hence, in this section, the conditions for its achievement are treated. To derive the basic relations for stability, let us consider the two surface model depicted in Figure 2.1.



Figure 2.1: Two surface model for static stability analysis [10]

The x-axis is positive towards the aft section of the aircraft. By examining the sum of moments acting on the aircraft center of gravity, it is possible to obtain the following relation, the so-called *pitching moment equation*:

$$C_{m_{cg}} = \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) [C_{L_{w}} + \eta \frac{S_{ht}}{S_{w}} C_{L_{ht}}] - \eta V_h C_{L_{ht}} + C_{m_{wb}}$$
(2.1)

In this equation there are various terms which require a more in-depth explanation. Firstly, $V_h = \frac{l_L S_L}{cS_w}$ refers to the tail volume coefficient; this term, which is independent of the c.g position, has been widely used in aircraft design with the purpose of sizing both horizontal and vertical tailplanes, based on statistical trends [2]. However, as it was discussed previously, this approach might lead to suboptimal designs.

Another important term is η , which is defined as the tail efficiency factor. The remaining factors are C_{m0_w} , or the zero-angle-of-attack pitching moment coefficient of the wing, C_{L_w} and $C_{L_{ht}}$, the lift coefficient of wing and horizontal tailplane respectively and finally C_{m_f} , which is the pitching moment contribution of the fuselage and nacelles. Note that all the coefficients are adimensionalized, and that the zero-angle-of-attack pitching moment coefficient of the tail is not present due to the assumption that its profile is symmetrical. The next step is to express the lift coefficients appearing in Equation 2.1 as dependent on the angle of attack:

$$C_{L_{\rm W}} = C_{L_{\alpha_{\rm W}}} (\alpha_{\rm FRL} + i_{\rm W} - \alpha_{0_{\rm W}}) \tag{2.2}$$

$$C_{L_{\rm ht}} = C_{L_{\alpha_{\rm ht}}} \left(\alpha_{\rm FRL} + i_{ht} - \epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha_{\rm FRL} \right)$$
(2.3)

Where α_{0_w} is the zero-lift angle of attack for the wing and ϵ_0 is the zero-angle-of-attack downwash angle. Likewise we can define the angle of attack of the complete aircraft at zero-lift as α_0 . With this, it is possible to express Equation 2.1 by defining the complete aircraft angle of attack from the zero-lift angle of attack and the angle of attack with respect to the fuselage reference line:

$$\alpha = \alpha_{\rm FRL} - \alpha_0 \tag{2.4}$$

Thus we obtain the following:

$$C_{m_{cg}} = C_{m_{0_{w}}} + \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \left[C_{L_{\alpha_{w}}} \left(i_{w} - \alpha_{0_{w}}\right) + \eta \frac{S_{ht}}{S_{w}} C_{L_{\alpha_{ht}}} \left(i_{ht} - \epsilon_{0}\right) \right] - \eta V_{h} C_{L_{\alpha_{ht}}} \left(i_{ht} - \epsilon_{0}\right) + C_{m_{\alpha}} \alpha_{0} + \left[\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) C_{L_{\alpha}} - \eta V_{h} C_{L_{\alpha_{ht}}} \left(1 - \frac{d\epsilon}{d\alpha}\right) + C_{m_{\alpha_{wb}}} \right] \alpha$$

$$(2.5)$$

This equation completely describes the equilibrium of moments around the center of gravity, and it can be seen to be in the form:

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha} \alpha \tag{2.6}$$

The term which is most interesting for static stability is the *pitch stiffness* C_{m_a} , which is found by comparing Equation 2.6 to Equation 2.5

$$C_{m_{\alpha}} = \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)C_{L_{\alpha}} - \eta V_h C_{L_{\alpha_{ht}}} \left(1 - \frac{d\epsilon}{d\alpha}\right) + C_{m_{\alpha_{wb}}}$$
(2.7)

Finally, we can define the neutral point with fixed controls as the position of the center of gravity which drives the pitch stiffness to 0. In mathematical terms we have:

$$\frac{x_{ac}}{\bar{c}} + \eta V_h \frac{C_{L_{\alpha_{\rm ht}}}}{C_{L_{\alpha}}} \left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{C_{m_{\alpha_{\rm wb}}}}{C_{L_{\alpha}}}$$
(2.8)

This point can be interpreted as the boundary between stable equilibrium and unstable equilibrium. In fact, should the position of the cg move aft of the neutral point, to a positive increase in angle of attack would correspond a pitch-up moment of the aircraft, which of course is undesirable. To formalize this statement we can then introduce the expression for the neutral point into Equation 2.7, and thus write:

$$C_{m_{\alpha}} = \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{np}}{\bar{c}}\right) C_{L_{\alpha}}$$
(2.9)

The main condition for static stability therefore follows from Equation 2.9 and can be stated as such: *in* order for an aircraft to possess static stability, the pitch stiffness $C_{m_{\alpha}}$ has to have a negative value. In other words, the location of the center of gravity of the airplane should always lie in front of the location of the neutral point stick-fixed.

This condition is best visualized in Figure 2.2. The solid line represent a situation of stable equilibrium, since to an increase in angle of attack corresponds a nose down pitching moment ($C_{m_{\alpha}} < 0$). Likewise, the dashed line describes a situation of unstable equilibrium, where to an increase in angle of attack follows a nose-up pitching moment ($C_{m_{\alpha}} > 0$). Point A in the graph defines the equilibrium angle of attack, at which the pitching moment about the cg of the aircraft is zero, and is therefore a *trim condition*.

A final remark can be made, by taking a closer look at an important term, which was already introduced in Equation 2.9, and that will be used extensively in later sections of this document:

$$S.M = -\left(\frac{x_{np}}{\bar{c}} - \frac{x_{cg}}{\bar{c}}\right)$$
(2.10)

This is the so-called *static margin*, which indicates the distance between the aircraft center of gravity and the neutral point. This is a key concept, since it can be used to immediately infer the degree of stability of an aircraft. As an example, some aircraft are designed with relaxed static stability (or RSS), which means that the static margin is driven to zero, or even to negative values. This approach has been applied to highly augmented fighter aircrafts, in order to enhance their agility [1, 37]. Usually, in commercial aviation, the static margin assumes values between 5% and 40% (expressed as percentage of the mean aerodynamic chord) [1, 2].



Figure 2.2: Pitching Moment Diagram [37]

2.3. Dynamic Stability

The second function that a horizontail tailplane fulfills is providing dynamics stability. It must be borne in mind that the latter cannot be accomplished without ensuring static stability first. This statement holds true for conventional aircrafts, without the presence of significant augmentation systems. Should such systems be present, the classical criteria for the evaluation of static stability are not valid, since it is accomplished artificially using control techniques [7]. This topic is going to be investigated in-depth in later parts of this report, so now the focus is going to be on the dynamics of unaugmented aircrafts, for which static stability is assumed.

In order to assess dynamic stability the aircraft equations of motion (EOM) are required. Usually, they are expressed in non-linear form, however in this report only the linearized uncoupled equations are considered. Hence the derivation of such equations is out of the scope this research, and will not be treated. For the interested reader, a complete desccription can be found in references [7, 36, 37].

2.3.1. Linearization

Since only the linearized equations of motion are used, it is deemed appropriate to provide a preliminary introduction to the theory behind linearization. It allows to obtain a more treatable version of the equations of motion and is widely employed to investigate stability and control problems [38]. It must be borne in mind that an equation can only be linearized around a state of equilibrium, or the initial point of linearization. For aircrafts this condition corresponds to trim [7, 36]. In practice, this operation is performed by expanding the non-linear terms in a Taylor series, and neglecting the higher order terms. A simple example, which can be found again in [36], is given considering a generic function Y, dependent on n-dimensional state and linearized about a point X_0 . The Taylor series can be then expressed as:

$$Y = f(X_0) + f_{x_1}(X_0)\Delta x_1 + f_{x_2}(X_0)\Delta x_2 + \dots + f_n(X_0)\Delta x_n$$
(2.11)

Which can then be arranged in a compact form in the following fashion:

$$Y = f(X_0) + \sum_{i=1}^{n} f_{x_i}(X_0) \Delta x_i$$
(2.12)

Once the linear model is obtained around a condition of equilibrium, it is then possible to derive information regarding the stability characteristics of the system at hand. Specifically, three conditions are possible:

- Unstable equilibrium
- Neutral equilibrium
- Stable equilibrium

Clearly, stable equilibrium is the most desirable condition.

2.3.2. Linearized Equations of Motion in Non-Dimensional Form

After having qualitatively described the procedure to linearize a given nonlinear function, it is now appropriate to introduce the linearized set of equations of motion. Furthermore, an operation of nondimensionalization is going to be performed, thus obtaining the standard form of the EOM which is going to be used throughout this report from now on. Before diving into it, however, a final assumption has to be presented: it is assumed that the aerodynamic forces acting on the longitudinal and lateral-directional plane are uncoupled. It allows to split the full system of equations into two distinct sets, the symmetrical equations of motion and the asymmetrical equations of motion. This step is of paramount importance since it gives the possibility to investigate the longitudinal and lateral-directional behavior of an aircraft separately. For the scope of this research, only the longitudinal set of equations of motion is discussed. A common form of expressing such equations is the following [36]:

$$\begin{bmatrix} C_{x_{u}} - 2\mu_{c}D_{c} & C_{X_{\alpha}} & C_{Z_{0}} & 0 \\ C_{Z_{u}} & C_{Z_{\alpha}} + (C_{Z_{\alpha}} - 2\mu_{c})D_{c} & -C_{x_{0}} & C_{Z_{q}} + 2\mu_{c} \\ 0 & 0 & -D_{c} & 1 \\ C_{m_{u}} & C_{m_{\alpha}} + C_{m_{\alpha}}D_{c} & 0 & C_{m_{q}} - 2\mu_{c}K_{Y}^{2}D_{c} \end{bmatrix} \begin{bmatrix} \frac{u}{V} \\ \alpha \\ \theta \\ \frac{q\bar{c}}{V} \end{bmatrix} = \begin{bmatrix} -C_{X_{\delta_{e}}} & -C_{X_{\delta_{t}}} \\ -C_{Z_{\delta_{e}}} & -C_{Z_{\delta_{t}}} \\ 0 & 0 \\ -C_{m_{\delta_{e}}} & -C_{m_{\delta_{t}}} \end{bmatrix} \begin{bmatrix} \delta_{e} \\ \delta_{t} \end{bmatrix}$$
(2.13)

Finally, the mathematical description needed to investigate the aircraft stability has been developed. It takes the form, for each set, of four simultaneous, constant coefficient, first order linear differential equations. It must be remembered that these equations are linearized around a condition of equilibrium, which often in practical applications is trimmed steady straight symmetric flight, and they retain their validity only for small perturbations around this point of equilibrium. However, this assumption is valid as long as the input to this system, which is the deflection of a control surface (for example δ_e), stays within a small range. In the next section, the solution of these equations is going to be discussed, along with how to obtain the aircraft stability characteristics.

2.3.3. Solution of the Equations of Motion

The solution of the full set of non-linear equations is possible only using numerical integration. This is however a lengthy and computationally expensive approach. The alternative lies in the linearized set of equations of motion, presented in the previous chapter, which are easily solved. A very important step in obtaining a solution is realizing that the stability of this system can be investigated by applying an input, i.e a control surface deflection, and assess the evolution of the system. It matters not what the deflection of a control surface is, as long as it stays within the bounds dictated by the assumption of small disturbances. By using this system of linearized equations, in the end one would pervene to the same results, in terms of stability. How could this concept be applied in order to obtain a solution of the set of equations then? Firstly, after having determined an equilibrium point for the system, given the independence of the system from the initial deflection, the EOM can be written in homogenous form. For sake of brevity, only the longitudinal (symmetric) set of equations is going to be investigated. However, the results are general and apply equally to the lateral-direction (asymmetric) set of equations. We thus obtain the following homogenous form:

$$\begin{bmatrix} C_{x_{u}} - 2\mu_{c}D_{c} & C_{X_{\alpha}} & C_{Z_{0}} & 0 \\ C_{Z_{u}} & C_{Z_{\alpha}} + (C_{Z_{\alpha}} - 2\mu_{c})D_{c} & -C_{x_{0}} & C_{Z_{q}} + 2\mu_{c} \\ 0 & 0 & -D_{c} & 1 \\ C_{m_{u}} & C_{m_{\alpha}} + C_{m_{\alpha}}D_{c} & 0 & C_{m_{q}} - 2\mu_{c}K_{Y}^{2}D_{c} \end{bmatrix} \begin{bmatrix} \frac{u}{V} \\ \alpha \\ \theta \\ \frac{d}{V} \\ \frac$$

Following the procedure outlined in [36], the solution to this homogenous equation will be in the form:

$$x = A_x e^{\lambda_c s_c} \tag{2.15}$$

with $s_c = \frac{V}{c}t$ and λ_c either real or complex. The latter term, for fixed A_c matrix, will be entirely responsible for the stability of the system. An explicit expression for λ_c which can then be introduced into Equations

2.14 is found by realizing that:

$$D_c x = \lambda_c x \tag{2.16}$$

and equally:

$$D_c^2 x = \lambda_c^2 x \tag{2.17}$$

Therefore, by rerranging Equations 2.15, 2.16 and 2.17 the following expression for the homogenous Equation 2.14 is found:

$$\begin{bmatrix} C_{x_{u}} - 2\mu_{c}\lambda_{c} & C_{X_{\alpha}} & C_{Z_{0}} & 0 \\ C_{Z_{u}} & C_{Z_{\alpha}} + (C_{Z_{\dot{\alpha}}} - 2\mu_{c})\lambda_{c} & -C_{x_{0}} & C_{Z_{q}} + 2\mu_{c} \\ 0 & 0 & -\lambda_{c} & 1 \\ C_{m_{u}} & C_{m_{\alpha}} + C_{m_{\dot{\alpha}}}\lambda_{c} & 0 & C_{m_{q}} - 2\mu_{c}K_{Y}^{2}\lambda_{c} \end{bmatrix} \begin{bmatrix} A_{u} \\ A_{\alpha} \\ A_{\theta} \\ A_{q} \end{bmatrix} e^{\lambda_{c}s_{c}} = \underline{0}$$
(2.18)

As the term $e^{\lambda_c s_c}$ can be omitted without any influence on λ_c , this equation in short-hand notation is found to be:

$$[\Delta]A = 0 \tag{2.19}$$

At last the values of λ_c , which here on are going to be referred to as *eigenvalues* of the characteristic matrix [Δ], can be computed by setting the determinant of this matrix to zero. Thus we obtain:

$$\begin{vmatrix} C_{x_{u}} - 2\mu_{c}\lambda_{c} & C_{X_{\alpha}} & C_{Z_{0}} & 0\\ C_{Z_{u}} & C_{Z_{\alpha}} + (C_{Z_{\alpha}} - 2\mu_{c})\lambda_{c} & -C_{x_{0}} & C_{Z_{q}} + 2\mu_{c}\\ 0 & 0 & -\lambda_{c} & 1\\ C_{m_{u}} & C_{m_{\alpha}} + C_{m_{\alpha}}\lambda_{c} & 0 & C_{m_{q}} - 2\mu_{c}K_{Y}^{2}\lambda_{c} \end{vmatrix} = 0$$
(2.20)

Computation of the determinant of this matrix will results in fourth order polynomial in the following form:

$$p_0\lambda_c^4 + p_1\lambda_c^3 + p_2\lambda_c^2 + p_3\lambda_c + p_4 = 0$$
(2.21)

Equation 2.21 is referred to as *characteristic polynomial*. Its roots are the eigenvalues of the characteristic matrix, and by examining them it's possible to extract information regarding the stability of the aircraft. Once they are computed, and introduced back into equation 3.41, they also allow for the investigation of the aircraft motion, from a condition of equilibrium, after a disturbance is applied. The next section will treat the state-space form of the linearized equations of motion, which will be used for the implementation of the flight dynamics model in the handling qualities optimziation module.

2.4. Equations of Motion in State-Space Form

Linear Time Invariant systems, which are widely used in flight dynamics analyses as well as control system design, are usually expressed in state-space form [7]. The state-space description provides the dynamics of a system as a set of coupled first-order differential equations, expressed in a set of internal variables known as state variables. Besides, a set of algebraic equations combines the state variables into physical output variables. From a practical point of view, it is possible to obtain a state-space form of Equations 2.13. The general mathematical description of an LTI system is the following:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{2.22}$$

$$\dot{\mathbf{y}} = C\mathbf{x} + D\mathbf{u} \tag{2.23}$$

(2.24)

The description of each term appearing in 2.22 can be found in the list of symbols. In order to achieve this form, firstly the differential operator D_c in Equation 2.13 is replaced by $\frac{\overline{c}}{V}\frac{d}{dt}$. Secondly the procedure involves separating the terms wich are time-dependent, thus obtaining:

$$P\dot{\mathbf{x}} = Q\mathbf{x} + R\mathbf{u} \tag{2.25}$$
with:

$$P = \begin{bmatrix} -2\mu_{c} & 0 & 0 & 0 \\ 0 & (C_{Z_{\dot{\alpha}}} - 2\mu_{c})\frac{\bar{c}}{V} & 0 & 0 \\ 0 & 0 & -\frac{\bar{c}}{V} & 0 \\ 0 & C_{m_{\dot{\alpha}}}\frac{\bar{c}}{V} & 0 & -2\mu_{c}K_{y}^{2}\frac{\bar{c}}{V} \end{bmatrix}, \quad Q = \begin{bmatrix} -C_{X_{u}} & -C_{X_{\alpha}} & -C_{Z_{0}} & 0 \\ -C_{Z_{u}} & -C_{Z_{\alpha}} & C_{X_{0}} & -(C_{Z_{q}} + 2\mu_{c}) \\ -C_{m_{u}} & -C_{m_{\alpha}} & 0 & C_{m_{q}} \end{bmatrix}$$
(2.26)
$$R = \begin{bmatrix} -C_{X_{\delta_{e}}} & -C_{X_{\delta_{t}}} \\ -C_{Z_{\delta_{e}}} & -C_{Z_{\delta_{t}}} \\ 0 \\ -C_{m_{\delta_{e}}} & -C_{m_{\delta_{t}}} \end{bmatrix}$$

Finally, a multiplication operation must be performed on the inverse of *P*, thus obtaining:

$$\dot{\mathbf{x}} = A P^{-1} \mathbf{x} + B P^{-1} \mathbf{u} \tag{2.27}$$

Which yields the desired form shown in Equation 2.22. Specifically, the various matrices matrices appearing in this state-space form are defined as:

$$A = \begin{bmatrix} x_u & x_\alpha & x_\theta & 0\\ z_u & z_\alpha & z_\theta & z_q\\ 0 & 0 & 0 & \frac{V}{c}\\ m_u & m_a & m_\theta & m_q \end{bmatrix}, \quad B = \begin{bmatrix} x_{\delta_e} & x_{\delta_t}\\ z_{\delta_e} & z_{\delta_t}\\ 0 & 0\\ m_{\delta_e} & m_{\delta_t} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0 \end{bmatrix}$$
(2.28)

A complete description of the terms that appear in the state and input matrices can be found in reference [36]. The roots of the *A* matrix univocally determine the stability characteristics of the system in terms of modal response parameters. Furthermore, the state-space formulation can be used to design stability augmentation systems, which will be introduced in the following Section.

2.5. Stability Augmentation Systems

In general, when a particular design is found to be deficient in terms of handling qualities, it is common practice to artificially modify, or augment, the stability characteristics of the bare airframe. The most convenient way to achieve this results is the introduction of negative feedback: in other words, motion sensors present on the aircraft dispatch signals that, after some processing, are used to operate the control surfaces to enhance the dynamic behavior of the aircraft. The stability augmentation system (SAS) is therefore used to produce aerodynamic moments and forces, proportional to the measured output signal, which will produce an effective damping of the motion of interest. An example of a pitch angle feedback system is shown in Figure 2.3.



Figure 2.3: Pitch angle feedback system [7]

The design of a stability augmentation system is therefore concerned with the identification of a suitable feedback gain K_{θ} , which yields the desired handling qualities, in terms of modal response characteristics.

In essence, the feedback gain will modify the locations of the poles of the system G(s), thus making the system artificially more stable with respect to its nominal configuration. When feedback is applied, the system is said to be *closed loop* and can be analyzed with classical and modern control techniques. For the sake of brevity the theory behind closed loop control will not be illustrated here, but the interested reader is referred to literature [7, 12, 16, 38].

It must be noted that SAS can be applied both to the longitudinal motions, as well as lateral-directional. This is done by decoupling the linearized equations motions, which allows for the investigation of the separate motions, and the implementation of suitable feedback systems. As stated in Sections 1.5 and 1.6, this research project is concerned only with the short period motion. Hence the output signals that will be considered are the angle of attack and the pitch rate (see Section 3.2.3). Consequently, the chosen feedback systems are indicated as an α feedback and a pitch damper (see Section 3.2.4).

2.6. Routh-Hurwitz Criterion

Before discussing the modified Routh-Hurwitz criterion, which forms the core and novelty of this research, it is deemed appropriate to introduce the classical Routh-Hurwitz criterion, thus allowing for a meaningful comparison between the two methods. The Routh-Hurwitz criterion is a mathematical test developed to assess whether a given Linear Time Invariant system is stable or not [39]. Let us consider the characteristic polynomial of a general system, in the form:

$$p(z) = p_0 z^n + p_1 z^{n-1} + \dots + p_n$$
(2.29)

The procedure allows for the determination of the number of roots of the polynomial which lie in the left half of the complex plane and therefore determine the overall stability of the system at hand. An example of the stability region of the complex plane is presented in Figure 2.4.



Figure 2.4: Routh-Hurwitz criterion - stability region

The most basic form of the test consists in evaluating the signs of the coefficients of the characteristics polynomial which describes the system at hand: if one or more of the coefficients are zero or have negative sign, the system is unstable. Taking the test one step further, it is possible to determine the exact number of roots which lie in the righ-hand region of the complex plane. This is done by means of the Routh array. Referring again to Equation 2.29 and considering a 5th order polynomial as example, the set up of the array is the following [16]:

$$R_{\text{array}} = \begin{cases} p_0 & p_2 & p_4 & \dots \\ p_1 & p_3 & p_5 & \dots \\ b_1 & b_2 & c_3 & \dots \\ c_1 & c_2 & c_3 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{cases}$$
(2.30)

With:

$$b_1 = \frac{p_1 p_2 - p_0 p_3}{p_1}, \quad b_2 = \frac{p_1 p_4 - p_0 p_5}{p_1}, \quad \dots$$
 (2.31)

$$c_1 = \frac{b_1 p_3 - p_1 b_2}{b_1}, \quad c_2 = \frac{b_1 p_5 - p_1 b_3}{b_1}, \quad \dots$$
 (2.32)

(2.33)

Now it is possible to count the sign changes in the first column of the array. Every change corresponds to an unstable root. The general rule is therefore the following: for all the roots to have negative real part, all the p_i need to be positive. Furthermore, the first column of the Routh array must be positive.

1

The effectiveness of this method is undisputed, however not much information can be gathered except for the determination of stability, or lack of it. Clearly, the algorithm as it is is not suitable for implementation in an optimization framework, as it does not offer the required level of control over the solution. This limitation can be overcome by a specialization of the Routh-Hurwitz criterion, as the one developed by Teofilatto [20, 21].

2.7. Modified Routh-Hurwitz Criterion

Considering again a system, which is described by a characteristic polynomial, it is possible to individuate new conditions on the coefficients of the characteristic polynomial with the purpose of locating a specific region of the complex plane, which corresponds to the desired modal response characteristics. The modified Routh-Hurwitz criterion has been developed as a means to test the degree of stability of a given aircraft configuration. Furthermore, in [21], this method was applied to lateral-directional stability to obtain a rudimentary indication on which geometrical parameters needed to be modified for achieving a specific level of handling qualities. The method was applied only with a limited scope (see Section 1.4), however, its full potential is believed to be achievable by going beyond the test nature and using the information it provides as design requirements. Furthermore, by including this method in an optimization framework, the number of variables could be increased substantially. Another major benefit lies in the general nature of the Routh-Hurwitz criterion: it can be applied to every Linear Time Invariant system. In other words, it can be used as a design procedure for optimizing just the airframe, as well as performing concurrent design of airframe and stability augmentation system. It is believed that it could also be applicable unconventional configurations, without extensive modifications. The modified criterion entails that two conditions may be identified in the imaginary plane:

- The real part of the eigenvalues λ is less than a prescribed value $x_1 = -c$.
- The solutions are inside a cone of amplitude $2\alpha^*$, such that the damping ratio ζ is within the values $(\cos \alpha^*, 1)$.

In other words, the method defines a precise area of the plane, as shown in Figure 2.4, in which the roots of the characteristic polynomial should be located to achieve the desired modal response characteristics. In other words, the methodology identifies the necessary and sufficient conditions for an aircraft to possess the desired handling qualities. An example of such a region is presented in 2.5.



Figure 2.5: Modified Routh-Hurwitz criterion - allowable pole locations

Hence α^* indicates the angle between the horizontal axis and each of the lines denoted with blue in Figure 2.5. The region in is then bounded within a cone of amplitude $2\alpha^*$ and a vertical line at $x_1 = -c$. To have a better understading of the modified Routh-Hurwitz criterion let us consider a characteristic polynomial in a general form:

$$p(z) = p_0 z^n + p_1 z^{n-1} + \dots + p_n$$
(2.34)

To satisfy the first condition, let us apply the classical Routh-Hurwitz criterion to the polynomial obtained by translating equation 2.34, such that $z = z_1 + x_1$:

$$p(z) = p_0(z_1 + x_1)^n + p_1(z_1 + x_1)^{n-1} + \dots + p_n$$
(2.35)

A Routh array similar to the one presented in Equation 2.30 is the created based on the new polynomial. If p(z) satisfies the conditions on the signs of the coefficients, then all the eigenvalues λ_i will be to the left of a straight line $x_1 = -c$ in the complex plane. The second condition, which is that the solution of equation 2.34 are within a cone of amplitude $2\alpha^*$, can be satisfied by referring to the following modified Routh array:

$$R_{\text{array,modified}} = \begin{cases} a_0 & a_1 & a_2 & \dots \\ b_0 & b_1 & b_2 & \dots \\ c_0 & c_1 & c_2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{cases}$$
(2.36)

With:

$$c_0 = \frac{a_0 b_1 - b_0 a_1}{b_0}, \quad c_1 = \frac{a_0 b_2 - b_0 a_2}{b_0}, \quad \dots$$
 (2.37)

For the sake of brevity the full mathematical derivation of this array is not present however, for the interested reader, the complete description can be found in [20]. In contrast with the Routh array in Equation 2.30, the first two rows are computed for i = 1, ..., n, as:

$$a_i = (-1)^i p_i \cos(n-i) \alpha^*$$
(2.38)

$$b_i = (-1)^{i+1} p_i \sin(n-i) \alpha^*$$
(2.39)

The second condition is satisfied by the usual test, or that $p_i > 0$ and, taking the first column of the modified Routhian 2.36, we have all entries with equal sign. If these two conditions are met, then the system has roots in a specific region in the complex plane, which can be chosen a priori as function of the characteristics of the system which are of interest. Specifically this is done by setting:

$$\zeta \ge \cos \alpha^{\star} \tag{2.40}$$

$$\omega_n \ge -\frac{x_1}{\cos \alpha^*} \tag{2.41}$$

Hence, it is possible to investigate whether a given aircraft configuration has roots in the region of the complex plane identified by the conditions on the signs of the polynomial 2.34. It must be noted that this methodology, just as the original Routh-Hurwitz criterion, still has the characteristics of a test. They can be used to assess the stability of a system, to a different extent respectively, and indeed the modified Routh-Hurwitz criterion has been applied as to give preliminary information on suitable design changes needed to fulfill a certain desired level of handling qualities. This can be seen in Figure 2.6.

In general, however, the scope of this method is still rather limited. Nevertheless, there is the potential to enhance this method, by translating its test nature into a design procedure which could be included in an optimization framework. This can be done, qualitatively, through the following procedure:

- 1. Choose the desired ζ and ω_n
- 2. Obtain the required α^* and x_1
- 3. Derive the two Routh arrays (Equations 2.30 and 2.36)
- 4. Implement the conditions on the signs of the polynomial coefficients as nonlinear inequality constraints on the design space

Hence, by casting the results of the modified Routh-Hurwitz criterion as constraints on the design space, it is possible to investigate only the configurations which satisfy the desired level of handling qualities. The practical implementation of the aforementioned procedure, devised by the author, is detailed in Section 3.2.5. The inclusion in an optimization framework allows for an increase in the design variables which can be considered, the only limitation being the eventual increment in the dimensionality of the problem. It must also be noted that there is no practical limit to the order of the polynomials that can be analyzed. The conditions on the coefficients of the characteristic polynomial are simple algebraic relations, which are readily solved. Hence this method is highly efficient in terms of computational time, as well as expandable to include higher order dynamics.



Figure 2.6: Lateral-direction handling qualities levels [21]

In this respect one drawback must be highlighted: this newly developed methodology is best suited for systems in which the poles of interest are on the lower end of the range of pole locations. This, for example, entails that some limitations are present when analyzing the full order longitudinal equations of

motion, since the poles of the phugoid, which are of less interest, are closer to the origin of the imaginary plane with respect to the short period poles.

This limitation is less marked for the lateral-directional set of equations of motion, since the poles of aperiodic spiral and dutch roll motion tend to be closer to each other [12, 36, 38]. Furthermore, this issue can be overcome by using approximations of the characteristics motion, such as the short period and phugoid approximation. The two modes can be analyzed separately, by applying the modified Routh-Hurwitz criterion to each system. In this case, each of the approximate models can be augmented with additional dynamics, such as actuators and filters, without incurring in any problem with respect to the pole placement.

Chapter 3

Methodology

In this Chapter, the methodology used to implement the module is presented. The nature of the problem at hand, which not only is highly multidisciplinary but also entails an optimization procedure, calls for a scrupulous partition of the discussion. The module itself is comprised of several parts. However, most of the methods are implemented in the NonLinConstraints.m file. Secondly, the objective functions which need to be minimized are found in the objective.m. The argumentation has been set up in such a way as to follow naturally from the structure of the module itself while ensuring an adequate level of detail.

Hence, in the first Section, an overview of the general optimization framework is going to be presented. This serves the purpose of putting the procedure into context, by having a broader look at the set-up of the problem. Subsequently, the objective function, which is comprised of three terms, is presented. In this respect, it will be argued that the chosen approach presents some issues, which calls for a more in-depth analysis in order to better understand the impact of the design variables on the objective function. As a consequence a sensitivity study was carried out preliminarily, to gain enhanced insights on the intrinsic behavior of the module. In section 3.1.4 an extensive discussion about the limitations of the presented optimization framework will be introduced.

After the introduction to the problem, with its limitations and advantages, the constraints on the optimization problem are presented in section 3.2. The bulk of the module implementation, such as the definition of the flight dynamics model (Section 3.2.3) and the stability augmentation system design (Section 3.2.4), is in fact found in the NonLinConstraints.m file. The pole placement methodology based on the modified Routh-Hurwitz criterion is also therein implemented, and treated in Section 3.2.5.

3.1. Optimization Framework

The appropriate set-up of the optimization stems from two main requirements which have to be met, namely:

- Computational efficiency
- Robustness

The need for computational efficiency is dictated by the requirements imposed by the Initiator in terms of computational time, as treated in Section 1.6; while a certain margin is allowed, nevertheless implementing a fast optimization is of capital importance.

The second consideration entails that the module has to cope with a variety of designs and configurations without incurring in problems that could halt the design procedure. This requirement is again due to the scope of this project, which aims at enhancing the capabilities of the Initiator. As such, a variety of aircraft configurations must be treated.

These considerations hold especially true in this case: the innovation of this research consist in casting the handling qualities requirements through a modified Routh-Hurwitz criterion as constraints on the design space. Clearly, this approach has potential drawbacks: certain configurations generated by the Initiator will not initially satisfy these constraints, which in other words means that the initial design vector is in the unfeasible region of the design space. This eventuality is not far-fetched, considering that many designs will not initially comply with the handling qualities requirements prescribed by the user. Such configurations might prove problematic, and while an unfeasible initial design vector does not necessarily entail a hindrance of the optimization procedure, it is certainly not a desirable contingency.

In an effort to enhance the robustness of the tool a two-step optimization framework has been devised, as will be described in the next section.

3.1.1. Optimization Structure

In figure 3.1 the high-level structure of the optimization is presented:



Figure 3.1: High level optimization architecture

As it can be seen, the optimization is divided into two steps: preliminary and main. The preliminary optimization, which is run using the solver *fmincon*, is mainly tasked with finding an initial design vector $x_{0_{init}}$ which is feasible. Thus only the nonlinear constraints have to be satisfied, while the objective function is set to zero. The overall purpose of this preliminary optimization is twofold. Firstly it ensures that a feasible design vector is fed into the main optimizer. This is done by modifying the design vector such that the non-linear constraints are satisfied. Secondly, it acts as a filter: should no feasible design vector be found, which entails that the configuration at hand cannot be optimized using the proposed methodology, a logical statement evaluates the *fmincon* exit flag and the optimization procedure is interrupted. The main optimization is then bypassed, while the baseline configurations are fed through to the subsequent modules in the Initiator. This approach was devised to make sure that the handling qualities module would not hinder the convergence of the Initiator, as well as making the module itself more robust and adaptable to different initial designs.

A flowchart of the preliminary optimization is presented in figure 3.2.



Figure 3.2: Preliminary optimizer architecture

The optimization problem can be therefore formally posed in the following way:

min	$J_i = 0$	i = 1, 2, 3
s.t	$c_{\mathrm{ineq}_j}(x_k) \leq 0$	j = 1,, m
	$lb_k \le x_k \le ub_k$	k=1,,n

Once the preliminary optimization has found a feasible design vector (which indeed could also simply be equal to the initial vector), the main optimization is run. It takes as input the design vector $x_{0_{init}}$ provided by the preliminary optimization and generates as output the final design vector x_{opt} . Furthermore, in contrast to the previous procedure, it not only satisfies the nonlinear constraints, but it also aims at minimizing an objective function comprised of three components, which will be detailed in subsection 3.1.2. In figure 3.3 the structure of the main optimization is presented.



Figure 3.3: Main optimizer architecture

It must be noted that for the main optimization the solver is different: for the preliminary optimization *fmincon* was used, while in the main optimization the solver is *fgoalattain*. It is a multi-objective opti-

mization solver, which allows for the definition of three separate performance indexes, instead of compounding all the components into one has required by *fmincon*. Weights on each performance index can be defined independently, depending on what is required by the Initiator user. Furthermore, a goal can be defined, which indicates the value of the objective function which the optimizer will strive to achieve. It must be noted that it is entirely possible to over-achieve or under-achieve that value. This contingency is described by the *attainment factor*: a value of 1 indicates that the goal has been achieved precisely. A value higher than 1, or lower than 1, indicate that the goal has been underachieved or overachieved, respectively. Should the goal be underachieved, it indicates that the required goal is not attainable, and it should be modified. On the other hand, an overachievement entails that the goal set was too conservative, and the optimization indeed achieved a minimum which exceeds the requirements. In either case, the optimization procedure itself is not affected, and will always strive to achieve a local, or global, minimum.

The main optimization problem can be therefore formally defined in the following way:

$$\min \gamma = \begin{cases} J_i(x_k) - \operatorname{weight}_i \cdot \gamma \le \operatorname{goal}_i & i = 1, 2, 3\\ c_{\operatorname{ineq}_j}(x_k) \le 0 & j = 1, ..., m\\ lb_k \le x_k \le ub_k & k = 1, ..., n \end{cases}$$

Where *n* indicates the number of design variables, *i* the number of objective functions and *j* the number of constraints on the design space. Weight and goals are defined for the respective objective functions.

3.1.2. Objective Functions

As stated in chapter 1, the novel approach of this thesis consists in implementing the handling qualities requirements as constraints on the design space, rather than as objective function, through a modified Routh-Hurwitz criterion. This allows for the definition of one or more performance-based objective functions, with the aim of achieving the required flight characteristics while concurrently seeking non-dynamic performance benefits.

As it was discussed in section 3.1.1 three distinct objective functions are used in the main optimization, namely:

- Horizontal tailplane weight Wht
- Horizontal tailplane induced drag C_{D_i}
- Horizontal tailplane zero-lift drag C_{D0}

To comply with the time constraints imposed by the Initiator the three objective functions are computed using handbook methods and semi-empirical relations. As it will be explained in section 3.1.3 this approach, however computationally inexpensive compared to more refined methods, presents some drawbacks and will directly impact the choice of the design vector as well as the results.

Regarding practical implementation, each of the objective functions is computed in the objective.m file within the module. A separate index is associated with each function and then fed to the optimizer, which will them aim at minimizing every component with respect to the desired goal and weight. In Figure 3.4 the general structure of the objective function calculation procedure is shown. The inputs consist of the aircraft geometry, center of gravity location and aerodynamic data. Furthermore, for the computation of the zero-lift drag, the skin friction coefficient $C_{D_{\rm f}}$ is required (see Section 3.1.2). Having described the general implementation of the objective functions in the optimization, it is now appropriate to outline each component more in detail, specifically regarding the methods used for computation. As stated previously, these are based on semi-empirical relations, as well as handbook methods.

Horizontail tailplane weight

The tailplane weight is computed using Torenbeek's method [2]. This choice was dictated by the need for consistency within the Initiator, in particular with the Class 2 weight estimation module. In fact the weight of the various components is computed using precisely this method. Clearly using a different estimation



Figure 3.4: Objective functions calculation - objective.m

procedure entails that a discrepancy might arise during the design convergence, which is an undesirable contingency. Torenbeek's method estimates the taiplane weight as function of taiplane area S_{ht} , aircraft dive speed V_D and tailplane half-chord sweep $\Lambda_{0.5_{ht}}$:

$$W_{ht} = K_h S_{ht} \left[3.81 \cdot \frac{S_{ht}^2 V_D}{1000 \sqrt{\cos(\Lambda_{0.5_{ht}})}} - 0.287 \right]$$
(3.1)

Furthermore, K_h is a correction factor for composite structures. In the current implementation, the presence of composites is not taken into account. Hence this terms is set to one. It is immediate to realize that this formulation is insensitive to several design parameters which would otherwise have a considerable impact on the weight of the tailplane, i.e. aspect ratio, and taper.

Horizontail tailplane induced drag

The second objective functions is the tailplane induced drag, which is computed using the well-known definition:

$$C_{D_i} = K C_{L_{ht}}^2 \tag{3.2}$$

where the factor *K* is defined as:

$$K = \frac{1}{\pi e A R_{ht}} \tag{3.3}$$

In equation 3.3 the term *e* is the Oswald factor, which in the module is determined through an empirical correlation prior to the optimization procedure.

The lift coefficient of the horizontal tailplane in cruise condition is obtained through the condition of longitudinal equilibrium and equality of weight and lift, as found in reference [2]. The equation, in non-dimensional form, is defined as follows:

$$C_{L_{ht}} = \frac{S}{S_{ht}} \frac{q}{q_h} \frac{\overline{c}}{L_{ht}} \left(C_{m_{ac,wb}} + C_{L_{wb}} \cdot \frac{x_{cg} - x_{ac,wb}}{\overline{c}} \right)$$
(3.4)

Where $\frac{x_{cg}-x_{ac,wb}}{c}$ is the static margin of wing plus fuselage. The location of the aerodynamic center is computed using a two-step method: firstly the aerodynamic center of the wing is found through Torenbeek's carpet plots [2], as a function of taper, sweep, and aspect ratio. Lastly, the presence of the fuselage is taken into account through a correction factor. Furthermore, the pitching moment of the wing is computed using the DATCOM method [40], and again a correction factor is used to determine the contribution of the fuselage [2]. It is immediate to realize that to a longer tail arm, larger tail area, and smaller aircraft minus tail pitching moment corresponds a lower lift coefficient and, hence, lower induced drag. It is important to note that the optimal location of the center of gravity for induced drag is highly dependent on the magnitude of the wing-fuselage pitching moment, as it will be treated in detail in Chapter 5.

Horizontail tailplane zero-lift drag

Lastly, the zero-lift drag of the horizontal tailplane has been estimated using the following empirical relation [1]:

$$C_{D_0} = R_{wf} R_{l_s} C_{D_f} \left(1 + L \cdot \left(\frac{t}{c}\right) + 100 \cdot \left(\frac{t}{c}\right)^4 \right) \frac{S_{ht_{wet}}}{S_{ht}}$$
(3.5)

The various terms appearing in equation 3.5 are detailed in the list of symbols. However, it must be noted that zero-lift drag is dependent on the Reynolds and Mach number through the tailplane turbulent flat plate coefficient $C_{D_{\rm f}}$. This term is computed within the parasitic drag estimation module in the Initiator. The remaining terms are mostly dependent on the geometry of the surface, namely the ratio between wetted area and reference area, sweep and thickness distribution. Furthermore, the presence of the fuselage, with the related interference effects, is also taken into account through the correction factor R_{wf} .

A remark must be made about the applicability of equation 3.5: it is generally valid for smooth surfces. Any roughness and imperfections will introduce additional drag, which however is not taken into account. It must be therefore borne in mind that the results are likely to be underestimated. Moreover the wetted area of the tailplane $S_{ht_{wet}}$ is defined as:

$$S_{ht_{wet}} = 2 \left[1 + 0.5 \cdot \left(\frac{t}{c}\right)_{max} \right] b\overline{c}$$
(3.6)

Each component is then normalized using the respective values for the baseline configuration, thus allowing for equal weighting in the *fgoalattain* solver. Hence the three components of the objective function can be defined in the following way:

J_1	J_2	J_3
$\frac{W_{ht}}{W_{ht_0}}$	$\frac{C_{D_0}}{C_{D_{0,0}}}$	$\frac{C_{D_i}}{C_{D_{i,0}}}$

Table 3.1: Objective function components

The methods illustrated in this section allow for the calculation of the objective functions in a computationally efficient way. However, speed comes at a price: the empirical nature of Equation 3.1,3.2 and 3.5 entails that only a few geometrical parameters of the horizontal tailplane are taken into account. Clearly, this presents some problems: as an example, an increase in tail aspect ratio definitely translates in an increase in weight, which however is not considered by the methods proposed. Hence, it is deemed mandatory to perform a preliminary sensitivity analysis on the optimization problem to assess how the various geometrical variables impact the objective functions, and consequently obtain to the final composition of the design vector. This analysis is treated in Section 3.1.3.

3.1.3. Sensitivity Analysis

In order to set-up the optimization procedure in a correct way, often a sensitivity analysis is performed. It is useful to determine the effect of the variation of the selected variable on the gradient of the objective function, such that a correct choice of the *fmincon* input parameters DiffMinChange and DiffMaxChange (see Section 5.1.1) is achieved. The aim is to render the optimization more robust, as well as increasing the chance of actually achieving a minimum in the objective function. Furthermore, another purpose of this sensitivity study is to investigate which design variables actually affect the performance indexes and, if so, to which extent. The usefulness of this approach is evident, insofar as it allows to avoid using extra design variables which have little to no impact, thus avoiding an unnecessary increase in computational time. In the present case, the sensitivity also serves the purpose of defining the final design vector which will be used during the optimization, by assessing whether key geometrical tailplane variables have an impact on the objective functions.

The design vector which was considered consists of the following elements:

x_1	x_2	<i>x</i> ₃	x_4	<i>x</i> ₅
AR _{ht}	S_{ht}	$\Lambda_{LE_{ ext{ht}}}$	λ_{ht}	x_{pos_w}

Table 3.2: Design vector for sensitivity study

These are the four variables required to univocally define the planform of the tailplane [2], plus the wing longitudinal position. The relevance of this last variable is clear considering the scope of this research, treated in Section 1.6. As it was stated before, the sensitivity study investigates the gradient of the objective functions respect to each design variables. Hence this investigation is only dependent on the formulation of the objective functions, and it can be used to characterize the optimization procedure in a general fashion. To be more precise, the results obtained in this section are valid whether the optimization is applied to a long-range conventional airliner as well as a flying wing configuration, as long as the objective functions remain in the same form as the one treated in Section 3.1.2.

Specifically, for this analysis, the input aircraft is an Airbus A320-200, generated by the Initiator. This is done for consistency, as this particular aircraft will be used as test case throughout this whole research project (see Chapters 4 and 5).

In practical terms, frequently the most used method involves the logarithmic sensitivity for the computation of the gradients, which is defined as [41]:

$$\frac{d_L f}{d_L x} = \frac{d(\log f)}{d(\log x)} = \frac{\frac{1}{f} df}{\frac{1}{x} dx} = \frac{x}{f} \cdot \frac{df}{dx}$$
(3.7)

The main advantage of this approach is that is dimensionless, and therefore allows for a quick comparison between different design variables. Furthermore, it is immediate to evaluate the relative importance, or order of influence, of the chosen parameters: in case the derivative of the response is greater than one then it can be inferred that it strongly affects the results, while the opposite can be said in case the derivative is appreciably lower than one. Hence this method was deemed the most suitable, as it gives an immediate indication of the relative impact of each of the variables presented in Table 3.2.

Several approaches exist for the implementation of a sensitivity study. For the interested reader, an extensive overview can be found in the article from Van Keulen [41]. For this particular case, the least complex method was chosen: the *global finite differences* approach. Given the form of the objective functions shown in section 3.1.2, and the fact that this study aims only at assessing the relative influence of each design variables in a preliminary way, this choice was deemed the most suitable in terms of a tradeoff between accuracy and complexity of the implementation. The finite differences have been computed in three different ways:

- · backward differences
- · forward differences
- central differences

The starting from the computation involves applying a forward perturbation h to a generic function f and expanding using Taylor series:

$$f(x+h) = f(x) + \frac{1}{1!}f'h + \frac{1}{2!}f''h^2 + \frac{1}{3!}f'''h^3 + \dots$$
(3.8)

It is possible to isolate the second term on the right-hand side, thus obtaining:

$$f'h = f(x+h) - f(x) - \frac{1}{2!}f''h^2 + O(h^3)$$
(3.9)

And finally, it' possible to obtain the expression for the first order forward finite difference:

$$f' = \frac{f(x+h) - f(x)}{h} - \frac{1}{2!}f''h + O(h^2)$$
(3.10)

Similarly, applying a backward perturbation and expanding again using Taylor series it's possible to pervene to an expression for the first order backward finite difference:

$$f' = \frac{f(x) - (f - h)}{h} + \frac{1}{2!}f''h + O(h^2)$$
(3.11)

By combining both perturbations, an expression for the firs order central finite difference is readily obtained:

$$f' = \frac{f(x+h) - f(x-h)}{2h} - \frac{1}{3!}f'''h^2 + O(h^3)$$
(3.12)

An example of the computation of finite differences for a generic function using the three approaches is presented in figure 3.5



Figure 3.5: First order forward, backward and central finite differences [42]

Hence the sensitivity study was performed for each of the design variables shown in table 3.2, considering the three performance indexes in a separate fashion. From a practical point of view, the design variables were varied one at a time with a logarithmic distribution, while the other four were kept fixed at the base-line values. The impact of each variable could be therefore isolated and analyzed in detail. As an example, for AR_{ht} with respect to all three objective functions, the results are presented in Figures 3.6 and 3.7.



Figure 3.6: Sensitivity study results - AR_{ht} vs. W_{ht} and C_{D_i}



Figure 3.7: Sensitivity study results - $AR_{ht} - C_{D_0}$

It is immediate to notice that the aspect ratio of the tailplane has almost no impact on both tailplane weight W_{ht} and tailplane parasitic drag C_{D_0} , since the value of the gradient is well below one. On the other hand, it can be seen how the aspect ratio strongly influences the gradient of the performance index concerned with tailplane induced drag. This should not surprise, as the aspect ratio is indeed of paramount importance for induced drag (see Equation 3.2). However it must also be noted that the aspect ratio has a strong influence on tailplane weight: unfortunately, this effect is not modeled by the chosen objective function, as testified by the very low gradient for W_{ht} .

In all three cases, considering the central finite differences approach, the gradient is constant for $10^{-5} \le dx \le 10^{-1}$ with a slight divergence towards 10^{-1} . This result gives a good indication of the choice of the settings for the optimization procedure discussed in section 3.1.1. Of course, given that the gradient is constant across a wide spectrum of dx, it is possible to select a smaller range, thus preventing the optimizer from selecting steps which are too small and hence reduce computational time. Similar results have been obtained for the remaining design variables, however, for the sake of concision, not all the plots will be illustrated here. The complete plots showing the results can be found in Appendix A.

$x \downarrow \frac{dF}{dx} \rightarrow$	$\frac{dW_{ht}}{dx}$	$\frac{dC_{D_f}}{dx}$	$\frac{dC_{D_i}}{dx}$
S _{ht}	1.19	1.01	-1.92
AR_{ht}	0.018	$-9.8 \cdot 10^{-3}$	-1.35
λ_{ht}	0.01	$-7.7 \cdot 10^{-3}$	-0.12
$\Lambda_{LE_{ m ht}}$	0.07	-0.08	-0.22
x_{pos_w}	0	0	30

The results are summarized in Table 3.3, which illustrates the relative strength of each design variable in terms of objective functions gradient.

Table 3.3: Gradient of objective functions for each design variable

It's immediate to see how the impact of several variables is unevenly distributed. The tail area has a decisive effect on all three objective functions, with a gradient which in every case is larger than one. On the other hand, the aspect ratio only affects induced drag in a significant way, while it has almost no effect on weight and parasitic drag. This situation not only does not correspond to a real-life scenario, but it also means that the aspect ratio will likely be increased as much as possible in order to reduce induced drag since no counterbalancing effect comes from the other two objective functions.

As a result, it is probable that the optimizer will place this variable on the upper bounds or, if no bounds are defined, it will increase it up to the point where the design becomes unrealistic. Similar conclusions can be drawn by inspecting the gradient for the other design variables.

This issue requires a more detailed discussion. The real problem lies in the fact the objective of this optimization is to generate configurations which are feasible, not only in an optimization sense but more broadly from an engineering point of view. By using the full set of geometrical parameters defined in Table 3.2 several problems can arise: as an example, the optimizer will have no notion that an increase in aspect ratio entails an increase in weight because this relation is not modeled in Equation 3.1. Hence, it might find a local minimum for both induced drag (which indeed depends on aspect ratio) and weight. However, more refined analyses at later stages would bring up a substantial increase in weight, caused exactly by this increase in aspect ratio. Clearly, this issue needs to be solved if a consistent optimization is to be implemented.

There is no doubt that that the choice of the objective functions, dictated mostly by the need for low computational cost, poses considerable limitations on the capability of the optimizer of achieving a realistic design, and hence the composition of the design vector itself needs to be evaluated carefully. This issue will be discussed more in depth in section 3.1.4.

3.1.4. Considerations On Sensitivity

Of the main design parameters which are needed to define a horizontal tailplane, at least three had little to no impact on one or more objective functions. This problem calls for a reconsideration on the composition of the design vector. In fact, not only the interactions between some design variables and the objective functions are not captured properly, but also other design aspects which need to be taken into account are neglected. Clearly, this predicament might have been avoided through the use of more sophisticated methods for the analysis of both the aerodynamics and the weight of the tailplane, which however would have entailed a higher computational cost and therefore might have exceeded the requirements of the Initiator, thus nullifying the scope of this research project.

It is, therefore, relevant to understand the effect of the design variables which have proved troublesome from a sensitivity point of view, and hence assess the loss in modeling accuracy that comes by including them, or not, in the design vector. To this end, only three of the five variables will be discussed: this choice is dictated by the fact the tail area and wing longitudinal location are fundamental parameters, since their combination determines the tail volume coefficient [1, 32], and are considered essential. Furthermore, the wing location, even though it was shown that it does not affect tail weight and skin friction drag (see Appendix 2), is essential to the scope of this research, and plays a major role in the creation of the X-plots [2]. Furthermore, it also drives controllability and stability requirements, which are of paramount importance in this methodology (see Section 3.2.6).

Aspect ratio

The tail aspect ratio is a fundamental parameter, which has a decisive impact on $C_{L_{\alpha}}$. Furthermore, while the $C_{L_{\text{max}}}$ is not very sensitive to it, it considerably affects the stall angle of attack, which is a fundamental parameter for tail design, especially in flight conditions which imply the deployment of flaps [32]. This is due to the change in downwash angle, which in turn could cause an excessive angle of attack at the tail, thus possibly leading to stall. For this limiting condition, a low aspect ratio would be beneficial. Clearly this effect is not taken into account in the proposed optimization framework, and therefore it could be speculated that the aspect ratio would increase in order to achieve a minimum induced drag, as well as increasing $C_{L_{\alpha}}$ for the benefit of handling qualities, with detrimental effects for high angle of attack behaviour.

Furthermore, tailplane weight is also directly affected by the aspect ratio, which is not accounted for by equation 3.1 [43].

Taper ratio

Taper ratio is found to have little effect on $C_{L_{\alpha}}$. It plays a major role in affecting the lift distribution of the tailplane, which in turn determines its stalling characteristics: increasing the taper ratio means that the

outboard section of the tailplane is producing more lift, while the section C_L is decreased. From an aerodynamic point of view this is beneficial in terms of avoiding tip stall, however, from a structural point of view, the increased load at the tip requires a heavier structure. The opposite happens with low taper ratio: the structural weight is reduced. However, the lift distribution is modified in such a way that the section C_L at the tip becomes higher. This, in combination with the reduced Reynolds number due to a smaller chord at the tip, might lead to tip stall, which of course is an eventuality that is best avoided. It can be therefore concluded that taper ratio indeed has a fundamental impact on both weight and aerodynamics, which is not captured by equations 3.1,3.2 and 3.3.

Leading edge sweep

Leading edge sweep also affects the lift distribution: is well known that a swept wing shows an increase in lift near the wing tip, matched by a decrease in lift at the root. Again, this effect contributes to an increase in structural weight and, in combination with low taper ratio, it has a significant impact on the stall characteristics of the tail [32]. Leading edge sweep also affects C_{L_a} , specifically decreasing it. On the other hand, it has the positive effect of increasing the stall angle of attack. From a physical point of view this can be explained considering that a swept leading edge sheds stable vortices which reduce the static pressure on the suction side and therefore produces additional lift, increasing $C_{L_{max}}$ and postponing stall [32]. This effect is enhanced in combination with a low aspect ratio, for which three-dimensional vortices are dominant. Another fundamental requirement which a tailplane has to satisfy is the capability of handling high Mach number without flow separation; to this end, a larger leading edge sweep is beneficial along with reduced relative thickness.

Clearly sweep is only taken into account in equation 3.1, while it has no impact on equations 3.2 and 3.3. Furthermore, no method to evaluate wave drag is implemented in the module, which is contrast to the previously mentioned requirements.

3.1.5. Design Vector Composition

From this qualitative analysis, it clearly emerged that the interaction between the various design variables is very complex, and the design must often be chosen as a tradeoff between various requirements. The objective functions in the form described by equations 3.1, 3.2, 3.3 clearly fails at capturing all these phenomena simultaneously. It could, therefore, be conjectured that the optimization might produce a feasible design, which however would prove not optimal to a more accurate analysis, and might even show considerable deficiencies in terms of design requirements.

A solution might be to define proper bounds on the design variables. However, this would call for an extensive preliminary analysis to effectively determine the allowable values of the bounds. Furthermore, even if this was the case, it can be hypothesized that the poor interaction between the design variables and the objective functions would produce solutions which are consistently on the bounds, which is not a desirable contingency. In the light of these considerations, it was decided to limit the geometrical design variables to the tail area S_{ht} and the wing longitudinal position x_{pos_w} , while the remaining variables will be set to their input value.

The downside to this approach is that the actual performance benefits achievable through the optimization will be diminished: as an example, fixing the aspect ratio entails that the induced drag might be lowered, but only to a certain extent. Furthermore $C_{L_{\alpha}}$ is highly dependent on it, and will be limited as well. The same applies to the other variables with respect to both the objective functions.

Nevertheless, this procedure was deemed necessary in order to achieve consistent designs which, even though perhaps sub-optimal, are feasible from an engineering perspective.

3.2. Module Implementation

This section of the report is devoted to summarizing the methods used in the implementation of the constrains on the design space of the optimization. Firstly a general description of the module is given, including its positioning in the context of the Initiator and a top-level workflow of the NonLinConstraints.m file. Secondly, the various submodules, which correspond to the different steps in the procedure, are outlined so that a clear overview of the inner workings of the tool can be provided.

3.2.1. General Description

As stated in chapter 1, the handling qualities optimization methodology is to be implemented in a module in the context of the Initiator. Consequently, an integral part of the development process was the determination of the position of this module within the general design framework. One of the driving considerations is the sheer amount of data which is required in order to perform an effective optimization, ranging from geometrical data to weight and inertia estimation. Furthermore, the design methodology should follow a logical procedure in terms of the sequence with which aircraft components need to be sized.

As a result of these considerations, it has been decided to implement the handling qualities optimization module at the very end of the Initiator class 2 convergence loop, before the Mission Analysis module. This position ensures that class 2 data for weight is available, as well as the complete baseline geometry of the aircraft including the initial wing location and horizontal stabilizer planform. Furthermore, reliable inertia data is required, and it was therefore decided to include the inertia estimation module in the class 2 convergence loop as well. It must be noted that this module was implemented in earlier version of the Initiator within the class 2.5 convergence loop. One could argue that a more suitable position for the handling qualities optimization module would be within the class 2.5 convergence loop, which makes use of more sophisticated methods for weight estimation, i.e EMWET. However, it must be borne in mind that the methods implemented in the present research are class 2, mainly due to computational time requirements, as stated in section 3.1.2. Therefore a discrepancy would arise between the methods implemented and the fidelity of the chosen convergence loop, which is not a desirable contingency.

Summing up these considerations, the updated N^2 chart describing the architecture of the Initiator is shown in figure 3.8:

Class 1 Weight Estimation	N	итом	мтом	MTOM, FF						J
	Wir Li	ng Thrust Dading	W/S, T/W			T/W		-		
			Geometry Modules	Geometry	Geometry		Geometry		Wing geometry	Fuselage geometry
OEM				Class 2 Weight Estimation	MTOM, FM, OEM	мтом	MTOM, c.g range	Range		
L/D, CD _{min} CL _{min}		Polar	Polar		Aerodynamic modules		Skin friction coeff.	Polar	Loading	Loading
SFC						Engine Model		SFC		
			Geometry				Handling Qualities			
	N	итом		MTOM, FF				Mission Analysis		
				Wing weight (OEM)					EMWET	
t				Fuselage weight (OEM)						Fuselage Weight Estimation

Figure 3.8: Updated Initiator N2 diagram

The orange limits define the class 1 convergence loop, while the blue and green limits define respectively

class 2 and class 2.5 convergence loops. The position of the handling qualities optimization module can be clearly seen in red.

It is now appropriate to introduce the structure of the methodology for the determination of the constrains, implemented in the NonLinConstraints.m file. Firstly, let us recall the general architecture of the main optimization, in order to provide some context. While Section 3.1 treated the general framework, as well as the definition of the objective functions and design vector, the focus of this section is shifted on the definition of the nonlinear constraints, as Figure 3.9 evinces. It must be noted that the bulk of the methodology is implemented in this sub part of the optimization framework.



Figure 3.9: General optimization structure - nonlinear constraints



Figure 3.10: Module architecture - NonLinConstraints.m

It can be seen in Figure 3.9 that only two submodules generate the set of constraints: regional pole placement, which is based on the modified Routh-Hurwitz criterion, and stability & control (see Sections 3.2.5 and 3.2.6). All together, these constraints define the design space, for which the solutions possess the desired level of handling qualities, along with the required characteristics in terms of controllability and static stability. The detailed workflow of the Nonlinear constraints block (Figure 3.9), is presented in Figure 3.10. It is possible to see that the module is divided into five main submodules. A logical statement is inserted to evaluate the presence of a stability augmentation system and modify the workflow of the methodology to account for it. The last two submodules, namely regional pole placement and stability and control, produce the set of twelve nonlinear inequality constraints which define the design space. These constraints are going to be discussed more in depth in Sections 3.2.5 and 3.2.6.

A remark must be made regarding the inputs to the module: while most of the parameters required are obtained from other Initiator modules there are several specific inputs which are defined externally. To allow for the selection of different inputs to the module without having to access the code itself, few settings have been implemented in the Initiator ProgramSettings file which are then fed directly to the module. These settings are shown in the following code extract:

```
1
   2
     <!-- HandlingQualitiesOptimization Module -->
3
     <setting>
4
     <!-- 1: Unaugmented Method 2: Augmented Method 1 3: Augmented Method 2
        -->
5
       <name>HQMethod</name>
6
       <value>1</value>
7
     </setting>
     <setting>
8
       <name >CAP </name >
9
10
       <value>0.3</value>
11
     </setting>
12
     <setting>
13
       <name>ShortPeriodDamping</name>
14
       <value>0.55</value>
15
     </setting>
```

The three options allow for the complete definition of the desired handling qualities, as well as the methodology which should be used during the optimization procedure. As it was stated in previous sections, the methodology ought to apply to both augmented and unaugmented designs, hence the definition of the option *HQMethod*. Referring to figure 3.10, selecting option 1 entails that the SAS design submodule is bypassed, and the results coming from the flight dynamics module are fed directly to the regional pole placement module. Option 2 and 3, on the other hand, modify the workflow of the module such that the SAS design submodule is included, receiving the results from the flight dynamics submodule. Furthermore, as it will be shown more in detail in section 3.2.4, the selection of either of the two options invokes a different method for the synthesis of the longitudinal stability augmentation system.

In the following sections a more accurate description of the various submodules outlined in figure 3.10 is given.

3.2.2. Aerodynamic Derivatives Estimation

The stability and control derivatives required for the implementation of the equations of motion are computed using a combination of the methods presented in references [44] and [45], which in essence are a simplified version of the DATCOM method [40]. Only a limited set of derivatives are calculated. Specifically the ones required to define the short period reduced order model, which will be introduced in section 3.2.3. These derivatives are computed with respect to user-defined flight condition, and are dependent mainly on position of the center of gravity and flight speed.

Derivatives with respect to angle of attack

One of the most important derivatives for an aircraft longitudinal motion is $C_{L_{\alpha}}$, or the lift curve slope. It is calculated in the following way:

$$C_{L_{\alpha}} = C_{L_{\alpha_{wb}}} + C_{L_{\alpha_{ht}}} \frac{q_{ht}}{q} \frac{S_{ht}}{S} \left(1 - \frac{d\epsilon}{d\alpha} \right)$$
(3.13)

where:

$$C_{L_{\alpha_{wb}}} = C_{L_{\alpha_w}} K_{wb} \tag{3.14}$$

$$C_{L_{\alpha_{w}}} = \frac{2\pi A R_{w}}{2 + \sqrt{A R_{w}^{2} \beta^{2} \left(1 + \frac{\tan^{2} \Lambda_{0.5_{w}}}{1 + \tan^{2} \Lambda_{0.5_{w}}}\right) + 4}}$$
(3.15)

$$C_{L_{\alpha_{ht}}} = \frac{2\pi A R_{ht}}{2 + \sqrt{\frac{A R_{ht}^2 \beta^2}{e_{hs}^2} \left(1 + \frac{\tan^2 \Lambda_{0.5_{ht}}}{\beta^2}\right) + 4}}$$
(3.16)

Recalling the considerations made in section 3.1.4, it is immediate to notice that $C_{L_{\alpha}}$ increases with the aspect ratio of the surface considered. The same effect can be achieved by reducing the half-chord sweep. Furthermore the flight condition directly impacts the lift curve slope, which specifically increases with the Mach number through the compressibility correction factor β . As stated in reference [40], equation 3.13 is quite accurate up to M = 0.6. The cruise Mach number of of the configurations considered in the Initiator is generally higher, which naturally introduces a source of error in the results of this methodology. Finally, the presence of the fuselage is taken into account through the correction factor K_{wb} .

Following from the lift curve slope it's possible to compute $C_{m_{\alpha}}$ through the following relation:

$$C_{m_{\alpha}} = \left(\frac{dC_m}{dC_L}\right) C_{L_{\alpha}} \tag{3.17}$$

where:

$$\left(\frac{dC_m}{dC_L}\right) = \frac{x_{cg} - x_{ac}}{\overline{c}} \tag{3.18}$$

Equation 3.17 is easily understood from a qualitative point of view considering that the aerodynamic center is defined as the point on which changes in lift ΔL act [36]. A sudden change in angle of attack will then generate a moment proportional to the arm $\frac{dC_m}{dC_L}$ and ΔL . The term $\frac{dC_m}{dC_L}$ is defined as the static margin, as it was previously discussed in Section 2.2.

Derivatives with respect to pitch rate

Derivatives with respect to pitch rate play a fundamental role in characterizing the longitudinal dynamics of the aircraft, in particular, the short period response. To better understand the meaning behind these derivatives, let's considered an aircraft subjected to a non-zero pitch velocity, which entails a change in the geometrical angle of attack along the lengthwise direction. Considering a positive pitching velocity the aircraft can be imagined to be in a curved flow field, with positive curvature and center of rotation along the vertical axis, in line with the center of gravity. Clearly, this curvature entails a varying angle of attack distribution, which will be higher at further positions from the center of gravity and lower at positions closer to it.

The first derivative which is computed is the change in lift due to pitch rate, namely C_{L_q} . It is calculated as the sum of the wing and tail contribution, while the presence of the fuselage is neglected.

$$C_{L_q} = C_{L_{q_w}} + C_{L_{q_{ht}}}$$
(3.19)

The wing contribution contains a correction factor multiplied by the wing $C_{L_{\alpha}}$ at zero Mach number. For the sake of concision the complete equations are not shown here, however for the interested read they can be found in reference [44]. The tail contribution is calculated with a similar procedure through the following equation:

$$C_{L_{q_{ht}}} = 2C_{L_{\alpha_{ht}}} \frac{q_{ht}}{q} \frac{S_{ht}L_{ht}}{S_w\overline{c}}$$
(3.20)

The most influential term is again the tailplane lift curve slope, along with the tail volume coefficient $\frac{S_{ht}L_{ht}}{S_{w}c}$.

The second derivative which is computed is the change in pitching moment due to pitch rate, which is of capital importance for handling qualities as it is also called the *pitch damping derivative*. It is comprised of two parts:

$$C_{m_q} = C_{m_{q_w}} + C_{m_{q_{h_t}}} \tag{3.21}$$

For aircrafts with a conventional configuration, the biggest contribution is given by the horizontal tailplane [36, 44], while the effect of the fuselage is generally neglected.

Similarly to the previous case, the wing contribution is obtained by determining the spanwise average value of the wing section lift curve slope, multiplied by two correction factors, which account for variation in wing aspect ratio, sweep and compressibility effects. The complete equations are not shown here. However, they can be found in reference [44]. The tail contribution, on the other hand, is computed in the following way:

$$C_{m_{q_{ht}}} = -2C_{L_{\alpha_{ht}}} \frac{q_{ht}}{q} \frac{S_{ht}L_{ht}^2}{S_w \overline{c}^2}$$
(3.22)

The change in angle of attack at the tailplane, therefore, determines an increment in normal force which generates a moment about the center of gravity of the aircraft.

Derivatives with respect to elevator deflection

According to the scope of this thesis, only the derivatives with respect to elevator deflection are computed. In reference [44] the control surfaces are treated as sealed gap plain flaps, for which the variation of lift coefficient with flap deflection $C_{L_{\delta_f}}$ is calculated. This method requires preliminary knowledge of the control surfaces geometry and two-dimensional aerodynamic characteristics. Once this factor is obtained then the derivative of lift coefficient with respect to elevator deflection can be computed as:

$$C_{L_{\delta_e}} = C_{L_{\delta_f}} \frac{S_{ht}}{S_w}$$
(3.23)

In essence the variation of lift coefficient with flap deflection is scaled with the tailplane area ratio. Having obtained this derivative is then possible to compute the variation of pitching moment with elevator deflection:

$$C_{m_{\delta_e}} = -C_{L_{\delta_e}} \frac{l_{ht}}{\bar{c}}$$
(3.24)

In this case, l_{ht} indicates the distance between the tailplane aerodynamic center and the center of gravity of the aircraft.

Derivatives with respect to angle of attack rate

Once an aircraft experiences a change in angle of attack ,the pressure distribution over it requires some time to adapt to the new condition. The effect of this delayed adjustment on stability is taken into account through the following derivatives:

$$C_{L_{\dot{\alpha}}} = C_{L_{\dot{\alpha}_w}} + C_{m_{\dot{\alpha}_{ht}}} \tag{3.25}$$

$$C_{m_{\dot{\alpha}}} = C_{L_{\dot{\alpha}_w}} + C_{m_{\dot{\alpha}_{ht}}} \tag{3.26}$$

As a general remark, it must be noted that this methodology is computationally very efficient and in general provides acceptable results in the subsonic domain [45]. However, it must also be borne in mind that it might prove less accurate compared to other methods, such as Vortex Lattice, and especially with respect to panel methods [46]. Since the methodology is based on the computation of the derivatives using this procedure, clearly a source of error is introduced. Even though the results, as it will be shown in chapter 4, are within an acceptable range still this discrepancy must be borne in mind when analyzing the results.

3.2.3. Flight Dynamics Model

As it was introduced in section 1, the scope of this work is to develop a tool capable of optimizing the handling qualities of an aircraft, specifically regarding the short period oscillation. To this end the linearized equations of motion (see Section 2.3.2) have been reduced using the short period approximation, which brings the number of states to just two, namely the angle of attack and the pitch rate. The hypothesis for this simplification are the following:

- $V_0 = constant$
- $\gamma_0 = 0$

The first assumption entails that $\hat{u} = 0$ and therefore the first equation may be omitted, since the forces in the X-direction remain in equilibrium during the entire motion. Furthermore this also mean that the first column of the system of equations (Equation 2.13) is cancelled out.

The second assumption states that the initial flight condition is considered to be level, which also implies that the term C_{X_0} is equal to 0, since there is no weight component in the X-direction. As a consequence in the equations for Z and M the pitch angle θ is neglected, which in turn allows for the omittance of the kinematic relation. This assumptions are consistent with the effective characteristics of the short period, which can be described as a highly damped oscillation around the aircraft center of gravity in which only the angle of attack and the pitch rate undergo significant changes. Hence the approximated equations of motion (EOM) can be expressed in the following form [36]:

$$\begin{bmatrix} C_{Z_{\alpha}} + (C_{Z_{\dot{\alpha}}} - 2\mu_c)D_c & C_{Z_q} + 2\mu_c \\ C_{m_{\alpha}} + C_{m_{\dot{\alpha}}}D_c & C_{m_q} - 2\mu_c K_y^2 D_c \end{bmatrix} \begin{bmatrix} \alpha \\ \frac{q\bar{c}}{V} \end{bmatrix} = \begin{bmatrix} -C_{Z_{\delta_e}} \\ -C_{m_{\delta_e}} \end{bmatrix} \delta_e$$
(3.27)

It must be noted that, in order to obtain this form of the EOM, the previously computed aerodynamic derivatives must be shifted from the stability reference frame to the body frame. This is done simply by considering a flight condition with $\alpha_0 \approx 0$, which allows for a quick translation of the obtained values. Rearranging Equations 3.27, it is possible to obtain a state space form of this reduced order model [36], following the procedure outlined in Section 2.4.

$$P\dot{\mathbf{x}} = Q\mathbf{x} + R\mathbf{u} \tag{3.28}$$

with:

$$P = \begin{bmatrix} (C_{Z_{\dot{\alpha}}} - 2\mu_c)\frac{\bar{c}}{\bar{V}} & 0\\ C_{m_{\dot{\alpha}}}\frac{\bar{c}}{\bar{V}} & -2\mu_c K_y^2 \frac{\bar{c}}{\bar{V}} \end{bmatrix}, \quad Q = \begin{bmatrix} -C_{Z_{\alpha}} & -(C_{Z_q} + 2\mu_c)\\ -C_{m_{\alpha}} & C_{m_q} \end{bmatrix}, \quad R = \begin{bmatrix} -C_{Z_{\delta_e}}\\ -C_{m_{\delta_e}} \end{bmatrix}$$
(3.29)

By premultiplying the right-hand side matrices with P it's possible to obtain the desired state space form:

$$\dot{\mathbf{x}} = A P^{-1} \mathbf{x} + B P^{-1} \mathbf{u} \tag{3.30}$$

Which finally yields:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{3.31}$$

$$\mathbf{y} = C\mathbf{x} + D\mathbf{u} \tag{3.32}$$

Where:

$$A = \begin{bmatrix} z_{\alpha} & z_{q} \\ m_{a} & m_{q} \end{bmatrix}, \quad B = \begin{bmatrix} z_{\delta_{e}} \\ m_{\delta_{e}} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
(3.33)

The calculated state space system will then be directly fed to the regional pole placement submodule in case the design is unaugmented. Otherwise, it will be passed on as input to the stability augmentation system design submodule, outlined in the following section.

3.2.4. Stability Augmentation System Design

Once the state space system of the plant has been obtained, it's possible to use state feedback to synthesize a stability augmentation system. Specifically, the SAS loop is made up of a pitch-rate damper and an angle of attack feedback, as shown in figure 3.11:



Figure 3.11: Block diagram for SAS loop

Two different methods have been implemented for the determination of the angle of attack and pitch rate feedback gains: the first one entails their inclusion in the design vector, while the second approach makes use the Linear Quadratic Regulator (LQR) for their computation. Two assumptions have been used during the implementation:

- Both states are measurable without noise.
- The actuators dynamics are neglected (ideal actuators assumption).

The first assumption is deemed acceptable at the conceptual level. However, it must be noted that reality might differ significantly, especially regarding the measurements of the angle of attack sensor which tend to be quite noisy due to turbulence [7, 38].

The second assumption entails that the input signal to the system is the elevator deflection angle δ_e . Clearly, this simplification neglects the influence of actuators on frequency and time response of the system. However, it was found to be consistent with similar methodologies found in open literature regarding optimization of handling qualities at the conceptual level [47–49].

In the following paragraphs, the two methods used for the implementation are described more in detail.

Method 1

This approach is devised in such a way that the feedback gains k_{α} and k_{q} are part of the design vector. Clearly the optimizer is then delegated with the taks of finding suitable values which satisfy the conditions imposed by the regional pole placement method in an automated fashion.

In particular, referring to figure 3.11, the control law for method 1 has the following form:

$$\mathbf{u} = -k\,\mathbf{x} + \delta_e = -\begin{bmatrix} k_\alpha & k_q \end{bmatrix} \begin{bmatrix} \alpha \\ \frac{q\bar{c}}{V} \end{bmatrix} + \delta_e \tag{3.34}$$

Equation 3.31 can then be rewritten as:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} = A\mathbf{x} + B\left(-k\mathbf{x} + \delta_e\right) = (A - Bk)\mathbf{x} + B\delta_e$$
(3.35)

The matrix $A_c = (A - Bk)$ is the plant matrix describing the closed-loop system, which will then be fed to the pole placement submodule.

A remark must be made: a basic requirement for the use of state feedback is state controllability, which is

defined as the ability of the input to move the internal states of a system from an initial state to a chosen final state. Referring to equation 3.31, the controllability matrix is given by:

$$\mathbf{R} = \begin{bmatrix} B & A B \end{bmatrix} \tag{3.36}$$

If the rank of *R* is equal to the number of state variables, then the system is controllable, and full state feedback can be applied. Clearly, the system at hand is controllable, since the input signal δ_e affects every state. To increase the robustness of the module and ensure its correct working within the Initiator across every configuration, a controllability check has been implemented and included in a logical statement which halts the design procedure if this condition is not met.

Method 2

For the LQR algorithm the control law is the same as the one implemented for method 1:

$$\mathbf{u} = -k\,\mathbf{x} + \boldsymbol{\delta}_e \tag{3.37}$$

The gains k are determined such that the control law minimizes the following quadratic performance index:

$$J(\mathbf{u}) = \int_0^\infty \left(\mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u} \right) dt$$
(3.38)

As opposed to the state feedback design, in which the gains are part of the design vector, for the LQR the design variables are the weighting factors for the matrices Q and R, namely q_1^*, q_2^* and ρ^* . This approach allows for the automatization of the design procedure, which usually requires manual tuning of these weighting factors in order to achieve the desired response [12]. Specifically, the matrices are defined as:

$$Q = \begin{bmatrix} q_1^{\star} & 0\\ 0 & q_2^{\star} \end{bmatrix} \quad R = \rho^{\star}[1] \tag{3.39}$$

The design of the controller is carried out using the *lqr* function in MATLAB, by firstly providing the matrices and the state space system of the model. The results is a 1×2 vector containing the required gains:

$$k = \begin{bmatrix} k_{\alpha} & k_{q} \end{bmatrix} \tag{3.40}$$

The augmented matrix is then formed using the same approach illustrated in equation 3.35 and passed on as input to the pole placement submodule. As a final remark, the LQR methodology required that *R* is positive definite and that *Q* is controllable, similarly to Section 3.2.4.

3.2.5. Regional Pole Placement - Modified Routh-Hurwitz criterion

In this section the practical implementation of the modified Routh-Hurwitz criterion, introduced in Section 2.7, is presented. This represents the very core of the optimization methodology, as it enables the achievement of the desired level of handling qualities.

The submodule accepts as input the state matrix, for both unaugmented and augmented designs. This matrix can be used to compute the determinant of the short period reduced order model, and hence obtain the characteristic polynomial of the system. For the two different options, augmented and unaugmented the determinant can be computed in the following way:

Unaugmented configuration
$$\rightarrow \det(\lambda \mathbf{I} - A) = p_0 \lambda^2 + p_1 \lambda + p_2 = 0$$
 (3.41)

Augmented configuration
$$\rightarrow \det(\lambda \mathbf{I} - A_c) = p_0 \lambda^2 + p_1 \lambda + p_2 = 0$$
 (3.42)

Note that the coefficients p_0 , p_1 and p_2 are function of the aircraft stability and control derivatives, as well as mass, inertia characteristics and, eventually, feedback gains. As it was shown in section 2.7, the coefficients must satisfy a specific set of conditions to ensure that the design complies with the required handling qualities. First, the amplitude of the semi-cone which determines the boundary of the allowable pole region in the complex plane is set, such that:

$$\alpha^{\star} = \arccos(\zeta) \tag{3.43}$$

This directly determines the minimum required short period damping ratio. Secondly, the lower limit for the real part of the eigenvalues is set, as function of the required Control Anticipation Parameter. To this end, the short period natural frequency corresponding to the required minimum CAP[17, 50] is obtained, such that:

$$\omega_{n_{\rm sp}} = \sqrt{\frac{\text{CAP}_{\min} \cdot \mathbf{V}_0}{T_{\theta_2} \cdot g}} \tag{3.44}$$

It must be noted that this definition of the CAP, found in reference [17] differs slightly from the basic form shown in Section 1.3.2. Nevertheless, it describes the same metric. It is therefore possible to compute the required real part of the eigenvalues as follows:

$$x_1 = -\omega_{n_{\rm sp}}\zeta_{\rm sp} = -\omega_{n_{\rm sp}}\cos\alpha^{\star} \tag{3.45}$$

A remark must be made about the incidence lag constant T_{θ_2} : it is obtained by implementing the full order longitudinal equations of motion in the flight dynamics submodule and extracting the pitch transfer function in the familiar zero-pole-gain form [50]:

$$\frac{\theta(s)}{\delta_{e}(s)} = \frac{k_{\theta} \left(s + (1/T_{\theta_{1}}) \right) \left(s + (1/T_{\theta_{2}}) \right)}{\left(s^{2} + 2\zeta_{ph} \omega_{n_{ph}} s + \omega_{n_{ph}}^{2} \right) \left(s^{2} + 2\zeta_{sp} \omega_{n_{sp}} s + \omega_{n_{sp}}^{2} \right)}$$
(3.46)

It is then possible to compute T_{θ_2} in order to obtain the required $\omega_{n_{sp}}$ in equation 3.44.

The next step entails the creation of the Routh matrices through the procedure detailed in Section 2.7, for both conditions ($Re(\lambda) \le -x_1$ and $\zeta_{sp} \ge \cos \alpha^*$). The final matrices have the following form:

$$R_{\operatorname{array},x_{1}} = \begin{bmatrix} p_{0} & a_{2} & a_{3} \\ b_{1} & b_{2} & b_{3} \\ c_{1} & c_{2} & c_{3} \end{bmatrix}, \quad R_{\operatorname{array},\alpha} = \begin{bmatrix} a_{0} & a_{1} & a_{1} \\ b_{0} & b_{1} & b_{2} \\ -c_{0} & c_{1} & c_{2} \\ -d_{0} & d_{1} & d_{2} \\ e_{0} & e_{1} & e_{2} \\ f_{0} & f_{1} & f_{2} \end{bmatrix}$$
(3.47)

Having defined all the required matrices is now possible to express the conditions on the polynomial coefficients in the form of inequality constraints. The first condition that must be met entails that the coefficients must all have equal sign, or in other terms:

$$c_{\text{ineq}}(s) = \text{sign}(p_0) = \text{sign}(p_s) \quad \text{for } s = 1, \dots, n \tag{3.48}$$

Which yields a number of constraints equal to the order of the polynomial considered. Secondly the sign conditions on the coefficients of the matrices can synthetized in the following way: in order for the poles of the system to be located inside the cone of amplitude $2\alpha^*$, the coefficients in the first column of the matrix 3.47 need to have the same sign as a_0 . In order for the poles to be located to the left of a straight vertical line at $x_1 = -c$, the coefficients in the first column of the matrix R_{array,x_1} (Equation 3.47) need to have the same applies to the first two rows.

In terms of inequality constraints, for the condition $Re(\lambda) \leq -x_1$, considering $p_0 > 0$ we have:

$$c_{\text{ineq}}(i) = -R_{x_1}(i+1,1) \le 0 \quad \text{for } i = 1,2$$
(3.49)

$$c_{\text{ineq}}(j) = -R_{x_1}(j,2) \le 0 \quad \text{for } j = 1,2$$
 (3.50)

For the condition $\zeta_{sp} \ge \cos \alpha$ a similar procedure applies:

$$c_{\text{ineg}}(\mathbf{k}) = -R_{\alpha}(\mathbf{k}+2,1) \le 0 \quad \text{for } \mathbf{k} = 1,...,4$$
 (3.51)

Finally the complete set of inequality constraints for the pole placement method is obtained: it consists of eight elements plus the two conditions on the coefficients shown in equation 3.48, or $c_{\text{ineq}} = c_{\text{ineq}}(s) + c_{\text{ineq}}(i) + c_{\text{ineq}}(j) + c_{\text{ineq}}(k)$. If the solution is feasible, it will automatically have the desired dynamic characteristics, or in some cases, especially for augmented designs, the design could also exceed the minimum requirements.

3.2.6. Stability and Control

In this section the methods applied to ensure that the optimized design meets controllability and stability requirements are discussed.

In particular, the approach is based on Torenbeek's X-plots [2]. While the required flight dynamic characteristics are guaranteed through the regional pole placement method discussed in the previous section, it must be ensured that the aircraft remains controllable and stable in all the relevant flight conditions. Specifically, the X-plot methodology is used to compute the minimum tail area which satisfies the required c.g range, obtained from the class 2 weight estimation module in the Initiator. The c.g range in given as percentage of the mean aerodynamic chord, hence the wing position is modified until an optimum is reached, which satisfies both controllability and stability requirements while yielding the minimum tail area.

The most aft position dictates the stability limit, or when the c.g location overlaps the aircraft aerodynamic center. Consequently a static margin is added, which is usally between 5% and 10% of the MAC. The most aft position is computed as follows:

$$\overline{x}_{cg_{NP,aft}} = \frac{x_{ac_{cruise}}}{\overline{c}} + \frac{C_{L_{\alpha_{ht}}}}{C_{L_{\alpha_{w,h}}}} \left(1 - \frac{d\epsilon}{d\alpha}\right) \frac{q_{ht}}{q} \frac{L_h}{\overline{c}} \frac{S_{ht}}{S_w}$$
(3.52)

While the center of gravity location including the static margin is simply:

$$\overline{x}_{cg_{NPaft_{cM}}} = \overline{x}_{cg_{NPaft}} - S.M$$
(3.53)

It must be noted that the aforementioned static margin should not be confused with the static margin in equation 3.18, which is computed using the actual c.g location at the chosen flight condition instead of the most aft location. To be more precise the margin that appears in equation 3.53 can be considered as a safety margin, insofar as that even with most aft c.g location the aircraft retains a small degree of static stability. Clearly, the determination of this static margin greatly influences the minimum achievable tail area. For aircrafts in which stability augmentation systems are present, this limit could be relaxed, allowing for a reduction in static stability at the chosen flight condition. It could even be envisaged to create instability, i.e a positive static margin, to obtain a further reduction in horizontal tailplane area. However, safety considerations must always be borne in mind. As an example, a question that might be asked is: in case the stability augmentation system should fail, which would most likely happen for sensor damage, with the c.g at its most aft location, what would happen to the aircraft's stability? Clearly, a conservative approach should always be kept. However, it can also be argued that, during normal operations, transport aircrafts rarely fly with the center of gravity at its most aft position. For these reasons, in case of unaugmented design the static margin with aft c.g is set at 5 % of the mean aerodynamic chord. If a stability augmentation system is present, the static margin is reduced to zero, to conserve at least a condition of neutral stability for the most aft position of the center of gravity.

The choice of two different static margins can be justified through the following considerations. Let us consider a situation in which the center of gravity is completely aft: for unaugmented designs, it is imperative that the aircraft retains a condition of static stability across the flight envelope. Should the static margin be set to zero, the aircraft would become quite arduous to control, requiring constant pilot input. On the other hand, the augmented designs possess both an angle of attack feedback and a pitch damper.

Let us imagine that a failure to the angle of attack sensor occurs: the handling qualities would be degraded. However, the presence of the pitch damper would still allow the pilot to retain some form of control over the aircraft. In other words, there is a layer of redundancy. The same logic applies in the case of failure of the pitch rate gyroscope. Hence, it is possible to set the static margin to zero while still being reasonably confident in the reliability of the design.

Clearly, this redundancy layer is not present for unaugmented designs, thus the need for a static margin with aft center of gravity, even though quite small.

The most forward c.g position is critical to determine whether the tailplane has enough control authority to reach the aircraft $C_{L_{max}}$ with full flaps down in landing configuration, while enough power is still available for maneuvering. The free body diagram depicting the stall condition with flaps down is illustrated in Figure 3.12:



Figure 3.12: Free body diagram for control stall [2]

The maximum fore c.g. location for control stall is evaluated in the following way [2]:

$$\overline{x}_{cg_{stall}} = \frac{S_h L_h}{S_w \overline{c}} \frac{\eta_h C_{L_{ht}}}{C_{L_{max}}} - \frac{C_{m_{0.25}}}{C_{L_{max}}} + 0.25$$
(3.54)

A remark must be made regarding the pitching moment at quarter-chord. It is computed by translating the pitching moment at the aerodynamic center through the following relation:

$$C_{m_{0.25}} = C_{m_{ac}} + C_L \left(0.25 - \frac{x_{ac}}{\overline{c}} \right)$$
(3.55)

Another condition, which usually is less limiting, is the availability of enough control power in order to rotate the aircraft at takeoff with a positive pitching velocity $\dot{\theta}$ of 2 degrees per second. A free body diagram which illustrates this condition is presented in the following figure:



Figure 3.13: Free body diagram for takeoff rotation [2]

By a balance of forces and moments the corresponding maximum allowable fore c.g location is obtained, as follows:

$$\overline{x}_{cg_{TO}} = \left\{ \left[\frac{S_{ht}L_{ht}}{S_w\overline{c}} - \frac{C_{L_R}}{C_{L_{ht}}} \left(\frac{x_g}{\overline{c}} - 0.25 \right) \right] \frac{\eta_h \eta_q C_{L_{ht}}}{C_{L_{max}}} - \frac{C_{m_{0.25}}}{C_{L_{max}}} \right\} \left(\frac{V_R}{V_{S_1}} \right)^2 + \frac{x_g - z_T \sum T/W}{\overline{c}}$$
(3.56)

Computing the aforementioned requirements for varying tail area ratios yields the lines which determines the minimum tail area for which the required c.g range is satisfied. An example of a complete Xplot produced by the module is presented in the following image:



Figure 3.14: Xplot - stability and control boundaries

The maximum aft and fore positions of the c.g are implemented as non-linear constraints in the optimization problem, adding to the previously derived ten constraints pertaining to the pole placement method:

$$c_{\text{ineq}}(11) = \overline{x}_{\text{cg}_{\text{aft,max}}} - \overline{x}_{\text{cg}_{\text{NP,aft}_{\text{S,M}}}} \le 0$$
(3.57)

$$c_{\text{ineq}}(12) = \max(\overline{x}_{\text{cg}_{\text{stall}}}, \overline{x}_{\text{cg}_{\text{TO}}}) - \overline{x}_{\text{cg}_{\text{fwd}, \text{max}}} \le 0$$
(3.58)

The implementation of these conditions as constraints on the design space implies that, if the solution is feasible, controllability and static stability are ensured.

Chapter 4

Verification and Validation

Verification and validation of the module are crucial to ensure the credibility of the results. Regrettably useful validation data regarding a handling qualities optimization methodology comparable to the one presented in this research was not found in open literature, and therefore the module in its entirety cannot be validated. Nevertheless, the separate parts of the module were checked extensively by hand calculations, to ensure the correct working. Furthermore, a qualitative verification was performed on both the aerodynamic derivative estimation procedure and flight dynamics model, to assess whether the results of these submodules behave as expected under known inputs (see Sections 4.0.1 and 4.0.2)

The third step entails validation of the aerodynamic derivatives estimation submodule using reference data (Section 4.0.1). Finally, the flight dynamics submodule is validated utilizing a multi-fidelity flight mechanics analysis and simulation tool developed at TU Delft, namely Phalanx (Section 4.0.2).

4.0.1. Aerodynamic Derivatives Estimation

Qualitative assessment

To verify the results of the aerodynamic derivatives estimation submodule, the general trend of the results with respect to varying initial inputs is qualitatively investigated. This approach allows to identify possible discrepancies in the calculations. In fact, many of the relations between the planform of a horizontal tail and the aerodynamic derivatives are well known. Hence it is immediate to assess whether the calculation procedure is flawed.

The methodology consists in isolating the submodule and providing a set of specific inputs. In this case, the derivatives were calculated for an Airbus A320-200 like aircraft, at cruise speed, altitude, and center of gravity locations. The data was obtained from the top level requirements of the Initiator and results from the class 2 weight estimation module. For the sake of brevity, the varying inputs shown here are the tail aspect ratio AR_{ht} and the tail leading edge sweep $\Lambda_{LE_{ht}}$. This choice might seem inconsistent, in the light of the fact the neither of these variables are included in the final design vector, as discussed in section 3.1.5. However, a justification can be found by considering that the other design variables, i.e tail area S_{ht} , wing position x_{pos_w} and tail taper λ_{ht} , either affect the results in a linear fashion or have almost no impact (such as tail taper), and therefore do not require extensive discussion. This can be clearly seen in the various equation presented in Section 3.2.2. Furthermore the impact of these parameters on stability derivatives $C_{La_{ht}}$ and $C_{Mq_{ht}}$ is evaluated. Concerning the choice of derivatives, the tail lift curve slope and pitching moment due to pitch rate drive the short period damping and natural frequency, and are therefore deemed crucial in the context of a verification process.

As a remark, these are the tail contributions to the respective derivatives, since the wing planform remains unchanged during the entire optimization. Subsequently the control derivatives $C_{L_{\delta_e}}$ and $C_{m_{\delta_e}}$ are presented, considering their importance for the motions of the aircraft. The remainder of the derivatives verification results can be found in Appendix B.



The calculation for $C_{L_{\alpha_{ht}}}$ and $C_{M_{q_{ht}}}$ yielded the following results:

Figure 4.1: Verification $C_{L_{\alpha_{ht}}} - C_{m_{q_{ht}}}$ vs AR_{ht} [-] - $\Lambda_{LE_{ht}}$ [deg]

The trends for both derivatives are as expected. Specifically, considering the variation in $C_{L\alpha_{ht}}$ with AR_{ht} in figure 4.1a, it is possible to see a sharp increase in value, which indicates a correct representation of the behavior of this derivative. This can be clearly understood by inspecting equation 3.14: the lift curve slope of a finite wing should indeed increase with increasing aspect ratio. Furthermore, the influence of $\Lambda_{LE_{ht}}$ is apparent. At low aspect ratios, the effect is almost negligible, while for higher values an increases in ten degrees of sweep can translate into 22% reduction in lift curve slope.

Focusing on the second derivative, the change in pitching moment due to pitch-rate, the results show a certain similitude. Firstly the derivative increases decisively with aspect ratio. Again, this should not surprise: recalling the qualitative description of $C_{m_{q_{ht}}}$ given in section 3.2.2 it is possible to infer that a horizontal tail with higher aspect ratio will entail a higher variation in lift and, consequently in pitching moment, in response to a change in angle of attack due to pitch rate. The sensitivity to leading edge sweep is slightly reduced with respect to the previous case, which can be explained considering that the eventual increase in lift is then multiplied by the horizontal tail arm L_{ht} , which becomes larger with increasing sweep, thus counteracting its adverse effect on the lift curve slope.

The next set of derivatives includes the change in lift due to elevator deflection $C_{L_{\delta_e}}$ and the pitching moment due to elevator deflection $C_{m_{\delta_e}}$. These terms are of paramount importance in the light of the fact that they drive the amplitude of the response following a control input. Moreover, $C_{m_{\delta_e}}$ plays a major role in trimming the aircraft in all flight regimes. Clearly a high elevator effectiveness entails a lower control surface deflection to achieve trim, with the associated reduction in trim drag. The results for varying tail aspect ratio and leading edge sweep are shown in figure 4.2.

Considering figure 4.2a, it is possible to see that $C_{L_{\delta_e}}$ is adversely affected by increasing aspect ratio, while it is insensitive to leading edge sweep. The reason for the first occurrence can be found in the calculation procedure that has been used. The method outlined in reference [44] treats the control surfaces as plain flaps, hence the effect of a deflection is mainly dependent on the elevator-to-stabiliser chord ratio and elevator span. These parameters are fixed within the Initiator and have not been included in the design vector, in order to avoid an increase in dimensionality of the problem. The other main factor which determines the charateristics of the control surface is the three-dimensional flap effectiveness, which is dependent on both the aspect ratio of the horizontal stabiliser and elevator-to-stabiliser chord ratio. Now, recalling that the latter is fixed, the only contribution can come from varying the aspect ratio, which specifically has an adverse effect, reducing the three-dimensional flap effectiveness for a given chord ratio. The insensitivity to sweep is due to the fact that, in the calculation procedure, this factor is never taken into account and therefore does not impact the results.



Figure 4.2: Verification $C_{L_{\delta_e}} - C_{m_{\delta_e}}$ vs AR_{ht} [-] - $\Lambda_{LE_{ht}}$ [deg]

The results for $C_{m_{\delta_e}}$ show an interesting trend, which at first would seem counterintuitive. Even though this derivative depends on $C_{L_{\delta_e}}$, it is seen to increase in magnitude with increasing aspect ratio. Recalling Equation 3.24 (Section 3.2.2), the reason behind this behaviour is readily found. $C_{m_{\delta_e}}$ is in fact dependent on the distance between the tailplane aerodynamic center and the aircraft center of gravity. Hence, to an increase in aspect ratio corresponds an increase in l_{ht} . This situation is best visualized in the notational diagram in Figure 4.3.



Figure 4.3: Increase of tail arm with tailplane aspect ratio

It is true that, by increasing AR_{ht} , the derivative $C_{L_{\delta_e}}$ is reduced, which in turn has a negative impact on $C_{m_{\delta_e}}$. However, the increase in tail arm outweighs this detrimental effect, with the net effect of increasing the magnitude of $C_{m_{\delta_e}}$. Furthermore, this phenomenon is magnified by the fact the trailing edge of the tailplane is kept at the original position, as shown in Figure 4.3. There is a simple consideration behind this choice: a repositioning of the horizontal tail could entail a blanketing of the rudder in high angle of attack conditions. This eventuality is, of course, to be avoided. However, no analysis method to account for this phenomenon is implemented in the present module. Hence, to avoid this occurrence, it has been decided to fix the trailing edge position of the tailplane at the baseline value, which should satisfy the requirements in terms of rudder blanketing. Clearly, a different approach would have had a different impact on $C_{m_{\delta_e}}$: as an example, let us consider a case in which the half root chord position is kept fixed. The location of the aerodynamic center would vary to a lesser extent, with the possible net effect of a reduction in the magnitude of $C_{m_{\delta_e}}$. Hence the behavior is seen in Figure 4.2 is pertinent only to this particular case, for which the trend is captured correctly. Lastly, the magnitude of the derivative is seen to increment with increasing $\Lambda_{LE_{hr}}$. This again is due to an increase in tail arm, and is to be expected.

Finally, from the previous analysis it can be inferred that the stability and control derivatives follow the expected trend for the variation in AR_{ht} and $\Lambda_{LE_{ht}}$. It can, therefore, be concluded that the derivatives estimation procedure is verified, at least in a qualitative fashion.

Validation

Once the procedure has been verified qualitatively, it is now appropriate to assess the validity of the numerical results. To this end, the stability and control derivatives are computed and compared against results found in open literature for two different aircrafts, namely Boeing B747-100 and Learjet 24 [44]. The conditions of the validation are presented in table 4.1:

Parameter	Unit	Boeing 747	Learjet 24
Altitude	m	12192	12192
Mach number	-	0.9	0.7
Reynolds number	-	$5.48 \cdot 10^7$	$1.07 \cdot 10^7$
Center of gravity (x_{cg}/\overline{c})	-	0.25	0.32
Mass	kg	288773	5897
$C_{\rm L}$	-	0.52	0.41
CD	-	0.045	0.0335

Table 4.1: Validation set-up	- Boeing 747-100 and Learjet 2	24
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The data is referred to cruise, for both aircrafts. The reference values for the geometry of the aircraftshave been obtained from [44] and implemented as external inputs to the submodule, in order to ensure consistency. This was deemed necessary in the light of the uncertainty regarding the methods used within the Initiator in relation with the actual designs. To be more precise, if the module was to be validated within the Initiator, the input aircraft geometry might not have been completely consistent with the actual geometry, thus preventing a meaningful comparison and hindering the validation. The results for the main derivatives are shown in figures 4.4 and 4.5:



Figure 4.4: Aerodynamic derivatives data - Boeing B747-100 [44]

Figure 4.5: Aerodynamic derivatives data - Learjet 24 [44]

The results are summarized in tables 4.2 and 4.3. The relative errors were calculated using the classical formula $e = (|x_{ref} - x_{calc}|/x_{ref}) \cdot 100$. Firstly, a general observation can be made for both the validation cases: it can be clearly seen how most of the derivatives are overpredicted, even though by different margins. This occurrence is difficult to justify: at first, the presence of a systematic error in the calculations was hypothesized. This eventuality, however, has been ruled out by accurate verification, which did not highlight any inconsistencies from an implementation point of view. A general reason might simply be found in the very nature of the methodology, which should supposedly apply to a variety of configurations and flight regimes [40]. Having such a wide scope is indeed very useful, at the expense however of a loss in accuracy possibly due to oversimplifying assumptions. However, it must be noted that, for the same derivatives,

Parameter	Unit	Reference	Calculated	Error %
$C_{L_{lpha}}$	[rad ⁻¹]	5.5	5.78	+5%
$C_{m_{\alpha}}$	[rad ⁻¹]	-1.6	-1.86	+16%
C_{L_q}	[rad ⁻¹]	7.8	8.65	+10%
C_{m_q}	[rad ⁻¹]	-25.55	-26.3	+2.9%
$C_{L_{\dot{lpha}}}$	[rad ⁻¹]	8	8.6	+7.5%
$C_{m_{\dot{lpha}}}$	[rad ⁻¹]	-9	-9.35	+3.9%
$C_{L_{\delta_e}}$	[rad ⁻¹]	0.3	0.4	+33%
$C_{m_{\delta_e}}$	[rad ⁻¹]	-1.2	-1.4	+16%

Table 4.2: Validation results - Boeing 747-100

Parameter	Unit	Reference	Calculated	Error %
$C_{L_{lpha}}$	[rad ⁻¹]	5.84	6.1	+4.4%
$C_{m_{lpha}}$	[rad ⁻¹]	-0.64	-0.73	+14%
C_{L_q}	[rad ⁻¹]	4.7	5.8	+23%
C_{m_q}	[rad ⁻¹]	-15.5	-16.4	+5.8%
$C_{L_{\dot{lpha}}}$	[rad ⁻¹]	2.2	2.4	+9%
$C_{m_{\dot{lpha}}}$	[rad ⁻¹]	-6.7	-7.1	+5.9%
$C_{L_{\delta_e}}$	[rad ⁻¹]	0.46	0.56	+21%
$C_{m_{\delta_e}}$	[rad ⁻¹]	-1.24	-1.4	+12%

Table 4.3: Validation results - Learjet 24

for both aircraft the relative error is approximately within the same range This gives a good indication of the consistency of the methodology, which would indeed seem to apply to different configurations with similar results. For lack of a valid general explanation regarding the trend of the results, a more in-depth discussion on the possible sources of discrepancies in the calculation of the various derivatives is deemed appropriate.

The lift curve slope is found to be calculated rather accurately in both validation cases. Considering the preliminary nature of the method, an error margin of 5% and 4.4% is deemed to be quite satisfactory. On the other hand, by inspecting the values for C_{m_a} , a larger discrepancy is found. Referring to equation 3.17 in section 3.2.2, the causes for error can be the following: firstly, the relative error in $C_{L_{\alpha}}$ trickles down to $C_{m_{\alpha}}$, being the latter dependent on the former. Secondly, only the center of gravity location is given in reference [44], while no data regarding the position of the aerodynamic center is provided. Hence, the computed static margin might differ significantly from the reference value, thus introducing an additional source of error which is not quantifiable. The same consideration applies to the Learjet 24, even though in this latter case $C_{m_{\alpha}}$ is underestimated. Considering that for both cases $C_{L_{\alpha}}$ is overpredicted, it can be inferred that the position of the Learjet's aerodynamic center is found in a more forward position with respect to the actual aircraft, thus producing an overall lower $C_{m_{\alpha}}$. For the B747-100 on the other hand the opposite applies, in the light of the fact that $C_{m_{\alpha}}$ is overpredicted, suggesting the presence of a slightly greater static margin with respect to the real aircraft.

Considering the derivatives with respect to pitch rate, it must be noted that C_{m_q} shows quite accurate results for both validation cases, with a maximum error margin of about 6% for the Learjet 24. C_{L_q} on the

other hand is overpredicted in both cases, with non-negligible error margins. The errors are likely to stem from the various correction factors that are introduced for the wing contribution, plus the inherent error caused by the dependency of the pitch rate derivatives on $C_{L_{\alpha}}$, for which the same considerations done for $C_{m_{\alpha}}$ apply.

The results for the control derivatives are overpredicted in both cases, with a maximum error of 33% for $C_{L_{\delta_e}}$ of the B747. Of all the derivatives, this set represents the one with the highest dependency on empirical correction factors. Therefore it should not surprise that the results show some of the highest error percentages. Furthermore the calculation of $C_{L_{\delta_e}}$, and consequently $C_{m_{\delta_e}}$ relies on accurate knowledge of the geometry of the horizontal stabilizer, as well as the elevator itself, which in this case is limited to what could be inferred from reference [44].

In order to assess whether the aerodynamic derivatives estimation methodology can be considered validated a few more words must be spent on analyzing the overall results, putting them into context concerning the scope of the author's work and the level of detail demanded at the conceptual design level. Firstly, it must be noted that $C_{L_{\alpha}}$ and C_{m_q} show quite an excellent agreement with the reference data. By remembering that the scope of the handling qualities optimization module is focused on the aircraft short period motion, the importance of these results is apparent: short period damping is driven for a good part by C_{m_q} , while short period frequency is highly dependent on the angle of attack derivatives. Surely the agreement between calculated and reference values for $C_{L_{\alpha}}$ and C_{m_q} enhances the confidence in the results produced by the module.

Other derivatives, on the other hand, show quite some discrepancy with respect to reference values. Even though no precise limit is given for validation in terms of percentage error, clearly some results would have to be considered unreliable. Then again, a consideration must be made on the context in which the module is implemented, which is the conceptual design stage of an aircraft. Having more accurate values would surely be beneficial, however, at this stage, such error margins could still be considered acceptable, with the thought of possibly improving the results and iterate the optimization procedure at later stages of the design process, when more accurate analysis tools are available.

4.0.2. Flight Dynamics Model

In this section, the flight dynamics model described in section 3.2.3 is analyzed and validated. At first, a qualitative investigation is carried out, in parallel with the approach followed in section 4.0.1. Finally, the results from the flight dynamics model with respect to three different aircrafts are validated against the results produced by Phalanx, an in-house multi-fidelity flight mechanics analysis tool developed at the Department of Flight Performance & Propulsion [52].

Qualitative assessment

The purpose of the flight dynamics model is to assess the modal response parameters of the chosen configuration and provide the inputs to the regional pole placement module, as discussed in section 3.2.1. The short period approximation has been used, which entails a simplification of the equations of motion to a 2-by-2 matrix, the states being the angle of attack α and the pitch rate $q\bar{c}/V$. The input to the system is the elevator deflection δ_e .

The assessment has been done with respect to a variation in all the derivatives, however in this section, only the results for variations in $C_{L_{\alpha}}$ and C_{m_q} are presented. Furthermore, a secondary analysis was performed to investigate the response of the model when subject to variation in maximum takeoff mass and inertia about the y-axis of the aircraft, which indeed affect handling qualities considerably. As in section 4.0.1, the analysis has been performed for an Airbus A320-200 like aircraft at cruise speed, altitude, and center of gravity locations. Furthermore, maximum takeoff weight and relative inertia are considered. Firstly, the response of the model in terms of short period damping ratio and natural frequency with respect to variations in $C_{L_{\alpha}}$ and C_{m_q} is presented in figures 4.6a and 4.6b.

In figure 4.6a it is possible to see that an increase in the negative value of C_{m_q} entails a higher damping ratio, as it was to be expected. Furthermore there is also a contribution of $C_{L_{\alpha}}$ to the damping, even though of a much smaller magnitude. Clearly this trend is consistent and captures correctly the impact of these


Figure 4.6: Verification of ζ_{sp} [-] and $\omega_{n_{sp}}$ [rad/s] against $C_{L_{\alpha}}$ [rad⁻¹] – C_{m_q} [rad⁻¹]



Figure 4.7: Verification of ζ_{sp} and $\omega_{n_{sp}}$ against I_{yy}/I_{yy_0} – MTOM/MTOM₀

derivatives on the short period damping ratio. A similar outcome is found for the resultant short period natural frequency in figure 4.6b, however it is possible to note that $C_{L_{\alpha}}$ assumes a more dominant role in determining the natural frequency with respect to C_{m_q} . From a physical perspective the results in figures 4.6a and 4.6b are readily explained. Let us consider a disurbance in angle of attack: the term C_{m_q} actually act as damper to the system, generating a moment which counteracts any disturbance which induces a varying ditribution of angle of attack in the longitudinal direction. A smaller contribution to this restoring pitching moment is given by $C_{m_{\alpha}}$, which in turn is dependent on $C_{L_{\alpha}}$ as shown in equation 3.17. Hence an increase in either of these values will entail a higher damping of the motion, which translates into the aircraft returning to a condition of equilibrium faster. Regarding the natural frequency, a similar logic can be applied. Considering again a disturbance in angle of attack, the airplane responds with an increase in lift through the term $C_{L_{\alpha}}$. A higher lift curve slope entails a more abrupt response to a given disturbance. Similarly, a lower lift curve slope will translate into a more sluggish response. Furthermore there is also a contribution of C_{m_q} , which augments this tendency.

In conclusion it can be said that the impact of varying derivatives on the modal response characteristics of the short period is captured correctly. This conclusion is corroborated by the results for the remaining derivatives, which are found in Appendix C. The next step of the qualitative assessment involves the investigation of the behaviour of the short period damping ratio and frequency with respect to varying aircraft mass and inertia. The results can be seen in figures 4.7a and 4.7b.

The analysis of these results requires a more accurate discussion, since weight and inertia are quantities which are deeply connected, and should, therefore, be treated in a coherent way. Focusing on figure 4.7a, it is possible to see that the short period damping ratio degrades with increasing MTOM and inertia. From a qualitative point of view, this phenomenon understandable in a quite intuitive way: such a configuration, with increased MTOM and I_{yy} and unvaried aerodynamic characteristics, will be perceived by a pilot as more sluggish, precisely due to the reduction in both damping ratio and short period natural frequency. To have a better overview of the caused behind this degradation, let us consider a perturbation in the angle of attack applied to an aircraft flying at constant forward speed. An increase in angle of attack would entail an increase in the vertical component of the velocity, or a vertical acceleration. Considering that the aerodynamic characteristics, i.e C_{L_a} , C_{m_q} , remain unvaried, an increase in mass will make this vertical motion more difficult to damp. Similarly, the natural frequency is degraded with increasing MTOM and I_{yy} . This again is explained using the same logic. Thus, while an increase in mass and inertia will render the aircraft more sluggish, for a lighter design, with reduced inertia, the opposite applies.

What is indeed interesting to note that for a fixed value of MTOM, an increase in I_{yy} does not entail a significant degradation of damping ratio. An opposite trend is found in figure 4.7b: for a given inertia an increase in mass does not imply an appreciable degradation in short period natural frequency.

A qualitative justification for this behavior can be found by considering a condition in which, for a given MTOM, I_{yy} is increased. This is equal to shifting mass to both ends of the aircraft, which clearly entails that the oscillation of the short period has a lower frequency, while the damping is affected to a lesser extent, thus explaining the trend seen in Figure 4.7b. On the other hand increasing MTOM with constant I_{yy} implies that the vertical motion during the short period is amplified, thus reducing the short period damping. These results are in agreement with open literature [10, 38], and are therefore considered reliable.

The analysis highlighted a correct qualitative behavior of the model with respect to known sets of inputs, which suggest that the methodology has been implemented correctly. It is now proper to introduce the validation of the results, which is discussed in section 4.0.2.

Validation

As stated previously, the validation of the flight dynamics model results is performed using Phalanx, a multi-fidelity flight mechanics analysis tool developed at the Department of Flight Performance & Propulsion [52]. This tool creates a non-linear model starting from a database of aerodynamic, structural and engine data. Once the model is generated, it is possible to trim it for the chosen flight condition, which results in a trim vector consisting of the states, i.e angle of attack, speed, angular rates and pitch angle, and controls required to trim, which in this case are control surfaces deflection and throttle setting. Once a trim condition is achieved, it is then possible to linearize the model using the *linmod* command. The procedure entails the perturbation of both states and controls around the condition of equilibrium, after which a state space model is obtained. The full model, consisting of the equations of motion plus the equations for actuators, if present, is then decoupled, in order to obtain the longitudinal and lateral-directional set of equations of motion. Clearly, considering the scope of this thesis, the interest is focused on the longitudinal system of equations.

Finally, a simple analysis of the eigenvalues of the obtained matrices allows for the determination of the modal response parameters of the model, such as damping ratio and natural frequency for both phugoid and short period, which can then be compared against the results obtained from the model outlined in section 3.2.3.

A proper comparison of the results requires the investigation of different aircraft sizes and configurations, to ensure consistency. Hence, a decision was made to consider three different designs, specifically a regional airliner, a mid-range single-aisle airliner and a long-range double aisle airliner. The chosen models are the Fokker 100, the Airbus A320-200 and the Boeing B747-100. The flight condition for all three configurations is cruise, at maximum takeoff weight. The reasons behind the choice of this flight condition will be discussed in depth in chapter 5.1. Hence three different input files for Phalanx containing the aerodynamic derivatives, structural data, and engine data were created. The reference data used for the validation procedure is presented in table 4.4.

As an example, the top level of the simulation model implemented in Simulink is presented in figure 4.8.

Parameter	Unit	A320-200	B747-100	F100
I _{yy}	$kg\!\cdot\!m^2$	$3.31 \cdot 10^6$	$8.52 \cdot 10^7$	$1.73 \cdot 10^{6}$
MTOM	kg	$8.02 \cdot 10^4$	$3.64 \cdot 10^5$	$4.40\cdot10^4$
Altitude	m	11280	10668	10668
Mach	-	0.78	0.84	0.74

Table 4.4: Validation set up data A320-200 - B747-100 - F100



Figure 4.8: Phalanx A320-200 simulation model

It can be seen that the inputs to the model are four, namely the elevator, aileron and rudder deflections plus the throttle setting. The outputs, on the other hand, are the Euler angles, the body rates, and finally the aircraft speed and position.

	A320-200		B747-100		F-100				
Param.	Initiator	Phalanx	Err.	Initiator	Phalanx	Err.	Initiator	Phalanx	Err.
ζ _{sp} [–]	0.41	0.4	1.7%	0.49	0.485	1%	0.66	0.64	3%
$\omega_{n_{sp}} [rad/s]$	1.23	1.22	0.8%	1.37	1.34	2.2%	3.87	3.75	3%
CAP [1/gs ²]	0.15	0.16	4.3%	0.1	0.09	3%	0.163	0.158	3%

A comparison of the results between the Initiator and Phalanx is shown in table 4.5.

Table 4.5: Validation results

As done in section 4.0.1, the error was calculated as $e = (|x_{ref} - x_{calc}|/x_{ref}) \cdot 100$. The results indicate that the flight dynamics model indeed captures the modal response characteristics of the system very accurately. It can be seen in table 4.5 that the maximum error is 4.3 % for the CAP of the A320. All the other values are consistently below this threshold. An explanation for this small discrepancy can be found considering that the flight dynamics model shown in equation 3.27 is an approximation, which entails some simplifying assumptions and therefore naturally introduces a small source of error [36, 38].

Chapter 5

Results and Discussions

In this chapter the handling qualities optimization module will be evaluated. To do that, in Section 5.1 a test case is set up with the aim of investigating all the capabilities and limitations of the tool, and an optimization is run for all three methods implemented. In Section 3.2.4 it was stated that the Stability Augmentation System design method makes use of the ideal actuators assumption, hence a preliminary study of the effect of the presence of said actuators on the optimized configurations is carried out in Section 5.2.1, in order to assess if any significant error is introduced by this simplification. Lastly, in Section 5.2, the Initiator is run for different short period damping ratios and Control Anticipation Parameters as to investigate the impact of handling qualities optimization on the converged designs, in terms of L/D_{max} and MTOM. The aim is to identify a general trend, if any is present, whence to extrapolate general conclusions on the design procedure.

5.1. Test Case

For the test case, a baseline aircraft is chosen: the Airbus A320-200. The optimization procedure is run one time for each of the user options which have been specified in section 3.2.1. As a reminder, the three options are:

- · Unaugmented design optimization
- · Augmented design optimization method 1
- · Augmented design optimization method 2

One of the main issues in the setup of the test case is the choice of the flight condition for which the configuration is optimized. A decision was made to consider only cruise, at maximum takeoff weight. It must be noted that the same applied to the validation case, as it was discussed in Chapter 4. This flight condition might not seem the most suitable. However, there is a rationale behind this choice. In fact, within the Initiator, no module for high-lift devices sizing is present. Their type, characteristics, and performance are hard coded and provided as input to the various modules. Given the lack of an accurate design procedure, and consequently the poor accuracy of the data regarding high-lift devices, it has been decided to perform all the analyses at cruise, for which their presence is not taken into account. It is no mystery that cruise itself is not the most interesting flight phase to perform handling qualities optimization, while landing would have been a more appropriate choice, insofar as it is the most demanding flight phase in terms of modal response characteristics [8], as shown in Table 1.4.

The choice of cruise as reference flight condition, therefore, entails the following drawback: an optimized design, both with and without stability augmentation system, would possess handling qualities which are likely to be inadequate for other flight phases. An attempt was made to overcome this issue precisely by considering a maximum takeoff mass configuration. Recalling the results shown in Section 4.0.1, an increase in MTOM brings about a degradation in short period damping ratio and undamped natural frequency. Clearly, an aircraft flying at cruise altitude and speed is not in MTOM configuration, due to the

fuel burn required to takeoff and climb to the required altitude. Hence, if a true cruise condition were to be considered, the weight and inertia would have to be lowered, thus implying an improvement in baseline handling qualities. Therefore, by considering maximum takeoff mass, the analysis is purposely conservative, which should increase the safety margin between a design optimized for cruise and one optimized for landing.

5.1.1. Optimization Setup

The general parameters required for the optimization are presented in this section. The setup is common to all three optimization cases. Firstly, following the previous discussion, the reference data for the flight condition is presented in Table 5.1:

Parameter	Unit	Value
Altitude	m	11280
Mach	-	0.78
MTOM	kg	$8.02\cdot 10^4$
Iyy	$kg\!\cdot\!m^2$	$3.31 \cdot 10^6$

Once the reference data has been presented, it is now appropriate to discuss the parameters which define the optimization procedure, i.e the inputs to the solver. This terms are of paramount importance, and have been selected by the rationale discussed in Section 3.1. As shown in Section 3.1.1 the main optimizer uses the *fgoalattain* solver, which requires extra inputs compared to *fmincon*. Complete data for both preliminary and main optimization are presented in table 5.2:

	Solver	Algorithm	DiffMin	DiffMax	TolX	TolFun	TolCon	Goal	Weight
Prelim.	fmincon	sqp	0.01	0.15	10^{-4}	10^{-4}	10^{-5}	-	-
Main	fgoalattain	sqp	0.01	0.15	10^{-4}	10^{-4}	10^{-5}	0.9	0.9

Table 5.2: Optimization input data - fmincon and fgoalattain

DiffMin and DiffMax determine the minimum and maximum change in design variables for finite-difference gradients. The values have been selected based on the preliminary sensitivity study discussed in Section 3.1.3. In this respect, the range of allowable differences is reduced in comparison with Figures 3.6 and 3.7. The aim of this modification is to ensure a faster convergence of the optimization procedure, by preventing unreasonably small steps, while still complying with the condition of constant objective function gradient (see Section 3.1.3). The choice of the *sqp* algorithm was dictated by the fact that it satisfies the bounds at every iteration, which for this particular optimization is of paramount importance. This ensures that the handling qualities constraints are not violated at any time. Furthermore it is capable of recovering from Inf or NaN results, which adds robustness to the framework [42, 53]. The tolerances on design vector, constraints, and objective function have been determined by a tradeoff between accuracy and computational speed. Too small of a tolerance could entail a higher run time, while a high tolerance, especially on the objective function, could give results which are sub-optimal. Furthermore, TolCon directly impacts the constraints concerned with the handling qualities characteristics. It is, therefore, crucial to keep a reasonable low tolerance, as to ensure the accuracy of the optimization procedure. As it is possible to see, in general, the settings tend towards the conservative side in terms of tolerances: this is because the accuracy of the optimization was deemed to have priority, even at the cost of a slight increase in computational time.

5.1.2. Baseline Aircraft

As stated previously, the baseline aircraft is an Airbus A320-200. Indeed throughout the whole project (see Chapter 4), this has been the preferred test case. There are two main reasons behind this choice: firstly the A320-200 is widely employed in commercial aviation, and its diffusion is predicted to increase in the next years [58]. It is therefore deemed an ideal candidate since the current research is focused on a new design procedure which could be applied to a generic modern airliner. Secondly, within the Initiator, this aircraft is found to be sized quite accurately, most notably in terms of MTOM and estimated horizontal tailplane area. Hence, it provides a solid baseline on which to apply the proposed design methodology. A top view of the baseline planform created in the Initiator is presented in Figure 5.1



Figure 5.1: Baseline Initiator A320-200 planform - top view and side view

The main geometrical parameters of the baseline configuration are presented in the Table 5.3. Since the optimization is concerned with the sizing of the horizontal tailplane and wing positioning only the relevant terms are present, namely the tail area, leading edge sweep, taper ratio, aspect ratio and wing longitudinal position with respect to the nose of the aircraft.

S _{ht,0}	$\Lambda_{LE_{ht,0}}$	$\lambda_{\rm ht,0}$	$\mathbf{x}_{\mathbf{pos}_{w,0}}$	AR _{ht,0}
26.3 [m ²]	32 [°]	0.35 [-]	11.47 [m]	5 [-]

Table 5.3: Main geometrical parameters baseline Initiator A320-200

As it can be seen, the horizontal tailplane is slightly more swept than the wing. This is done to comply with the requirements discussed in Section 2.1. The tail area ratio, and consequently the tail area itself, is determined through Torenbeek's Xplot methodology in the Initiator Horizontal Stability Estimation module. The computed tail area ratio is 0.2, which was determined by satisfying the same controllability and stability requirements shown in Section 3.2.6.

As it was discussed in Section 3.1.4, only two of these geometrical parameters are used as design variables during the optimization, namely tail area S_{ht} and wing longitudinal position x_{pos_w} , while the remainder are kept fixed at their baseline value. In Section 3.1.2 the objective functions to be minimized in the optimization have been presented. These are the tailplane weight, zero-lift drag, and induced drag. The values for these quantities for the baseline configuration are presented in Table 5.4:

W _{ht,0}	$C_{D_{0,0}}$	$C_{D_{i,0}}$
7014 [N]	0.0016 [-]	$2.1 \cdot 10^{-4}$ [-]

Table 5.4: Baseline taiplane weight, zero-lift drag and induced drag - Initiator A320-200

It must be noted that the values for zero-lift drag and induced drag are calculated for the flight condition presented in Table 5.1. Now that all the relevant parameters for the baseline configuration have been described, the three optimization cases are discussed and the results presented.

5.1.3. Unaugmented Optimization

The first option of the module available to the user allows for handling qualities optimization purely through resizing the horizontal tailplane and repositioning the wing. For all the considerations made in Section 3.1.4, the only design variables are the tail area S_{ht} and the wing longitudinal position x_{pos_w} . Leading edge sweep, tail taper, and tail aspect ratio are fixed to the baseline values. In the following tables the initial design vector is presented:

Variable	Value	Bounds
$S_{ht}/S_{ht,0}$	1	$0.1 < S_{ht}/S_{ht,0} < +\infty$
$x_{pos_w}/x_{pos_{w,0}}$	1	$0.95 < x_{pos_w} / x_{pos_{w,0}} < 1.05$

Table 5.5: Design variables and bounds - unaugmented optimization

The design variables are normalized with respect to the baseline values, presented in Table 5.3. The bounds on the tail area are quite relaxed: a hard upper bound is not needed since the optimizer will inherently try to minimize it due to its impact on the objective functions. The lower bound is just needed to avoid the eventuality that the tail area goes zero, thus generating NaN within the code, for example in the computation of tailplane induced drag. It is indeed no more than a safeguard since the actual minimum value is limited by the controllability and stability requirements described in section 3.2.6. The bounds on the wing position, on the other hand, required a more in-depth reasoning. In the Initiator no dedicated module exists for the landing gear sizing, which is highly dependent on the wing location. Hence, if no hard bounds are put on this design variable, it could results in unfeasible designs within the convergence of the Initiator. It was therefore tentatively decided to limit the excursion of the wing to 5 % of the original distance from the nose, which results in an absolute maximum shift of 0.5 meters, both fore and aft. It is entirely possible that this value is too conservative, thus unnecessarily limiting the design space. However, it was deemed preferable with respect to the eventuality of an unfeasible wing and landing gear position.

Once the design variables are set, the last required inputs are the ones regarding the regional pole placement method, based on the modified Routh-Hurwitz criterion, which define the desired modal response characteristics and CAP. As it will be discussed in-depth in Section 5.2, there is a bound to the actual achievable level of handling qualities with unaugmented optimization without incurring in a heavy degradation of aircraft performance (see Section 5.2.2), especially considering the limited amount of design variables at disposal.

Such considerations, therefore, need to be implemented by the user through a careful evaluation of the maximum desired handling qualities. Clearly, this does not apply to augmented designs, for which the desired damping and CAP are reached mainly through the design of a suitable stability augmentation system. An in-depth discussion about this issues is found in Section 5.2.2.

Following this logic, for the unaugmented case, the inputs have been set to a value which is deemed reasonable: enough to assess the capabilities of the module, but not so high that the design becomes unreasonable. The chosen values for the regional pole placement submodule are presented in Table 5.6.

Variable	Value
CAP _{min}	0.3 [-]
$\zeta_{ m sp_{min}}$	0.55 [-]

Table 5.6: Regional pole placement inputs - CAP and ζ_{sp}

The modified Routh-Hurwitz criterion will therefore impose constraints on the design space such that

the roots of the characteristics polynomial shown in Equation 3.41 are within a cone with semi-angle $\alpha^* = \arccos(0.55) = 56^\circ$, and the real part of the eigenvalues is such that the corresponding $\omega_{n_{sp}}$ satisfies a Control Anticipation Parameter of 0.3, as described in Section 3.2.5. Hence, in this case a minimum increase of 100 % in CAP and of 34 % in short period damping ratio is required.

The optimization procedure for unaugmented design ran in a total time of 12 seconds on a laptop with the following specifications: Intel i7 2.4 GHz processor with 8 Gb RAM 1200 Mhz.

The optimized design variables are shown in Table 5.7:

Variable	Initial value	Optimized value	Dimensional value
$S_{ht}/S_{ht,0}$	1	1.43	37.65 [m ²]
$x_{pos_w}/x_{pos_{w,0}}$	1	0.95	10.82 [m]



As it can be seen the optimized tail area is 43 % larger than the baseline, while the wing has been shifted forward (towards the nose) by 5 % of the original position, which puts this variable on the lower bound.

To have a better overview of the geometrical changes a comparison between the optimized and the baseline configurations is presented in figure 5.2.



Figure 5.2: Optimized and baseline configuration A320-200

Two features are immediately recognizable: the fore shift (towards the aircraft nose) of the wing and the substantial increase in tail area. Furthermore, it must be noted that the trailing edge root position of the tailplane is kept constant. Hence an increase in root chord caused by a higher tail area translates in a forward shift of the root apex. This clearly reduces the tail arm, which partly explains the notable shift of the wing position that aims at counterbalancing this effect. In addition the increase in tail arm has a positive impact on derivatives such as C_{m_q} and $C_{m_{\delta_e}}$, as it can be seen in Section 3.2.2. Recalling the qualitative analysis in Section 4.0.1, an increase in the magnitude of the former brings about an increase in both short period damping ratio and natural frequency, while an increase in the latter entails enhanced controllability of the aircraft, due to a surplus of control power. A similar analysis can be done for C_{m_a} : a larger tail entails an aft shift of the aircraft aerodynamic center, which in turn determines a larger static margin, therefore increasing the magnitude of this derivative (Equation 3.17).

As shown in 3.4, the required tail lift coefficient is driven mainly by the wing-fuselage pitching moment

as well as the distance between the center of gravity location and the wing-fuselage aerodynamic center, namely the wing-fuselage static margin. Hence, moving the wing forward has the purpose of counterbalancing the nose down pitching moment $C_{m_{ac,wb}}$, precisely by increasing this arm (see Equation 3.4), and therefore producing a moment which opposes the wing-fuselage contribution and reducing the required tailplane lift coefficient.

At any rate, the increase in tail area and horizontal tail arm, caused by the forward wing shift, are mainly attributed to the handling qualities requirements, which translates into a demand for improved aerodynamic stability and control derivatives. With such an increase in these variables, controllability and stability requirements are not a sizing condition, and therefore do not pose active constraints on the design space. The active constraints for the unaugmented optimization are indeed the ones related to the handling qualities.

Objective function	Initial value	Optimized value	Dimensional value
$J_1 = W_{ht} / W_{ht_0}$	1	1.43	10035 [N]
$J_2 = C_{D_0} / C_{D_{0,0}}$	1	1.5	0.0024 [-]
$J_3 = C_{D_i} / C_{D_{i,0}}$	1	0.3	$6.3 \cdot 10^{-5}$ [-]

Shifting the focus on the objective functions, the optimized values are shown in Table 5.8:

Table 5.8: Optimized objective functions - unaugmented design

The optimized design shows an increase of 43 % of horizontal tailplane weight, along with an increase of 50 % of zero-lift drag. This should not surprise: as shown in section 3.1.2 the equations for these objective functions are chiefly dependent on tailplane area. With respect to the goal defined in Table 5.2, which was 90 % of the initial objective function values, or in other words a desired reduction of 10 %, the final results are under-achieved by 66 % for zero-lift drag and 58 % for the weight. On the other hand induced drag has decreased dramatically, specifically by 70 %, thus over-achieving the goal by 66 %. This reduction in induced drag has been achieved by the increase in tail area, along with a decrease in the wing-fuselage static margin due to the increase in horizontal tail arm. All these factors contribute to lowering the tailplane lift coefficient which in turn entails a reduction in induced drag.

The results in terms of modal response and CAP are the following:

Parameter	Initial value	Optimized value	Diff. %
$\zeta_{ m sp}$	0.41 [-]	0.55 [-]	+34 %
$\omega_{ m n_{sp}}$	1.23 [rad/s]	1.72 [rad/s]	+40%
CAP	$0.15 [1/gs^2]$	$0.3 \ [1/gs^2]$	+100 %

Table 5.9: Optimized modal response parameters and CAP - unaugmented design

Both short period damping and natural frequency have increased substantially. Furthermore, a doubling of the Control Anticipation Parameter has occurred. It must be remarked that the optimization achieved precisely the desired damping ratio and CAP, thus satisfying the constraints. Since for the unaugmented case every increase in demanded ζ_{sp} and CAP directly translates into a larger tail area, it is only natural that no attempt by the optimizer has been made to over-achieve the minimum requirements since it would lead to a design which is less optimal in terms of objective functions values. Generalizing these results, it can be inferred that, for the unaugmented case, the pole placement methods loses its regional characteristics. In other words, the allowable region of the complex plane, as shown in figure 2.5, reduces to the poles which generate the selected damping ratio and natural frequency. This, of course, entails that a very high precision concerning pole location is achieved, while the downside is that the optimizer can search within a smaller design space. In particular, the possible configurations are restricted to the ones that just satisfy the constraints. Considering also the limited number of design variables, it is immediate to

see that the achievable benefits in terms of tailplane weight and drag are limited. These results confirm the considerations made in section 3.1.3 regarding the composition of the design vector. On the other hand, as it will be discussed in Section 5.2, it is entirely possible to achieve a reduction in the objective functions for short period damping ratio and CAP closer to the baseline values.

In figures 5.3 and 5.4 a comparison of the time response for non-zero initial conditions and step input for both baseline and optimized configuration is presented:



Figure 5.3: Comparison of response for initial condition $\Delta \alpha = 0.1$ [rad]



Figure 5.4: Comparison of response for step input $\Delta \delta_e = -0.05$ [rad]

Inspecting Figures 5.3a and 5.3b, it is possible to analyse the response of the aircraft to an initial perturbation $\Delta \alpha = 0.1$ [rad]. The higher damping of the optimized configuration is evident for both the angle of attack and the pitch rate. The response decays faster, and the maximum amplitude of the response for the angle of attack is substantially lower. Regarding the pitch rate response, the higher damping is evident as well. Furthermore, the higher frequency of the oscillations is immediately recognizable, as it was expected considering that the configuration has been optimized for higher CAP. In a nutshell, the time response following an initial perturbation in the states confirms the improved stability characteristics of the optimized aircraft. The response of the model to a step input of amplitude $\Delta \delta_e = -0.05$ [rad] shows a similar trend. The higher peak amplitude is explained by the increase in tail area, which in turn entails an increase in elevator effectiveness. Hence, following an input, the pitching moment around the center of gravity generated by the elevator is much higher, thus determining the peak that can be seen in Figures 5.4a and 5.4b.

To have a deeper understanding of the response following a step input, the main time response parameters for the angle of attack and pitch rate are presented in Tables 5.10 and 5.11

Configuration	Settling Time [s]	Peak Time [s]	Rise Time [s]
Baseline	6.5	2.73	1.19
Optimized	3.2	2.16	1
Diff %	-50 %	-20.8 %	-16 %

Table 5.10: Comparison of step input time response parameters - angle of attack (unaugmented)

Configuration	Settling Time [s]	Peak Time [s]	Rise Time [s]
Baseline	7.9	1.36	0.27
Optimized	4	0.81	0.12
Diff %	-49 %	-40 %	-55 %

Table 5.11: Comparison of step input time response parameters - pitch rate (unaugmented)

The improvement in time response for both states is evident. Most notably, the settling time has halved. This metric is often used to characterize the time response of a system [38, 50]. It must be noted that in this case, the settling time refers to the condition in which the response reaches 3 % of the steady state value. Furthermore, even though the aircraft was not optimized for time response, the final configuration comes close to achieving level 1 handling qualities with respect to this parameters, insofar as the military specifications set the limit for level 1 as a settling time of 3 seconds to reach 3 % of the steady state value [8]. It can, therefore, be concluded that by optimizing for short period damping ratio and Control Anticipation Parameter there is a beneficial side effect in terms of time response as well, which was to be expected, even though not quantifiable a priori.



Figure 5.5: Control Anticipation Parameter comparison - unaugmented design

In figure 5.5 the Control Anticipation Parameter is illustrated, in order to show clearly the improvement from baseline to optimized configuration. It can be seen that the baseline configuration already satisfied level 1 handling qualities for CAP, as it was to be expected from a modern airliner. Nevertheless, the improvement for the optimized configuration is remarkable, having obtained a 100 % increase in CAP with a 34 % increase in short period damping ratio. Clearly, these results came at a price, specifically in terms of tailplane weight, which increased by 43 %, and zero-lift drag which was found to be 50 % higher than the baseline value.

5.1.4. Augmented Optimization - Method 1

The second option for the module entails the concurrent design of a pitch damper and α -feedback to augment the stability of the baseline aircraft. This approach includes the feedback gains in the design vector, as described in section 3.2.4. The geometrical parameters that define the horizontal tailplane remain unvaried with respect to the previous case. Hence, the design vector now consists of four elements as it can be seen in Table 5.12:

Variable	Value	Bounds
$S_{ht}/S_{ht,0}$	1	$0.1 < S_{ht}/S_{ht,0} < +\infty$
$x_{pos_w}/x_{pos_{w,0}}$	1	$0.95 < x_{pos_w} / x_{pos_{w,0}} < 1.05$
k_{α} [rad/rad]	1	$-1 < k_{\alpha} < 0$
k_q [rad/rad/s]	1	$-50 < k_q < 0$

Table 5.12: Design variables and bounds - augmented optimization method 1

The bounds for tail area and wing position are unvaried. On the other hand k_{α} and k_{q} need to be bounded since an excessive value of the gains would entail a saturation of the control surfaces, for example in response to an intense gust. This eventuality, of course, is to be avoided at all costs. Furthermore, the greater the gains, the smaller the bandwidth of the system [7, 38]. Hence, the frequency response of the system could be degraded. The bounds were therefore determined by referring to open literature, in which suggested values for maximum allowable feedback gains with respect to similar stability augmentation systems were in the range of -1 [rad/rad] for α -feedback and -2 [rad/rad/s]. In this respect, a remark must be made regarding the value of k_q presented in Table 5.12. The high value on the bounds is related to the fact for the state space system considered in Equation 3.31, the pitch rate is scaled with mean aerodynamic chord and speed, namely $q\bar{c}/V$. The bounds of -50 and 50 are therefore imposed accordingly. To obtain a value similar to k_{α} the pitch rate gain must then be multiplied by a factor V/\bar{c} . Hence the bound values are consistent with the reference gain magnitude found in literature, as discussed before. Furthermore, it is possible to see that the upper bound on k_{α} and k_q is zero. This is because negative feedback is required to augment the stability of the aircraft, while in some cases the optimizer could opt for not closing one of the loops, thus setting one of the gains to zero.

With respect to the unaugmented case, it is possible to ask more to the optimization in terms of short period damping, while maintaining the same Control Anticipation Parameter. This is true in the light of the fact that the bulk of the effort to improve the handling qualities is now delegated to the stability augmentation system, while in the previous case it was done only through a modification of the geometry. Hence, the inputs to the regional pole placement method are now the following:

Variable	Value
CAP _{min}	0.3 [-]
$\zeta_{ m sp_{min}}$	0.76 [-]

Table 5.13: Regional pole placement inputs - CAP and ζ_{sp} method 1

Clearly, such a short period damping positions the optimized design consistently in level 1 handling qualities, as can be seen in Figure 5.9. The optimization procedure for augmented design using the first method ran in a total time of 16 seconds on the same machine used for the previous case. The computational time increased slightly, mainly due to the two extra design variables that are added. The optimized design variables are shown in Table 5.14.

Variable	Initial value	Optimized value	Dimensional value
$S_{ht}/S_{ht,0}$	1	0.78	20.7 [m ²]
$x_{pos_w}/x_{pos_{w,0}}$	1	0.97	11.2 [m]
k_{α} [rad/rad]	1	-0.3	-
k_q [rad/rad/s]	1	-31	-

Table 5.14: Optimized design vector - augmented design method 1

The tail area has been reduced by 22 %. Moreover, the wing has been shifted forward by 3 %. It is interesting to note the difference with the previous case, were the wing position was on the lower bound. In order to have a clearer overview of the modifications to the geometry a comparison between the baseline and the optimized configuration is presented in Figures 5.6a and 5.6b.



Figure 5.6: Optimized and baseline configuration A320-200 - augmented method 1

The wing shift has been evidently reduced with respect to the previous case. An explanation for this choice of the optimizer can be found by analyzing the interaction between the stability and control requirements discussed in Section 3.2.6 and the objective functions. As it was stated previously, there is a hard limit on the most aft center of gravity location attainable, specifically selected to ensure at least neutral stability for safety reasons. Inspecting Equation 3.52 it is possible to see that $\overline{x}_{cg_{NP}}$ is dependent on tail arm and tail area. Clearly, an increase in tail arm is beneficial, while a reduction in tail area has detrimental effects. The reason for this is twofold: firstly to a lower tail area corresponds a forward shift of the aircraft aerodynamic center, which in turn reduces the static margin. Secondly, the tail area in itself gives a contribution to the allowable aft center of gravity position, and the larger the area, the better for stability. The contribution of the wing position is slightly more complex since it has both a negative and a positive effect: on the one hand, it increases the horizontal tail arm, which is beneficial. On the other hand, it also shifts the aerodynamic center more forward, thus further reducing the static margin. Clearly, there is a tradeoff between the two design variables with respect to static stability.

A complete picture of the situation is obtained by taking into account also the objective functions: for tailplane weight and zero-lift drag a lower tail area is beneficial. However induced drag behaves in a different fashion since it dependent on tail arm, wing-fuselage aerodynamic center location, and tail area. Specifically, a fore shift of the wing position entails an increase of the wing-fuselage stability margin, which reduces the required lift coefficient at the tail by counteracting the nose-down pitching moment $C_{m_{ac,wb}}$. In this respect, the same considerations on the accuracy of the pitch down moment at the aerodynamic center made for the unaugmented case apply. It also increases the horizontal tail arm, reducing $C_{L_{ht}}$ even

more. A reduction in tail area has a twofold effect: it increases the tail lift coefficient, thus increasing induced drag. However, it also gives a small contribution to the horizontal tail arm, which is beneficial. Nevertheless, the overall impact was found to be detrimental. Hence the solution to this problem is found by selecting a suitable combination of the two design variables, such that tailplane weight, zero-lift drag, and induced drag are minimized as much as possible, up until the point where neither the tail area nor the wing position can be modified without violating the constraint on stability. Hence, the solution achieved by the optimizer is justified by a tradeoff between these two conflicting goals. Indeed an inspection of the X-plot of the optimized configuration showed that the most aft center of gravity position overlaps the aircraft aerodynamic center, thus providing a condition of neutral stability as it was intended. There is no doubt that, by relaxing the requirements on the stability with aft c.g, more benefits could be reaped concerning the objective functions. However, it was deemed reasonable to maintain a certain safety margin in this sense. This phenomenon was not found in the previous case simply because the tail area increased due to handling qualities requirements, therefore shifting the aircraft aerodynamic center aft.

Having discussed and justified the choices made by the optimizer, the objective functions pertaining to the final configuration are presented in Table 5.15.

Objective function	Initial value	Optimized value	Dimensional value
$J_1 = W_{ht} / W_{ht_0}$	1	0.78	5218 [N]
$J_2 = C_{D_0} / C_{D_{0,0}}$	1	0.74	0.0013 [-]
$J_3 = C_{D_i} / C_{D_{i,0}}$	1	0.45	$9.45 \cdot 10^{-5}$ [-]

Table 5.15: 0	ptimized ob	iective functions -	 augmented d 	esign method 1
		J		

Tailplane weight has been reduced by 22 %. Furthermore, both zero-lift drag and induced drag have been reduced, by 26 % and 55 % respectively. It is immediate to see that the optimizer greatly overachieved the goal selected in Section 5.1.1, which was 0.9. With respect to the unaugmented case, the decrease in J_1 and J_2 is remarkable, and mainly due to the reduction in tail area, on which both objective functions are dependent. It is interesting to note that J_3 , the tailplane induced drag, has been decreased considerably. However, the optimized value is still 50 % higher with respect to the unaugmented case. This result confirms the considerations made previously on the results of the optimizer in terms of design variables. Nevertheless, even with this limitations, the augmented design has reached considerable improvements in terms of objective functions with respect to both the baseline and the unaugmented configuration. More so, the benefits of including a stability augmentation system are highlighted in the results for CAP and short period damping ratio, shown in Table 5.16.

Parameter	Initial value	Optimized value	Diff. %
$\zeta_{ m sp}$	0.41 [-]	0.76 [-]	+85 %
$\omega_{ m n_{sp}}$	1.23 [rad/s]	1.72 [rad/s]	+40~%
CAP	$0.15 [1/gs^2]$	$0.3 [1/gs^2]$	+100 %

Table 5.16: Optimized modal response parameters and CAP - augmented design method 1

The increase in ζ_{sp} and CAP is evident and has been achieved through the selection of optimal gains, as shown in Table 5.14. The negative sign on k_{α} and k_q indicates that the optimizer has opted for negative feedback, as it was expected. Furthermore, neither of the variables is on the bounds, which indicates that there is still room for improvement in terms of handling qualities. However, due to the limitations discussed previously, the optimization procedure did not overachieve the minimum requirements regarding short period damping ratio and CAP. Similarly to the unaugmented case, it is possible to derive a general consideration on the behavior of the optimization for this method. The procedure aims at minimizing the objective functions to the greatest extent, while the stability augmentation system is tuned in such a way to maintain ζ_{sp} and CAP at the minimum level required, without any attempt to go further. It is hypothesized that this behavior stems from the choice of the limited set of geometrical design variables. Should be more variables be included, it is reasonable to think that the optimizer would search into a larger design space, in which optimal results might entail an overachievement of the minimum requirements on handling qualities. Nevertheless, this behavior could be just due to the moderately high requirements that are imposed, while for lower required short period damping ratio and CAP the selected gains might be selected such that these minimum values are surpassed. The regional pole placement method achieves a high precision on the location of the poles and consequently on the optimized ζ_{sp} and CAP, while simultaneously achieving remarkable benefits with respect to the objective functions. To assess the characteristics of the stability augmentation system, the time response of the augmented aircraft with respect to an initial perturbation and a step input is presented in Figures 5.7 and 5.8.



Figure 5.7: Comparison of response for initial condition $\Delta \alpha = 0.1$ [rad]

The time response shows a remarkable improvement with respect to the baseline configuration. Both the angle of attack and the pitch rate are highly damped, and return to a condition of equilibrium in less than 5 seconds. Moreover, by comparing the response around 2.5 seconds, it is possible to see that the oscillations around the equilibrium that characterize the baseline configuration have almost disappeared thanks to the stability augmentation system, while the maximum peak is also reduced for both the states. The benefits of the pitch damper and α -feedback are therefore apparent, as far as the response to an initial perturbation goes. To gain more insight in the performance of the augmented aircraft it is interesting to evaluate the response to an elevator step input of amplitude $\Delta \delta_e = -0.05$ [rad], shown in Figures 5.8a and 5.8b.



Figure 5.8: Comparison of response for step input $\Delta \delta_e = -0.05$ [rad]

Qualitative analysis of the step response reveals an improvement with respect to both the baseline configuration as well as the unaugmented case. The response is highly damped and settles within a few seconds from the initial input. Furthermore both for $\Delta \alpha$ and Δq the peak amplitude is greatly reduced, as opposed to what happened to the unaugmented optimized aircraft. The presence of the stability augmentation system clearly has an influence, however it must also be noted that, given the reduction in tail size, the elevator effectiveness has reduced and for the same $\Delta \delta_e$ a smaller pitching moment around the aircraft center

Configuration	Settling Time [s]	Peak Time [s]	Rise Time [s]
Baseline	6.5	2.73	1.19
Optimized	1.96	2.8	1.36
Diff %	-70.7 %	+2.56 %	+14.2 %

of gravity is generated, thus partly justifying the smaller peak in the response. A quantitative analysis of the step response indeed reveals the improvements achieved, as shown in Tables 5.17 and 5.18.

Table 5.17: Comparison of step input time response parameters - angle of attack (method 1)

Configuration	Settling Time [s]	Peak Time [s]	Rise Time [s]
Baseline	7.9	1.36	0.27
Optimized	2.76	0.8	0.12
Diff %	-65 %	-41 %	-55.5 %

 Table 5.18: Comparison of step input time response parameters - pitch rate (method 1)

The settling time has been reduced by 70 % for $\Delta \alpha$ and 65 % for Δq with respect to the baseline. Comparing this results with the unaugmented optimized case, a further reduction of 20 % and 16 % is found respectively. The benefits of including the stability augmentation systems are therefore apparent, more so considering that, for both states, the settling time is consistently rated as level 1 in the military specifications [8]. In general, the response is seen to be faster for the pitch rate, which is confirmed by the reduced peak time and rise time. On the other hand, considering $\Delta \alpha$, it is interesting to note that these two terms are increased for the augmented configuration when compared to the baseline, specifically by 2.56 % and 14.2 %. In other terms, the response with respect to the angle of attack is slightly slower, especially considering the rise time. The difference between the response of $\Delta \alpha$ and Δq can be justified in the following way: the reduction in tail size brings about a lower elevator effectiveness, as discussed previously, and consequently the rise time and peak time are reduced due to a decrement of control authority. Introducing proportional control in the system, on the other hand, has the beneficial effect of making the response faster [7, 12]. Simply put, the magnitude of the α feedback gain is not enough to counterbalance the reduced control authority, with a net effect of slowing down the response. On the other hand, the pitch rate gain k_q has been increased substantially mainly due to damping ratio requirements, enough to



Figure 5.9: Control Anticipation Parameter comparison - augmented design (method 1)

overcome the lower elevator effectiveness and determine an overall faster response, comparable with the one presented for the unaugmented optimized configuration. It must be remembered that, for almost the same pitch rate rise time and peak time, the unaugmented configuration showed a 43 % increase in tail area, with the associated increase in elevator surface and effectiveness.

Finally, a visual representation of the achieve Control Anticipation Parameter is presented in Figure 5.9. The improvement is evident and places the optimized configuration consistently in level 1 handling qualities.

In conclusion, the optimization procedure for augmented aircraft using method 1 has proved successful, obtaining precisely the desired level of CAP and short period damping ratio. Both of these values were increased, the former by 100 %, while the latter by 85 %. The optimized design simultaneously achieved a reduction in tailplane weight, zero-lift drag, and induced drag, respectively by 22 %, 26 % and 55 %, thus proving the merit of including stability augmentation systems in the design procedure.

5.1.5. Augmented Optimization - Method 2

The last user-defined option for the optimization involves the design of the same stability augmentation system as in the previous case. However, it makes use of the Linear Quadratic Regulator to compute the feedback gains. The design vector is then augmented with the weighting factors required for the matrices *Q* and *R* defined in Section 3.39. Clearly, with respect to the previous case in which the gains were part of the design vector, there is less control over the final solution. The Linear Quadratic Regulator aims at minimizing a time-domain performance index. Hence the gains will not be selected only with the purpose of satisfying the required short period damping ratio and Control Anticipation Parameter, but also to achieve a satisfactory time response of the augmented system.

Variable	Value	Bounds
$S_{ht}/S_{ht,0}$	1	$0.1 < S_{ht}/S_{ht,0} < +\infty$
$x_{pos_w}/x_{pos_{w,0}}$	1	$0.95 < x_{pos_w} / x_{pos_{w,0}} < 1.05$
q_1^\star [-]	1	$0.001 < q_1^{\star} < +\infty$
q_2^{\star} [-]	1	$0.001 < q_2^{\star} < +\infty$
ρ* [-]	1	$0.001 < \rho^{\star} < +\infty$

In Table 5.19 the initial design vector, consisting of five elements, is presented:

Table 5.19: Design variables and bounds - augmented optimization method 2

As in the previous case, the initial value and bounds for the tail area and wing position remain unvaried. There is no need for a hard upper bound on the weighting factors, as they do not pose problems to the optimization in terms of maximum values. On the other hand, a lower bound is needed, as to avoid the occurrence of a singularity in the matrices Q and R, which would halt the optimization. In fact, the Linear Quadratic Regulator requires that the matrix R is definite positive, as discussed in Section 3.2.4. Furthermore, a lower value of ρ^* entails larger control inputs, which might then lead to an excessive feedback gain with the risk of saturating the control surfaces. Conversely, a high value of ρ^* suppresses the control signals in the closed loop system, that is less control effort is allowed. This also means that the controls are less effective in suppressing undesirable oscillations of the system.

Similarly, a large value of q_1^* and q_2^* implies that the respective states are weighted more in the procedure. Hence the gains will be computed as to minimize the excursion of α and $q\overline{c}/V$ in the closed loop system.

The inputs to the regional pole placement method remain unvaried with respect to the previous case, thus allowing for a meaningful comparison between the two methods for the computation of the feedback gains.

Variable	Value
CAP _{min}	0.3 [-]
$\zeta_{ m sp_{min}}$	0.76 [-]

Table 5.20: Regional pole placement inputs - CAP and ζ_{sp} method 2

The optimization using method 2 is the most costly in terms of computational time; using the same laptop as for the previous two case the procedure took 22 seconds, which is almost double with respect to the unaugmented case. This is readily explained considering the increased number of variables in the design vector and, more importantly, the addition of the Linear Quadratic Regulator which in itself represents a small scale optimization and therefore requires higher computational time.

Variable	Initial value	Optimized value	Dimensional value
$S_{ht}/S_{ht,0}$	1	0.78	20.7 [m ²]
$x_{pos_w}/x_{pos_{w,0}}$	1	0.97	11.2 [m]
q_1^\star [-]	1	0.05	-
q_{2}^{\star} [-]	1	80	-
$ ho^{\star}$ [-]	1	0.1	-

The optimized design vector is shown in Table 5.21.

Table 5.21: Optimized design vector - augmented design method 2

The main feature that can be recognized is that both the optimized tail area and wing position values are the same as in the previous case, which used method 1. Clearly, then the considerations made previously on the tradeoff between stability requirements and minimizing the objective functions apply here as well. The final values of the weighting factors for the Linear Quadratic Regulator correspond to the feedback gains shown in Table 5.22.



(method 1)

Figure 5.10: Optimized and baseline configuration A320-200 - augmented method 2

Gain	Value
k_{lpha}	–0.5 [rad/rad]
k_q	-40.6 [rad/rad/s]

Table 5.22: Computed feedback gains with Linear Quadratic Regulator

Comparing these values with the ones found for method 1 it is possible to see an increase in both k_{α} and k_q . This result can be attributed to the Linear Quadratic Regulator, which obtained higher gains by minimizing the time-domain performance index in Equation 3.38. This has a profound effect on the modal response characteristics and CAP of the configuration, as it will be discussed shortly.

A comparison between the baseline and the optimized geometry is shown in Figure 5.10a and 5.10b. It can be seen the geometrical modifications to the baseline configurations are identical. Furthermore, the final values of the objective functions are equals to the previous case as well, as it is shown in Table 5.23.

Objective function	Initial value	Optimized value	Dimensional value
$J_1 = W_{ht} / W_{ht_0}$	1	0.78	5218 [N]
$J_2 = C_{D_0} / C_{D_{0,0}}$	1	0.74	0.0013 [-]
$J_3 = C_{D_i} / C_{D_{i,0}}$	1	0.45	$9.45 \cdot 10^{-5}$ [-]

Table 5.23: Optimized objective functions - augmented design method 2

Clearly being the objective functions dependent only on the tail area and wing position, the situation is unvaried.

Even though no notable changes can be found in the geometry of the optimized configuration, it is interesting to analyze the effect of the Linear Quadratic Regulator on the modal response parameters and Control Anticipation Parameter.

Parameter	Initial value	Optimized value	Diff. %
$\zeta_{ m sp}$	0.41 [-]	0.76 [-]	+85 %
$\omega_{ m n_{sp}}$	1.23 [rad/s]	1.95 [rad/s]	+58 %
CAP	$0.15 [1/gs^2]$	$0.38 [1/gs^2]$	+150 %

Table 5.24: Optimized modal response parameters and CAP - augmented design method 2

In Table 5.24 it is possible to see that the short period damping ratio is exactly at the value selected as input and equal to the previous case. On the other hand $\omega_{n_{sp}}$ has increased substantially. Even more so, the minimum CAP has been overachieved by 50 %, which is only natural considering it is dependent on the undamped natural frequency. These results show the capabilities of the Linear Quadratic Regulator, which managed to achieve improved handling qualities with respect to method 1 while maintaining the same benefits in terms of objective functions.

To assess qualitatively the characteristics of the optimized configuration, the time response to an initial condition and step input are presented in Figure 5.11, similarly to the previously analyzed cases. The response is similar to the method 1 augmented design. However, a noticeable increase in response speed can be seen. Specifically, referring to Figure 5.11b, the time to reach the peak amplitude of the pitch rate is seen to be reduced. Moreover, the oscillations decay more promptly. The response shows almost no oscillations around the equilibrium, as it is to be expected for $\zeta_{sp} = 0.76$. By analyzing the response to a step input in Figure 5.12 a more quantitative description of the time response of the aircraft can be given.



Figure 5.11: Comparison of response for initial condition $\Delta \alpha = 0.1$ [rad]



Figure 5.12: Comparison of response for step input $\Delta \delta_e = -0.05$ [rad]

The peak amplitude is clearly reduced, which can again be ascribed to the reduced tail surface. Furthermore, as opposed to the case discussed in the previous section, the peak time for $\Delta \alpha$ is reduced with respect to the baseline configuration, while the rise time is still greater, as found in the previous augmented design, but to a less extent. This behavior can be explained by the increase in the magnitude of the feedback gains, which in turn hastens the time response. On the other hand, the peak time and rise time for pitch rate are further reduced. It must be remarked that the settling time of both $\Delta \alpha$ and Δq has decreased with respect to the values shown in Tables 5.17 and 5.18.

Configuration	Settling Time [s]	Peak Time [s]	Rise Time [s]
Baseline	6.5	2.73	1.19
Optimized	1.75	2.52	1.21
Diff %	-73 %	-7.7 %	+1.6 %

Table 5.25: Comparison of step input time response parameters - angle of attack (method 1)

Configuration	Settling Time [s]	Peak Time [s]	Rise Time [s]
Baseline	7.9	1.36	0.27
Optimized	2.4	0.67	0.09
Diff %	-70 %	-50.7 %	-66 %

 Table 5.26:
 Comparison of step input time response parameters - pitch rate (method 1)

Finally, in Figure 5.13 the achieved CAP level is presented, which confirms the better performance with respect to the baseline configuration as well as both the previously discussed unaugmented and augmented designs.



Figure 5.13: Control Anticipation Parameter comparison - augmented design (method 2)

5.2. Performance Study

In this section, two aspects of the proposed design methodology are investigated. As discussed in Section 3.2.4, the stability augmentation system makes use of the ideal actuators assumption. Hence, it is deemed crucial to determine whether the presence of actuators implies an appreciable degradation of the performance of the aircraft in terms of time response.

Subsequently, a study of the Initiator convergence is carried out, to assess the impact of the handling qualities optimization module on the performance of the final designs, in terms of maximum lift over drag ratio and maximum takeoff mass.

5.2.1. Actuators

Actuators play an important role in determining the frequency and time response of an aircraft. Even though the design methodology based on the modified Routh-Hurwitz criterion is perfectly capable of taking their presence into account, it has been decided to use the ideal actuators assumption (see Section 3.2.4). To have a complete overview of the robustness of the methodology, it is necessary to investigate how the presence of actuators affects the optimized designs. To this end, the state space system defined in Section 3.2.3 is augmented with first order linear actuator dynamics. The transfer function of such actuators is defined as follows:

$$H_{servo}(s) = \frac{K_{servo}}{1 + T_{servo}s}$$
(5.1)

In Equation 5.1, the most important parameter is the time constant T_{servo} , which depends on the type of actuator. In general, for electric actuators, which tend to be slower, $T_{servo} \approx 0.25$ while fast hydraulic actuators can achieve $T_{servo} \approx 0.1$ [7, 12, 38].

It is immediate to understand that a fast actuator will impact the dynamics of the aircraft to a lesser extent, while a slow actuator will introduce a lag component, which could significantly alter its response characteristics. For this analysis $K_{servo} = 1$, while the time constant is varied. Furthermore, only the augmented design with method 1 is considered. This choice is justified by arguing that the interaction between the stability augmentation system and the actuators could prove more crucial with respect to the unaugmented case. The results in terms of time response following an elevator step input $\Delta \delta_e = -0.1$ [rad] is shown in Figure 5.14.



(a) $\Delta \alpha$ response with actuators - step $\Delta \delta_e = -0.1$ [rad] (b) Δq response with actuators - step $\Delta \delta_e = -0.1$ [rad]

Figure 5.14: Time response of optimized aircraft with actuators - method 1

The condition of $T_{servo} = 0$ corresponds to the ideal actuators assumption. Hence the results are seen to be identical to Figure 5.8. From a qualitative point of view, the time response is generally slower, as it was to be expected. The actuator indeed causes the output to lag behind the input, due to the additional dynamics introduced within the system. Nevertheless, no substantial degradation in the response is seen for neither of the states. To have a better overview of the effects of including the actuators, a comparison of the characteristics of the response for $T_{servo} = 0$ and $T_{servo} = 0.25$ is presented in Tables 5.27 and 5.28.

Time Constant	Settling Time [s]	Peak Time [s]	Rise Time [s]
$T_{servo} = 0$	1.96	2.83	1.36
$T_{servo} = 0.25$	2.07	2.94	1.38
Diff %	+5.6%	+3.9 %	+1.47%

Table 5.27: Comparison of step input time response parameters - pitch rate (method 1)

Considering the response with respect to the angle of attack it is possible to see that, even with the largest time constant, which corresponds to the worst performing actuators, the degradation is limited. The settling time shows the highest increase, which indeed is of just 5.6 %. On the other hand, the response with respect to pitch rate presents a slightly different scenario.

Time Constant	Settling Time [s]	Peak Time [s]	Rise Time [s]
$T_{servo} = 0$	2.76	0.8	0.12
$T_{servo} = 0.25$	2.88	0.91	0.18
Diff %	+4.34%	+13.75 %	+50 %

Table 5.28: Time response characteristics with actuators - $\Delta \alpha$

In a relative sense, the response with respect to pitch rate has degraded quite substantially, mainly in terms of rise time and peak time. It must be noted however that, in an absolute sense, the results are still considered quite satisfactory. In fact, for both angle of attack and pitch rate, the settling time is well within the bounds of level 1 time response handling qualities. In general, it can be said that, by including first order linear actuators in the system, the performance of the optimized design is still rather acceptable. No study has been performed on second order linear actuators, hence no conclusion can be drawn in this respect. It can be hypothesized that the dynamics of such actuators would considerably interact with the system, possibly degrading the time response even further. Nevertheless, at the conceptual design stage, the use of first order linear models is deemed appropriate to preliminarly investigate the effect on the optimized designs.

5.2.2. Initiator Convergence

In this section, the impact of the newly developed handling qualities optimization module is assessed with respect to the overall performance of the optimized aircraft. The objective is to get the big picture in terms of the complete design procedure: in order to do that, the Initiator has been run to convergence on the aforementioned A320-200 aircraft for varying short period damping ratios and Control Anticipation Parameters. Since the final purpose of the module is to be included at the conceptual design stage, is meaningful to assess the impact that the different optimization methods have on the final configurations, and to extrapolate general trends which can be used to characterize the overall design methodology. Specifically, the convergence study has been performed fro $\zeta_{\rm sp_{min}}$ ranging from 0.41, which is the baseline value, to 0.7. Furthermore, two different CAP levels have been investigated: 0.15, i.e the baseline value, and 0.3. For each design point, the $L/D_{\rm max}$ and MTOM of the converged aircraft are obtained and evaluated with respect to the baseline values. Firstly, the unaugmented optimization is investigated. The results are shown in Figure 5.15:



Figure 5.15: Initiator convergence study - unaugmented optimization

It must be noted that the y-axis values are normalized with respect to the baseline configuration. Before any discussion of the results, it is appropriate to introduce a term which appears in Figure 5.15a and 5.15b: the breakeven point. In this context, this denomination was defined to indicate the short period damping ratio values for which, at a given CAP, the L/D_{max} and MTOM of the optimized configuration are equal to the respective baseline values. In other words, it marks the maximum handling qualities level, intended broadly as combination of ζ_{SD} and CAP, which can be achieved without incurring in a degradation of the performance of the aircraft in terms of lift-over-drag and maximum takeoff mass. It must be pointed out that in the Initiator the payload remains constant. Hence an increase in MTOM translates in an increase in fuel and structural weight, which of course is detrimental. With this concept in mind, it is now possible to assess the results. Firstly, considering the study for CAP = 0.15, it can be seen that the two breakeven points, L/D_{max} and MTOM respectively, occur at $\zeta_{sp} = 0.675$ and $\zeta_{sp} = 0.67$. It can be therefore inferred that, by optimizing an aircraft purely through geometrical design, it is possible to increase the short period damping by 63 % without degradation in aircraft performance. Furthermore, it is interesting to note that, by running the optimization at the baseline damping ratio ζ_{sp} = 0.41, an increase of 1 % in L/D_{max} and a reduction of 1.3 % in MTOM can be achieved. This is an important result, insofar as it demonstrates that the proposed optimization framework is not only capable of improving the handling qualities of a given configuration but also obtain performance benefits while maintaining the baseline short period damping ratio and CAP.

Figure 5.15b depicts a quite different situation. Increasing the required CAP by 100 % has a substantial effect. Most notably, the breakeven points have shifted to lower short period damping ratio. Specifically, for L/D_{max} the breakeven point occurs now at $\zeta_{\text{sp}} = 0.52$, while for MTOM the value is now $\zeta_{\text{sp}} = 0.51$. This results should not surprise, as the handling qualities requirements in terms of CAP have become more stringent and therefore require larger tail areas to be satisfied for the same damping ratio. As an example

let us consider again the baseline damping ratio value, or $\zeta_{sp} = 0.41$. For CAP = 0.3 there is an increase in L/D_{max} of 0.8 % and a reduction in MTOM of 0.6 %. Comparing these results with the ones obtained previously for CAP = 0.15 it is possible to see that the performance benefits have been reduced. This difference increases in magnitude as the short period damping ratio increase, up to the point where there is a decisive degradation for CAP = 0.3 while for CAP = 0.15 there are still benefits margins. For the highest considered damping ratio it is possible to notice a reduction of 2.1 % in L/D_{max} and an increase of 1.8 % in MTOM. To put these values in perspective, for the considered aircraft (A320-200), this would translate into an increase in MTOM of 1418 kg.

The definition of the breakeven points gives valuable insight on the effects of the integration of the handling qualities optimization module in the Initiator. A general trend can be therefore identified, with respect to the unaugmented case: if a configuration is to be optimized by keeping the CAP and ζ_{sp} at the baseline values, it is possible to achieve substantial performance benefits by reducing the tail size and repositioning the wing to an optimal position in order to achieve higher aerodynamic efficiency and lower maximum takeoff mass. On the other hand, if a higher CAP is required, the breakeven points get closer to the baseline configuration, thus limiting the achievable short period damping ratio before a degradation in performance is introduced.

Having discussed the unaugmented optimization method, it is now relevant to assess what happens if the configuration is optimized by taking into account the presence of a stability augmentation system, like the one used in Sections 5.1.4 and 5.1.5. For this analysis, only method 1 was investigated. There is a simple reason for this: as shown in the previous sections, including the feedback gains in the design vector provides with excellent precision with respect to the achievable short period damping ratio and CAP. On the other hand method 2, which uses the Linear Quadratic Regulator, has been shown to overachieve the minimum requirements. Hence, in order to perform a meaningful comparison with respect to the unaugmented case, a decision was made to consider only method 1. Nevertheless, as it will be discussed further on, the results apply to both methods all the same and are therefore general in character.



Figure 5.16: Initiator convergence study - unaugmented optimization

In Figure 5.16a the outcome of the analysis in terms of L/D_{max} and MTOM for the optimized augmented configuration is presented. One feature stands out: the performance of the aircraft is independent of the desired level of damping ratio for CAP = 0.3. This is a remarkable result, which gives profound insights on the effect of including stability augmentation systems in the conceptual design process. Clearly, the optimizer minimizes the tail area and positions the wing as to obtain the highest improvements in terms of performance while the desired level of handling qualities is achieved just by adjusting the feedback gains accordingly. This explanation is confirmed by the results shown in Figure 5.16b: it is possible to see that, for $\zeta_{sp} = 0.41$, k_q is at its minimum, while k_{α} is at its highest in terms of magnitude. With increasing damping ratio the pitch-rate feedback is enlarged, while the α feedback is reduced. This trend can be justified by considering that k_{α} impacts preeminently the short period undamped natural frequency, while it tends to reduce the short period damping ratio [7, 12, 38]. On the other hand k_q affects mainly ζ_{sp} . Hence the

decrease in magnitude of the former and the consequent increase of the latter is indeed to be expected.

The plot for CAP = 0.15 is not presented, since it shows the same trend, while only the gains magnitude varies with respect to the one presented in Figure 5.16b. Hence, another general rule can be inferred by the evaluation of the Initiator convergence: if an aircraft is to be optimized for handling qualities by simultaneously designing a stability augmentation system, the final configuration will achieve the maximum benefits, in terms of performance, independently of the required short period damping ratio and CAP. In other words, the optimizer will reduce the tail area and reposition the wing as to minimize weight and drag to the full extent right from the start, while the compliance with the handling qualities requirements is delegated to the stability augmentation system. At this point however the question that must be asked is the following: what is the limiting factor that establishes the extent of the savings in terms of L/D_{max} and MTOM seen in Figure 5.16a? This query can be readily answered by recalling the discussion introduced in Section 5.1.4, insofar as the minimum achievable tail area and the optimized wing position are constrained by stability requirements. Hence, another important conclusion can be presented: all the augmented configurations investigated in the convergence study have the same final geometry, in terms of wing position and tail area. This also confirms the results found for method 2 in Section 5.1.5, and further justifies the choice of running the convergence only for method 1. Should method 2 be investigated, it will be found that the only the magnitude of the gains vary, while the geometry remains unchanged. Besides, it can be inferred that the achievable performance benefits can be varied by modifying the aft center of gravity static margin described in Section 3.2.6 (Equation 3.53). By introducing a positive static margin, it is possible to increase L/D_{max} and MTOM, however, the safety considerations presented in Section 3.2.6 must be always kept in mind.

Finally, it can be hypothesized that there is a limit for the augmented configuration to the achievable short period damping ratio and CAP as well, dictated by the bounds on the gains, for which there is the risk of saturating the control surfaces, as discussed in Section 5.1.4. Should the handling qualities requirements be higher than this limit, the gains could not be further modified and the geometry should therefore be altered to comply with the required ζ_{sp} and CAP, most likely entailing a degradation in performance. It is however not clear if a breakeven point could be actually found for the augmented configurations, unless the handling qualities requirements became incredibly stringent, determining a combination of a very high CAP and damping ratio.

Since the level of CAP does not affect the performance of the aircraft, from a quantitative point of view, it is possible to compare the baseline configuration, for which the CAP is 0.15 to the augmented configuration, even though the CAP in this case is 0.3. It can be seen in Figure 5.16a that for the baseline damping ratio $\zeta_{sp} = 0.41$ there is an increase of 2 % in L/D_{max} and a decrease of 1.6 % in MTOM, when the stability augmentation system is included. A final comparison is made between the augmented configuration and the unaugmented ones, for both levels of CAP. In Figure 5.17 the comparison for CAP = 0.15 is presented. It is possible to see that the augmented design performs better right from the start, at the baseline damping ratio. The results then diverge, up to $\zeta_{sp} = 0.7$, where the difference in L/D_{max} is about 2 %. Considering MTOM on the other hand the difference is around 1.8 %.

Clearly then the more is asked in terms of handling qualities, the higher the difference in terms of achievable performance between the two methods. However, it is interesting to note that, for the baseline value of damping ratio, the difference between unaugmented and augmented configurations is much less marked. Specifically, a difference of 0.9 % in L/D_{max} and of 0.3 % in MTOM is found. This result is very interesting insofar as a designer could opt for optimizing just the aircraft's geometry instead of implementing a stability augmentation system, achieving similar benefits in terms of performance for the baseline level of handling qualities and avoiding extra costs introduced by the need for the flight control system. Thus, by using the handling qualities optimization methodology, the flexibility during the design process is greatly enhanced.

In Figure 5.18 the trend of the augmented and unaugmented configuration for CAP = 0.3 is presented. In this particular case, the difference is marked throughout the range of damping ratios. Specifically, considering the baseline short period damping ratio value, the augmented configuration achieves a relative increase of 100 % in aerodynamic efficiency as well as a relative decrease of 100 % in MTOM with respect



Figure 5.17: Comparison of augmented and unaugmented designs - CAP = 0.15



Figure 5.18: Comparison of augmented and unaugmented designs - CAP = 0.3

to the unaugmented values. This should not surprise, as an increase of 100 % in CAP is required, which has the effect of shifting the breakeven points to the left, as discussed previously in this section. Nevertheless the fact the methodology is indeed capable of obtaining performance benefits even for such level of handling qualities is remarkable, and again a designer could opt for the geometrical optimization, should the presence of a stability augmentation system to be required, for example by a potential customer. In general, however, it has been proved that the integration of such control systems is indeed quite beneficial.

Chapter 6

Conclusions and recommendations

6.1. Conclusions

The main goal of the current research was the creation of a handling qualities optimization module to be included in the Initiator, a conceptual design tool developed at the Department of Flight Performance and Propulsion of TU Delft. The module is focused on conventional aircraft configurations, specifically on the sizing of the horizontal tailplane and positioning of the wing. In terms of handling qualities, the module is concerned with the improvement of the short period motion. It must be capable of optimizing both unaugmented and augmented designs, in the latter case by the inclusion of a suitable stability augmentation system. To this end, a general stability criterion was required. Hence, a regional pole placement methodology based on a modified Routh-Hurwitz criterion was developed and implemented as a set of constraints on the design space. This choice was dictated by the general character of this criterion, which makes it suitable for a variety of aircraft configurations. Furthermore, it has the possibility of being expanded to take into account higher order dynamics, which is of remarkable importance in perspective of future developments. Besides achieving optimal handling qualities, the module systematically searches for a solution which entails a reduction in tailplane zero-lift drag, induced drag, and weight. Moreover, controllability and stability are ensured in all flight regimes by imposing constraints on the minimum tail area and wing position.

The two research question posed in Section 1 are now recalled, to finally provide a meaningful answer, which follows naturally from the previously discussed investigations. The first research question is the following:

Can an optimization methodology based on the modified Routh-Hurwitz criterion consistently achieve the desired handling qualities, for both unaugmented and augmented designs?

The first question can be answered positively. The optimization routine has been tested on a baseline aircraft, namely an Airbus A320-200, for all three user specified options. For the unaugmented design, the procedure achieved a short period damping of 0.55, and a Control Anticipation Parameter of 0.3, as required. This corresponds to an increase of 34 % in ζ_{sp} and of 100 % in CAP. The time response of the optimized configuration showed a decisive improvement, specifically by reduced settling time in response to an elevator step input. When an initial perturbation is applied to the model, the response is seen to decay promptly, reaching a condition of equilibrium faster than the baseline aircraft due to the increased damping.

The augmented optimization using method 1 again achieved precisely the required short period damping ratio and CAP, which were $\zeta_{sp} = 0.76$ and CAP = 0.3. This result corresponds to an increase in ζ_{sp} of 85 % with respect to the baseline configuration and 38 % compared to the unaugmented optimization. The time response of the model was improved with respect to both the baseline configuration and the unaugmented optimized design. Settling time in response to an elevator step input has been further reduced compared to the unaugmented optimization. The oscillations around the condition of equilibrium are

almost entirely damped out, thus proving the goodness of the stability augmentation system. Lastly, the augmented optimization with method 2 was run. The procedure achieved the desired $\zeta_{sp} = 0.76$, while it actually overachieved the required CAP of 0.3 by 50 %. The LQR computed the feedback gains to minimize a time-domain performance index. The result is not only an overachievement of the required CAP but also a substantial improvement in terms of time response to an elevator step input, with an ulterior reduction in settling time with respect to the previously discussed configurations. The response to an initial perturbation in angle of attack improved as well, achieving a slightly faster decay towards the equilibrium condition.

Hence, it can be said that the handling qualities optimization module based on the modified Routh-Hurwitz criterion proved quite successful, consistently achieving the minimum requirements imposed regarding short period damping ratio and Control Anticipation Parameter for all the investigated cases. This entails that the methodology is indeed capable of coping with unaugmented and augmented design all the same. The second and final research question which needs to be addressed is the following:

What are the effects of handling qualities optimization on the performance of the aircraft, in terms of aerodynamic efficiency and maximum takeoff mass, both in case of unaugmented and augmented designs?

The answer to the second questions requires a more articulated discussion. In fact, the performance benefits have been analyzed at two different stages, which are deeply connected: firstly, the results in terms of objective functions within the optimization procedure. Secondly, the effect of the integration of the module itself within the Initiator. Clearly the former has a direct impact on the latter. Hence, only the converged results will be used to draw conclusions. This approach is deemed reasonable insofar as it allows to characterize the overall methodology, and the benefits (or drawbacks) of including it at the conceptual design stage. Being the analysis qualitative in nature, it is therefore deemed possible to obtain general conclusions, which could be extended to similar conventional aircraft configurations. The convergence study was carried out for two different CAP levels and a range of short period damping ratios.

A new definition has been introduced: the breakeven point. This term indicates the valued of ζ_{sp} for which, at a given CAP, the L/D_{max} and MTOM of the optimized configurations coincide with the baseline. Firstly, the unaugmented optimization has been investigated. It was found that, by keeping ζ_{sp} at the baseline value, the optimized design achieves higher L/D_{max} and lower MTOM both for CAP = 0.15 and CAP = 0.3. The breakeven points are found to be in the range of $\zeta_{sp} \approx 0.67$ for the former case and $\zeta_{sp} \approx 0.52$ for the latter. Hence, a general conclusion can be drawn for the unaugmented design: for a given range of short period damping ratios, increasing the required Control Anticipation entails a shift of the breakeven points towards the baseline values. There is a limit to the achievable ζ_{sp} before incurring in a degradation of aircraft performance. This limit is found to be chiefly dependent on the required CAP.

The investigation of the augmented design revealed a rather different behavior. Both L/D_{max} and MTOM were found to be insensitive to changes in CAP and short period damping ratio. In other words, no breakeven point has been identified. From a quantitative point of view, the performance benefits have been found to be considerably higher with respect to the unaugmented case and, more importantly, constant with varying ζ_{sp} and CAP. The compliance with the increasingly stringent handling qualities requirements is achieved by suitable changes in feedback gains, while the aircraft geometry is modified to achieve minimum drag and weight and kept constant throughout the convergence. It was found that the factor which limits the achievable performance benefits is the stability requirement, embodied by the static margin with aft center of gravity. Thus another general conclusion can be derived, regarding the augmented optimization: by including a stability augmentation system in the design, the geometry is modified in such a way as to obtain the maximum performance benefits right from the baseline values of ζ_{sp} and CAP and up to the upper bound of the investigated range. Optimal handling qualities are achieved purely by selecting suitable feedback gains. Hence, no breakeven point is deemed to exist, alas handling qualities requirements become so stringent that the gains are pushed to the bounds. In this case, to avoid saturation of the control surfaces, it is hypothesized that the geometry would have to be modified, thus entailing a reduction in performance benefits. Nevertheless, this behavior has not been encountered in the investigated range of CAP and ζ_{sp} . Finally, the limiting factor which determines the minimum tail area and consequently the achievable values of L/D_{max} and MTOM is the static margin with aft center of gravity. Hence, by relaxing this requirement, higher performance benefits could be reaped.

In conclusion, it has been showed that, by including these handling qualities considerations at the conceptual design stage, it is possible not only to achieve designs with better flight characteristics, but also to obtain performance benefits. This holds true for both augmented and unaugmented designs.

6.2. Recommendations For Future Work

The proposed design procedure based on the modified Routh-Hurwitz criterion has proven successful in optimizing the handling qualities of both augmented and unaugmented aircrafts. However, the methodology could be refined to become more efficient and insightful. To organize all suggestions, the recommendations have been divided into the main topics which, in the author's view, need addressing.

Analysis Tools The limitations of the empirical relations used for the computation of the objective functions have been extensively discussed in Section 3.1.4. By implementing more refined methods for the evaluation of induced drag and for the assessment of the tail weight would substantially increase the capabilities and accuracy of the tool. Furthermore, the design vector could be expanded, thus allowing for the exploration of a larger design space. The main concern in this respect is the increase in computational time. It would be therefore beneficial to investigate possible analysis methods, to be integrated within the module, which provide the required sensitivity and computational efficiency.

Flight Phases The analysis performed is limited to cruise condition, as discussed in Section 5.1. To overcome this limitation, and therefore paving the way for the optimization of handling qualities over the complete flight envelope, a module dedicated to the design and sizing of high-lift devices would have to be implemented in the Initiator. Furthermore, by having this additional information available, it would be possible to perform gain scheduling, thus designing an optimal stability augmentation system suitable for most flight phases, expanding the scope of the present module.

Handling Qualities Evaluation The present research is focused on the optimization of handling qualities with respect to two metrics: the modal response parameters provided in the military specifications MIL-F-8785C, and the Control Anticipation Parameter. Even though it has been proven that, by using this rationale, the time response of the various configurations, specifically regarding settling time, has improved substantially, it would be beneficial to take into account other handling qualities evaluation methods (see Section 1.3). In particular, frequency response characteristics should be addressed, to achieve an overall improved design. These considerations become even more crucial when actuators are introduced in the system.

Higher Order Dynamics This suggestion is directly linked to the considerations on handling qualities evaluation. Since the modified Routh-Hurwitz criterion can be easily augmented to consider higher-order characteristic polynomials, it would be compelling to include the dynamics of actuators in the system. In combination with frequency response analysis, it would, therefore, be possible to optimize the various configurations in a more complete and synergistic fashion.

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Appendix A

Sensitivity Study Results



The complete results of the sensitivity analysis discussed in Section 3.1.3 are presented.

Figure A.1: Sensitivity study - W_{ht} vs. S_{ht} - $\Lambda_{LE_{ht}}$



Figure A.2: Sensitivity study - W_{ht} vs. x_{pos_w} - λ_{ht}







Figure A.4: Sensitivity study - C_{D_0} vs. x_{pos_w} - λ_{ht}



Figure A.5: Sensitivity study - C_{D_i} vs. S_{ht} - $\Lambda_{LE_{ht}}$



Figure A.6: Sensitivity study - C_{D_i} vs. x_{pos_w} - λ_{ht}

Appendix B

Aerodynamic Derivatives Estimation Verification

The complete results for the aerodynamic derivatives estimation verification procedure are hereby presented.



Figure B.1: $C_{m_{\alpha}} - C_{L_q}$ verification



Figure B.2: $C_{L_{\alpha}} - C_{m_{\alpha}}$ verification



Figure B.3: $C_{L_q} - C_{m_q}$ verification



Figure B.4: $C_{L_{\delta_e}} - C_{m_{\delta_e}}$ verification

Appendix C

Flight Dynamics Model Verification

The flight dynamics model has been verified in terms of short period damping ratio and undamped natural frequency for $C_{L_{\alpha}}$, C_{m_q} , I_{yy} and MTOM. The control derivatives $C_{m_{\delta_e}}$ and $C_{L_{\delta_e}}$ have no impact, so they are not shown. The results for the remaining derivatives, $C_{m_{\alpha}}$ and C_{L_q} are presented in Figure C.1.



Figure C.1: ζ_{sp} - $\omega_{n_{sp}}$ vs. $C_{m_{\alpha}}$ - C_{L_q} verification