SHIP VIBRATION

BY

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Synopsis.—*The paper gives the results of research on the hull vibration of passenger vessels, the contribution made by the big superstructures in such ships being one of the more doubtful items in the calculation of hull frequency.*

1. *Introduction*

SHIP vibration has been one of the approved subjects of research at the Ship Division of the National Physical Laboratory since 1928. The principal object of the work has been to develop methods whereby the natural frequencies of a vessel's hull can be calculated from the drawings while she is still in the design stage. At first this was done by the use of a simple formula involving only the principal dimensions of the ship and an empirical coefficient derived from a similar ship[1]. This method is never very exact, because there are always some differences between the prototype and the new vessel, and the frequency depends on the condition of loading, the shape of the under-water hull and many other features which cannot be taken into account in a simple formula of this type.

A ship forms an elastic girder of varying cross-section, with a varying load along the length, due to the uneven distribution of weight and of the entrained water in which she floats. It is possible, if the distribution of all these factors is known, to calculate the natural frequencies of such a beam from first principles, without the use of any empirical coefficient. Such a method was developed in 1933, using results for the inertia of the surrounding fluid obtained mathematically by Lewis in America[2]. The acid test of any such calculation is, of course, to see whether or not it gives the correct answer. The method was therefore applied to thirteen ships for which the two-node vertical frequency had been observed at sea. Except for two cases, where special conditions were known to exist, the agreement was remarkably good, the calculated and observed values for the other eleven ships being generally within about 3·0 per cent.[3]

This was a great step forward, for it was now possible to calculate the two-node vertical frequency for any ship from the drawings, without the necessity of having measured frequencies on a similar ship, and for any distribution of cargo, shape of under-water form or distribution of strength in the cross-section.

Two things should be noted about these results: the modulus of elasticity of the steel was taken at its test-piece value of 30×10^6 lb/sq. inch and most of the ships were tankers (8 out of 13) while the cargo ships had in general only short erections—the longest erection covered 36 per cent. of the length, and for this ship the calculated frequency, giving the long bridge its full value in the moment of inertia distribution, was 5 per cent. higher than the observed figure, which suggested that some allowance should be made by tapering off the inertia at each end of the bridge.

(1) See Bibliography for references.

Further calculations made at the same time indicated that the distribution of inertia for the main girder towards the ends was of minor importance, the difference between using the correct distribution and assuming it to be constant throughout the length and equal to that at midships being in a typical case only some two per cent., whereas the correct distribution of weight and of entrained water was essential in order to obtain accurate results.

The three aspects of the problem which remained unsolved by this work were the frequencies of the higher modes of vertical vibration, the frequencies of horizontal vibration, and the effect of superstructures of different lengths upon the calculated frequencies.

The research has been pursued as and when opportunity occurred, and further results have been published from time to time[4][5]. The present paper gives the measured and calculated frequencies for a number of ships on which experiments have been carried out either just before the outbreak of war in 1939, or in the last year or so, during which it has been possible to resume this particular branch of our work. While the research has been directed in general towards vessels with long superstructures, the opportunity has not been lost of doing similar work upon other ships when facilities were offered.

2. *Experimental Procedure*

The vibration has in most cases been measured by means of the Cambridge Low Period Vibrograph, described in an earlier paper[1]. It records either vertical or horizontal vibration, the trace being obtained on celluloid, which is very durable and weatherproof. Simultaneously, records are also marked on the film, of time and engine revolutions, the latter signal being obtained from a contact on the propeller shaft. The record is subsequently projected on a screen and the amplitude, frequency and shaft revolutions measured. The instrument has a lowest frequency of about 28 per minute, and so can be used to measure frequencies as low as 60 per minute without excessive dynamic magnification. It has been accurately calibrated, and the necessary correction due to frequency is applied to the records before any plotting is made. When the ship is moving in a seaway the instrument fails because of the large movements of the hull, and under such conditions a Cambridge Accelerometer is used. This records both horizontal and vertical acceleration and time on a celluloid strip. While it overcomes the pitching and rolling interference, it is not of much use for low frequency vibration, since the accelerations involved are so small, but it is a useful adjunct to the larger instrument. The ideal vibrograph for use on board ship has yet to be designed, although experimental ones involving electronic control or recording are in use.

On any particular ship, records are first taken at a fixed position—preferably on the stern—while the engine revolutions are slowly increased by small steps. If any resonance is observed, the engines are then kept at the requisite speed while records are taken along the deck in order to obtain the vibration profile and so to determine the mode of vibration. It may happen, of course, that the natural frequencies lie outside the range of engine revolutions, or that the unbalanced forces in the engines are insufficient to excite them. In such cases it is sometimes possible to measure the natural frequency of the two-node vertical vibration during anchor trials.

3. *Description of Ships*

The two-node vertical criticals have been measured on thirteen ships, the principal particulars of which are shown in Table I. They are numbered 13 to 25 in succession to those ships described in earlier papers. Nos. 13 to 16 have been briefly referred to previously in paper 5. All but four are passenger and cargo ships with substantially long erections. The moulded depths at side to the uppermost continuous deck are shown, and also to each

TABLE 1

Principal Particulars of Ships on which Vibration Tests have been carried out

Ship No.	Type	Moulded Dimensions: Length B.P.	Moulded Dimensions: Beam mld.	Moulded Dimensions: Depth feet	Length of Super-structure as % of L.B.P.	Draught mid-ships d	Displace-ment in tons Δ	% of full load	V.I.F.	B/d	Two-node Vertical Frequency per minute: Calculated	Two-node Vertical Frequency per minute: Observed	Observed 2-node horizontal	Observed 3-node vertical	Type of engines
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
13	T.S. pass. and cargo	445′	61′	35·5 to Upper Deck 44·0 to Br. Deck 52·5 to Boat Deck	100% 43% 35·5%	16·64	8,365	60%	2·396	3·66	to Upper Dk. 79·7 to Br. Deck 110	102			Steam
14	T.S. pass. and cargo	450′	65′	44·5 to B Deck 53·0 to A Deck 61·5 to Sports Deck	100% 59% 45%	17·67	7,776	74%	2·255	3·68	to B Deck 110·5 to A Deck 126·5	124			Diesel
15	T.S. pass. and cargo	460′	66′	35·5 to Shel. Deck 43·5 to Br. Deck 51·5 to Prom. Deck 60·5 to Boat Deck	100% 75% 62% 48%	19·42	10,300	79·4%	2·235	3·40	— to Br. Deck 95·6 to Prom. Deck 109·3 to Boat Deck 118·5	112		269	Diesel
16	T.S. pass. and cargo	585′	76′	45·5 to C Deck 54·5 to B Deck 62·5 to A Deck 71·0 to Boat Deck	100% 81·5 or 60% 74·1 or 48·7% 48·7%	24·46	19,918	85%	2·251	3·11	to B Deck 69·2 to A Deck 78·5	81	108		Steam Turbine
17	Q.S. pass. and cargo	650′	84·5′	48·5 to B Deck 56·5 to A Deck 64·5 to Prom. Deck 72·5 to Lounge Dk. 82·5 to Games Dk.	100% 63·5% 63·5% 47·2% 36·3%	24·37	24,867	100%	2·249	3·394	to Prom. Dk. 78·3 to Lounge Dk. 87·8	79	108		Diesel
18	T.S. cargo	495·5′	68·0′	41·5 to Upper Deck 49·75 to Br. Deck	100% 46·8%	22·42 27·04 16·91	14,500 18,130 10,390	80% 100% 57%	2·162	3·07 2·51 4·01	to Upper Dk. 80·4 to Br. Deck 95·5	89 102	122 114 128	225 209 243	Diesel
19	S.S. pass. and cargo	375′	54·5′	27·0 to Upper Deck 34·75 to Br. Deck 42·50 to Prom. Dk. 50·50 to Boat Deck	100% 84% 40% 40%	20·0 14·79	7,683 5,376	89·5% 62·6%	2·033 2·305	2·72 3·69	to Br. Deck 100·5 to Prom. Dk. 118·0 to Br. Deck 110·6 to Prom. Dk. 129·4	117·5 133			Diesel
20	S.S. pass. and cargo	425′	58·0′	34·5 to Upper Deck 44·0 to Br. Deck	100% 36·4%	12·71	6,070		2·66	4·65	to Upper Deck 99·5 to Br. Deck 120·8	105·5			Diesel
21	S.S. cargo coaster	180′	32·83′	20·5 to Upper Deck	100%	8·23	887		2·34	3·99	to Upper Dk. 261	243			Diesel
22	S.S. cargo collier	257′	39·33′	22·5 to R.Q. Deck 18·75 to Main Deck	70% 30%	16·75	3,371	95·5%	1·91	2·35	including hatch coamings 147	140	196	320	Single screw driven by Twin Diesels
23	S.S. coaster	210′	36·5′	21·66 to Upper Dk. 14·08 to 2nd Deck	100% 100%	14·375	2,178	100%	2·012	2·54	—	157+			Steam Recip-rocating
24	S.S. cargo	469·5′	64·5′	42·0 to Shel. Deck 50·0 to Br. Deck 58·0 to Boat Deck	100% 24·2% 24·2%	24·17	14,454			2·66	—	90			Turbine
25	T. Steam tanker	382′	62·5′	18·75 H br. Deck. 27·00 Trunk Deck		9·87 16·37	5,159 8,985	57·5% 100%		6·33 3·81	— —	78 69½			Steam Recipro-cating

of the superstructure decks, the percentage of length covered by each of these latter being also given. The moment of inertia of the midship section has been calculated to each of these decks in turn. Outline profiles and sections are shown in Fig. 1(a) and (b).

4. *Method of Calculating the Two-node Vertical Natural Frequency*

The method of calculating the two-node vertical natural frequency has been described in some detail in the appendix to paper 3, to which reference may be made. It is sufficient to point out here where some departures have been made in the present calculations.

The added virtual mass curve is obtained in the manner described in that paper, and added to the weight curve to give the total load curve. The vibration profile is then assumed to be the same as that for a uniform, free-free beam, and on the assumption that the vibration is of the simple harmonic, isochronous type, this profile also represents the acceleration to some scale. Thus the product of the ordinates of the total weight curve and the acceleration curve at any station represents the dynamic load at that point, and in this way a dynamic-load curve for the whole ship is obtained. Integration of this curve gives the shear force curve. This does not, in general, close, and the base of the acceleration curve must be moved until it does, thus ensuring the necessary condition that the centre of gravity of the ship remains at rest during the vibration. A second integration, of the s.f. curve, gives the bending-moment curve, and again, in general, this will not close. To correct this means going back to the assumed profile or acceleration curve and rotating it about the centroid of the curve of total mass (i.e., including the virtual mass of the water). This, in turn, upsets the s.f. curve again, and in the past a number of calculations had to be done until the residual bending moment was quite small, when the curve of b.m. was closed by drawing a new base line, and two more integrations then gave the derived profile. From this and the assumed acceleration curve the frequency can be determined.

In 1932 a method of determining the vertical shift and the rotation of the base line of the assumed profile, to ensure that both the s.f. and b.m. curves would close, was described by Schladofsky, and a translation of this work has recently been made available to the Authors by Captain H. E. Saunders, U.S.N., until recently Director of the David Taylor Model Basin, Washington, for which courtesy they wish to extend to him their thanks[6]. This method is briefly described in Appendix I of the present paper, and has been used in the calculations for the more recent ships, and results in a great saving of time and labour.

In all the calculations, the moment of inertia has been assumed to be constant along the whole length of the ship and equal to the value amidships, treating the deck as being complete right across the ship, and thus ignoring hatch openings, and coamings. Thin engine-room casing and deckhouse sides have been omitted. The only exception to this is vessel 22, which had very wide hatches, the coamings of which were continuous and formed a substantial part of the ship's structure.

In every case, the calculated frequency has been corrected to take account of the deflection due to shear by using the approximate method described before this Institution by Lockwood Taylor in 1927.[7]

5. *Comparison between Calculated and Measured Frequencies*

A complete detailed calculation has been made of the two-node, vertical natural frequencies for eleven of the ships in the condition in which the actual frequencies were measured. In making these, the moment of inertia has in general been calculated to the uppermost continuous deck, and then to each superstructure deck in turn. These calculated frequencies are shown in column 12 of Table 1, and may be compared with those observed on the ships, as shown in column 13.

In two of the ships (Nos. 21 and 22) there were no appreciable superstructures above the uppermost continuous deck. Results for a number of other vessels of this kind have already been published(3) and are reproduced for reference in Table 2.

TABLE 2.—*Calculated and Observed Values of Two-node Vertical Frequencies for Vessels with no Substantial Superstructures*

Ship No.	Frequency per minute		Percentage Difference (+for calculated above observed)	Reference letter of ship in paper 3.
	Calculated	Observed		
1	74·2	78·5	−5·0	B
2	107·5	105·5	+2·0	D
3	102·5	104·5	−1·8	H
4	121·0	115·0	+5·0	K
5	91·3	90·5	+0·8	M
6	95·8	98·5	−2·8	O
8	81·5	80·0	+1·8	S
10	112·2	109·0	+2·9	G
12	77·5	78·5	−1·2	N
21	261	243·0	+7·2	—
22	147	140·0	+5·0	—

Considering the complexity of structure in a ship and the difficulties of allowing for all the discontinuities of decks in way of hatches, engine-room casings, etc., the agreement is considered to be good. The greatest differences of 5 per cent. or so occur generally with the smaller vessels.

The vessels with superstructures fall into two classes: those with long upper works covering 60 per cent. or more of the length, and those with short bridges. There are three of this latter class in the present selection of ships—Nos. 13, 18 and 20. In all these cases the side shell is carried up to the bridge deck throughout its length, and the latter is therefore of substantial construction. On the other hand, the bridge only covers 43, 46 and 36 per cent. of the length respectively in the three cases, and therefore does not cover the nodes. A comparision of calculated and observed frequencies is shown in Table 3.

TABLE 3.—*Calculated and Observed Values of Two-node Vertical Frequencies for Vessels with Short Bridges*

Ship No.	Frequency per minute			Percentage of length covered by bridge	Percentage difference (+ for calculated above observed)	
	Calculated to uppermost continuous deck	Calculated to Bridge deck	Observed		to 100% deck	to Bridge deck
13	79·7	110	102	43	−20·8	+ 8·5
18	80·4	95·5	89	46	−10·6	+ 7·2
20	99·5	120·8	105·5	36	− 5·7	+14·5

In ships 18 and 20, there is no further superstructure deck above the bridge deck, and it is obvious that the bridge is playing an important part in the stiffness of the girder against vibration, but that this effect decreases with decrease in length of the bridge. This is to be expected, since when the bridge is very short, as in tankers, it ceases to act as part of the hull girder and becomes practically speaking only a concentrated load.

No 13, on the other hand, had a boat deck, covering 35 per cent. of the length of the ship, above the bridge deck, and this makes the comparison on a basis of the percentage covered by the bridge rather misleading. The figures for calculated and observed frequencies suggest that this boat deck is providing some stiffness, and that in consequence the length of bridge should be virtually increased in this ship for comparative purposes.

The remaining ships were all of the passenger or passenger and cargo type with long superstructure decks.

An examination of the observed frequencies with those calculated to different decks suggests that any deck covering 60 per cent. or more of the length of ship, and therefore covering also the nodes, is fully effective as far as the stresses in vibration are concerned. The data for these ships are shown in Table 4.

TABLE 4.—*Calculated and observed Values of Two-node Vertical Frequencies for Vessels with Long Superstructures*

Ship No.	Frequency per minute		Percentage of length covered highest deck used in calculation	Percentage difference in frequency (+ for calculated above observed)
	Calculated to highest deck covering 60% or more of vessel's length	Observed		
14	126·5	124	60	+ 2·0
15	109·3	112	62	− 2·2
16	78·5	81	74	− 3·0
17	78·3	79	63	− 0·8
19	100·5	117·5	84	−14·5

On the above basis, the agreement between calculated and observed frequencies is very good for the first four vessels. Nos. 14, 15 and 16 each had one further deck above that included in the calculations, which was in general a light boat or sports deck covering about 47 per cent. of the ship's length. No. 17 had a lounge deck covering 47 per cent. and above this a very light games deck covering 36 per cent., which would, it is thought, have little if any effect on the natural frequency. No. 19 had a promenade deck and above it a boat deck each covering 40 per cent. of the length, and these have both been ignored in the calculation because they did not cover 60 per cent. of the ship's length. If we assume that the presence of these two decks has virtually the effect of lengthening the promenade deck and we include it in the inertia calculation, then the calculated frequency becomes 118, in good agreement with the observed figure.

6. *Approximate Formulae*

While it is believed that to take account of all the factors in a new ship it is necessary to make a complete calculation such as that described above, the Authors recognize the great convenience to the naval architect of having a simple formula which will give the natural frequency with a minimum of calculation from data which are available in the early design stages.

The first such formula was given by Schlick some sixty years ago :

$$N = \phi \sqrt{\frac{I}{\Delta L^3}} \qquad \text{(1)}$$

where N = frequency per minute of two-node vertical vibration;

I = moment of inertia of midship section in inch ^{2} feet ^{2} units;

Δ = displacement in tons,

and L = length b.p. in feet.

ϕ was an empirical coefficient to be derived from actual observations on ships.

Two major difficulties arise in using this formula. First, it ignores the effect of the virtual mass due to the surrounding water, and secondly in vessels with superstructures above the topmost continuous deck there is always considerable doubt as to what material should be included in the calculation of I. In 1935, Burrill suggested a similar formula, but incorporating two factors to take account of the surrounding water and the shear correction respectively(8).

The results given in the present paper have been analysed, and plotted in Fig. 3, on the basis of a modified parameter $\sqrt{\frac{I}{\Delta_1 L^3}}$, where Δ_1 is the displacement including the added virtual mass due to the surrounding water. The latter can be calculated from the shape of the underwater hull, making certain assumptions, in the manner described in paper (4). The ratio $\frac{\Delta_1}{\Delta}$ is called the " virtual inertia factor " and its values for the present series of ships are given in column 10 of Table 1.

The moment of inertia I has been calculated to different decks in turn, according to the general arrangement of the particular ship in question, and the spots in Fig. 3 have been arranged to show the effect of including the superstructures of varying length.

If this diagram is examined carefully, the Authors believe it will be agreed, that the line drawn there is a reasonably good mean of the spots derived from those ships having no appreciable superstructures, i.e., tankers and cargo ships with only poop, very short bridge and forecastle. Examining the spots for the other vessels and their relationship to this line, it appears that in general most satisfactory results for ships with long superstructures will be obtained by including in the moment of inertia all decks covering 60 per cent. or more of the ship's length, while in certain cases, depending on the particular arrangement of the ship, decks covering between 40 and 60 per cent. of the length must also be included in the calculation of I. In other cases, marked on Fig. 3, there were in addition light decks such as sports and boat decks which have not been included in the calculation. In the case of vessel 20, which had only one superstructure deck above the top continuous deck, a bridge covering 36 per cent. of the ship's length and with side shell carried up in way of the bridge, it is obviously necessary to allow for this to some extent, to obtain reasonable agreement with the suggested average line.

In all, there are results for twenty-two ships plotted on Fig. 3, and apart from five exceptions, the observed frequency is within 5 per cent. of that given by the drawn line, and in most cases the difference is very much less. For some of the exceptions, no explanation can be given, but No. 22 was a vessel of peculiar construction, and in No. 23 the actual frequency was not quite reached because it lay just above the maximum permissible engine revolutions—the real frequency is somewhat above that plotted, as indicated by an arrow in the figure.

The Schlick formula, as modified above, involves both the calculation of the moment of inertia of the cross-section amidships and also of the virtual inertia due to the surrounding fluid, which necessitates a knowledge of the actual lines of the vessel. Both these calculations take time, and it would be a great convenience to naval architects and engineers if a simpler approximate formula could be evolved which would give results of comparable accuracy and yet avoid the necessity for these calculations. Such a formula was, in fact, proposed by one of the present Authors in 1931 (1), the value of I being assumed proportional to BD^3, and for tankers and cargo ships having no substantial erections, and in which, therefore, there was no question of the appropriate value of D to be used, it gave very promising results.

As originally stated, it was of the form

$$N = \beta \sqrt{\frac{B\,D^3}{\Delta\,L^3}} \quad \text{.........(2)}$$

where B = the breadth moulded in feet
and D = the depth moulded at side, in feet, to topmost continuous deck. The other symbols are as previously defined. Values of β have been given for a number of ships in papers (1) and (4), and it was concluded that empirical formulae of this type were only useful for comparing vessels of closely similar type in the same general condition of loading.

When we come to consider vessels with long superstructures, we meet the added difficulty of knowing the correct depth D to use in such a formula in order to make some allowance for their various lengths. Several methods were tried to find an equivalent depth for the ship to allow for the different lengths and heights of superstructures. Finally, that first proposed by Lundberg in 1932[9] was found to be the most effective, and it has been developed to take account of more than one tier of upper works.

If we have a vessel of length L with, say, two superstructure decks of length L_1 and L_2 respectively, the depths to the topmost continuous deck and to the superstructure decks being D, D_1 and D_2, respectively (see Fig. 4), then the equivalent depth of ship has been expressed as

$$D_E = \sqrt[3]{D^3(1-x_1) + D_1^3\,(x_1-x_2) + D_2^3 \cdot x_2} \quad \text{.........(3)}$$

where $x_1 = \dfrac{L_1}{L}$ and $x_2 = \dfrac{L_2}{L}$

This can obviously be extended to more decks as necessary.

Short forecastles and poops and bridges such as those in oil tankers have been neglected as being too short to influence the stiffness of the girder.

To avoid the second calculation, that of the virtual mass of the surrounding water, all the calculated inertia factors have been plotted in Fig. 2 to a base of beam to draught ratio B/d.

The virtual inertia factor is the ratio of the total displacement, including the entrained water, to the actual ship displacement, and in the notation used above, is equal to the ratio $\dfrac{\Delta_1}{\Delta}$.

It will be seen from Fig. 2 that the expression

$$\Delta_1 = \Delta\left(\frac{1}{3} \cdot \frac{B}{d} + 1{\cdot}2\right) \quad \text{.........(4)}$$

gives a very good approximation to the values calculated by the detailed method.

Replacing D and Δ, in equation (2) by the modified values given in (3) and (4), we have

$$\mathrm{N} = \beta \sqrt{\frac{B.\,D_E^3}{\Delta_1\,L^3}} \quad \text{.........(5)}$$

The values of the observed frequency N have been plotted on this basis in Fig. 5.

The available data seem to indicate that on this basis of plotting, tankers with a longitudinal system of construction must be treated separately from cargo and passenger ships. Two mean lines have been drawn for these two classes of ship, and these indicate that for the same value of $\sqrt{\dfrac{B.\,D_E^3}{\Delta_1\,L^3}}$ the tankers are stiffer and give frequencies about 10 per cent. higher than the cargo and passenger ships. Results for some 9 tankers are shown in Fig. 5, and for 8 of these the departure from the mean line never exceeds 3 per cent. The exception is No. 6, which does not plot on Fig. 3 either, although the detailed calculation gave a result within 3 per cent. of the observed figure. There is no obvious explanation of this difference.

Tankers Nos. 2, 3, 4 and 5 were small and of trunk-deck type used for feeder services in the West Indies. They all had narrow harbour decks, and the depth used in the above equation has been measured to the top of trunk. No. 25 was an ocean-going tanker and had a very wide harbour deck along each side (12 feet). For this vessel D_E has been calculated to make allowance for this section.

In drawing the mean line for the cargo and passenger ships, primary consideration was given to those vessels with no appreciable superstructures, viz: 10, 11, 12, 18, 21 and 22. For vessels with long superstructures, the effect of including or neglecting the topmost decks when these are of light construction, and therefore of using different equivalent depths D_E, is clearly shown in Fig. 5.

Considering the variation in types of ships and in the extent and arrangement of their superstructures, the results have plotted extraordinarily well. The Authors believe that the intelligent use of this diagram in association with the profiles shown in Figs. 1(a) and (b) will enable designers to make a very close estimate of the two-node vertical frequency for a new ship before the information required for a detailed calculation is available.

7. *Effect of Change in Displacement*

For three ships results have been measured for two different draughts. Two of them were cargo-passenger types, Nos. 18 and 19, and one a tanker, No. 25.

Calculating Δ_1, from the same approximate formula (4), these can be plotted and are shown in Fig. 6 together with the mean lines already drawn on Fig. 5. It will be seen that with decreasing displacement there is a tendency for the frequency to increase rather more rapidly than would be expected from the slope of the mean lines.

8. *Horizontal Vibration*

The two-node horizontal natural frequency has been observed on four ships, and the results are listed in Table 5.

TABLE 5

Ship	2-node vertical frequency per minute	2-node horizontal frequency per minute	Ratio of $\frac{\text{horizontal frequency}}{\text{vertical frequency}}$
16	81	108	1·34
17	79	108	1·37
18	89	122	1·37
22	140	196	1·40

For the vessels in reasonably loaded condition, the average value of the ratio of the horizontal to vertical two-node frequencies is about 1·37.

In addition to the above figures, the two frequencies were also measured in a lighter condition (57 per cent. of load displacement instead of 80 per cent.) and the ratio was then found to be 1·25, the frequencies being 102 and 128 per minute. This appears to be the opposite of what would be expected since due to the virtual inertia, the horizontal frequency would be expected to increase more rapidly than the vertical with decrease in draught.

9. *Three-node Vertical Frequencies*

The three-node vertical natural frequency was measured on three ships (Table 6):

TABLE 6

Ship	2-node vertical frequency per minute	3-node vertical frequency per minute	Ratio 3-node frequency / 2-node frequency
15	112	269	2·40
18	89 (80% load)	225	2·53
	102 (57% load)	243	2·38
22	140	320	2·28

For a uniform beam the ratio is 2·76, while for one of uniform depth but consisting of two wedges in plan, it is 2·26. The above ratios therefore appear to be reasonable. For very fine ships such as destroyers there is evidence that the ratio approaches 2·0.

10. *Conclusions*

It is believed that the results given in this paper represent a further step towards assessing the strength value of superstructures in vibration problems. The detailed method of calculation remains the only one possible for a new design where measured results for similar ships are not available. In using it, discretion must be used in making allowance for the superstructure decks, but the results given here would suggest that any deck covering more than 60 per cent. of the vessel's length may be taken as fully effective. The inclusion of any higher decks of shorter length will depend very much on the arrangement of the individual ship, and some guidance on this point can be obtained from the profiles in Fig. 1 and the remarks in the text and tables.

The use of the method of correcting the vibration profile base line due to Schladofsky has proved very useful in saving time and reducing the labour involved in the detailed calculation.

A further refinement of this calculation would be to use a moment of inertia curve showing the actual distribution along the length rather than a uniform value equal to that amidships. This, however, would involve a great increase in the work, since it would mean calculating I for perhaps 10 or 12 sections along the length, and there has not been time to carry this out even if all the relevant information were available. In any case, calculations to find the effect of such a process have shown that it is of a secondary character (paper 3).

Two approximate formulae have also been developed. The first of these involves the detail calculation both of the moment of inertia of the midship section and of the amount and distribution of the entrained water. In the second, the moment of inertia has been assumed to be proportional to BD_E^3, where D_E is an equivalent depth designed to allow for the varying lengths of superstructure, while the amount of entrained water has been assumed to depend on the ratio of beam to draught. The approximation to the inertia of midship section will only be expected to apply to vessels built to a common strength standard, such as, for example, the classification societies rules, and special care would have to be taken where owners ask for additional scantlings, the vessel is strengthened for ice, or similar cases.

Either formula appears to form a good basis for approximate estimates of the two-node vertical frequency. The first, which includes I, involves the use of a certain amount of judgment because it does not intrinsically take any account of the lengths of the superstructure decks. In the second, this is allowed for on an empirical basis. It would appear from the second plotting that the cross-sections of tankers are somewhat stiffer than those of cargo and passenger ships for the same beam and equivalent depth, because it is necessary to discriminate between these two classes and draw separate lines for them. This difference exists even between tankers and cargo ships when the latter also have no substantial superstructures and is presumably a real difference between longitudinally and transversely framed ships.

The measurement of higher frequencies is much less easy. The amplitudes are very small and it is very often extremely difficult to decide from the distribution along the deck just what is the proper vibration profile, i.e., whether it has three or four or more nodes. The figures given in the paper for higher frequencies are believed to be correct for the modes of vibration, stated, although it was not possible to measure the profiles. A large number of measured frequencies of these higher types are required in order either to compare them on an empirical basis or with detail calculations, and it is hoped that with the return of normal trial procedures more opportunities will occur to continue this work.

11. *Acknowledgments*

The work described above has been carried out as part of the research programme of the National Physical Laboratory and this paper is published by permission of the Director of the Laboratory.

The Authors wish to express their thanks to the shipbuilders, engine builders and shipowners concerned, not only for permission to carry out the tests on their ships, but also for the very willing and generous help that has always been given during the fitting up of the apparatus and the conduct of the experiments.

They would also like to acknowledge the assistance rendered by Mr. F. Gridley of the Ship Division Staff, who has attended most of the trials and has suggested, and made, many improvements to the recording instruments.

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APPENDIX 1

In a paper given by Todd in 1933 (3) a method was detailed for calculating the two-node vertical frequency when all the necessary information was available. This method has been adhered to in all the calculations made for this publication. It was, however, soon discovered that in spite of the correction for closing the dynamic-shear-force curves, the integration of this s.f. curve invariably failed to close and in many cases left a residual bending moment at the fore perpendicular which was far too great to correct by simply joining the ends of the bending-moment curve as shown in Fig. 7.

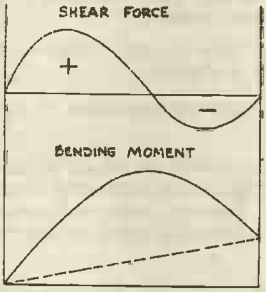

Fig. 7

The following method was adopted by the Authors to ensure that both the s.f. diagram and bending-moment diagram closed, with a minimum of calculation. The method is due to E. Schadlofsky and was first published in 1932 (6) but the Authors believe it has not yet been published in this country.

By using two simple formulae it is possible to predict from a single performance of the standard calculation, combined with a calculation of the longitudinal moment of inertia of the total mass curve about its own axis, the vertical parallel movement of the vibration profile base in order to close the shear-force curve, and the rotation of the base required to close the bending-moment diagram.

(1) The parallel shift of the base

y_s = Residual Dynamic Shear Force at Fore Perpendicular / Total Mass

where the residual dynamic shear force is the value of the s.f. ordinate at the f.p. and the total mass is the sum of the ship's weight and that of the entrained water.

(2) The rotation of the base at the f.p. about the centre of gravity of the total mass curve

$$y_{o\,F.P.} = \frac{L_2}{J(y_{1\,F.P.} - y_s)} \left[RM^{F.P.} - L_2 RS^{F.P.} \right]$$

where L_2 is the distance in feet from the f.p. to the centre of gravity of the total mass curve.

J is the longitudinal moment of inertia of the total mass curve about its own axis.

$Y_{1F.P.}$ is the original ordinate at the f.p. of the assumed vibration profile (which is 1·0).

$RM_{F.P.}$ is the residual bending moment at the fore perpendicular (i.e. the amount by which the b.m. diagram fails to close).

$RS_{F.P.}$ is the residual dynamic shear force at the f.p. (*i.e.* the amount by which the s.f. diagram fails to close).

After the two corrections have been applied to the original vibration profile the ordinate of this curve at the fore perpendicular is equated to unity. The whole vibration calculation is repeated and both the dynamic s.f. and the dynamic bending moment curves should close. If there are any discrepancies they are usually so small as to be negligible.

Several points must be borne in mind regarding signs, and the following rules must be observed :

(1) The area below the base is negative.

(2) The shifting of the base y_s is given the negative sign when it is downwards.

(3) The residual dynamic s.f. is to be made negative when the lower parts of the dynamic load curve, *m.y.*, are greater than the upper.

(4) The residual dynamic bending moment will be negative when the dynamic mass moment curve ends below the base assuming that its ordinates at the after perpendicular take an upward course.

(5) The rotation of the base must be anti-clockwise at the forward perpendicular when the residual moment is negative.

These facts can be verified easily by inspection of the conditions applying to each individual case.

The ends of the tilted vibration profile are joined by a straight line, and the maximum ordinate between this line and the profile, measured perpendicular to the original base, is the maximum deflection. The end deflection can then be obtained by simple proportion and the frequency then calculated as described in detail in the appendix to an earlier paper (3).

Several calculations have been done by the Authors using this method and it has been found to be very satisfactory.

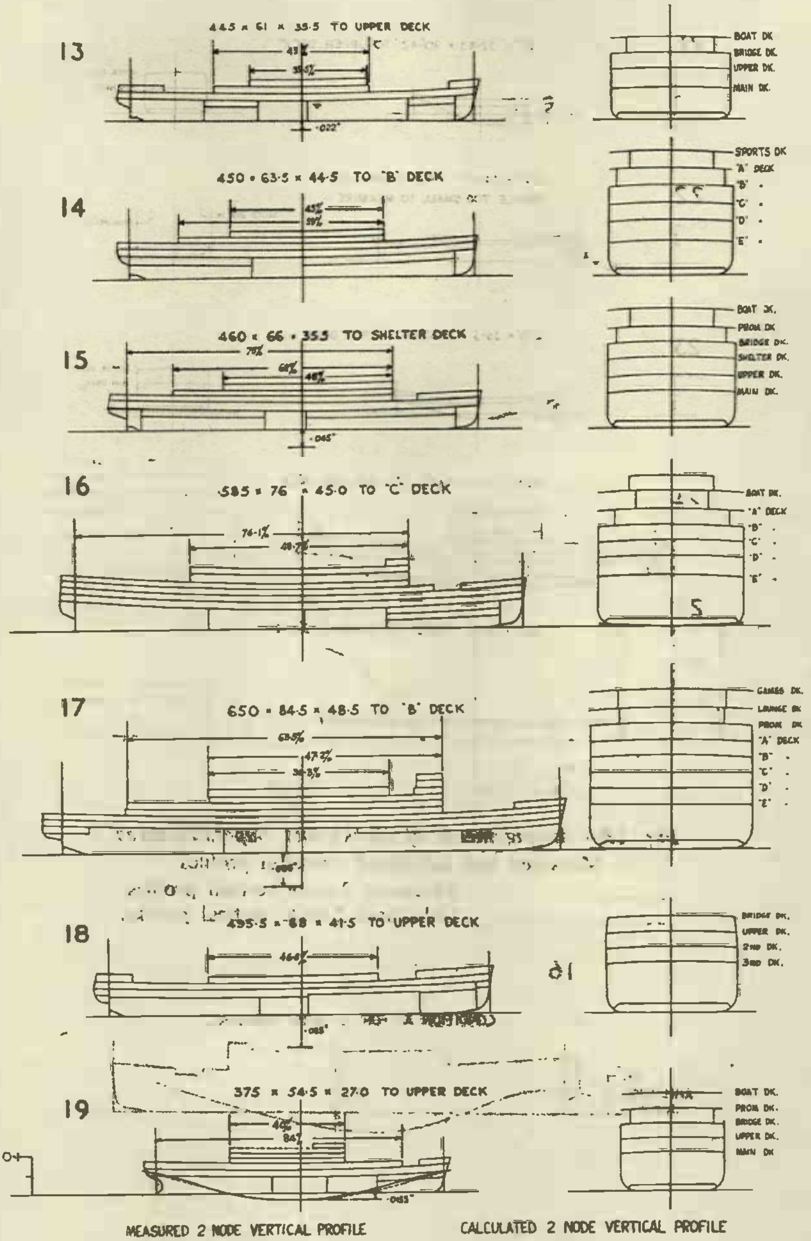

Fig. 1A—Arrangements of Decks and Superstructures

Measured and calculated vibration profiles

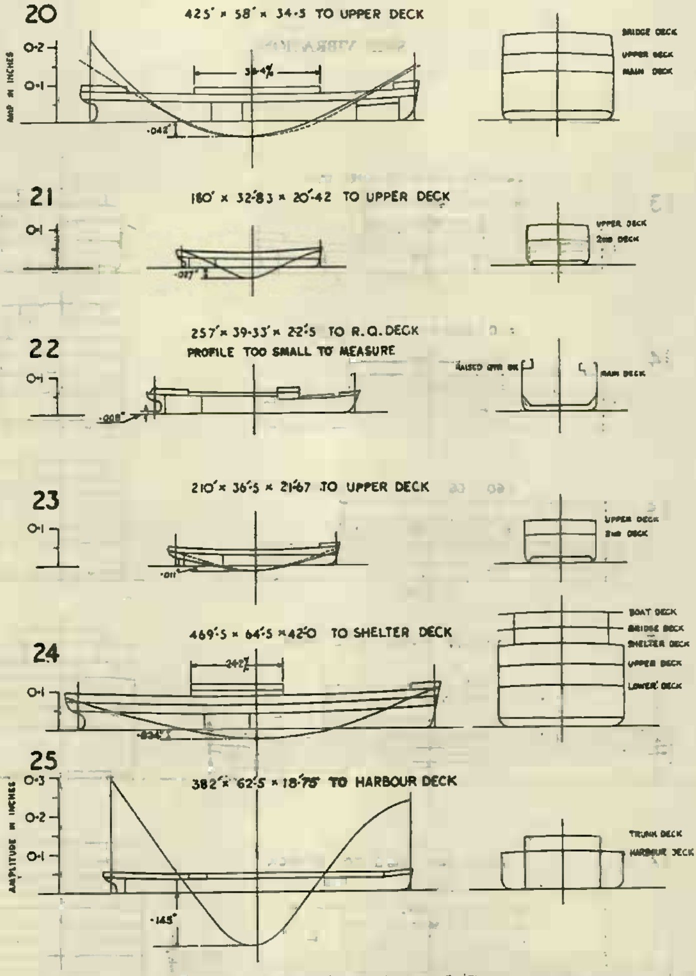

Fig. 1B—Arrangements of Decks and Superstructures
Measured and calculated vibration profiles
________________ Measured 2-node vertical profiles
- - - - - - - - - - - - - - - Calculated 2-node vertical profiles

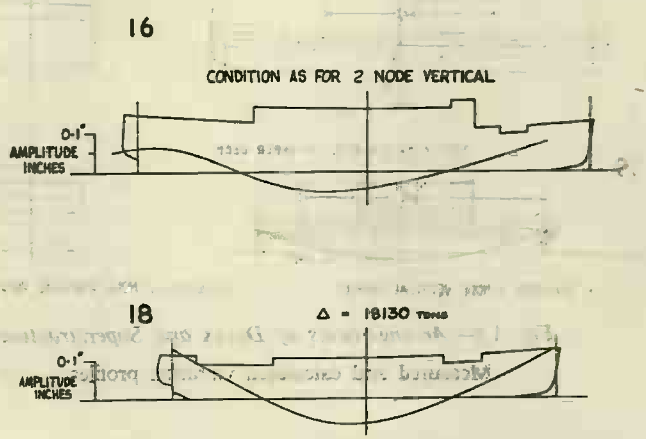

Fig. 1C—Measured Vibration Profiles
2-node horizontal

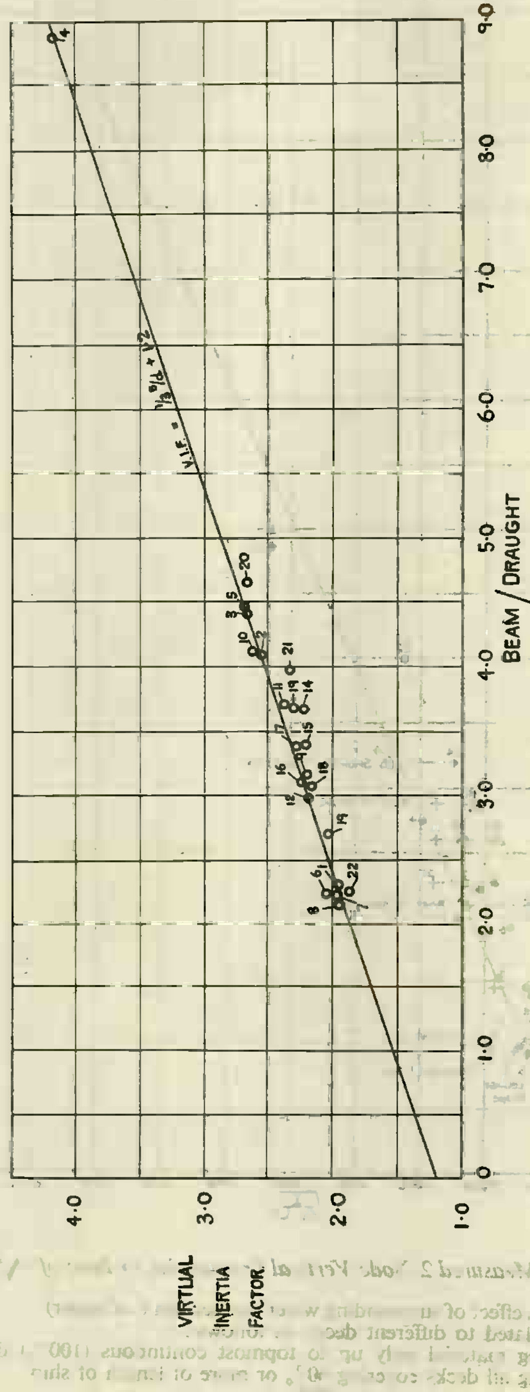

Fig. 2—Virtual Inertia Factors

$$\text{Virtual Inertia Factor} = 1 + \frac{\text{Added virtual mass}}{\text{Displacement}}$$

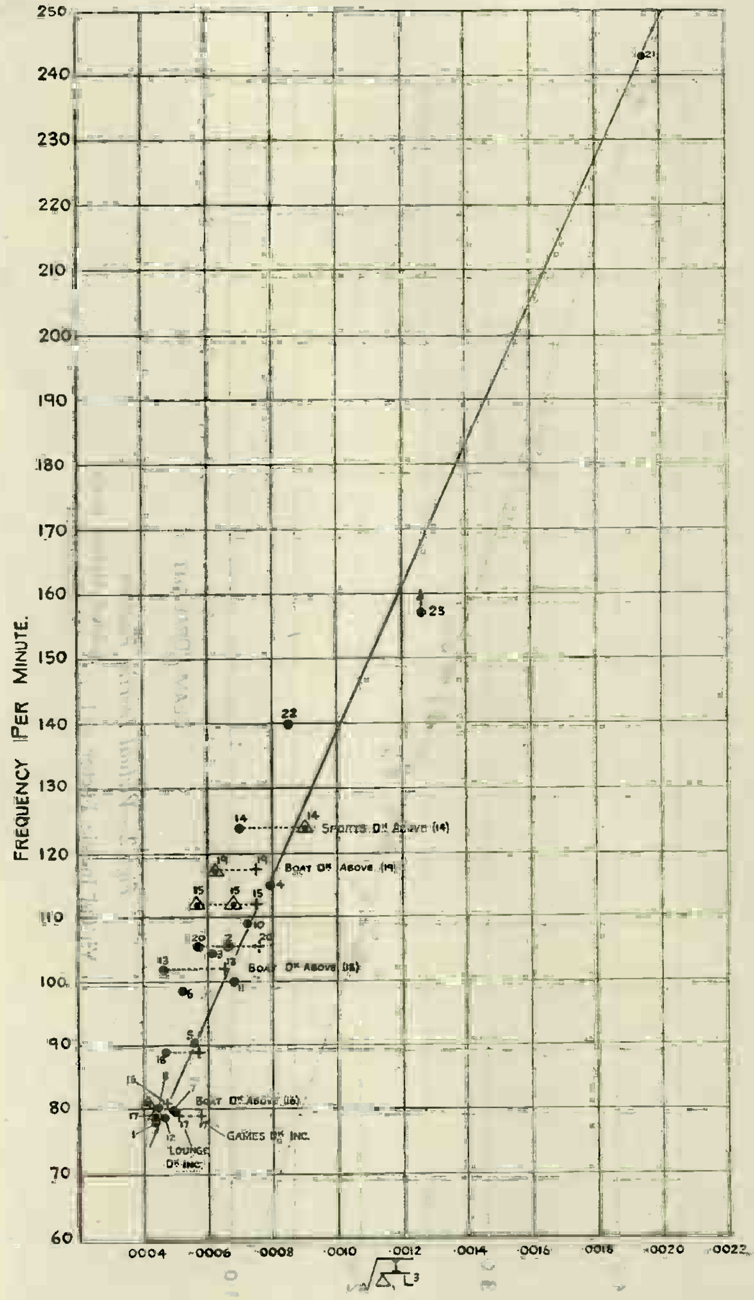

Fig. 3—*Plot of Measured 2-Node Vertical Frequencies to Base of* $\sqrt{\frac{I}{\Delta_1 L^3}}$

Δ_1 Includes effect of surrounding water (see section 6 of paper)
I Is calculated to different decks as follows :—
A Including material only up to topmost continuous (100%) deck
• Including all decks covering 60% or more of length of ship
+ ,, ,, ,, 40% ,, ,, ,,

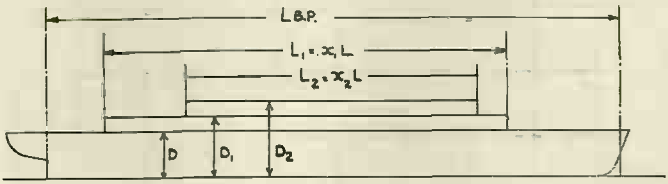

Fig. 4

For Fig. 5 see p. 210

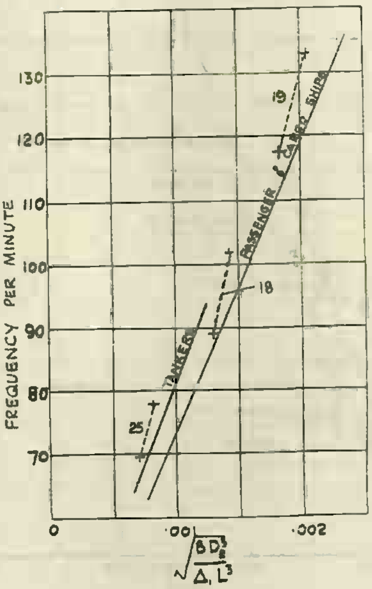

Fig. 6—Plot Showing Effect of Change of Load on 2-Node Vertical Frequency

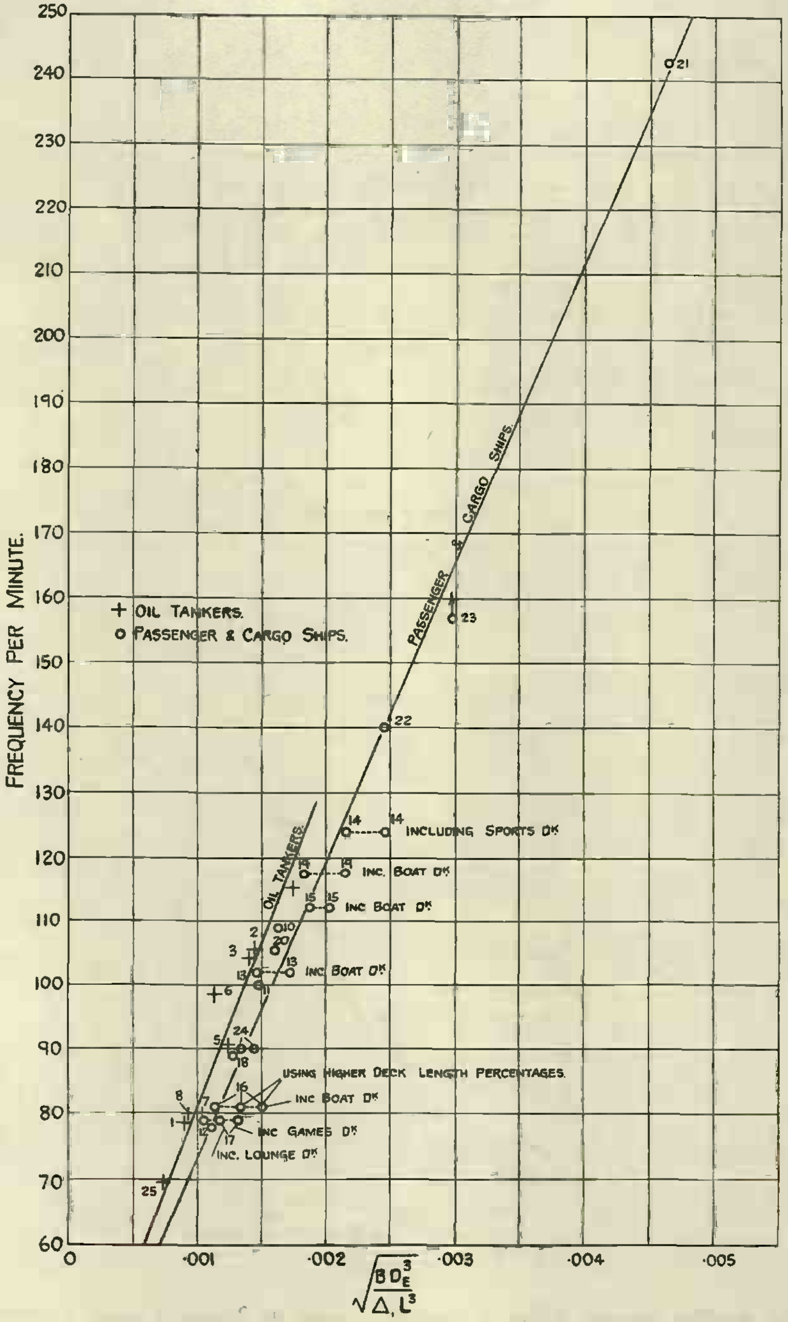

Fig. 5—Plot of Measured 2-Node Vertical Frequencies to Base of $\sqrt{\frac{B\,D_E^3}{\Delta_1\,L^3}}$

Where D_E is the equivalent depth (see equation 3 in text)

" Δ_1 = includes effect of surrounding water (see section 6 of paper)

" B = breadth

" L = length b.p.

DISCUSSION ON "SHIP VIBRATION"*

Sir WESTCOTT ABELL, K.B.E., *Fellow*: I am afraid I am rather out-of-date and have no intimate knowledge of the problem of vibrations such as are described in Dr. Todd and Mr. Marwood's paper. I have rather a simple idea about this problem; it is to me rather like a simple pendulum: that is to say the frequency is inversely as the square root of deflection! I ask myself, is there anything in the deflection of the ship under the loading system which I ought to look into first?

I may tell you one story from which I derived, I suppose by intuition, a considerable amount of kudos. We received a message in Lloyd's office that a ship on trial was shaking herself to bits. They asked what they should do about it, and what were Lloyd's doing in this matter. This was in St. Nazaire, if I remember rightly. We only got this warning that the ship was shaking herself to pieces. We looked at the dimensions of the ship; there was nothing in the problem that seemed unusual, and it occurred to me it was the loading of the ship. When I got the reply it seemed the builders were under contract to put the ship on trial at her load displacement, and they thought the cheapest possible way of loading a ship to displacement was to get sand from the river and pour it into the hold *amidships*. I immediately jumped to the conclusion that the deflection was enormous and brought down her frequency to something that would agree more or less with reciprocating-engine revolutions and I sent back to tell them to take out the load and distribute it more evenly over the whole ship. There was no more trouble.

A French naval architect arrived at the time they were re-trying the ship and he tried to persuade me that the ship was wrong. I said all that was wrong was that some way or another they had increased the deflection beyond what might reasonably be expected since the load was not evenly distributed.

On the question of stiffness with oil tankers, I think I am right in saying there is increased stiffness and less deflection in a tanker with its longitudinal bulkheads than with the ordinary ship. Therefore, if one could estimate in some way the difference of deflection between the systems of construction, a direct numerical correction between the ordinary type and the one with longitudinal strengthening, could be made.

I had a lot of trouble with a certain ship and went through the usual process. We crept up the revolutions of the engines one at a time, and finally the clapper on the ship's bell on the forecastle head began to ring violently. It came time to go to lunch and I said we must leave it. After lunch we returned, started below the revolutions and went up; the ship's bell rang again. It was a question of balancing certain weights and when balanced out, at the next trial, we stepped up the revolutions and the ship's bell did not ring so we knew we were much better. I forgot to add that I found someone had muffled the clapper!

Mr. HARRY HUNTER, O.B.E., *Fellow*: I would agree with the Authors as to the danger of relying on simple empirical formulae—while it is true that in the majority of cases the observed natural frequency is in line with such calculations yet exceptions can and do arise with difficulties for all concerned and consequent trouble is not helped by the fact that it is an exception. Surely the calculations relating to vibration are equal, or nearly equal, in importance to those relating to stability and propulsion and therefore the naval architect should accept the necessity for proper investigation.

On the matter of ship vibration, broadly there are two prime factors involved: the ship which may vibrate and something which may cause it to vibrate—in fact it is the very common case of the hull being a "bell" which will certainly "ring" if struck by a suitable "clapper." Some bell-clapper problems are best solved by going after the bell and others by going after the clapper and others again demand simultaneous action in respect of both sides; in my view ship vibration belongs to this latter class just as does torsional vibration of shafting and for the same reason that a violin ("bell") gives its best performance when it and the bow ("clapper") are under one control.

Dr. T. W. F. Brown in his 1939 paper before this Institution† clearly sets out the benefits and some principles of "parallel treatment" in the section "The Ship and her Main Engines". This view of parallel treatment has also been adopted by the British Shipbuilding Research Association in setting up the Ships' Vibration Committee and one of the first actions of the committee was to order an electrically driven vibration

* Paper by F. H. Todd, B.Sc., Ph.D., *Associate Member* and W. J. Marwood. See p. 193, *ante*.

† "Vibration Problems from the Marine Engineering Point of View," Vol. 55.

exciter for the purpose of investigating ship vibration generally and including the effect of varying magnitude and location of excitation. This machine has been in use in several ships and has given valuable information.

The present paper, as with the vast majority of previous papers on the same subject, deals entirely with the "bell" side of the problem and the only reference to the "clapper" is on p. 194, "It may happen . . . that the unbalanced forces in the engines are insufficient to excite them", (i.e. hull vibration). That one sentence puts my main point in a nutshell, namely, that we engineers are most anxious to know what magnitude, type and location of excitation is acceptable.

Incidentally, the Authors rather imply that the main engines usually provide the excitation, yet two or three of his ships, (16, 24, and 13 (?)) are turbine driven and, therefore, presumably some other "clapper" is at work. Have the Authors any information on this point?

Any reciprocating engine, steam or Diesel, or propeller rotating in the disturbed wake, sets up various forces some of which are under ready control in the design stage but much more difficult to deal with when the vessel and machinery are completed. Unfortunately very little information is available on this "clapper" end and I would propose to put the sort of requirements in the form of a *pro forma* request to the Authors for further information while realizing that the information requested is far more than can be expected in the reply to a discussion.

In case of each vessel, can the Authors state the designed propeller r.p.m. at service speed; also in case of reciprocating engines the number of cylinders and whether the Diesels are 2-stroke or 4-stroke? From the paper one assumes that in each case when vibration occurred the propeller r.p.m. coincided with the frequency of vibration; was this so?

The location of main engine in relation to the nodes may well be of importance since at the node one might expect a couple to be a more effective "clapper" than a force, and vice versa if engine is at an antinode a force should be avoided. Since in engine balance one can, in the design stages, often ring the changes between forces and couples, some definite information as to their relative importance would be most helpful. Further, the magnitude of any engine forces or couples at the critical speed would be most valuable information.

Apart from the main engines there are, of course, other possible "clappers", for instance, those arising from the propeller working in the disturbed wake. This may well set up horizontal and vertical forces acting at the stern-tube bush—perhaps a good position for setting up vertical or horizontal vibration. Also there will be a varying thrust acting, well away from the neutral axis and capable of exciting, one thinks, vertical and also possibly in a twin-screw ship, horizontal vibration; and so it goes on.

While it is not expected that the Authors can answer all of the foregoing requests it is suggested that future papers on this subject of ship vibration should include some reference to the "clapper" on the lines indicated.

Prof. L. C. BURRILL, *Member of Council*:

From the naval architect's point of view, there are four main aspects of the ship-vibration problem. First of all, there is this question of carrying out a long calculation which is worth while, and we have heard to-night that it takes nearly a fortnight to work out such a calculation. It was not stated that it may take two or three weeks to assemble the information to start the calculation, to prepare a proper loading diagram for a given condition and to make a detailed moment-of-inertia calculation. It is obvious, therefore, that before undertaking such a calculation in a shipyard it is necessary to be sure that a satisfactory answer will be obtained.

I think we can now say that a satisfactory result can be obtained, if sufficient care is taken and a sufficient length of time is spent on the calculation. The principle of balancing the strain energy in the ship structure in its extreme bent position to the kinetic energy when the ship is passing through the neutral position has proved very satisfactory, and I think it is a matter of congratulation that most of the developments of this method are due to students of King's College, and not least to the work of Dr. Todd who has pursued this subject for many years.

In the second place, there is the problem of establishing a short calculation method which will give a reasonably accurate answer at an early stage in the design. The principle of this method goes back to Schlick who said in effect that the frequency of vibration of a free-free rod is given by $F=C\sqrt{\frac{I}{WL^3}}$, where I is the moment of inertia of the cross-section, W is the weight, L is the length and C is a constant, and that therefore for ships of a given type it should be possible to substitute another value of C which takes into account the variations from a uniform beam. With this in mind, he made some tests on various ships towards the end of last century, and found that constant was in fact nearly the same as for a uniform beam having a moment of inertia equal to that of the midship section and the same total weight. As a result he published three constants, for use with different types of vessels, but it was later found by experience that it was somewhat difficult to choose a suitable value of the constant.

This Schlick method has been developed by various observers, and it now takes two principal forms, namely:

$$F=C_1\sqrt{\frac{I}{WL_1^3}} \text{ and } F=C_2\sqrt{\frac{BD^3}{W_1L^3}}$$

In the first of these expressions the constant C_1 takes into account the effect of entrained water, the effect of sheer-strain energy and the effect of distribution of mass and moment of inertia along the length, and in the second the item W_1 is the sum of W and the entrained water. This inclusion of the effect of entrained water was, I think, an important step forward, and the results obtained from such a formula should be correct to within five to seven per cent. The use of BD^3 in place of I can, however, be very misleading, and I would join with Mr. Hunter in stressing this point. The accuracy of this formula using BD^3 depends entirely upon the assumption that I is a constant times BD^3, and I think that the diagram at the end of the paper shows that the variation in the useful range is such that there is quite a wide range of choice in putting a mean line through the diagram. There was one instance about two years ago in which the shipbuilders used a formula of this type and estimated a frequency of about 117 whereas the actual frequency on trial was about 77 per minute and there was considerable vibration as this was near the working revolutions. Other similar instances have occurred and I would accordingly recommend to shipbuilders that they use this form of the expression with great caution.

The third problem is the question of collecting as many *ad hoc* records from actual ships concerning their frequencies of vibration in the lower nodes, for comparison with the calculated values. It is in this direction, I think, that the Authors are to be congratulated on being able to place before us a good deal of new information which represents the work of about twelve years or so. It is a long time now since we had a paper giving us new data concerning the actual frequencies measured on ships at sea. In my view, there is room for many investigators in this field, and any naval architect who can interest himself in the collection and publication of reliable frequencies for actual ships will be helping towards the final solution of this vibration problem. It would, in fact, be highly desirable that we should reach a stage in which we could plot the frequencies for different classes of vessels to a base of length or displacement, as this would enable a very rapid decision to be made concerning the engine revolutions or type of engine which might give trouble in any particular instance.

On p. 195 Dr. Todd has drawn attention to the base line corrections suggested by Schladlofsky in 1932 and it is suggested in the paper that these corrections were unknown in this country. I think I should correct this suggestion, in that quite a number of investigators who have been interested in the vibration problem not only knew of Schladlofsky's method of dealing with this matter but have also applied it in carrying out such calculations. The method is, for example, referred to in my 1935 paper on ship vibration.*

It is true that Schladlofsky's paper has not appeared in English and that it is not our habit to translate many of the valuable papers published on the Continent on technical subjects of this kind, but I am glad to note that through *The Shipbuilder* and *Marine Engineer* translations of current foreign papers of interest are beginning to be available for readers in this country. In particular I would refer to the very valuable paper entitled "The Vertical Vibration of Ships", which was read before the Association Technique Maritime et Aéronautique last year by Professor Prohaska of Copenhagen University, which appeared in *The Shipbuilder* not long ago. This paper represented a very important advance in connexion with the estimation of ship-vibration frequencies, particularly in connexion with the corrections for entrained water.

The fourth and final aspect of the ship-vibration problem is, in fact, this question of the effect of entrained water. At the present time we have to rely mainly on the theoretical work of Professor Lewis, together with a partial verification by experimental means obtained by Messrs. Moullin and Brown. So far as their work on the vibration of free-free beams of various cross-sections is concerned, it would appear that the practical values of the entrained mass effect are about 90 per cent. of the calculated values. There is room for a great deal of further careful investigation on this aspect of the problem, and experiments are at present being carried out at King's College towards this end, under the auspices of the British Shipbuilding Research Association.

The present paper by Dr. Todd carries the general investigation a considerable stage forward in that it deals mainly with the effect of erections on the frequencies of vibration in the fundamental mode. The present position can, I think, be summed up as follows. For ships having no erections, a reasonably satisfactory answer can be obtained by applying existing methods, and for ships having a fairly long set of erections, the same applies; but there is a transition region between

* "Ship Vibration: Simple Methods of Estimating Critical Frequencies," Vol. 51.

these two types where at present it is difficult to obtain a really satisfactory answer.

The new information and the methods suggested by Dr. Todd in this paper will certainly help the designer in estimating where a proposed new design lies in this transition region. There is no doubt this subject must be pursued further and we shall welcome a further paper on the same subject in due course.

The Authors, on p. 194, state that they usually take their records at the stern of the vessel, whereas I have always thought it most convenient to take such records at the forward end. There is no doubt that the two free ends give the best records, but I would say that the local influence of engines and propellers is liable to be greater at the after end than at the extreme forward end of the ship.

Mr. H. G. YATES, *Member*:

Have any measurements been made at speeds near resonance sufficiently accurate to determine the degree of damping present in the system?

As Mr. Hunter says, what we want to know is the exciting force which the ship can stand without dangerous or unpleasant vibration. That can only be determined by a knowledge of its reaction to frequencies above and below resonant frequency.

With reference to Mr. Hunter's query on the point of damping in the region where the exciting force is acting, I think one can say with reasonable certainty that it does not matter where the damping is for a given mode of vibration, it has the same effect whether it acts near the exciting force or somewhere else. The exciting force may come from the engine amidships or aft or from the screw, but in all cases it will be possible to make a reasonable determination of the exciting force, and only the damping is necessary to give the resulting velocity. The amplitude can be determined as soon as the frequency is known.

VOTE OF THANKS

On the motion of the PRESIDENT (Mr. H. B. Robin Rowell, A.F.C., D.L.) a vote of thanks was accorded to Dr. Todd and Mr. Marwood for their paper.

CORRESPONDENCE

Mr. N. CARTER, *Member*:

This paper gives a very simple method for a preliminary assessment of the two-node vertical frequency. The following list gives the results of a few ships on which the frequency was observed and which agree reasonably with the curves given in the paper:—

Type of Ship	Length b.p.	$\sqrt{\frac{I}{\Delta_1 L^3}}$	$\sqrt{\frac{B.D^3}{\Delta_1 L^3}}$	Observed frequency
Tanker	490	·000391	·00085	77½
Tanker	483	·000402	·00088	72½
Tanker	460	·000405	·00088	73
Tanker	420	·000451	·00096	83
Tanker	500	·000365	·000787	69
Tanker	460	·000406	·000865	76
Cargo	433	·000625	·00158	104
Cargo	412	·000522	·00134	88

The two cargo ships are complete superstructure types with midship house about 20 per cent. long, the figures being given to the uppermost continuous deck.

It is significant that the ships mentioned in the paper without erections, and those with long erections, give consistent results, while the ships with erections between 30 per cent. and 60 per cent. are not so good. The Authors state that discretion must be used in making allowance for superstructure decks. Is it logical to assume a mean depth or mean I to cover these shorter erections? The effect of these erections will be to stiffen up the girder amidships and thereby change the shape of the deflection curve and this suggests that a modification should be made to the length rather than to the depth. The spots can be brought more into line by using a depth D_1 in conjunction with a length L_c.

where $L_c = \text{l.b.p.} \times \sqrt[3]{\frac{D1}{D2}}$

D_2 = depth to top erection 30 per cent to 60 per cent.

D_1 = depth to highest deck over 60 per cent.

For example No. 19 ship in Table 1.

$$D^2 = 50\cdot50 \quad L = 375$$

$$D^1 = 34\cdot75 \quad L_c = 375 \sqrt[3]{\frac{34\cdot75}{50\cdot50}} = 331$$

$$\text{Then} \sqrt{\frac{BD^3_1}{\Delta 1\, L^3_c}} = \frac{54\cdot5 \times 34\cdot75^3}{15600 \times 331^3} = \cdot0020$$

No. 17 ship should not be included in Fig. 5 which uses depth as a basis, as an expansion joint was fitted in the games deck also the decks from the promenade upwards were overhanging the normal breadth of ship.

Dr. J. F. C. CONN, *Member*:

Thanks to Dr. Todd's earlier work, calculations of the two-node vertical frequency give reliable results for ordinary vessels. The present paper deals with the effects of long erections and the higher modes of vibration.

So far as structural rigidity is concerned, it is already clear that the two-node frequency depends mainly on the inertia value about amidships, but with higher modes this will not remain true.

Further refinements in frequency calculation must depend upon the experimental results available, and the Authors have given useful additions to the amount of published information. It appears to me that only extensive series of tests on vessels of different types, where vertical and horizontal vibration of fundamental and higher modes are produced by means of an exciter, can provide the required data.

The Authors will probably agree that the correction for shear in ship vibration calculations is an appreciable, if not a large, one. It is extraordinary that while Dr. Lockwood Taylor's shear correction is commonly applied in vibration calculations, the same shear corrections as applied to stress and deflection are not yet commonly accepted in structural strength calculations.

Mr. R. W. L. GAWN, O.B.E., *Member*:

The title of the paper while commendably brief appears too comprehensive and would, it is suggested, be more representative if expanded to include "The Natural Frequency and Amplitude of Main Hull Vibration". The particular emphasis in the paper is rather on the degree of exactitude of prediction of these characteristics of vibration from design drawings of ships. This is indeed a sufficiently wide and important subject. The complexities of elasticity and hydrodynamics involved are reflected in the many excellent contributions to the subject that have been made in the past in which company the present paper finds a very good place indeed.

The Authors rightly draw attention to the immense labour involved in ship calculations of this type from design drawings. Their solution which reduces the labour effort and gives predictions of primary natural frequency ranging from $+7\frac{1}{2}$ per cent. to -5 per cent. of the measured frequency will be welcomed as an important achievement by all interested in the detail of vibration problems.

The accuracy of prediction of amplitude by calculation brought out in Fig. 1*B* while not so close on the whole as for the frequency can nevertheless be regarded as extremely satisfactory in view of the complexity of the problem. It is, however, disconcerting to find that amplitudes as large as about 300 thousandths of an inch are recorded for ship 25. This large vibration may possibly be explained by the reciprocating Diesel machinery but even with the two turbine-driven ships considered in the paper, the amplitude is 90 thousandths for ship 24 and 60 thousandths for ship 16. The amplitude is large for other ships and in fact the only record of a small vibration is for ship 22, the amplitude then being 8 thousandths. This is a shorter ship, the length being 257 ft., but even the smaller ship 21 has an amplitude aft of about 50 thousandths of an inch associated naturally with a high frequency.

If these large vibrations occur within the operational range of speed of the ships concerned they must be unpleasant. The consequential reactions on the efficiency of a radar set and possibly wireless or other equipment might be serious if local resonances occur. It would be of interest if the Authors could give an explanation of the excessive amplitudes and remark on the operational aspect including speed at which the large vibrations occur. It would also be appreciated if particulars could be given of the engine and shaft revolutions, ship speed, hull clearance and number of propeller blades, to permit of further consideration of this important aspect. There would appear to be scope for improvement, possibly by modification of propellers.

The first report on vibration of H.M. ships completed at Admiralty Experiment Works, Haslar, is dated 1889. A vibrograph was specially designed and made for the trials. The vibration was recorded as satisfactory, the amplitude of movement of the deck of *Gleaner* being 150 thousandths of an inch at 20 knots and 375 thousandths for *Medusa*. These trials were primarily to ascertain whether vibration prejudiced structural strength in a seaway and it is in that sense that the vibration was recorded as unimportant. The vibration measured on these ships sixty years ago is as satisfactory as many of the ships dealt with in the paper. The present-day requirements for vibration of H.M. ships are governed by many considerations other than structural strength and very refined standards are required.

It is necessary to evaluate the moment of inertia for strength estimates so that the objection raised by the Authors to Schlick's formula that the moment of inertia is unknown will surely not apply to ships for which longitudinal strength calculations have been made. Vibration formulae can accordingly be readily applied to these cases. Fig. 3 of the paper can be compared with Schlick's formula. The frequency locus as drawn does not, however, pass through the origin, and it would be interesting if the Authors could remark as to any explanation of this. The locus appears to have been partly governed by the results for ships 21 and 23. Ship 21 is a single-screw cargo coaster of 180 ft. length and ship 23 a single-screw coaster of 210 ft. length. The majority of the ships considered are of cargo and passenger type of length ranging from 375 to 650 ft. I have drawn a new frequency line through the origin, averaging approximately the measured frequencies excluding the two short ships referred to. This line gives a frequency of about 150,000 times the abscissa parameter. The coefficients quoted originally by Schlick are 156,850 for destroyers, 143,500 for liners and 127,900 for full-cargo ships but Schlick's parameter did not allow for entrained water. If the Authors' parameter is modified to disregard the effect of entrained water then the coefficient of 150,000 is reduced to about 100,000. The extent to which this falls below Schlick's coefficient is worthy of some consideration because Schlick stated that his formula gave generally reliable values and this claim is substantiated for certain classes of warship. If in working out the coefficient from Fig. 3 regard is paid to the parameter for which the moment of inertia included material only up to the topmost continuous deck the coefficient is increased to about 127,000. This approximates to that quoted by Schlick. The implication appears to be that Schlick's formula could be applied to give a fairly close estimate of the primary frequency of ships of the type dealt with in the paper with the exception of short ships 21 and 23 which will require some reduction in coefficient. The Authors' remarks on this point would be much appreciated.

The paper brings out the advantage of combining fundamental theory with experiment and the considerable effort involved in ship vibration problems. The effort is greatly intensified by the numerous records required of local vibration to supplement the hull vibration dealt with in the paper. Avoidance of resonance by a good margin is the one method of obviating or reducing the vibration of ships. The Authors' work makes an important contribution to this aim in ship design and thanks are due to them for this important advance.

Mr. F. McALISTER, *Associate Member*:

The great value of such papers as this is the undoubted contribution to the store of data available on the subject. The papers on vibration listed by Dr. Todd form in themselves a large and extensive store of authentic data without equal in our records.

Although no doubt it is valuable to examine in post-mortem fashion such cases of vibration as are investigated in detail in the paper, the supreme test of the data so analysed must lie in their application to new designs and for that purpose little time is available at that stage for determining the critical revolutions by involved investigation.

The design of any ship revolves round weights and to determine machinery weights the r.p.m. must be known. This is at a very early stage when even the erections are not fully determined. The preliminary investigation for critical r.p.m. should then be determined by an approximate application of known data, and one useful formula is

$$\text{Primary Critical r.p.m.} = \frac{L^{1\frac{1}{2}}}{K}$$

where K is a constant from similar ships and L the length in feet. The constant, of course, varies with the type of ship, displacement, disposition of structure and many other factors, yet in similar types of ship this formula gives a very good guide in the initial stages of the design.

For example, for oil tankers K is 75 and for trunk-type tankers, coasters, etc., K is about 54 to 58, K rises 110-115 in intermediate passenger vessels and up to 130 or more in large vessels.

At a later stage in the design detailed calculations can be made to confirm that the fundamentals of the design are sound.

Still searching on this point of the approximate solution of the problem Dr. Todd's allowance for superstructures is in the right direction and is, in fact, much the same equivalent depth as that used by naval architects in estimating their K.G's. Most of my own weight data and centre of gravity data are based on a rather more elaborate form of equivalent depth and I endorse Dr. Todd's formula (3) as a useful basis to adopt.

I do think, however, that instead of approximating the critical two-node vertical frequency and comparing it with the observed critical, it might be more powerful to analyse the actual critical back to the equivalent depth and plot the excess depth above that to the uppermost continuous deck on a basis of percentage of erections or some other suitable parameter.

Mr. R. G. MANLEY, *Associate Member*:

First, it is noted that the experimental results were obtained by using the main engines of the ships as exciters. Has it yet been possible to use artificial vibrators to excite the vibrations? Admittedly the ships' engines and the propellers are the principal sources of vibrations, but there might be circumstances in which it would be desirable to see the effect of exciting from some other part of the structure, and again it might be more convenient at times to carry out vibration tests without calling upon the engine-room for co-operation.

Is it a practical proposition to perform vibration tests on structural models? There are several obvious reasons why such a procedure would be difficult to employ usefully, but it would be interesting to learn whether the Authors know of any successful work on these lines.

Amongst the vibration records obtained during a typical test, what proportion are reasonably pure sine-waves? What is the Authors' preferred method for analysing those that are not?

It is noted that in Fig. 1*C* on p. 206 the measured vibration profile for ship No. 16 turns back towards the zero line at the stern very much more markedly than is the case with the other profiles shown. Is there any known reason for this? Unless there is a very great concentration of mass at the stern the appearance is rather odd.

The Authors use the term " moment of inertia ", and occasionally simply " inertia", for the quantity proportional to BD^3 which determines the stiffness of the structure. This seems to me to be unfortunate, as it leads to statements such as that at the top of p. 194: " . . . the distribution of inertia for the main girder towards the ends was of minor importance . . . whereas the correct distribution of weight . . . was essential." The vibration characteristics of a beam depend upon the distribution of mass and stiffness. The engineer would prefer to talk of weight or of inertia instead of mass, but it is important to note that the *inertia* which affects the issue is associated with the *mass*. The BD^3 quantity, which is properly if horridly termed " second moment of area ", is a purely geometric quantity which is independent of the density and the physical properties of the material, and it is misleading to refer to it as " inertia ". It is, after all, merely coincidence that the mathematics of the situation makes the second moment of area of a cross-section something similar in form to a moment of inertia. My objection could perhaps be justifiably termed a mere quibble were it not for the fact that true inertia operates in a manner quite opposite to the so-called " moment of inertia ", for an increase in mass inertia decreases the natural frequencies whereas an increase in the area moment increases them. I should like to see the term " moment of inertia " deleted from all the textbooks and papers except where it means what it says, namely, the rotational analogue of mass in equations of motion.

Mr. J. M. MURRAY:

This paper continues the records of ship vibration which Dr. Todd has given to the Institution and is of value both on account of the specific cases which he furnishes and the general formula which he has evolved. The approximations which he gives are exceedingly useful, but it must be borne in mind that, as the Authors point out, they are only approximations and in applying them a certain amount of judgment must be used. The results given in Figs. 3 and 5 have been applied to several cases which have come within my experience with very good results. It is of interest to quote two of these cases—(1) a tanker 460 feet long and (2) a passenger ship 570 feet long.

For the tanker very complete records of vibration in three conditions were available and the frequency at load draught coincided with that given for an oil tanker in Fig. 5; at lighter draughts the divergence from the curve noted in Fig. 6 was confirmed. The ratio of the three-node vibration to two-node vibration was 2·47, which is in accordance with the results given in Table 6. Since tankers are more or less of a standard design and do not differ much in proportions from ship to ship, it is not surprising that such close agreement was obtained with Figs. 5 and 6.

The case is altered considerably when we come to passenger vessels, for here there may be a wide variation in the characteristics of different ships. Nevertheless, in the case of the 570-foot passenger ship, with two tiers of erections, there was also coincidence with the results derived from Fig. 5. At the same time, it is not certain that such a favourable result would be obtained in every case for I have not found that the inertia is proportional to BD^3 irrespective of the proportions of the ship. Dr. Todd, in his 1931/32 paper to the Institution,* has given a diagram which seems to demonstrate that the inertia coefficient does not vary with L/D. I have not had this experience and have found that, in general, there is a very definite variation with the proportions of the ship. Some confirmation of this point of view may be obtained from a comparison between Fig. 3 and Fig. 5. If the frequency is plotted with respect to inertia, as is done in Fig. 3, it will be observed that tankers, passenger and cargo ships fall on the same line. When the inertia is related to BD_E as in Fig. 3, two curves are necessary, and it is suggested that this is due as much to the difference in proportions of the two classes of vessels under consideration as to differences in type or construction of ship.

It is considered that the method of indicating the characteristics of the ships in Table 1 and Fig. 1*A* and *B* is extremely useful and should be of considerable assistance in determination of the natural frequency of a ship in the initial stages of design.

Arising from this paper, though not directly concerned with it, there is one point to which reference might be made, and that is that general or even local vibration is not a sign of weakness in a ship as it is sometimes considered to be, and that it is not often that the introduction of additional material can have any sensible effect.

Prof. C. W. PROHASKA, *Member*:

The aim of the paper has obviously been to find the effect of superstructures on frequency, and although Fig. 3 gives some idea of this effect, it seems difficult from this diagram to draw definite conclusions.

* " Some Measurements of Ship Vibration," Vol. 48.

I think it should be possible to obtain closer agreement between calculated and observed data than that generally borne out by the Authors' investigation.

The reasons for the rather confusing differences present in some cases, are, in my opinion, mainly due to the following causes:

1. Moment of inertia has been taken as constant,
2. Full moment of inertia has been used for top decks, not decks to the hull,
3. Shear correction is too approximate, and
4. Virtual inertia factor is probably over-estimated, if observed frequencies correspond to deep-water condition.

But apart from that, it is good to remember, that an uncertainty of one or two per cent. is quite normal in observed frequencies.

1. On p. 201 it is stated that the effect of variation of moment of inertia along the length of the vessel is of secondary character. In this I disagree, and I should like to refer to a paper read before the Association Technique Maritime and Aeronautique in Paris* last year, in which it is shown that the necessary correction in some cases amounts to 20 per cent. or more. In my opinion, therefore, it is absolutely essential to make this correction for practically all vessels, except tankers. The correction factor to be used is greater or smaller than unity according to the I value introduced in the calculation.

2. For top decks of light scantlings and supported at the sides of the ship by stanchions only, the wing parts of the deck are less effective than those rigidly supported by deck-house sides, and will carry but a part of their theoretical load. It therefore seems logical to introduce a reduction factor for the moment of inertia to such decks.

3. The shear correction has been estimated in accordance with Dr. Taylor's approximate method. This method is based on theoretical deductions for a box-vessel with uniform thickness of plating, but the curvature for pure bending is related to the corresponding deflection by an empirical and rather arbitrary constant.

Dr. Taylor gives: $\frac{N_{S+B}}{N_B} = \frac{1}{\sqrt{1+r}}$, with

$$r = 3{,}5 \cdot \frac{D^2}{L^2} \cdot \frac{3a^3 + 9a^2 + 6a + 1 \cdot 2}{3a + 1};$$

$a = \frac{B}{D}$. If the proportion of deck and bottom thickness to that of the side shell is λ, this expression can be generalized into:

$$r = 3 \cdot 5 \times \frac{D^2}{L^2} \times \frac{3a^3\lambda + 9a^2\lambda^2 + 6a\lambda + 1 \cdot 2}{3a\lambda + 1};$$

When we deal with ships with several decks and a double bottom, it is obvious that a value of $\lambda > 1$ must be introduced, which results in an increase in r, and consequently in the shear correction on frequency. For the box girder of non-uniform thickness, it can be shown (loc. cit.), that:

$$\zeta = \frac{\rho^2}{D^2} = \frac{I}{A \times D^2} = \frac{1}{12} \times \frac{3a\lambda + 1}{a\lambda + 1}, \text{ and:}$$

$$C = \frac{a\lambda + 1}{(3a\lambda + 1)^2} (3a^3\lambda + 9a^2\lambda^2 + 6a\lambda + 1 \cdot 2)$$

$$\text{where: } C = \theta \times \frac{AG}{Q} = \frac{A}{Q} \int \tau^2 dA = \frac{A}{I^2} \int \frac{St^2}{\delta} ds$$

(A = cross section area of material, Q = shear force, G = modulus of rigidity, θ=shear angle, τ=shear stress, St=statical static moment about neutral axis of material outside the point of the section considered, δ = thickness of plating, and s = girth co-ordinate). Introducing c and ζ in Dr. Taylor's modified r value, the following general expression is obtained:

$$\frac{N_{S+B}}{N_B} = r_2 = 1 \div \sqrt{1 + 42C\zeta \times \left(\frac{D}{L}\right)^2}$$

The variation of r_2 is graphically given in Fig. 8. As the constant 42 is arbitrary, these curves cannot be expected to give the true value of r_2. Schadlofsky dealt with the question on similar lines, but used instead of Dr. Taylor's arbitrary constant, the figure 28·6, which is correct for the uniform free-free beam. Curves showing Schadlofsky's correction are also given in Fig. 8.

Solving Timoshenko's complete differential equation for the vibration of a homogeneous bar and adjusting the results to be fit for the non-uniform ship-girder, I have produced the diagram in Fig. 9 (loc. cit.), which shows corrections somewhat intermediate between those found from the two sets of curves in Fig. 8. Taylor's correction, therefore, might be acceptable, if the proper values of $c\zeta$ were used. The Authors, I suppose, used Taylor's original data corresponding to $\lambda = 1$, which give too low a correction. This may be seen from Fig. 10, in which the variation of $c\zeta$ with $a = B/D$ and λ is shown, for rectangular sections of non-uniform thickness. The four points marked $A-D$ correspond to Schadlofsky's calculations for four ship sections all with double bottom, but with one, two, three and five decks respectively. As Schadlofsky's sections have uniform thickness of plating, the values indicated are somewhat higher than

*C. W. Prohaska "Vibrations Verticales du Navire." *Bulletin de l'ATMA*, Paris 1947. English extract in *The Shipbuilder and Marine Engine-Builder*, Oct., 1947. (See also: Prohaska: Lodrette Skibssvingninger med 2 Knuder, Copenhagen, 1941.)

those found for actual ships. I therefore recalculated ship *B* with correct Lloyd's scantlings and found the $c\zeta$ value marked B_1, which is much closer to the line for $\lambda = 1$, but still gives a 2 per cent. higher reduction on the frequency than according to Taylor. More calculations of exact $c\zeta$ values will have to be carried out, before it is advisable to use this graph for ships, but it gives some indication of the variation of $c\zeta$.

water, but it is suggested that the Authors include such data in their reply to the discussion, to make the paper still more valuable for future exploration.

I should have liked to try my suggested correction on all the ships given in the paper, but it is only for ship No. 15 that it is possible from the data given to calculate the moments of inertia corresponding to all the superstructure decks. I therefore have to content myself with this single

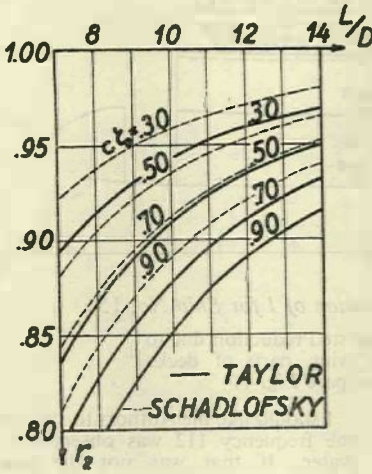

Fig. 8—Shear Correction Factor: r_2 derived from Lockwood Taylor's and Schadlofsky's Investigations

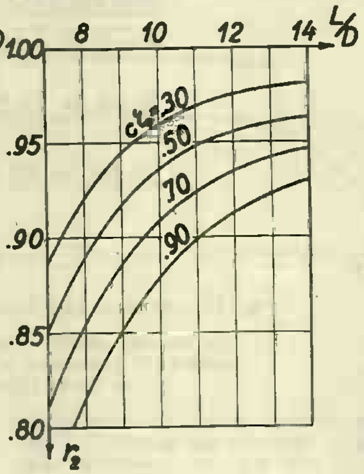

Fig. 9—Shear Correction Factor: r_2 (Prohaska, loc. cit.)

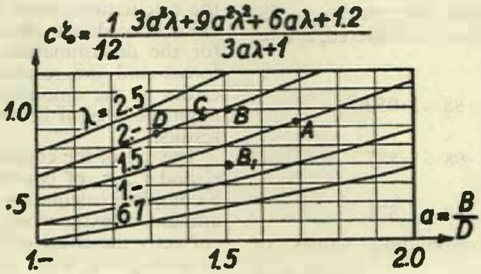

Fig. 10—A–D indicates Schadlofsky's Ship Sections of Uniform Thickness, B_1 ship B re-calculated with Correct Scantlings

4. The virtual inertia of entrained water has been assessed according to the method developed by Lewis. In my opinion, the correction for three-dimensional flow should be performed in accordance with the results found by Lockwood Taylor and not by means of Lewis's more approximate formula. But it is still more important to take into consideration the depth of water, as the virtual inertia factor increases rapidly when depth decreases (loc. cit.). No reference is made in the paper to depth of

calculation, which is to be considered more as an example than as a proof of my method:

From an enlarged photograph of Fig. 3, I read off the values of $\sqrt{\dfrac{I}{\Delta_1 \times L^3}}$, and thus for ship No. 15 found the following values of I for the three top-decks: $1\cdot27 \cdot 10^6$, $1\cdot05 \cdot 10^6$ and $\cdot73 \cdot 10^6$. I for the shelter-deck is not given, but is, due to the extent of the superstructures, without great importance, and can be estimated to about $\cdot 3$ to $\cdot 4 \cdot 10^3$. Fig. 11 depicts the approximate variation of I. The dotted lines indicate an arbitrary reduction for the upper decks due to the above-mentioned causes. Calculating now the reduction factor r_1 by means of a method I have described previously (loc. cit.), I found: $r_1 = 1 \cdot 26$ when using the bridge-deck moment of inertia in the calculation of the frequency. Similarly was found: $r_1 = 1 \cdot 065$ and $r_1 = 0 \cdot 985$ for the promenade and boat-decks respectively. With $c\zeta = \cdot 60$ and $\frac{L}{D}$ to boat deck $= 7 \cdot 6$, $r_2 = \cdot 85$ was lifted from Fig. 9. With a further correction for transverse stresses (loc. cit.): $r_3 = \cdot 98$, I get for the three cases considered, a total correction of:

I to bridge deck:
$R = 1 \cdot 26 \times \cdot 85 \times \cdot 98 = 1 \cdot 050$

I to prom. deck:
$R = 1 \cdot 065 \times \cdot 85 \times \cdot 98 = \cdot 887$

I to boat deck:
$R = \cdot 985 \times \cdot 85 \times \cdot 98 = \cdot 821$

The Authors' corresponding values of $\frac{1}{\sqrt{1+r}}$ amounts to: $\cdot 901$ $\cdot 887$ and $\cdot 870$. Thus their calculated frequencies of: $95 \cdot 6$, $109 \cdot 3$ and $118 \cdot 5$, multiplied by $R \cdot \sqrt{1+r}$ and by the square root of the proportion between the reduced and the actual values of moment of inertia give:

Bridge deck: $95 \cdot 6 \times \frac{1 \cdot 050}{\cdot 910} \times \sqrt{\frac{\cdot 73}{\cdot 73}} = 111$

Prom. deck: $109 \cdot 3 \times \frac{\cdot 877}{\cdot 887} \times \sqrt{\frac{1 \cdot 03}{1 \cdot 05}} = 108$

Boat deck: $118 \cdot 5 \times \frac{\cdot 821}{\cdot 870} \times \sqrt{\frac{1 \cdot 20}{1 \cdot 27}} = 109$

These values are in good accordance mutually, but fall a bit short of the observed frequency: 112. This might be due to the v.i.f., which according to the Authors was taken as $2 \cdot 235$. Had Lockwood Taylor's reductions been used instead of Lewis's, v.i.f. would have been $2 \cdot 07$, changing the above calculated figure of 109 into 113. No correction for depth of water was used.

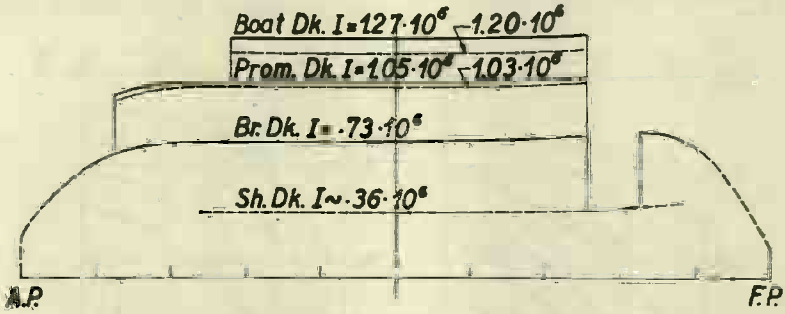

Fig. 11—Approximate Variation of I for Ship No. 15

Dotted lines indicate suggested reduction due to ineffective connexion of wing parts of decks to the hull (compare Fig. 1)

I should like the Authors to state whether the frequency 112 was observed in deep water. If that was not the case, the agreement might be less perfect. As far as I can see from this paper and from Dr. Todd's earlier papers, the vibration profile used in the calculations was in all cases taken as that of the uniform bar. If the calculations were not repeated to give close accordance between the curve used for the determination of the acceleration forces and the resulting deflection curve, errors to the amount of several per cent. might be present in some of the calculated results.

The use of the correction factors, such as r_1 and r_2, is, of course, unnecessary when a complete calculation is performed. The amount of extra work involved, to include correctly the effect of variation of moment of inertia and shear deflection, is not prohibitive (loc. cit.).

I agree, however, with the Authors that approximate methods are of value at the design stage, but I do not think the modified Lundberg formula fit to take account correctly of variation in moment of inertia. In Fig. 12 curves of r_1 derived from this formula, for the case of one step only in the moment of inertia curve, are compared with r_1 curves found by direct calculation (loc. cit.) for a uniform distribution of load. It is seen that the formula underestimates the correction, except, of course, at zero length of bridge. For usual ship-distributions the differences become even greater.

In the Appendix the Authors have dealt

at length with the necessary shift of zero-line for the vibration profile, and attributed the method to Schadlofsky. This is not quite correct. The method dates back at least to Poisson, and has been described in English several times before. (Barfoed: "The Natural Vibration of Ships", *The Motor Ship*, Feb., 1926; Lewis: "Vibration and Engine Balance in Diesel Ships", *Trans. Soc.N.A. & M.E.* 1927, *The Shipbuilder and Marine Engine-Builder*, Oct., 1947, p. 543-544).

Finally, I would draw attention to the calculated figures for ship No. 14, which I think are not in mutual accordance, or not in accordance with the corresponding data of Fig. 3, as $\dfrac{\sqrt{\dfrac{I}{\Delta_1\cdot L^3}}}{N\cdot\sqrt{1+r}}$, which ought to be the same for the two decks, differs about 15 per cent.

drawn through the mean of each pair of spots will cover more ships than any straight line, and such a curve (to work in the reverse as it were) may lead the way to a more generally correct approach to the effect of superstructures.

It is noted that the mean straight line is made to coincide with the 180 ft. coaster No. 21, while the small ships No. 22 and 23 do not conform to the line. Small ships are, in the nature of things, more liable to differences of type, and it is suggested that the mean line or curve need not necessarily agree with No. 21.

Dr. J. LOCKWOOD TAYLOR

The general conclusions as to the possibility of calculating from first principles the two-mode vertical vibration seem to tally with those of my 1930 I.N.A. paper.* I believe, however, that the three-dimensional correction to the virtual inertia,

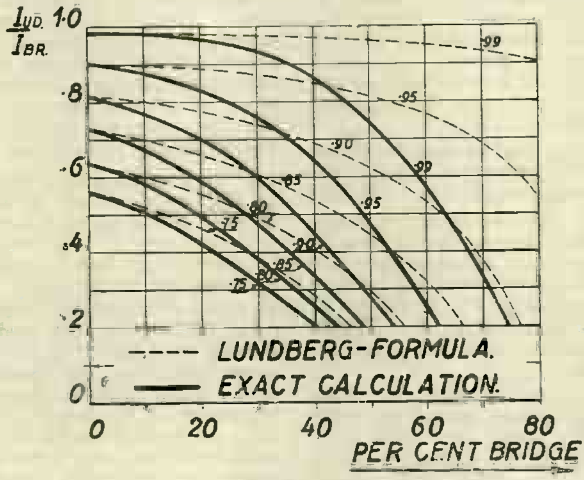

Fig. 12

Mr. S. B. RALSTON

Some of the earlier papers dealing with vibration have tended to include factors which cannot be predicted before the ship is actually constructed, so that the conclusions were of little assistance to the harassed naval architect. The Authors of this paper are to be congratulated on their practical approach to the problem and appear to be justified in their belief that their work will enable designers to make very close estimates at an early stage before full information is available.

The Authors have examined the effect of superstructures, and Figs 3 and 5 show pairs of spots for each ship with and without the superstructure. The mean line is drawn as a straight line, but must this line be straight? I suggest that a curve

which I based on mathematical results for an ellipsoid, should be applied. I was able also to get agreement for the three-mode, between observation and calculation.

The apparent discrepancy for the two-mode horizontal seems to require investigation. Further calculations of virtual inertia for different sections allowing for the free-water surface and of the effect of adding mass at an offset from the shear axis of the structure suggest themselves as subjects worth looking into. As I pointed out in an earlier paper†, there is always some torsion combined with lateral vibration.

*"Vibration of Ships," Vol. 72.

†"Ship Vibration Periods," Vol. 44, *N.E.C. Inst.*

Prof. G. VEDELER, *Member* :

It is usually instructive to compare calculated or measured values with known results for simple bodies. To start with Fig. 2 it may be of interest to note that the virtual inertia factor for a rectangular box with fairly good approximation within the usual range of B/d may be given by $(\frac{1}{3}B/d + 1\cdot 2)$ which lies well above the straight line for actual ships.

For a homogeneous rod the coefficient ϕ in Schlick's formula (1) is 140,000. This should be unaltered when the rod is partly immersed when replacing Δ by Δ_1, at least for fairly long rods for which it should be sufficiently correct to assume the added virtual mass to be evenly distributed over over the length. Now the equation of the straight line drawn in Fig. 3 is $N = 110{,}000x + 29$, when x is the abscissa. The line does not pass through the origin and will be cut by the line for the homogeneous rod at about $N = 132$.

Also in this figure it seems natural to draw a separate line for the tankers, *e.g.* through a point midway between 2 and 3 and another point midway between 7 and 8. This line will lie entirely above and parallel to the line for the homogeneous rod $N = 140{,}000x$.

If the solid rod is of rectangular shape, it has $I = BD^2/12$, which means that it has a coefficient of equation (5) of $\beta = 140{,}000/\sqrt{12} = 40{,}400$. The corresponding line in Fig. 5 lies a good distance to the right of all the points from actual ships. The straight lines drawn by Dr. Todd seem to have the equations : $N = 46{,}500x + 27$ for passenger and cargo ships, $N = 52{,}200x + 28\cdot 8$ for tankers, where x is now the abscissa of Fig. 5.

Oil tankers having a moment of inertia of the midship section divided by LBD^2 some 10 to 15% larger than other ships without superstructures and some 20 to 25% larger than ships with superstructures*), it is quite natural that they should fall outside the line suitable for other ships in Fig. 5. Supposing that the classification societies require the same moment of inertia for transverse framing as for longitudinal framing, the type of framing should have no influence unless the vessel is so heavily loaded that some part of the structure has been stressed beyond the elastic limit.

The critical stress for deck plates in compression is undoubtedly lower for transversely framed ships than for longitudinally framed ones. But it would not be reasonable to expect that the critical stress under normal circumstances has been reached in any part of the plating when the natural frequency is being measured in port.

There is, however, a fundamental similarity between the calculation of natural frequencies of vibration and the calculation of critical stress in elastic stability. In both problems similar sets of homogeneous differential equations appear. In such a complex structure as a ship the higher modes of vibration must have some influence on the two-mode vibration just as in the elastic stability problem terms which describe a higher number of half-waves influence also the magnitude of the lower critical load through the requirement that the factor determinant must be zero. The influence may not be large, but theoretically it should be there. This leads to the conclusion that the number of bulkheads may have some influence on the frequency, in a similar way to the influence of lattices on the critical load of a latticed strut. The influence of the bulkhead is much less, however, because the hull proper is rather strong with only few bulkheads. Nevertheless the large number of bulkheads in oil tankers may have some influence on their frequency, say the difference noted in Fig. 3 between tankers and other ships.

It is astonishing that the straight lines in Fig. 5 fit as well as they do. The abscissa of this figure has been arrived at by substituting I in the abscissa of Fig. 3 by αBD^3, which may be correct for a solid rectangle, but not for a ship. The moment of inertia of the midship section should be substituted by the value used by the classification societies. This should be nearly proportional to the bending moment, which again is proportional to the displacement by the length. If the distance from the neutral axis to the upper deck is considered to be proportional to D it might be worth while trying the abscissa $\frac{I}{L}\sqrt{\frac{D}{1\cdot 2 + \frac{1}{3}B/d}}$ with an eventual correction due to the permissible stress increasing with the length of ship.

The curves given by professor Prohaska (loc. cit.) show that the midship moment of inertia I divided by LBD^2 has a minimum at a ship's length between 350 and 400 feet and rises considerably for smaller lengths. Ships with lengths below 300 feet built to Lloyd's rules must, therefore, be relatively much stronger than ships with lengths over 400 feet. It would, therefore, not be surprising if the frequency of small ships should fall outside the curve of frequencies for larger ones when tried against the abscissa suggested above.

*See Prohaska: "Vibrations verticales du navires." *Ass. Techn. Mar. et Aeronautique*, 1947, Fig. 12.

AUTHORS' REPLY

Reply to Sir Wescott Abell

Sir Wescott Abell in his remarks has illustrated the very great effect which loading can have on the natural frequency of a vessel's hull. Calculations done on a tanker and published in an earlier paper[3] have shown that assuming the load to be equally distributed along the length there is a difference in calculated frequency of over 20 per cent. from that calculated using the correct curve of load distribution. It is interesting to hear that Sir Wescott's practical experience bears out the calculations in this respect.

Reply to Mr. Hunter

The Authors are glad to find that Mr. Hunter is in agreement with them on the importance of making as correct a calculation as possible of natural hull frequencies and on the dangers of the use of empirical formulae. It is, of course, true that any hull vibration must in the first instance be excited by some out-of-balance force, either in main engines, propellers, or auxiliary machinery. From our experience the main cause of a ship's excitation has generally been the main engine. In the two turbine-driven ships to which Mr. Hunter calls special attention, the frequencies measured indicate that the out-of-balance force is in the main propelling machinery and of one-per-revolution frequency. This may, of course, be due to an out-of-balance force either in the main engines, shafting, or propellers. In the latter event it means that either the centre of gravity of the propeller is not on the axis, or one of the blades is badly out of pitch. In the case of vessel No. 16, which was turbo-electric, most exhaustive tests were done in dock with each auxiliary machine at a time, and it was proved beyond doubt that the exciting force was either in the main motors, shafting or propellers. These forces can only be dealt with from the marine engineering side by keeping them to the very minimum possible, but there remains those hydrodynamic forces resulting from the rotation of the propeller in the disturbed wake. Some disturbing force must be accepted here, but every endeavour should be made to reduce its magnitude by careful attention to such matters as clearance between propeller blade and hull or fin and to the shaping of shaft webs, bossings, A-brackets and the endings of the waterlines at the after end, in single-screw ships.

Reply to Prof. Burrill

Professor Burrill has given a very lucid summary of the present position of our knowledge regarding ship vibration. As one of the Authors* was responsible for introducing the use of a formula involving the BD^3 to which Professor Burrill takes some exception, we should like to make it quite clear that we did not intend that it should be preferred to the use of one involving the correct value of the moment of inertia I. It was more on account to provide the practitioner with an approximate formula which he could use when the information necessary for the more detailed calculation of I was not available to him. We ourselves have found it quite useful on many occasions and cannot believe that the difference between 77 and 117 per minute quoted by Professor Burrill can have been due to the reasonable use of this formula. We are quite sure some other factor must have been present to account for this big difference.

It is very gratifying to note that experiments are to be carried out at King's College to investigate the effect of entrained water, and we shall await the results of the B.S.R.A. work with great interest.

It has been the Authors' custom to take records at the stern rather than at the bow or midships. At midships it is quite possible to miss certain of the critical frequencies since, for example, a three-node vibration would give no amplitude at that point. Admittedly the stern is subject to certain local vibration due to propeller and possibly auxiliary machinery, but it has the advantage of being in general easier of access to the engine-room for communication purposes, and it has also been found on more than one occasion that the amplitude at the fore end of the ship is less than would be expected from the general trend of the vibration profile.

Reply to Mr. Yates

Curves showing the relation between amplitude and frequency over the resonant range have been published for ships from time to time, and a sample may be found in the literature on the subject, in particular Nos. 1 and 5 of the Bibliography. These should enable Mr. Yates to make some estimates of the degree of damping.

Reply to Mr. Carter

Mr. Carter's contribution to the discussion is extremely useful and gratifying, since he gives the results of another eight ships which can be added to the data already given in the paper. It will be found

* Dr. Todd

on comparing these with Fig. 5 that the observed frequencies fall very closely along the lines there drawn. There are, of course, many ways of attempting to allow for the different length of superstructure by some empirical factor, and an obvious alternative to assuming an equivalent depth is to alter the length of the erections, since their strength effect will taper off gradually into the main strength of the hull. The Authors will examine Mr. Carter's suggested methods of using different length *Lc* and at the same time they are grateful to him for pointing out that ship No. 17 had an expansion joint fitted into the depth, which means in effect that the spot in Fig. 5 which shows the results of including the games deck should not have been included, and it will be seen that taking the equivalent depth only to the lounge deck it is in fact in very good agreement with the main line.

Reply to Dr. Conn

Undoubtedly the most satisfactory way of obtaining really reliable vibration data is to use an exciter on the ship, as Dr. Conn suggests. This is, however, a costly and a time-consuming process, and the Authors feel that while this may be done in certain chosen cases with a resultant steady accumulation of information, there will still for a considerable time be a place for those observers who are able to attend ship trials with recording instruments with the hope of being able to pick up critical frequencies from time to time. We would agree with Dr. Conn on the importance of the shear correction in all vibration or strength calculations. In the former its inclusion in the calculations may alter the frequency by as much as 10 per cent. and it would certainly seem advisable that it should be included also in strength and deflection calculations.

Reply to Mr. Gawn

Mr. Gawn has drawn attention to what at first sight appears to be satisfactory agreement between measured and predicted amplitudes. The Authors must, however, disclaim any such credit and apologize to Mr. Gawn if he has been misled by the results shown in Fig. 1B. The measured profiles shown there are, of course, correct and are measured on actual ships. In making the calculations, however, an arbitrary scale is assumed for the acceleration curve, and no attempt has been made to calculate the actual amplitude. In order to compare the shape of calculated and observed profiles, however, the calculated profile has been superposed upon the measured one in such a way as to make amplitudes amidships agree with that actually measured. The comparison shown in the figure, therefore, extends no further than to the shape of the curve. The amplitudes measured on these ships are quite normal, and on no less than three occasions amplitudes of over one inch have been measured[1]. The Authors' experience suggests that an amplitude of ·05 inches or more is sufficient to attract general attention on a ship, but to anyone interested in this question of vibration an amplitude of ·02 can quite definitely be felt without the use of any instruments.

Reply to Mr. McAlister

The Authors are very glad to have Mr. McAlister's endorsement of their suggested use of an equivalent depth in the empirical formula which they have suggested in the paper. They also look forward to applying his own formula to the results given in the paper in order to determine the appropriate values of K. This formula is, of course, an extremely useful one, since it involves nothing more than the length of the ship and a single empirical factor varying with type of ship and which can in fact be easily memorized.

Reply to Mr. Manley

From time to time vibration exciters have been used on board ship in order to determine critical frequencies. Such exciters, however, are very heavy and need elaborate electrical control gear in order to ensure not only a fine graduation in speed but also absolute constancy at any given speed. Their use necessitates devoting the ship to vibration measurements for some days while the exciter is fitted and afterwards removed from the ship. This is costly and needs the resources of some large organization behind it.

Vibration tests have been carried out on structural models—as long ago as 1908, Professor J. B. Henderson carried out such tests with models of the *Lusitania* and *Pathfinder*. These models consisted of flat bars of steel of constant thickness, the width being proportional to the moment of inertia of the cross-section of the ship at any point. The beam was loaded with strips of lead to represent the distribution of weight without contributing to the stiffness of the bow. In these experiments, Henderson neglected the effect of the surrounding water, and concluded from his experiments that the value of E for the steel of the ship's structure must be taken as 10,000 tons per square inch rather than the test-piece value of 13,000. More recent tests have been carried out on actual metal models of ships at the David W. Taylor Basin in Washington.

In analysing the records the envelope method has been generally used and was in fact illustrated in paper[1] as long ago as 1931. The reduction in amplitude at the extreme end of a ship to which Mr. Manley draws attention has been observed several times and is obvious not only in No. 16 of this paper but also in No. 25.

While one might in theory agree with Mr. Manley's objections to the use of the words "moment of inertia", this has become so much a part of the literature of naval architecture that we doubt whether it would be wise to change it now.

Reply to Mr. Murray

It is interesting to have Mr. Murray's confirmation of the usefulness of Fig. 5 as shown by the agreement between the results for the two ships which he has stated. In searching for a suitable parameter on which to plot the results of these vibration tests, the Authors had in mind the possibility of variation in moment of inertia with L/D, and indeed it was one of the methods of plotting which was tried among many others before adopting that used in Fig. 5. With more knowledge of the subject, it may be possible in the future to take account of this variation by adding another term to the formula in equation (5).

Mr. Murray's statement that vibration is not a sign of weakness in a ship draws attention to an aspect of the subject which is still rather widespread, particularly among owners. To alter the frequency of a ship's hull by say 5 per cent. to remove a condition of resonance would mean in effect adding some 10 per cent. to the moment of inertia of the cross-section, which is almost impossible in a completed ship, and, therefore, this method is not one which is generally available in dealing with such problems.

Reply to Prof. Prohaska

Professor Prohaska's remarks form an exceedingly valuable addition to the paper. The Authors were aware of his paper before the Association Technique Maritime and Aeronautique but when this came into their hands the calculations for the second paper were all complete and it was not possible in the time available to make any comparison with those which might have been found using Professor Prohaska's method. Detailed calculations involving the use of an inertia curve along the length of the hull would probably give better agreement if the actual inertia distribution were correctly known rather than by the use of a constant value equal to that amidships, but it is in fact not possible to use a correct inertia curve because of the effect of the end of the superstructure decks. It is obvious in fact that the inertia of the decks must be graded up into the inertia of the main hull at their ends, but any such grading would have to be an approximation made without true knowledge. On the other hand we have found that using the amidship inertia gives results very close to those found from the ship, and certainly have no knowledge of differences as large as the 20 per cent. quoted by Professor Prohaska.

The same reasoning applies to the second point raised in this discussion, namely the effect of the top decks in vibration. In the Authors' experience even quite light decks play a considerable part in the strength of the hull structure and certainly we believe that insufficient is known at present to enable any reduction of strength to be made other than by the use of some overall empirical factor such as is included in equation (5).

Professor Prohaska's remarks on the shear correction are most interesting and the Authors hope to make use of his work in future papers on vibration. He is right in his assumption that we used Taylor's original data in making the calculations of the present paper.

As regards the allowance for entrained water, the ship results were always obtained in relatively deep water, and no allowance was considered necessary for any depth effect.

Professor Prohaska states that the virtual inertia factor increases rapidly when depth decreases, and this is what one would expect. On the other hand, certain information which we have obtained recently on a large ship in varying depths of water in a river estuary have not shown any such effect, although the tests were designed specifically to find it. This result was very surprising and the experiments have been repeated on a sister vessel with much the same result.

The frequency of 112 per minute for vessel 15 was obtained in deep water.

In making the calculations, the original assumed vibration profile was always that for a uniform beam, but the calculations were repeated as necessary, in each case using the derived profile as the new assumed profile until very close agreement was obtained between the two of them.

Reply to Mr. Ralston

There is no fundamental reason why the mean lines through the spot need be straight since the parameter on which they are plotted cannot be expected to take account of every factor which influences the hull frequency. On the other hand, there is perhaps insufficient evidence in Fig. 5 to justify more than a straight line at present. As more data become available it may well be that some curves will be found to fit them more closely than the lines at present drawn.

Reply to Dr. Taylor

The Authors agree that further work is required on the effect of a free water surface upon the virtual mass before the accuracy of calculation for horizontal vibration can approach that for vertical.

Reply to Dr. Vedeler

It was very interesting to have Dr. Vedeler's comparisons between our results and those for simple prismatic bodies. These seem in a general way to bear out what we have found for the different types of ship.

His suggestion that small ships are relatively much stronger than large ones is in fact borne out by experience with vibration and the small ship is a notoriously difficult one to deal with.