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# Balancing Waste Water Treatment Plant Load Using Branch and Bound

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**Abstract.** The problem of smoothing dry weather inflow variations for a Waste Water Treatment Plant (WWTP) that receives sewage from multiple mixed sewer systems is presented, together with a first rough solution algorithm. A simplification followed by a naive translation into a zero-one linear programming problem results in 1152 inequalities for 480 binary variables.

**Keywords:** Water Management, Branch and Bound

## 1 Introduction

In 2015 representatives of two water boards and one municipality decided to look into a long standing problem. The Garmerwolde Waste Water Treatment Plant (WWTP) receives water from several pressurized pipelines. These in turn receive sewage from local sewer systems through pumping stations. The WWTP plant was designed to process sewage as it arrives in the plant, there are no buffers. The supply varies roughly sinusoidally over a 24 hour period and the pumps are under local control (no coordination). This leads to extremely uneven supply to the WWTP, which in turn leads to high costs for chemical additives and air injection. Moreover, at times three large pumping stations may be using the same pressurized line, which wastes energy as well. Ideally, the flow to the WWTP should be approximately constant, the number of pump starts and stops should be limited, and the sewage in local sewer systems should not be stationary for long periods to avoid silting up of the pipes. The pumping stations themselves cannot deliver all flows between 0 and their maximum capacity, there may be “holes” in the flow range. In fact some of them may be limited to on/off operation, so this has some of the properties of a mixed integer problem. More information on the design and operation of Dutch combined sewer systems can be found in [2].

## 2 Abstract problem formulation

We have  $m$  reservoirs and an  $n$  time step inflow forecast for each reservoir. Time step length is  $\Delta t$ ,  $t_k$  is the start of time step  $k$ . For each reservoir there is a time

dependent regulated outflow. We introduce the following functions (lower case: scalars or vectors, upper case: closed finite intervals):

- $v_i(t)$  volume stored in reservoir  $i$  at time  $t$ , non-negative real number;
- $\tau_i(t)$  time since last state change of pumping station  $i$ ;
- $\tau_{e,i}(t)$  time since reservoir  $i$  was last considered empty;
- $V_i(k)$  limits on storage use for reservoir  $i$  at  $t_k$ , a non-negative, non-empty real closed finite interval;
- $v_{e,i}$  level for which reservoir  $i$  is considered empty;
- $q_{in,i}(t)$  actual inflow into reservoir  $i$  at time  $t$ , non-negative real number;
- $q_{fc,i}(k)$  average inflow into reservoir  $i$  forecast for time step  $k$ , non-negative real number;
- $c_i$  number of different flow ranges available for pumping station  $i$ ,  $c_i \in \mathbb{N}$ ,  $c_i > 0$ ;
- $C_i(k)$  set of flow ranges accessible to pumping station  $i$  in time step  $k$ , integer,  $\{0\} \subseteq C_i(k) \subseteq \{0, 1, \dots, c_i\}$ ;
- $j_{out,i}(t)$  actual flow range in use for pumping station  $i$  at time  $t$ , this is a piecewise constant function;
- $j_i(k)$  selected flow range for pumping station  $i$  for time step  $k$ , non-negative real number;
- $Q_{i,j}(k)$  with  $j = 0, 1, 2, \dots, c_i(k) - 1$ , non-negative, non-empty disjoint real closed finite intervals, for all  $i$  and  $k$  we have  $Q_{i,0}(k) = [0, 0]$  which is used when the pump is off;
- $q_{out,i}(t)$  actual discharge from pumping station  $i$  at time  $t$ ;
- $q_i(k)$  discharge setting for pumping station  $i$  for time step  $k$ , non-negative real number;
- $\tau_{min,i}$  minimum time between pump range switching moments for pump at reservoir  $i$ ;
- $\tau_{e,max,i}$  maximum time between times that  $v_i(t) \leq v_{e,i}(k)$  for reservoir  $i$ ;
- $Q_{tgt}(k)$  range of allowed total flows to the WWTP in time step  $k$ .

### 3 Problem to be solved

The simplest version of the problem is the following. Given the initial state  $v_i(t_0^-)$ ,  $\tau_{e,i}(t_0^-)$ ,  $\tau_i(t_0^-)$ ,  $j_{out,i}(t_0^-)$  for  $i = 1, 2, \dots, m$ ; fixed parameters  $\tau_{min,i}$ ,  $v_{e,i}$  for  $i = 1, 2, \dots, m$  and the constraints up to the time horizon,  $q_{in,i}(k)$ ,  $C_i(k)$ ,  $Q_{i,j}(k)$ ,  $Q_{tgt}(k)$  for  $i = 1, 2, \dots, m$  and  $k = 0, 1, \dots, n-1$ . The constraint  $V_i(k)$  is available for  $i = 1, 2, \dots, m$  and  $k = 0, 1, \dots, n$ . Assume that  $v_i(t_0^-) \in V_i(0)$ . Determine  $j_i(k)$  and  $q_i(k)$  for  $i = 1, 2, \dots, m$  and  $k = 0, 1, \dots, n-1$  such that

$$j_i(k) \in C_i(k), q_i(k) \in Q_{i,j_i}(k)$$

$$\sum_{i=1}^m q_i(k) \in Q_{tgt}(k)$$

$$v_i(t_{k+1}) \in V_i(k+1)$$

and as “soft” constraints

$$\begin{aligned}\tau_{e,i}(t_{k+1}) &\leq \tau_{e,\max,i} \\ \tau_i(t_k) &< \tau_i(t_{k+1}) \text{ or } \tau_i(t_k) \geq \tau_{\min,i}\end{aligned}$$

where for  $t_k < t \leq t_{k+1}$

$$\begin{aligned}v_i(t) &= v_i(t_k) + (t - t_k)(q_{\text{in},i}(k) - q_i(k)) \\ \tau_i(t) &= \begin{cases} (t - t_k) & : j_i(k) \neq j_i(t_k^-) \\ \tau_i(t_k) + (t - t_k) & : j_i(k) = j_i(t_k^-) \end{cases} \\ \tau_{e,i}(t) &= \begin{cases} 0 & : v_i(t) \leq v_{e,i} \\ t - t_{\text{last empty}} & : v_i(t) \leq v_{e,i} \text{ and } v_i(t) > v_{e,i} \\ \tau_i(t_k) + (t - t_k) & : v_i(t_k) > v_{e,i} \text{ and } v_i(t) > v_{e,i} \end{cases}\end{aligned}$$

So we have  $2 \times (5 + 1)$  inequalities per time step (2 for total discharge and 2 per pumping station for storage) and 5 variables (the discharges) per time step. These variables might be either integer or continuous or continuous with gaps in the allowed value range. In addition we have the soft constraints on run times and on emptying the system. For a simplified problem without run time or emptying constraints with on/off pumps, a 15 minute time step and 24 hour look-ahead we get 1152 inequalities for 480 binary variables.

## 4 Typical input data

In the Garmerwolde case there are five pumping stations. The inflow exhibits a roughly periodic sinusoidal pattern with a length of 24 hours with a minimum around 6 AM. Time step length is 15 minutes. Minima and maxima for the inflow patterns are given in Table 1. Note that the dry weather hourly inflows

Pumping station	min	max
	m <sup>3</sup> /h	m <sup>3</sup> /h
Groningen (GR)	454	1179
Selwerd (SE)	126	423
G. Huizinga (GH)	144	600
Haren W. (HW)	80	155
Lewenborg (LE)	2	335

**Table 1.** Minima and maxima of hourly inflow

for all but Groningen are always below the minimum pumping capacity. Two examples for the  $V$  and  $Q$  intervals are given in Table 2. A typical target flow range would be  $Q_{\text{tgt}} = [1700, 2200]$ . Even with  $O(10)$  possible combinations of pumps per time step, the number of possible selections in the unconstrained

Pumping station	$v_{e,i}$	Volumes (m <sup>3</sup> )		Pumps (m <sup>3</sup> /h)	
		Example 1	Example 2	Example 1	Example 2
Groningen (GR)	1000	[95,3957]	[95,6800]	[1800,2200]	[1000,4250]
Selwerd (SE)	121	[38,296]	[38,3200]	[1620,1980]	[1060,2000]
G. Huizinga (GH)	176	[150,300]	[150,4900]	[330,400]	[1600,3280]
Haren W. (HW)	27	[22,50]	[22,1378]	[1020,1320]	[300,1200]
Lewenborg (LE)	30	[9,239]	[9,4100]	[1440,1760]	[550,1900]

**Table 2.** Data used in program

problem with 15 minute time step and 24 hour look ahead is  $O(10^{96})$ . For the problem with variable flows there is the added complexity of selecting the flow for each pump. If the pumps were simple on/off pumps then fitting them into the range of target flows would be remarkably like the two dimensional cutting stock problem [1]. The problem also has points in common with the problem addressed in [4], but that approach does not allow for variable part supply rates. A result on the existence of solutions for a very special case is discussed in [3].

## 5 A simple greedy algorithm

We do a depth first search for a path of length  $n$  in a tree where the nodes correspond to system states at times  $t_k$  and the edges correspond to choices of flow ranges. At each node we generate all possible combinations of pump flow states, filter out those that are certain to lead to volume or target flow constraint violations, order the remainder so that we will first try those, that respect both “minimum run time” and “time since empty” constraints, then those that respect only “minimum run time”, followed by those that respect only “time since empty” and finally those that do not respect either of these constraints. Within each group the flow state combinations where the pumps for districts furthest from empty are active are taken first. The first path to reach the horizon is used. For the selection of flow ranges in the first step along the path specific discharges are calculated. The main purpose of the search is to cope with the periodic variation in the inflow. Once a path is found the selection of flow ranges for the first step needs to be translated into actual pump flows.

### 5.1 Implementation with interval arithmetic

For narrow pump ranges  $Q$  and a narrow target range it makes sense to work in interval arithmetic for the system state. For wide ranges further investigations are needed. For narrow ranges we proceed as follows.

$$V_{fc,i}(t_0) = v_i(t_0^-)$$

A time step is processed as follows. For each  $\mathbf{j} \in C_1(k) \times C_2(k) \times \dots \times C_m(k)$  we apply the following accept/reject process. We calculate

$$Q''_{j_i}(k) = \left( \frac{V_i(k+1) - V_{fc,i}(t_k)}{\Delta_k} \right) \cap Q_{j_i}(k)$$

$$Q'_{j_i}(k) = Q''_{j_i}(k) \cap \left( Q_{\text{tgt}}(k) - \sum_{u=1, u \neq i}^m Q_{j_u}(k) \right)$$

and then test

1.  $(\sum_{i=1}^m Q'_{j_i}(k)) \cap Q_{\text{tgt}}(k) \neq \emptyset$
2.  $(V_{fc,i}(t_k) + \Delta_k Q'_{j_i}(k)) \cap V_i(k+1) \neq \emptyset$

If both tests then we define

$$V_{fc,i}(t) = (V_{fc,i}(t_k) + (t - t_k) Q'_{j_i}(k))$$

$$V_{fc,i}(t_{k+1}) = (V_{fc,i}(t_k) + (t - t_k) Q'_{j_i}(k)) \cap V_i(k+1)$$

Next we calculate  $\tau_i(t)$  as before, but we set

$$\tau_{e,i}(t) = \begin{cases} 0 & : v_{e,i} \in V_{fc,i}(t) \\ t - t_{\text{last empty}} & : v_{e,i} \in V_{fc,i}(t_k) \text{ and } V_{fc,i}(t) > v_{e,i} \\ \tau_i(t_k) + (t - t_k) & : V_{fc,i}(t_k) > v_{e,i} \text{ and } V_{fc,i}(t) > v_{e,i} \end{cases}$$

## 6 Plans

First trials with the algorithm showed a potential problem with sedimentation due to long stays of sewage in the urban sewer system. Probably a slowly varying target flow (changes of up to 100m<sup>3</sup>/h per hour will not adversely affect the WWTP) would be a better option. Interval arithmetic is probably desirable as we are constantly checking constraints on calculated values, but it might be a good idea to preselect much narrower intervals for the calculation of  $V_{fc,i}(t_{k+1})$  from  $V_{fc,i}(t_k)$ .

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