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ABSTRACT

To facilitate the shift from conventional to electric buses, the required charging infrastructure must be deployed. This study models the charging station location selection problem for fixed-line public transport services consisting of electric buses. The model considers the deadheading time of electric buses between the final stop of their trip and the locations of the potential charging stations with the objective of minimizing vehicle running costs. The problem is solved at a strategic level; therefore, several parameters of day-to-day operations, such as deadheading distances, are included as aggregate data considering their average values. In addition, it considers different charger types (slow and fast), which are subject to a day-ahead scheduling of the charging sessions of the buses. The developed model is a mixed-integer nonlinear program, which is reformulated as a mixed-integer linear program and can be solved efficiently for large networks with more than 1940 bus trips and 336 charging installation options. The model is applied in the Athens metropolitan area, demonstrating its potential as a decision support tool for selecting charging station locations and charger types in large public transport networks.

1. Introduction

As cities worldwide strive to reduce carbon dioxide (CO_2) emissions, there is a growing shift in the public transport sector towards transitioning from conventional vehicles to electric ones. Numerous urban centers have introduced electric bus fleets, and various public transport authorities have established ambitious targets for accelerating the electrification of their bus fleets. In Europe, over 80 cities have signed the Clean Bus Declaration Act, with ambitious targets for exclusive purchases of electric buses by 2025 in some cases (e.g., Athens, Paris) and even achieving full electrification by 2030 in others (e.g., Oslo, Copenhagen) [1]. In the U.S., the California Air Resources Board has aims to achieve a fully zero-emission bus fleet by 2040, while the city of Boston envisions to achieve this by 2030 [2]. In the U.K., the electric bus fleet is anticipated to experience a nearly threefold expansion by 2024, while cities in South America, including Santiago and Bogotá have already deployed several electric buses [3]. In China, electric buses comprised roughly 60% of the country's public transport fleet in 2021, under a continuously growing electrification trend [4]. Clearly, public transport electrification is gradually becoming the norm worldwide, calling for new strategic planning and design models.

More specifically, this transition necessitates and largely depends on the development of adequate charging infrastructure. Initially, when electric buses were first introduced, charging stations were predominantly located near large bus depots. Nevertheless, space limitations and geographical factors may render bus depots unsuitable for charging station deployment [4]. With the advent of new and more accessible types of electric bus vehicle charging infrastructure, there has been increased flexibility in choosing the placement of charging stations at several locations of a given electric bus network [5]. The different types of charging infrastructure can be distributed across different areas of the city, such as bus stops or even transient, adaptable sites where mobile charging units may be temporarily situated. In addition, the advent

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of fast charging technology allows for locating chargers at bus stops or charging stations which may be shared among several providers [6,7]. These types of operational strategies allow for improved management of infrastructure resources and can facilitate the transition to electric bus systems under financial constraints. However, utilizing centralized charging stations typically involves covering substantial deadhead distances. For instance, in Beijing, this translated to approximately 30,000 kilometers of deadhead travel, while it was estimated that more than 1000 additional drivers were required daily in order to carry out deadhead trips to charging stations [4]. Therefore, apart from the initial investment cost, charging station deployment may result in increased operational costs for public transport agencies.

Associated charging location decisions are anything but straightforward, as operators must determine optimal locations along with infrastructure requirements and also ensure that charging activities do not interfere with the schedule of the bus services. For example, while a charging site near an electric sub-station may have lower construction costs than a charger at a bus stop, it is essential to consider that vehicles may not be able to reach the former while adhering to their planned schedules. Additionally, the deployment of shared chargers, either among several bus lines or among various operators, necessitates the allocation of exclusive charging slots to vehicles in order to prevent queuing and delays [7]. Hence, the challenge of charger placement is intricately connected to scheduling decisions, necessitating the use of decision support tools.

In this context, the objective of this study is to address the problem of selecting optimal locations for charging stations and determining the charger types and the schedule of charging requests at these locations for a fleet of fixed-line public transport services comprising electric buses. In particular, in formulating the problem, we take into consideration the average deadheading time, which refers to the time electric buses spend traveling without passengers between their final stop and potential charging stations. The aim is to minimize the operational costs associated with vehicle running times, including these deadheading times. To address this problem, a mathematical model is proposed to optimally locate charging stations and allocate vehicles to charging time slots, considering limited financial resources. The proposed formulation aims to determine the set of charging stations that would be required to cover vehicle energy needs while minimizing the deadheading times, given certain resource constraints. The model considers two types of charging stations (slow and fast) and determines the assignment and scheduling of charging requests at charging stations. The set of potential charging time slots enables the formulation of the problem given a daily (or day-ahead) charging scheduling approach according to the bus fleet recharging needs. In this way, it is ensured that each vehicle is assigned to an empty (available) charging slot, which becomes occupied and cannot be used by another vehicle at the same time. This mechanism is equivalent to consideration of charging queues because vehicles in need of charging cannot use the occupied charging slots and have to wait for an appropriate empty slot.

By applying this modeling approach, this study contributes to the efficient planning and decision-making processes involved in the selection of charging station locations and charging scheduling optimization for electric bus fleets in public transport systems. The findings can provide valuable insights for policymakers and transportation authorities to optimize the placement and management of charging infrastructure, thereby ultimately facilitating the adoption of electric vehicles and supporting the goal of reducing CO_2 emissions in urban areas.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature, focusing on the charging station location problem and the combined charging station location and vehicle charging problem. Section 3 presents the mathematical formulation of our model, and its reformulation to a mixed-integer linear program. Section 4 presents experimental results and performance analysis on a set of benchmark instances and in the network of the Athens metropolitan area. Finally, in the concluding remarks section, the paper discusses potential future research directions.

2. Literature review

This section provides a brief overview of studies related to the optimal selection of charging stations and the optimal scheduling of charging requests. The section concludes by outlining the knowledge gap which is addressed in this study.

2.1. Charging Station Location Problem

The Charging Station Location Problem (CSLP) involves strategically locating static charging infrastructure for a bus network. The main objective is to efficiently provide electric power to buses under considerations related to the installation cost and the energy requirements of electric vehicles. In the past few years, several mathematical programming models have been proposed to determine the optimal charging station locations for a bus route network, considering different charging configurations.

Among the early approaches, Jang et al. [8] presented a mixed-integer programming model for minimizing the initial investment cost of charging infrastructure, with the goal of determining the number of stationary chargers located at terminal stops along with the optimal battery size. Adopting the same objective, Wang et al. [9] presented integer linear programming models for locating chargers at terminals and bus stops along with efficient heuristics to handle the complexity of the problem. A group of studies optimized both charger locations and battery costs, considering various cost components associated with electric bus fleets. In the same vein, Kunith et al. [10] presented a mixed-integer linear programming (MILP) model to determine both the optimal placement of charging stations and the appropriate battery capacity for each bus line. Bi et al. [11] presented a multi-objective framework for locating wireless chargers in a multi-route electric bus system, considering both system-level costs but also life cycle greenhouse gas emissions and energy consumption over the entire lifetime of the system. A genetic algorithm (GA) was used to solve the optimization model and evaluate the trade-offs between installation and life cycle costs. Considering the use of energy storage stations, He et al. [12] developed a MILP to minimize the total cost of batteries, terminal and opportunity charging stations, energy storage units and electricity demand charges. Similarly, Lotfi et al. [13] identified the optimal charging system configuration and battery capacity for various electric bus types and charging infrastructures, using an MILP model that minimized the total cost of ownership (TCO) of the fleet. Lin et al. [14] presented a mixed-integer nonlinear programming model (MINLP), representing the problem of sizing and locating plug-in charging stations for electric buses as a multi-step planning model and explicitly modeling the interactions with the power grid. Under a similar context of a gradual transition to electric bus fleets, He et al. [2] presented a bi-objective MINLP to determine the locations of en-route and depot chargers, along with the number of vehicles to be purchased and bus lines to be electrified at each planning period.

A successive stream of studies sought to incorporate more realistic considerations into the CSLP, such as uncertainty related to energy supply or consumption and the formation of queues at charging stops. Incorporating energy consumption uncertainty in the fast-charging station location problem, Liu et al. [15] developed a robust optimization MILP model to define the number and type of fast chargers at each stop as well as the battery size of buses. An [16] devised a stochastic integer program to concurrently optimize the placement of charging stations and the bus fleet size, taking into account unpredictable variations in charging demands and time-of-use electricity tariffs. Considering energy supply uncertainty, Iliopoulou and Kepaptsoglou [17] developed a robust integer programming model for locating chargers at bus stops under power variability at charging stations. Incorporating grid-related considerations, Wu et al. [18] presented a two-stage nonlinear programming model to determine the capacity and locations of charging stations. An Affinity Propagation algorithm was employed to generate potential charging sites first, and a Binary Particle Swarm Optimization method was then used to find the locations and numbers of corresponding stations. Tzamakos et al. [19] developed a MILP model with the aim of minimizing the investment cost associated with the deployment of opportunity and terminal charging facilities under waiting time constraints, using an M/M/1 queuing model to handle bus queuing.

2.2. Charging station location and scheduling

A branch of the related literature has formulated comprehensive models that optimize both location and scheduling decisions for electric bus charging. In this line of research, Rogge et al. [20] jointly examined vehicle scheduling and charger location under depot charging, presenting a grouping GA and a MILP model for each task, minimizing the TCO for an electric fleet. Liu and Ceder [21] introduced a model based on the deficit function theory, along with a mathematical programming model, to establish schedules for an electric bus fleet while optimizing the number of fast-chargers deployed at terminal stations. Stumpe et al. [22] presented a MILP to concurrently optimize both the opportunity charging infrastructure and schedules for an electric fleet and presented a Variable Neighborhood Search-based (VNS) algorithm to solve the model. Using a VNS algorithm, Olsen and Kliewer [23] explored the integration of depot charging planning and electric bus scheduling with the aim of minimizing the overall cost, accounting for depot charger installation, vehicle expenses, and operational costs. Adopting a similar objective, Yao et al. [24] designed a hybrid GA to address the Multi-Depot Electric Vehicle Scheduling Problem with multiple types of vehicles, concurrently minimizing the number of chargers installed at depots. Li et al. [25] addressed the problem of finding the optimal assignment of trips to vehicles and the allocation of battery chargers to charging stations under a partial charging policy and variable electricity prices. The authors presented an adaptive GA to minimize the total investment cost of the public transport system. Considering uncertainty in passenger demand and travel times, Hu et al. [26] presented a robust optimization model to optimize the location of en-route fast chargers and charging schedules under time-varying electricity prices and waiting costs.

Several studies incorporated battery sizing decisions in their model formulations, along with practical constraints, with the objective of minimizing annualized costs. In this line of research, He et al. [27] presented a MILP model to concurrently optimize charger deployment, on-board battery capacity, and charging schedules. Subsequently, a charging scheduling model was introduced, and a rolling horizon approach was applied to optimize the real-time charging schedules for electric buses. A branch of studies considered variability in input parameters, in an effort to more realistically capture planning decisions. Wang et al. [28] developed a MILP model for opportunity charger deployment and bus scheduling that simultaneously determines the optimal battery capacity, fleet size, charger locations and the number of chargers installed at the central terminal under variable ridership, dwell time, and travel times. Considering energy-related variability, Foda et al. [29] presented an integer linear optimization model that determined the location and capacity of charging infrastructure, battery capacity, and charging schedules under variable electricity prices, grid constraints and trip-level energy consumption rates. Similarly, aiming to enhance the realism of modeling-related decisions, Wang et al. [4] focused on resolving charging conflicts between multiple bus lines, mitigating battery degradation resulting from overcharge and over-discharge, and ensuring the seamless continuity of the charging process. The authors presented two MILP models for en-route charging station location, battery sizing and charging scheduling optimization. In the only approach incorporating deadheading costs in the objective function, McCabe and Ban [3] introduced a MILP model to determine the locations and quantity of chargers, as well as the location, duration, and sequence of charger visits for each bus.

Several studies extend the integrated problem by incorporating considerations related to the energy grid and energy consumption dynamics. For example, in a recent survey, Foda and Mohamed [30] introduced three alternative optimization models to achieve the optimal configuration of electric buses' charging infrastructure, including chargers and Energy Storage Systems (ESS). Liu et al. [31] extend the problem to include solar-powered bus charging infrastructure by integrating Photovoltaic and Energy Storage Systems (PESS) to enhance public transport resilience during charging service disruptions or in cases when the charging capacity is compromised due to charging service degradation. Another work by Liu et al. [32] proposes an optimization model for electric bus charging infrastructure that considers power matching between chargers and batteries as well as the seasonal effects on battery performance. Using a surrogate-based approach, the model significantly impacts bus scheduling and charging efficiency, emphasizing the need to incorporate these factors in infrastructure planning.

2.3. Study contribution

Several mathematical programming models have been proposed for the CSLP over the past years which vary in terms of the planning horizon, decisions, and the optimization objectives considered. A few of these have addressed both location and scheduling decisions. Table 1 shows the contribution of our study in relation to the stream of studies on optimal charging location and scheduling in detail.

In Table 1, the term aggregated means that the constraints do not refer to scheduling conflicts for a specific charger but rather ensure that the number of charging vehicles is less or equal to the number of available chargers during a charging slot. The term scheduling decision refers to the types of decisions that are considered for the optimization of different types of schedules (i.e. vehicle-to-trip scheduling, vehicle charging scheduling, or crew-to-vehicle scheduling). We also note that bus stops also include terminal stops.

As can be seen in Table 1, the vast majority of relevant models considered charging to take place at depots, terminals or bus stops, overlooking the possibility of exploiting multi-use charging stations. The only exceptions are the studies by Rogge et al. [20] and McCabe and Ban [3], which, however, considered a single charger type with the objective of minimizing a weighted cost value comprising various cost components. In contrast, we assume that different charger types can be deployed, as the number of charging requests assigned at a given location may not warrant the installation of more expensive, fast chargers. Further, we minimize deadheading costs, treating investment costs through a budgetary constraint, in line with electrification subsidization policies. Moreover, we utilize binary variables to explicitly handle charging scheduling and prevent conflicts, allocating trips to specific chargers and charging time slots. Methodologically, in the following section, we propose a concise MILP model that exploits an efficient mapping scheme to match charging stations to distinct power levels, thereby reducing the number of variables needed. This results in an effective formulation that can be solved to global optimality for large problem instances, despite the NP-Hardness of the problem.

3. Formulation as a minimum deadheading problem

3.1. Mathematical program

The problem at hand entails determining the optimal charging installation options, where a charging installation option is associated with a physical location and a charger type (slow/fast). The problem also entails the optimal scheduling of vehicle charging requests at the selected charging installation options. In practical applications, a given budget is available for installing chargers, while different types of chargers (i.e., slow or fast) can be selected depending on the frequency of charging requests at each location. Furthermore, charging requests

Table 1
Literature summary.

Reference	Charging location	Scheduling decision	Multiple charger types	Charger scheduling constraints	Deadheading to chargers	Optimization goal	Solution approach
Rogge et al. [20]	Charging stations	Vehicle Charging		explicit	✓	TCO	GA and exact
Liu and Ceder [21]	Terminal stations	Vehicle Charging				Purchase cost	Heuristic
Stumpe et al. [22]	Bus stops	Vehicle Charging				TCO	VNS
Olsen and Kliewer [23]	Bus stops and depots	Vehicle Charging				TCO	VNS
Li et al. [25]	Terminal stations	Vehicle Charging		explicit		TCO	Adaptive GA
Yao et al. [24]	Depots	Vehicle Charging			to depot	TCO	GA-based heuristic
Hu et al. [26]	Bus stops	Vehicle Charging		explicit		TCO	exact
He et al. [27]	Terminal stops and depots	Charging	✓			TCO	exact
Wang et al. [28]	Bus stops	Vehicle Terminal Charging		aggregated		TCO	exact
Foda et al. [29]	Bus stops and depots	Charging	✓	aggregated		TCO	exact
Wang et al. [4]	Bus stops	Charging	✓	aggregated		TCO	exact
McCabe and Ban [3]	Charging stations	Charging		aggregated	✓	Purchase and deadheading cost	exact
Foda and Mohamed [30]	Charging stations, depots	Charging Station, Vehicle Charging		explicit		TCO and emissions	exact
Liu et al. [31]	Depots	Charging station, Vehicle Charging		explicit		Operating cost, service robustness	exact
Liu et al. [32]	Depots	Charging station, Vehicle scheduling		explicit		Operating cost, passenger waiting time	heuristic
This study	Depots and charging stations	Charging station, Charger Type, Vehicle Charging	✓	explicit	✓	deadheading cost	exact

are typically managed in advance through a reservation policy that mandates operators to reserve charging slots for vehicles at charging stations. As such, the charging station location selection and scheduling problem is cast as follows:

"*Given a limited budget that allows to install a certain number of charging stations with varying power levels and installation costs, a pool of $\mathcal{V}$ potential charging station locations, a pool of $\mathcal{N}$ potential charging station installation options, and $\mathcal{K}$ electric bus trips that require charging, select the optimal types and locations of chargers from the pool of charging station installation options and assign bus trips to charging slots in order to reduce the overall deadheading costs.*"

The main assumptions of the problem are the following:

1. Lines are not associated with dedicated charging locations. That is, a bus trip can be freely assigned to any charging station if deemed necessary.
2. Different types of chargers can be installed, i.e., slow and fast charging.
3. The time needed to recharge after a trip varies depending on the vehicle's remaining battery level and the type of the corresponding charger (slow or fast).
4. Vehicles leave charging stations after being recharged up to their maximum allowed battery level.
5. A charging time slot is fully occupied by a vehicle, regardless of the actual charging duration of the vehicle. That is, the corresponding charger is considered occupied for the entire duration of the time slot, even if the bus finishes charging before the end of the slot.
6. The consideration of charging needs takes place according to a daily charging schedule, considered to be known a day ahead. In this way, the charging scheduling modeling and time slot availability can be subject to a day-ahead booking system and queuing at the chargers is taken into consideration in case of unavailable time slots.

In this problem definition, we have a pre-selected pool of potential charging station locations $\mathcal{V}$. At each charging station location, we may have several chargers of different types (slow/fast). This results in the set of all possible charging installation options $\mathcal{N}$. Note that $\mathcal{V} \subseteq \mathcal{N}$ because multiple charging installation options might be offered at the same physical location. The set $\mathcal{N}$ of possible charger installations results in an expanded set of potentially available charging slots $\mathcal{F}$. Given the above, we have to allocate $\mathcal{K} = \{1, 2, 3, \ldots\}$ vehicle trips that require charging to the available charging slots $\mathcal{F}$ such that the deadheading times of all trips are minimized subject to budgetary constraints. The deadheading times are average values (considering an average bus speed [33] and the distance between the final stop and the potential charging station).

The set of potential charger installations $\mathcal{N}$ can be decomposed into the set of slow charging installation options $\mathcal{N}_1$ and the set of fast charging installation options $\mathcal{N}_2$, respectively. These charger installation options can be located at any point of the set $\mathcal{V}$, which represents the set of all possible physical locations of charging stations and is a sub-set of $\mathcal{N}$. In addition, each charger $j \in \mathcal{N}_1$ can be used multiple times during the day resulting in a set of time slots $\mathcal{F}_1$. The same holds for any charger $j \in \mathcal{N}_2$, resulting in time slots $\mathcal{F}_2$.

The set of all bus trips is $\mathcal{M}$, and the available time slots for slow and fast charging are $\mathcal{F}_1$ and $\mathcal{F}_2$, as defined above. From the set of all available bus trips, $\mathcal{M}$, we indicate with $\mathcal{K}$ the subset of trips that need charging ($\mathcal{K} \subseteq \mathcal{M}$). These $|\mathcal{K}|$ vehicle trips should be assigned to the available charging time slots $\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2$, as described above. This assignment of trips to charging time slots can be viewed as an unbalanced assignment problem [34], where the number of charging time slots should exceed the number of trips in set $\mathcal{K}$ to guarantee the feasibility of the problem.

Considering the parameters of the problem, the state of charge of each trip k that has finished its operations and requires charging is SOC_k. The time that trip k is completed and needs to drive to a charging station is τ_k. In addition, its minimum allowed state of charge is $SOC_k^{\min}$. The starting time for charging time slots for slow and fast charging stations are $c^s_{f_1}$ and $c^s_{f_2}$, respectively. Given the state of charge of trip k when it requires charging, its latest time threshold to start charging at a slow charger is p^s_k and at a fast charger is p^h_k, respectively.

The battery consumption per traveled distance is e and the fixed cost of installing a charger $j \in \mathcal{N}$ is b_j. The total available budget for installing charging stations is $b^{\max}$. Regarding the traveled distances, the minimum travel distance between the last stop of trip k and a potential charger location j is d_{kj}, resulting in a $|\mathcal{K}| \times |\mathcal{N}|$ matrix. Similarly, the estimated deadhead time from the last stop of trip k and a potential charger j is t_{kj}. Finally, we introduce binary parameters a_{kj}, where $a_{kj} = 1$ if charger j is reachable from the last stop of trip k given its minimum state of charge constraints; and 0 otherwise.

The decision variables of the problem include:

- $x_j \in \{0,1\}$, where $x_j = 1$ if we decide to construct charger $j \in \mathcal{N}$, and 0 otherwise. We note that $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ resulting in a simultaneous selection of charging location and type.
- $q_{kj} \in \{0,1\}$, where $q_{kj} = 1$ if trip $k \in \mathcal{K}$ is assigned to charger $j \in \mathcal{N}$ and 0 otherwise.
- $u^s_{kjf_1} \in \{0,1\}$, where $u^s_{kjf_1} = 1$ if trip k starts charging at the time slot $f_1 \in \mathcal{F}_1$ at the slow charger $j \in \mathcal{N}_1$.
- $u^h_{kjf_2} \in \{0,1\}$, where $u^h_{kjf_2} = 1$ if trip k starts charging at the time slot $f_2 \in \mathcal{F}_2$ at the fast charger $j \in \mathcal{N}_2$.
- $y_k \in \mathbb{R}_{\geq 0}$ which indicates the deadheading time of each trip $k \in \mathcal{K}$.

The notation of this problem is summarized in Table 2.

Using this nomenclature, we know in advance the state of charge SOC_k of each trip $k \in \mathcal{K}$ that has finished its operations and requires charging. Knowing also the minimum allowed state of charge $SOC_k^{\min}$ of each trip k and the exact location of each potential charger $j \in \mathcal{N}$, we can derive all potential chargers which are reachable by trip k. These charger installation options are all options that can be reached by trip k without its state of charge falling below $SOC_k^{\min}$. That is, a charger j is reachable by trip k if, and only if,

$$SOC_k - e d_{kj} \geq SOC_k^{\min} \tag{1}$$

If the above condition is met for a charger j, we set $a_{kj} = 1$. If not, $a_{kj} = 0$. Performing this check for all other pairs of trips $k \in \mathcal{K}$ and potential chargers $j \in \mathcal{N}$, we pre-compute the adjacency matrix:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1|\mathcal{N}|} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2|\mathcal{N}|} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kj} & \cdots & a_{k|\mathcal{N}|} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{|\mathcal{K}|1} & a_{|\mathcal{K}|2} & \cdots & a_{|\mathcal{K}|j} & \cdots & a_{|\mathcal{K}||\mathcal{N}|} \end{bmatrix}$$

where each element a_{kj} takes the value of 1 if the respective charger j is reachable by trip k and 0 otherwise. To ensure that we will construct chargers so that any trip $k \in \mathcal{K}$ will be served by at least one charger $j \in \mathcal{N}$, one can introduce a subset $\mathcal{N}_k \subseteq \mathcal{N}$ which contains all charger options that are reachable by trip k (that is, $a_{kj} = 1 \ \forall j \in \mathcal{N}_k$) and construct at least one charger from the options in set $\mathcal{N}_k$. That is, force that $x_j = 1$ for at least one $j \in \mathcal{N}_k$. Mathematically, this can be enforced by the following inequality constraint:

$$\sum_{j \in \mathcal{N}_k} x_j \geq 1 \quad \forall k \in \mathcal{K} \tag{2}$$

or, equivalently,

$$\sum_{j \in \mathcal{N}} a_{kj} x_j \geq 1 \quad \forall k \in \mathcal{K} \tag{3}$$

Table 2
Nomenclature.

Sets	
$\mathcal{V}$	set of all possible charging station physical locations.
$\mathcal{N}$	set of all possible installation options for chargers, where $\mathcal{V} \subseteq \mathcal{N}$.
$\mathcal{N}_1$	set of slow charger installation options.
$\mathcal{N}_2$	set of fast charger installation options.
$\mathcal{F}_1$	set of time slots for slow chargers.
$\mathcal{F}_2$	set of time slots for fast chargers.
$\mathcal{K}$	set of trips that need charging.
Parameters	
SOC_k	state of charge of trip k after its completion.
$SOC_k^{\min}$	minimum allowed state of charge of trip k.
$c^s_{f_1}$	starting times for charging time slots of slow charging stations.
$c^h_{f_2}$	starting times for charging time slots of fast charging stations.
τ_k	time when trip k is completed.
p^s_k	latest time threshold for trip k to start charging at a slow charger.
p^h_k	latest time threshold for trip k to start charging at a fast charger.
M	a very large positive number.
e	battery consumption per traveled distance.
b_j	fixed cost of installing charger j, i.e., if close to inverter, the cost is smaller.
$b^{\max}$	total amount of installation budget.
d_{kj}	minimum travel distance between the final stop of trip k and the potential charger location j.
t_{kj}	estimated deadhead time from the last stop of trip k and the location of the potential charger j.
a_{kj}	binary parameter that equals to 1 if there exists a charger $j \in \mathcal{N}$ which is reachable from the last stop of trip k given its minimum state of charge, and 0 otherwise.
Decision Variables	
$\mathbf{x}$	$\mathbf{x} = [x_1, \ldots, x_j, \ldots, x_{\lvert N \rvert}]^\intercal$, where $x_j = 1$ if we decide to construct $j \in \mathcal{N}$ and $x_j = 0$ if not.
Q	0-1 matrix, where $q_{kj} = 1$ if the trip $k \in K$ is assigned to charger j and 0 otherwise.
$u^s_{kjf_1}$	binary variables, where $u^s_{kjf_1}$ if trip k starts charging at slow charging time slot f_1 at charger $j \in \mathcal{N}_1$.
$u^h_{kjf_2}$	binary variables, where $u^h_{kjf_2} = 1$ if trip k starts charging at fast charging time slot f_2 at charger $j \in \mathcal{N}_2$.
Variables	
$\mathbf{y}$	$\mathbf{y} = [y_1, \ldots, y_i, \ldots, y_{\lvert K \rvert}]^\intercal$ deadheading time of trip $k \in \mathcal{K}$.

Note that the above formulation is also used for the traditional coverage problem [35].

In addition, each trip $k \in \mathcal{K}$ should charge at exactly one charger $j \in \mathcal{N}$. That is, each trip k should be assigned to exactly one charger j. This can be achieved by using the binary variables q_{kj} – where $q_{kj} = 1$ if the trip $k \in \mathcal{K}$ is assigned to charger j and 0 otherwise – and enforcing that:

$$\sum_{j \in \mathcal{N}} q_{kj} = 1 \quad \forall k \in \mathcal{K} \tag{4}$$

Because a trip k can be assigned to a charger j if, and only if, we have decided to construct this charger, the previous constraints should be accompanied by the following:

$$q_{kj} \leq x_j \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N} \tag{5}$$

Constraints (5) indicate that a trip k cannot be assigned to charger j if it has not been constructed ($x_j = 0$). Because a trip k cannot be assigned to a charging station j if this charger is out of reach, we also have that:

$$q_{kj} \leq a_{kj} \quad \forall k \in \mathcal{K}, \ \forall j \in \mathcal{N} \tag{6}$$

Using the above constraints that assign each vehicle k to exactly one charger j, the deadheading time of a trip k from its last stop to its charger is defined as:

$$y_k = \sum_{j\in\mathcal{N}} t_{kj} q_{kj} \quad \forall k \in \mathcal{K} \tag{7}$$

where t_{kj} is a parameter indicating the estimated deadhead time from the end stop of trip k to charger j.

Because we need to minimize the overall deadheading time of all trips when traveling to the locations of their chargers, the objective function of our problem is:

$$\min_{\mathbf{y}} \sum_{k\in\mathcal{K}} y_k \tag{8}$$

which is a linear function. Chargers are available in two levels of charging speed: slow or fast charging. In this manner, slow chargers belong to subset $\mathcal{N}_1 \subseteq \mathcal{N}$ and fast chargers belong to subset $\mathcal{N}_2 \subseteq \mathcal{N}$, where $\mathcal{N}_1 \cap \mathcal{N}_2 = \emptyset$ and $\mathcal{N}_1 \cup \mathcal{N}_2 = \mathcal{N}$. The use of subsets for indicating the different types of chargers circumvents the need to consider a separate decision variable to capture the power level of chargers, thus simplifying the formulation without altering the problem.

Because a charger can be used multiple times during the day, we consider that the time horizon is discretized into $\mathcal{F}_1$ (for slow chargers) and $\mathcal{F}_2$ (for fast chargers) homogeneous time intervals, which represent charging time slots. We assume that vehicles arrive at chargers based on an advance reservation policy [36]. Slot allocation and scheduling must be hence performed prior to the start of the service. Consequently, if a vehicle is assigned to a charger j after completing trip k, that is $q_{kj} = 1$, there exists a slot f_1 or f_2 such that $u^s_{kjf_1} = 1$ or $u^h_{kjf_2} = 1$, based on the type of charger (slow/fast) that the vehicle is being assigned to. Depending mainly on the deadhead distance and the arrival time at the depot, a vehicle can either occupy a slow charging slot f_1 at a respective charging option ($j \in \mathcal{N}_1$) or a fast charging slot f_2 at a fast charging option ($j \in \mathcal{N}_2$). A vehicle can only be assigned to a charging slot f if it can arrive on-time at the start of the charging period at the corresponding charger j, while also maintaining its battery level above the minimum state of charge. Advance slot allocation reflects common practice by operators since vehicle charging schedules must be known at the start of daily operations. Moreover, this approach allows us to reduce the number of variables in the model. Note that we do not aim to track the exact energy level of the vehicles but only to ensure that the allocated charging time under the reservation policy suffices for recharging the battery from the minimum to the maximum state of charge. This is ensured through the selection of appropriate values for the duration of charging slots and battery level thresholds, which are problem parameters that can be pre-computed.

Considering the above, the charging station location selection and scheduling problem with slow and fast charging options, which strives to minimize the overall deadheading times, is summarized as follows:

$(\tilde{Q})$:

$$\min_{\mathbf{y}} \sum_{k\in\mathcal{K}} y_k \tag{9}$$

$$\text{s.t.:} \sum_{j\in\mathcal{N}} a_{kj} x_j \geq 1 \quad \forall k \in \mathcal{K} \tag{10}$$

$$y_k = \sum_{j\in\mathcal{N}} t_{kj} q_{kj} \quad \forall k \in \mathcal{K} \tag{11}$$

$$\sum_{j\in\mathcal{N}} x_j b_j \leq b^{\max} \tag{12}$$

$$q_{kj} \leq x_j \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N} \tag{13}$$

$$\sum_{k\in\mathcal{K}} q_{kj} \geq x_j \quad \forall j \in \mathcal{N} \tag{14}$$

$$\sum_{j\in\mathcal{N}} q_{kj} = 1 \quad \forall k \in \mathcal{K} \tag{15}$$

$$\sum_{f_1\in\mathcal{F}_1} u^s_{kjf_1} + \sum_{f_2\in\mathcal{F}_2} u^h_{kjf_2} \leq q_{kj} \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N} \tag{16}$$

$$\sum_{f_1\in\mathcal{F}_1} \sum_{j\in\mathcal{N}_1} u^s_{kjf_1} + \sum_{f_2\in\mathcal{F}_2} \sum_{j\in\mathcal{N}_2} u^h_{kjf_2} = 1 \quad \forall k \in \mathcal{K} \tag{17}$$

$$\sum_{k\in\mathcal{K}} u^s_{kjf_1} \leq 1 \quad \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{18}$$

$$\sum_{k\in\mathcal{K}} u^h_{kjf_2} \leq 1 \quad \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{19}$$

$$q_{kj} \leq a_{kj} \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N} \tag{20}$$

$$u^s_{kjf_1} c^s_{f_1} \geq (\tau_k + t_{kj}) q_{kj} u^s_{kjf_1} \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{21}$$

$$u^h_{kjf_2} c^h_{f_2} \geq (\tau_k + t_{kj}) q_{kj} u^h_{kjf_2} \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{22}$$

$$u^s_{kjf_1} c^s_{f_1} \leq (p^s_k + t_{kj}) q_{kj} u^s_{kjf_1} \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{23}$$

$$u^h_{kjf_2} c^h_{f_2} \leq (p^h_k + t_{kj}) q_{kj} u^h_{kjf_2} \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{24}$$

$$x_j \in \{0,1\} \quad \forall j \in \mathcal{N} \tag{25}$$

$$y_k \in \mathbb{R}_{\geq 0} \quad \forall k \in \mathcal{K} \tag{26}$$

$$q_{kj} \in \{0,1\} \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{N} \tag{27}$$

$$u^s_{kjf} \in \{0,1\} \quad \forall k \in \mathcal{K}, \forall f \in \mathcal{F}, \forall j \in \mathcal{N} \tag{28}$$

$$u^h_{kjf} \in \{0,1\} \quad \forall k \in \mathcal{K}, \forall f \in \mathcal{F}, \forall j \in \mathcal{N} \tag{29}$$

The objective function (9) seeks to minimize the deadheading time. Constraints (10) ensure that the constructed chargers are such that any trip $k \in \mathcal{K}$ can reach at least one of them. Equality constraints (11) compute the deadheading time of each trip $k \in \mathcal{K}$. Constraints (12) ensure that the total installation cost of all chargers does not exceed the total budget, $b^{\max}$. Constraints (13) ensure that a trip can be assigned to a charger $j \in \mathcal{N}$ only if we decide to construct this charger ($x_j = 1$). Constraints (14) ensure that if there are no assigned trips to charger j, then this charger is not constructed ($x_j = 0$). Constraints (15) ensure that each trip is assigned to exactly one charger. Constraints (16) guarantee that if one assignment of bus trip k to charger j takes place ($q_{kj} = 1$), then this trip k has to be assigned to at least one of the charging time slots f_1 or f_2 at charger j. At the same time, if $q_{kj} = 0$ for any k and j, an assignment of trip k to charging time slots cannot take place (left hand-side equals to zero). Constraints (17) ensure that each trip k is assigned either only to a slow charger $j \in \mathcal{N}_1$ at a specific time slot $f_1 \in \mathcal{F}_1$ or only to a fast charger $j \in \mathcal{N}_2$ at a specific time slot $f_2 \in \mathcal{F}_2$, i.e., the two options are mutually exclusive. Constraints (18) ensure that for each slow charger $j \in \mathcal{N}_1$ and for each slow charging time slot $f_1 \in \mathcal{F}_1$, we can assign at most one bus trip k. Constraints (19) ensure that for each fast charging station $j \in \mathcal{N}_2$ and for each fast charging time slot $f_2 \in \mathcal{F}_2$, we can assign at most one bus trip k. Constraints (20) ensure that we cannot assign a trip k to a charger j if the latter is not reachable.

Constraints (21) to (24) are introduced in order to ensure that the scheduling of bus trips to charging time slots adheres to the operational limitations imposed by their itineraries. Constraints (21) ensure that if a trip k is assigned to a slow charger $j \in \mathcal{N}_1$ at charging time slot f_1, that is $u^s_{kjf_1} = 1$, then the time slot $f_1 \in \mathcal{F}_1$ that is selected must start after the arrival time of the trip at the final stop summed with the travel time (denoted by t_{kj}) between the final stop k and the selected charger j. Notice that if trip k is not assigned to time slot f_1, constraints (21) are satisfied by default since the left-hand side and the right-hand side of the inequality constraint are equal to 0 because both of them are multiplied by $u^s_{kjf_1} = 0$. Similarly, constraints (22) ensure that if a trip k is assigned to a fast charger $j \in \mathcal{N}_2$, then the charging time slot $f_2 \in \mathcal{F}_2$ must start after the arrival time of the trip at the final stop and the travel time between the final stop and the charger. Hence, constraints (21) and (22) ensure that a trip k cannot start its charging before its arrival at the respective charger.

Correspondingly, constraints (23) ensure that if a trip k is assigned to a slow charger $j \in \mathcal{N}_1$, then the charging time slot $f_1 \in \mathcal{F}_1$ must start before the assumed latest charging time threshold p_k^s summed with the travel time between the final stop and the slow charger. Equivalently, constraints (24) ensure that if a trip k is assigned to a fast charger $j \in \mathcal{N}_2$, then the charging time slot $f_2 \in \mathcal{F}_2$ must start before the assumed latest time threshold p_k^h and the travel time between the final stop and the fast charger.

3.2. Reformulation to a mixed-integer linear program

Mathematical program $(\tilde{Q})$ is non-convex because constraints (21)–(24) are nonlinear. The nonlinearity emerges at the right-hand sides of these constraints, where variables q_{kj} are multiplied by variables $u^s_{kjf_1}$ or $u^s_{kjf_2}$, respectively. These nonlinearities in the constraints, prohibit the computation of a globally optimal solution (see Gkiotsalitis [37]). To address this, we introduce a big positive number $M \ggg 0$ and we replace the nonlinear constraints (21)–(24) of $(\tilde{Q})$ by the following linear constraints:

$$(1-u^s_{kjf_1})M + u^s_{kjf_1} c^s_{f_1} \geq (\tau_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{30}$$

$$(1-u^h_{kjf_2})M + u^h_{kjf_2} c^h_{f_2} \geq (\tau_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{31}$$

$$-(1-u^s_{kjf_1})M + u^s_{kjf_1} c^s_{f_1} \leq (p^s_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{32}$$

$$-(1-u^h_{kjf_2})M + u^h_{kjf_2} c^h_{f_2} \leq (p^h_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{33}$$

Notice that constraints (30)–(33) and (21)–(24) are equisatisfiable. We can thus cast $(\tilde{Q})$ as the following mixed-integer linear program $(\hat{Q})$:

$(\hat{Q})$:

$$\min_{\mathbf{y}} \sum_{k \in \mathcal{K}} y_k \tag{34}$$

s.t.: Equations (10)–(20), (25)–(29), (30)–(33). (35)

Theorem 3.1. *Provided that the problem is feasible, the continuous relaxation of the mathematical program $(\hat{Q})$ has a globally optimal solution.*

Proof. Mathematical program $(\hat{Q})$ in Eqs. (10)–(20), (25)–(29), (30)–(33) is a mixed-integer linear program. Its continuous relaxation has a feasible region which consists of affine functions in the form of equality and inequality constraints, resulting in a polyhedron. The objective function is a linear function, and thus the continuous relaxation of the problem is both convex and concave. Consequently, any locally optimal solution of the continuously relaxed problem is also a globally optimal solution. ■

Remark 1. From Theorem 3.1, it follows that we can solve our mixed-integer linear program $(\hat{Q})$ to global optimality by employing the branch-and-bound solution method which exploits the solutions of the continuous relaxation of $(\hat{Q})$ to compute lower bounds.

Theorem 3.2. *Mathematical program $(\hat{Q})$ is NP-Hard.*

Proof. One can cast the optimization problem $(\hat{Q})$ as a decision problem by checking whether the objective function $\sum_{k \in \mathcal{K}} y_k$ is not greater than a value ψ for the problem instance y_k.

This decision problem is a nondeterministic polynomial time problem (NP) because given a certificate y_k, which constitutes a solution to the problem, a deterministic Turing machine can check in polynomial time whether

$$\sum_{k \in \mathcal{K}} y_k \leq \psi$$

and whether the constraints of the problem are satisfied. Note that the constraints of the problem increase polynomially with the size of the problem, and this is why all conditions in (10)–(29) can be checked in polynomial time.

We will further prove that our decision problem is NP-complete. Our decision problem is a generalization of the coverage problem of Toregas et al. [35], which is an NP-complete decision problem. The coverage problem is also poly-time reducible to $(\hat{Q})$ since we can use a polynomial time algorithm that translates problem instances of the exact cover decision problem to problem instances of $(\hat{Q})$ in such a way that the instance of one decision problem has a 'yes' answer if, and only if, the translated instance of the other decision problem has also a yes answer (see Gkiotsalitis et al. [6]). Thus, $(\hat{Q})$ is also NP-complete. Finally, $(\hat{Q})$ is NP-Hard when expressed as an optimization problem because its decision problem counterpart is NP-complete. ■

4. Numerical experiments

In our numerical experiments, we illustrate the model's application across three distinct cases. Firstly, we showcase its effectiveness using a simplified network model based on synthetic data derived from Athens, Greece. Subsequently, we delve into a series of experiments aimed at assessing the computational complexity of the model by solving problem instances of varying sizes. Lastly, we present a practical case study involving real-world data from the bus network of Athens. This study is particularly motivated by the imminent transition of the city's public transport authority to an electric bus fleet, commencing in 2024. Given this imminent shift, our focus lies on determining optimal locations for charging stations, charger types, and charging slots for the bus trips requiring charging.

Across the three applications of the model, there are several common considerations. First, an area that is serviced by 280 bus lines is considered, which corresponds to the greater Athens metropolitan area. Out of the total number of bus lines under consideration, a subset of $\mathcal{K}$ services are selected for the transition to an electric bus fleet. Bus trips of set $\mathcal{K}$ primarily operate between starting and ending stops within the Athens Municipality, the central administrative region of the metropolitan area, or directly adjacent municipalities, because the electric buses in Athens will operate only in these areas. Regarding the candidate charging station locations $\mathcal{V}$, for each application of the model we consider that multiple chargers can be placed at each location, that may be slow (set $\mathcal{N}_1$) or fast chargers (set $\mathcal{N}_2$). Thus, each potential charging station location offers several charger types and charging time slots. Depending on the analysis conducted, the number and type of chargers that can potentially be installed varies. Notwithstanding, across all model applications we consider that chargers have a predetermined number of charging time slots available, that are accessible at specific times within the day. The mathematical model introduced in this study allows for flexibility in the specification of chargers (slow/fast).

In addition to spatial and charging-related input factors, the mathematical model also accounts for the timing of charging and assigns charging sessions to specific *time slots*. Within the daily time horizon that is considered for this model, the first charging *time slot* is available at 10 a.m., and the last charging *time slot* ends at 10 p.m. (charging is completed for the last vehicle if charging option is utilized). The slow charging stations are considered to provide six *time slots* within the day for each charging option, with a duration of one hundred and twenty minutes each. In addition, the fast charging stations are considered to have 12 charging *time slots*, which are available every sixty minutes, given that the duration of charging for fast charging stations is equivalent to sixty minutes as well. This modeling of the

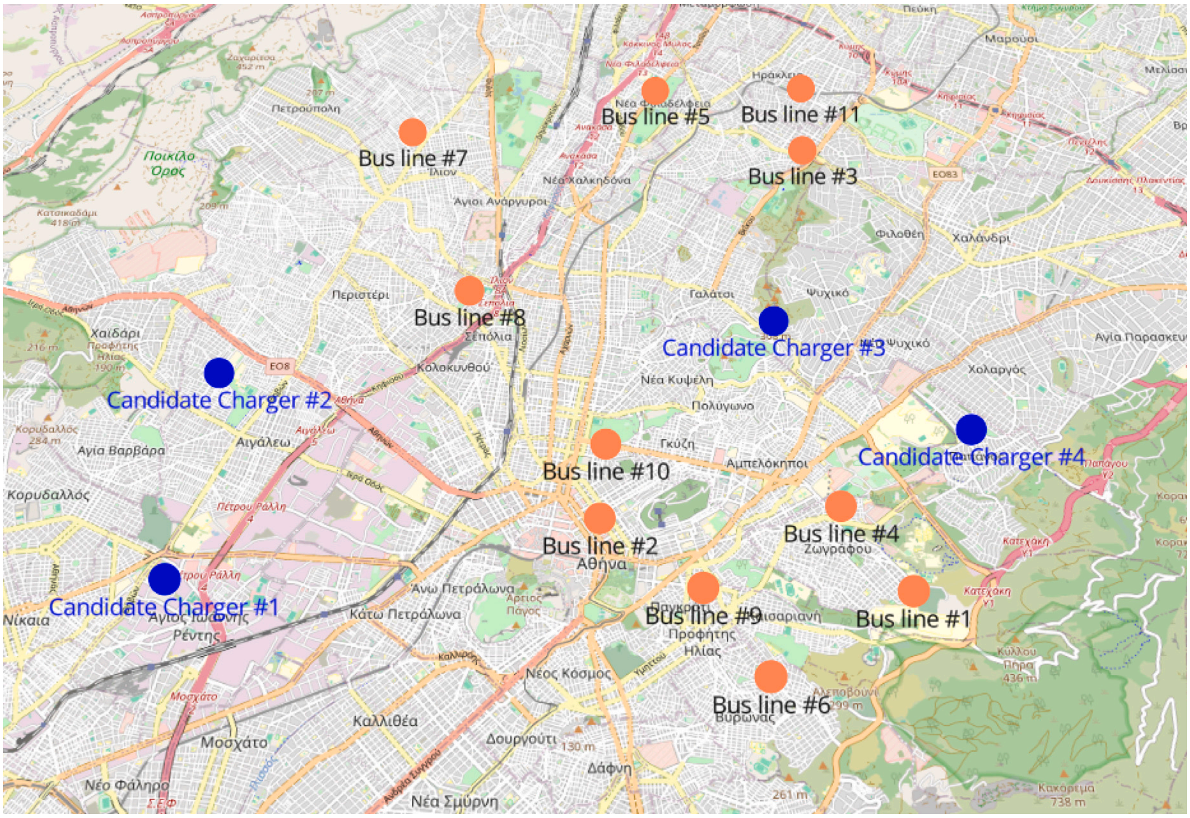

Fig. 1. Toy bus network example. The final stops of the eleven trips that require charging are presented in orange. The candidate charger installation options are presented in blue.

charging time slots enables the charging session scheduling based on a day ahead booking system for the charging of electric buses. This, in turn, ensures that each vehicle is assigned to an empty charging slot and does not allow the utilization of the occupied charging slots. This mechanism considers queuing and helps minimize it since vehicles in need of charging cannot use the occupied charging slots and have to wait for an appropriate empty slot.

All of the three cases of model application are solved using the same program code implementation of the mathematical model presented in Section 2. Python 3.7 is used, along with many of its standard libraries, while the Branch-and-Cut solution method implemented in Gurobi is used for the solution of the model. The Python code is made publicly available along with the necessary documentation on the GitHub repository [38]. The experiments are carried out on a conventional computer machine with a 2.0 GHz processor and 8 GB of RAM.

4.1. Demonstration on the toy network

To facilitate the reproduction of our model, we start our experimentation with a demonstration of our model's application in a small-sized scenario with synthetic data from the central area of Athens—see Fig. 1. For simplicity reasons, we consider four candidate chargers in this area. Each is at a different location, thus $\mathcal{V} \equiv \mathcal{N}$. From the pool $\mathcal{N}$ of the four candidate chargers, two are slow, and two are fast. More specifically, candidate chargers #1 and #2 are slow (belong to set $\mathcal{N}_1$), and candidate chargers #3 and #4 are fast (belong to set $\mathcal{N}_2$). That is, $\mathcal{N}_1 = \{1,2\}$ and $\mathcal{N}_2 = \{3,4\}$. Furthermore, in Fig. 1, one can notice the final stops for the trips of the bus lines that require charging (orange markers). For this small-sized example, we consider one trip per bus line ($|\mathcal{K}| = 11$). This input decision is based on the preliminary planning of the electric bus lines by the public transport authority, which requires only one charging detour from the original trip schedule during the daily operating hours (also discussed as partial re-charging). However, one can consider more trips per line without loss of generality. Finally, all eleven trips require charging when they arrive at their final stop, and the exact coordinates of the locations accompanying this example can be found at the GitHub repository.

Table 3
Travel times t_{kj} in minutes between the end stop of each bus trip $k \in \mathcal{K}$ and the potential charger installation option $j \in \mathcal{N}$.

line $k \in \mathcal{K}$	Charging station option $j \in \mathcal{N}$			
	1	2	3	4
1	22.8	23.0	10.2	5.9
2	13.4	13.3	9.1	12.2
3	25.8	21.2	6.9	11.8
4	20.9	20.3	6.9	5.0
5	24.0	18.2	10.1	16.6
6	18.5	20.1	12.2	10.6
7	18.0	11.2	14.3	21.6
8	13.6	8.5	10.1	16.9
9	16.4	17.1	9.7	10.2
10	14.2	12.7	7.1	11.6
11	27.6	22.5	9.6	14.3

Before solving the model and determining where to install the chargers with the respective slow or fast charging options, we first need to compute the travel distance between the end stop of each trip and the respective charger, resulting in the 11 × 4 matrix of Table 3.

Along with the spatial data (bus stop locations, candidate chargers locations) and the distances data, some synthetic temporal data must be considered. These are the bus arrival times τ_k, as well as the latest charging time thresholds p_k^s and p_k^h for slow and fast chargers, as reported in Table 4. In addition, in Table 5, we give the starting times $c_{f_1}^s$ and $c_{f_2}^h$ for charging slots for slow and fast chargers accordingly. Given that we are modeling the electric bus operations on a daily

Table 4
Values of temporal parameters for the toy bus network example: Arrival Time at the last stop τ_k, and latest time threshold for slow or fast chargers p_k^s and p_k^h (in minutes past midnight, assuming continuous time representation).

	1	2	3	4	5	6	7	8	9	10	11
τ_k	656.4	710.7	807.1	890.5	876.8	971.2	892.5	918.9	1125.3	1160.6	1024.0
p_k^s	776.4	830.7	927.1	1010.5	996.8	1091.2	1012.5	1038.9	1245.3	1280.6	1144.0
p_k^h	716.4	770.7	867.1	950.5	936.8	1031.2	952.5	978.9	1185.3	1220.6	1084.0

Table 5
Starting times of charging *time slots* for the toy bus network example for slow chargers $c_{f_1}^s$, and fast chargers $c_{f_2}^h$ (in minutes past midnight, assuming continuous time representation)

	1	2	3	4	5	6	7	8	9	10	11	12
$c_{f_1}^s$	600	720	840	960	1080	1200	–	–	–	–	–	–
$c_{f_2}^h$	600	660	720	780	840	900	960	1020	1080	1140	1200	1260

horizon, all of these temporal parameters are expressed in continuous time representation in minutes past midnight (e.g. 19:04 is represented by 1144.0).

In more detail, the bus arrival times τ_k refer to the time the bus trip k reaches its final stop and is available to its journey towards a charger location v to charge a candidate charger $j \in \mathcal{N}$. Regarding p_k^s and p_k^h, these are two time thresholds defining the time window when each bus can start its charging session at one of the candidate chargers n. These time thresholds and the respective time window would have to include the starting time of the charging slot $c_{f_1}^s$ or $c_{f_2}^h$ for a bus to start charging at that specific slot at any charger n.

For example, for this toy network and bus line $k = 5$, according to Table 4, the bus arrives at each final stop at $\tau_k = 876.8$. To start charging at any charger $j \in \mathcal{N}$ and according to the linearized constraints (30)–(33), which also account for the travel time, the bus $k = 5$ would have to reach the candidate charger $j \in \mathcal{N}$ after $\tau_5 + t_{kj} = 876.8 + t_{5j}$, depending on the travel time t_{5j} to each charger. While this serves as the earliest bound of a time window in which a charging session can start, there are also the latest thresholds or bounds. These are formed based on p_k^s and p_k^h. For bus line $k = 5$, the charging session must start before $p_5^s + t_{5j} = 996.8 + t_{5j}$, if it is a slow charger, and before $p_5^h + t_{5j} = 936.8 + t_{5j}$ if it is fast charger. Given the travel time values for bus line $k = 5$ ($t_{51} = 24.0$, $t_{52} = 18.2$, $t_{53} = 10.1$ and $t_{54} = 16.6$, a subset of the initial F_1 and F_2 times slots are available at the different chargers $j \in \mathcal{N}$ based on the emerging time windows. In this way, inspired by practices followed by real-world public transport operators, the model is incentivized to assign buses to chargers and charging time slots near the ending locations of the trips but also close to the finish time of the service for that trip. Still, the assignment process must account for the maximum latest time thresholds p_k^s and p_k^h for slow and fast, guaranteeing that the buses do not wait and queue up at the chargers.

Based on this data for the toy network instance, the model's optimal solution indicates that the installed chargers should be charger #2 (slow charging), as well as #3 and #4 (fast charging). This was computed with the Branch-and-Cut solution method of Gurobi, and results in an optimal total deadhead time of 95.73 min for our eleven bus trips.

Table 6 presents the optimal assignment of bus trips to chargers. Regarding the optimal values of the three-dimensional binary variables $u_{kjf_1}^s$ and $u_{kjf_2}^h$ which indicate the assignment of bus trips to charging time slots, the non-zero values are: $u_{7,2,4}^s = u_{1,4,3}^h = u_{2,3,3}^h = u_{3,3,5}^h = u_{4,4,6}^h = u_{5,3,6}^h = u_{6,4,8}^h = u_{8,3,7}^h = u_{9,3,10}^h = u_{10,3,11}^h = u_{11,3,9}^h = 1$. Namely, trip 7 is assigned to the 4th time slot of charger 2, trip 1 to the 3rd time slot of charger 4, etc. The detailed assignment to time windows is presented at Table A.13 for slow chargers and Table A.14 for fast chargers in Appendix section.

Given the optimal solution derived and the respective assignments of bus trips k to charger options j, as well as time slots f_1 and f_2, one can notice that for this small network the assignment yielded by the model is aligned with expectations: all bus trips charge at the nearest candidate charger location, with only one exception. By examining the

Table 6
Values of q_{kj} for the assignment of bus trips $k \in \mathcal{K}$ to chargers $j \in \mathcal{N}$ in the toy network case study.

q_{kj}	Charger option $j \in \mathcal{N}$			
	1	2	3	4
1	0	0	0	1
2	0	0	1	0
3	0	0	1	0
4	0	0	0	1
5	0	0	1	0
6	0	0	0	1
7	0	1	0	0
8	0	0	1	0
9	0	0	1	0
10	0	0	1	0
11	0	0	1	0

values of t_{kj} for this toy network instance as well as the values for $u_{7,2,4}^s = u_{8,3,7}^h = 1$, it can be noticed that bus line #8 does not get assigned to the slow charger at location #2, which is the nearest one to its final stop, but rather gets assigned to the fast charger at location #3. When examining the solution and the input parameters τ_k, p_k^s and p_k^h in Table 4, as well as the values of $c_{f_1}^s$ and $c_{f_2}^h$ of Table 5, one can notice that bus lines #7 and #8 compete for the same time slots at charger location #2, and more specifically the one described by variable $u_{k,2,4}^s$. Considering the travel times for bus lines #7 and #8 to locations #2 and #3, as they can be seen from the Travel time matrix (Table 3), the optimization model correctly chooses line #7 to be assigned to charger #2, saving 1.5 min from the total deadhead time, in comparison to the scenario that bus trip #8 would be assigned the slow charging option at location #2 and its fourth charging slot.

This first demonstration on the toy network illustrates the functionality of the model, with the model successfully assigning bus trips k from set $\mathcal{K}$ to slow or fast chargers $j \in \mathcal{N}$, as well as charging time slots $f_1 \in \mathcal{F}_1$ and $f_2 \in \mathcal{F}_2$. In this toy example, a competition for a limited resource, namely charger availability, is only manifested for trips #7 and #8, which compete over slot $u_{k,2,4}^s$, but this inherent property of the problem becomes increasingly challenging as the size of the network grows.

4.2. Numerical experiments for exploring the computational complexity

In this numerical analysis, we investigate the computational complexity of our model. To this end, we further develop a Python module to generate synthetic instances of the problem for the Athens region. The Python module, given the values for the sizes of sets $\mathcal{K}$, $\mathcal{V}$ and $\mathcal{N}$, generates the remaining of the input parameters.

The strategy for defining the size of the synthetic problem through the sizes of sets $\mathcal{K}$, $\mathcal{V}$ and $\mathcal{N}$ mainly focuses on increasing the size of the problem by increasing the number of bus trips considered in set $\mathcal{K}$. Sets $\mathcal{V}$ and $\mathcal{N}$ are modified to facilitate the charging needs of fleet $\mathcal{K}$ for each problem instance, while considering the ability to obtain a feasible

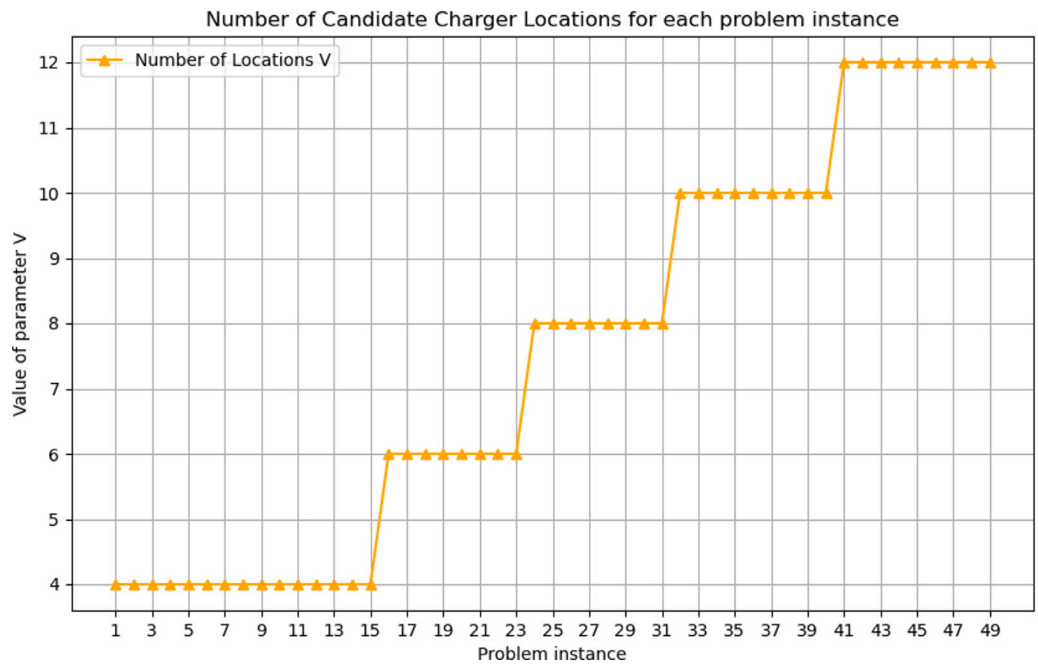

Fig. 2. Number of charging station physical locations $|\mathcal{V}|$ considered for each of the 49 problem instances.

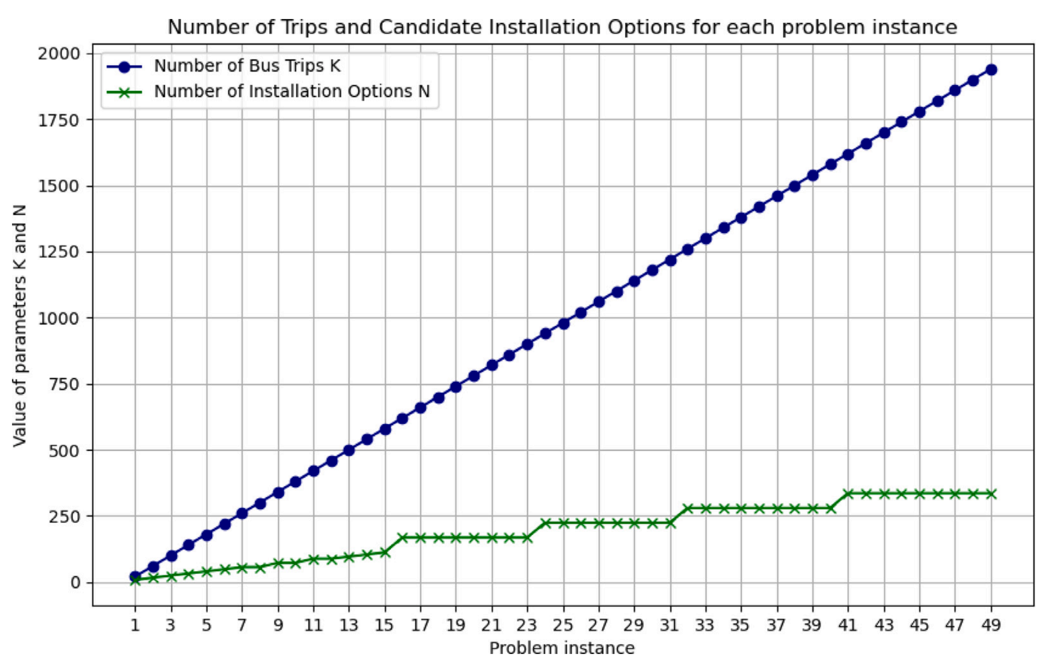

Fig. 3. Number of bus trips $|\mathcal{K}|$ and the number of charger installation options $|\mathcal{N}|$ for each of the 49 problem instances.

solution. That is, if the number of considered trips $\mathcal{K}$ is increased up to the point that the provided charger installation options are not enough to facilitate the charging demand, we increase the sizes of sets $\mathcal{V}$ and $\mathcal{N}$. This experimental design strategy can be conceptualized based on a 'supply' and 'demand' dynamic, which is created between the chargers that provide the required charging and the bus trips which generate the demand for charging. We adopted the following procedure: when a problem instance becomes infeasible due to an increase in set $\mathcal{K}$, we first raise the number of charger options $\mathcal{N}$ up to a certain number (i.e., 30 charger options per candidate location), and next when that threshold is reached, the number of physical locations for charging stations (set $\mathcal{V}$) is increased.

The computational complexity results are reported for 49 synthetic problem instances. We start from a problem instance of $\mathcal{K} = \{1, 2, \dots, 20\}$ trips that require charging, $\mathcal{V} = \{1, 2, 3, 4\}$ physical locations for charging stations and $\mathcal{N} = \{1, 2, \dots, 8\}$ charger installation options. We then increase the sizes of the instances progressively until going up to the size of $\mathcal{K} = \{1, 2, \dots, 1940\}$, $\mathcal{V} = \{1, 2, \dots, 12\}$ and $\mathcal{N} = \{1, 2, \dots, 336\}$. Figs. 2–4 report the related results for each problem instance, considering different combinations of values for sets $\mathcal{K}$, $\mathcal{V}$ and $\mathcal{N}$. The same results are given in matrix format in Table A.11 in Appendix.

The results table in A.11, as well as Figs. 2–4, show that the MILP model is solved within relatively short computational times, given its size. For problems of smaller sizes (i.e., Problem 5) with up to 180 bus

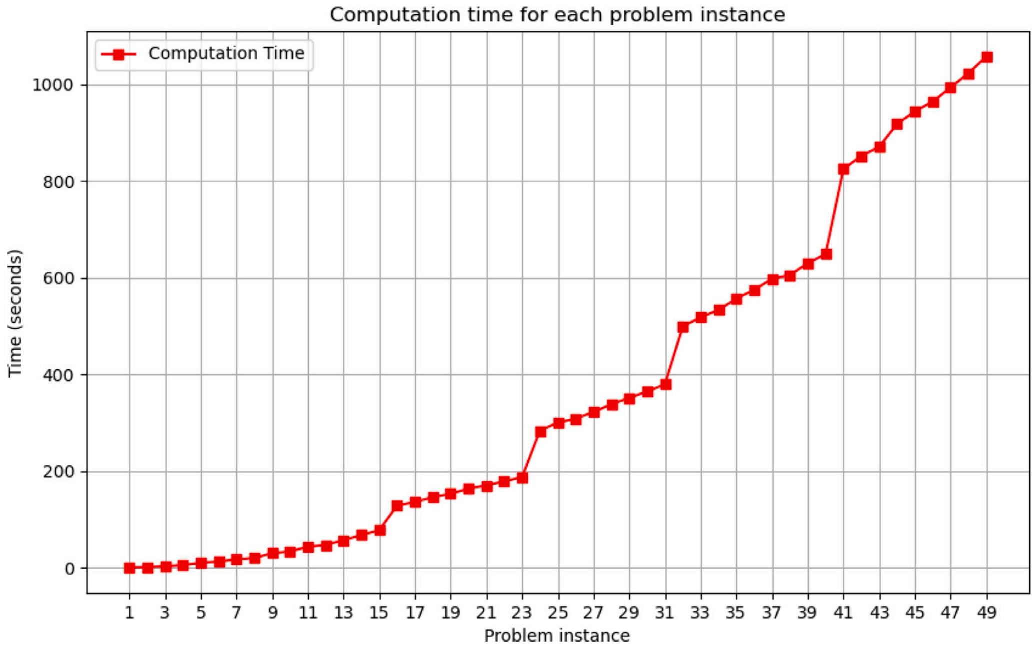

Fig. 4. Computation time for each of the 49 problem instances.

trips, 4 candidate charging station physical locations and 40 charger installation options, the problem can be solved in less than 10 s in a conventional computing machine. As the synthetic problem instances grow, one can notice that even challenging problem instances, such as Problem 26 with 1020 bus trips, 8 possible physical locations for chargers and 224 charger installation options, are solved in a little over 5 min, which is still an acceptable computational time for an offline scheduling problem. In the context of this numerical analysis, we solve the problem for up to 1940 bus trips, 12 candidate physical locations and 336 charger installation options in approximately 17 min. Beyond this point, the model could not be solved within a reasonable time by the conventional computer machine used for this experiment.

Based on the results of this experimentation, one can conclude that our problem formulation can be effectively solved using exact methods (Branch-and-Cut) within reasonable computation times, thereby providing solutions applicable to real-world problems, i.e., up to medium-sized municipal areas within urban environments.

4.3. Case study on the bus network of central Athens

Our real-world model application utilizes actual data from the central area of Athens. In this area, we consider nine candidate charging station physical locations, each having two potential charger options, one slow and one fast. If the model selects a candidate location, both or either one of the two chargers may be installed. Thus, we have a pool $\mathcal{V}$ of nine locations and a pool $\mathcal{N}$ of eighteen candidate charger installation options: nine slow chargers and nine fast chargers. More specifically, charger installation options #1, #3, #5, #7, #9, #11, #13, #15, and #17 refer to slow charging options (set $\mathcal{N}_1$), and #2, #4, #6, #8, #10, #12, #14, #16 and #18 refer to fast charging options (set $\mathcal{N}_2$). Hence, $\mathcal{N}_1 = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$ and $\mathcal{N}_2 = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$. The locations considered as candidate locations for charging stations correspond to the actual locations of bus depots because the public transport operator in Athens is willing to install chargers only at the locations of the bus depots. However, our model can consider more charging station locations which are not close to bus depots, given our generalized formulation.

For the electric bus network, we consider 10 different lines with electric buses. These 10 lines were chosen from Athens' extensive network of 280 bus lines because their conventional buses will be replaced by electric ones in 2024. The selection criteria were based on the geographical positioning of the lines' terminal stops, ensuring that they fall within the boundaries of the Athens Municipality. Fig. 5 illustrates the specific bus network layout considered for this case study. Because these 10 bus lines are not long, the electric buses operating in these lines will have to charge only once during the day. That is, we have a total number of 10 trips that require recharging (one trip per line).

Before attempting to solve the model, the travel distance between the terminal stop of each line and each of the candidate charging stations needs to be computed. The result is a 10×18 matrix presented in Appendix (Table A.12).

Similarly to the Athens toy network, in addition to the spatial and distance parameters, some extra parameters are considered for the temporal attributes of the Athens city network instance (Tables 7 and 8).

The bus arrival times τ_k indicate when bus trip k finishes its itinerary and is ready to head to a charger location v to charge at a candidate charger $j \in \mathcal{N}$. The parameters p_k^s and p_k^h refer to two time limits that define the period during which a bus can start charging at a charger $j \in N$ at either charging slots $c_{f_1}^s$ or $c_{f_2}^h$ depending on its type (fast or slow). As in the Athens toy network, all times are represented in continuous time representation in minutes past midnight (e.g. 1114.0 represents 18:34).

Based on this input data, the model's optimal solution (Table 9) indicates that four charging options should be selected (i.e. #1, #2, #15, #16) and chargers should be installed at locations #1 and #8. The total deadhead time for this optimal solution is 50.23 min for the ten bus trips of the respective lines.

Regarding the values of the three-dimensional binary variables $u_{kjf_1}^s$ and $u_{kjf_2}^h$ that handle the assignment of bus trips to charging slots, the non-zero values are: $u_{1,15,3}^s = u_{2,15,2}^s = u_{3,1,3}^s = u_{6,15,4}^s = u_{7,1,5}^s = u_{8,1,4}^s = u_{9,15,6}^s = u_{4,2,7}^h = u_{5,16,7}^h = u_{10,16,10}^h = 1$. The detailed assignment to time windows is presented in Appendix section at (Table A.15) for slow chargers and (Table A.16) for fast chargers.

One can notice that only two charging station locations are selected out of the 9 available locations: locations #1 and #8. Bus trips 3, 4, 7 and 8 are assigned to charging station location #1, while the rest of the bus trips are assigned to charging station location #8. Given the optimal solution derived and the respective assignments of bus trips k to charging options $j \in \mathcal{N}$, as well as time slots $f_1 \in \mathcal{F}_1$ and $f_2 \in \mathcal{F}_2$,

Table 7
Values of temporal parameters τ_k, p_k^s and p_k^h for the toy network example (in minutes past midnight, assuming continuous time representation).

	1	2	3	4	5	6	7	8	9	10
τ_k	723.0	699.0	783.0	950.0	892.0	851.0	987.0	955.0	1114.0	1090.0
p_k^s	843.0	819.0	903.0	1070.0	1012.0	971.0	1107.0	1075.0	1234.0	1210.0
p_k^h	783.0	759.0	843.0	1010.0	952.0	911.0	1047.0	1015.0	1174.0	1150.0

Table 8
Values of the sets $c_{f_1}^s$ and $c_{f_2}^h$ for the Athens city network example (in minutes past midnight, assuming continuous time representation).

	1	2	3	4	5	6	7	8	9	10	11	12
$c_{f_1}^s$	600	720	840	960	1080	1200	–	–	–	–	–	–
$c_{f_2}^h$	600	660	720	780	840	900	960	1020	1080	1140	1200	1260

Table 9
Values of q_{kj} for the assignment of bus lines $k \in \mathcal{K}$ to charging station options $j \in \mathcal{N}$ for the Athens case study with two types of charging stations.

	Charging station option $j \in \mathcal{N}$																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

Table 10
Values of q_{kj} for the assignment of bus lines $k \in \mathcal{K}$ to charging station options $j \in \mathcal{N}$ for the Athens case study with a single type of charger.

	Charging station option $j \in \mathcal{N}$																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

one can notice that in the Athens Municipality area, all ten bus trips can charge at the nearest candidate charging location.

To demonstrate the relevance of the proposed model compared to existing approaches, an additional experiment was conducted on the Athens network, incorporating exclusively a single type of charger—specifically, the slow chargers ($N \equiv N_1$). In the initial application of the model, we considered up to two candidate chargers per location—one slow charger and one fast charger. In this second application, however, we experimented with two candidate chargers of the slow type at each location. Consequently, the model may select only slow chargers at any given location in this scenario.

The results differ substantially when the decision space is limited to slow chargers compared to when both slow and fast chargers are incorporated into the analysis. More specifically, the model selects an optimal solution where the total deadhead time is 55.13 min, and an extra charger is required at location #3. The optimal values for q_{kj} can be viewed at Table 10, while the optimal assignments of buses to time slots are as follows: $u^s_{1,6,3} = u^s_{2,15,2} = u^s_{3,1,3} = u^s_{4,2,4} = u^s_{5,16,4} = u^s_{6,15,4} = u^s_{7,1,5} = u^s_{8,1,4} = u^s_{9,16,6} = u^s_{10,15,6} = 1$. Similarly to the rest of the case studies, in Appendix Table A.17, the detailed assignment of buses to slow chargers and time windows can be found.

Overall, this case study on the Athens city network demonstrates the functionality of the model using realistic data from the 10 bus lines that operate within the Municipality of Athens. The model successfully assigns bus trips to potential charging options $j \in \mathcal{N}$ of nine candidate charging station locations. Following the same approach, one can easily extend the application of our model in several medium-sized cities and parts of wider metropolitan regions. We finally note that the 10 selected lines in Athens are short, and the electric buses operating in these lines will have to recharge only once during their daily operations, resulting in 10 trips that need recharging. Notwithstanding this, our model can also be applied to longer lines for which electric buses might require multiple rechargings during their daily operations.

Concluding remarks

The study proposes a novel mathematical optimization model for the Charging Station Location Problem (CSLP), given a network of electric buses while considering charger scheduling constraints and multiple charger types. The model accounts for the minimization of deadhead time, defined as the travel time between the last stop of each trip and the location of the charging station. The deadheading times are average values considering the distance mentioned above and the mean speed of each bus. At the same time, the model accounts for several constraints, such as the duration of charging and the number of charging slots per day. This modeling of charging slots enables the electric bus charging session scheduling as a day-ahead booking system. In this way, buses cover their charging needs based on our day ahead

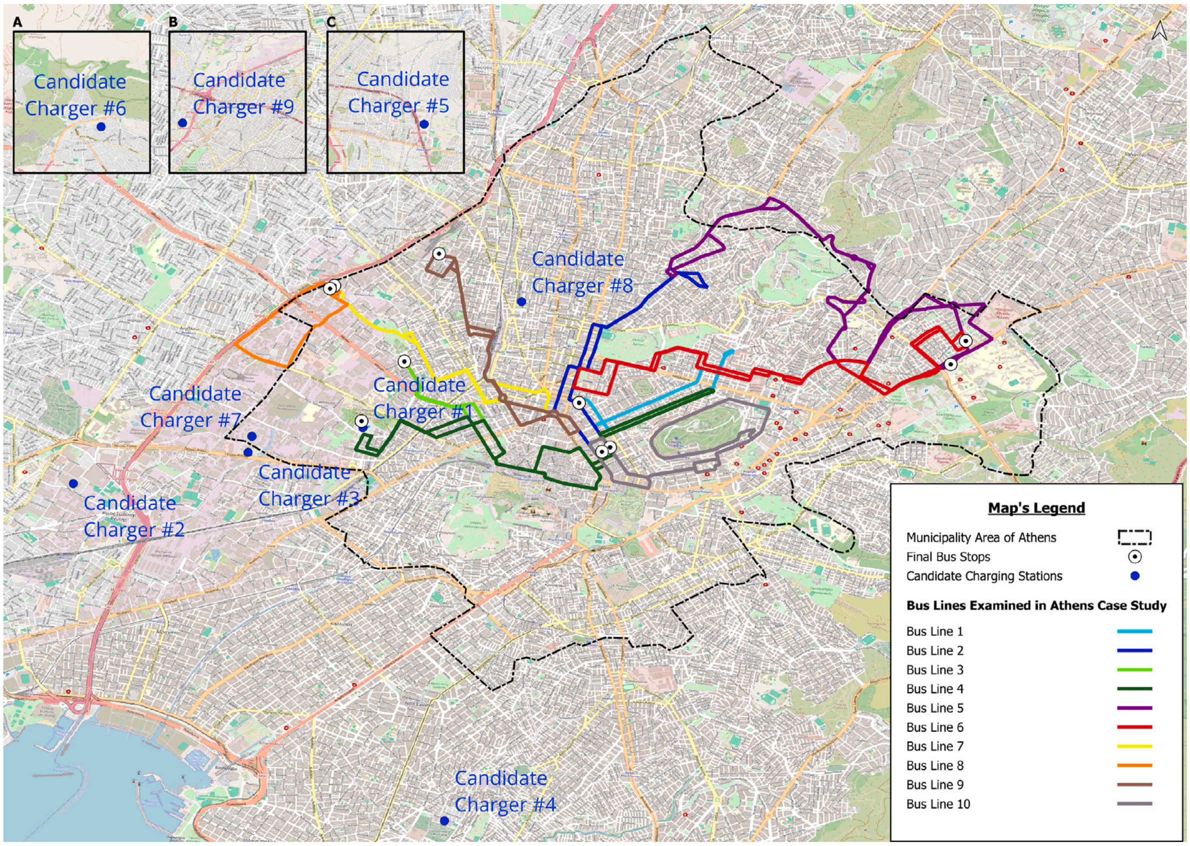

Fig. 5. Athens network of the electric bus lines considered for the study, together with the physical locations candidates for the charging stations. Locations A, B and C on the upper left corner represent depots in the Ano Liosia, Nea Philadephia and Anthousa regions.

knowledge of their charging needs, which helps avoid queuing for charging at occupied charging slots.

Following the problem formulation, the model is applied in three cases: (i) a toy network, with synthetic data from the metropolitan region of Athens, Greece, (ii) a computational complexity numerical analysis, where problems with up to 1940 bus lines, 12 candidate charging station locations and 336 charging installation options are also solved with synthetic data from the aforementioned region, and, finally, (iii) a case study with real-world data from the bus lines that operate in the central Athens, Greece.

The main contribution of this study is twofold: first, we introduce a mathematical model which is formulated as a MILP and can be solved to global optimality for instances with considerable size, thereby enabling the provision of support towards strategic decisions related to the transition of medium-sized cities to public transport networks that include electric buses. On a second level, the study provides insights from the real-world network of Athens, Greece, where the model is applied to select specific locations for charging stations, and respective charging station types, out of a wider set of charging station installation options. Considering the assumptions of our model and the respective limitations, future research directions could include:

- The consideration of more than two charging type options (slow/fast) in the model formulation.
- The consideration of the possibility to charge a vehicle before the end of its trip.

Future research directions should focus on addressing the abovementioned limitations. Additionally, further investigations could enrich our understanding of three interconnected areas: the interaction between charging station networks and the power grid, the integration with the Vehicle Scheduling Problem (VSP) and the consideration of the Crew Scheduling Problem (also known as rostering). Regarding the first research direction, future studies should examine the capacity and constraints of power grids near charging stations' locations. This investigation is crucial as the existing infrastructure might require upgrades to support increased charging demands. Several scenarios could be examined where these upgrades and up-front investment costs can be counterbalanced with demand forecasting and smart energy management systems. Regarding the second research direction proposed, the optimal solution of the CSLP for a fleet of electric buses is inherently linked with the timetable of the services (i.e. trips) and the assignment of the set of available buses in the fleet to the trips of the timetable. Therefore, it is proposed that solutions to the CSLP should be co-analyzed along solutions to the VSP, which could lead to the optimization of both charging station utilization and the buses' operation as a whole. Lastly, electric bus network planning approaches should consider the human factor by integrating the crew scheduling problem, which includes parameters like work hour limits and mandatory breaks between shifts. The VSP, which creates blocks for the vehicles, must be aligned with the corresponding driver shifts and breaks, requiring both to be analyzed together.

CRediT authorship contribution statement

Konstantinos Gkiotsalitis: Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Conceptualization. **Dimitrios Rizopoulos:** Writing – original draft, Visualization, Validation, Software, Formal analysis. **Marilena Merakou:** Writing – review & editing, Writing – original draft, Visualization, Software, Formal analysis. **Christina Iliopoulou:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization. **Tao Liu:** Writing – review & editing, Validation, Methodology. **Oded Cats:** Writing – review & editing, Supervision, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

See Tables A.11–A.17.

Table A.11
Computational analysis for the 49 synthetic problem instances.

	Number of bus lines K	Number of candidate CS locations V	Number of candidate Charging Stations $\mathcal{N}$	Computation time (s)
Problem 1	20	4	8	0.25
Problem 2	60	4	16	1.04
Problem 3	100	4	24	2.69
Problem 4	140	4	32	5.04
Problem 5	180	4	40	9.975
Problem 6	220	4	48	12.33
Problem 7	260	4	56	16.94
Problem 8	300	4	56	19.60
Problem 9	340	4	72	29.62
Problem 10	380	4	72	33.25
Problem 11	420	4	88	42.92
Problem 12	460	4	88	46.70
Problem 13	500	4	96	56.72
Problem 14	540	4	104	67.073
Problem 15	580	4	112	77.161
Problem 16	620	6	168	128.36
Problem 17	660	6	168	135.66
Problem 18	700	6	168	145.55
Problem 19	740	6	168	152.64
Problem 20	780	6	168	163.38
Problem 21	820	6	168	170.08
Problem 22	860	6	168	178.63
Problem 23	900	6	168	187.24
Problem 24	940	8	224	283.35
Problem 25	980	8	224	300.19
Problem 26	1020	8	224	308.15
Problem 27	1060	8	224	321.95
Problem 28	1100	8	224	337.98
Problem 29	1140	8	224	351.28
Problem 30	1180	8	224	364.23
Problem 31	1220	8	224	379.61
Problem 32	1260	10	280	498.92
Problem 33	1300	10	280	518.05
Problem 34	1340	10	280	533.24
Problem 35	1380	10	280	556.63
Problem 36	1420	10	280	575.05
Problem 37	1460	10	280	597.80
Problem 38	1500	10	280	605.60
Problem 39	1540	10	280	630.14
Problem 40	1580	10	280	648.92
Problem 41	1620	12	336	825.33
Problem 42	1660	12	336	852.06
Problem 43	1700	12	336	871.10
Problem 44	1740	12	336	918.16
Problem 45	1780	12	336	944.49
Problem 46	1820	12	336	964.59
Problem 47	1860	12	336	993.93
Problem 48	1900	12	336	1023.45
Problem 49	1940	12	336	1057.79

Table A.12
Travel times t_{kj} in minutes between the end stop of each bus line $k \in \mathcal{K}$ and the potential charging station options $j \in \mathcal{N}$.

line $i \in \mathcal{N}$	Charging station option $j \in \mathcal{N}$																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	5.66	5.66	13.38	13.38	8.41	8.41	13.88	13.88	29.02	29.02	29.28	29.28	8.57	8.57	3.51	3.51	17.76	17.76
2	6.25	6.25	13.77	13.77	8.86	8.86	12.65	12.65	28.67	28.67	30.86	30.86	9.11	9.11	5.12	5.12	19.11	19.11
3	2.46	2.46	9.6	9.6	4.89	4.89	14.7	14.7	33.22	33.22	27.47	27.47	4.8	4.8	3.59	3.59	18.03	18.03

(continued on next page)

Table A.12 (*continued*).

line $i \in \mathcal{N}$	Charging station option $j \in \mathcal{N}$																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
4	0.25	0.25	7.86	7.86	2.92	2.92	12.91	12.91	34.58	34.58	29.35	29.35	3.01	3.01	5.66	5.66	20.24	20.24
5	15.2	15.2	22.9	22.9	17.94	17.94	19.69	19.69	19.48	19.48	31.3	31.3	18.11	18.11	10.99	10.99	17.01	17.01
6	15.72	15.72	23.44	23.44	18.47	18.47	20.51	20.51	18.98	18.98	30.85	30.85	18.62	18.62	11.28	11.28	16.46	16.46
7	4.44	4.44	9.41	9.41	5.69	5.69	17.12	17.12	34.83	34.83	25.19	25.19	5.32	5.32	4.78	4.78	17.01	17.01
8	4.5	4.5	9.42	9.42	5.73	5.73	17.18	17.18	34.87	34.87	25.14	25.14	5.35	5.35	4.82	4.82	16.99	16.99
9	6.02	6.02	12.26	12.26	8.08	8.08	18.15	18.15	32.11	32.11	24.08	24.08	7.83	7.83	2.56	2.56	14.52	14.52
10	6.37	6.37	13.9	13.9	8.98	8.98	12.74	12.74	28.54	28.54	30.84	30.84	9.23	9.23	5.12	5.12	19.03	19.03

Table A.13
Assignment of bus trips $k \in \mathcal{K}$ to chargers $j \in \mathcal{N}_1$ to time windows $f_1 \in \mathcal{F}_1$, for slow chargers in the toy network case study.

Charger	Trip assignment to time windows for slow chargers					
	10:00-12:00	12:00-14:00	14:00-16:00	16:00-18:00	18:00-20:00	20:00-22:00
2	–	–	–	7	–	–

Table A.14
Assignment of bus trips $k \in \mathcal{K}$ to chargers $j \in \mathcal{N}_2$ to time windows $f_2 \in \mathcal{F}_2$, for fast chargers in the toy network case study.

Charger	Trip assignment to time windows for fast chargers											
	10:00–11:00	11:00–12:00	12:00–13:00	13:00–14:00	14:00–15:00	15:00–16:00	16:00–17:00	17:00–18:00	18:00–19:00	19:00–20:00	20:00–21:00	21:00–22:00
3	–	–	2	–	3	5	8	–	11	9	10	–
4	–	–	1	–	–	4	–	6	–	–	–	–

Table A.15
Assignment of bus trips $k \in \mathcal{K}$ to chargers $j \in \mathcal{N}_1$ to time windows $f_1 \in \mathcal{F}_1$, for slow chargers in the Athens case study.

Charger	Trip assignment to time windows for slow chargers					
	10:00–12:00	12:00–14:00	14:00–16:00	16:00–18:00	18:00–20:00	20:00–22:00
1	–	–	3	8	7	–
15	–	2	1	6	–	9

Table A.16
Assignment of bus trips $k \in \mathcal{K}$ to chargers $j \in \mathcal{N}_2$ to time windows $f_2 \in \mathcal{F}_2$, for fast chargers in the Athens case study.

Charger	Trip assignment to time windows for fast chargers											
	10:00–11:00	11:00–12:00	12:00–13:00	13:00–14:00	14:00–15:00	15:00–16:00	16:00–17:00	17:00–18:00	18:00–19:00	19:00–20:00	20:00–21:00	21:00–22:00
2	–	–	–	–	–	–	4	–	–	–	–	–
16	–	–	–	–	–	–	5	–	–	10	–	–

Table A.17
Assignment of bus trips $k \in \mathcal{K}$ to chargers $j \in \mathcal{N}_1$ to time windows $f_1 \in \mathcal{F}_1$, for the second experimentation in the Athens case study for a single type of charges (slow).

Charger	Trip assignment to time windows for slow chargers					
	10:00–12:00	12:00–14:00	14:00–16:00	16:00–18:00	18:00–20:00	20:00–22:00
1	–	–	3	8	7	–
2	–	–	–	4	–	–
6	–	–	1	–	–	–
15	–	2	–	6	–	10
16	–	–	–	5	–	9

Data availability

We share out data in a GitHub repository. The link for the repository is provided in our manuscript.

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