Design Aspects of Multiple Driven Belt Conveyors

Ashley Nuttall

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Design Aspects of Multiple Driven Belt Conveyors

Proefschrift

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Preface

In 1992 Dunlop-Enerka BV, located in Drachten, the Netherlands, started to develop a pouch shaped closed belt conveyor system called the Enerka-Becker System (or E-BS). It featured a revolutionary method of supporting the belt and included a multipoint drive system. Although multiple drives were integrates into the design of this conveyor system, little was actually known about how to coordinate the spatially distributed drive stations. As this system was only built with relatively small belt lengths (under 500 meters) and a small motor spacing, the belt stress caused by the introduction of drive forces was relatively low and not a limiting factor for the system. Therefore, a simple control method sufficed. However, the question was if larger systems would still work with this simple control method. Gabriël Lodewijks, who had also been involved with part of the development of the E-BS, also identified this issue. As little was known with respect to the implementation of multiple drives not only in the E-BS, but also in more conventional belt conveyor systems, he formulated a research proposal for the coordinated control of multiple driven belt conveyor system. This proposal formed the base for my PhD research. Initially, the focus was put on the coordinated control of the multimotor layout. However, as little was know about how the current simple control method adopted in the E-BS would perform in a large scale application, the focus was shifted to identifying problems that can be expected when adapting this method of control.

My special thanks goes to Gabriël Lodewijks, who inspired me with his fascination of applying scientific research on belt conveyor systems and who encouraged me throughout my research. Without his enthusiasm and drive this thesis would not have been possible. In addition I would like to thank Ton Klein Breteler for his ideas and guidance during my PhD work.

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I would also like to thank all the colleagues from Transport Engineering and Logistics for their interest in my research, sharing ideas on different subjects and the pleasant working environment.

Last but not least I would like to thank my mum, dad and family for their love and support, and my friends just for being who they are.

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1 Introduction

Worldwide belt conveyors are used to transport a great variety of piece goods as well as bulk solid materials, ranging from chunks of coal or iron ore to finer materials such as wood pulp and cement, in a wide array of applications. They are the most favourable transport system when it comes to moving bulk solid materials overland, especially in areas where infrastructure, such as road and railway tracks, is underdeveloped or non-existent. Although the basic belt conveyor configuration, with an endless belt spanned between a head and a tail pulley and supported by idlers, plates or air, is a well proven concept, the ever increasing bulk transport requirements continually press belt conveyor technology to its very limits. The desire to carry higher tonnages over longer distances and more diverse routes, while keeping exploitation costs as low as possible, has not only fuelled technological advances in the field of technical design, but in the field of monitoring system health, and dynamic analysis and simulations for optimal design as well. An interesting development in the recent past is the distribution of drive power along the path of the belt conveyor.

1.1 Multiple driven belt conveyors

Initially, the idea of distributing drive power along the length of the belt conveyor found its way into the mining industry. Due to a growing trend in mining efficiency production capacity and transport distances, the required drive power and belt strength surpassed that of what had ever been used underground before. This development combined with the fact that the mining equipment is continually moved with the progressing mining face gave rise to a number of conveyance concerns. Problems included the large size of high power drives and the inability to move them around. Although belting technology could have handled the increased strength requirements, it also meant moving from fabric to steel reinforced belting that was much harder to handle and required time consuming vulcanised splicing to connect new sections of belt to the system [Alspauch, 2003]. Similar problems occur with Tunnel Boring Machines (or TBM's). With the introduction of intermediate drives the required belt strength and drive unit size was reduced and by combining this with a belt take-up device with integrated belt storage, belt conveyor equipment had become flexible enough to solve the before mentioned problems. Figure 1.1 shows examples of drive configurations that turn the conventionally single driven flat or troughed conveyor into a multiple driven system.



Figure 1.1: Different concepts of multiple driven belt conveyors

The first configuration in Figure 1.1A is the tripper. In this configuration the belt is wrapped around two additional drive pulleys in the carrying strand of the conveyor. To create a sufficiently large wrap angle (which is the angle with which the belt is wrapped around the drive pulley and is required to apply the desired drive force) while keeping the belt travelling in the same direction, the belt has to discharge the bulk solid material onto itself. This additional transfer is also the downside of the tripper drive configuration as it is a source for dust, spillage, belt surface wear and it takes power.

The belt-on-belt drive configuration, see Figure 1.1B and C, where the main carrying belt rests on a smaller dedicated drive belt, does not interfere with the flow of material on the belt because it requires no transfer. In this case the weight of the bulk solid material and carry belt pressing down on the drive belt help to generate a friction based drive force. Figure 1.1B and C present two different variants of the belt-on-belt principle. The first variant is the booster belt configuration that has a number of small belt conveyors built inside the larger conveyor. In the second variant the double function a conveyor belt normally has to fulfil, both carrying the bulk solid material and transmitting the drive force, is effectively separated. This is achieved by placing a single drive belt or power strip within the main conveyor. Like the booster belt the power strip has a width that is less than the length of the central carrying idler. With the power strip spanning a large part of the system length, the main carry belt is subjected to very low drive forces and can have a much lighter construction. In this configuration multiple drive stations can be introduced without interfering with the material flow on the carry belt, as shown in Figure 1.1C.

A more recent development is the design and construction of special belt conveyors with highly flexible layout capabilities that inherently feature intermediate drive stations. Due to the special characteristics involved in such systems, they have been developed with the utilisation of multiple drives in mind, making it possible to offer systems with relatively light standardised belts, compact drive units and a light support structure. Figure 1.2 shows examples of these special belt conveyor systems.



Figure 1.2: Special conveyors featuring multiple drives

The first system, proposed by Bekel [1992] and shown in Figure 1.2A, consists of a more conventional trough shaped belt conveyor that has a special drive strip vulcanised to its underside, which acts as a drive rail. To apply traction, two motors with drive wheels press onto the belt at each side of the strip. The main idea behind this configuration is to have a trough shaped belt with a relatively low strength, so it can flex easily in horizontal curves, making it possible to negotiate sharp radii.

The second and third system are two different types of pouch shape belt conveyors that completely enclose the conveyed material. In the Enerka-Becker System (or E-BS), shown in Figure 1.2B, triangular profiles have been vulcanised to the edges of a flat belt. When the belt is folded and closed at the top, the profiles form the running surface for the supporting rolls or idlers as well as the drive wheels. With this configuration the drive units can be placed at virtually any location along the belt.

The drive units in the Sicon pouch belt conveyor system, shown in Figure 1.2C, cannot be placed as freely. The edges of this belt overlap at the top when the belt is folded and form a double v-shaped profile on one side of the pouch. This double v-shaped profile is used to wrap the belt around a drive or horizontal cornering pulley. Due to the fact that the Sicon belt has to be wrapped around such a drive pulley, the placement of drive units directly affects the layout of the system and has to be taken into account when designing the system.

Three different methods of creating a drive force in a belt conveyor system can be distinguished from the previous examples. They are presented in Figure 1.3. Figure 1.3A, B, and C show, respectively, the conventional drive pulley with the belt wrapped around it, drive

wheels that press onto the belt surface and the belt-on-belt principle where the weight of the carry belt and its load press down on a drive belt. In all cases a friction based drive force is generated by applying a normal force perpendicular to the contact surface in the form of a surface pressure σ_n , normal force F_n or distributed normal force F'_n , but the way with which the normal force is applied differs.



Figure 1.3: Different methods of creating drive forces

With the pulley configuration the tensions in the belt (T_1 and T_2) pull the belt onto the pulley and create a contact stress on the contact surface between the belt and pulley. To guarantee a maximum effective tension T_e or drive force, which is the difference between the tight side tension T_1 and the slack side tension T_2 , a minimum slack side tension always has to be present, as indicated by the Euler-Eytelwein equation:

$$T_{e} = T_{1} - T_{2} = T_{2} \cdot (e^{\mu \cdot \theta} - 1)$$
(1.1)

where μ is the friction coefficient between the belt and pulley and θ is the wrap angle. In the drive wheel configuration a normal force F_n is applied directly to the belt surface and directly affects the maximum effective traction as follows:

$$T_{e} = 2 \cdot F_{n} \cdot \mu \tag{1.2}$$

where the friction coefficient μ is not always constant, but can be a function of the normal force F_n [Bekel, 1992]. In the belt-on-belt configuration the normal force per unit belt length F'_n, caused by the weight of the carry belt and the bulk solid material on it, affects the maximum effective traction:

$$T_{e} = F'_{n} \cdot l_{cl} \cdot \mu \tag{1.3}$$

where l_{cl} is the contact length between the carry and drive belt. This shows that the drive wheel and belt-on-belt configuration do not require a slack side tension to drive the conveyor and can therefore be used in systems with a very low pretension and they interfere less with the conveyor layout than drive pulleys.

1.2 Distributed drive power and tension control

When distributed drive power is applied to a belt conveyor system, the system designer gains the opportunity to control and to reduce the tension in the belt. The most important result of this ability to influence the belt tension is that it enables the system designer to reduce the belt's strength requirements. The reduction in belt strength allows:

- A lighter and cheaper belt construction and support structure.
- The possibility to standardise system components. As the system length and power requirements increase the same type of belt can still be used by adding more drive stations.
- An increase in layout flexibility. With the lighter belt construction the belt can negotiate tighter turns and the smaller drive units occupy less space.

The actual reduction of the required belt strength does, however, depend on the application because the conveyor belt has a double function; firstly, the belt has to be strong and stiff enough to support a specified volume of bulk solid material without exceeding sag limits and secondly it has to be strong enough to transfer the required drive forces. As the belt length and power requirements increase, the potential benefits to be gained from the application of a distributed drive system also become larger.

To illustrate the principle behind the implementation of distributed drive power, Figure 1.4 shows a tension diagram of a conventional flat belt conveyor. In this diagram the belt tension is plotted as a function of the belt coordinate, which starts at the tail, where bulk material is loaded onto the belt, and increases in the transport direction.



Figure 1.4: Effect of distributed drive on the belt tension

The solid line represents the reference situation with a single drive pulley positioned at the head of the belt. Working from the tail, the tension increases as a result of the accumulation of resistance forces from the supporting rollers. It reaches its peak at the drive pulley because here the total drive force is applied to the belt. After passing the drive pulley the tension drops by an amount that is equal to the drive required to run the conveyor and it levels out at the pretension generated by the gravity take up device. As the belt moves back to the tail of the conveyor system the tension increases again, but at a much lower rate than in the carrying strand because there is no bulk material present on the return strand, resulting in much lower resistance forces. From this graph it is clear that the peak at the head pulley has a direct impact on the required belt strength when the safety factor and the belt width remain the same.

Splitting the drive power and dividing it over two separate locations results in a large reduction in belt tension, of about 40% in this example, as illustrated by the dashed line. In this case an intermediate drive unit is placed halfway down the carrying strand, causing an additional drop in tension and reducing the peak previously found at the head pulley. A further reduction of the peak value has also been made possible by the fact that the pretension can be reduced because the drive force generated by the head pulley has become smaller. However, this only applies to conveyor systems where the belt is wrapped around a drive pulley, for which the Euler-Eytelwein equation shows that the maximum transferable drive force is directly dependent on the wrap angle and the slack side tension of the pulley. To illustrate what happens when more drive units are added, the tension profile for a system with two intermediate drives is indicated by the dash-dotted line. In this case a further reduction of 15% is reached, assuming that the drive motors equally share the load.

From this example it is clear that a lighter belt construction can be used when smaller drive units are implemented and placed at those locations where power is needed the most. It also shows that the potential reduction also depends on the required pretension and that the attainable reduction diminishes with an increasing motor count.

In practice the optimum number of drive units is determined by the overall system cost rather than the belt strength. Although the required belt strength and thus the cost of belting decreases as the number of drive units increases, the cost of the drive system increases steadily with the increasing complexity. Figure 1.5 illustrates this correlation between the belting and drive costs, and the number of drive units. As both these component costs make up the majority of the total system costs, especially in large scale applications, the optimum number of drives is reached when the overall costs are minimised. Note that the drive size effects the dash-dot line.



Figure 1.5: Optimum number of drive units in relation to total cost

1.3 Research questions and scope of study

To fully exploit the principle of distributed drive power in a belt conveyor system, system designers require a good insight, and proper tools and guidelines. With these elements in hand it should enable them to attain the potential benefits such an approach has to offer and sufficiently compensate the added complexity and costs of using small spatially distributed drive units. However, as most existing design rules and practices are based on the more conventional single drive system, the question arises how usable these existing design rules and practices are for systems with multiple drives. Therefore, this study investigates what aspects of existing design rules and practices are applicable to multiple driven belt conveyor systems and, if necessary, what changes have to be made.

The main idea behind this study is based on finding the right balance between the locally applied drive power and the occurring resistances, so the belt tension stays within the safety margins in a controlled manner. To find the right balance during both normal operation, when the belt is running at its design capacity and at a constant speed, and transient conditions, encountered while starting, stopping, loading and unloading, the main focal points and their related questions are:

- The calculation of occurring resistances along the belt: What calculations are necessary to calculate the resistance, not only for the whole system, but also for each section?
- The transfer of mechanical power to the belt: What phenomena are involved to transfer the power and how much power can be transferred in relation to the resulting wear rate of the belt surface?
- The coordination of the drive units during transient conditions: What kind of starting and stopping procedures should be used and how long should it take to reach the new steady state?

• The scalability of multiple driven belt conveyor systems: Is there a need to modify the design rules when the system length is increased and more drive units are added?

To find the answers to these questions the Enerka-Becker System, with its pouch shaped belt and spatially distributed drive units, is used as a base for the analysis in this study. Although this is not a conventional belt conveyor system, it does feature the main characteristics of a multiple drive system, where the coordination of the drive motors and the control of the belt tension plays a vital role. Furthermore, the E-BS's drive units can be placed at virtually any location along the belt and the distribution of drive power greatly affects the belt tension because the pouch shape and the small idler spacing allow a relatively low pretension without causing excess sag. Due to the low pretension extra care also has to be taken to prevent the tension force in the belt becoming compressive during braking for example. Too low a tension force or a compressive force will cause the normally closed pouch shape to open and in the worst case the belt will run out of the idlers, causing major system failure.

Furthermore, scalability is also currently an important issue for the E-BS. With the longest system having a total belt length of around 500 meters, while using relatively small power units that do not make use of the full belt strength (including safety margins), questions arise with regard to belt dynamics and motor coordination when much longer systems are desired.

1.4 Outline of thesis

Chapter 2 takes a broader view with regard to distributed driven systems. A definition of multiple driven systems is formulated and used to compare multiple driven belt conveyors with other systems featuring distributed drive power. With this comparison the question is analysed if design and control approaches from other system can be applied to multiple driven belt conveyor systems.

Chapter 3 reviews which parts of existing models, standards and guidelines developed for conventional single driven belt conveyors are applicable to the multiple driven case. Attention is also paid to where further development is required in order to model most elements of a multiple driven belt conveyor system.

Chapter 4 looks closer at motion resistances. In this chapter an existing model for indentation rolling resistance is expanded to include a curved belt surface. A model is also presented to calculate resistances for tight horizontal curves, as found in the E-BS. Finally, an existing dynamic friction model is presented to include frictional effects, such as breakaway force and stiction, during dynamic simulations where a belt conveyor is started or stopped.

Chapter 5 investigates the mechanical transfer of drive force to the belt surface. The main focus is on the relationship between traction, slip and belt wear, occurring in a wheel driven belt conveyor. A method based on this relationship is also presented to determine the minimum number of drive stations that are needed to reach the belt's guaranteed lifetime.

Chapter 6 deals with the dynamic behaviour of a multiple driven belt conveyor during starting and stopping procedures and loading and unloading. To investigate the dynamic behaviour a dynamic model is used to carry out simulations, where the effect of the start-up time, speed ramp up shape, delayed start-up procedures and belt loading are analysed.

Finally, Chapter 7 summarises the conclusions of the research and formulates recommendations for future work.

2 Multiple driven transport systems

Recent technological developments in control and power electronics have made sophisticated drive technology more accessible and economically viable for many different applications. In the field of mechanical transport systems it has attributed to a rise in the number of state-ofthe-art systems featuring distributed drive power. The belt conveyor system is just one example into which multiple drive technology has been used. Other examples are transport systems such as high-speed trains, chain conveyors and the paper transport system in printing presses. Although multiple driven belt conveyors have been successfully implemented, no specific theories have been developed for these systems. Therefore, the goal of this chapter is to seek out other multiple driven systems that are comparable to belt conveyors systems and to see if the design theories for these other systems are adaptable to belt conveyor systems. To highlight the unique character of belt conveyors, section 2.3 presents a number of examples of other multiple driven system, which is followed by a comparison in section 2.4 that explores the similarities and differences between the systems. This comparison is based on a categorisation, where the link that connects the individual drive locations plays an important role. Before the different systems are compared a definition of a multiple drive system is given in section 2.1. To clarify the main differences, a categorisation is given in section 2.2 based on the character of the link connecting the distributed drive points.

2.1 Mechanical drive configurations

To get a load to its desired location a controllable drive force has to be applied directly to the load or the carrying element, making the drive unit an important component in every mechanical transport system. Based on the number of drive units and drive application points, a distinction can be made between the most basic configuration, where a single drive unit is used and more complex configurations, where multiple drive forces are applied at various locations in systems.

2.1.1 Single drive configuration

A system with just one element that converts energy from a certain source to mechanical energy, is considered a single drive system. Figure 2.1 illustrates the main elements of such a drive configuration. A load carrying element, moving with a velocity v, is supported by a

stationary base frame. To overcome friction and other resistances, and control the motion, a drive force is applied at a contact point between the load carrying element and the base frame. The drive unit converts energy from an internal or external energy source, like fuel or an electric power line for example, to the mechanical energy required for the motion. The drive unit usually consists of an actuator, which converts the source's energy to mechanical energy, and a transmission element, which converts the actuator's output speed and torque (or force) and matches it with the element's motion resistance and operational speed range.



Figure 2.1: Schematic of a self-propelled single drive systems

Figure 2.1 also shows a lorry as an example of a single drive system. In this case the lorry is the driven element that has its own energy source (diesel fuel) and drive unit (diesel engine and drive train) on board. The road supporting the lorry forms the base frame and the drive application point is positioned between the lorry's rear wheel and the road surface.

Different variants of single drive configurations are possible. One possible variation is the placement of the drive unit, which depends on the intended application and the available energy source. The drive unit can be either part of the driven element as illustrated in Figure 2.1 or fixed to the base frame as illustrated in Figure 2.2A, which shows a single driven belt conveyor as an example. In this case the belt forms the load carrying driven element.

Figure 2.2B presents another possible variation where the drive unit applies forces to multiple contact points. In this case it is the transmission that distributes the torque (or force) produced by a single actuator. The lorry shown in Figure 2.1 already is an example of a single drive configuration with multiple drive application points. The differential that is part of its drive train divides the engines torque over both rear wheels. An example of a system with a single drive application point is a motorbike that has only one driven rear wheel. If two driven

wheels, as is the case for the lorry, do not deliver a sufficient traction force, it is also possible to drive all four wheels, as the example of the four wheel driven car shows in Figure 2.2B.

Systems that resemble one of the presented variants are considered single drive systems. They can be identified by the fact that they have just one distinguishable drive unit that can be part of the moving system or the stationary base frame. The single drive unit has one actuator with one transmission element that can apply the driving force through one or more contact points between the driven element and its base frame.



A. Stationary drive unit

B. Multiple drive application points

Figure 2.2: Single drive variants

2.1.2 Multiple drive configuration

In a multiple driven system the total drive power available for the driven element is spread over a number of drive units. Each drive unit has its own transmission that applies a drive force to one or more points. Figure 2.3 presents the main elements of such a multiple driven system. The picture is similar to that of the single driven system, but instead of one large single drive unit the system is equipped with a number of smaller units that are spatially distributed along the system. Like the single driven system the drive units can be located in the driven element or in the base frame.

Figure 2.3 also shows a high-speed passenger train [Kurz, 1997] as an example of a multiple driven system. The driven element is the train that has an external power source (overhead power line) supplying multiple drive units. The track on which the train runs forms the base frame. Eight carriages, of which four are equipped with traction motors, make up the whole train. The powered carriages each have four electric motors that are individually connected to an axle in its bogies, making a total of 16 drive units and 32 drive application points contributing to the movement of the train as a whole.



Figure 2.3: Schematic of a multiple driven system

In this example the distributed motor layout offers two main advantages over a single driven system. A higher traction force can be applied to the track and the smaller motor and transmission combination takes up less space in height under the train's cabin floor. With more traction available the train is able to speed up faster and go up steeper inclines. The reduced space requirements make it possible to integrate the motors into the bogies and maximise cabin space.

Other reasons also exist for implementing distributed drives. If the total amount of installed drive power is kept the same, while the number of drive units is increased, smaller drive forces are applied at each drive application point. This puts less stress on the parts in the driven element and base frame that are exposed to the drive forces and helps to reduce the required technical requirements of these parts. In the case of a belt conveyor system for example, an increased number of drive stations can lead to lighter and cheaper belt and support construction.

The use of more than one drive unit can also contribute to an increased operating reliability. If a drive unit fails and the control systems permits it, other units that are still in working order can take over a part of the workload. Because part of the total drive power has gone offline, the system might have to run at a lower speed or lower capacity, but it will still be operational.

The implementation of a multiple drive configuration also sets new technical challenges. As more drive units are added, the complexity of the system increases. With more drive units the motor coordination becomes an issue and it puts greater demands on the infrastructure, like the increased amount of cabling required to supply partially distributed electric motors for example. There are more variables to control while the system is running and special attention has to be paid to starting, stopping and speed changing procedures to prevent undesired oscillations or instabilities.

2.2 Categorisation of multiple driven systems

A number of transport systems exist that feature multiple drive units. For these systems different drive configurations and control strategies can be identified because each application has a unique set of specifications and characteristics. To analyse and compare these different systems, a categorisation is introduced that is based on the properties of the elements or links connecting the drive application points. These links couple the state (consisting of quantities like position, speed and force) of each drive application point to the states of the surrounding points to which they are linked. If, for example, a drive force is applied at one point, the speed and position of the linked points is influenced. As Figure 2.4 illustrates, two types of physical links can be distinguished if the strength property of the interaction between the drive application points is considered.



Figure 2.4: Flexible and rigid links connecting drive application points

The rigid link forms a direct kinematic coupling between the drive application points, making it appear as though the connected points belong to one rigid body. When a displacement or speed change occurs in one point, as a result of the application of a drive force, it causes the same displacement and speed change at the linked point. With the flexible link the coupling is physical, but not as direct. Through its spring like behaviour, the drive points can move with respect to each other and if the system components surrounding the drive points have a significant mass, dynamics come into play, which may cause significant delayed reactions and oscillations. In the high speed passenger train shown in Figure 2.3 both rigid links and flexible links are present. Each powered bogie frame forms a rigid link between the two drive motors it holds, while the couplings between the carriages are flexible.

A multiple driven system is not limited to the configuration shown in Figure 2.4. Both link types can appear in the driven element, but it is also possible to have a system consisting exclusively of either rigid or flexible links. Another possible variation is the location of the drive units. The drive units can either be located in the driven element as in Figure 2.4 or in the base frame, as illustrated in Figure 2.5. The multiple driven belt conveyor is an example of a system with its drive units located in the base frame, while the belt forms the flexible element in the driven part.



Figure 2.5: Multiple driven system with drives in base frame

On the base frame's side both rigid and flexible links can also appear. However, in most landbased applications the base frame can be considered as rigid because they have the earth as a base. Examples of non-land-based multiple driven system, which are outside the scope of this study are multi-engine planes and ships, where the base frame consists of air and water respectively.

Apart from the multiple driven transport systems, containing the presented rigid and flexible links that are observable in the real world, systems also exist where the states of the drive application points are linked by a non physical or virtual element. As Figure 2.6 illustrates, these systems usually consist of two or more single driven elements, that are not physically connected, but that have to work as a whole to accomplish their common transport goal. The element linking the individual systems together does not exist in the real world, but it is more of a rule that force the individual drives to synchronise their efforts.



Figure 2.6: Virtual links connecting drive application points

An example of a multiple driven system with virtual links is a set of automatically guided vehicles running in a platoon. As they follow each other in a tight formation, an algorithm inside the controller of each vehicle continually adjusts the drive power to maintain the appropriate distance. In this case the distance keeping algorithm forms the virtual element.

Figure 2.7 illustrates another example of a system with virtual links. It is a special transport mechanism in a copying machine's sheet feeder, consisting of several independently driven

sections that transport the sheets from the feeder trays to an image transfer section, where the image is transferred onto the sheet. The sub system has been introduced to correct positional errors of the sheets leaving the feeder trays and match the velocity of the sheets to the constant velocity of the image transfer section, [Cloet et al, 1999 and 2001]. There is no direct link between the driven rollers. Again the linkage between the drives is virtual because they have to work together and get the sheet to the image transfer section at the right time and at the correct velocity, while keeping the sheets spaced at the desired distance.



Figure 2.7: Sheet feeder of a copying machine with paper spacing system

From the examples it is clear that virtual links exist as a result of the need to synchronise the movement of a number of individually driven sub systems. Although there are no real interaction forces present between the subsystems, the algorithms forming the virtual links make it appear as though they are physically connected. Due to this difference with the physical links, the design challenges surrounding the coordinated control of the drive units are also of a different nature and therefore systems with virtual links are put in a different category.

Figure 2.8 illustrates the resulting categorisation, with the main distinction between physical and virtual links. For both systems with physical and virtual links a subdivision is made of rigid and flexible links. Virtual links are rigid if a strict synchronisation is required between the driven elements and flexible if the speed and distance between the linked elements is allowed to vary. The set of automatically guide vehicles running in a platoon is an example of a system with flexible virtual links. To allow other vehicles to merge with the platoon, for instance, space has to be created in the platoon. The category with physical flexible links is highlighted as the area of interest for this study because multiple driven belt conveyor systems fall into this group. For these systems the drive stations are located in the base frame and the belt forms the flexible link between the drive application points.



Figure 2.8: Categorisation of multiple driven system

2.3 Existing multiple driven transport systems with flexible links

Within the category with flexible links, belt conveyors are not the only type of transport systems that feature multiple drive stations. Other systems can be distinguished such as printing presses, trains and an all-wheel drive fuel cell car. A description of each system is given to illustrate how close the design and control issues of these systems are actually related.

2.3.1 Belt conveyors

Traditionally belt conveyors are driven by a single drive unit positioned at the head of a primarily straight or incline conveyor, or at the tail of a decline conveyor. Although this is a simple and proven configuration, it does not always offer a workable or optimal drive solution. This is caused by the fact that the belt has to fulfil a double function. The belt does not only have to support the load without excessive sag occurring, but it also has to transfer the drive forces, which are required to overcome the motion resistance forces. Since the motion resistance generally increases when a longer system is desired, the total drive force that has to be applied to the belt also increases. In the single drive case this directly results in an increased peak in the belt tension at the drive station because the required drive force is applied at a single point along the belt. As a result, a stronger and heavier belt will be required, when the system length is increased. One alternative is to cascade a number of smaller conveyor systems to span the desired transport distance. Although this makes it possible to span longer distances with the same belt strength specification, transfer points have to be used between the cascaded conveyors that are a source for dust and belt wear and require power to lift the load before it is discharged on the next conveyor.

Another alternative is to use a multiple driven approach, where a number of drive stations are placed along the conveyor belt. As presented in the introductory chapter this creates an opportunity to take control of and reduce the maximum drive tension in the belt, offering possibilities to cut belt cost, increase layout flexibility and standardise system components. In special conveyor systems that inherently require low belt tensions, like the Enerka-Becker System (or E-BS) for instance, the use of multiple drive stations is already integrated into the design concept.

Apart from the benefits a spatially distributed driven approach has to offer, the system designer also has to take additional issues into account because the system designer will face new questions involving the total number of drive stations to install, how to distribute the stations along the system and how to control and coordinate each station. Regarding the distribution of drive power the system designer will aim to put the drive stations at those locations where the belt tension would otherwise exceed a set maximum belt tension. This way optimal use is made of the available belt strength by locally matching the installed power with the resistance occurring in each belt section.

For the control and coordination of the drive stations, the dynamic behaviour of the flexible conveyor belt will also have to be taken into account. During transient situations, such as starting and stopping, the combination of the belt's flexibility and its distributed mass cause delayed interactions between the drive stations. If, for instance, the drive force is suddenly increased at one drive station, an acceleration wave will start travelling outwards from this point. It takes some time before this action is noticeable to the surrounding drive stations because the acceleration wave has a finite speed.

Currently multiple driven systems like the E-BS are controlled using a load sharing approach. This is accomplished by equipping each drive station with identical AC induction motors and supplying them with the same power signal. Consequently, each drive station will more or less have the same speed-torque characteristic at every instance in time. As long as the system is started and stopped in a smooth fashion, the dynamical response of the elastic belt will be small. As a result, the belt velocity will be virtually the same for each point along the conveyor system. Therefore, the total load is equally shared among the drive stations, regardless of the fact whether the belt is partially or fully loaded because each station will be running round about the same velocity and thus be applying the same drive force to the belt [Cowie, 1999][Paulson, 1998].

The system designer also has to be attentive with regard to the occurrence of compressive forces in the conveyor belt, especially in systems with a relatively low pretension force, like the E-BS. As the belt is pulled into a drive station on one side and pushed out the other side, the belt tension will drop while passing a drive station. At the beginning of a start-up procedure, the tension can drop below the pretension and cause compressive forces. This can result in excessive belt sag or belt spilling out of the system. Therefore, the system designer has to prevent compressive forces occurring in the system during transient situations, such as starting, stopping, loading and unloading.

2.3.2 Printing presses

Industrial printing presses are composed of many rotating components through which a web of paper has to pass. To give an idea of the components involved, Figure 2.9 shows an example layout of a printing press. The printing is done on the central impression drum. A number of print stations consisting of drums are placed around the central drum. They transfer the different colours of an image onto the paper. The positional accuracy of these printing drums with respect to each other and the impression drum determines the quality of the printed image. To prevent material failures in the sections before and after the impression drum, the tension and speed of the web needs to be regulated. Too high a tension and the



paper might break and too low a tension and the paper may crease and spill out of the machine.

Figure 2.9: Example of a industrial rotary printing press layout (Source: Siemens)

Traditionally, the components of a mechanical printing press are synchronised with a line shaft. As Figure 2.10 illustrates, gears, gearboxes and clutches distribute the motion of the shaft to the individual rotating components. As each part of the mechanical synchronisation system has its own tolerance, the position errors accumulate because the connections between the driven components are linked end-to-end. The largest errors occur during accelerating and decelerating of the mechanical line shafted press. During the acceleration phase oscillations caused by mechanical play and flexibility in the system create considerable waste until the production speed is reached and the mechanical gears and shafts spring back to a continuous speed state.



Figure 2.10: Printing press with a mechanical line shaft (Source: Rexroth)

A new development is the shaft-less printing press design, which has been successfully introduced and has set new standards in the printing industry [Hulman, 1999]. In this shaft-less press, each printing station is individually controlled by AC servo drives and all printing operation are electronically synchronised by a master motion controller. The absence of any

mechanical line shaft means that torsional twist and backlash are not transmitted down the driveline. As a result of eliminating the mechanical tolerance build up, it is possible to achieve greater synchronisation or machine stiffness. Synchronisation is maintained even during acceleration and deceleration ramps. Figure 2.11 shows how the mechanical line shaft is replaced by an electronic substitute. It features independent motors placed throughout the press.

In a shaft-less printing press the mechanical line shaft is replaced by a virtual electric line shaft to synchronise the printing units [Brandenburg et al, 1999]. The electronic line shaft consists of a motion controller that is connected to a digital information bus. The information bus cross connects all the individual motor controllers (or servo drives) that in turn control the position and output of the motors.

What information is sent along the bus and how the drives act up on this information depends on the implemented control structure. An example [Anderson et al, 2001] is a control structure that emulates the mechanical line shaft in compliant shaft machines. It allows a coordinated operation of different axes, even during severe load disturbances or torque/speed saturation.



Figure 2.11: Printing press with an electronic line shaft (Source: Rexroth)

The electronic line shaft does not only offer possibilities of synchronising the position and speed of the motors. It also makes tension control possible through velocity and torsion adjustments, keeping the tension in the paper web within the prescribed limits. Different theories on the control of web tension have been developed, like modelling, regulation [Shin, 2000] [Boulter, 1997], robust control [Koç et al, 2002] [Nagarkatti et al, 2000] and active disturbance control rejection [Hou et al, 2001] for web tension control.

2.3.3 Trains

In trains and locomotives multiple drives have been implemented to increase the amount of available drive force by attaching drive units to a number of wheel axles. This is a result of the fact that dynamic track forces limit the vertical load on each axle. Since the traction force that can be generated by each axle is dependent on both the friction coefficient between the wheel and rail surface and the vertical axle load, it is advantageous to maximise the tractive effort by powering multiple axles [R.J. Hill, 1994].

Classically trains are composed of a single locomotive pulling a number of railway carriages. Upfront the locomotive forms a single drive unit with a primary power source. Within the

locomotive the engine power is spread over the locomotives drive wheels. Figure 2.12 shows an example of a main line diesel-electric locomotive that does not have a mechanical type transmission. Instead it has a generator that is connected to the engine and converts the torque of the diesel engine to an electric current that is fed to the electric traction motors [Brenneisen et al, 1973]. Since all motors are identical, which means they all have the same torque characteristics, and they are connected to the same power supply, the total load is shared equally among them.

Although this is a multiple driven system with a physical link between the drive application points, it lies outside the area of interest of this study because the train chassis that keeps the drive motors firmly in place is considered as a rigid link. A more relevant configuration is used in passenger trains, where the drive motors have been integrated into the bogies of the carriages. By also suspending the drive electronics underneath the carriage, the passenger cabin space can be maximised and the multiple axle drive offers higher accelerations.



Figure 2.12: Main line diesel-electric locomotive [British Railways, 1962]

This configuration is often found in passenger trains that have to travel short distances between stations [Joachimsthaler, 1972] [Nakagawa, 1995] and therefore have to speed up and slow down more frequent on their journey. But it can also be found in high-speed intercity trains [Koller et al, 1973] [Mochinizuki, 1982]. Figure 2.13 shows an example of a modern high speed train that has bogies equipped with electric motors.

The standard train has a total of eight cars of which half are equipped with powered axles [Kurz, 1997]. The powered cars each have four motors driving each axle. They carry their own power converters to which al four motors are connected in parallel. A transformer carried by the carriage placed between two motor equipped cars feeds the converters.

Coordination between the converters controlling the motors is necessary to equally distribute the total load of the train along the motors and prevent longitudinal oscillations of the carriages during acceleration and deceleration. The carriages of the train can be modelled as a number of masses that are connected to each other by a flexible element consisting of a parallel placed spring and damper assembly. Therefore, this configuration can be considered as a multiple driven system with flexible links.



Figure 2.13: Passenger train with distributed drive motors (Source: Siemens)

2.3.4 Chain conveyors

Chain conveyors are mainly used for internal transport in plants and distribution centres. As Figure 2.14 illustrates, the chain is supported by wheels in a suspended track and load carrying hooks are connected to the chain at regular intervals.



Figure 2.14: Overhead conveyor chain and track (Source: OmniMetalcraft)

The whole system is usually driven by a single drive station, where the chain is wrapped around a toothed disc. Another option is a caterpillar type drive unit that can be placed anywhere along a straight section and consists of a short driven secondary chain that hooks into the main conveyor chain. In the single drive configuration the total driving force is applied at a single point on the chain. As a result, the required chain strength depends on the drive tension generated at this drive point. To reduce the drive tension and the technical requirements of the chain, more drive stations can be introduced. Figure 2.15 shows an example of a possible multiple driven chain conveyor layout.



Figure 2.15: Overhead chain conveyor with dual drive (Source: Electro Sensors)

Although the links in the chain can be considered as rigid, these multiple driven systems are classed as systems with flexible elements. This is caused by the fact that tensioning units are used, which use pre-loaded springs to take up the play in the chain links. As a result, the states of the drive application point are not directly linked and when controlling the drive motors this flexibility will have to be taken into account to prevent unwanted oscillations during operation.

2.3.5 Fuel cell car concept

In the car industry a new development has emerged that also involves multiple drives. Concepts of fuel cell cars have been presented that have an electric drive motor for each wheel [Lovins and Cramer, 2004] [Burns et al, 2002]. As Figure 2.16 shows, the fuel cells are incorporated into a modular chassis design that it is made as flat as possible, so the same rolling base can be used for a number of car designs.



Figure 2.16: Four-wheel driven fuel cell car (Source: General Motors)

Instead of using a single motor and a transmission, taking up space in an engine bay, multiple drive motors are placed in the wheels of the vehicle, making this concept a distributed driven system. In this case the links between the drive application points are not considered rigid because the wheel speed can differ through slippage and while taking corners.

How the coordination of these motors will be accomplished has not been published. A good control strategy will be required to make the motors equally share the load but directional stability will also have to be guaranteed. In a straight line the motors on the one side produce the same amount of force at the road wheel interface as their counterparts on the other side. For corners the coordination will be become more complex, because the wheels turn at different speeds while cornering.

2.4 Comparison of systems

To investigate what similarities exist between the design and control issues of multiple driven belt conveyors and the other presented transport systems a comparison is made, using the properties of the elements that form links between the drive stations. Table 2.1 lists the main properties that are compared and indicates the similarities and difference between the systems. The first row in this table summarises the parts that form the flexible links between the drive application points. It shows that all systems have an elastic part in the connecting links. However, in the chain conveyor the main flexibility in the system does not come from the chain links themselves, but from the play between them and the movement of the tensioning unit.

| | Belt Conveyors | Shaft-less presses | Trains | Chain conveyors | Fuel cell car |
|---------------------------------------|--|---|--|--|-------------------------------------|
| Flexible part | Elastic conveyor belt | Elastic paper web | Elastic carriage coupling | Play in chain tensioning unit | Flexible suspension |
| Pretension | Required | Required | Not required | Required | Not required |
| Compressive forces | Belt cannot take up comp. forces | Paper cannot take up comp. forces | No problem | Dead band through play in linkages | No problem |
| Inertia | Relevant | Negligible | Relevant | Relevant | Relevant |
| Load distribution | Varies while loading and unloading | Constant | Varies little while moving | Varies while loading and unloading | Constant |
| Drive station count and spacing | Related to total drive force, belt tension and wear | Related to the number of components in the press | Related to total drive force and traction per axle | Related to total drive force and belt tension | Equal to the number of wheels |

Table 2.1: Comparison of the presented multiple driven systems

The second and third row indicate whether the element has to be preloaded and if it can withstand compressive forces. Both the conveyor belt and the paper web require a pretension because for both systems compressive forces will result in excessive sag and possibly cause the belt or the paper to spill out of the system. In the case of a pulley driven belt conveyor the pretension also has to guarantee a minimum slack side tension to be able to generate the specified maximum drive force without causing excessive slippage. In chain conveyor systems a pretension is also applied, but in this system it is used to take up the play in the links. Although the chain conveyor can transfer compressive forces it is undesirable to have these forces occurring in the system because the play in the chain links causes a dead band in the movement during transitions between tensile and compressive forces. To prevent this erratic behaviour, the chain has a pretension force.

The fourth and fifth row involve the mass of the moving medium. When considering the inertia, the shaft-less printing press is the only system where the mass of the moving medium is negligible because the mass of the paper web is very small compared with the inertia of the drive components. In the other systems the inertia of the carrying medium and the mass of the load have a strong influence on the dynamical behaviour of the system. In two of these systems, which are the belt and chain conveyor, an additional effect has to be taken into account due to the movement of the load relative to the drive station locations. Varying load distributions are caused by a varying material flow at the loading station. Therefore, this effect is especially noticeable while the system is being filled or emptied.

The last row indicates which factors have the greatest influence on the total number of drive stations and the spacing between them. For both the belt and chain conveyors the drive station count is usually dependent on the allowable belt or chain tension and the motion resistances. Since the resistances accumulate along the conveyor system, the required drive tension also rises. As a result, drive stations are placed there where the tension would otherwise exceed the allowable value. This creates a direct relationship between the number of drive stations and the total required drive force. Another factor that affects the number of drive stations in a belt conveyors system is the belt wear resulting from the application of drive forces. This is especially the case for wheel driven conveyors systems like the Enerka-Becker System. To be able to attain the guaranteed belt life, the wear rate has to remain below a predefine value, putting a limit on the maximum allowable drive force that can be generated by each drive station.

For the other systems the maximum occurring drive forces in both the base frame and the flexible links have less influence on the number of drive stations and the spacing between them. In these systems the available locations where drive stations can be placed play a more important role. In a shaft-less printing presses, for instance, each component that would otherwise have been driven by the mechanical line shaft is connected to its own drive motor. This is even more apparent for the fuel cell car concept, where the drive motors are mounted in the wheels. In this configuration the motors can be mounted in only two or all four wheels. In trains the number of available axles also determines where the drive units can be placed, but in this case the total number of drive units is also influenced by the maximum amount of traction that can be produced by each axle.

Although the comparison shows that the multiple driven systems with flexible links can be very similar in some areas, there are also a number of differences between them, requiring different design approaches. If the multiple driven belt conveyor system is compared with the other systems, special attention will have to be paid to the belt dynamics during transient situations, such as starting and stopping procedures. This is caused by the fact that the flexible belt and distributed mass cause acceleration waves to travel trough the system, which do not occur in the other systems. Another issue is the varying load distribution. Although belt

conveyor systems share this issue with chain conveyor systems little is known about what effect this will have on the belt tension when using a considerable amount of drive stations. The last issue involves the total number of drive stations and the spacing between them. Again there is a similarity with chain conveyor systems because both the belt and chain tension have to remain below the set limit. However, as the soft conveyor belt is driven with a rolling contact, wear becomes an issue that will have to be taken into account to be able to guarantee the belt's life time. Based on these issues further investigations are made.
3 Existing models for belt conveyor systems

From the previous chapter it is clear that multiple driven belt conveyors have a unique set of characteristics compared to other transport systems. They stand out as a result of the dynamic belt behaviour, the pretension requirement and the varying load distribution along the system, when the inflow of bulk solid material to the system is not constant. Therefore, the design rules and practices formulated for other multiple driven transport systems cannot be used for multiple driven belt conveyors. A different approach is required. The approach taken in this study uses the existing design rules and practices developed for conventional belt conveyors as a base and investigates what changes are necessary for the multiple driven case. To compare the differences in running behaviour between a multiple driven belt conveyor and a conventional single driven system, especially during transient situations such as starting, stopping, loading and unloading, a dynamic model needs to be constructed that includes the main belt dynamics and features multiple drive units. Using the Enerka-Becker System (or E-BS) as a test case this chapter investigates which parts of the model are already available and which parts need to be developed. Before the modelling is discussed, section 3.1 gives a description of the main components in the EB-S to indicate which parts are of importance for the study of the belt behaviour. The following sections present the element that are required to model the multiple driven system as a whole. Section 3.2 discusses the model for the belt dynamics, section 3.3 summarised the existing models for calculating motion resistances and section 3.4 presents a model for the drive stations.

3.1 Main components of the Enerka-Becker System

As Figure 3.1 shows the E-BS is a special type of closed belt conveyor, a so called pouch conveyor. Other closed belt conveyors include the Sicon and pipe conveyor. The E-BS' main feature is the pouch shaped belt that isolates the bulk solid material from the surrounding environment, offering advantages such as dust free operation and preventing ambient conditions affecting the conveyed material. Another special feature of this system is the fact that it uses multiple drive units that are placed along the route of the belt. This keeps the tension in the belt low and makes it possible to use the same light belt construction irrespective of the overall system length. It also uses drive wheels that press into the belt instead of a drive pulley to generate drive forces on the belt.



Figure 3.1: The Enerka-Becker System (Source: Fenner-Dunlop)

3.1.1 Belt

The heart of the E-BS is the pouch shaped conveyor belt. As Figure 3.2 shows, the pouch shaped belt is made up by a folded standard flat belt, which is reinforced by two fabric polyester-polyamide plies. At both edges of the belt two solid rubber triangular profiles are warm vulcanised to it. They form a larger triangle when the belt is folded into the pouch shape, effectively sealing the bulk solid material from the surrounding environment. In the closed form the triangles act as a running surface for both the support rolls and the drive wheels.



Figure 3.2: The pouch shaped belt of the E-BS

The type of belt used for the E-BS is indicated as an EP 250/2 belt, where the first number stands for the standardised unit strength of the belt, also called belt rating, and the second number states that it is reinforced by two plies. The standardised unit belt strength k_N defines the belt's breaking strength per unit belt width and is expressed in N/mm. If this value is combined with the belt width and the safety factors, the allowable belt tension during operation can be determined. In the DIN standard 22101 safety factors are defined for both non-stationary and stationary conditions by S_A and S_B respectively. Although this standard is

only intended for conventional belt conveyor systems it serves as a guideline for the E-BS as no specific standards are available for closed belt conveyors. Table 3.1 shows the safety factors for fabric reinforced belts.

| Carcass Material | Operational condition | S_A | S_B | |
|------------------|-----------------------|-------|------------|--|
| B (Cotton) | Light | ≥4.8 | ≥6.7 | |
| P (Polyamide) | Normal | ≥5.4 | ≥ 8.0 | |
| E (Polyester) | Heavy | ≥6.0 | ≥9.5 | |

Table 3.1: safety factors according to DIN 22101

According to DIN 22101 the actual safety factor S along the belt has to remain above S_A during starting and stopping and above S_B during steady state operation. The actual safety factor is defined as follows

$$S = \frac{k_N B}{T}$$
(3.1)

where T is the belt tension and B is the belt width. Two different belt widths of 800 mm and 1400 mm are available for the E-BS. Under normal operational conditions and taking the 800 mm as an example, this leads to a maximum allowable belt tension of 25 kN during normal operation and 37 kN during starting and stopping.

Figure 3.3 presents the stress-strain curve of the E-BS belt. The solid line clearly shows the non-linear stiffness characteristic of the belting material, which is primarily caused by the behaviour of the belt's carcass. However, under normal operational conditions the maximum belt strain is smaller than 2%. In this range it has a linear characteristic, which is represented by the dashed line in Figure 3.3. Therefore, under normal operational conditions, the belt's stiffness can be described by a constant modulus of elasticity E_b .



Figure 3.3: Stress-strain curve of E-BS belt [Twaalfhoven, 2004]

The slope of the dotted line represents the value of E_b , which in this case is equal to 280 N/mm². This value is calculated over the entire cross sectional area of the belt, including the carcass and belt covers and it is used when modelling the belt dynamics.

3.1.2 Belt support

The belt is supported in a gallery of support rolls or idlers that are mounted on brackets. Figure 3.4 shows this support configuration for a bracket. Each bracket suspends two rolls and it is connected to a main support pipe that can be mounted to a wall, ceiling or on the ground.



Figure 3.4: Idler set configuration

The running surface on the rubber triangles is curved to prevent high peak stresses occurring at the edges of the idler [Lodewijks, 2000]. This keeps the contact stress below an acceptable value and prevents the belt from jamming between the wedge shaped idlers configuration, reducing both rolling resistance and belt wear. Additionally, the idler set is tilted in the direction of travel by about 2°. This creates a lifting force on the belt that also prevents it from wedging between the idlers. Although the tilting angle reduces the friction associated with the wedging effect, it may also introduce an extra resistance force as a result of lateral creep.

3.1.3 Drive units

A drive unit in the E-BS is composed of two identical AC motors, as Figure 3.5 illustrates. Both motors power a drive wheel that is pressed onto the belt to generate a drive force. Apart from the weight of the belt and the bulk solid material in it, an additional clamping roll at the top of the profile presses the drive wheels into the belt, which helps to generate a higher normal force than would be produced by the weight of the belt and its load. To allow small variations in the thickness of the triangular rubber profiles, each motor is connected to the mounting bracket by a hinge, while a spring presses the drive wheel into the belt. This ensures that the normal contact force between the wheel and belt stays close to the desired value.



Figure 3.5: Drive station configuration

In this configuration a drive unit can replace any idler set in the system, making it possible to place it virtually anywhere along the belt conveyor. Contrary to drive stations in conventional flat belt conveyors systems, where the belt is wrapped around a drive pulley, the E-BS drive unit requires no slack side belt tension to generate drive forces, as discussed in chapter 1. Therefore, the pretension in the belt can be kept relatively low. Although the lower pretension helps to reduce the stress in the belt, it also narrows the margin within which the belt tension has to stay during normal operational conditions. If the belt tension comes too close to the compressive state, the pouch shaped E-BS belt will open and in the worst case it will spill out of the system, making the compressive state an undesirable condition.

Compressive forces are a concern in multiple driven belt conveyors because most drive stations that are placed along the belt strand have no tensioning unit located near them to guarantee a minimum slack side tension. As Figure 3.6 illustrates, a drop in tension occurs when a drive unit applies a drive force to the belt, which is independent of the implemented drive configuration. If the equilibrium of the forces in the belt's longitudinal direction is considered at a particular drive station, the difference between the magnitude of the tight side tension T_1 and the slack side tension T_2 is equal to the total drive force F_d applied by the drive station or

$$T_2 = T_1 - F_d$$
 (3.2)

So for drive units with no tension device placed after them to keep T_2 constant, compressive forces are generated when a greater drive force is applied than the local belt tension. This situation is most likely to occur at the beginning of the starting procedure of an E-BS. In this case drive forces are introduced to the belt while the belt tension is still at the pretension level. In the E-BS with its low pretension this can be a point of concern and therefore care should be taken when selecting the start up ramp for this system.



Figure 3.6: Tension drop in the belt at a drive station

3.1.4 Loading station

At the system's loading station the belt has to be opened to fill the E-BS' pouch shaped belt with bulk solid material. Figure 3.7 illustrates how slider plates are used to only open the top part of the pouch shape and support the belt. When the belt enters the loading station, it is opened in the trough out section and loaded in the following straight section. After the belt is loaded it is closed in the trough in section. With this configuration there is no need to open the belt completely. Therefore, the loading section can be placed at any location along a straight section.



Figure 3.7: Loading station configuration

Compared to a normal straight section additional resistance forces occur at the loading station due to four effects. Firstly, the sliding contact surface between the belt and the slider plates causes extra frictional forces. Secondly, the bulk solid material that is loaded onto the belt has to be accelerated to the belt speed. During this acceleration phase the bulk solid material exerts a resistance force on the belt. This effect occurs in all belt conveyor systems where there is a speed difference in the transport direction between the material flowing into the system and the belt speed. Thirdly, the opening and closing of the pouch shape causes hysteresis losses in the belting material. Fourthly, when the loaded belt is closed in the trough in section, the bulk solid material is forced into the new shape, causing internal friction.

3.1.5 Discharge and belt turnover

At the head of the system the belt is opened and lead over a pulley to discharge the bulk solid, as Figure 3.8 shows. The belt is opened just before it reaches the head pulley. When the belt leaves the head pulley upside down it is folded back into the pouch shape before it enters the turnover section. In the turnover section the support rolls form a helix shaped trajectory, so they twist the belt 180° about its neutral axis parallel to the belt speed.



Figure 3.8: Discharge point and belt turn over section (Source: Fenner-Dunlop)

3.2 Belt dynamics

The belt dynamics play an important role when analysing the belt behaviour during transient conditions such as starting and stopping. This is especially true for long distance and/or high capacity belt conveyor systems. As a result of the combination of the belt's elasticity and distributed mass, acceleration waves travel through the system during transient situations. During starting for example the belt starts moving gradually, and as the acceleration wave propagates along the belt, its successive sections are put into motion [Zür, 1986].

Failure to include the transient response into the design can result in operational problems such as excessive large displacement of the weight of the gravity take-up device, premature collapse of the belt, mostly due to the failure of the splices and destruction of the pulleys and major damage of the idlers [Lodewijks, 2002]. Therefore, finite element models of belt conveyor systems have been developed [Nordell and Ciozda, 1984] [Schulz, 1985] [Ellis and Miller, 1987] [Lodewijks, 1992] that take the dynamic belt behaviour into account. Although these models only determine the longitudinal response of the belt by mainly using truss like elements, they have been quite successful in predicting the elastic response of the belt during starting and stopping. To also include the transverse vibrations, the finite element models have been further extended with special beam elements by Lodewijks (1996).

In this study the main interest is on the longitudinal elastic response because it is primarily focused on how the drive tensions in the belt react in a multiple driven configuration. Furthermore, in the E-BS, which is used as test case, the transverse oscillations are not as dominant as in a flat conventional conveyor, due to the small idler spacing that prevents the pouch belt from opening and the fact that the pouch shape has a much higher bending

stiffness than the flat or troughed belt. Therefore, the models including only the longitudinal elastic response will suffice in this case.

In the finite element approach for the longitudinal elastic response the distributed mass of the belt and the bulk solid material is divided over a finite number of elements, as Figure 3.9 illustrates. The mass of each finite belt part is divided over the adjacent nodes, while a spring element represents the belt stiffness, which can consist of a combination of linear springs and dashpots [Nordell and Ciozda, 1984].



Figure 3.9: Finite element model of belt conveyor

At the tensioning device the belt is split, forming an open string of elements. The first and last nodes represent the locations where the belt leaves and enters the pulley of the tensioning weight respectively. To incorporate the behaviour of the tensioning weight, its static gravitational force of the tensioning weight is equally divided over the first and last node, while its dynamic mass is divided over both nodes using the following relationship for the tensioning pulley

$$y_{t} = \frac{1}{2} (u_{1} - u_{n})$$
(3.3)

where y_t represents the vertical displacement of the tensioning weight and u_1 and u_n are the displacements of the first and last node respectively. Here the first node is located on the left side of tensioning device's pulley in Figure 3.9 and the last node is located on the right side. If a drive station is placed directly next to the tensioning device, both components can be considered as one mass element. This is acceptable due to the fact that the belt section between the drive station and tensioning device is very short compared to the main belt section.

To keep the element stationary with respect to the support structure, the displacement of each node is expressed relative to the displacement of the first node. This effectively fixes the first node to a stationary position, while the remaining nodes can only move relative to this point as the belt is stretched. Although this approach neglects the influence of the belt speed on the

longitudinal response because the belt and the bulk solid material on it travel trough the fixed element grid, it will have little effect on the calculated results. This is caused by the fact that the longitudinal wave propagation speed is far greater than the belt speed. For the E-BS, for example, the maximum operational belt speed is 5 m/s and its wave propagation speed c_1 can be calculated using the following equation [Lodewijks, 1996]

$$c_1 = \sqrt{\frac{E_b A}{m_b' + m_1'}}$$
 (3.4)

where m'_b and m'_1 are the mass of the belt and the load per unit length respectively and A is the area of the belt's cross section. Taking the lighter type 800 mm wide E-BS belt, as an example, which is 6 mm thick and weighs 11 kg/m the wave propagation speed equates to 360 m/s. In the loaded situation this belt can carry 40 kg/m, which results in a reduced wave propagation speed of 170 m/s.

Although the calculated wave propagation speeds for the E-BS belt are relatively low compared to conventional conveyor belts, where c_1 ranges from 750 to 1500 m/s, the belt speed still has little effect on the axial vibration. To assess the magnitude of this effect, the natural frequencies are considered of a moving belt element spanning the distance L between two idler sets. If the belt element is modelled as a string the following equation result for the natural frequencies of the axial vibration [Lodewijks, 1996]

$$\omega_{n} = \frac{n\pi}{L} c_{1} \left(1 - \frac{v_{b}^{2}}{c_{1}^{2}} \right), \quad n = 1, 2, 3, \dots$$
(3.5)

If the belt speed is neglected by setting v_b to zero, the maximum error in the frequency of the first harmonic is smaller than 0.1% for a fully loaded belt travelling at 5 m/s. Therefore, it is possible to consider the belt stationary in this model without causing a noticeable error.

Multiple drive units can easily be added to this model because external forces acting on the belt, including resistance and drive forces, are allocated to the corresponding nodes. In a single drive case with a head drive pulley, for example, the drive force is only applied to the last node, assuming that the belt length between the drive pulley and the tensioning device is negligible compared to the overall system length. For a multiple driven system the additional drive units can be added by directly applying drive forces to the nodes where the drive units are located. This also influences the size of the belt elements because the element lengths have to be chosen such that the nodes coincide with the locations of the drive units.

3.3 Motion resistances

In the finite element approach the motion resistances, which vary along the belt, are classified as external forces that can be applied locally to the corresponding nodes. To be able to calculate how the resistances are distributed along the system and how they vary in different operational conditions, a number of models and calculation methods exist that describe the resistances occurring in belt conveyor systems. Using the DIN22101 standard as a guideline the following resistance classes can be distinguished:

- Main resistances: Occurring along the whole length of the belt
- Secondary resistances: Occurring only locally at the loading and unloading locations
- Slope resistances: Resulting from the lifting or lowering of the bulk solid material and the belt on incline conveyors, which can be either positive or negative
- Special resistances: Occurring in some belt conveyor systems like the belt turn over

With this classification the existing resistance models and calculation methods are discussed.

3.3.1 Main resistances

The main resistances are resistances that occur along the whole length of the belt conveyor. Apart from the possible slope resistance, these resistances play an important role in large scale belt conveyor applications because the energy consumed during the operation of a long horizontal belt conveyor is primarily due to the resistances that occur along the length of the conveyor. In the DIN 22101 standard the main resistances are generalised into a resistance factor f_i . With this factor the total main resistance force $F_{M,i}$ in each belt section with length l_i is calculated as follows

$$F_{M,i} = f_i \cdot l_i \cdot g \cdot \left(m'_{r,i} + \left(m'_b + m'_{l,i} \right) \cdot \cos \delta_i \right)$$
(3.6)

where $m'_{r,i}$ and δ_i represent the reduced mass of the idlers (which is the rotational inertia of the idlers expressed as a mass moving in the direction of the belt) and the section's inclination angle respectively. Note that the index i indicates the different sections of the belt conveyor system. Although the DIN 22101 standard prescribes different values of f_i for different operational conditions, the accuracy of this approach is not able to meet with modern belt conveyor requirements. To make a more accurate approximation, models have been developed for the specific resistances that contribute to the main resistances, which are:

- Resistance due to the indentation of the belt cover by the idlers
- Resistance due to the recurrent flexing of the belt and bulk material
- Friction of the idler bearings and seals

Every time the belt passes over an idler set, the belt cover (or rubber triangle in the case of the E-BS belt) is temporary indented due to the weight of the belt and the bulk solid material. As the belt cover is a viscoelastic material, having the rigidity of a solid material and the ability to flow and to dissipate energy like a viscous fluid, the cyclic deformation causes indentation rolling resistance. As the belt passes over an idler roll, the belt cover compresses and relaxes in short succession. Due to the viscoelastic properties of the cover material the relaxation part will take some time, which results in an asymmetric stress distribution between the roll and the belt. Figure 3.10 shows a typical asymmetric distribution that causes the resulting contact force F_n . The fact that the line of action of F_n lies in front of the roll's centre causes a resistance moment opposite to the roll's rotation.



Figure 3.10: Asymmetric stress distribution between belt and idler roll

A number of researchers have studied the contact phenomena of a rigid cylinder rolling on a viscoelastic surface, which has resulted in different formulations of the indentation rolling resistance problem. Lodewijks (1995) made a comparison of the different methods described by May et al. (1959), Hunter (1961), Jonkers (1980) and Spaans (1991) that are applicable to belt conveyor systems. In contrast to the first two methods the last two assume that the indentation of the belt by the roll is symmetrical to the roll's centre line. However, as the belt speed increases, the indentation profile becomes more asymmetrical. Wheeler (2003) also notes that Spaans (1991) and Jonkers (1980) model the belt indentation as a linear analysis accompanied by hysteresis, while Hunter (1961) and May et al. (1959) apply stress strain relations for linear isotropic viscoelastic materials. Further, the comparison shows that the indentation rolling resistance factors calculated by Jonkers (1980) and Spaans (1991) are significantly higher.

Lodewijks (1995) also derived a formula for the indentation rolling resistance factor that is similar to the one derived by May et al. (1959). In this model the layer of the belt cover is modelled as a Winkler foundation, consisting of spring elements that do not interact with each other. Since there is no interaction between the springs, the shear between adjacent elements is ignored. To account for the fact that in the Winkler foundation shear forces between adjacent elements are neglected, Lodewijks (1995) introduced a correction factor based on the difference between the outcome of models described by Hunter (1961) and May et al. (1959). This is possible because Hunter's (1961) model includes the shear forces while the model by May et al. (1959) does not.

As Figure 3.11 shows, each spring element in the Winkler foundation is comprised of two springs and a dashpot, which is also known as the three parameter Maxwell model. The three parameter Maxwell model, which is a simplification of the generalised model that can have any number of spring dashpot assemblies, is used to include the viscoelastic behaviour. The simplified model with its single relaxation time suffices in this case because the contact patch between the belt and the idler has a constant contact length. With the contact length constant along the length of the idler, the belt cover material is excited with a single frequency f_e , which is calculated as follows

$$f_e = \frac{V_b}{l_c}$$
(3.7)

where l_c is the contact length. With only one frequency of excitation, it is possible to match the model with the behaviour of the viscoelastic surface by tuning the single relaxation time to the excitation frequency.



Figure 3.11: Winkler foundation (left) with three parameter Maxwell model (right)

Wheeler (2003) constructed a model for the indentation rolling resistance by implementing a finite element method derived from the work of Lynch (1969) and Batra et al. (1979). To verify the different models, Wheeler (2003) also conducted experiments and concluded that the results predicted by the finite element analysis and those predicted by May et al. (1959) compare favourably to the experimentally measured values.

The discussed models describing the indentation rolling resistance apply only to cases where the contact length is constant along the idler roll, such as the line contact found in conventional belt conveyors. However, due to the curvature of the belt's running surface in the E-BS, a contact patch is formed at each idler with a varying contact length. Therefore, an adaptation is required to include the effect of the curved running surface. Based on the model derived by Lodewijks (1995), a modified model for the indentation rolling resistance is presented in chapter 4.

When bulk solid material is transported on a belt conveyor, both the belt and the bulk solid undergo transverse and longitudinal displacements between successive idler sets due to the sag of the belt. This causes flexure resistance through the hysteresis losses associated with the induced bending of the conveyor belt, the internal friction of the bulk solid and friction at the belt and bulk solid interface.

Spaans (1991) modelled the flexure resistance by approximating the sag profile by two radii of curvature, limiting the application of the model to flat belts. To predict the flexure resistance for the more common troughed belt configuration more accurately, Wheeler (2003) adopted an orthotropic plate model to predict the deflection of the troughed belt. The orthotropic plate model was also used by Harrison (1984) to analyse the bending vibration of steel cord belts. However, as noted by Wheeler (2003) the classical method applied by

Harrison (1984) is not applicable for the calculation of belt deflections with interactions between belt and bulk solid. Therefore, Wheeler (2003) used a numerical approach and modelled the belt span between idlers with a finite difference method. Figure 3.12 shows the resulting mesh of discrete nodes.



Figure 3.12: Plate mesh used by Wheeler (2003) to calculate flexure resistance

In conventional belt conveyor systems the bulk solid flexure resistance is consistently the second largest of the main resistances and may exceed the indentation rolling resistance in the case of wide conveyor belts [Wheeler, 2003]. However, in the E-BS the belt does not open and close as a troughed belt passing from one idler set to the next. Therefore, the bulk solid material's cross-sectional shape remains the same and prevents the friction generating cyclic expansion and contraction of the bulk material in the transverse direction. In the longitudinal direction the expansion and contraction of the bulk material due to belt sag is also limited. This is caused by the fact that the pouch shape offers a much higher stiffness to bending compared to a troughed shape, further limiting the effect of bulk solid flexure resistance. The higher rigidity of the pouch shape also results in a lower belt flexure resistance compared to conventional throughed belts. As Wheeler (2003) concluded that the flexure resistance of a

to conventional throughed belts. As Wheeler (2003) concluded that the flexure resistance of a conventional belt is small compared to the other components of the main resistance, this effect will be even smaller in the E-BS and it is therefore negligible.

The friction of the idler bearings and seals mainly depends on the type and specification of the bearings used in the belt conveyor system. For each bearing the resistance is made up of the rolling and sliding friction between the rolling elements and the cage guiding surfaces and the friction of the lubricants. These components are normally given by the bearing manufacturer. In a belt conveyor system the resistance moment M_b of a single bearing can be written as

$$M_{b} = M_{rr} + M_{sl} + M_{seal}$$
(3.8)

where M_{rr} is the rolling moment, M_{sl} is the sliding moment and M_{seal} is the friction moment of the seals. Using this resistance moment the total bearing and seal resistance idlers in each belt section can be expressed as the resistance factor f_b . This factor is a part of the total resistance

factor f_i and is defined as the ratio between the bearing resistance force F_b of an idler set and the total vertical load F_1 or

$$f_b = \frac{F_b}{F_1}$$
(3.9)

This can also be expressed in terms of the bearing resistance moment M_b and the belt and bulk solid mass as follows

$$\mathbf{f}_{b} = \frac{\mathbf{M}_{b} \cdot \mathbf{n}_{b}}{\mathbf{R}_{i} \cdot \mathbf{L} \cdot \mathbf{g} \cdot (\mathbf{m}'_{b} + \mathbf{m}'_{1,i})}$$
(3.10)

where n_b represents the number of bearings installed in each idler set and R_i is the idler's outer radius.

3.3.2 Secondary resistances

The secondary resistances are resistances that occur only locally at the loading and unloading locations. In the DIN 22101 standard the influence of these resistances is approximated by the factor C, which decreases exponentially with increasing belt length. With this factor the secondary resistances are calculated as follows

$$\mathbf{F}_{\mathbf{S}} = (\mathbf{C} - 1) \cdot \mathbf{F}_{\mathbf{M}} \tag{3.11}$$

Table 3.2 indicates what values are prescribed for C by DIN 22101. Note that these values are only applicable to conventional belt conveyor systems.

Table 3.2: indicatory value of the C factor as defined by DIN 22101

| Head to tail pulley distance (m) | 100 | 200 | 500 | 1000 | 1500 | \geq 2000 |
|-------------------------------------|------|------|------|------|------|-------------|
| С | 1.78 | 1.45 | 1.20 | 1.09 | 1.06 | 1.05 |

A more accurate approximation of the secondary resistance can be made by splitting it into the separate resistance generating effects, which are:

- Inertial and frictional resistances due to the acceleration of the material at the loading section
- Resistance due to the friction on the sidewalls at the loading station
- Resistance due to friction between belt and pulley cleaners

At the loading station the bulk solid material is accelerated to the belt speed. As a result of the speed difference between the inflowing bulk solid material and the belt, a friction force F_c is exerted on the belt, as illustrated in Figure 3.13. When the impulse equilibrium is considered

in the direction of the belt speed the friction force F_c is calculated as follows [Jonkers and Rademacher, 1990].

$$F_{c} = Q_{c} \left(v_{b} - v_{c} \cos \phi_{c} \right)$$
(3.12)

where Q_c , v_c and ϕ_c represent the mass flow, speed and angle of the inflowing bulk solid material flowing at the loading section respectively.

Skirt boards are usually added to the sides of a loading station or transfer point in conventional belt conveyors. The purpose of these skirt boards is to keep the load on the conveyor, preventing the material from spilling over the belt edge until the load is settled and has reached the belt speed [Swinderman et al, 2002]. The sliding of the bulk solid material along the skirt boards generates a friction force that is classed as a secondary resistance.



Figure 3.13: Inflow of bulk solid material at the loading station

As Figure 3.14 illustrates, skirt boards are unnecessary in the E-BS due to its pouch shaped belt. However, instead of having skirt boards to prevent spillage, slider plates are used at the loading station to open the top of the pouch, which generate friction forces on the belt. As the belt purely slides along the plates, the total sliding friction E generated at the loading.

As the belt purely slides along the plates, the total sliding friction F_{sp} generated at the loading station can be calculated as follows

$$F_{sp} = 2 \cdot \mu_{sp} \cdot F_{p} \tag{3.13}$$

where μ_{sp} is the friction coefficient of the rubber surface sliding over the metal plates and F_p represents the total normal force acting on each side of the belt that is required to keep the pouch shape open.



Figure 3.14: Forces on the E-BS belt at the loading station

With F_1 representing the total vertical load of the belt and the bulk solid material in the loading section, the contact force F_p is determined by considering the equilibrium of the forces acting on the belt, which results in

$$F_{p} = \frac{1}{2}F_{l} \cdot \tan\alpha \qquad (3.14)$$

where α is the angle of the idlers in the E-BS. To determine the total vertical load F₁ in the loading station, it is assumed that in the loading section the belt is gradually filled with bulk solid material, giving a linear loading profile as presented in Figure 3.15. As the loading profile has an average loading degree of 50 % along the length l_{sp} of the slider plates, the vertical load F₁ is equal to

$$F_{l} = l_{sp} \cdot g \cdot \left(m'_{b} + \frac{1}{2}m'_{l}\right)$$
(3.15)

In conventional belt conveyor systems belt and pulley cleaners are installed after the discharge pulley to minimise residual material adhering to the belt and being carried back with the belt's return strand. Too much residual material carry back can be hazardous because it can become dislodged by the vibration of the return rollers. The particles will eventually fall off the belt, accumulating in piles under idlers and pulley [Swinderman et al, 2002]. Typically, cleaners are made with blades that peel or scrape off the residual material by pressing onto the belt. The combination of the pressure and the sliding motion creates a resistance force on the belt.

The E-BS requires no belt cleaners because the belt is closed and turned over directly after it leaves the discharge pulley. This effectively seals in the residual material and prevents it from falling from the belt and from accumulating to the idlers in the return strand. Therefore, no resistance has to be accounted for belt cleaning in the E-BS.



Figure 3.15: Loading profile of bulk solid material in the loading station

3.3.3 Slope resistances

The slope resistance is not a direct result of a frictional effect, but it represents the forces that are related to the lifting and lowering of the belt and the bulk solid material in incline or decline conveyor sections. Therefore, it is positive for incline and negative for decline section. According to the DIN22101 standard the slope resistance $F_{Sl,i}$ for each incline or decline section can be calculated as follows

$$F_{Sl,i} = h_i \cdot g \cdot (m'_b + m'_{l,i})$$
(3.16)

where h_i represents the elevation change of the related belt conveyor section. This calculation of the slope resistance also applies to the multiple drive case.

3.3.4 Special resistances

Special resistances are only present in some systems. For the E-BS the drag resistance due to the forward tilt of the idlers in the direction of motion and the belt turnover section falls into this category.

3.4 Drive stations

As discussed in section 3.1.3 each drive station in the E-BS consists of two motor pairs that each power a drive wheel through a gearbox. Figure 3.16 shows a schematic representation of the driveline of each motor in a drive station.



Figure 3.16: Schematic of the driveline in the E-BS

3.4.1 Induction motor and inverter

Like most belt conveyor systems the E-BS is fitted with squirrel cage type induction motors. To control the speed of the belt, each motor is equipped with an inverter. The function of the inverter is to draw power from the fixed-frequency constant-voltage mains and convert it to a variable frequency and voltage for driving the induction motor. Variable frequency inverter fed induction motor drives are used in ratings up to hundreds of kilowatts. Standard 50 Hz or 60 Hz motors are usually employed, and the inverter output frequency typically covers the range from around 5 to 10 Hz to 120 Hz. This is sufficient to give at least a 10:1 speed operating range [Hughes, 1990].

For its operation an induction motor relies on the induction of voltages and currents in its rotor circuit from the stator circuit. Because the induction of voltages and currents in the rotor circuit of an induction motor is essentially a transformer operation, the equivalent circuit of an induction motor is very similar to the equivalent circuit of a transformer [Chapman, 1998].

Figure 3.17A shows the equivalent circuit that is used to derive the induction motor's output or induced torque T_{ind} as a function of the input voltage frequency ω_s and magnitude U_s . In this figure R_1 and X_1 represent the resistance and inductance of the stator coils and X_2 and R_2/s represent the inductance and resistance of the rotor and X_m represents the mutual inductance between both stator and rotor coils. Note that the rotor resistance is a function of the motor slip and the inductances X_1 , X_2 and X_m are established for a certain supply frequency [Beaty and Kirtley, 1998].

If the machine is operated at variable frequency these inductances depend on the synchronous motor shaft speed ω_{sync} as follows

$$X_1 = \omega_{sync} L_s , \quad X_2 = \omega_{sync} L_r , \quad X_m = \omega_{sync} L_m$$
(3.17)



Figure 3.17: Equivalent circuit of an induction motor

The synchronous speed is a function of the supply frequency f_{s} and the number of pole pairs p and is calculated as follows

$$\omega_{\rm sync} = 2\pi \cdot \frac{f_{\rm s}}{p} \tag{3.18}$$

To solve this circuit, it is simplified by replacing the input circuit to the left of X_2 by its Thevenin equivalent. Thevenin's theorem states that any linear circuit that can be replaced by two terminals from the rest of the system, can be replaced by a single voltage source in series with an equivalent impedance [Chapman, 1998]. The resulting simplified circuit is presented in Figure 3.17B, where the magnitude of the Thevenin voltage is defined by

$$U_{th} = U_{s} \frac{X_{m}}{\sqrt{R_{1}^{2} + (X_{1} + X_{m})^{2}}}$$
(3.19)

and where the impedances Rth and Xth of the replacement resistance and inductance are

$$R_{th} = \frac{R_1 \cdot X_m^2}{R_1^2 + (X_1 + X_m)^2}$$
(3.20)

$$X_{th} = X_{m} \cdot \frac{R_{1}^{2} + X_{1} \cdot (X_{1} + X_{m})}{R_{1}^{2} + (X_{1} + X_{m})^{2}}$$
(3.21)

With this approach the following expressions are found for the current I_2 and the rotor-induced torque [Chapman, 1998]

$$I_{2} = \frac{U_{th}}{\sqrt{(R_{th} + R_{2}/s)^{2} + (X_{th} + X_{2})^{2}}}$$
(3.22)

$$T_{ind} = \frac{3 \cdot U_{th}^{2} \cdot R_{2} / s}{\omega_{sync} \cdot \left((R_{th} + R_{2} / s)^{2} + (X_{th} + X_{2})^{2} \right)}$$
(3.23)

With the shaft speed ω_r known, the motor slip is defined by

$$s = \frac{\omega_{sync} - \omega_{r}}{\omega_{sync}}$$
(3.24)

To calculate the torque curve, the parameters of the equivalent circuit have to be determined. If the inductances and the resistances are given by the manufacturer the Thevenin equivalents can be calculated with equations (3.19), (3.20) and (3.21), which can be inserted into equation (3.23) to yield the torque curve. However, motor catalogues do not usually state the inductances and resistances. For a 3kW AC motor that is comparable to the motor used in the E-BS, for example, the motor catalogue states the following values.

Table 3.3: SEW catalogue data for the E-BS drive motors

| Quantity | Value | Unit | |
|------------------------|-------|------|--|
| Nominal voltage | 230 | V | |
| Nominal frequency | 50 | Hz | |
| Nominal shaft speed | 1415 | rpm | |
| Nominal torque | 20.3 | Nm | |
| Nominal current | 6.7 | А | |
| Pull out torque torque | 54.7 | Nm | |

In order to calculate the maximum torque or pull out torque, the slip value s_{max} has to be determined first. This is accomplished by calculating the slip value where the torque-speed curve reaches its peak, which results in [Chapman, 1998]

$$s_{max} = \frac{R_2}{\sqrt{R_{th}^2 + (X_{th} + X_2)^2}}$$
(3.25)

With this information the motor parameters can be found by fitting the curves for both the motor current and torque as defined in equation (3.22) and (3.23) on to the given data for the nominal torque, nominal current and starting torque. Table 3.4 presents the parameter set that was found after fitting the curves onto the data.

| Parameter | Value | Unit |
|----------------|-------|------|
| R ₁ | 0.375 | Ω |
| R ₂ | 1.34 | Ω |
| Ls | 4.82 | mH |
| Lr | 24.5 | mH |
| L _m | 11.8 | mH |

Table 3.4: Calculated motor parameters for the E-BS drive motors

Figure 3.18 shows the resulting torque-speed and current characteristic for this motor type at different supply frequencies. For supply frequencies below the nominal value of 50 Hz the ratio between the supply voltage and frequency is kept constant, to keep the magnetic flux in the motor approximately constant. This results in a family of curves that shift to the left if the frequency is reduced.



Figure 3.18: Nominal torque-speed and current characteristic for E-BS motor

The pull out torque and torque stiffness (i.e. the slope of the torque-speed curve in the normal operating range) is more or less the same at all points below the nominal frequency, except at low frequencies where the effect of the rotor resistance in reducing the flux becomes is very pronounced [Hughes, 1990]. Beyond the nominal frequency the ratio between the supply voltage and frequency decreases because the supply voltage remains constant. As a result, the pull out torque reduces significantly and the torque-speed curve becomes less steep.

3.4.2 Gear reduction box and drive wheel

The drive motor's output torque T_{ind} is an input to the gearbox that in turn drives the drive wheel. In this drive line most of the motor power will be applied locally to the belt in the shape of the drive force F_d . Part of the power is lost through efficiency losses in the gearbox or is stored or released when the inertias of the rotor J_r , gears J_g and drive wheel J_d are accelerated or decelerated respectively. Looking from the motor shaft, as seen in Figure 3.16, the relationship between the angular acceleration, the motor torque and drive force is as follows

$$\left(\mathbf{J}_{r}+\mathbf{J}_{g}+\frac{\mathbf{J}_{d}}{i^{2}}\right)\cdot\dot{\boldsymbol{\omega}}_{r}=\mathbf{T}_{ind}-\mathbf{F}_{d}\cdot\frac{\mathbf{r}_{d}}{i\cdot\boldsymbol{\eta}_{g}}$$
(3.26)

where i is the reduction ratio of the gearbox with efficiency η_g and r_d is the radius of the drive wheel.

An additional relationship is required to solve equation (3.26) for the applied drive force F_d . In the dynamic belt model used by Lodewijks (1996) no traction model is included to model slippage between the drive pulley and the belt. With no slip occurring between the drive pulley and the belt, the motor shaft speed ω_r is directly related to the belt speed v_b , which yields the following additional equation

$$\omega_{\rm r} = v_{\rm b} \cdot \frac{i}{r_{\rm d}}$$
(3.27)

However, when modelling the system behaviour of a multiple drives belt conveyor, like the E-BS for example, a traction model that includes slip as a result of the application of the drive force has a greater preference. Such a traction model makes it possible to model situations where the applied drive forces approach the friction limit, for example in the case that too much power is applied at certain drive stations causing high levels of slip through an imbalance of applied motor power. To include the relationship between traction and slip a traction model is presented in chapter 5.

In chapter 5 attention is also paid to the wear of the belt's contact surface, resulting from the slippage between the drive wheel and belt. The relationship between traction and wear rate is of interest in multiple drive belt conveyors because it determines what maximum value of traction still gives an acceptable belt life and it gives an indication of the minimum number of drive stations that are required in the system.

From this chapter it is clear that the existing dynamic belt model discussed in section 3.2, which is usually used for single drive conveyors, can be easily adapted to the multiple drives case. However, to be able to model the behaviour of a system like the E-BS a couple of sub models have to be developed further. Therefore, in chapter 4 models are presented for the indentation rolling resistance in the E-BS and the resistance in its horizontal curves, and in chapter 5 the relationship between traction, slip and wear is investigated for wheel driven belt conveyors.

4 Modelling resistance in the E-BS

As discussed in the previous chapter a number of researchers have investigated the resistances occurring in belt conveyor systems. In their quest to improve the predictability of the total resistance, which is vital when determining the required belt strength and overall drive power, and to reduce friction, they focussed on phenomena such as indentation rolling resistance, flexure resistance and bearing resistance. Although the models resulting from these investigations are developed for troughed and flat belt conveyor systems, most of the models and theories are also applicable to the multiple drives system like the E-BS. However, for the indentation rolling resistance an adaptation is required to incorporate the belt's curved running surface in contact with the idlers. Therefore, an extension to an existing model is presented in section 4.1 to account for the curved running surface. Another aspect that is different in the E-BS is the fact that the E-BS belt can negotiate much sharper horizontal curves. As the occurring friction at each idler in the curve is proportional to the local belt tension, the belt tension is amplified through the curve. This effect is investigated in section 4.2. Finally, in section 4.3 a dynamic friction model is introduced to the dynamic belt model to include frictional effect such as a breakaway force, stiction and stick slip.

4.1 Indentation rolling resistance in the E-BS¹

To prevent the triangular profiles of the E-BS' pouch shaped belt from jamming between the idlers, the belt's running surface is curved, as discussed in section 3.1.2. As a result, existing models for indentation rolling resistance developed for conventional flat and troughed belt conveyors cannot be directly applied in this case because in conventional belt conveyors a line type contact occurs between the support rollers and the belt. Therefore, an adaptation of an existing model is presented to incorporate the effect of the curved running surface.

4.1.1 Modelling the belt's viscoelastic surface

As the belt cover passes an idler the rubber surface compresses and relaxes in quick succession. Due to the viscoelastic properties of the rubber cover material the relaxation will take some time. As discussed in section 3.3.1, this causes an asymmetrical stress distribution that results in a resistance force. To incorporate the viscoelastic properties for the calculation of the indentation rolling resistance, a number of researchers have used the Maxwell model to

¹ This section is based on [Nuttall et al., 2006a]

quantify the energy dissipation of a cylinder rolling on a viscoelastic surface [Johnson, 1985] [May et al., 1959] [Hunter, 1961] [Lodewijks, 1996], which is comparable to a conveyor belt passing over an idler. The model described by Lodewijks (1996) is very similar to the one described by May et al. (1959). In both cases the derivation of the formula for the resistance force is the same, but the formulations of the problem differ. Lodewijks (1996) prescribes the vertical load, whereas May et al. (1959) prescribe the indentation depth.

Lodewijks (1996) combines the Maxwell model in its three parameter form with a Winkler foundation or 'mattress' consisting of springs that do not interact with each other, see Figure 3.11. Because shear forces between adjacent spring elements are not considered the model becomes less complex. Despite this simplification results presented by Wheeler (2003) show that this representation of the belt cover behaviour generates results that are close to experimentally measured values. Therefore, the combination of the Maxwell model and Winkler foundation will serve as starting point for the analysis of the indentation rolling resistance of the E-BS' curved running surface.

Figure 4.1 shows the contact patch that exists between the curved belt surface and the idler. In this situation the curved viscoelastic surface with radius r_2 is indented by a rigid cylinder with radius r_1 , representing the idler. The rubber surface passes underneath the rolling cylinder with speed v_b and the cylinder is considered rigid because, compared to the rubber surface, the idlers are relatively stiff.

To accommodate for the E-BS' elliptically shaped contact patch, the model described by Lodewijks (1996) is expanded by increasing the number of Maxwell elements, allowing a model with more than three parameters. As noted by Lodewijks (1996), the three parameter Maxwell model with its single relaxation time is sufficient for modelling the rubber surface of a flat belt, if the investigated belt speed range is not too large. In that case the relaxation time must be chosen equal to the time it takes a point on the belt cover to pass the contact zone between roll and belt. However, for a curved contact surface, as found in the E-BS, the contact time varies with the contact width along the length of the idler. Therefore, a model with a single relaxation time will not suffice and a more generalised approach allowing more Maxwell elements is used to be able to match the model with the real rubber behaviour throughout the contact patch.



Figure 4.1: Cylinder rolling over a curved surface

4.1.2 Maxwell model

Figure 4.2 shows the Maxwell model that is used to approximate the viscoelastic behaviour of the contact surface. It consists of a spring set in parallel with a number of spring and dashpot assemblies, the Maxwell elements. In the ideal case the model has an infinitely large number of Maxwell elements, which is know as the generalised Maxwell model. For the rubber belt surface the ideal representation is simplified by limiting the total number of elements to m.



Figure 4.2: Maxwell model with m elements

The total stress in this model is composed of the stress on the spring with stiffness E_{0} and the stresses on each spring-damper assembly, or

$$\sigma = \sigma_0 + \sum_{i=1}^{m} \sigma_i$$
(4.1)

The stress σ_0 generated by the first spring is directly related to the total ε strain on the model

$$\sigma_0 = E_0 \cdot \varepsilon \tag{4.2}$$

The stress on the remaining spring and dashpot elements is directly related to the local strains of the individual element

$$\sigma_{i} = E_{i} \cdot \varepsilon_{i}^{E} \tag{4.3}$$

$$\sigma_{i} = \eta_{ni} \cdot \dot{\varepsilon}_{i}^{\eta} \tag{4.4}$$

where ϵ^{E} and ϵ^{η} represent the local strain of the spring and dashpot elements respectively. The sum of the local strains is equal to the total strain on the element. Taking the time derivative of the strains leads to

$$\dot{\varepsilon} = \dot{\varepsilon}_i^{\rm E} + \dot{\varepsilon}_i^{\eta} \tag{4.5}$$

The time derivatives of ϵ^{E} and ϵ^{η} can be found from equation (4.3) and (4.4)

$$\dot{\varepsilon}_{i}^{E} = \frac{\dot{\sigma}_{i}}{E_{i}}$$
(4.6)

$$\dot{\varepsilon}_{i}^{\eta} = \frac{\sigma_{i}}{\eta_{ni}}$$
(4.7)

Combining equation (4.6) and (4.7) with equation (4.5) results in a relationship between the total strain and the stress on each spring dashpot assembly.

$$\dot{\sigma}_{i} + \sigma_{i} \frac{E_{i}}{\eta_{ni}} = E_{i} \dot{\varepsilon}$$
(4.8)

This leads to a set of equations that describes the behaviour of the Maxwell model that is used to determine the stress and shear forces in the contact plane. It also serves as a base for calculating the complex modulus of elasticity that is used to match the model parameters with measured viscoelastic material properties.

4.1.3 Pressure distribution and rolling friction due to hysteresis

A cylinder rolling on a viscoelastic surface, like an idler on a conveyor belt, will encounter rolling friction due to hysteresis. Energy losses occur because the energy required to indent the viscoelastic material in the front part of the contact surface is not fully regained in the latter part, where the material is allowed to relax. The surface needs some time to regain its original shape. This effect manifests itself in an asymmetrical pressure distribution, which results in a residual torque working opposite to the direction of movement.

The model used to calculate the rolling friction is shown in Figure 4.3. A cylinder rolls on a viscoelastic layer with thickness h with angular velocity ω . The cylinder is pressed into the layer with force F_z , causing a reaction force F_n at its base and the cylinder's centre remains stationary while the rubber passes underneath with speed v_b . In this situation the rubber material, represented by the Maxwell model, makes first contact with the cylinder at the leading edge where x = a. Subsequently, it passes through the contact plane and loses contact at the trailing edge where x = -b.

If the deformation in the contact zone is known, it is possible to solve the individual differential equations in (4.8) and find the pressure distribution. The deformation can be found by assuming that the material in the viscoelastic layer is the only material that deforms and that the contact zone is small compared to the curvatures of the cylinder and rubber surface, so $x \ll r_1$ and $y \ll r_2$.



Figure 4.3: Cylinder rolling on a viscoelastic layer

If the distance with which the cylinder is pressed into the moving surface is set to z_0 , then the deformation of the contact plane can be described as follows

$$w(x, y) = z_0 - \frac{x^2}{2 \cdot r_1} - \frac{y^2}{2 \cdot r_2} \quad -b \le x \le a, \text{ with } z_0 = \frac{c^2}{2 \cdot r_2}$$
(4.9)

where point C represents the intersection of the x-axes and the boundary of the contact plane, see Figure 4.1. If only the steady state of the rolling cylinder is considered, the time derivative of equation (4.8) can be eliminated. As a result, the differential equation of each Maxwell element can be written as

$$\frac{\partial \sigma_{i}}{\partial x}\frac{dx}{dt} + \sigma_{i}\frac{E_{i}}{\eta_{ni}} = E_{i}\frac{\partial \varepsilon}{\partial x}\frac{dx}{dt}$$
(4.10)

or, under steady state conditions and using the deformation equation (4.9) in the x-y plane

$$\frac{\partial \sigma_i}{\partial x} - \sigma_i \frac{E_i}{\eta_{ni} \cdot v_b} = -E_i \frac{x}{h \cdot r_l}$$
(4.11)

with $\frac{dx}{dt} = -v_b$ the steady state condition, and $\varepsilon = \frac{w(x, y)}{h}$, so $\frac{\partial \varepsilon}{\partial x} = -\frac{x}{h \cdot r_1}$ derived from (4.9).

Although the conveyor belt does not always have a constant speed during operational situations, such as starting and stopping for example, the changes in belt speed in the localised contact plane will be perceived as very gradual. Therefore, the rolling indentation resistance calculated at different belt speeds will be converted in to a friction curve that will serve as a base for the dynamic friction model presented in section 4.3.

Differential equation (4.11) can be solved by setting the stress at the first point of contact equal to zero or $\sigma(a, y) = 0$. At the first point of contact no deformation has occurred yet, which means that all Maxwell elements are in the unloaded state. The solution combined with equation (4.1) reveals the following pressure distribution in the contact plane

$$\sigma(\mathbf{x}, \mathbf{y}) = \frac{E_0}{2 \cdot r_1 \cdot h} \left(a^2 - x^2 \right) + \sum_{i=1}^m \frac{E_i \cdot k_i}{h \cdot r_1} \left(x - a + \left(a + k_i \right) \left(1 - \exp\left(\frac{x - a}{k_i}\right) \right) \right),$$
(4.12)
where $k_i = \frac{\eta_{ni} \cdot v_b}{E_i}$ and $a = f(\mathbf{y})$

With the stress distribution in the contact plane given, the total vertical contact force can now be calculated by integrating the stress distribution over the whole contact region.

$$F_n = 2 \cdot \int_{0}^{c} \int_{-b(y)}^{a(y)} \sigma(x, y) dx dy$$
(4.13)

To complete the first part of the integration in (4.13), the contact surface's leading and trailing edge, given by a(y) and -b(y) respectively (also see Figure 4.1), need to be determined. The upper boundary a(y) describes the distance from the cylinder's centreline to the line of first contact and is found as follows

$$w(a(y), y) = 0 \Longrightarrow a^{2}(y) = 2 \cdot r_{1} \cdot \left(z_{0} - \frac{y^{2}}{2 \cdot r_{2}}\right)$$
(4.14)

The lower boundary -b(y), representing the distance between the trailing edge and the centreline, is found by setting $\sigma(x, y)$ equal to zero.

The torque M_y resulting from the asymmetric pressure distribution is calculated by integrating the moment about the centre of the rolling cylinder using the same contact boundaries as in (4.13)

$$M_{y} = 2 \cdot \int_{0}^{c} \int_{-b(y)}^{a(y)} x \cdot \sigma(x, y) dx dy$$
(4.15)

The resistance torque is usually represented by a rolling resistance force equal to

$$F_{\rm r} = \frac{M_{\rm y}}{r_{\rm l}} \tag{4.16}$$

To be able to calculate the indentation rolling resistance factor f_r as defined in the DIN 22101, the ratio between the total resistance encountered at each idler set and the vertical load F_1 carried by each set has to be determined or

$$f_r = \frac{2 \cdot F_r}{F_l}$$
(4.17)

where the factor 2 is added for the fact that an indentation force F_r is generated by both idler rolls.

In the next step the vertical load force F_1 is expressed in terms of the contact force F_n . This makes it possible to compare the indentation rolling resistance occurring in the E-BS with that occurring in a flat belt conveyor system because in the latter case the friction factor is directly dependent on the ratio between the resistance force F_r and the contact force F_n . Figure 4.4 shows the situation at an idler set in the E-BS with the contact forces F_n acting on the inclined surfaces of the belt and the vertical force F_1 pulling the top of the belt into the idler rolls. The force F_1 represents the weight of both the belt and the bulk material between successive idler sets.



Figure 4.4: Normal contact forces at an idler set

If the equilibrium of the forces in Figure 4.4 are considered, the vertical load can be expressed in terms of the contact forces F_n and the idler angle α as follows

$$F_1 = 2 \cdot F_n \cdot \cos \alpha \tag{4.18}$$

Finally, the friction factor f_i is obtained by substituting equation (4.18) into (4.17).

$$f_{\rm r} = \frac{F_{\rm r}}{F_{\rm n} \cdot \cos\alpha} \tag{4.19}$$

Note that from equation (4.19) the effect of the inclined contact surface is expressed by the term $\cos\alpha$. For a flat belt conveyor the idler angle is zero, giving the lowest friction factor for a given type of rubber belt and idler roll. If the idler angle is increased, as is the case for the E-BS, it will cause a greater friction factor.

4.1.4 Finding the Maxwell parameters

Viscoelastic properties of a material like rubber can be measured by conducting oscillatory experiments, where the material is subjected to sinusoidal varying stresses and strains [Gent, 2001][Lodewijks, 2004]. Results are typically expressed as the storage modulus E', loss modulus E'' and loss factor tan δ , which together represent a complex modulus of elasticity. The results are related to the complex modulus as follows

$$E^* = E' + i \cdot E''$$
 (4.20)

$$\tan \delta = \frac{E''}{E'}$$
(4.21)

The next step consists of setting the parameters of the Maxwell model, so its reactions to varying stresses are comparable to the real material. To accomplish this, a set of parameters matching the measured viscoelastic properties of a real belt needs to be chosen.

One possibility is to fit the properties of the model onto the measured data. This is done with the aid of the following equations that relate the Maxwell parameters to the storage modulus E' and loss modulus E''.

$$E' = E_{0} + \sum_{i=1}^{m} \frac{\omega^{2} \eta_{ni}^{2} E_{i}}{\omega^{2} \eta_{ni}^{2} + E_{i}^{2}}$$

$$E'' = \sum_{i=1}^{m} \frac{\omega \eta_{ni} E_{i}^{2}}{\omega^{2} \eta_{ni}^{2} + E_{i}^{2}}$$
(4.22)

The number of Maxwell elements m to be used in the model depends on the required accuracy of the complex modulus of elasticity in a desired frequency range. With a possible operational belt speed of 1.6 - 10 m/s and an approximated contact length of 0.02 m, the frequency of excitation ranges from 80 to 500 Hz. The accuracy generally increases when more elements are added. However, with more elements the model also becomes more complex, making computations more time consuming and the search for starting conditions that give a good convergence of the optimisation routine during the matching procedure increasingly difficult. Furthermore, due to the implemented least squares approach in the matching procedure, the maximum number of elements is physically limited by the amount of experimentally measured data. It is impossible to fit a model with more parameters than data points. Figure 4.5 shows how the model fits onto the measured viscoelastic properties of the E-BS when different numbers of Maxwell elements are used.



Figure 4.5: Measured and approximated viscoelastic properties

The figure clearly illustrates the difference between the simplest model with one element (or three parameters) that gives an unsatisfactory approximation between 10 and 1000 rad/s and a model with three elements (or seven parameters) with an improved accuracy. The 7 parameter model is chosen for further calculations because it gives the better match. Table 4.1 presents the values found for the 7 parameter model.

| Parameter | E ₀ | E_1 | η_{n1} | E_2 | η_{n2} | E ₃ | η_{n3} |
|-----------|----------------|-------|--------------------|-------|---------------------|----------------|------------------|
| Value | 4.6 | 8.0 | $1.8 \cdot 10^{6}$ | 3.3 | 3.1·10 ⁴ | 6.2 | $5.1 \cdot 10^3$ |
| Unit | MPa | MPa | Nm/s | MPa | Nm/s | MPa | Nm/s |

Table 4.1: Values of the 7 parameter Maxwell model

With the parameters of the Maxwell model known, the next step consists of determining the model's layer thickness h. For a conventional flat belt the layer thickness is chosen equal to the thickness of belt's bottom cover. However, in the E-BS the layer thickness of the triangular running profile varies along the length of the idler roll. To be able to use the presented model, the layer thickness h is set equal to the average thickness of the rubber triangle. Figure 4.6 illustrates how this average thickness is found. With an idler angle α of 65° and a centre of contact occurring 44 mm from the bottom of the rubber triangle the average layer thickness equates to 21 mm.



Figure 4.6: Finding the average layer thickness h

4.1.5 Results

Figure 4.7 shows the calculated results of the indentation rolling resistance due to hysteresis, using the described 7 parameter Maxwell model with data from the E-BS. The radius r_1 of the rolling cylinder is set to 44.5 mm, which is equal to the radius of an idler roll, and the radius r_2 of the rubber contact surface is 0.5 m. In the left diagram the rolling resistance factor f_r is set out against the belt speed v_b for an empty belt and for a fully loaded belt. In the right diagram the resistance factor is plotted against the vertical load for two different operational speeds.



Figure 4.7: Indentation resistance versus speed (left) and vertical load (right)

The calculated results show that the indentation rolling resistance factor reduces when the belt speed increases beyond 3 m/s. This is caused by the fact that the loss factor tan δ of the E-BS' cover material decreases at higher frequencies of excitation. The results also show that the

resistance factor increases continuously when the vertical load F_1 increases. However, at higher loads the resistance factor increases less progressively.

To determine the influence of the belt's curvature, Figure 4.8 presents the calculated resistance factor for different values of the surface radius r_2 . In this figure the left diagram presents the values for an empty belt (with $F_1 = 86$ N) and the right diagram presents the values for a belt travelling at a constant operational speed of 1.6 m/s.



Figure 4.8: Influence of surface radius r₂ on the indentation rolling resistance

The graphs in Figure 4.8 indicate that the resistance factor decreases when the belt's curvature r_2 is increased. As the radius r_2 increases the contact patch between the roll and the belt start to take on the shape also found for a flat belt. Consequently, the calculated results also start to approach the values for a flat belt. From this observation it can be concluded that the application of a curved belt surface, as found in the E-BS, also increases the occurring indentation rolling resistance. Note however, that the calculation of the indentation rolling resistance does not take the wedging effect, discussed in section 3.1.2, into account. Therefore, it is undesirable to choose a very large value for r_2 . To find the optimal value for r_2 , the value will have to be determined that accomplishes the best compromise between the wedging effect and the indentation rolling resistance.

To determine the occurring resistances, tests were conducted on a pilot E-BS installation with an empty belt. With these results an indication is made of the magnitude of the indentation rolling resistance. The most applicable measurements were carried out on a straight section situated between two motor pairs because no other sources of resistance, such as inclines and curves, were present in this section. With only the bearing, seal and indentation rolling resistance present the main resistance factor of this section f_i is equal to the ratio between the measured main resistance $F_{M,i}$ and the total vertical load of this section or

$$f_{i} = \frac{F_{M,i}}{l_{i} \cdot m'_{b} \cdot g}$$
(4.23)

With an overall length of 36 m and a measured resistance force of 58 N, the main resistance factor of the empty belt section equates to 0.015. With this result it can be concluded that the calculated results as displayed in Figure 4.7 and Figure 4.8 are in the same order of magnitude. However, to further verify the model, more accurate measurements are required. This involves carrying out more extensive experiments in a more controlled environment, so it is easier to isolate the actual occurring indentation rolling resistance. On the model's side more attention will also have to be paid to the measurement of the actual viscoelastic cover properties because as noted by Lodewijks (2004) the measurement of these properties is not trivial and can lead to significant variations in the outcome.

4.2 **Resistance in horizontal curves**

When the E-BS belt passes through a horizontal curve, the tension forces present in the belt pull the belt onto the inner rolls of the idler sets. This generates an addition force on each idler set that points inward towards the centre of the curve and causes an increased motion resistance. This effect is amplified as the belt travels through the corner because the tension in the belt increases as it passes an idler set, causing an even greater resistance force at the next idler set.

4.2.1 Modelling the curve resistance

Figure 4.9 illustrates the situation of the belt passing through a curve with a sweep angle α_c and radius R_c . In this figure the belt is modelled as a string that is pulled against the idler rolls on the inside of the curve. When a particular idler roll is considered, as presented on the right of the figure, the belt tension $T_{c,i}$ before the idler and the increased tension $T_{c,i}+\Delta T_i$ after the idler both contribute to the resultant contact force $F_{n,i}$ acting on the idler roll, which is calculated as follows

$$F_{n,i} = \left(2 \cdot T_{c,i} + \Delta T_i\right) \cdot \sin\left(\frac{1}{2}\beta\right)$$
(4.24)

where β is the angle between two idler sets and i identifies a particular idler set in the curve. The angle β can also be expressed in terms of the curve radius R_c and the idler spacing L as follows

$$\sin\left(\frac{1}{2}\beta\right) = \frac{L}{2 \cdot R_{c}}$$
(4.25)



Figure 4.9: Additional contact forces on the inner idler rollers in a curve

In equation (4.24) the increase in tension after the belt has passed the idler is represented by ΔT_i and its value is equal to the resistance occurring at the idler set. Assuming that the resistance force is proportional to the contact force, the following relationship holds for the increase in belt tension

$$\Delta T_i = f_c \cdot F_{n,i} \tag{4.26}$$

where f_c is the resistance factor for the curve. Note that this factor is not part of the factor f_i as defined the DIN 22101 standard because f_c is not related to the vertical load of the belt. By combining (4.25) and (4.26) with (4.24) the contact force $F_{n,i}$ is expressed as

$$F_{n,i} = T_{c,i} \cdot \frac{L}{R_c - \frac{1}{2}f_c \cdot L}$$
(4.27)

With the aid of equations (4.26) and (4.27) it is now possible to determine the increased belt tension $T_{c,i+1}$ after the belt passes an idler set, which is

$$T_{c,i+1} = T_{c,i} + \Delta T_i = T_{c,i} \cdot \left(1 + \frac{f_c \cdot L}{R_c - \frac{1}{2}f_c \cdot L} \right)$$
(4.28)

From equation (4.28) it is clear that the belt tension in a curve is amplified by the same factor each time an idler set is passed. So, if the belt enters the curve with a tension $T_{c,i}$ and passes n idlers in the curve, the tension $T_{c,n+1}$ at the end of the curve is equal to

$$T_{c,n+1} = T_{c,1} \cdot \left(1 + \frac{f_c \cdot L}{R_c - \frac{1}{2}f_c \cdot L}\right)^n, \text{ with } n = \frac{\alpha_c \cdot R_c}{L} + 1$$
(4.29)

With both belt tensions before and after the curve known, the total curve resistance $F_{\rm c}$ is calculated as follows

$$F_{c} = T_{c,n+1} - T_{c,1} = T_{c,1} \cdot \left(\left(1 + \frac{f_{c} \cdot L}{R_{c} - \frac{1}{2}f_{c} \cdot L} \right)^{n} - 1 \right)$$
(4.30)

Note that the curve resistance is directly proportional to the belt tension at the beginning of the curve. While optimising the system design it is therefore important to keep the belt tension as low as possible before the belt enters a curve to minimise the resistance. This can be accomplished by placing a drive station in front of the curve, for example.

4.2.2 Results

angles.

With the experiments conducted on the pilot E-BS installation [Twaalfhoven, 2004] it is possible to give an indication of the actual corner resistance factor f_c . Table 4.2 presents the dimensions and measured values for two curves present in the pilot installation. Both curves have about the same radius, but the first curve has a sweep angle of 90°, while the second curve has a sweep angle of about 205°.

 Table 4.2: Corner resistance measured in E-BS pilot installation

| Parameter | R _c [m] | n [-] | T _{c,1} [N] | $T_{c,n+1}[N]$ | L [m] | f _c [-] |
|-----------|--------------------|-------|----------------------|----------------|-------|--------------------|
| Curve 1 | 8.8 | 17 | 3276 | 3870 | 0.8 | 0.11 |
| Curve 2 | 8.2 | 37 | 2080 | 3524 | 0.8 | 0.15 |

At the end of Table 4.2 the calculated curve resistance factor is also presented for both curves. This curve factor is calculated by rearranging equation (4.29), which results in

$$f_{c} = \frac{2 \cdot R_{c}}{L} \cdot \frac{\sqrt[n]{T_{c,n+1}} - \sqrt[n]{T_{c,1}}}{\sqrt[n]{T_{c,n+1}} + \sqrt[n]{T_{c,1}}}$$
(4.31)

Looking from the systems designer's point of view, where it is important to keep the curve resistance to a minimum, it is clear from equation (4.30) that a low belt tension and a small sweep radius will lead to a low resistance. However, the influence of the curve radius R_c is not as clear because the number of idlers n is linear dependent on the curve radius for a set curve sweep α_c . On the one hand a larger curve radius reduces the resistance generating force $F_{n,i}$ on each idler set, as is clear from equation (4.27), while on the other hand more idler sets will have to placed in the curve, resulting in more points where the belt tension is amplified. To investigate the influence of the curve radius on the curve resistance, the total factor by which the belt tension is amplified in a curve, see equation (4.29), is calculated as function of the curve radius. Figure 4.10 presents the results of this exercise for two different curve sweep


Figure 4.10: Influence of curve radius on curve resistance

The results in Figure 4.10 show that if the curve radius is increased, the curve resistance reduces significantly up to a radius of 8 meters. However, beyond this value the resistance reduces only very slightly. For curve radii beyond 8 m, the system designer can only obtain a marginal gain, with respect to resistance, by increasing the curve radius. Note that these results are based on the measurements conducted on the pilot E-BS installation. Depending on the system configuration the radius value can differ where the reduction in resistance becomes marginal.

The results presented in Figure 4.10 also show that the amplification factor approaches a value larger than 1 when the radius is increased. Even if the radius is hypothetically increased to an infinitely large value, the amplification factor remains above 1. As a result, the belt tension always increases in a horizontal curve irrespective of the chosen curve radius and a resistance force can be expected from the friction generated by the inward facing cornering forces.

4.3 Adding a dynamic friction model to the dynamic belt model

In the dynamic belt model described by Lodewijks (1991) the main resistance force $F_{M,i}$ acting on each node is derived from the DIN 22101 and uses both Coulomb and viscous friction parts. In this model the resistance force on each belt node is described as follows

$$\mathbf{F}_{\mathrm{M},i} = \mathbf{C} \cdot \mathbf{f}_{i} \cdot \mathbf{g} \cdot \mathbf{m}_{i} \cdot \left(\mathbf{c}_{\mathrm{vo}} + \mathbf{c'}_{\mathrm{v}} \cdot \dot{\mathbf{u}}_{i}\right)$$
(4.32)

where m_i is the lumped mass of the belt and its load, and where c_{vo} and c'_v represent the static and viscous friction component respectively. Although this model works satisfactory in simulations where the belt is started directly at the beginning of the simulation, problems arise during stopping procedures and stationary conditions. If the static friction component c_{vo} is present and the belt is stopped with the motors switched off, the resistance force does not disappear, causing the modelled belt to move backwards. To improve the description of the friction effects, the LuGre friction model [Canudas de Wit et al., 1995] [Olsson et al. 1998] is introduced to the dynamic belt conveyor model. This dynamic friction model does not only offer the possibility to include friction effects, such as stick-slip, stiction and friction lag, but it is also continuously differentiable, making it computationally friendly for simulations. Figure 4.11 illustrates the main idea behind the LuGre friction model, which is visualised by two rigid bodies contacting each other through elastic bristles.

When the bodies move with respect to each other, the bristles deflect like a spring and generate a tangential force in the contact plane. This is a highly random phenomenon due to the very irregular form of the contact surfaces at a microscopic level [Canudas de Wit et al., 1995]. Therefore, the LuGre model uses an average bristle deflection to model this effect, which is described by [Olsson et al. 1998]

$$\dot{z}_{i} = v_{i} - \sigma_{0} \frac{\left|v_{i}\right|}{g(v_{i})} \cdot z_{i}$$
(4.33)

where v_i is the relative velocity between the surfaces, σ_0 is the stiffness of the bristles and $g(v_i)$ is a positive function that depends on many factors such as material properties, lubrication and temperature.



Figure 4.11: Bristles between contact surfaces in the LuGre model

The index i has been added to equation (4.33) to be able to incorporate the LuGre model in the dynamic belt model, discussed in section 3.2. This index indicates the individual bristle state of each node in the belt model. As a result, the state of each node now consists of the local belt displacement u_i , belt speed \dot{u}_i and bristle defection z_i . Also note that in the dynamic belt model the relative speed v_i in equation (4.33) is equal to the nodal speed \dot{u}_i .

With the bristle deflection defined, the main resistance generated from the bending of the bristles is described by

$$\mathbf{F}_{\mathbf{M},i} = \boldsymbol{\sigma}_0 \mathbf{z}_i + \boldsymbol{\sigma}_1 \dot{\mathbf{z}}_i + \mathbf{F}_r(\mathbf{v}_i)$$
(4.34)

where σ_1 is a damping coefficient. The last term accounts for the indentation rolling resistance, which is a function of the belt speed and is based on the results calculated in section 4.1.5.

When friction occurs in mechanical systems like belt conveyors, mechanical energy is mainly converted to heat. As this is an irreversible process, mechanical energy will dissipate out of the system at the locations where friction is generated through sliding or rolling contacts. Since the LuGre model is a dynamic model there may be phases where friction gives back energy. To guarantee that the friction force calculated with (4.34) always dissipates energy the damping coefficient σ_1 has to comply with the following relationship [Olsson et al. 1998]

$$\sigma_1 < 4 \cdot \frac{g(v_i)}{|v_i|}$$
(4.35)

From equation (4.35) it is clear that the dampening factor is velocity dependent. Therefore, Olsson et al. (1998) proposed the following function

$$\sigma_1(v_i) = \sigma_1 \cdot e^{-(v/v_d)^2}$$
(4.36)

Note that in the dynamic belt model the resistance is expressed in terms of the resistance factor f_i , as defined by the DIN 22101 standard. To obtain the resistance factor, equation (4.34) is scaled as follows

$$\mathbf{f}_{i} = \boldsymbol{\sigma}'_{0} \cdot \mathbf{z}_{i} + \boldsymbol{\sigma}'_{1} \cdot \dot{\mathbf{z}}_{i} + \mathbf{f}_{r}(\mathbf{v}_{i}), \quad \text{where } \boldsymbol{\sigma}'_{0} = \frac{\boldsymbol{\sigma}_{0}}{\mathbf{m}_{i} \cdot \mathbf{g}} \text{ and } \boldsymbol{\sigma}'_{1} = \frac{\boldsymbol{\sigma}_{1}}{\mathbf{m}_{i} \cdot \mathbf{g}}$$
(4.37)

At low velocities the force necessary to set the belt in motion, or so called breakaway force, is often larger than the dynamic friction that is generated when the belt is moving. This is mainly caused by the lubricant in the idler bearings that pushes the contact surfaces apart when the bearing starts rotating. Stribeck (1902) investigated this effect and observed that at low velocities the friction force decreases continuously from the static value to the dynamic value when the velocity increases. To included this phenomenon Canudas de Wit et al. (1995) proposed the following parameterisation of $g(v_i)$

$$g(v_i) = F_C + (F_B - F_C) \cdot e^{-(\frac{v_i}{v_s})^2}$$
 (4.38)

where F_C is the dynamic or Coulomb friction level, F_B is the level of the breakaway or stiction force and v_s is the Stribeck velocity. Figure 4.12 presents the resulting curve and illustrates the transition from the static to the dynamic friction level.



Figure 4.12: Friction curve for Stribeck effect

To determine the parameters of the LuGre model, the relationship between the steady state velocity and friction is considered. At a constant velocity the bristle deflection is constant, or $\dot{z} = 0$. By combining equation (4.33), (4.34) and (4.38) the following steady state friction factor is derived.

$$F_{ss,i} = g(v_i) \cdot sng(v_i) + F_r(v_i)$$
(4.39)

The first term in this equation describes the level of static and dynamic friction. The dynamic part in this term is mainly determined by the load dependent friction caused by the ball bearings in each idler set. This value is usually provided by the bearing manufacturer. For the static friction value Wheeler (2003) also notes that the breakaway torque for deep groove ball bearings is approximately twice as high as the load dependent friction moment.

The last term is already known because it represents the indentation rolling resistance, which was discussed in section 4.1.5. For fast computation during the dynamic belt simulation, a look up table is created using the results from section 4.1.5 that holds the calculated friction factor for a range of speeds and vertical belt loads.

Figure 4.13 presents the resulting friction factor when the LuGre model is in the steady state, which was calculated with the aid of equation (4.39). In this figure the solid line indicates the total friction factor, including both the bearing and rolling indentation rolling resistance. The dashed and the dotted lines in Figure 4.13 represent the individual contributions of the bearing and indentation rolling resistance respectively.

In this case the dynamic bearing friction factor is chosen as 0.005, the static or breakaway friction factor is twice as high and the Stribeck velocity v_s is 0.25 m/s. The dotted line for the indentation resistance is retrieved from a look up table that contains the indentation rolling resistance friction factor calculated in section 4.1.5 with a belt surface curvature r_2 of 0.5 m and a vertical load of 100 N.



Figure 4.13: Resistance factor in steady state condition

To illustrate the dynamic character of the LuGre friction model, Figure 4.14 presents the model's response, using equation (4.37). During this simulation the speed is ramped up to 4 m/s in 5 seconds and ramped back down to zero in the following 5 seconds.



Figure 4.14: Dynamic response of the friction model

At the start of the simulation (indicated by t_{start} in Figure 4.14) there is no friction force present because the initial bristle deflection is zero. As the conveyor belt starts to move, the bristles in the model deflect and create an increasing friction force. The friction force increases rapidly until the bristles' friction limit is reached, in which case the curve starts to follow the steady state curve presented in Figure 4.13. The rate at which the friction force approaches the steady state curve is mainly determined by the scaled bristle stiffness σ'_0 and

dampening factor σ'_1 . In this case a scaled bristle stiffness of 10 m⁻¹ and dampening factor of $2 \cdot 10^{-3}$ s/m were used, which gives a small bristle deflections while producing an acceptable time step during the numerical simulations.

After the speed is ramped back down to zero and the belt is held in place, the friction does not return to zero. Instead, a residual friction force remains at the end of simulation (indicated by t_{end} in Figure 4.14) because the bristles remain deflected. This indicates that a holding force would have to be exerted on the belt to keep it in place. If the belt is released it would move back very slightly until the bristles reach the non deflected state again.

As discussed in chapter 3 it is clear that most resistances occurring in multiple drive systems like the E-BS can be predicted and calculated using existing models and guidelines for conventional belt conveyor systems. In this chapter two additions have been made, involving the indentation rolling resistance and curve resistance, that are specific for the E-BS. In contrast, the LuGre is not a model that helps to predict the motion resistance, but it is a model that gives an opportunity to insert main resistances into the dynamic belt model and include frictional effects such as a break away force and stiction.

5 Traction in a wheel driven belt conveyor

Unlike drive stations found in conventional belt conveyor systems, where the belt is wrapped around a drive pulley, the E-BS' drive stations have drive wheels that press into the belt to generate the required drive force. Due to the fact that the belt is not wrapped around a drive pulley, the contact phenomena between the E-BS belt and the wheel surface are of a different nature. Therefore, a rolling contact model is presented in 5.1 to investigate the relationship between slip and traction, which also includes the viscoelastic properties of the belt cover. This relationship is also used to include the transfer of power to the belt in the dynamic belt conveyor model.

Another related aspect is the belt wear caused by the application of a drive force to the belt surface. As a drive wheels or pulley applies a drive force to the belt, the slippage occurring between the contact surfaces results in belt wear. In pulley drive belt conveyors this is usually not an issue because the wear is negligibly small due to the pulley's large contact surface. However, for wheel driven belt conveyor systems like the E-BS, where drive forces are applied through significantly smaller contact patches, the wear rate will also be significantly higher and no longer negligible. Therefore, attention will have to be paid to this type of belt wear to prevent the belt from wearing out before its guaranteed lifetime. In section 5.2 a model is presented that relates the belt wear to the applied drive force, making it possible to give an estimation of the minimum required drive stations in a wheel driven belt conveyor system like the E-BS.

5.1 Traction versus slip²

When a drive wheel applies a force to an elastic surface, like the cover of a conveyor belt, a speed difference occurs between the drive wheel's outer diameter and the belt. This apparent speed difference or creep is a result of both the deformation and sliding in the contact surfaces, due to the applied shear force. This speed difference is expressed as the creep ratio δ as follows

$$\delta = \frac{\mathbf{v}_{\mathrm{b}} - \boldsymbol{\omega}_{\mathrm{d}} \cdot \mathbf{r}_{\mathrm{l}}}{\left|\mathbf{v}_{\mathrm{b}}\right|} \tag{5.1}$$

² This section is based on [Nuttall and Lodewijks, 2006b]

where ω_d is the angular velocity of the drive wheel and R_1 is the wheel diameter.

If the drive force applied by the wheel is within the traction limit, as described by equation (5.2), stick and slip zones exist in the contact surface.

$$\mathbf{F}_{d} < \boldsymbol{\mu} \cdot \mathbf{F}_{n} \tag{5.2}$$

In the stick zone the normal stress acting on the contact surface is sufficient to prevent the surfaces from sliding due to the applied drive force and creep occurs through the elastic deformation of the rubber surface. In the slip zone the friction limit is reached. As a result, the wheel's surface now also slides over the rubber surface. To distinguish the stick and slip zones, the Amontons-Coulomb law is used to model friction in the contact zones

$$\left|\tau(\mathbf{x},\mathbf{y})\right| \le \mu \cdot \sigma(\mathbf{x},\mathbf{y}) \tag{5.3}$$

where τ and σ are the shear and normal stresses acting in the contact zone and μ is the friction coefficient. To solve equation (5.3), the normal stress distribution $\sigma(x, y)$ acting on the curved viscoelastic surface is determined, using the same method as presented in section 4.1. In order to use this calculated stress distribution, it is assumed that the shear stress does not influence the normal stress distribution, which has also been used by Johnson (1985) when establishing the relation between traction and slip.

5.1.1 Shear stress distribution

The next step is the calculation of the shear stress distribution $\tau(x,y)$ in both stick and slipzone. In the stick zone no sliding takes place between the contact surfaces. Therefore, in this zone the creep ratio is related to the shear angle by the following equation

$$\frac{\partial \gamma}{\partial x} = -\frac{\delta}{h}$$
(5.4)

To establish a relationship between the creep ratio and shear stress distribution in the stick zone, the Maxwell model, presented in section 4.1, is combined with a brush model that represents the shearing effects. Comparable brush models are also used as a simplified representation of the rubber tread behaviour on car tyres [Pacejka, 1995], but in that case the deflecting brush elements are connected to the circumference of the tyre.

Figure 5.1 illustrates the brush model for the wheel driven belt conveyor, which consists of rigid elements that hinge at their base. Torsion springs hold the elements in place, which create a resistance torque when the brush elements are deflected. To include the viscoelastic behaviour of the belt cover, the stiffness of the torsion springs is represented by a Maxwell model analogous to the spring element presented in Figure 4.2.



Figure 5.1: Brush model of wheel drive belt cover

By replacing the modulus of elasticity E, stress σ and strain ε in equations (4.1), (4.2) and (4.8) with the shear modulus G, shear stress τ and shear angle γ respectively equations are derived that describe the behaviour of the brush elements. Under steady state conditions and using the deformation equation (5.4) the differential equation describing the shearing of each Maxwell element can be written as

$$\frac{\partial \tau_{i}}{\partial x} - \tau_{i} \frac{G_{i}}{\eta_{si} \cdot v_{b}} = -G_{i} \cdot \frac{\delta}{h}$$
(5.5)

To derive the viscoelastic shear parameters, additional oscillatory experiments should be conducted where the rubber test sample is subjected to shear stresses and strains. However, due to the fact that no results of shear experiments were available, the shear parameters were derived from the normal stress experiments and converted with the aid of the following equation, which relates the materials shear modulus G to the modulus of elasticity E

$$G = \frac{E}{2 \cdot (1 + \nu)}$$
(5.6)

If it is assumed that the stick zone starts at the leading edge of the contact plane, a solution to differential equation (5.5) can be found, yielding the shear stress in the stick zone

$$\tau_{\text{stick}}(\mathbf{x}, \mathbf{y}) = \frac{\delta}{h} \cdot \mathbf{G}_0 \cdot (\mathbf{a} - \mathbf{x}) + \sum_{i=1}^{m} \left[\frac{\delta \cdot \eta_{si} \cdot \mathbf{v}_b}{h} \left(1 - \exp\left(\frac{\mathbf{G}_i \cdot (\mathbf{x} - \mathbf{a})}{\eta_{si} \cdot \mathbf{v}_b}\right) \right) \right]$$
(5.7)

The contribution of both the stick and slip zone can now be calculated by integrating the calculated shear stress in each zone separately

$$F_{d} = \int_{-c}^{c} \left(\int_{-b(y)}^{t_{1}(y)} \sigma(x, y) dx + \int_{t_{1(y)}}^{a(y)} \tau_{stick}(x, y) dx \right) dy$$
(5.8)

where $t_1(y)$ represents the transition line separating the stick from the slip zone. It represents the edge where the shear stress reaches the friction boundary and it can be found by solving

$$\tau_{\text{stick}}(t_1, y) = \mu \cdot \sigma(t_1, y)$$
(5.9)

5.1.2 Correction factor

A correction factor f_t is introduced to compensate for the fact that the Winkler foundation does not incorporate the shearing effect between adjacent spring elements and to match the stiffness of the model with the actual stiffness of the layer. Under the condition that the speed difference between the drive wheel and the belt is small, the slip region at the trailing edge becomes negligible. As there is virtually no slip in the contact region, the occurring speed difference or creep is predominantly determined by the layer stiffness. The corresponding limit for the creep ratio, as derived by Johnson (1985) using a half space approximation, is

$$\delta = \frac{\mathbf{a} \cdot \mathbf{F'_d}}{2 \cdot \mathbf{r} \cdot \mathbf{F'_n}} \quad \text{or} \quad \mathbf{F'_d} = \frac{2 \cdot \mathbf{r} \cdot \mathbf{F'_n}}{\mathbf{a}}$$
(5.10)

where F'_d and F'_n are measured per unit length of the contact width.

The normal force F'_n can be expressed as a function of the distance to the leading edge of the contact zone. Bekel (1992) derived the following equation, using the Hertz formulas

$$\mathbf{a} = \sqrt{\frac{8 \cdot \mathbf{F'}_{n} \cdot \mathbf{r} \cdot (1 - v^{2})}{\pi \cdot \mathbf{E}}}$$
(5.11)

where E is the statically measured modulus of elasticity. With this equation the normal force F'_n is eliminated from equation (5.10). To match the stiffness of the brush model, the tangent at the start of the model's traction curve has to match the creep curve described by equation (5.10), which is calculated with

$$\lim_{\delta \to 0} F'_{d} = f_{t} \int_{-b}^{a} \tau_{stick} dy = f_{t} \frac{\delta}{h} \left(\frac{G_{0}}{2} (a+b)^{2} + v_{b} \sum_{i=1}^{n} \eta_{si} \left(a+b-k_{si} \left(1-\exp\left(-\frac{a+b}{k_{si}}\right) \right) \right) \right)$$
(5.12)

where $k_{si} = \frac{\eta_{si} \cdot v_b}{G_i}$ and f_t is the correction factor

Elimination of F'_d by combining equation (5.10), (5.11) and (5.12) gives the following correction factor:

$$f_{t} = \frac{a \cdot \pi \cdot E \cdot h}{4 \cdot (1 - v^{2}) \cdot p},$$

$$p = \frac{G_{0}}{2} (a + b)^{2} + v_{b} \sum_{i=1}^{n} \eta_{si} \left(a + b - k_{si} \left(1 - \exp\left(-\frac{a + b}{k_{si}}\right) \right) \right)$$
(5.13)

The stiffness of the model is compensated by scaling the Maxwell parameter with the factor of equation (5.13).

5.1.3 Experimental validation

Experiments were conducted to measure the actual relationship between traction and slip at a drive station in the E-BS and to validate the presented model. For this purpose a special test installation was constructed. Figure 5.2 shows the top of this installation.



Figure 5.2: Test installation to measure traction forces

The main components of the installation consist of two drive wheels that are each connected to an electric drive motor. One all steel wheel represents the drive wheel that applies a drive force to a second wheel, with a rubber layer vulcanised to it. This 30 mm thick rubber layer represents the belt cover and therefore has the same rubber compound as used in the E-BS. A sub frame supports the drive wheel and is connected to the installation's main frame by a hinge, so the drive wheel can be pulled onto the brake wheel to create an adjustable contact force.

The drive force applied to the rubber layer is controlled by accurately adjusting the speeds of both wheels. Initially the speeds of both wheels are synchronised. In this state no drive force is present. When the drive wheel is slowed down, slip occurs in the contact surface, which results in an increasing drive force. To keep the speed of the brake wheel constant the electric

motor connected to the rubber layered wheel now acts as a brake. For this goal it is equipped with a controller that is capable of running the motor in both the braking and driving quadrant. Figure 5.3 presents a schematic of the test installation with the forces acting on both wheels.



Figure 5.3: Schematic of test installation

The drive force F_d is measured with strain gauges that are placed on the drive motor shaft. These strain gauges directly measure the torque T_d that is applied to the drive wheel. With the radius of the drive wheel known, the traction force is calculated as follows

$$F_{d} = \frac{T_{d}}{r_{d}}$$
(5.14)

Initially strain gauges were also placed on the brake motor shaft to measure the brake torque T_b applied to the rubber layer wheel. As both the brake and drive wheel have an identical diameter, a difference between the measured brake and drive torque would indicate the level of bearing friction in the installation. However, after calibrating both strain gauges it was concluded that the bearing losses were negligibly small because within the measurement resolution of the implemented strain gauges, the measured torques were virtually identical. Therefore, data was only collected from the strain gauges on the drive wheel shaft during the experiments.

An adjustable spring pulls the drive wheel onto the brake wheel to create the normal contact force between the wheels. This contact force is controlled by changing the pretension in the spring, which is measured with a strain gauge located between the spring and the frame of the test installation. As Figure 5.4 illustrates, the hinge between the sub and main frame is not located on the working line of the drive force F_t .

As a result, the normal force F_n is a function of both the measured load cell force and the drive torque T_d and is calculated as follows

$$F_n = \frac{T_d}{L_1} + \frac{L_1 + L_2}{L_1} \cdot F_{\text{loadcell}}$$
(5.15)



Figure 5.4: Forces acting on drive wheel sub frame

The diameters of the drive and brake wheel were chosen such that the contact patch in the test installation is comparable to the patch found at the drive wheel in the E-BS. Using the Hertz formulation (Johnson, 1985) of the contact phenomenon the contact patches are similar if

$$\frac{1}{D} = \frac{1}{D_{d}} + \frac{1}{D_{b}}$$
(5.16)

where D is the E-BS' drive wheel diameter, and D_d and D_b respectively are the drive and brake wheel diameter used in the test installation. With the E-BS' drive wheel diameter equal to 250 mm D_d and D_b were both set to 500 mm. Furthermore, the rubber layer vulcanised to the brake wheel is also given the same curvature r_2 (see Figure 4.1) as found on the E-BS' belt running surface to prevent jamming.

At the start of each experiment the contact force and the drive wheel speed are set to the required test values. To compensate for a decrease in brake wheel diameter due the indentation and wear of the rubber layer, the speed of the brake wheel is adjusted just below synchronous speed until the drive torque reduces to zero. From this point, where the measured traction is zero, a traction slip curve is created by successively decreasing the brake wheel speed and measuring the resulting increase in traction.

Figure 5.5 presents the results for normal contact forces ranging from 500 to 1500 N and a speed of 1.6 m/s together with the results of the presented Maxwell model. The results show that the presented Maxwell model gives a good match with the measured values for low contact forces. As the contact force increases, the model starts to underestimate the actual applied drive force.

Figure 5.5 also shows curves that were calculated with an elastic half space approach as used by Bekel (1992), which he used to describe the traction slip relationship for a wheel driven rubber strip. Bekel (1992) used a similar half space approach as described by Johnson (1985) for a line contact involving completely elastic material, which results in

$$\delta = \frac{\mathbf{a} \cdot \boldsymbol{\mu}}{\mathbf{r}} \cdot \left(1 - \sqrt{1 - \frac{\mathbf{F}_{d}}{\boldsymbol{\mu} \cdot \mathbf{F}_{n}}} \right) \quad \text{, with } \frac{1}{\mathbf{r}} = \frac{1}{\mathbf{r}_{1}} + \frac{1}{\mathbf{r}_{2}}$$
(5.17)

where the contact length a is calculated with equation (5.11). From the graphs presented in Figure 5.5 it is clear that values calculated with (5.17) slightly overestimate the measured values.



Figure 5.5: Comparison of experiments and model ($v_b = 1.6 \text{ m/s}$)

To assess the influence of the viscoelastic properties on traction, different curves were also calculated with the Maxwell model and measured for different speeds. Figure 5.6 presents the calculated (left) and measured (right) results for a contact force of 1000 N. For the calculated curves the speed ranges from the E-BS's standard belt speed of 1.6 m/s to a potential high speed application with a belt speed of 10 m/s. For the measured results the speed is limited to 4 m/s because the test installation cannot run faster than this speed.

The calculated curves on the left of Figure 5.6 suggest that traction decreases with increasing speed, with the greatest reduction occurring in the middle part of the slip range. However, this effect seems very small in the feasible speed range of a belt conveyor. From this it can be concluded that the viscoelastic properties of the rubber have little influence on the relationship between traction and slip. The measured curves on the right of Figure 5.6 also confirm this because they also show small differences that have the same order of magnitude as the measurement error.

The knowledge gained from the relationship between traction and slip will be a valuable asset for the system designer, when choosing the number of drive stations to install in a belt conveyor system like the E-BS. As both traction and slip generated by the drive wheels influence the overall belt wear, the system designer will have to determine the minimum number drive stations that will give an acceptable wear rate or belt life.



Figure 5.6: Traction-slip curves for different speeds with F_n=1000 N

5.1.4 Adding the traction model to the dynamic belt model

A simple traction model was presented in section 3.4 to link the motor model with the dynamic belt model. In this traction model, described by equation (3.27), it is assumed that no slip exists between the drive wheel and the conveyor belt. This makes solving the equation of motion for the drive motor (3.26) a straightforward operation because the motor's rotor speed is directly linked to the belt speed. With the rotor speed known and the induced motor torque calculated with equation (3.23), the drive force F_d can be calculated directly. Although this works well if the drive forces are low, this simple traction model cannot simulate the excessive slippage that occurs when the drive station reaches its friction limit.

To also model this phenomenon, a traction model is presented that includes a similar traction slip curve as measured with the test installation. In this case equation (3.26) also forms the main equation of motion for the drive station. However, the rotor speed has now become a separate state in the system, turning the rotor acceleration in equation (3.26) into a variable that is dependent on both the produced motor torque and the applied traction force. To solve the equation of motion, the drive force is calculated with a traction slip curve that is based on equation (5.17). After calculating the creep ratio δ between the drive wheel and belt surface, the drive force is calculated as follows

$$\begin{cases} F_{d} = \frac{F_{n} \cdot r}{a} \cdot \frac{\delta}{|\delta|} \cdot \left(2 \cdot |\delta| - \delta^{2} \cdot \frac{r}{a \cdot \mu}\right), & |\delta| < \frac{a \cdot \mu}{r} \\ F_{d} = \mu \cdot F_{n} \cdot \frac{\delta}{|\delta|} & , & |\delta| \ge \frac{a \cdot \mu}{r} \end{cases}$$
(5.18)

With this set of equations a drive wheel generates drive or brake forces when the drive wheel is running respectively faster or slower than the belt. When the drive motor produces a torque that is larger than the friction allows, the excess torque will accelerate the drive wheel and cause an increasing wheel slip without an increase in drive force.

To be able to match the traction curve to the measured values presented in Figure 5.5, the first equation in (5.18) is simplified as follows

$$F_{d} = -C_{1} \cdot \delta^{2} + C_{2} \cdot \delta$$
, $C_{1} = F_{n} \cdot \frac{r^{2}}{a^{2} \cdot \mu}$, $C_{2} = F_{n} \cdot \frac{r}{a}$ (5.19)

Table 5.1 presents the parameters for this equation after matching it with the measured values. As example Figure 5.7 illustrates the resulting relationship between the traction and slip when the drive wheel is pushed onto the belt with a normal contact force of 1000 N.

 Table 5.1: Matched parameters of the simplified traction equation



Figure 5.7: Traction curve used in the dynamic belt model

5.2 Traction versus wear

As discussed in the previous section, a small amount of slip or creep always exists when a drive pulley applies a drive force to a conveyor belt. Although it can be expected that this slippage causes abrasive wear of the rubber cover, conveyor system designers and belt manufacturers pay relatively little attention to this effect because the abrasive action of the bulk solid material on the carrying side of the belt presents a far greater problem with regard to the belt's expected life time. However, in multiple driven belt conveyor systems that are driven by drive wheels, like the E-BS, the abrasive wear caused by drive forces becomes a greater issue. Due to the fact that the contact surface between the belt cover and the drive

wheels is much smaller compared to a conveyor system with a drive pulley and the fact that the belt will encounter more drive cycles in a multiple drives system, the abrasive wear caused by the application of a traction force becomes more critical. As little attention has been paid to this effect in the past, hardly any guidelines are available that incorporate the relationship between wear and traction.

To gain a better insight into this phenomenon, the wear mechanism is investigated and a method is presented to determine the minimum number of drive stations in a multiple drives system that are required to be able to reach the belt's guaranteed lifetime. It is based on observations and measurements obtained from the test installation presented in section 5.1.3.

5.2.1 Wear mechanism

During wear experiments, with a drive wheel, from the E-BS, abrasion marks were discovered on the rubber running surface running perpendicular to the direction of movement. This is contrary to the wear pattern commonly found on metals and plastics where scratches caused by abrasive asperities run parallel to the direction of movement. Figure 5.8 illustrates the difference between typical wear pattern found on metal and plastic and that found on the rubber running surface.



Figure 5.8: Different abrasion pattern for rubber

Closer inspection of the pattern on the worn rubber surface shows that the marks are regularly spaced ridges. Figure 5.9 shows an idealised representation of the abrasion pattern with ridges that are undercut at their base and that face the direction of travel. Although little is known about this deviating wear pattern of rubber in the field of belt conveying systems, it has already been encountered in other applications [Southern and Thomas, 1979] [Pulford, 1985] [Grosch, 1992].

The mechanism illustrated by Figure 5.9 presents the main idea behind a proposed model [Champ et al., 1974] that simplifies the complex abrasion process and accounts for the quantitative behaviour found in experimental observations. In this model a link is also made between abrasion and crack growth. Despite the fact that a razor blade is chosen as the abrading device in this model, the abrasion patterns closely resemble those produced by a multi-asperity surface. Therefore, it should also be applicable to the profiled surface of the drive wheels found in the E-BS.



Figure 5.9: Abrasion pattern and wear process

The pattern is thought to originate from stick- slip and sliding [Schallamach, 1952] and once formed it becomes a major source of rubber loss. As a ridge passes the razor blade abrader in this model, the traction force peels it back, causing the crack at the base to grow with a distance dc and at an angle θ_E . After a number of load cycles a ridge will break off through mechanical fatigue, which creates debris on the surface in the form of small rubber particles. This process results in a self-perpetuating pattern that moves opposite to the direction of the rubber surface.

The fracture mechanics approach used in the proposed model is based on the tearing energy T_E , which is the mechanical energy released when the crack grows in increments of dc. Under the assumption that the razor blade abrasive peels the ridges back in such a way that the traction force F_d is applied directly to the crack tip, the tearing energy is given by [Southern and Thomas, 1979]

$$T_{E} = \frac{F_{d}}{W} \cdot \left(1 + \cos \theta_{E}\right)$$
(5.20)

where w is the width to which the drive force F_d is applied. From studies into the crack growth behaviour of rubbers under repeated stressing the following relationship between the crack growth per load cycle and the energy release rate was found [Southern and Thomas, 1979]

$$dc = B_{\rm F} \cdot T_{\rm F}^{\ \alpha_{\rm E}} \tag{5.21}$$

where constant B_E and the exponential α_E are empirical constants that are related to the fatigue properties of the rubber sample. With this expression for the crack growth it is possible to determine the wear depth h_w per cycle as follows

$$h_{w} = dc \cdot \sin \theta_{E}$$
 (5.22)

By combining equations (5.20), (5.21), and (5.22) the wear depth is expressed as a function of the applied traction force F_t as follows

$$\mathbf{h}_{w} = \mathbf{B}_{E} \cdot \sin \theta_{E} \cdot \left(\frac{\mathbf{F}_{d}}{\mathbf{w}} \cdot \left(1 + \cos \theta_{E} \right) \right)^{\alpha_{E}}$$
(5.23)

Due to the fact that this mechanism also applies to multiple asperity abrasives, like the profiled drive wheel of the E-BS, this expression is used to determine the minimum required amount of drive station in the E-BS.

5.2.2 Wear experiments

To be able to use equation (5.23), wear experiments were conducted with the test installation presented in section 5.1.3. The wear rate was determined by letting the installation run for longer periods of time at a set speed and with a constant drive force. To determine the actual wear, the layer thickness was measured before and after each test run with the wheel stationary and the rubber at the ambient temperature. During each run sensors continually measured the wheel speeds, motor torques, rubber surface temperature and the force applied by the spring and the build up of rubber particles resulting from the wear process was removed periodically.

The results of the experiments for two different normal force setting are presented in Figure 5.10.



Figure 5.10: Results of wear tests

The markers represent the measured wear rates, while the solid lines represent a match with the following simplified form of equation (5.23)

$$h_w = C_E \cdot F_d^{\alpha_E}$$
, where $C_E = B_E \cdot \sin \theta_E \cdot \left(\frac{1 + \cos \theta_E}{w}\right)^{\alpha_E}$ (5.24)

With the wear rate expressed in mm per million cycles, Table 5.2 presents the parameters that correspond with the solid lines in Figure 5.10. The results show that the exponent α_E varies little when the normal contact force F_n is changed. It was also expected that this parameter would remain the same as only one type of rubber compound was used. According to Southern and Thomas (1979) α_E varies from about 2 for natural rubber to 4 or more for non crystallising unfilled rubbers.

| F _n [N] | $\alpha_{\rm E}$ | $C_{\rm E}$ |
|--------------------|------------------|-----------------------|
| 1000 | 4.06 | 4.0·10 ⁻¹¹ |
| 1500 | 4.56 | 8.0·10 ⁻¹³ |

Table 5.2: Measured wear parameters

In contrast, the parameter C_E varies significantly when the normal force is altered. As C_E is also a function of B_E and θ_E and the contact width w is constant, this suggests that the parameters B_E and /or θ_E are dependent on the contact force. This is possibly caused by the increase in contact surface when the normal force is increased.

5.2.3 Required number of drive stations

With the relationship between traction and wear determined by equation (5.24), it is possible to make an estimation of the overall belt wear. As equation (5.24) returns the depth that wears of each time the belt passes a drive stations the total wear can be estimated by

$$\mathbf{h}_{\text{tot}} = \mathbf{n}_{\text{cycle}} \cdot \mathbf{h}_{\text{w}}$$
(5.25)

where n_{cycle} represents the total number of times the belt has passed a drive station. With the operational belt speed v_b , the total operational time t_{op} , the belt length l_b , and the number of drive station n_{drive} , the total number of cycles is calculated as follows

$$n_{cylce} = \frac{v_b \cdot t_{op}}{l_b} \cdot n_{drive}$$
(5.26)

By combining equation (5.25) with, equation (5.24) and (5.26) the total wear is obtained as follows

$$h_{tot} = \frac{v_b \cdot t_{op}}{l_b} \cdot n_{drive} \cdot C_E \cdot \left(\frac{F_{tot}}{2 \cdot n_{drive}}\right)^{\alpha_E}$$
(5.27)

where F_{tot} is the total mechanical resistance force in the belt conveyor system. Note that F_{tot} is divided by twice the number of drive stations because in the E-BS each drive station consists of two motors that act as a pair driving the belt on either side of the pouch.

To illustrate how this expression can be used to determine the minimum number of drive stations, Figure 5.11 presents the results of a wear rate calculation for a test E-BS layout. The overall belt length of this system is 240 m and when running fully loaded at an operation speed of 2.5 m/s it has an estimated resistance of 7.5 kN. The results are generated by assuming that the system will run at full capacity for 46 weeks a year and 40 hours a week, using the experimentally determined wear parameters.

With this graph it can be concluded that in order to guarantee that the belt's running surface does not wear more than 5 mm after 5 years (or 1 mm per year) at least 10 drive stations are required in the current situation, where the drive wheels are pressed onto the belt by a contact force of 1000 N. By increasing the contact force to 1500 N with a clamping roller, it is possible to reduce the drive station count to 8.



Figure 5.11: Predicted annual belt wear versus the number of drive stations

For the E-BS belt with a width of 800 mm the maximum allowable tension during normal operation is 25 kN (see section 3.1.1). With a pretension force of 1.5 kN present in the belt, this would implicate that a maximum drive force of about 23 kN could be applied by each station. However, when taking the maximum allowable wear into account it is clear from the presented test case that the drive force per station is limited to about 1 kN, which is far smaller than the possible 23 kN. So for the E-BS the belt wear presents a far greater limit on the maximum allowable drive force than the limit imposed by the belt strength.

With the model presented in this chapter it is possible to predict the relationship between both traction and slip, and between traction and wear. This makes it possible to predict the interaction between the drive wheels and the belt surface and to estimate the minimum required number of drive stations that will guarantee the belts lifetime in a multiple driven system configuration.

6 Dynamics of multiple driven belt conveyors

During starting and stopping procedures of a belt conveyor system, the belt does not respond instantaneously to the changing drive forces. Instead, a delayed reaction takes place due to the belt's flexibility and distributed mass. Acceleration waves travel outward from each drive location when the drive force changes, causing longitudinal oscillations and additional dynamic stresses in the system. To keep the conveyor belt's response smooth and to keep the peaks in stress within the safety margins, guidelines and dynamic models have been developed for single drive belt conveyor systems that help to prescribe the shape of speed profiles, and the minimum transition time for speed changes [Funke, 1973][Harrison, 1983][Nordell, 1985][Singh, 1994]. However, the question is if these guidelines also apply to multiple driven belt conveyor systems. To investigate the dynamic response in the multiple drive case a dynamic model has been developed that combines the existing dynamic belt and motor model, discussed in chapter 3, with the modified models for calculating friction and traction forces from chapters 4 and 5. The resulting combined system model is described in appendix A. In section 6.1 this model is used to compare the dynamic response of a single drive belt conveyor system with that of a multiple driven system and find the main differences and possible similarities between both drive schemes. With the knowledge gained from this comparison different possibilities are explored in section 6.2 to improve the dynamic response and the effect of the bulk solid material travelling on the belt is analysed in section 6.3. Finally, the effects of braking presented in section 6.4.

6.1 Comparison of belt behaviour during starting

As already discussed above, sufficient knowledge is currently available for system designers to successfully develop state of the art large scale belt conveyor systems in a single or dual drive configuration. However, for large scale multiple drive configurations system designers have little information to work with. This is especially true for systems that inherently feature relatively close drive station spacing, like the E-BS for example. For this multiple driven belt conveyor the current challenge is to increase the system length beyond the 1 km mark. Up until now a maximum system length of 500 m has been achieved. When the E-BS is extended beyond 1 km, different challenges have to be faced as the overall system complexity increases and belt dynamics start to play a more dominant role during starting and stopping procedures. As more knowledge is available with regard to the starting and stopping behaviour of single

drive belt conveyors then for multiple driven types a comparison is made between the dynamic belt behaviour of single and multiple driven systems, using the E-BS as test case. The goal is to investigate in which areas the standards and guidelines developed for the starting of single drive systems are extendable to multiple driven systems and in which areas further research is required. For the comparison the start up response of a single driven belt conveyor system is analysed first, followed by that of a multiple driven system with a comparable layout.

6.1.1 Single drive belt conveyor

Figure 6.1 presents the single drive layout that will be used as a base for the comparison. It is a straight belt conveyor system with a drive station located at the head of the system and a tensioning device located in the return strand near the head pulley. A tension weight in the tensioning devices provides the required pretension in the belt. This layout is comparable to that of a conventional straight and level overland belt conveyor, which does not negotiate height variations.



Figure 6.1: Single drive configuration

Normally such a system would be driven by a powered head pulley. However, as the multiple driven layout that will be compared with the single drive layout is based on the E-BS, the belt and drive properties of the single drive layout will also be based on of the E-BS. This makes it possible to directly compare the results of both layout types. Therefore, an E-BS type drive station is presumed in the modelled single drive layout instead of the more conventional drive pulley.

As presented on the right side of Figure 6.1 and described in chapter 3, the E-BS type drive station consists of two drive wheels that are pressed onto the belt's surface to generate the friction based drive force. This implies that the maximum applicable drive force is independent of the slack side tension of the pulley, as defined by the Euler-Eytelwein equation (1.1). Only the friction coefficient and the normal force acting between the drive wheel and the belt determine the maximum applicable drive force.

The belt properties of the single drive layout are also based on the E-BS. Table 6.1 lists the properties for the both belt and drive system. These parameters are based on a pilot installation that was constructed in Almere, the Netherlands, see Figure 6.2 [Gielisse, 2002][Twaalfhoven, 2004]. The purpose of this installation was to test, further develop and present the E-BS concept. The test installation has a total belt length of 240 meters and is powered by 6 drive stations, each consisting of two 3 kW AC-motors. As a result, the average

distance between the drive stations is equal to 40 m. The system requires a relatively large number of drive stations due to the fact that the secondary resistances play a dominant in the relatively short belt conveyor system [Twaalfhoven, 2004].

| Belt parameters | | | Drive station parameters | | |
|---------------------------|-----------------------|----------|--------------------------|----------------|------------------|
| Parameter | Symbol | Value | Parameter | Symbol | Value |
| Belt stiffness | Е | 290 MPa | Nominal motor power | Pn | 3 kW |
| Belt width | W | 800 mm | Nominal phase voltage | Un | 230 V |
| Thickness | Т | 6.2 mm | Nominal phase current | In | 6.2 A |
| Empty belt mass | m _e | 11 kg/m | Nominal frequency | f_n | 50 Hz |
| Full belt mass | m' _f | 56 kg/m | Number of pole pairs | Р | 2 |
| Reduced idler mass | m'r | 3.5 kg/m | Nominal speed | N | 1400 rpm |
| Tension weight mass | m' _t | 300 kg | Drive wheel diameter | d _w | 250 mm |
| Static resistance factor | $\alpha_0 + \alpha_1$ | 0.005 | Gear reduction | Ι | 4.665 |
| Dynamic resistance factor | α_1 | 0.004 | Traction constant 1 | C ₁ | $4.0 \cdot 10^4$ |
| Viscous resistance factor | α ₂ | 0.00275 | Traction constant 2 | C ₂ | $1.2 \cdot 10^4$ |

Table 6.1: System parameters for the test cases



Figure 6.2: E-BS test installation (Source: Spaans Bulk Handling Systems)

A larger motor spacing is used for the dynamic belt simulations of both single and multiple driven layouts. This is done for two reasons. Firstly, a large scale multiple driven system with a belt length beyond the 1 km mark would require a vast number of drive motors, making it a complex and expensive system. Therefore, to reduce complexity and increase competitiveness with conventional large scale belt conveyor systems, the belt will have to bridge larger distances between successive drive stations. For a large scale E-BS it is expected that this will be possible because a longer system will also have significantly longer straight sections compared to the investigated E-BS test installation. As the rolling resistance of a straight belt section is significantly lower then the secondary motion resistance, the average drive station distance will increase as the total system length increases.

An estimation of the maximum motor spacing can be determined by rewriting the formula for the main resistance given by the DIN 22101 standard, see equation (3.6). If a level conveyor

belt section is assumed and the available drive force is set equal to the main resistance this equation can be used as follows

$$l_{i} = \frac{F_{M,i}}{f_{i} \cdot g \cdot (m'_{r,i} + m'_{b} + m'_{l,i})}$$
(6.1)

With a drive force of 750 N per station, taken from the wear calculation from section 5.3.2 and the measured rolling resistance factor f_i of 0.015, calculated in section 4.1.5, for example, it should be possible to bridge a maximum distance of about 110 m between drive stations for a fully loaded belt strand and about 340 m for an empty return strand.

Secondly, with a distance of 40 m the dynamic belt response would be negligible during starting and stopping procedures. Even with the lowest longitudinal wave propagation speed of 170 m/s, calculated in section 3.2, the time required for a wave to travel between drive stations or from head to tail, in the single drive case, is only 0.24 s. With this information and existing starting rules, often applied to single drive belt conveyor systems, an approximation can be made of the required start up time. One such rule states that the start-up time should at least be five times the time it takes a longitudinal acceleration wave to travel from head to tail [Lodewijks, 1996] or

$$T_a \ge 5 \cdot \frac{L_{conv}}{c_1}$$
(6.2)

This leads to an approximated start-up time of 1.2 s. In practice the 'one minute per km of conveyor length' is also often applied, which usually results in a less steep acceleration profile during starting. Even in this case the recommended start-up time for a 40 m section would only be 2.4 s. This is very small compared to the starting periods usually applied to large scale system, which are more in the order of minutes rather than seconds. Therefore, it is expected that a significantly longer start-up time would be implemented in practice then the minimum time suggested by the existing start-up rules for a system with a motor spacing of 40 m. This would make the dynamic behaviour a less dominant factor during starting and stopping procedures.

To get a realistic motor spacing for a large scale system with relevant dynamics, the belt length of the single drive belt conveyor, shown in Figure 6.1, is set to 250 m, producing a system with a centre to centre distance L_{conv} of 125 m. In this case a single E-BS style drive station provides the required drive power. Initially, the belt is considered empty, which keeps the system simple and makes the initial comparison straightforward.

Besides the belt length of 250 m, a single drive system with an overall belt length of 1,000 m is also simulated to analyse scale effects. In this configuration the drive station can deliver four times the amount of power and traction of a single E-BS style drive station. This single large drive station is comparable to four very closely spaced E-BS drive stations. Multiple drive stations would be required to guarantee sufficient contact surface between the belt and the drive wheels and to generate the required drive force without excessive slippage occurring between the belt and the drive wheels.

During the starting procedure the belt speed is ramped up gradually to reduce belt jerk, which is the second derivative of the belt speed with respect to time. This prevents excessive belt oscillations and limits the transient belt tension. Both Harrison (1983) and Nordell (1985) recommend speed based profiles for the starting procedure, based on simulations and experimental verification. Both profiles are very similar in shape, although small variations in acceleration occur at different points in the proposed curves. While Harrison's speed profile, using a cosine function, shows slightly higher acceleration at the start and end of the starting procedure, Nordell's profile, using second order polynomials, has a higher acceleration at the halfway point. In this section Harrison's profile is chosen for the simulated start-up of the modelled single and multiple driven belt conveyor system because for single drive systems it has been proven that it gives better results [Lodewijks, 1996]. In this case the belt speed v_b is ramped up during starting time T_a as follows

$$v_{b}(t) = \frac{v_{b,t}}{2} \left(1 - \cos\left(\frac{\pi \cdot t}{T_{a}}\right) \right), \quad 0 \le t \le T_{a}$$
(6.3)

where $v_{b,t}$ represents the target speed. In section 6.2.1 variants of both Harrison's and Nordell's profiles are compared.

For the simulations the belt speed profile is converted to a ramp up curve for the AC drive motor's supply frequency. This is required because the simulated belt conveyor system is controlled in an open loop manner. In this simple control strategy, often applied to belt conveyor systems, the synchronous speed of the drive motor increases linearly with the applied supply frequency. Due to the stiff torque-slip characteristic of the AC drive motor and the gradual speed increase, the drive motor will closely follow the synchronous speed. Figure 6.3 presents the resulting starting profiles for the 250 m (dashed line) and 1,000 m (solid line) belt lengths with a start up time of 30 and 60 seconds respectively.



Figure 6.3: Starting curves with start time 30 (dashed) and 60 (solid) seconds

Initially, the start up times for both system lengths were chosen according to the 60 seconds per 1 km of belt length rule. For the 250 m system this results in a recommended start time of

15 seconds. For the 1,000 m system this is 60 seconds. However, when a start up time of 15 seconds is applied to the 250 m modelled conveyor system, the friction limit of the drive wheels is reached during the simulation. Figure 6.4 presents the results of this simulation. The drive force applied by the drive station is shown on the left and the belt acceleration at the drive station on the right. Based on the traction slip curve for a normal force of 1,000 N, measured in section 5.1.3, each drive wheel can apply a maximum drive force of approximately 850 N to the belt. With two drive wheels present in each drive station the maximum applicable drive force per station becomes 1.7 kN.

Figure 6.4 shows that during the execution of the 15 seconds start up profile the drive station reaches its traction limit after 5 seconds. As this causes excessive wheel slippage and uncontrolled acceleration waves to travel through the system, the start up time for the 250 m belt conveyor system is increased to 30 seconds. Also note that the system is not able to reach the end speed with in the 15 second start up period. This shows that for short belt lengths the minimum required start up time is not primarily determined by the dynamics of the conveyor belt because other factors such as the maximum applicable traction may come in to play for short start up times.



Figure 6.4: Start up of a 0.25 km belt with a single drive station in 15 s

Figure 6.5 illustrates what happens when start up time is increased to 30 seconds. In this case the diagram on the left shows that the drive force stays below the friction limit. As a result,

the start up behaviour is improved, giving a smooth transition of the belt tension. This can be seen in the diagram on the right of Figure 6.5 that presents the belt tension at two different locations.

The solid line represents the belt tension measured just before the belt passes the drive station, while the dashed line represents the belt tension after it has passed the drive station. In effect the latter location is positioned directly between the drive station and the gravity take up device. At this location the belt tension only deviates from the pre-tensioning force due to the up- and downward acceleration of the tensioning weight and hysteresis in the take up device can actually be $\pm 10\%$ of the tension in reality.



Figure 6.5: Start up of 0.25 km belt with a single drive station in 30 s

When a start up time of 30 seconds is applied no distinguishable oscillations occur. Both the drive force and belt tension roughly follow the belt's start up acceleration profile. To see what happens when the system length is increased, the single drive system with a belt length of 1 km is simulated with the start up time of 60 seconds. Figure 6.6 presents the results from this simulation. In this case the 60 seconds per kilometre start up rule does not cause the drive wheels to exceed the friction limit, as can be seen on the left of the figure.

Note that in this configuration four E-BS style drive stations are assumed to be present at the drive location. Consequently, the friction limit is four times as high as the previous configuration. In this case a maximum drive force of 6.8 kN can be applied to the belt. In the diagram the total drive force stays just below 3.3 kN, which is well below the friction limit.



Figure 6.6: Start up of 1 km belt with a single drive station

Although the friction limit is not reached, oscillations are present in both diagrams, causing an undesirable increase of the applied drive force and belt tension. This is a result of the belt dynamics that play a more dominant role in this case. To give an idea of the dynamics involved, a comparison is made with a simpler model where the belt is considered to be a rigid element. If the belt does not flex, it can be considered as a lumped mass. Therefore, the drive force for the rigid model can be calculated as follows during the start up

$$F_{d} = l_{b} \cdot \left(m'_{b} + m'_{r}\right) \cdot \left(g \cdot \left(\alpha_{0} + \alpha_{1} + v_{b} \cdot \alpha_{2}\right) + \frac{\pi \cdot v_{b,t}}{2 \cdot T_{a}} \cdot \sin\left(\frac{\pi}{T_{a}} \cdot t\right)\right)$$
(6.4)

Figure 6.7 shows the comparison between the rigid and flexible belt model. The greatest noticeable difference is located at the beginning of the start up procedure. While the rigid model directly applies a drive force to the belt, this is not the case for the flexible model. This is a result of the motor model that is present in the flexible model rather then the belt's flexibility because the modelled induction motor hardly delivers any torque at low supply frequencies. As the supply frequency is ramped up the motor torque increases rapidly, causing the applied drive force to overshoot the value calculated with equation (6.4).



Figure 6.7: Comparison of flexible (solid) and rigid (dashed) 1 km belt model

After the initial overshoot the flexibility and distributed mass of the belt cause the drive force to oscillate around the start up profile of the rigid model. Consequently, the drive force peaks slightly higher as the oscillation does not dampen out that fast. In this case the small increase in belt tension is not a cause for concern and the start up time of 60 seconds can be considered acceptable.

6.1.2 Multiple drives belt conveyor

To compare the belt behaviour in a single and multiple driven belt conveyor system, the single drive configurations, discussed in the previous section, are expanded both in length and in drive station count. The overall belt length is increased, while keeping the motor spacing constant by adding more drive stations. For a good comparison the motor spacing of 250 m and 1000 m are taken from the previously discussed examples. Figure 6.8 presents the multiple driven belt conveyor configurations with a motor spacing of 250 m.



Figure 6.8: Belt conveyor system with 16 and 32 motors spaced at 250 m intervals

In the first case 16 drive stations power a conveyor belt with an overall length of 4000 m. In the second case both the belt length and the number of drive stations have been doubled, creating a system with an overall belt length of 8000 m with 32 drive stations. These two different belt lengths are also used to analyse the scalability of a multiple driven system.

Note that the same amount of power is installed per drive station as in the single drive case and an empty belt is considered. Therefore, the multiple driven configurations can be seen as a number of linked single drive systems, but with a single gravity take up device. Due to this analogy, the 30 second start up time from the single drive case, is also used for the multiple driven configuration shown in Figure 6.8. Figure 6.9 presents the results of the start up procedure of the configuration with 16 drive stations after it was simulated with the dynamic belt model.



Figure 6.9: Start up of a 4 km belt with 16 drive stations

Looking at the figure on the left the drive forces applied by each drive station generally follows a similar profile. At the start of the graph an initial overshoot can be distinguished, similar to the single drive case that dampens out directly. After this overshoot the acceleration profile, imposed by the s-shaped starting curve, mainly governs the magnitude of the drive force. Although the drive force profiles generally look the same, the drive stations do not deliver the same amount of work. The first drive station located after the tensioning device applies the most drive power, while the last drive station at the head of the system applies the least drive power. The initial overshoot is also much more apparent for the first drive station than the last station. After the 30 second start procedure the drive forces equalise, indicating that the drive stations equally share the load when the belt conveyor system operates at a constant speed.

The imbalance in drive force between the drive stations is caused by the fact that during acceleration phase each drive station does not accelerate the same amount of belt length. Near the centre of the system drive stations receive assistance from the drive stations positioned directly next to them. Consequently, each drive station accelerates a section of belt that is equal in length to the drive station spacing. The situation is different for stations D₁ and D₁₆ located near the gravity take up device because the tensioning device reflects oncoming acceleration waves. Due to this phenomenon they only receive assistance from one direction. As a result, drive station D₁ has to accelerate the complete section between it and the tensioning device and half of the section to station D₂, while drive station D₁₆ only has to

accelerate half of the belt length between it and station D_{15} . The deviant behaviour of D_1 and D_{16} also affects the graphs of neighbouring drive station. The effect lessens the more the drive stations are located towards the centre.

The solid lines in the right diagram of Figure 6.9 reveal a similar profile shape for the belt tension T_1 on the tight side of the drive wheels. As can be expected from the fact that the first drive station delivers the most work, the highest tight side tension occurs near this drive station. For drive stations located further away from the tensioning device the tight side tension reduces. However, towards the head of the system the tension increases again, but it is still below the values found at the first drive station.

To get a better idea of the tension distribution during the start up procedure Figure 6.10 presents the stress along the belt after 20 seconds have elapsed. At this point in time the belt stress at the tensioning device is approximately 2 N/mm. It is also possible to see that D_1 is applying a higher drive force than D_{16} because the fall in belt stress at D_1 is greater than the one at D_{16} . The higher drive force applied by the drive stations near D_1 and the lower drive force applied by those located near D_{16} cause a dip in belt tension towards the centre of the system. This explains why the belt's tight side tension is higher near the tensioning device than towards the centre of the system.



Figure 6.10: Stress along the 4 km belt with 16 drive stations after 20 seconds

Looking at the graph with the highest belt tension, it can be concluded that the belt section between the tensioning device and the first drive station has the most dominant belt behaviour. Furthermore, this behaviour is very similar to that found in the single drive case, see Figure 6.5.

The dashed lines in the right diagram of Figure 6.9 show that the low tensions T_2 found on the slack side of the drive wheels fall below the pretension force. This is a result of the fact that the tensioning device can only guarantee a minimum belt tension for the drive station located at the head of the system. As the belt is pulled on one side and pushed forward on the other side of each drive wheel, the belt tension falls below the pretension force at the drive stations located away from the tensioning device. This phenomenon can also occur in single drive decline belt conveyor systems, where the drive station is positioned at the tail while the tensioning device is located at the head of the system. Care should be taken to prevent the belt

tension from becoming too low, which could lead to excessive belt sag or in the case of the E-BS to the opening of the pouch shape and spillage of the bulk solid material.

Figure 6.11 presents the results when both the belt length and the motor count are doubled. Both diagrams for the applied drive force and the tight side belt stress are very similar to the previous results. Near the tensioning device the response of the drive stations and the belt remain virtually unaltered. The lengthening of the system only produces more diagrams that are comparable to the profiles produced by the drive stations located towards the centre of the system. In the left diagram of Figure 6.11 the start profiles of the applied drive station are much denser in the centre of the area occupied by the graphs. In the right diagram the belt stress graphs for the tight side of the drive wheels are much denser at the lower end of the graph band. From this observation it can be concluded that the presence of the gravity take up device causes an end effect that influences the behaviour of a number of drive stations. In this case the effect is measurable at about 8 drive stations from the tensioning device. The remaining drive stations all react very similar to each other.



Figure 6.11: Start up of an 8 km belt with 32 drive stations

The right diagram in Figure 6.11 shows that the slack side tension in the longer system falls further below the pretension force for a longer period of time. This occurs at the drive stations located in the centre section of the system. At these drive stations the slack side belt stress graph is similar to the one of the tight side stress when it is mirrored with respect to the pretension force.

Figure 6.12 presents the distribution of the belt stress for the system with 32 drive stations at 20 seconds. It also shows the end effect near the tensioning device. In the centre section the average belt tension levels out towards the pretension force. Looking at this result, it is expected that a further increase in system length will lead to a tension distribution with a longer centre section, while the section influenced by the end effects remains the same.



Figure 6.12: Stress along the 8 km belt with 32 drive stations after 20 seconds

Like the single drive system with a belt length of 250 m both multiple driven configurations of 4 km and 8 km show little dynamic belt behaviour. There is an initial overshoot, but it dampens out straightaway. To analyse a multiple driven belt conveyor system, where belt dynamics are more pronounced, the drive motor spacing is increased to 1000 m. This is accomplished in the same manner as done with the single drive system that was increased from a belt length of 250 m to 1000 m. Each drive station now has 4 times the installed power compared to the previous multiple driven systems. As a result, the systems with a belt length of 4 km and 8 km now only require 4 and 8 drive stations respectively, where each drive station is equipped with 4 E-BS style drive motor pairs. Figure 6.13 illustrates the altered configurations.



Figure 6.13: Belt conveyor system with 4 and 8 motors spaced at 1000 m intervals

The start up procedure was simulated for both configurations, using the same belt properties as in the previous cases. Similar to the simulated single drive system with a belt length of 1 km a start up time of 60 seconds was implemented. Figure 6.14 presents the results for the first configuration with 4 drive stations.



Figure 6.14: Start up of a 4 km belt with 4 drive stations

The diagrams for the drive force and belt stress show that oscillations mainly occur at drive station D_1 , which is the first drive station the belt passes after leaving the tensioning device. As also seen with the multiple driven configurations with a drive station spacing of 250 m, station D_1 applies the highest drive force and it produces the highest tight side belt tension near its drive wheels. Some oscillations are also noticeable at the other drive stations, but these are not as pronounced. From this observation it is concluded that the most dominant dynamic behaviour occurs in the section between the tensioning device and the first drive station. As a result, the dynamics in this section will determine the start up time that is still acceptable.

If the dynamic behaviour of this section is compared with the single drive case with a belt length of 1000 m, it is apparent that the tight side belt tension approximately follows the same profile. The oscillation is slightly larger in the multiple driven case, but the peak stress is about the same. A bigger difference occurs when the applied drive force is considered. In the multiple driven case the peak drive force is noticeably higher. Consequently, it is possible to approximate the dominant belt behaviour of a multiple driven system with a model of a single drive system when the single drive belt length is equal to implemented drive station spacing. This will give an acceptable prediction of the peak belt stress. However, care should be taken when a single drive belt model is used to predict the peak drive force because it will be underestimated.

The right diagram in Figure 6.14 also shows lower minimum belt tension compared to the results of the 4 km system with a motor spacing of 250 m, see Figure 6.9. In this case the minimum belt tension falls far below the initial pretension force. The situation becomes even more critical when the system length is increased and more drive stations are added.
Figure 6.15 presents the results for the system configuration where the belt length is increased to 8 km and 4 more drive stations are added. The increase in system scale leads to the same effects as seen with the multiple driven configurations with a motor spacing of 250 m. In the drive force diagram more graphs appear in between the profiles generated by the first and last drive stations. In the right diagram more graphs appear at the lower end graph range for both the slack and tight side belt stresses. As the end effects have become less influential at the centre of the system, the minimum belt tension falls even further than observed in the 4 km belt configuration. Therefore, special attention should be paid to this phenomenon when a multiple driven belt configuration with a relatively large drive station spacing is scaled up in length.



Figure 6.15: Start up of an 8 km belt with 8 drive stations

6.2 Optimising starting behaviour

From the previous section it is clear that in a multiple driven belt conveyor system the most dominant belt behaviour occurs in the belt section between the tensioning device and the first drive station. In this section different possibilities are analysed to improve this dominant belt response and optimise the starting behaviour. This is accomplished by changing the pre described start up profile, starting the drive stations in sequence rather then all at the same time and altering the locations of the drive stations.

6.2.1 Start curve shape

In the previous section only one type of single start up of curve was implemented based on Harrison's (1983) start up profile. With this profile the acceleration changes continually, creating a smooth transition to the belt conveyor's operational speed and at the halfway point of the start up procedure the acceleration reaches its highest peak. This peak in acceleration also causes a peak in drive force and belt tension at about the same time.

To analyse whether it is possible to reduce these peak values, Harrison's profile is altered and split into three parts. In the first part speed is ramped up smoothly with a cosine function. This is followed by the second part where the acceleration remains constant. In the last part the acceleration is smoothly ramped down again. In Harrison's case the first part is directly

followed by the last part at the halfway point of the start procedure. In the altered case the formulas for each part of the speed profile are setup in such a manner that the duration of the acceleration smooth in and out can be altered. This leads to the following equations for the ramp up of the stator supply frequency f_s

$$f_{s}(t) = \begin{cases} c_{1} \left(1 - \cos\left(\frac{t}{2 \cdot t_{1}} \pi\right) \right) &, & 0 \le t < t_{1} \\ c_{2}t + c_{3} &, & t_{1} \le t \le t_{2} \\ c_{4} - c_{5} \cos\left(\frac{T_{a} - t}{2 \cdot (T_{a} - t_{2})}\right), & t_{2} \le t < T_{a} \end{cases}$$
(6.5)

where t_1 represents the smooth in time and t_2 indicates at which point in time the smooth out starts. The values of constants c_1 to c_5 depend on both t_1 and t_2 and they are calculated based on the fact that the acceleration and speed profile at t_1 and t_2 has to be a continuous function and that at T_a the speed profile has to reach the target speed. This leads to a set of 5 equations that can be solved linearly.

To investigate the effect of the start curve's shape, a similar approach is applied to Nordell's (1985) start up profile. This second order polynomial used to describe this start up curve in this case is also split into three parts, which results in the following set of equations

$$f_{s}(t) = \begin{cases} c_{1}t^{2} & , & 0 \le t < t_{1} \\ c_{2}t + c_{3} & , & t_{1} \le t \le t_{2} \\ c_{4}t^{2} + c_{5}t + c_{6} & , & t_{2} \le t < T_{a} \end{cases}$$
(6.6)

This set of equations requires an additional condition to find the appropriate values for the constants c_1 to c_6 . This condition states that the acceleration should be zero at the end of the start up procedure. Note that when t_1 and t_2 are both set to $\frac{1}{2}T_a$ the original Nordell curve is revealed and when t_1 is set to zero and t_2 to T_a a linear curve results.

Figure 6.16 illustrates a number of examples that are produced when different values are chosen for t_1 and t_2 in equation set (6.5) and (6.6). The solid line in the left diagram represents Harrison's original profile, while the one in the right diagram represents Nordell's profile. Also note that both original profiles have a steeper maximum gradient than the derived curves of which the linear curve has the smallest gradient. This indicates that the original curves have the highest peak acceleration. In both diagrams a start up time of 60 seconds is employed because the multiple driven configuration with a belt length of 4 km and a drive spacing of 1 km is used to simulate the effect of the different start up profiles.

The start up of the 4 km belt was simulated with t_1 and t_2 ranging from 0 to 60 seconds where t_2 is always larger than t_1 . For each simulated result the minimum and maximum belt stress values were picked out and stored together with the peak belt acceleration and the applied drive force.



Figure 6.16: Start curve shapes with a cosine (left) and polynomial (right)

Figure 6.17 presents the result for the Harrison based cosine start up profiles. In this figure the horizontal axis of each diagram indicates the magnitude of the smooth in time t_1 , while the vertical axis indicates the smooth out time, which is equal to the time interval between t_2 and T_a .



Figure 6.17: Effect of varying smooth in and out time for a cosine based start profile

The lowest peak acceleration occurs when the smooth in time is maximised, indicated by the dark area on the right side of the maximum acceleration diagram. This is a result of the fact that a more gradual ramp up at the beginning of the start up reduces the initial overshoot,

which occurs when the stator frequency reaches the value where the drive motors overcome the static friction. This effect is also favoured when the minimum belt tension is considered because it drops the least when the smooth in time is large. However, this is not the case when the maximum drive force and belt stress are considered. The bottom diagrams in Figure 6.17 show that the lowest peaks for these quantities occur when the length of the smooth in and out time are balanced. This is indicated by the dark areas that occur near the line where the smooth in and out time are equal. Consequently, Harrison's original start up profile, where both t_1 and t_2 are equal to 30 seconds, generates one of the most desirable system responses.

Figure 6.18 presents similar results for the polynomial start up profiles. However, in this case the diagrams show a slightly higher maximum acceleration, drive force and belt stress. This is a result of the fact that the polynomial based profile curve has a higher peak acceleration compared to the curves derived from Harrison's start up profile. Further, Figure 6.18 also shows that the most optimal situation with regard to the belt acceleration and minimum belt tension does not occur at the maximum smooth in time. In this case the optimal point lies somewhere in between a smooth in time of 35 and 40 seconds. Looking at the maximum drive force and belt tension the lowest values are also found for a balanced smooth in and out time, but now the darker area has shifted more towards the point where both times are shorter. As a result, the polynomial defined by Nordell, where t_1 and t_2 are both set to 30 seconds, does not produce the most optimal system response in this case. The system response improves when a smooth in time of approximately 20 seconds is chosen together with a smooth out time of 10 seconds. Note that these settings depend on the characteristics of the system and the chosen start up time. A different selection of the smooth in and out time may produce better results for other cases.



Figure 6.18: Effect of varying smooth in and out time for a polynomial start profile

From the presented results it can be concluded that alterations made to the shape of the start up profile can improve a belt conveyor system's response. Slight improvements can be made with profiles like those defined by Harrison and Nordell by altering the smooth in and out time. Although Harrison's original profile already produces a satisfactory result, different values are recommended for Nordell's profile when it is used to start up the 4 km belt conveyor system with a drive spacing of 1 km. The simulation results also show that Nordell's profile causes a slightly higher peak drive force and belt tension compared to Harrison's profile. Lodewijks (1995) also observed this phenomenon and he attributed it to the higher peak acceleration present in Nordell's speed profile.

6.2.2 Sequenced starting

The simulations presented up until this point show that it is possible to successfully start up a multiple driven belt conveyor system in an open loop manner by applying a speed profile to each drive station. During the acceleration phase each drive station receives exactly the same power input signal. With this simple control strategy the maximum applied drive force and belt stress does not change significantly when the system length is increased, while keeping the same drive spacing.

However, the simulations also show that the minimum belt tension decreases when the system scale increases. This is especially true for multiple driven systems with large motor stations. To prevent the belt tension from falling below the minimum required operational tension, it is possible to increase the pretension in the belt. Opting for this solution does have a downside because the maximum belt stress will also increase when the pretension is increased. Another option is to implement a sequenced start up procedure.

The idea behind the sequenced start up procedure is that a drive station is not started until the acceleration wave generated by the previous drive station has reached it. As the initial acceleration wave passes a drive station that is not started, the local belt tension tends to increase. Consequently, if this drive station is started exactly at the time when the belt tension would start to increase, it is possible to compensate the fall in slack side belt tension. The drive station located directly next to the gravity take up device has to initiate the acceleration because the tensioning device will guarantee a near constant minimum belt tension. In the case of the multiple driven configuration with 4 km and 4 drive stations this would mean that drive station D_4 is started first. As it is started, an acceleration wave begins travelling towards drive station D_3 . When this acceleration wave reaches D_3 , this drive station is also started. Similarly, drive station D_2 and D_1 are started sequentially as the wave also passes them in turn.

Initially, each drive station has to follow the same speed up ramp when they are started. Due to the fact that the previous drive station has already been running for a while before the following drive station is started, a speed difference will occur between the different stations, with D_4 running the fastest and D_1 the slowest. To get them all running at a synchronous speed at the end of the start up procedure, a special sequencing strategy is adopted.

Figure 6.19 illustrates the principle behind this strategy. In this figure the left diagram shows how the initial acceleration wave travels through the system. The sequence starts with the direct speed ramp up of drive station D_4 . The initial acceleration is half of what will become the final acceleration. As a result, an acceleration wave with magnitude $\frac{1}{2}a$ starts to travel towards the other end of the system. When this wave reaches the other drive stations they also start accelerating at $\frac{1}{2}a$. After the wave passes the last drive station D_1 and it reaches the tensioning device at time T_e , it is reflected and it travels back through the system. To synchronise the drive station, the acceleration of each drive station is increased to its final value a when the reflected wave passes each drive in reverse order. The right diagram illustrates the resulting frequency profiles that will be applied to each drive station. This diagram clearly shows that the initial sequenced start with half the end acceleration causes an offset between the profiles. This difference disappears when the acceleration is further increased as the reflected wave passes each drive station. After the reflected wave reaches drive station D_4 again, all drive stations will be accelerating at the same rate until the end of the start up procedure, where the target speed is reached and the acceleration is removed.



Figure 6.19: Start up ramp for a simple sequenced start

With the aid of the diagram on the right of Figure 6.19 it is possible to determine the magnitude of the final acceleration, so the target frequency $f_{s,t}$ is reached at the end of the start up procedure. For this sequenced strategy the acceleration is calculated as follows

$$a = \frac{f_{s,t}}{(T_a - T_e)}$$
(6.7)

Note that the start up time T_a has to be at least twice as long as T_e . This guarantees that all drive stations are synchronised before the end of the start up procedure is reached.

To be able to use this sequenced start up, the acceleration wave has to occur the moment the supply frequency of drive station D_4 is increased. However, as observed in previous simulations, the drive stations do not directly apply the required drive force to initiate the acceleration of the belt. As a result, a delay occurs before the belt is set into motion. This delay is undesirable when adopting a sequenced strategy because such a strategy relies on a precise timing between the passing of the initial acceleration wave and the start of each drive station. To compensate for this delay, the stator frequency of all drive stations is initially ramped up quickly to a low frequency, so the drive stations will already start applying the minimum force required to set the belt in motion. After a short delay that allows the system to settle down from the step in drive force, the main sequence is started.

Figure 6.20 presents the results of the sequenced starting procedure for the 4 km multiple driven configuration with a drive spacing of 1 km. The upper left diagram shows the stator frequency supplied to each drive station. It clearly illustrates the initial increase in frequency before the starting sequence is applied after 7 seconds. In the upper right diagram drive station D_4 , which is started first, delivers the most work during the starting procedure. It produces an even higher peak tension than was found when the drive stations were started simultaneously, see Figure 6.14.



Figure 6.20: Simulation results of a simple sequenced start

Drive station D_4 clearly delivers the most work and the more a drive station is removed from D_4 the smaller the delivered effort. The last drive station D_1 only applies a larger drive force after its acceleration is increased to the final value. This is contrary to the initial expectation that drive station D_1 would increase its effort directly after it is commanded to start acceleration at $\frac{1}{2}a$. However, as the initial acceleration wave, emanating from the previous drive stations, already forces the belt to accelerate at $\frac{1}{2}a$, drive station D_1 hardly has to apply any additional effort to guarantee its imposed start up profile. Up until drive station D_1 also increases its effort drive station D_4 and D_3 are mainly accelerating the belt.

As the lower right diagram of Figure 6.20 shows, this imbalance in drive force causes a significantly higher belt tension than observed in previous simulations. The minimum belt tension now also increases rather than decreases when the sequence starts, so as expected low belt tensions should pose no problems in this case. A small dip in tension only occurs before the sequence due to the required initial frequency offset.

The lower left diagram shows the belt acceleration along the belt as a function of time. In this diagram the initial acceleration wave is clearly visible. After the initial frequency offset it

appears at drive station D_4 the moment the main starting sequence begins. At 20 seconds the wave arrives at the other end of the belt where it reflects and travels back through the system. After the reflected wave reaches drive station D_4 again, the belt accelerates at a constant rate until the end of the start up procedure. At the end the acceleration is removed suddenly, which causes residual oscillations. A smooth out could be implemented to prevent this from happening.

From the presented results it can be concluded that a sequenced start up procedure will prevent the belt tension from falling below the minimum required belt tension. However, the adopted sequencing strategy also generates a significant imbalance of the work delivered by each drive station. The drive station that initiates the acceleration wave applies the most drive force because the other drive stations have to add little extra power when they are started with the same acceleration as the wave passing them. As a result of the imbalance in drive power the peak belt tension increases undesirably. To prevent the imbalance in power, while keeping the positive effect for the minimum belt tension, an alternative sequenced start up is proposed.

Figure 6.21 presents the principle idea of this alternative strategy. Instead of having an initial acceleration wave that does not change in magnitude as it travels through the system, the acceleration is increased each time it passes a drive station. As the left diagram in Figure 6.21 depicts, drive station D_4 initially starts accelerating at a rate of $\frac{1}{4a}$. Each time the generated acceleration wave reaches a drive station the acceleration is increased with $\frac{1}{4a}$. The idea behind this is that each drive station will be forced to apply at least the drive force to achieve the additional acceleration. However, the increase in acceleration also generates an acceleration wave in the opposite direction of $\frac{1}{4a}$. These waves are indicated by the dashed lines. To account for these additional waves, the acceleration is further increased with $\frac{1}{2}a$ each time one of the dashed lines crosses the location of a drive station. After the initial acceleration wave has reflected at the tensioning device the magnitude of the wave is decreased in steps of $\frac{1}{4}a$ when it passes drive station D_3 again. This allows a gradual and controlled reduction of the wave's magnitude.



Figure 6.21: Start up ramp for a complex sequenced start

At the end of the start up procedure drive station D_4 will have increased its acceleration 5 times, first with $\frac{1}{4}a$ than 3 times with $\frac{1}{2}a$ and finally with $\frac{1}{4}a$ again. Therefore, the final acceleration is equal to 2a. Drive station D_1 also reaches the same final acceleration, but it accomplishes this with two equal steps. The right diagram in Figure 6.21 presents the resulting speed profiles for each drive station. Note that the speed differences in this case are significantly smaller than for the simple sequenced start in Figure 6.19 and that the required acceleration expressed in the stator frequency is now calculated as follows

$$a = \frac{f_{s,t}}{2 \cdot (T_a - T_e)}$$
(6.8)

Figure 6.22 presents the results for the complex starting sequence. The upper left diagram shows the applied stator frequency profiles with the initial offset that was also used for the simple sequence to get a well defined start of the acceleration wave. The improvement that is obtained with these frequency profiles is observable in the upper right diagram because it shows that the drive stations apply about the same amount of work during the start up procedure. Therefore, the imbalance has become significantly smaller and the belt stress peaks much lower than with the simple starting sequence in the lower right diagram. The minimum tension does also not fall as much. However, in this case the belt tension does drop below the pretension force at the end of the start up procedure, but it does not fall below the minimum tension caused by the initial frequency offset applied at the beginning of the procedure.



Figure 6.22: Simulation results of a complex sequenced start

The results show that the more complex starting sequence can guarantee the minimum tension, while it only produces a slightly higher belt tension compared to the case where an scurve is applied simultaneously, see Figure 6.14. It is even expected that the difference in peak tension between both cases can be decreased by also adding a smooth out period at the end of sequenced start up profiles.

To analyse the scalability of the sequenced start up strategy, different belt lengths, ranging from 2 to 8 km, were simulated with a motor spacing of 1 km. For each simulation the peak drive force and belt stress were analysed and stored. Figure 6.23 present the results of this exercise for the simultaneously applied Harrison profile, the simple sequenced and complex more balanced sequenced starting strategy.



Figure 6.23: Performance of starting strategies for different belt lenghts

The result shows that the simple sequenced starting strategy already starts experiencing difficulty when the system becomes longer than 3 km because from this point on both the peak drive force and belt stress start to increase. At a belt length of 6 km it even causes the drive force to reach the friction limit. Note that the left diagram only shows the maximum occurring drive force. Not all drive station will reach the friction limit and it will only be reached during a certain time. For systems longer than 6 km the severity of the occurring friction limit will still increase as the system length increases. Therefore, the belt tension shown in the right diagram will still rise.

The balanced sequenced starting procedure performs nearly as good as the simultaneously applied s-curve with respect to the peak drive force and belt tension. However, for belt lengths beyond 5 km both values start to increase, but at a lower rate compared to the simple sequenced start up. Although both sequenced starting strategies do cause an increase in the maximum belt stress, the minimum occurring belt stress remains constant contrary to the falling stress in the simultaneous controlled approach. In conclusion, the balanced sequenced start up could have an advantage up to a belt length of 5 km, beyond this length it will have to be considered whether the increase in belt tension falls within an acceptable range. If the tension rise is too large, either the starting sequence will have to be improved or the simultaneous controlled strategy will have to be adopted, most probably with an increased pretension force to guarantee the minimum required belt tension.

6.2.3 Layout alterations

Apart from changing the start up profiles to improve the belt's start up response, as discussed in the previous sections, it is also possible to influence the dynamic belt behaviour by altering the layout of the multiple driven belt conveyor configuration. Section 6.1.2 also revealed that during a simultaneous start up procedure the most dominant belt behaviour occurs in the belt section between the tensioning device and the first drive station, regardless of the belt length. This is due to the fact that the drive station at this location has to accelerate the longest section of belt. Therefore, an attempt is made to reduce the dominant behaviour in this part by shortening the distance between the gravity take up device.

Figure 6.24 illustrates a possible solution to shorten the belt section near the gravity take up device. The figure shows the 4 km multiple driven belt system with most drive stations spaced at 1 km, except for the two located after the gravity take up device. For these drive stations the spacing is halved together with their installed drive power. Where the other stations have 8 times the nominal power of an E-BS drive motor, these drive stations only have 4 times the nominal power of a single motor. This is sufficient because these drive stations now only have to drive 500 m of belt during normal operation. As a result, the redistribution of the installed drive power has effectively halved the distance between the gravity take up device and the first drive station.



Figure 6.24: Layout with two smaller drive stations near the gravity take up device

Figure 6.25 presents the simulation results of the altered configuration, when it is started in 60 seconds with Harrison's speed profile. If these results are compared with those of the unaltered configuration, see Figure 6.14, it is clearly visible that there are no oscillations anymore. The peak drive force and belt stress that occurred at drive station D_1 have now also gone and have been replaced by the much lower values produced by the smaller drive stations. The unaltered drive stations deliver about the same effort in both cases. Consequently, the removal of the dominant belt behaviour causes a significant reduction in peak belt stress together with a smoother start up procedure. Instead of having a peak belt stress of approximately 6 N/mm the peak stress now only just passes 5 N/mm. Although the system becomes slightly more complex due to the higher motor count and the use of a large and small drive station type, the resulting reduction in required belt strength could compensate for the additional costs, making this modified configuration an interesting alternative.



Figure 6.25: Simulation results for a layout with a helper drive

Another alternative is to shift all drive stations towards the tensioning device, so the belt section between the first drive station D_1 and the tensioning device becomes equal to the distance between the last drive station D_4 and the head pulley. Figure 6.26 presents the resulting system configuration. In the system with a motor spacing of 1 km the length of the belt section before and after the tensioning device is now equal to 500 m. This effectively halves the length of the section with the dominant belt behaviour and actually creates a symmetrical system. With no drive station located directly next to the tensioning device, special measures have to taken to prevent the belt tension from falling too far during the start up procedure. When no sequenced start is adopted, a fall in tension is inevitable. Therefore, the pretension is increased to keep the minimum tension at a desired level. To see how much the pretension has to increase, the results from the simulations with a belt length of 8 km and a drive spacing of 1 km can be used, see Figure 6.15. These simulations showed that at the centre of the system the minimum belt stress drops about 2 N/mm. Consequently, the pretension is increased with this value.



Figure 6.26: Layout with the gravity take up device centred between drive stations

Figure 6.27 presents the results with the shifted drive station locations and the increased pretension. In this case each drive station virtually delivers the same amount of work during the whole start up procedure. This causes the overlap of the graphs for the applied drive force and belt stress. Although the pretension has been increased, the maximum belt stress is

actually slightly lower than the value found during the start up simulation of the unaltered case, see Figure 6.14. The transition is also much smoother with no oscillations.



Figure 6.27: Results with the tension weight centred between drive stations

As expected the belt tension drops at the slack side of the drive stations. However, due to the increased pretension, it does not drop below the previously implemented pretension force. Even if the system length is increased, while keeping the drive spacing the same, the belt tension will not drop further because it is expected that all drive stations will still generate the same drive force and belt stress graphs. Consequently, this type of multiple driven configuration, where the first and last drive station are offset half a motor spacing away from the tensioning device and the head pulley respectively, is very appealing. Furthermore, due to the symmetry of this configuration, it does not matter in which direction the belt conveyor is operated because in both directions the start up response will be exactly the same. Note that although this layout performs better than the original layout presented in section 6.1.2, the original layout is still considered in the following sections to keep the overall discussion consistent.

6.3 Influence of belt loading

In the previous sections an empty belt was used during the simulation to keep the system configuration and analysis simple. With this simplification a constant motor spacing can be implemented through out the system because the friction forces are evenly distributed along the system. Although the empty system gives results that are easier to interpret and compare, it is not very realistic. In practice a belt conveyor system usually has a loaded carry strand and an empty return strand.

A loaded carry strand generates more rolling resistance. Therefore, this section requires more drive power than the return strand. In a multiple driven system this difference in power demand can be accomplished with a varying motor spacing. Figure 6.28 presents how this is accomplished for the 4 km belt system when the top half of the system becomes the carrying strand that moves bulk solid material from the tail to the head and the bottom half is the return strand. In this model the mass of the belt with load is four times as heavy as an empty belt. Therefore, it also generates four times as much friction. To account for this fact, the drive

spacing in the carrying strand has been reduced to 250 m. In the return strand the belt is always empty, so in this section the original empty belt spacing of 1 km suffices. In total 2 drive stations now power the empty section, while 8 other stations power the loaded strand.



Figure 6.28: Belt conveyor with load on the carry strand

To be able to compare the fully loaded configuration, as depicted in Figure 6.28, with the empty multiple driven system with a belt length of 4 km and a motor spacing of 1 km, the start up of the loaded system was simulated with a similar start up profile. Like the simulations presented in section 6.1.2, the start up speed profile was based on Harrison's prescribed curve with a start up time of 60 seconds. Figure 6.29 presents the results of this simulation.



Figure 6.29: Simulation results with a full carry strand

In this figure the left diagram shows that most drive stations react very similar and relatively smoothly. However, as also observable in the empty configuration, the first drive station D_1 still delivers more work than the rest and it applies the drive force with some oscillations. Note that it actually generates a graph very similar to the one shown in Figure 6.14. The drive station positioned at the head of the system, which is D_{10} in the loaded system and D_4 in the empty system, also produces very similar graphs. Compared with Figure 6.14 more graphs are present between those produced by the first and last drive station. The left diagram also shows that each drive station delivers approximately the same drive force at the end of the start up procedure. This indicates that the chosen motor spacing causes a balanced sharing of the load when the system is operating at a constant speed.

The right diagram in Figure 6.29 also shows that the dominant belt behaviour during the start up procedure occurs at drive station D_1 . The graph of the tight side tension at this station is actually virtually identical to the one observed in the empty belt configuration, see Figure 6.14. Although the belt now has a loaded section, it is still the 1 km long empty belt section between the tensioning device and the following drive station D_1 that mainly determines the start up behaviour. Therefore, the dominant belt behaviour of the loaded belt configuration is also comparable to the start response of a single drive belt conveyor with a belt length of 1 km.

The right diagram also shows that the minimum belt tension also drops slight further compared to the 4 km empty belt configuration. Although this loaded configuration has more drive stations than the longer 8 km empty system with 8 drive stations, the belt tension does not drop al the way to zero. Consequently, the combination of the total number of drive stations and the overall system length determine the maximum drop in belt tension.

In practice a large scale belt conveyor system is mainly started in the empty state. The system is filled with bulk solid material only after the belt has reached its operational speed. A fully loaded start up usually only occurs after an emergency shut down or drive failure. To analyse the more common empty start up situation, the previous system layout, illustrated in Figure 6.28, was also simulated with an empty carry strand.

Figure 6.30 presents the results for this situation, using the same start up procedure as before. The start up of an empty system gives a very different picture. From the left diagram in Figure 6.30 it is now apparent that the drive stations apply less force compared to the loaded situation, while drive station D_1 still delivers most of the work. Although the reduction in drive force can be easily explained with the reduction in friction, the increase in belt tension, shown in the right diagram, is not as obvious. The tight side belt stress at drive station D_2 now exceeds the values found near station D_1 . Also note that the tight side belt stress near drive station D_1 is still very similar to the loaded situation shown in Figure 6.29. The maximum belt stress now exceeds the peak value found in the loaded case by approximately 15%.



Figure 6.30: Simulation results with an empty carry strand

The cause of the increase in belt tension has little to do with the belt dynamics because the high tension remains when the system reaches its operational speed. The actual reason for the

increase lies with the load sharing behaviour of the drive motors. In the considered multiple driven belt configuration each drive station is identically equipped with a pair of induction motors, which all receive the same power signal. When the belt conveyor reaches its operational speed, the belt speed will have equalised throughout the system. As each induction motor has the same torque curve and they are all running at the same speed, each drive station will apply the same amount of drive force to the belt. This is also designated as load sharing and it works very well when the motion resistance along the belt matches the distribution of installed power. However, when the multiple driven belt conveyor system that is designed to have a loaded carry and empty return strand is running completely empty, an imbalance occurs, which leads to an increase of the belt tension. For example, with 8 drive stations in the empty carry strand and only 2 in the return strand 80% of the total drive power is applied to the empty carry strand while it is only generating 50% of the total motion resistance. Therefore, 30% of the force applied to the carry strand also drives the empty strand. This causes the slack side tension of drive station D₂ to rise by 30% of the total drive force. Earlier simulations showed that in each empty belt section the motion resistance causes an increase in belt stress of 2.8 N/mm per kilometre, so for the simulated 4 km system the total required drive force, expressed as belt stress, is equal to 11.2 N/mm. Consequently, the slack side stress at drive station D₂ rises 3.4 N/mm above the pretension of 1.8 N/mm. On the tight side of drive station D₂ the belt stress the applied drive force adds another 10% of the total applied drive force. Therefore, the maximum belt tension during normal operation is equal to 6.3 N/mm (= 1.8 + 3.4 + 1.1 N/mm). This value is also observable after the start up procedure in the right diagram of Figure 6.30.

During the filling and emptying stage of a belt conveyor system the situation changes again because the distribution of the motion resistance now also changes with time. To investigate how this shift in load distribution effects belt stress in a multiple driven system, additional simulations were carried out for both stages. In the filling simulation the belt is initially running empty at it operational speed. At the start bulk solid material is continually loaded onto the belt at the tail of the system, while the power signal to each drive station remains unchanged. With an operational belt speed of 4 m/s the bulk solid material completely fills the 2 km carry strand after 500 seconds.

Figure 6.31 presents the results of the filling simulation. The left diagram shows that the drive stations gradually apply more drive force as the load progresses along the belt. At the beginning and end of the simulation all drive stations are equally sharing the total load. However, during the period where the motion resistance increases along the carry strand, an imbalance occurs between the applied drive forces. Just before the bulk solid material fills the complete carry strand, half the drive stations generate a peak in drive force. The highest peak occurs at drive station D_{10} , which is positioned at the tail of the system.

The right diagram in Figure 6.31 shows that the local belt tension in the carry strand rises and decreases as the system gradually fills with bulk solid material. The belt tension at each drive station actually rises until the first bulk solid material passes the station's location. In this case it is also the imbalance between the distribution of the load resistance and the installed drive power that causes the peaks in belt tension. As the first bulk solid material passes drive station D_6 , which is positioned in the centre of the carry strand, the imbalance between motion resistance and drive power is most unfavourable because at this point the tight side belt stress reaches the highest peak. The belt stress now even exceeds the highest belt stress observed

during the start up of the empty system. Consequently, compared to a single drive belt conveyor system a system designer will have to pay extra attention to this phenomenon when determining the required belt strength for a multiple driven system that incorporates equal load sharing.



Figure 6.31: Filling of a conveyor running at constant speed

In the emptying simulation the situation is reversed. Initially, the carry strand is loaded and the belt is running at the operational speed. At the start the loading of bulk solid material is stopped, which results in a completely empty system after 500 seconds. Figure 6.32 shows that the response of the system is now also reversed. From the left diagram it is clear that when the inflow of bulk solid material is stopped, the drive force applied by each drive station gradually reduces. Similar to the filling stage, an imbalance occurs between each station, but in this case the drive force applied by station D_1 drops near the end of the emptying stage. After the drive force has reached a minimum and the belt conveyor system is completely empty, the imbalance disappears again.

The right diagram of Figure 6.32 also shows the reversed response. Instead of an increase in belt stress, as observed during filling, the belt stress at each drive station now drops before the last bit of bulk solid material passes the station's location. The lowest value now also occurs in the centre of the carry strand near drive station D_6 . The large fall in slack side belt tension now even results in a compressing force in the belt. As this is an undesirable situation, this should be prevented. The solution to this problem would be to increase the pretension to compensate for the tension drop. Consequently, the occurring imbalance between the motion resistance and installed power distribution during the filling and emptying operation of a multiple driven belt conveyor system can significantly increase the required belt strength.

From the simulations presented in this section it can be concluded that when not each drive station applies the exact amount of drive power to overcome the motion resistance in the preceding belt section, an undesirable rise or fall in belt tension can occur. In a multiple driven belt conveyor system where drive stations inherently share the load equally this may lead to an increased belt strength requirement. This phenomenon is caused by the fact that all drive stations receive the same power signal. This links the delivered torque to the installed drive power rather than the local occurring motion resistances.



Figure 6.32: Emptying of a conveyor system running at constant speed

If it is possible to predict the motion resistances along the belt at each moment in time, it is possible to match the locally applied drive force with the required drive power. For example, one method could be to monitor the incoming mass flow of bulk solid material and predict the movement of mass trough the system. With this predicted mass distribution existing resistance models for belt conveyor systems can be used to estimate the actual distribution of motion resistances. When a reliable prediction is available, the next step is to alter the power signal, so the applied force can be adjusted according to the predicted resistances.

One option is to reduce a drive stations stator voltage if the load in the preceding section is lower than the local design capacity. As presented in section 3.4.1 the torque produced by an induction motor depends on the square of the applied stator voltage, see equation (3.23). This makes it possible to scale the torque curve of each drive stations according to the occurring motion resistances, while keeping the synchronous speed of each motor identical. With each drive station still receiving the same supply frequency, each induction motor will have the same amount of slip, when the system is running at steady state because the belt speed will be the same at each drive station. To analyse the effectiveness of this control strategy, the start up simulation with an empty carry strand was rerun. In this case the stator voltages of the drive stations in the carry strand were reduced to compensate for the empty system state. As 4 times as much drive power is installed in the carry strand than the return strand, the stator voltage supplied to the drive stations in the carry strand. Figure 6.33 presents the results of this strategy.

The results in the left diagram of Figure 6.33 show that, as expected, the drive stations in the empty strand apply four times the drive force applied by each drive station in the return strand. The responses of both drive station D_1 and D_2 are actually very similar to the one found when the system was started fully loaded. This is also reflected in the right diagram. Again the dominant dynamic belt behaviour of the belt section between the tensioning device and drive station D_1 is visible, which was also observed in the single drive case. As the locally applied drive force now matches the occurring motion resistances, the belt stress does not rise undesirably as found in the case when voltage reduction was not applied.



Figure 6.33: Start up of an empty conveyor with stator voltage reduction

From these results it can be concluded that the control strategy with voltage reduction can give an effective means to match the local drive power with the resistance and keep the belt stress below the desired level. However, a sound prediction model will have to be developed to reliably determine the motion resistances along the whole system. Invertors will also have to be selected that can regulate the output voltage of each drive station in the carry strand independently from the output frequency. If these requirements can be met, it is expected that this strategy will also work satisfactorily during the filling and emptying of the belt.

6.4 Stopping

Apart from starting a multiple driven system without exceeding the minimum and maximum belt tension, it should also be possible to bring the system to a controlled stop in different scenarios. After the belt conveyor has completed its task the deceleration can be very gradual, but in an emergency situation the system is required to stop in a very short time. To investigate the behaviour of a multiple driven belt conveyor system, both these scenarios have been simulated with the 4 km empty belt system with a drive spacing of 1 km, which is illustrated in Figure 6.13. For the gradual deceleration of the system a reversed Harrison's speed profile was implemented, which starts at the operational speed and reduces to zero. During the simulation of the gradual stopping routine the speed was gradually ramped down in 60 seconds. Figure 6.34 presents the resulting belt behaviour.

The left diagram shows that the drive stations gradually reduce the applied drive force during the stopping procedure. As the motion resistances also reduce with the slowing belt speed, drive stations D_1 , D_2 and D_3 even have to start braking to be able to follow the imposed speed profile. Note that in order to accomplish this braking procedure in a practical situation, the drive controllers will have to able to absorb the braking power, either by dissipating it with braking resistors or by adopting regenerative braking. Similar to the starting procedure station D_1 reacts strongest because it receives the least help from surrounding stations. Therefore, it has to decelerate the greatest belt mass. As the deceleration is applied, there is no delay as observed at the beginning of the start up procedure, which was caused by the fact that a minimum stator frequency is required to generate a sufficiently large drive force to overcome

the static friction. As a result, the drive stations' direct reaction prevents the oscillation that was present during the starting procedure.



Figure 6.34: Stopping of a 4 km belt in 60 seconds

The right diagram also shows a smooth transition of the belt tension. It is only the tight side tension at drive station D_1 that drops below the pretension force due to the braking force. Unlike the starting procedure, there is no maximum peak in drive tension and the slight dip in tension does not pose a problem. As a result, the gradual stopping procedure does not affect the required belt strength. As long as the drive stations are capable of applying a braking force this is an acceptable stopping method. However, a different picture unveils when an emergency stop is executed. Figure 6.35 shows how the belt behaviour changes when the system is forced to stop in an emergency within 20 seconds.



Figure 6.35: Stopping of 4 km belt in 20 seconds

From the left diagram it is clear that the drive stations are capable of stopping the belt in time. Due to the over dimensioning of the installed power to limit slip and wear in the contact zone between the drive wheels and the belt, the drive stations apply a large braking force. As also observed in the gradual stopping procedure drive station D_1 creates the largest negative peak, but in this case the imbalance between the first and last station is much larger.

The strong braking action has a detrimental effect on the belt system because the right diagram shows that on the normally tight side of drive station D_1 the belt tension falls extremely low. It actually drops that far that a large compression force is created for a fair amount of time. In a real system this will not only cause the E-BS belt to sag and open, but it will also lead to the belt running out of the supporting idlers near drive station D_1 . After such a scenario the system cannot be restarted before it has been thoroughly inspected and the necessary repairs have been carried out.

If an emergency stop is an important requirement, alterations will have to be made to the system's layout. The most straightforward solution would be to increase the weight of the gravity take up device. However, this will lead to an increase of the belt strength requirement of approximately three times the strength that is required under normal operating conditions. Another possibility is to adopt the system layout illustrated in Figure 6.26 where the tensioning device is centred between the first and last drive station. This should eliminate the imbalance in the system and reduce the large tension drop at station D_1 . Based on the idea of the helper drive, depicted in Figure 6.24, it is also possible to place a mechanical brake just after the tensioning device, to help drive station D_1 decelerate the belt. However, the coordination between the brake and drive stations is beyond the scope of this thesis. Therefore, it is recommended that the proposed layouts are investigated in future work to improve the multiple driven belt conveyor system's behaviour for emergency stop scenarios.

7 Conclusions

Compared to the more conventional belt conveyor system, with a single drive station positioned at the head or tail of the system, a multiple driven layout with drive stations distributed along the whole length of the belt can offer a number of advantages. If the total required drive force is distributed over a number of drive stations, the system designer gains more control over the drive tension occurring in the belt. Through a reduction in belt strength requirement, this control offers the opportunity to implement both a lighter and cheaper belt construction and support structure, while increasing the layout flexibility and giving the possibility to standardise system components. However, to make a multiple driven configuration a good alternative to a single drive system, the system designer will have to utilise these benefits to be able to compensate for the increased system complexity, as it requires additional power and control cabling along its whole length. The full potential of a multiple drives system can only be realised if the right balance is found between the locally applied drive force and the resistances occurring along the system. Only then can the belt tension be kept below the desired value, so a lighter and cheaper belt construction can be implemented. As little standards and guidelines are currently available for multiple driven belt conveyors in particular, the aim of this thesis was to expand existing models developed for single drive belt conveyor systems and analyse to which degree existing standards and guidelines can be adapted to large scale multiple driven applications.

The EB-S with it pouch shaped belt and multipoint drive system was used as a base for this analysis because it faces a number of fundamental questions in order to scale its belt length beyond the 1 km mark. The fact that the drive power is transferred through drive wheels gives the system designer the possibility to place drive stations at virtually any location along the system. This makes it easier to match the local drive power with the occurring motion resistances and unlike conventional drive pulleys, where the belt requires a minimum wrap angle, the maximum applicable drive power does not depend on the belt tension. However, the light belt construction and the low pretension force requirement will make it more critical to keep the belt tension under control. This is especially true for large scale systems because as the total required drive power increases with the system length, while the belt strength remains unchanged, it will become a greater challenge to keep the local tension within the allowable tension range.

This thesis shows that a belt conveyor system is not the only transport system where the drive power has been spatially distributed. Examples of other successfully operating systems are high speed trains with a large number of powered axles, overhead chain conveyors and shaftless printing presses. Of the systems that were compared the shaft-less printing press comes closest to the multiple driven belt conveyor system. They both feature a flexible medium that connects the spatially distributed drive units, require a pretension force and cannot take up compressive forces. However, closer inspection reveals that the inertia and the changing mass distribution of a conveyor belt and the bulk solid material on it play an important role, while the inertia of the paper web in a printing press is small compared to the drive motor inertia. This leads to the conclusion that drive strategies developed for other multiple driven transport systems are not directly applicable to belt conveyor systems. Therefore, this thesis was focussed on the belt behaviour and the control of the belt tension in different operational situations to investigate the limitations of the current control method.

To be able to conduct this analysis, a dynamic model had to be constructed for a multiple driven belt conveyor system, such as the EB-S. Although little research exists specifically for multiple driven systems, most components of single driven belt conveyors have already been analysed and modelled. For the proposed multiple driven analyses the models describing the belt dynamics and the occurring motion resistances are of most interest. When these tried and tested models are combined with an existing model for induction motors, almost all ingredients are available to model the belt behaviour of the multiple driven E-BS. To complete the overall multiple driven system model, expansions made in this thesis with regard to the rolling resistance and the mechanical transfer of power.

Unlike a conventional conveyor belt the EB-S belt has a curved running surface to prevent wedging of the belt at the support rollers. Therefore, an existing viscoelastic rolling contact model, originally developed to calculate the indentation rolling resistance for flat conveyor belts, was modified to incorporate the curved running surface of the E-BS belt. This modified model shows that a curved belt surface increases the occurring rolling resistance at the belt's support rollers. On the one hand, the radius of the belt surface should be large enough to keep the indentation rolling resistance down, while on the other hand there has to be a radius present to prevent jamming or wedging of the belt between the support rollers.

A model was also constructed to analyse the rolling resistance in the sharp horizontal curves that are possible in the E-BS. This analysis shows that a curve radius of at least 8 m is recommended because for smaller radii the resistance increases significantly. For greater radii the ratio between the belt tension after and before the curve remains about the same, but the greater the sweep angle of the curve the higher the ratio.

Another special feature of the E-BS is that it is equipped with drive wheels, which press into the belt's running surface. Due to the relatively small contact surface between the drive wheels and the belt, the creep that occurs in the contact surface is comparatively high. To analyse this phenomenon a viscoelastic rolling contact model was also constructed to describe the relationship between the drive force applied by one of the wheels and the resulting creep. Due to the fact that this model uses a Winkler foundation, where the rubber surface is represented by a layer of independent spring elements, it requires a correction factor to incorporate the shearing factor between adjacent spring elements. Experimental results show that with this correction factor the predicted relationship between the applied drive force and the occurring slip corresponds well with the measured values. Further investigation also shows that the viscoelastic behaviour of the rubber contact surface has a negligible small effect on the relationship between traction and slip.

As a result of the small contact area between the E-BS belt and its drive wheels, the wear of the belt is more critical compared to a conventional system where the belt is wrapped around a belt wide pulley. Experimental results show that the current E-BS drive stations can only apply a drive force to the belt that is under 5% of the allowable belt tension during normal operation to guarantee a reasonable belt life. If the drive wheels would apply a higher drive force, they will cause an unacceptable amount of wear on the belt surface. Therefore, it is proven that the belt wear presents a far greater limit on the maximum allowable drive force than the limit imposed by the belt strength. To limit the number of drive stations and complexity of the E-BS in a large scale application, this belt wear issue will have to be addressed by increasing the contact area with the belt.

To analyse the belt behaviour in a multiple driven belt conveyor system, the expanded friction and traction models were incorporated in an existing model that describes the longitudinal belt dynamics. Different start up simulations with this model show that the dynamic belt behaviour of a single driven system is comparable to the dominant behaviour found in a multiple driven system, when the drive station spacing is set equal to the belt length of the single driven system. Irrespective of the overall system length the dominant belt behaviour occurs in the belt section between the point where the belt leaves the tensioning device and the first drive station. Therefore, it is possible to use existing start up rules and guidelines that have been developed for single driven belt conveyors for the multiple driven case. This applies to the case where the drive motors are speed controlled in an open loop manner.

However, when the belt length of the multiple driven belt conveyor system is increased while keeping the drive station spacing constant, the minimum belt tension can drop below the pretension and even become compressive. This is a result of the fact that only one drive station can be positioned near the tensioning device. Therefore, care should be taken in large scale applications to prevent the local belt tension from falling below the minimum required tension. If the belt tension falls too far, the pretension will have to be increased.

Similar to a single driven belt conveyor system the dynamic start up behaviour of a multiple driven system can be optimised by altering the shape of the start up speed profile. Favourable are those suggested by Harrison (1983) and Nordell (1985), which are based on a sine and second order polynomial function respectively. Further optimisation of these curves for the simulated E-BS reveals that the belt's response to Nordell's curve improves when both the smooth in and out times are reduced. For Harrison's curve the optimum already is at the point where both smooth in and out times are equal to half the start up time. Although the belt's response to Nordell's curve still remains marginally better.

For a multiple driven belt conveyor system more possibilities are explored in this thesis to optimise the belt's start up response, which are the implementation of a sequenced start procedure and changes in the drive layout. With the sequenced start procedure a drive station is started only when the initial acceleration wave reaches it and the local belt tension starts to increase. This effectively prevents the fall in local belt tension, but it requires a complex sequence to prevent the maximum belt tension from rising excessively when the belt length is increased. Furthermore, to be able to use this starting method, the belt's longitudinal wave

propagation speed has to be known quite accurately. Therefore, the sequenced start will be difficult to implement if the belt conveyor system has to be started in a partially loaded state.

In a multiple driven layout where one drive station is positioned directly next to the tensioning device, the dominant belt dynamics can be improved when the distance to the next drive station on the other side of the tensioning device is reduced. This effectively reduces the belt length of the section where the dominant belt behaviour occurs. One option that is introduced in this thesis is to place a small helper drive in the middle of this section, which removes the worst oscillations, and the highest peaks in drive force and belt tension without affecting the behaviour in other locations of the belt conveyor system. Another option that is introduced is to centre the tensioning device between two drive stations. Although this requires an increase in pretension, it creates a symmetrical drive layout where all belt sections have a comparable length. This results in a balanced response of all sections and for the set drive spacing it gives the most optimal belt response. Despite the fact that the pretension is increased, the actual maximum peak in tension is not higher than found in the original layout. With the symmetrical drive layout the minimum belt tension can also be guaranteed irrespective of the overall belt length and due to the symmetry the system performs just as well under braking.

In a multiple driven belt conveyor system that is controlled in an open loop manner the belt tension is at its optimum value when the installed drive power matches the locally occurring motion resistances. However, as the distribution of bulk solid material on the belt can vary during loading and unloading the optimum situation is not always achieved. As a result, higher peak and lower minimum belt tension can occur than initially expected. Simulations show that in a multiple driven belt conveyor system, where the drive spacing has been optimised for a fully loaded carry strand and an empty return strand, higher peek stress occur when it is started empty than in the fully loaded state. This is caused by the fact that relatively more power is installed in the carry strand than in the return strand, while the motion resistance is equally distributed along the system when it is empty. In effect the drive motors in the carry strand are powering the return strand. Also during the filling process the local belt tension rises locally when the first part of the load travels towards a point along the belt. When this point is passed by the oncoming load, the belt stress reduces again. During emptying the situation is reversed with the local tension initially falling before it rises again as the last bit of bulk solid material passes a point along the belt. Consequently, if a system designer has designed the drive layout to suite a loaded carry strand and empty return strand, a check will have to be performed of the local belt tension in the empty, filling and emptying situations to see if the belt tension stays within the design specifications.

An option to prevent the imbalance between locally applied power and motion resistance is to reduce the local drive power when the resistance in the local belt section is lower than the maximum design value. If the load distribution and the resulting motion resistance can be predicted accurately, it is possible to accomplish this by reducing an AC motor's stator voltage when it has to produce relatively less drive power than the other motors.

With this approach the drive stations are still controlled in an open loop manner. Therefore, it can be sensitive to parameter variations. A less sensitive method would be to incorporate a closed loop system where the local drive power reacts to measured changes in belt tension. However, as this is beyond the scope of this thesis, it is recommended that in future work the

possibilities of such a closed loop control strategy is investigated. It will have to find a method of directly or indirectly measuring the belt tension or the occurring resistance and adopting a multiple variable feedback loop that will produce a sufficiently fast and stable response of the multiple driven belt conveyor system.

Appendix A: Dynamic conveyor belt model

An existing finite element model has been adapted in this thesis to model the dynamic belt behaviour in a multiple driven belt conveyor during starting and stopping procedures. This model was implemented with the aid of MATLAB®. The constructed model is based on research work of Lodewijks (1991) (1992), where he describes a dynamic belt model capable of simulating the propagation of longitudinal waves in a single drive belt conveyor system. To describe his model he starts with a simple belt conveyor configuration with a gravity take up device and drive pulley positioned at the head of the system, see Figure A.7.1.



Figure A.7.1: Single drive belt conveyor configuration for the dynamic model

In the first step the belt is divided into a number of finite elements. Figure A.7.2 illustrates how the belt is split into a number of finite elements. Both the nodes and elements are numbered in sequence starting from the tensioning device. The numbers increase in the belt's moving direction. Note that the arced belt sections on the end pulleys are not divided into a fine mesh because only the global longitudinal behaviour of the belt is of interest. If it is included, relatively small elements are required at the pulleys, which increases the computational load due to the fact that the number of elements increases and the time step during the numerical integration has to be reduced to accommodate for the smaller element sizes.



Figure A.7.2: Belt model split into finite elements

In the final step the conveyor belt is considered as a one dimensional system that can only move and it is loaded in a horizontal direction. Figure A.7.3 shows this horizontal representation of the belt. At the gravity take up device, where the endless belt has been split to form the horizontal model, half of the tensioning mass' weight is applied to both ends of the belt. In the single drive case the drive pulley is combined with the gravity take up device. Therefore, drive force F_d is applied to the last node. For a multiple driven layout the model is easily adaptable because the drive forces can be applied at different nodes along the belt.



Figure A.7.3: One dimensional model of a single drive belt conveyor system

With this horizontal representation of the conveyor belt the belt would move to the right when it is set in motion. To compensate for this phenomenon, the displacement of all nodes is expressed relative to the displacement of node 1. This effectively fixes the first node while the other nodes can move relatively to this point as a result of the strain in the conveyor belt. Furthermore, the first and last node are linked to make the model act as an endless belt. This is accomplished with the equation that describes the displacement of the tensioning weight.

The belt elements themselves are modelled as rod like elements that have a stiffness and a distributed mass. As Figure A.7.4 illustrates, each element consists of two nodes and between these nodes a linear displacement field is presumed. If a dimensionless coordinate is chosen, which is -1 in x_1 and 1 in x_2 , each point on the non-deformed element can be described as follows

$$\mathbf{x}(t) = \frac{1}{2} \left((1 - \eta) \cdot \mathbf{x}_1(t) + (1 + \eta) \cdot \mathbf{x}_2(t) \right)$$
(A.1)



Figure A.7.4: Description of a one dimensional rod element

Similarly the displacement of each point along the element can be described as function of the nodal displacement u_1 and u_2 , or

$$u(\eta, t) = \frac{1}{2} \left((1 - \eta) \cdot u_1(t) + (1 + \eta) \cdot u_2(t) \right)$$
(A.2)

To be able to simulate the belt's dynamic behaviour, equations (A.1) and (A.2) are combined with the principle of virtual work to form a set of equations for the whole conveyor belt. The system's internal work is solely contributed to the stiffness of the belt elements. For each belt element this leads to

$$\delta W_{in,i}^{B} = \frac{E \cdot A}{l_{i}} \cdot \delta \overline{u}_{i}^{T} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \overline{u}_{i}$$
(A.3)

where $W_{in,i}$ represents the internal work of element number i and \bar{u}_i is a vector, holding the deformations of both the element's nodes.

The work related to the acceleration mass of the belt and bulk solid material is classed as external work. For a single element it is calculated as follows

$$\delta W_{ex,i}^{B} = -\int_{L} \left(\delta u \cdot \left(m'_{b} + m'_{l,i} + m'_{r,i} \right) \cdot \ddot{u} \right) dl$$
(A.4)

$$\delta W_{\text{ex},i}^{\text{B}} = -\frac{\left(m'_{\text{b}} + m'_{\text{I},i} + m'_{\text{r},i}\right) \cdot l_{i}}{6} \cdot \delta \overline{u}_{i}^{\text{T}} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \ddot{\overline{u}}_{i}$$
(A.5)

Note that this equation includes the belt and bulk solid material's mass per unit length and also the reduced mass of the idlers. After adapting the principle of lumped mass, where the element's mass with load is concentrated in its nodes, the following diagonal mass matrix is created

$$\delta W_{ex,i}^{mb} = -\frac{\left(m'_{b} + m'_{1,i} + m'_{r,i}\right) \cdot l_{i}}{2} \cdot \delta \overline{u}_{i}^{T} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \ddot{\overline{u}}_{i}$$
(A.6)

Due to the fact that an element remains stationary, while the mass of the belt and the bulk solid material flows trough it, the impulse of the flowing mass has to be accounted for. Expressed as virtual work the impulse effect of a single belt element is calculated as follows

$$\delta W_{ex,i}^{imp} = -\int_{V} \left(\delta u \cdot \frac{\partial (m'_{b} + m'_{1,i}) \cdot \dot{u}^{2}}{\partial x} \right) dV$$
(A.7)

$$\delta W_{ex,i}^{imp} = -\frac{(m'_{b} + m'_{1,i})}{3} \cdot \delta \overline{u}_{i} \begin{bmatrix} 2 \cdot \dot{u}_{i}^{2} - \dot{u}_{i} \cdot \dot{u}_{i+1} - \dot{u}_{i+1}^{2} \\ \dot{u}_{i}^{2} + \dot{u}_{i} \cdot \dot{u}_{i+1} - 2 \cdot \dot{u}_{i+1}^{2} \end{bmatrix}$$
(A.8)

As illustrated in Figure A.7.3, the mass in the gravity take up device m_{tw} applies a pretensioning force to both ends of the belt. Consequently, the virtual work term that can be attributed to the vertical movement y of the tensioning mass is equal to

$$\delta \mathbf{W}_{\mathrm{ex}}^{\mathrm{tw}} = \delta \mathbf{y} \cdot \mathbf{m}_{\mathrm{tw}} \cdot \mathbf{g} = \left(\delta \mathbf{u}_{1} - \delta \mathbf{u}_{N}\right) \cdot \mathbf{m}_{\mathrm{tw}} \cdot \mathbf{g}$$
(A.9)

where the index 1 and N indicate the first and last node respectively. Apart from the constant force applied by the gravity take up device at each end of the belt, the inertia of the tensioning mass also contributes to the external virtual work as follows

$$\delta W_{ex}^{mtw} = -\delta y \cdot m_{tw} \cdot \ddot{y} = \frac{1}{4} m_{tw} \cdot (\delta u_1 - \delta u_N) \cdot (\ddot{u}_1 - \ddot{u}_N)$$
(A.10)

Note that the impulse equation (A.8) combined with the virtual work equations for the gravity take up device (A.9) and (A.10) form the connecting equations that turn the horizontal belt model into an endless system.

The drive forces are directly applied to the nodes where the drive stations are located. Therefore, the contribution of a drive station located at node j to the virtual work is equal to

$$\delta W_{ex,j}^{d} = \delta u_{j} \cdot F_{d,j}$$
(A.11)

The drive force is calculated with the aid of the models for the electric drive motors and gearboxes described in section 3.4 and the traction model described in section 5.1.4. The input for these models is the belt speed at each drive station's node and the power signal supplied to the stator of the drive motor.

The motion resistances generated in each belt element is equally divided over its nodes, which gives the following equation for the virtual work

$$\delta W_{ex,i}^{f} = \frac{1}{2} \left(\delta \overline{u}_{i} + \delta \overline{u}_{i+1} \right) \cdot f_{i} \cdot \frac{\left(m'_{r,i} + \left(m'_{b} + m'_{l,i} \right) \cdot \cos \delta_{i} \right) \cdot l_{i} \cdot g}{2}$$
(A.12)

where f_i is the resistance factor, which is described in section 3.3.1, and δ_i is the belt section's inclination angle.

Finally, the resistance generated at the loading station is applied to the node where the bulk solid material is discharged onto the belt. When the bulk solid material flows onto the belt with a mass flow Q_c and velocity v_c at node q, its contribution to the virtual work is calculated as follows

$$\delta W_{ex}^{c} = \delta u_{q} \cdot Q_{c} \cdot \left(\dot{u}_{q} - v_{c} \cdot \cos \phi_{c} \right)$$
(A.13)

where φ_c represents the angle at which the bulk solid material flows onto the belt.

When the derived terms for the internal and external virtual work are combined, the system equations for the belt conveyor model are revealed. The system is in equilibrium when the sum of the internal virtual work of the belt elements is equal to the sum of the terms for the external virtual work, or

$$\sum (\delta W_{in}) = \sum (\delta W_{out})$$
(A.14)

This leads to the following set of differential equations

$$M(t) \cdot \overline{\ddot{u}}(t) + K(t) \cdot \overline{u}(t) = \overline{F}_{imp} + \overline{F}_{f} + \overline{F}_{c} + \overline{F}_{d}$$
(A.15)

where M and K are the mass and stiffness matrix respectively. The vector \bar{u} holds the displacement of each node. On the right side \overline{F}_{imp} represents the impulse effect of the flowing mass, and \overline{F}_{f} , \overline{F}_{c} and \overline{F}_{d} are the vectors for the motion resistance, loading resistance and drive forces respectively.

To simulate the belt behaviour as a function of time, a numerical integration routine was implemented. With equation (A.15) the nodal accelerations were calculated at each time step and the nodal positions and velocities for the next step were determined with an existing numerical integration procedure.

Nomenclature

| Capitals | Unit | Description |
|---------------------------|-------------------|---|
| А | m^2 | Belt cross-sectional area |
| В | mm | Belt width |
| \mathbf{B}_{E} | - | Energy release rate |
| С | - | DIN factor for main and secondary resistances |
| D | m | Diameter |
| D_b | m | Brake wheel diameter |
| D_d | m | Drive wheel diameter |
| E | N/m^2 | Modulus of elasticity |
| E' | N/m^2 | Storage modulus |
| E'' | N/m^2 | Loss modulus |
| Ei | N/m^2 | Modulus of elasticity of the generalised Maxwell model |
| F _B | Ν | Breakaway friction force |
| F _C | Ν | Dynamic friction force |
| F _d | Ν | Drive force |
| F _n | Ν | Normal force |
| F'n | N/m | Distributed normal force |
| $F_{M,i}$ | Ν | Main resistance |
| F_1 | Ν | Vertical load of belt and bulk solid material |
| Floadcell | Ν | Load cell force |
| F _p | Ν | Normal force on slider plate |
| Fr | Ν | Indentation rolling resistance force |
| Fs | Ν | Secondary resistance |
| F _{Sl,i} | Ν | Slope resistance |
| F _{sp} | Ν | Friction generated by the slider at the loading station |
| F_t | Ν | Traction force |
| F't | N/m | Traction force per unit width |
| F _{tot} | Ν | Total drive force |
| F_z | N | Vertical load on roll |
| G | N/m^2 | Shear modulus |
| Gi | N/m^2 | Shear modulus of the generalised Maxwell model |
| I_1 | А | Stator current |
| I ₂ | A | Rotor current |
| J _d | kg/m ² | Drive wheel inertia |
| J_{g} | kg/m ² | Gearbox inertia |

| J _r | kg/m^2 | Rotor inertia |
|---------------------------|----------------|--|
| L | m | Idler spacing |
| L _{conv} | m | Conveyor length |
| L _m | Н | Magnetising inductance |
| L _{mp} | m | Motor pitch |
| L _r | Н | Rotor inductance |
| Ls | Н | Stator inductance |
| M _b | Nm | Total bearing and seal resistance moment |
| M _{er} | Nm | Bearing rolling friction moment |
| M ₁₁ | Nm | Frictional moment of seals |
| M 1 | Nm | Rearing sliding friction moment |
| M | Nm | Indentation rolling resistance torque |
| N ly | ka/s | Mass flow at the loading station |
| Qc P. | к <u>е</u> / 5 | Stator resistance |
| K] D | <u>52</u> | Deter registence |
| R ₂ D | 52 | Curre redius |
| К _с D | | Curve factors |
| K _{th} | 52 | Sefete feeten fennen stationeren merine helt |
| S _A | - | Safety factor for non-stationary moving beit |
| S _B | - | Safety factor for stationary moving belt |
| 1 T | N | Belt tension |
| T_1 | N | Tight side tension |
| T_2 | Ν | Slack side tension |
| T_a | S | Start up time |
| T _b | Nm | Brake torque |
| T _{c,i} | Nm | Belt tension in curve |
| T _d | Nm | Drive torque |
| $T_{\rm E}$ | N/m | Tearing energy |
| T _e | Ν | Effective transmissible tension |
| T _{ind} | Nm | Induced motor torque |
| Us | V | Stator supply voltage |
| U _{th} | V | Thevenin equivalent supply voltage |
| W _{ex} | J | External virtual work |
| W _{in} | J | Internal virtual work |
| X ₁ | Ω | Stator reactance |
| X_2 | Ω | Rotor reactance |
| X _{th} | Ω | Thevenin equivalent input reactance |
| Xm | Ω | Magnetising reactance |
| | | |
| Non-capitals | Unit | Description |
| a | m | Centre distance to leading edge |
| b | m | Centre distance to trailing edge |
| C | m | Centre distance to contact edge |
| C1 | m/s | I ongitudinal wave propagation speed |
| | - | Viscous friction component |
| \mathbf{v}_{V} | | v isous menon component |

Static friction component

Curve resistance factor

Crack growth per cycle Bearing and seal resistance factor

Contact surface excitation frequency

 $c_{\rm v0}$

dc

 $f_{b} \\$

 $\mathbf{f}_{\mathbf{c}}$

 \mathbf{f}_{e}

_

-

_

Hz

mm
| \mathbf{f}_{i} | - | DIN main resistance factor |
|-----------------------|---------|--|
| f _r | - | Indentation rolling resistance factor |
| f_s | Hz | Stator supply frequency |
| f _{s,t} | Hz | Target stator frequency |
| ft | - | Correction factor for traction force |
| g | m/s^2 | Gravitational acceleration |
| h | m | Thickness of viscoelastic layer |
| h _i | m | Elevation change of an incline or decline belt section |
| h _{tot} | m | Total wear depth |
| h _w | mm | Wear depth per load cycle |
| i | - | Gear ratio |
| k _N | N/mm | Standardised unit belt strength |
| 1 _b | m | Total belt length |
| l _c | m | Contact length of rolling contact patch |
| l _{cl} | m | Contact length between drive and carry belt |
| l _i | m | Length of a belt section |
| 1 _{sp} | m | Length of loading section |
| m' _b | kg/m | Belt mass per unit length |
| m'ı | kg/m | Load mass per unit length |
| m'r | kg/m | Reduced weight of the idlers |
| m _{tw} | kg | Mass of tensioning mass |
| n _b | - | Number of bearing per idler set |
| n _{cvcle} | - | Number of drive cycles |
| n _{drive} | - | Number of drive stations |
| р | - | Number of pole pairs |
| r | m | Radius |
| r ₁ | m | Roll radius |
| r ₂ | m | Radius of belt curvature |
| r _d | m | Radius of drive wheel |
| r _p | m | Radius of drive pulley |
| r _R | m | Combined radius of belt and roll |
| S | - | Motor slip |
| S _{max} | - | Motor slip at pullout torque |
| t | S | Time |
| t_1 | m | Centre distance to slip zone start |
| t _{op} | S | Total operational time |
| ui | m | Horizontal displacement of i th belt node |
| V | m/s | Velocity |
| Vb | m/s | Belt velocity |
| v _{b,t} | m/s | Target belt velocity |
| Vc | m/s | Speed of bulk solid material flow at the loading station |
| Vd | m/s | Bristle damping fall of velocity |
| Vs | m/s | Stribeck velocity |
| W | m | Contact width |
| y _t | m | Vertical displacement of tension weight |
| Z ₀ | m | Indentation depth |
| Zi | m | Bristle deflection |

| Greek Symbols | Unit | Description |
|----------------------|-----------------|---|
| α | 0 | Idler angle |
| α_0 | - | Static resistance factor |
| α_1 | - | Dynamic resistance factor |
| α_2 | - | Viscous resistance factor |
| α_{c} | rad | Curve sweep radius |
| $\alpha_{\rm E}$ | - | Energy release exponent |
| β | rad | Angle between idler sets in a curve |
| γ | rad | Shear angle of brush model |
| δ | - | Creep ratio |
| δ_i | 0 | Belt section inclination angle |
| 3 | - | Strain |
| $\epsilon_i^{\rm E}$ | - | Strain of spring element |
| ϵ_i^η | - | Strain of dashpot element |
| η_{g} | - | Gearbox efficiency |
| η_{ni} | $N \cdot s/m^2$ | Damping factor of Maxwell model |
| η_{si} | $N \cdot s/m^2$ | Damping factor of Maxwell model for shearing |
| θ | rad | Wrap angle |
| $\theta_{\rm E}$ | rad | Direction of crack growth |
| μ | - | Friction coefficient |
| $\mu_{ m p}$ | - | Friction coefficient between belt and slider plates |
| ν | - | Poissons ratio |
| τ | N/m^2 | Shear stress |
| $	au_i$ | N/m^2 | Shear stress in Maxwell element |
| σ | N/m^2 | Stress |
| σ_0 | N/m | Bristle stiffness |
| σ_1 | N/m | Bristle damping |
| σ_i | N/m^2 | Stress in Maxwell element |
| σ_n | N/m^2 | Normal contact stress |
| ϕ_c | deg | Charge angle at the loading station |
| ω | rad/s | Angular cylinder speed |
| ω_d | rad/s | Drive wheel speed |
| ω _n | rad/s | Natural frequency of axial string vibration |
| ω_r | rad/s | Motor shaft speed |
| ω _{sync} | rad/s | Synchronous motor shaft speed |

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Summary

The implementation of distributed drives are a relatively new development in the belt conveyor industry. Conventionally, large scale belt conveyor systems, which are designed to transport bulk solid material, are equipped with a drive station positioned at the head or tail of the system. Sometimes drive stations are placed at both ends. However, as all the drive force, required to overcome the motion resistances of the belt conveyor, is applied at only one or two locations along the system, the belt has to be capable of withstanding these high loads. Therefore, the strongest and heaviest belt types are usually installed in large scale overland belt conveyor systems. Not only does the belt become a costly component in such systems, but it also requires a heavy support structure and horizontal curves can only be laid out with very large radii.

The implementation of a multiple driven belt configuration, where drive stations are placed at different locations along the belt to spread the total required drive force, can offer a number of benefits in large scale applications. In such a system the maximum occurring belt tension can be reduced. As a result, a lighter belt construction can be implemented that is more flexible and can negotiate tighter curves. This not only makes it possible to generate a much more flexible layout, but it can also have a lighter supporting structure. Furthermore, the system length can be increased without the need for a heavier belt by letting the motor count grow with the increase in belt length. This makes it possible to standardise system components. However, as more drive stations are added to the system its overall complexity also increases, which goes hand in hand with cost increases for components other than the belt. A trade off will have to be made between the reduced belt costs and the increased costs due to the increased complexity.

To make a multiple driven belt conveyor system competitive with respect to the more conventional single driven system and to compensate for the increased complexity, the system designer will have to take full advantage of the benefits a distributed driven layout has to offer. For a system designer this is not a straightforward task because little standards and guidelines are available specifically for multiple driven belt conveyor systems. To increase the knowledge for this type of system, this thesis investigates a number of aspects of multiple driven systems, using the Enerka-Becker System (or EB-S) as a base. The EB-S is a pouch belt conveyor system that is inherently equipped with spatially distributed drive stations and which is currently only used in small to medium scale applications with belt lengths up to 500 m. It also features drive wheels instead of drive pulleys, so the drive stations can be

placed at any location along the conveyor belt. To be able to turn the EB-S into a large scale system, with a belt length spanning well beyond the 1 km mark, a number of challenges lie ahead that are directly related to the coordination of the drive stations.

An important aspect will be to take control of the local belt tension, so it remains below the belt specification. This entails that the locally applied drive forces have to be kept in balance with the occurring motion resistances, not only during normal operation, where the belt is running at a constant speed, but also during transient situations such as starting and stopping, loading and unloading. To get the balance right or as good as physically possible, this thesis focuses on the prediction of motion resistances, maximum applicable traction, wear, and the dynamic belt behaviour during starting and stopping of a multiple driven belt conveyor system. For the prediction of motion resistance a large number of models already exist that have been developed for single drive systems. Specially, for the EB-S with its curved running surface on the belt an existing model for indentation rolling resistance is expanded to incorporate the curvature on the viscoelastic rubber belt surface. Furthermore, in relation to friction an existing friction model that can describe stick-slip behaviour is also added to an existing dynamic belt model, which is used for the simulation of the belt behaviour during starting and stopping starting and stopping behaviour is also added to an existing and stopping procedures.

Due to the fact that the E-BS uses drive wheels that have a much smaller contact area with the belt than a drive pulley, the relationship between traction, slip and belt wear is also investigated. To this end the viscoelastic model, which is also used for the calculation of the indentation rolling resistance, is expanded to incorporate shear forces. A comparison with traction experiments shows that it is possible to use this model to predict the relationship between the applied traction and the resulting slip. During the experiments it was discovered that excessive wear of the belt starts to occur when the slip between the drive wheel and the belt's running surface exceeds 4%. At this level of slip a wear pattern can be observed that has already been discovered and modelled in other systems, where a traction force is applied through a rubber surface, such as car tyres for example. Wear experiments with the rubber used in the E-BS show that the belt wear behaves similar to the wear model that coincides with the wear pattern, making it possible to estimate the minimum number of drive stations that are required to prevent the belt from wearing out before its guaranteed lifetime.

With the aid of a dynamic belt model the behaviour of a multiple driven belt conveyor is investigated during starting and stopping procedures. When these results from the simulations are compared with those of a single driven system, where the belt length is chosen equal to the distance between successive drive stations in the multiple driven layout, it is revealed that the dominant belt behaviour of a multiple driven system is similar to the single drive case. In the multiple driven layout the dominant behaviour, which causes the highest peak in belt tension, occurs in the section from where the belt leaves the gravity take up device to the location of the first drive station. This phenomenon is not affected by the overall belt length and motor count as long as the drive spacing is kept the same.

Although the maximum peak in belt tension does not change significantly, the minimum belt tension does decrease when the system length is increased together with the motor count. To prevent the belt tension from dropping below the minimum allowable value, a simple and complex sequenced starting procedure are introduced, where the speed of the drive stations is ramped up one at a time instead of all at once. Results show that if the propagation speed of the longitudinal acceleration waves can be predicted accurately, it is possible to prevent a drop in belt tension with a sequenced starting procedure without significantly increasing the

maximum belt tension. However, as the belt length passes a certain limit, the maximum belt tension does start to increase. For the simple procedure this limit occurs earlier than for the complex starting sequenced starting procedure.

Apart from optimising the belt's dynamic response with the duration and shape of the start up curves it also possible to influence the belt behaviour by altering the drive layout in a multiple driven belt conveyor system. Improvements can be made by equalising the distances between the gravity take up device and the first and last drive station in the system. One possibility is the implementation of an extra helper drive in the section where the belt leaves the gravity take up device. This improves the belt's response and reduces the maximum peak tension, but does not prevent the belt tension from dropping excessively in long belt conveyor systems. A better option is to centre the gravity take up device between two drive stations and increase the tensioning mass to compensate for the fact that no drive station is placed directly next to it. With this approach it is possible to create a symmetrical system that produces the optimum dynamic starting behaviour and that can perform just as good under braking as during starting. Although the pretension has been increased to guarantee a minimum belt tension, a lower peak tension is generated.

Simulations with a multiple driven belt conveyor layout, which is designed to have a loaded carry strand and an empty return strand, and therefore has a closer drive spacing in the carry strand, show that the dominant belt dynamics actually occur in the return strand. The dominant starting behaviour found in the return strand is comparable to a single drive system with a overall belt length equal to the largest motor spacing in the multiple drive case. However, if the multiple driven system that is designed to have a loaded carry strand is started empty, higher peak tensions can occur than in the fully loaded case due to the imbalance between the local motion resistances and the installed drive power. This imbalance also causes a temporary peak in belt tension during the loading procedure and a dip in tension during emptying of the belt. In the case where electric AC drive motors are implemented, a possible solution is to equalise the imbalance by reducing the stator voltage in the drive stations near the belt sections where the local resistance is lower than the nominal design value.

Samenvatting

Gedistribueerd aandrijven is een relatief nieuwe ontwikkeling in de bandtransporteurindustrie. Gebruikelijk worden in grootschalige transportbandsystemen, die ontworpen zijn om stortgoed te transporteren uitgerust met een aandrijfstation aan de kop of staart van het systeem. In sommige gevallen worden aandrijfstations aan beide uiteinden van het systeem geplaatst. In een dergelijk systeem wordt echter de gehele aandrijfkracht, die nodig is om de bewegingsweerstand van de transportband te overwinnen, op één of twee plaatsen op de band aangebracht, met als gevolg dat de band sterk genoeg moet zijn om deze krachten te kunnen weerstaan. Hierdoor worden vaak de sterkste en zwaarste banden gebruikt in grootschalige transportbandsystemen. Dit maakt de band niet alleen een dure component, maar het vereist ook een zwaarder ondersteuningsframe en horizontale bochten kunnen alleen met grote stralen worden gemaakt.

De toepassing van een geditribueerd aangedreven band, waar aandrijfstations op verschillende locaties langs de band zijn geplaatst om de geheel benodigde aandrijfkracht te verspreiden, biedt een aantal voordelen in grootschalige toepassingen. In een dergelijk systeem kan de maximaal optredende trekkacht in de band worden gereduceerd. Als gevolg kan er een lichtere band worden gebruikt, die flexibeler is en scherpere bochten kan nemen. Dit maakt het niet alleen mogelijk om een flexibelere lay-out te creëren, maar het verreist ook een minder zwaar ondersteunigsframe. Verder kan de systeemlengte worden vergroot zonder dat er een zwaardere band hoeft te worden gebruikt door het aantal motoren te laten groeien met de lengte. Dit maakt het tevens mogelijk om systeemcomponenten te standaardiseren. Als echter het aantal motoren wordt vergroot, neemt ook de complexiteit van het systeem toe, wat samengaat met een kostenverhoging van de componenten rond de band. Er zal een compromis worden gemaakt tussen de verlaagde bandkosten en de verhoogde kosten, die gepaard gaan met de toegenomen complexiteit.

Om een gedistribueerd aangedreven bandtransporteur te kunnen laten concureren met de conventionelere enkel aangedreven transportband en te compenseren voor de toegenomen complexiteit, zal de systeemontwerper de voordelen van een gedistribueerd aangedreven systeem volledig moeten benutten. Voor de systeemontwerper is dit niet een vanzelfsprekende taak, omdat er nauwelijk normen en richtlijnen specifiek voor gedistribueerd aangedreven systemen te vinden zijn. Om de kennis op het gebied van dergelijke systemen te vergroten, onderzoekt dit proefschrift een aantal aspecten van gedistribueerd aangedreven systemen met het Enerka-Becker Systeem (of E-BS) als leidraad. Het E-BS is een buidelband systeem dat

inherent uitgerust is met ruimtelijk gedistribueerde aandrijfstations en dat tot nu toe alleen in kleine tot middel schalige toepassingen wordt toegepast met band lengtes tot 500 m. Het maakt ook gebruik van aandrijfwielen in plaats van aandrijftrommels, waardoor de aandrijfstations op willekeurige locaties langs de band kunnen worden geplaats. Om van het E-BS een grootschalige toepassing te kunnen maken, met een bandlengte langer dan 1 km, liggen er een aantal uitdagingen in het pad, die direct aan de coordinatie van de aandrijfstations gerelateerd zijn.

Een belangrijk aspect is het beheersen van de bandspanning, zodat het binnen de band specificaties blijft. Dit houdt in dat de lokaal aangebrachte aandrijfkrachten in balans moet worden gehouden met de optredende bewegingsweerstand, niet alleen tijdens normaal bedrijf, wanneer de band met constante snelheid draait, maar ook tijdens overgangssituaties zoals starten en stoppen. Om de balans zo goed mogelijk te krijgen, concentreert dit proefschrift zich op het voorspellen van bewegingsweerstanden, de maximale tractie die kan worden aangebracht, slijtage en het dynamische gedrag van de band tijdens starten en stoppen van een gedistribueerd aangedreven bandtransporteur.

Voor de voorspelling van bewegingsweerstanden bestaan er al een groot aantal modellen die ontwikkeld zijn voor transportbanden met een enkel aandrijfstation. Speciaal voor het E-BS, dat een gekromd bandcontactoppervlak met de ondersteuningsrollen heeft, is een bestaand model voor indrukrolweerstand uitgebreid, zodat de kromming van het rubberen viscoelastische contactoppervlak kan worden geïntegreerd. Tevens is in relatie tot frictie een model, dat stick-slip gedrag kan beschrijven, toegevoegd aan een bestaand dynamisch bandmodel, dat gebruikt wordt om het bandgedrag tijdens starten en stoppen te kunnen simuleren.

Als gevolg van het feit dat het E-BS aandrijfwielen gebruikt, die een veel kleiner contact oppervlak hebben dan aandrijftrommels, is ook de relatie tussen tractie, slip en slijtage onderzocht. Hiervoor is het viscoelastische model uitgebreid, dat ook is gebruikt voor de berekening van de indrukrolweerstand, zodat schuifkrachten ook kunnen worden verrekend. Een vergelijking met tractieexperimenten toont aan dat het met dit model mogelijk is om de relatie tussen de aangebracht tractie en de resulterende slip kan worden voorspeld. Tijdens de experimenten is geconstateerd dat veel slijtage optreedt, als de slip tussen het aandrijfwiel en de band boven 4% komt. Bij deze mate van slip wordt er een slijtage patroon zichtbaar, dat al ondekt en gemodelleerd is in andere systemen, waar tractie door een rubberen oppervlak wordt aangebracht, zoals bijvoorbeeld op autobanden. Slijtageproeven met het rubber uit het E-BS tonen aan dat de bandslijtage vergelijkbaar is met het slijtagemodel, dat overeenkomt met het slijtagepatroon. Met dit model is het mogelijk om een schatting te maken van het minimum aantal aandrijfstations, dat nodig is om te voorkomen dat de band versleten is voor de gegarandeerde levensduur.

Met behulp van een dynamisch bandmodel is het gedrag van een gedistribueerd aangedreven transportband tijdens starten en stoppen onderzocht. Als de resultaten van de simulaties vergeleken worden met die van een centraal aangedreven systeem, waar the bandlengte gelijk is gekozen aan de afstand tussen opeenvolgende aandrijfstations in het gedistribueerd aangedreven systeem, blijkt het dominante bandgedrag van het gedistribueerde systeem overeen te komen met dat van het centraal aangedreven geval. Het dominante gedrag, dat de hoogste piek in bandspanning veroorzaakt, treedt op in de sectie waar de band het spanmechanisme verlaat en de locatie van het eerste aandrijfstation. Dit fenomeen wordt niet beïnvloed door de totale lengte en het aantal aandrijfstations zo lang de stationsafstand ongewijzigd blijft.

Ondanks het feit dat de maximum bandspanning niet veel verandert, neemt de minimum bandspanning wel af wanneer het aantal aandrijfstations toeneemt. Om te voorkomen dat de spanning onder de minimum toelaatbare waarde daalt, zijn een simpele en complexe opstartprocedure geïntroduceerd, waar de snelheid van de aandrijfstations één voor één in plaats van allemaal tegelijk wordt verhoogd. Resultaten tonen aan, dat als de voortplantingsnelheid van de versnellingsgolven voorspeld kan worden, het mogelijk is om de daling in bandspanning te voorkomen zonder de maximum spanning te beïnvloeden. Bij een bepaalde bandlengte neemt de maximum bandspanning echter wel toe. Voor de simple startprocedure treedt dit eerder op dan bij de complexe procedure.

Los van het optimaliseren van het bandgedrag door de lengte en vorm van de startcurve te wijzigen, is het ook mogelijk om het bandgedrag te beïnvloeden door de aandrijflayout in een gedistribueerd systeem te wijzigen. Verbeteringen kunnen worden doorgevoerd door de afstand tussen het spanmechanisme en het eerste en laatste aandrijfstations gelijk te maken. Eén mogelijkheid is het gebruik van een extra hulp aandrijfstation in de sectie waar de band het spanmechanisme verlaat. Dit verbetert het bandgedrag en reduceert de maximale spanningspiek, maar het voorkomt niet dat de spanning te ver daalt in lange transportbandsystemen. Een betere optie is om het spanmechanisme te centreren tusses twee aandrijfstations en het spangewicht te vergroten om te compenseren voor het feit dat er geen aandrijfstation meer direct naast ligt. Met deze aanpak is het mogelijk om een symmetrisch systeem te genereren, dat een optimaal dynamisch startgedrag oplevert en dat even goed presteert tijdens remmen als tijdens starten. Ondanks het feit dat de voorspanning moet worden opgeschroefd om de minimum bandspanning te garanderen, daalt de maximale piekspanning.

Simulaties met een gedistribueerd aangedreven layout, dat ontworpen is met een beladen heengaand en leeg teruglopende bandpart, en dus meer motoren in het heengaand dan in het teruglopende part heeft, tonen aan dat het dominante bandgedrag in het teruglopende part optreedt. Als gevolg is het dominante startgedrag, dat in het teruglopende part is gevonden, vergelijkbaar met een centraal aangedreven systeem met een bandlengte gelijk aan de grootste afstand tussen de aandrijfstations in het gedistribueerd aangedreven systeem. Als dit systeem, dat ontworpen is voor een beladen toestand, echter leeg wordt gestart, kunnen er hogere bandspanningen optreden als gevolg van de onbalans tussen de lokale bewegingsweerstanden en het geïnstalleerde vermogen. Deze onbalans zorgt ook voor een tijdelijke spanningspiek tijdens het beladen van de band en een spanningsdaling tijdens het legen van de transportband. In het geval er asynchrone motoren zijn toegepast, is hiervoor het reduceren van de statorspanning in de aandrijfstations, waar de lokale bewegingsweerstand onder de nominale ontwerpwaarde ligt, een mogelijke oplossing.

Biography

Ashley Nuttall was born on April the 8th 1977 in Rotterdam, the Netherlands. After graduating from secondary education (VWO) at the Einstein Lyceum in 1995, he studied Mechanical Engineering at Delft University of Technology. In October 2001 he received his master degree (cum laude) on a research project concerning the design of an automatic handling device for semi-automatic twistlocks. This master assignment was carried out at Nelcon in Rotterdam. After completing a research paper on his research subject he left the university and started work at ASMI in Bilthoven. In September 2002 he returned to the Transport Engineering and Logistics section at Delft University of Technology where he started his PhD research project, which was supervised by Gabriël Lodewijks. During this project he cooperated with Fenner Dunlop B.V. (Drachten, the Netherlands), supervised a number of students on belt conveyor related subjects and was a lecturer at three international conferences. Currently Ashley is employed as design engineer at Huisman-Itrec B.V. in Schiedam. This company develops and builds heavy offshore equipment such as cranes and pipe laying systems.

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