

## TRANSIENT RESPONSE OF SHIP HULLS TO WAVE IMPACT

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*In heavy seas ships are subjected to the impact of waves on the bottom and forward sections. These loads induce a transient vibratory response of the ship hull which behaves like a free-free beam interacting with the water. The relative motion between the ship and waves is determined from a strip theory, and the impact force is calculated from the rate of change of momentum of the fluid. The hydrodynamic coefficients are calculated with two-dimensional singularity functions. The vibratory response of the ship hull is calculated by modelling it with two-dimensional Timoshenko beam finite elements with a consistent mass formulation, which is used to determine the mode shapes and natural frequencies. The response is obtained by modal superposition and each modal response is obtained using a Newmark integration scheme. Results are presented of example calculations on a cargo ship.*

### **1. Introduction**

The impact loads induced by waves on ships are, as a rule, concentrated at their forward sections. These slamming loads can consist of bottom impacts or of the development of sudden forces at bow-flared sections. The interest on slamming has been often raised from the point of view of the structural strength of the hull and from the ship operationally

standpoint.

From the structural side, slamming is looked as a possible cause of structural damage and also as a significant component of the primary stresses that the hull is subjected to.

The wave slamming originates a whipping vibratory response of the hull, which is mainly in the fundamental two noded mode, inducing significant stresses at the midship section. Damages can occur near the midship section due to the high stresses induced by the whipping vibration of the hull. They can be the buckling of panels [1] or the development of cracks at points of high stress intensity [2]. The damage can also have a more localized nature and affect only the zone where most of the load is acting [3,4], i.e. the bow and the bottom forward.

Full-scale measurements [5] have shown that the whipping stresses can attain values similar to the wave induced bending stresses. Although these stresses have a transient character, it has been shown [6] that their long-term predictions result in values comparable to the wave and still-water bending stresses. However, for the long-term prediction methods, it is necessary to account for operational conditions [7,8], being the voluntary speed reduction one of the main considerations.

The criteria specified by Classification Societies for the structural design of ships is based on predicted long-term values of primary stresses which should include slamming stresses. For this purpose, the overall consequences of slamming are more important than its localized aspects, and were given the primary attention herein.

## 2. Prediction of the slamming load

Some authors [8], have presented predictions of the slamming load based on a description of the pressure distribution on the hull. The information used for the shape of the pressure distribution is based on theoretical and experimental studies, but, no definite statement can be made about their accuracy. Theoretical studies have contemplated mostly two-dimensional structures [9,10]. Although reasonable results have been obtained, comparisons with experiments on three-dimensional bodies



[11] have shown some discrepancies. Recently an apparently accurate procedure has been developed [12] showing the very large dependence of the slamming procedure on small changes in the angle of impact, and thus, justifying the large scatter in the experimental data for small deadrise angles.

When considering the overall response of the ship structure to the sudden and localized slam load, it is more important to have a prediction of the total load acting on the ship than to describe accurately the time and spatial variation of the pressure distribution. A global treatment of the problem will neglect the details of the physical problem and had proved to yield adequate results in different studies [13-15].

The basic formulation of the problem can be found on the theory of hydrodynamic impact originally presented by Von Karman. He considered a body with constant velocity impacting on an undisturbed water surface. The body will accelerate the fluid and the impact force developed is given by the rate of change of momentum. This approach has been successfully used for two-dimensional wedge-shaped bodies with moderate deadrise angles. It has also been successfully applied to circular cylinders [15] representing a good basis for a generalisation to ship impact problems. For very small deadrise angles other effects such as air entrapment, water compressibility and structural elasticity, come into effect, requiring a different type of analysis [12].

The determination of the slamming load on a ship is more involved than the uniform velocity, normal impact of a cylinder or a wedge. Indeed, besides being a three-dimensional body with varying cross-section, the ship has also a forward velocity and is impacting a disturbed free-surface, with a combination of a translatory and rotational motion. The approach commonly adopted to tackle the ship problem has been to use the concept of the strip in the theory of ship motions. It considers the ship divided in several transverse strips and looks at each as a two-dimensional body impacting an undisturbed free-surface with a velocity equal to the relative velocity between ship and water [13,14]. The total load is then obtained by adding the contribution of each strip.

The same idea is used here. The linear theory of ship motions [16] is used to predict the motion of the ship relative to a system of regular

waves. Whenever any ship section leaves and re-enters the water, the slam loads are calculated on the basis of the relative velocity and acceleration. The linear theory has limitations in its applicability to high Froude numbers and low frequencies of encounter [17]. Slamming usually occurs with high Froude numbers but also with head seas which imply a high frequency of encounter and give some confidence to the accuracy of the linear theory.

The theoretically correct approach to study the slamming problem would be to model the non-linear ship motions. However, recent results have indicated that only for small ships are these effects relevant [18,19].

The slamming force per unit length of a strip of the ship hull is given by the rate of change of momentum and by the buoyancy force at each moment:

$$F(x,t) = \frac{D}{Dt} (A_{33} \dot{z}_r) + \rho g S \quad (1)$$

where  $\frac{D}{Dt} = \left( \frac{\partial}{\partial t} - V \frac{\partial}{\partial x} \right)$  is the substantial time derivative,  $A_{33}$  is the two-dimensional added mass of the section,  $\dot{z}_r$  is the vertical velocity of the section relative to the water,  $V$  is the ship speed and  $S$  is the value of the sectional area corresponding to a given value of draft.

The formulation related with this force assumes that the inertial effects are dominating over the gravitational ones which implies that the added mass values to be used in the force calculation should correspond to the limit of infinite frequency. To calculate the values of added mass for the different sections, use can be made of Lewis [20] or of the Frank close-fit method [21]. The first would provide computationally simpler results but, is not appropriate for non-regular sections such as bulbous bows. Therefore, the more versatile sink and source Frank close-fit method has been used for the added mass calculations.

The force expression can be further developed to:

$$F = \left( \frac{\partial A_{33}}{\partial z} \dot{z}_r - V \frac{\partial A_{33}}{\partial x} \right) \dot{z}_r + \left( \ddot{z}_r - V \frac{\partial \dot{z}_r}{\partial x} \right) A_{33} + \rho g S \quad (2)$$



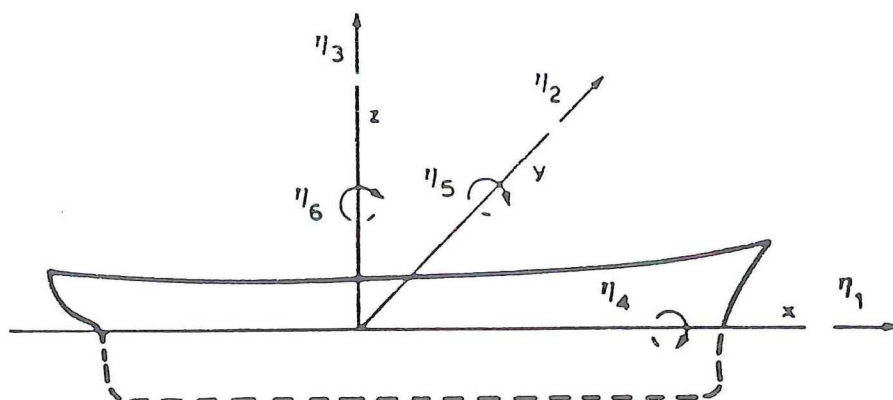


Figure 1. Sign convention for the translatory and angular displacement of a ship.

which is the format actually used in the present calculations.

The force calculation needs the previous solution of the relative motion problem. Defining the coordinate axis as in Figure 1, the vertical displacement and velocity relative to the water surface is given by:

$$z_r = \eta_3 - x\eta_5 - \xi \quad (3)$$

$$\dot{z}_r = \dot{\eta}_3 - x\dot{\eta}_5 - V\eta_5 - \dot{\xi} \quad (4)$$

where  $\eta_3$  and  $\eta_5$  are heave and pitch motions which are harmonic in time:

$$\eta_3 = \bar{\eta}_3 \sin(\omega_e t - \epsilon_3) \quad (5)$$

$$\eta_5 = \bar{\eta}_5 \sin(\omega_e t - \epsilon_5) \quad (6)$$

with  $\epsilon_3$  and  $\epsilon_5$  as phase lags. The motions will be executed at the encounter frequency between ship and waves:

$$\omega_e = \omega + \frac{\omega^2}{g} V \quad (7)$$

where  $\omega$  is the circular frequency of the incoming wave, whose elevation is given by:

$$\xi = \bar{\xi} \sin(\omega t + kx) \quad (8)$$

where  $k = 2\pi/\lambda$  is the wave number and  $\lambda$  the wave length.

The solution of the harmonic motion problem is obtained by the use of a program based on the method of Salvesen, Tuck and Faltinsen [6]. That provides the amplitudes and phase angles in Equations (5) and (6). Substitution of Equations (5), (6) and (8) in Equations (3) and (4) will provide the time history of the relative motion and velocity. A slam will occur when the relative motion of the section is equal to the still-water draft of the section and at the same time the relative velocity is negative, i.e. the ship enters the water. The other condition to be satisfied is that on the previous time step the section must have been out of the water. The slam is terminated when the relative motion is zero again.

Some authors define a slam to occur only when the relative velocity is higher than a certain value [8]. It is thought, however, that this definition is only associated with the definition of slams that can be felt by the ship master and does not have relevance for the physical problem. The present definition of slamming is not contemplating bow-flare forces which occur without bow emergence. However, the calculation procedure is exactly the same and other definitions can easily be accounted for.

Use of Equations (3) and (4) allows the definition of the sections where slamming is occurring at each time instant. At each time instant the force developed at each section is obtained by using Equation (2). The total force acting on the ship is obtained by integrating these forces along the sections where they are acting. Contrary to [19], no assumption is made as to the slamming length. Its value depends on the severity of the slam and results from checking the time history of Equation (3) at various stations along the ship.

A computer program has been developed to perform these calculations, consisting of four main blocks which are interpolation of



functions, added mass calculations, relative motion simulation and the slamming force calculation.

The calculation of the slam force in Equation (2) makes it necessary to have the values of the sectional area and of the added mass and its longitudinal and vertical derivatives at any draught and ship station. As these values will be calculated in a time simulation it is not economical to use the added mass subroutines each time a value is needed. Therefore, the approach chosen has been to calculate the added mass for a given number of draughts of each station and afterwards the relevant values were obtained by interpolating between these calculated values.

The interpolation uses the Lagrange method with a third degree polynomium. The derivatives are calculated from analytical derivation of the interpolated polynomium. Derivatives are needed to perform the vertical and longitudinal differentiation of the added mass indicated in Equation (2). The added mass calculations were performed with the Frank close-fit method.

The motion characteristics (Eqs. (5), (6)) are predicted by the ship motions program of Salvesen, Tuck and Faltinsen [16]. The time simulation of the relative motion between ship and waves (Eqs. (3), (4)) is performed in the program for slam load calculation. Whenever a slam occurs, the force (Eq. (2)) is calculated making use of the interpolated values of added mass at each time step.

### 3. Prediction of the structural vibratory response

It has been shown at different occasions that a simple beam model can model adequately the hull vibratory behaviour, specially if only the lower modes are of interest [13,19,22]. It has also become clear that the vibratory response to wave loading is an overall behaviour which is dominated by the lower modes. Engine and propeller induced vibrations are phenomena which involve higher frequencies and sometimes have a more localized nature. In these situations more complex models of two- and three-dimensional finite elements have been used [23,25]. However, for the situation of concern here, a Timoshenko beam formulation has

been found adequate since it is not expected that modes higher than the sixth could have any dominant effect [22].

The method often used in similar studies has been a modal analysis with the response of each mode calculated by a convolution integral [19]. Another technique of more recent development consists in the use of direct integration methods. One starts from the dynamic equilibrium equations and the initial conditions and performs a step-by-step numerical integration of the equations. Basically this consists of satisfying the equilibrium equations at discrete points in time and to assume a certain variation of displacements, velocities and accelerations within each time interval.

The main advantage of these methods is that they can be used in non-linear problems either due to material or geometric non-linearity while the modal superposition techniques are restricted to linear problems. In linear problems both approaches are equally accurate being the choice dictated only by computational effectiveness. The approach adopted in the present work is to use direct integration methods because, although, dealing only with a linear problem, the same formulation can be adjusted later on to deal with any non-linearity.

The time-integration schemes can be divided in explicit and implicit types. In the first one, the solution at time  $t_{n+1}$  is obtained from the equilibrium conditions at time  $t_n$ . In the implicit methods the solution at time  $t_{n+1}$  is obtained from the equilibrium at time  $t_{n+1}$ . The implicit methods are unconditionally stable and the maximum time step to be used is only governed by the desired accuracy in the response description. However, at each time step a larger number of operations is required, as compared with explicit methods. The explicit methods are usually only conditionally stable, requiring a smaller time step, sometimes one order of magnitude smaller.

The assessment of which methodology is computationally more economical is problem-dependent. However, as a general guideline explicit techniques are often preferred for wave propagation type of problems and implicit procedures are indicated for inertia dominated situations, like the present problem [26]. Therefore, in the present study an implicit technique was adopted. There are different implicit



methods available and each of them introduces specific distortions in the way they represent the system response. Therefore, the choice of the method to be adopted must be made such that the combined effect of the integration technique and of the finite element structural modelling will partially cancel out.

Examples of implicit techniques are the Newmark constant acceleration method, the Wilson and the Houbolt method. All of them introduce a frequency distortion by lowering the real frequencies and in addition introduce a damping which increases with frequency. Therefore, to counteract this frequency distortion, the spatial discretization should be done with consistent mass instead of a lumped mass model since the first increases the time frequency. The combination chosen in this work has been the Newmark constant acceleration method ( $\gamma = 1/2$ ,  $\beta = 1/4$ ) and a consistent mass formulation.

The direct integration methods can also be used together with a modal analysis. In linear problems this results in uncoupled equations and decreases substantially the computational effort of satisfying the equilibrium equations at each time step. When dealing with non-linear systems the equations obtained are still coupled but have a much smaller band-width. Therefore, whenever long-time histories are of interest, as is the case here, it is worthwhile the extra effort of determining the modal shapes and performing the model transformation. With this in mind, a modal superposition approach was also adopted here.

The basis of the finite element method are well-known and documented elsewhere as for example in [27]. The present formulation adopts Timoshenko beam elements and a consistent mass matrix which is obtained by using the same interpolation functions as for the element stiffness matrix. When the elements are assembled, a system equation is obtained in the following form:

$$\underline{M} \ddot{\underline{U}} + \underline{C} \dot{\underline{U}} + \underline{K} \underline{U} = \underline{F}(t) \quad (9)$$

where the underbar indicates matrices.

The mode shapes are determined from the solution of the free-vibration problem with no damping, i.e.:

$$\underline{M} \ddot{\underline{U}} + \underline{K} \underline{U} = 0 \quad (10)$$

which is casted in the following form:

$$(\underline{K} - \underline{\Omega}^2 \underline{M}) \underline{\phi} = 0 \quad (11)$$

having a sinusoidal response of circular frequency  $\omega$  for each mode  $\phi$ . Several methods exist to solve this generalised eigenvalue problem [27]. The algorithm adopted here is based on the Householder factorization whose main advantage is its stability.

Solution of the eigenvalue problem yields the  $\phi$  and  $\Omega$  matrices, being the last one a diagonal matrix. Performing now a modal transformation, will have

$$\underline{U} = \underline{\phi}^T \underline{x} \quad (12)$$

where T indicates the transpose of the matrix. Substituting this expression in the system equations and pre-multiplying by the modal matrix results in

$$\underline{\hat{M}} \ddot{\underline{X}} + \underline{\hat{C}} \dot{\underline{X}} + \underline{\hat{K}} \underline{X} = \underline{\hat{F}} \quad (13)$$

where:

$$\underline{\hat{M}} = \underline{\phi}^T \underline{M} \underline{\phi} \quad (14a)$$

$$\underline{\hat{C}} = \underline{\phi}^T \underline{C} \underline{\phi} \quad (14b)$$

$$\underline{\hat{K}} = \underline{\phi}^T \underline{K} \underline{\phi} \quad (14c)$$

$$\underline{\hat{F}} = \underline{\phi}^T \underline{F}. \quad (14d)$$

In linear systems the system modal matrices are diagonal, being possible to solve each equation independently.

Using the Newmark method, the velocity and displacement at time  $t+\Delta t$  are given by:



$$\dot{\underline{x}}_{t+\Delta t} = \dot{\underline{x}}_t + \left\{ (1 - \gamma) \ddot{\underline{x}}_t + \gamma \ddot{\underline{x}}_{t+\Delta t} \right\} \Delta t \quad (15)$$

$$\underline{x}_{t+\Delta t} = \underline{x}_t + \dot{\underline{x}}_t \Delta t + \left\{ (1/2 - \beta) \ddot{\underline{x}}_t + \beta \ddot{\underline{x}}_{t+\Delta t} \right\} \Delta t^2. \quad (16)$$

The acceleration at time  $t+\Delta t$  is obtained by solving the equilibrium equation:

$$\hat{\underline{M}} \ddot{\underline{x}}_{t+\Delta t} + \hat{\underline{C}} \dot{\underline{x}}_{t+\Delta t} + \hat{\underline{K}} \underline{x}_{t+\Delta t} = \hat{\underline{F}}_{t+\Delta t}. \quad (17)$$

The initial conditions are given by:

$$\underline{\underline{X}}_0 = \underline{\phi}^T \underline{\underline{M}} \underline{\underline{U}}_0. \quad (18)$$

After the system equations have been transformed to the modal form of Equation (13), the time response of each mode is obtained by the solution of Equation (17), where  $\hat{\underline{F}}_{t+\Delta t}$  is the value of the modal load function at  $t+\Delta t$ . After obtaining the response of all the mode shapes of interest, the nodal displacements are obtained by adding the contribution of all modes, i.e. Equation (12).

A computer program was developed to perform the numerical calculations. It is divided in one part that determines the mode shapes and frequencies and another that computes the response. The stiffness and mass matrices are assembled and the relevant boundary conditions are applied and, afterwards, the eigenvalues are calculated. The system coordinates are then transformed to the modal ones. The second part of the program calculates the model response to the exciting force which is given by Equation (14d).

#### 4. Example calculations

Calculations have been performed for a Mariner ship using the information about ship geometry, weight and stiffness distribution given in [18]. Simulations of relative motion have been made for several headings

number of ship section

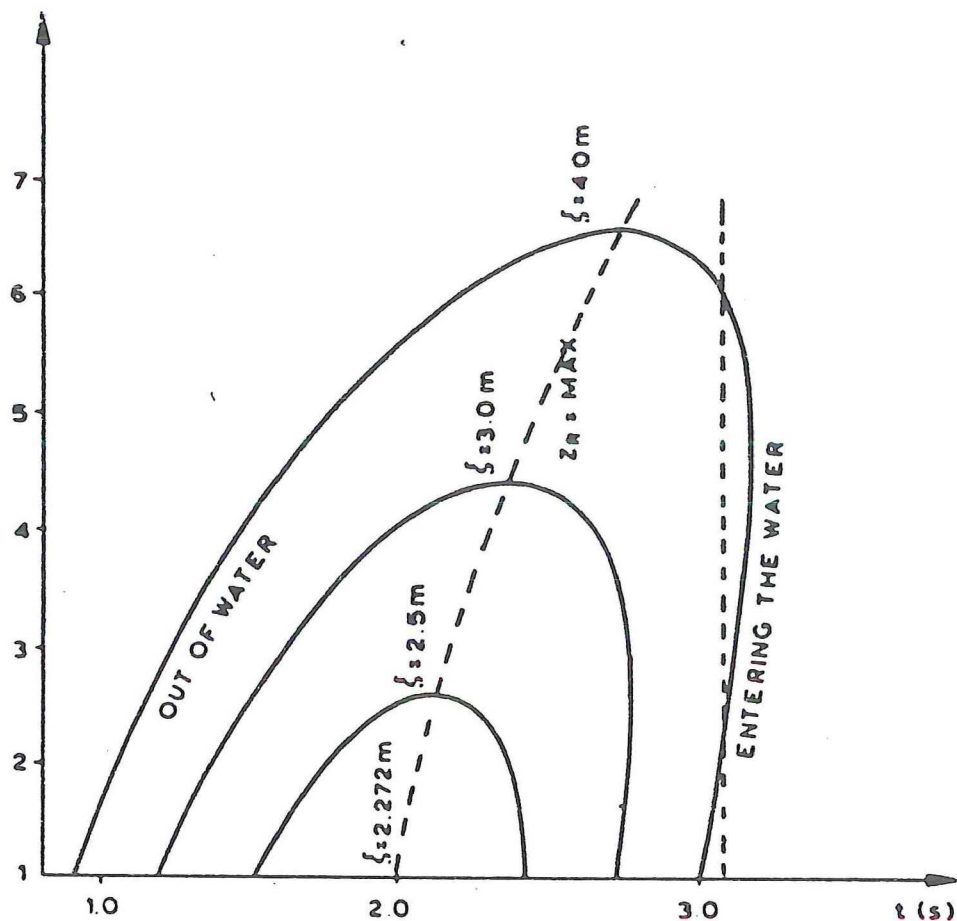


Figure 2. Relative motion of the ship in waves indicating the time when the ship sections go out of the water and enter again in the water. For regular waves of amplitude  $\xi$ , the forward perpendicular is section 1 and midship is section 11. The vertical axis indicates the number of the transverse ship section, with section 1 at forward perpendicular and section 20 at aft perpendicular.

from bow sectors at various speeds. A typical result is shown in Figure 2, which indicates the instant in time that the various ship sections go out of the water and re-enter it. It was found that in most cases the slam initiates at a section aft of the bow and propagates forward, although, sometimes it starts also forward and propagates aft. This situation is indicated in Figure 2 for the case of a wave amplitude of 4.0 m.



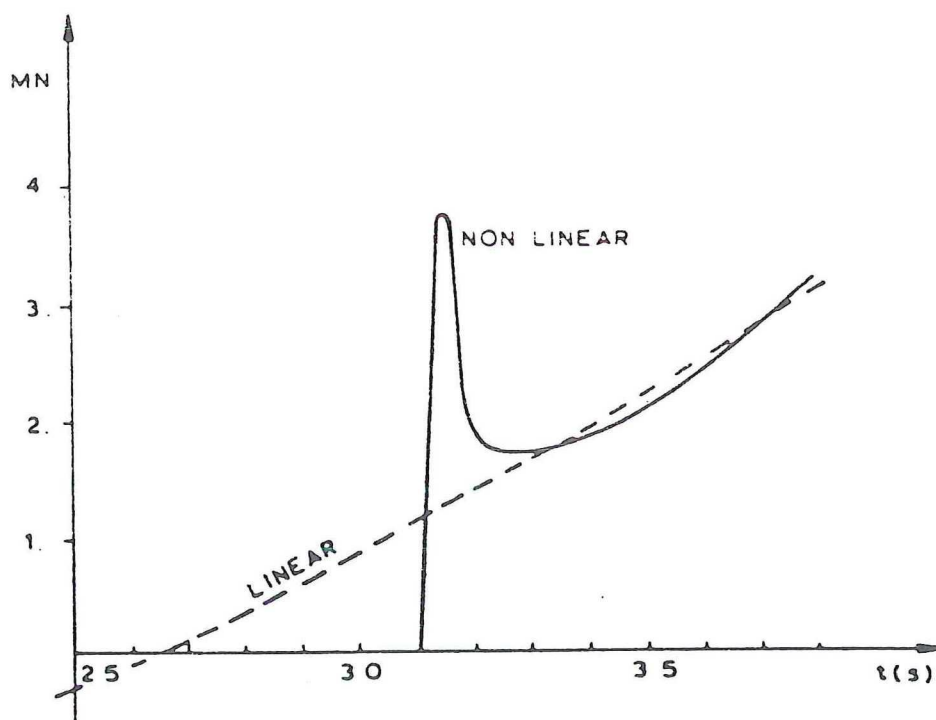


Figure 3. Wave exciting force predicted by the linear theory that does not account for the section emergence of the water and the transient non-linear force due to the wave slam.

The slamming load as predicted by Equation (2) has been calculated and compared with the load that is predicted by linear theory. Since ship motions are the result of the linear theory, the load that has not been accounted yet, and which will excite the transient response, is the difference between the two. Figure 3 compares both indicating that the transient load has a duration of about 0.25. The main contribution to the pressure peak results from the first term in Equation (2) which is proportional to the velocity squared. This term decreases very quickly, but after some time the last term of Equation (2), which represents the buoyancy, increases quickly and makes the non-linear force approach the linear term.

The dynamic amplification provided by the first term only occurs whenever the section goes out of the water. This can be shown by calculations in which the ship section just touches the sea surface, in one

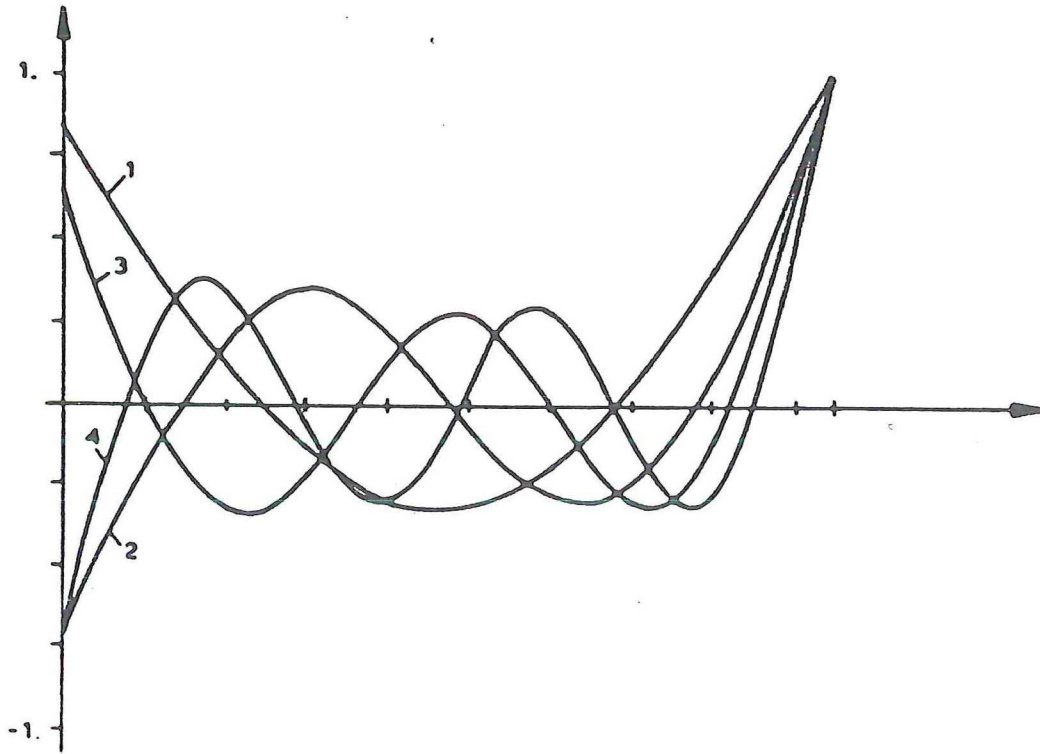


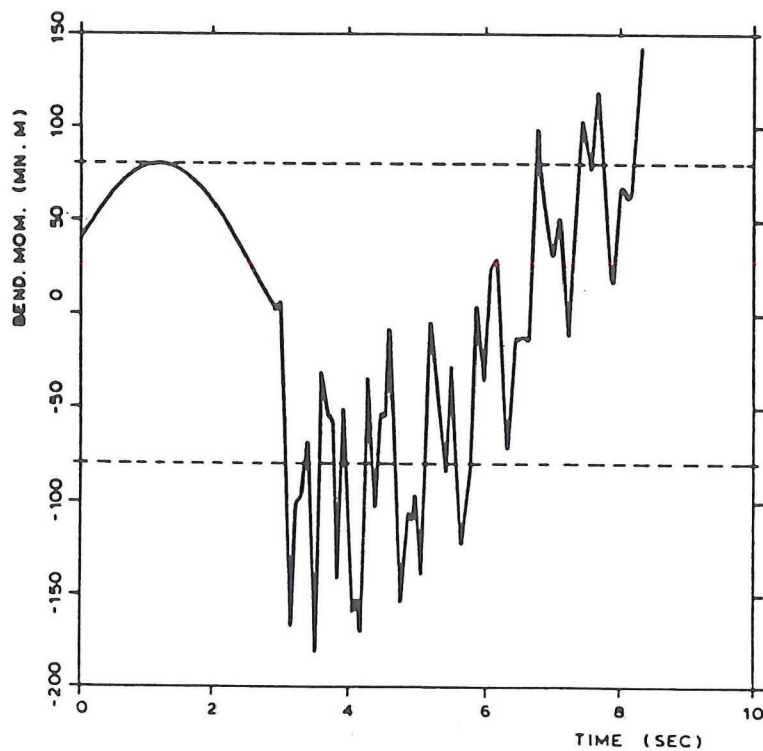
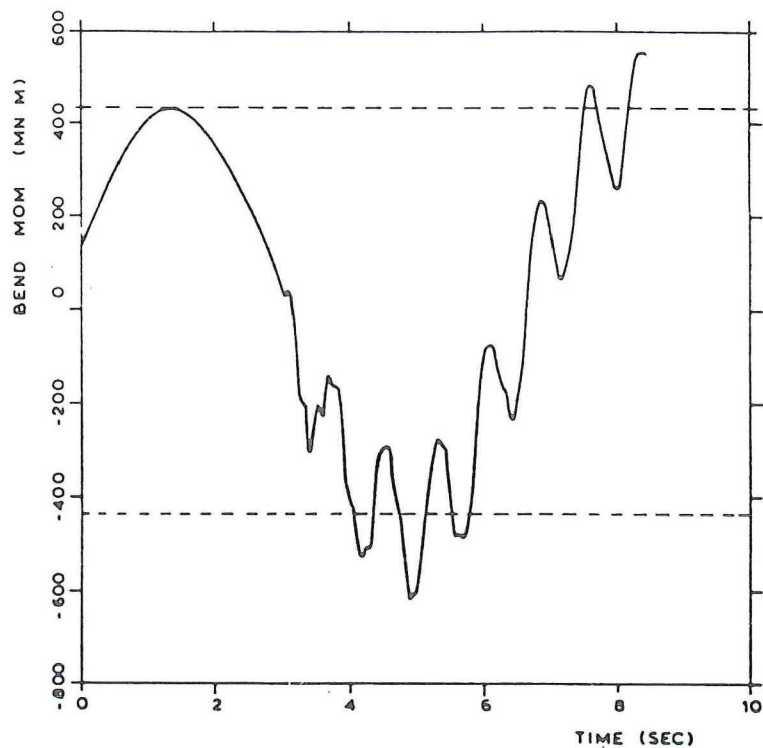
Figure 4. Four lower elastic mode shapes of the Gopher Mariner.

case going out of the water and in other not. In the latter, the force predicted by Equation (2) varies gradually without any sudden change.

The slamming load is represented as several time series applicable in various sections of the ship. The hull was represented by 20 beam elements and the buoyancy provided by the water was represented by spring elements. The mode shapes that have been determined with this model are indicated in Figure 4 for the four lower elastic modes. The corresponding natural frequencies are 8.14, 18.84, 26.86 and 40.26 rad/s, respectively. The frequencies corresponding to the rigid body motions of heave and pitch were .93 and .97 rad/s, which are higher than the values of .84 and .86 that have been predicted from the calculations of ship motions. The elastic natural frequencies are slightly higher than the ones reported in [28] but the first mode is very close.

To calculate the hull response, values for the modal damping were assumed as a percentage of the critical damping. The values adopted for





Figures 5 and 6. Time history of the wave induced bending moment amidships and in section 5 from the bow, showing the high order components in the latter case and the exceedance of the response above the linear case whose amplitude is shown dotted.

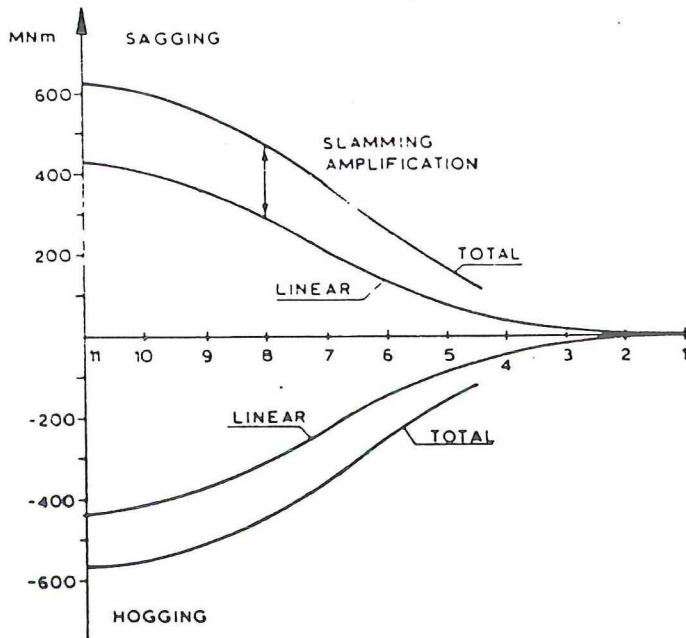


Figure 7. Amplitude of total response and of the linear component of wave induced bending moments in various ship sections from the forward perpendicular (1) to midships (11) for a regular wave system with an amplitude of 3.5 m.

the four lower elastic modes were respectively .003, .008, .013 and .020. A typical response is shown in Figures 5 and 6 corresponding to points close to the midship and in the quarter length forward. It can be observed that in the midship the response is basically in the lower mode while towards the bow higher modes are present.

The initial part of the response indicated in Figures 5 and 6, corresponds to the bending moments induced during motions predicted by the linear theory. The effect of the slamming can be represented as the increase in the amplitude of the response over the linear response, which is indicated with dotted lines in Figures 5 and 6. This amplitude varies along the length of the ship, as indicated in Figure 7 for one case considered. The effect of the slamming depends also on the amplitude of the wave, increasing with it, as indicated in Figure 8.

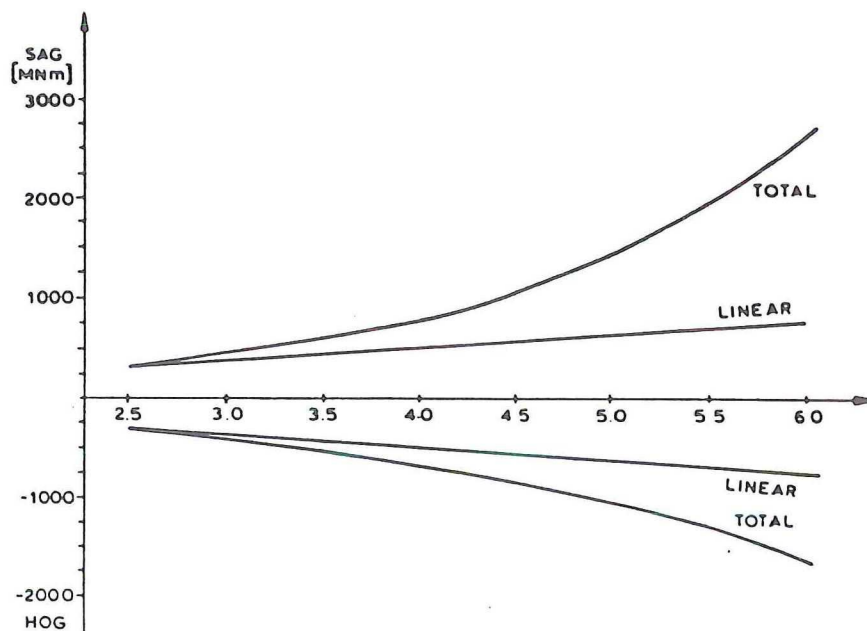


Figure 8. Comparison of the amplitude of midship wave induced bending moments predicted by the linear theory and the vibratory response with regular waves with amplitudes varying from 2.5 m to 6.0 m.

## 5. Concluding remarks

Computer programs have been developed to simulate the relative motion between a ship and waves, to calculate the hydrodynamic impact forces that arise when ship sections impact the waves, and to calculate the structural response to those forces. Results of some calculations have been reported here indicating important features of the three aspects of the problem. The program can be used further, in parametric studies to generate guidelines for structural design rules.



### Acknowledgements

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