
Advancing the roll-out of energy networks by embedding cost differentiation in
modeling approaches which minimize assessed construction costs

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1 Introduction

1.1 Energy networks and their relevance to society

Energy networks, like the power grid and oil and gas pipelines, form the backbone of our energy system. These networks are essential to society as they supply utilities. Society, however, faces challenges stemming from decreasing energy reserves and pollution caused by harmful emissions [21]. A shift in the way in which energy is produced, transmitted and used is therefore needed. Existing grid infrastructures might not be up to cope with that challenge. They can be too old and fragmented [34, p. 3] or it cannot be feasible to adapt them. As a consequence, new energy infrastructures have to be constructed. By developing and rolling-out these networks, the rate of decline in energy reserves might decrease and adverse effects from pollution might be reduced.

When talking about the development of completely new energy networks, investment costs play an important role [11, p. 29], [10]. A study of the European commission estimated in 2010 that around one trillion euros must be invested in the energy system until 2020 in order to meet policy objectives and climate goals. About half of this amount will be needed for networks, including storing, transmitting and distributing electricity and gas [34, p. 11]. Part of the required investment will be provided by private sector investment. This involvement will continue to be insufficient and a gap between required and anticipated investment is expected to remain [18], [34, p. 11]. This is partly caused by “projects with positive externalities and wider European benefits, but no sufficient commercial justification [ibid.]”. This gap might be closed by proposing energy networks with lower estimated building costs, but which have the same functionality as their more expensive counterparts. Lower up-front costs will lead to a higher return on private sector investments and will make these investments more attractive.

A large part of the investment costs of new energy networks are determined by design and construction costs [21]. In order to minimize the construction costs associated with building a new energy network, two different modeling approaches have been developed in literature [14]. Both a top-down and bottom-up approach allow to connect a single source node to multiple consuming nodes in an area consisting of allowed and no-go regions [ibid.]. In the future, planners involved in the development of an energy network might make use of tools based upon these approaches. Given particular input information, these approaches allow a planner to determine network configurations while minimizing construction costs.

The assumptions underlying these modeling approaches provide a scope for being relaxed. Assuring that a particular assumption is in better agreement with reality is of central importance in this thesis. The modeling approaches have been developed to design a lay-out for an energy network with minimal construction costs. Actual construction costs of an energy network can be represented by the estimated costs plus an error in the estimated costs. Minimizing actual construction costs therefore implies minimizing both the estimated costs and the accompanying error. The latter can be achieved by assuring that input information is reliable, i.e. in agreement with reality. Given a more accurate assessment of reality, the internal aim of this thesis is to minimize the former term.

To be more specific, when a new energy network is developed in a particular area, this area might comprise of 'different cost regions' instead of allowed and no-go areas. Accounting for regions that have different construction costs is in better agreement with reality [14]. If this additional characteristic is precisely known by a planner, then the question arises which network configurations minimize building costs. In turn, addressing this question might enable the roll-out of valuable energy networks. The goal of this thesis is to examine how the two modeling approaches can be extended such that they are able of coping with different cost regions. This thesis aims to be a contribution to the development of new energy networks by presenting and discussing ways to minimize assessed construction costs associated with these networks. It does this by providing an extension of already existing methods to estimate building costs.

1.2 Research questions

Based on these observations, the main research question is: *How can estimated construction costs of new energy networks be minimized by embedding cost differentiation in existing modeling approaches to advance the roll-out of these networks?* Corresponding to this main question, the following sub questions are addressed in this thesis.

1. What does an energy network comprise of?
2. Which stages can be distinguished during the development of an energy network?
3. Why are assessed construction costs relevant for the completely new development of energy networks?
4. What is the importance of different cost regions for the construction costs of energy networks?
5. Which factors determine the actual construction costs associated with the completely new development of energy networks?
6. Which methods can be used to minimize construction costs and in which manner can cost differentiation be implemented in the existing modeling approaches?
7. What are typical modeling results obtained from this implementation?
8. How can the results be tested and how do these extended approaches compare on assessed construction costs when challenged with different situations?

It is expected that the insights obtained from extending the modeling approaches allows us to formulate recommendations that improve the roll-out of new energy networks. The to be formulated recommendations are aimed at planners associated with producers, consumers and governments involved in the completely new development of energy networks. The reason for formulating these recommendations is to advance the roll-out of these networks.

1.3 Thesis structure

As mentioned at the start of this introduction, energy networks are considered in this thesis. What do these networks comprise of and which phases can be distinguished during their development? The first phase, their completely new development, provides an opportunity to introduce networks with beneficial properties [15], [11]. We already stated that assessed construction costs determine whether these networks will be realized. Why are these costs relevant for the building of new energy networks? And what is the role of regions that have different construction costs in this respect? The answers to these questions establish a foundation for the subsequent research questions.

Having examined the relevance of construction costs, the question arises which factors determine construction costs. A planner associated with the development of a network has to deal with these factors. In turn, the factors and their effects on construction costs determine whether an energy network is eventually built. Coping with all factors is beyond the scope of this thesis. Therefore, it is assumed that the initial steps of the planning process are finalized and that particular input information is precisely known. This input allows the two modeling approaches to be applied.

The answers to the previous research questions form a preamble to extending the modeling approaches developed in literature. A substantiation of the importance of construction costs, which are partly influenced by the factor 'different cost regions', raises the following question: in which manner can cost differentiation be implemented in the modeling approaches developed in literature? And as a follow-up question, how do these two extended methods compare on assessed construction costs when challenged with different situations?

Corresponding to the research questions, the outline of this thesis is as follows. The second chapter will be aimed at the characteristics of energy networks. We will also address the different stages that can be resolved during the evolvement of these networks. Furthermore, it is examined why construction costs are relevant for the realization of new energy networks and what the influence of different cost regions is. Subsequently, will focus on facts relevant to the construction costs associated with building energy networks 'from scratch'. A distinction will be made between factors that will be addressed in this thesis and factors that are assumed to be known.

Two different methods, which are both capable of addressing the routing of networks, are considered in the third chapter. These routing algorithms comprise of a prescribed set of statements via which it is attempted to construct a layout for a networked infrastructure while satisfying certain constraints. In this theoretical section, it is described what is already known from literature on the geometric graph method (GGM) and agent-based modeling (ABM). Next, the GGM and ABM are refined by incorporating cost differentiation. Examples of cost minimizing networks will be looked at and a comparison of modeling results be performed. Lastly, recommendations for advancing the construction of new energy networks are considered. Moreover, recommendations for future research will be formulated.

2 Energy networks

Firstly, it is described in this chapter what energy networks are. We will subsequently address the different stages that can be distinguished during the development of these networks. This allows us to specify to which stage this thesis aims to contribute. As already mentioned in the introduction, advancing the development of *new* energy networks by minimizing estimated construction costs of energy networks is of central importance in this thesis. Thirdly, the significance attributed in this thesis to the construction costs of energy networks and 'different cost regions' is substantiated. The latter can be encountered while constructing energy networks in a particular area comprising of different regions. Lastly, the factors determining construction costs are discussed.

2.1 Development stages of energy networks

The nature of energy networks is socio-technical, which means that they have both a social and physical structure [21]. The social structure is related to the organizations that design, operate and make use of these networks. From a physical point of view, these infrastructures enable distribution of energy. Distribution is established by suppliers and consumers which are physically connected. The term networked energy infrastructure is used to emphasize the physical aspect of energy networks [13]. The focus of this thesis is on the design of the physical structure of energy networks. This structure is conceptually represented as a network consisting of links and nodes. The links enable the flow of energy carriers between the nodes, where these carriers are processed [21]. The power grid facilitates the transport of electricity from producers to consumers for instance.

Due to the socio-technical nature of an energy network, it needs to evolve over time. Organizations involved in a particular network might sense growth opportunities for instance and therefore pursue expansion of the operational network. An example is household access to electricity and gas in the Western part of the world. The relevant networks have expanded considerably during the twentieth century. The accompanying S-shaped adoption curve are, amongst others, displayed in Figure 1 [13, p. 69]. The ability of especially the power grid to adapt to an increase of its use has been impressive in this respect.

Different stages can be distinguished during the development of energy networks. The evolvement of a energy network starts with the *development* phase. The energy infrastructure has only a very limited reach during this phase. Concerning gas networks, for example, countries like Vietnam and Indonesia are currently in this phase [8]. Next, the *growth* phase might set in. The infrastructure is being built out and networks are fixated around large-scale supply [ibid.]. Lastly, consolidation begins and interlinkages between compatible energy networks are constructed during the *mature* phase. A schematic overview of network development is also represented by the adoption curves depicted in Figure 1 as development of networks goes hand in hand with adoption.

From these established phases of network evolvement, the development phase is most strongly related

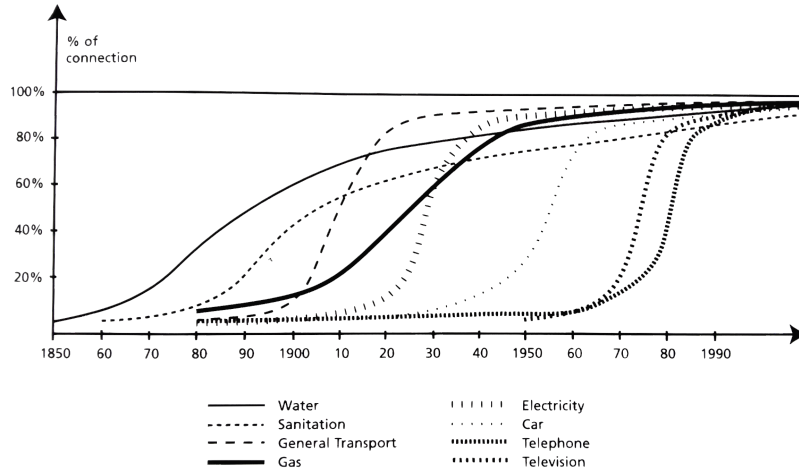


Figure 1: Adoption curves of household connection to urban infrastructures in the Western part of the world [13, p. 69].

with developing a completely new network. Developing a network from scratch is relatively costly, but there might be substantial benefits [8]. An exemplary location where new energy networks are rolled-out is the development of Maasvlakte 2 by the Port of Rotterdam [26]. This is a new stretch of land with ample opportunity for large ships to moor and new energy networks are being developed.

A new network might also be placed on top of or near an existing stretches of infrastructures. The reason for superimposing infrastructures is the cost lowering effect of synergy between the construction of different infrastructures [13, p. 30]. Transportation and energy infrastructures, for example, are often placed over another [13, p. 30]. Electric cabling have been laid alongside rights of way and infrastructures conduits which enable physical movement [13, p. 31]. The superimposition of different networks can be well understood as a large part of the costs of network providers are caused by constructing the network. An example of such a situation is considered later on in this thesis, for which we refer to Section 5.1. This observation already indicates the importance of network construction costs, which is examined next.

2.2 Relevance of network construction costs and different cost regions

An examination of why network construction costs and different cost regions are of importance for new energy infrastructure is performed in this section.

Firstly, the construction costs of networks have become more important due to two contemporary developments in infrastructures: *liberalization* and *internationalization*. Most networked infrastructures in developed countries have become accessible for private sector participation due to economic reform in the 1980s and 1990s [20]. Public and private monopolies have been replaced by profit-driven markets. A liberalized infrastructure captures this notion of multiple providers competing directly with each other to provide utilities or services within a regulatory framework [13, p. 152]. The undermining of monopolistic powers has been taken place around the world, although at very different rates and in very different

contexts [13, pp. 13-14]. There has been, however, a notable shift towards the liberalization of national and local infrastructure monopolies [20]. In addition, privatization has shifted the ownership of networked infrastructures from the public to the private sector [13, p. 151].

Alongside this development, internationalization of these infrastructure has taken place from the mid-1980s onwards. Internationalization is mainly concerned with the regional integration of infrastructures in the developing and developed world [20]. These developments enabled the formation of large electricity supply companies like RWE and EDF [ibid.]. A large part of the costs incurred by network providers can be attributed to constructing and maintaining the relevant networks. As these companies are operating in profit-driven markets, network construction costs are of importance.

Secondly, there is a variety of factors which can significantly effect construction costs at particular regions. At a country-specific level, the productivity and costs of labor, skills and experience, the climate, prices of material and equipment, overhead and profit can be important [32]. Further, remoteness and distance from major cities or supply centers might be of effect on a regional level. At a site-specific level, geographical location has been identified as a key cost driver [22]. The acquisition or rental cost of construction land can give rise to different cost regions in this respect. Last, but certainly not least, constructing connections such as pipelines through a watery region is more expensive than over land [1]. Therefore, geographical location has a significant influence on construction costs and is dependent on the scale which is considered.

The development of location cost-adjustment factors (LCAFs) also hints at the importance of different cost regions. A LCAF, which is also called simply location or area cost factor, is used to adjust project costs for different geographical locations. It recognizes “differences in productivity and costs for labor, engineered equipment, commodities, freight, duties, taxes, procurement, engineering, design and project administration” [22]. Most notably, the cost of land is usually not included. LCAFs are applicable to all types of vertical construction projects, but the same factors seem to be of importance for mainly horizontal projects like energy networks.

The different level specific factors, as discussed above, are also encountered in projects of this type. Dependent on the level that is considered, macro, meso or micro, different factors are of importance. These factors can have significant different values at particular regions. In turn, this can considerably affect construction costs in these regions [32]. At a regional level, for instance, area cost factors have been estimated. Exemplary countries are the United States and Australia, for which we refer to respectively [25] and [3]. Although location factors are difficult to estimate due to the multitude of factors involved, they are helpful to planners for estimating construction costs. In turn, this can streamline early decision making on construction projects [32].

Let us now summarize the previous observations. Both suppliers and consumers are involved in an energy network. From a technical point of view, an energy network consists of links and nodes. The connections between producers and consumers enable the distribution of energy. An energy network is therefore considered as a system which enables these suppliers and consumers of energy to physically

connect. Transportation of a commodity is made possible via the connections of a particular network. The different stages that can be distinguished during the development of networks have also been discussed in this section. This thesis will from now on focus on advancing the development of new projects as there is a potential to construct energy networks with beneficial properties [8], [11]. This implies that the development stage is considered most important.

Moreover, the preceding analysis emphasized the importance of the costs associated with constructing networks. Energy networks may need to be developed in an area comprising of different cost regions. Synergy between the construction of different infrastructures in cultivated regions and geographical location give rise to such regions. Due to liberalization and internationalization, the cost aspect of a project has witnessed increased attention. The introduction and employment of location cost-adjustment factors also hints at this development. Given these observations on the relevance of construction costs, let us look more specifically at the factors which determine the construction costs of new energy networks.

2.3 Factors determining construction costs of new energy networks

This subsection will focus on factors relevant to the construction costs associated with the development of completely new energy networks. Which facts are relevant during the development stage of evolution? As already mentioned in the introduction, a large part of the investment costs of new energy networks is determined by design and construction costs [21]. These costs are affected by a wide variety of factors. These factors are discussed since this provides insights into the interdependent ways in which construction costs are affected. However, this thesis is focused on a particular subset of these factors. Agreement on input information for the modeling approaches has to be reached before these can be used sensibly.

2.3.1 Planning uncertainty

Let us recall that an energy network is a physical infrastructure which enables the exchange of an energy carrier between suppliers and consumers. As a consequence, decisions on the development of these networked infrastructures have to be taken in a multi-actor context. As pointed out in [16], the involved parties face a chicken-and-egg problem. The utility provider would like to have a reasonable amount of certainty about potential demand before getting involved in the development of a network. On the other hand, clients prefer to have a good indication of the utility provided by a network before making the commitment to join that particular infrastructure. It might therefore take a substantial amount of time before potential suppliers and consumers commit to a particular network. In turn, the number of suppliers and clients that are willing to commit may be a decisive factor for the success of a project. When a substantial amount of consumers would like to get involved, investment costs can be spread out over these consumers. Furthermore, it might be more beneficial for producers to join. Investment costs are therefore partly determined by the number of suppliers and consumers that are to be connected by the networked infrastructure.

Moreover, the locations of producing and consuming nodes might not be known when a new network

is developed. Producers and suppliers can be separated by a large distance or can be situated relatively closely together. Thirdly, the decision to commit to a network is related to the required capacities of the connections. These capacities determine how much and at what speed a commodity can be provided. A producer could want to supply in large quantities in order to attain economics of scale. An other reason might be that it is expected that the price of the commodity, which is transported by the future network, will increase. In addition, it could be beneficial to construct a network with connections having larger capacities than required at this moment. This enhances the adaptivity of a network and ensures that it is able to cope with future growth.

Lastly, the roll-out of a network might be constrained in terms of costs by the presence of a river or densely populated area [16]. It is more expensive to build a pipeline through a watery region than over land for instance [1]. A planner might not have complete information on the construction costs associated with a particular region. This factor in conjunction with uncertainty about the number of suppliers and consumers, their respective locations and the required capacities gives rise to *planning uncertainty*.

A planner, which can be related to a producer, consumer or government, associated with the development of an energy network must be able to cope with this uncertainty. In other words, he or she has to plan under uncertainty. The planner has to consider the effect of uncertainty on its plans and attempt to select plans that avoid uncertain outcomes. If the latter cannot be accomplished, contingency plans against different possible outcomes can be constructed [2]. In short, the planner has to take decisions while future developments are uncertain [17].

Some of the planning uncertainty might be resolved through collaboration. For example, the commitment of renowned suppliers provides a clear signal for consumers. The initiators of the Gate terminal project, which is an import terminal for LNG situated in the Netherlands, have been able to gain notable consumers [35]. Since commitment of parties can be established over time and through collaboration, the investment costs associated with this factor are of indirect nature. On the other hand, the locations of the participators and required capacities have a direct effect on investment costs. These is also an interdependent aspect: the commitment of consumers can be dependent on to be provided capacity by the network. Moreover, it might be difficult to arrive at accepted expectations on future commodity prices through collaboration for instance.

These factors related to planning uncertainty will *not* be further considered in this thesis. These have been discussed to provide an overall picture on factors related with investment costs. The to be considered methods require some of the uncertainty involved to be resolved before an analysis can be performed. It is not feasible to examine the entire space of plans as the number of possible network configurations can grow very large. Therefore, it is assumed from now onwards that the supplier and consumers, their locations, required capacities and the costs of the different cost regions are fully known. The initial phases of the decision making process should resolve uncertainty about these aspects before the modeling approaches can be applied.

2.3.2 Routing problems

Given this input information, the planner now faces a *routing problem*. How should the suppliers and consumers be connected such that the required capacities are satisfied? As argued in Section 2.2, the construction costs of a network are a relevant constraint in this respect. In order to reduce these costs, it might be beneficial to join particular connections and to introduce auxiliary points in a network. Moreover, the fact that a particular area might comprise of different cost regions causes the number of relevant routing configurations to grow considerably. Dependent on how expensive it is to build in a particular region, it can be beneficial to partly go through or to circumvent that region. If no different cost regions are present, then no benefit is obtained from constructing a roundabout route. As a consequence, a planner faces a more difficult job when an area comprises of different cost regions.

The factors of joining connections, introducing auxiliary points in networks and different cost regions partly determine investment costs. A planner has to resolve the routing problem caused by these factors before the construction of a completely new network is able to begin. Furthermore, these factors have a direct influence on investment costs. These three factors will be addressed in this thesis. The first two factors already have been dealt with by the literature on the to be considered methods, see [14] and [15]. The focus will therefore be on addressing routing problems in areas comprising of different cost regions. Addressing this third factor extends the existing methods. In addition, it has the potential to provide network designs that are less expensive to realize.

To recapitulate, a distinction is made between investment determining factors related to planning uncertainty and factors related to routing problems. An overview of the relevant factors is displayed in Table 1. The factor related to planning uncertainty will not be further considered in this thesis while directing networks along specified courses will be. This implies that the initial phases of the decision making process should resolve uncertainty about these aspects before the modeling approaches, which are discussed next, can be applied.

Table 1: Investment determining factors related with planning uncertainty and routing problems. The former are required input information for addressing the routing problem.

Planning uncertainty	Routing problems
Commitment of suppliers and consumers	Joining connections
Location of producing and consuming sites	Introducing auxiliary points
Capacity of connections	Different cost regions
Actual construction costs associated with particular regions	

3 Theory on the geometric graph and agent-based methods

3.1 Introduction

Two different approaches that are both capable of addressing routing problems are discussed in this section. These approaches belong to two different classes, which are named the geometric graph and agent-based methods. A description will be given of what is already known from literature on the geometric graph and agent-based methods. These procedures are employed to minimize the estimated construction costs of networked infrastructures. The reason for considering these methods is that these seem to provide valuable insights into how networks can be designed cost effectively [14]. In addition, these methods are able to cope with networked infrastructures from different sectors and are therefore to a certain extent generic. These methods assume that a commodity is transported from a supplier to various consumers. Energy infrastructures are considered in this thesis.

Before discussing these methods, let us further consider which features of energy networks are taken into account by the existing methods developed by Heijnen, Chappin and Nikolic [14]. A networked infrastructures allows for the physical exchange of a commodity or service between suppliers and consumers. In this context, a source is defined as a node where the commodity is produced or supplied and a sink as a consuming node. The connections between these nodes are also called edges. Heijnen et al. make the following modeling assumptions concerning a network:

1. A network consists of one producing and several consuming nodes. A commodity is transported via the connections between the source and sink nodes. It is assumed that network connects all nodes and that the source can meet the total demand of all consuming nodes.
2. The costs of building a network are mainly determined by the physical distance that has to be bridged by the connections.
3. The construction costs can also partly be determined by the required capacities of the edges. The capacities of these connections satisfy the demand of the consuming nodes.
4. Networks might have to be built in an area comprising of regions that bar some of the routing possibilities [15].

The first assumption might not be a good representation of reality and research is to be performed to relax this assumption [ibid.]. The other characteristics are assumed to be realistic for a network. These four assumptions are currently incorporated in both the geometric graph and agent-based methods [14]. The theoretical concepts underlying these methods are discussed next.

3.2 Geometric graph method

Over the last decades, several related methods have been developed to incorporate some of the above described characteristics in routing algorithms, see [12], [33] and [15] for instance. Let us recall from

the Introduction that routing algorithm is a prescribed set of statements via which it is attempted to construct a layout for a networked infrastructure while satisfying certain constraints. In this section, a recapitulation of the relevant graph theoretical notions is given. Moreover, the algorithm developed by Heijnen et al. [15], which addresses the four characteristics, is described in parallel.

The objective of the geometric graph method (GGM) is to find a network that minimizes construction costs while satisfying the described characteristics. The total costs $C(N)$ of a network N is the sum of the costs encountered at all edges:

$$C(N) = \sum_{e \in E} l_e f(q_e), \quad (1)$$

where E is the set of edges of N , l_e is the length of an edge e and $f(q_e)$ is the cost per unit length of constructing a connection e with a flow capacity of q_e . The latter function is assumed to be given by

$$f(q_e) = q_e^\beta \quad \text{with} \quad 0 \leq \beta \leq 1, \quad (2)$$

in which β is called the capacity cost exponent. The lower the cost exponent, the more beneficial it is in terms of construction costs to merge connections. However, this is an empirical parameter and a typical value for it is 0.6 [16]. Figure 3 demonstrates the influence of the cost exponent on a cost minimizing network.

The GGM starts off by defining an initial network using a non-crossing spanning tree which connects all nodes and has the required capacities as weights on the connections [15]. Together with a set specifying the edges and the accompanying weights a weighted graph is defined. In [15], the initial network is chosen to be a minimal length spanning tree or star network. The former is a tree which connects all sources with a total length less than or equal to the length of every other spanning tree. The latter is a network in which the nodes are directly connected to the source.

The initial network is then locally changed if a network is found with lower total building costs. A network is locally minimal if it satisfies a certain angle constraint, see [33]. If this is not the case, then costs can be reduced by partly joining two adjacent connections. This occurs when the angle between those connections is relatively small. The relevant angle constraint has been specified by Thomas and Weng [ibid.]. This inequality is dependent on the cost per unit length $f(q_e)$. If a splitting point is introduced, then the location of this point can be determined by using the geometric method developed by Heijnen et al. [15]. For determining this location, the angles of incidence to the so-called Steiner point have to be calculated amongst others. This can be achieved by solving the force equilibrium as displayed in Figure 2 [16]. After introducing the relevant Steiner points, a network is found with local minimal building costs. An example of such a network is given in Figure 3. Additionally, no-go regions can also be accounting for in this algorithm by rerouting various connections to allowed regions.

3.3 Agent-based modeling

A fundamentally different method to construct costs minimizing networks is an agent-based model. In contrast to the geometric graph method, agent-based modeling is a bottom-up approach. A specified

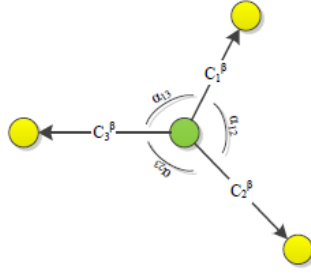


Figure 2: Force equilibrium that needs to be solved for obtaining the edges incident to a Steiner point [16]. The relevant capacities are named C_i , β is the capacity cost exponent and the angle between edge i and j is named α_{ij} .

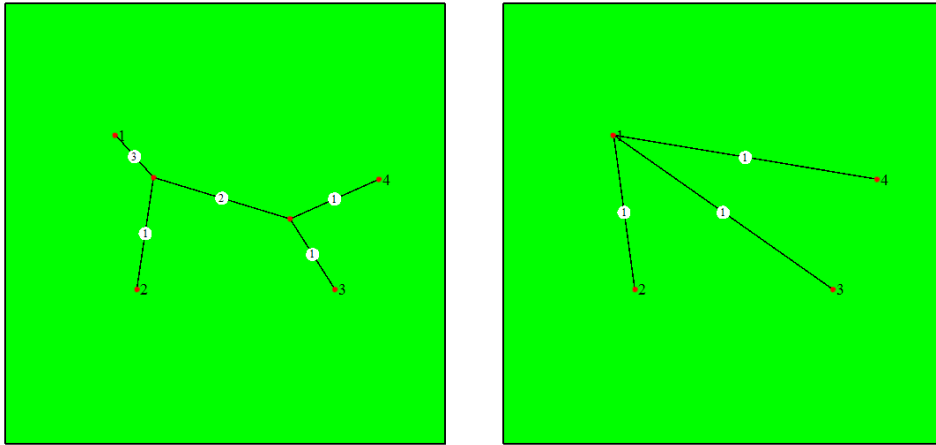


Figure 3: Cost minimizing networks when the capacity cost exponent is 0.65 and 0.99. The edge labels indicate the capacity of the connections. The network on the left contains two Steiner points while the network on the right is also called a star network.

number of agents interact with each other and with the encountered environment in an agent-based model. Their behavior is also partly determined by parameter values set by the model maker. These agents make decisions and change their actions based on these interactions [9]. One is then interested in how and which macro phenomena are emerging from this micro level behavior among the interacting agents [19]. A particular way to implement an agent-based model is to employ ant colony optimization (ACO) algorithm, which is discussed next.

An ant colony is, from a biological point of view, a community of ants living close to together. Some ant species are able to collectively find the shortest path between the nest of the ant colony and a food source [30]. When the path is obstructed or destroyed, alternative paths are found. An ant deposits a pheromone trail on the ground while it moves around and is also able to detect the intensity of such a pheromone trail. Initially, ants choose their path randomly. After some time, however, the probability

of choosing a direction will depend on the intensity of pheromone trails on the ground [30]. In turn, subsequent ants will reinforce the pheromone trail with their own pheromone and positive feedback occurs. This principle is illustrated in Figure 4 [5]. Almost all ants will eventually choose the shorter path. Some ants may not follow the highest pheromone trail and thus exhibit exploratory behavior [31].

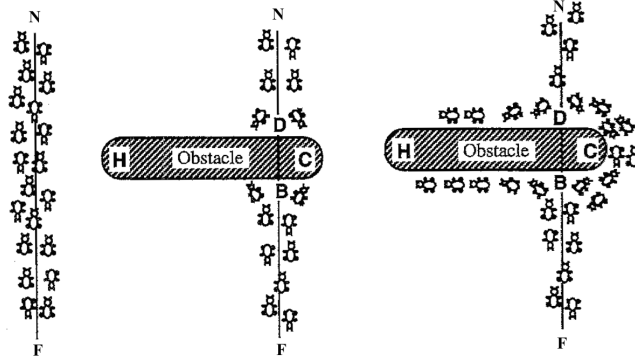


Figure 4: An example of ants which move around while suddenly an obstacle is placed on the shortest path from the nest N to the food source F [5].

These characteristics can be modeled in an optimization algorithm. The behavior of ants, as previously described, is a main source of inspiration for the development of ant colony optimization (ACO). An ACO algorithm is a multi-agent system in which the agents are artificial ants [36]. These artificial ants build solutions to a considered optimization problem [4] and communicate in a comparable way to real ants. Choices have to be made concerning, for example, the updating, evaporating and diffusing of pheromones. Different ACO algorithms have therefore been constructed, see for example [6]. Some features of artificial ants have been implemented in such a way that they do not correspond with those of real ants any longer.

An ant colony optimization algorithm can be used in order to construct cost minimizing networks. Such an algorithm has been developed by Heijnen, Chappin and Nikolic. Ants move across a particular area, lay pheromones and search for food in this model [28]. This is a bottom-up approach to find “a connected set of discretized patches that link all consumer nodes with the source at the lowest network cost” [14, p. 4]. A lowest cost network is built in steps as ants record the capacity required by the encountered consumer node and the set of patches on their return path to the source. These ants are attracted to the nest by the scent of this source [14]. The various sinks are connected to the source one by one after a specified number of ants have returned from a particular sink to the source without reducing construction costs. The barring of some regions can also be incorporated in such a model by assigning high building costs to patches located outside the allowed region. It is very unlikely that these patches become part of the network in this way.

In this section, we have made a review of geometric graph method (GGM). This is a top-down approach via which it is attempted to deterministically construct a totally new network from scratch with minimal costs. A review on the main concepts of agent-based modeling (ABM) was also given.

This is a bottom-up, stochastic approach. Having discussed the theory, manners to incorporate different cost regions are examined next. The reader less interested in the technical details of these methods can skim through this section by paying attention to the various figures.

4 Modeling cost differentiation in the GGM and ABM

When an energy network has to be developed in a particular area, regions can be encountered that differ in terms of construction costs. As we argued in Section 2.2, accounting for these regions while using the considered modeling approaches has the potential to provide insights into how such a network can be developed cost effectively. Therefore, the focus of this section is on incorporating cost differentiation in the geometric graph method (GGM) and the agent-based model (ABM).

4.1 Implementing cost differentiation in the GGM

Let us first focus on a manner to account for different cost regions while using the geometric graph method. A specific type of problem, the weighted region shortest path problem, is relevant for constructing networks in different cost regions. A justification of this is given below. This type of problem, however, cannot be solved exactly as pointed out by De Careful et al. [24]. After having defined the considered cost differentiation problem in networks, we therefore focus on an algorithm which can construct an approximate solution.

4.1.1 Problem definition

Specific information is required in order to construct a particular network from scratch within regions having different construction costs. For applying both the geometric graph and agent-based methods to a considered area, the following information has to be (approximately) known:

- the physical location in the Euclidean plane of the producing and consuming nodes
- the required capacities demanded by these nodes and
- the construction costs of the different regions comprising the considered area.

This input information coincides with the factors related to planning uncertainty, which are contained in Table 1. Notice that no-go regions are equivalent to regions with very high building costs. The characteristic concerned with no-go areas, as previously discussed, becomes partly redundant in this way.

Assuming that this information is available, the total construction costs of a particular configuration of a network can be calculated. In order to so, we introduce the following notation. Let the set A contain the physical location of the producing and consuming nodes. These nodes can also be called terminals. We assume that a_1 is the source (sink) node while the subset $\{a_2, \dots, a_n\}$ consists of the consuming nodes. In addition, auxiliary points might be introduced in the network which can reduce network costs. These are called Steiner points and their location is assumed to be contained in the set B . A particular network configuration consists of connections between the elements from $D = A \cup B$. A connection or edge e can be defined by two physical points from D and these are named e_1 and e_2 . These edges of a network N are collected in the set E and q_e is the capacity assigned to a particular edge e .

The total costs $C(N)$ of a network N is the sum of the costs encountered at all edges:

$$C(N) = \sum_{e \in E} C(e) = \sum_{e \in E} \int_{e_1}^{e_2} h(q_e, x, y) dl. \quad (3)$$

The costs of constructing edge e is called $C(e)$ and can be calculated with the use of a path integral between points e_1 and e_2 . The accompanying integrand h is a function of required capacity of edge e and actual position. This function represents construction cost per unit length and it is assumed that it can be expressed as a product of two factors. Firstly, a weight function $g(x, y)$ can be defined on the considered area Ω :

$$g(x, y) : \mathbb{R}^2 \rightarrow \mathbb{R}^+ \quad \text{for } (x, y) \in \Omega \quad (4)$$

Given an arbitrary standard region, this function represents how many times more expensive it is to build at position $(x, y) \in \Omega$. Secondly, the other factor can be represented by an unweighted (or standard) cost per unit length of an edge to which a flow capacity of q_e is assigned. This factor is represented by $f(q_e)$. Throughout this thesis, it is assumed that

$$f(q_e) = q_e^\beta \quad \text{with } 0 \leq \beta \leq 1. \quad (5)$$

The influence of the cost exponent on a cost minimizing network can be seen from Figure 14. Cost advantages can be attained by building high capacity connections if $0 < \beta < 1$ [15].

Since the latter function is independent of position, the total network costs can be calculated as

$$C(N) = \sum_{e \in E} \int_{e_1}^{e_2} h(q_e, x, y) dl = \sum_{e \in E} f(q_e) \int_{e_1}^{e_2} g(x, y) dl. \quad (6)$$

If $g(x, y) = 1$ for all $(x, y) \in \Omega$, then this expression reduces to $C(N) = \sum_{e \in E} l_e f(q_e)$ where l_e is the length of edge e . As to be expected, this result coincides with Equation (1). The aim is to find a network that connects the specified nodes in such a way that that the required capacities of these nodes are satisfied while minimizing the cost $C(N)$.

The weighted region shortest path problem is related to developing networks in different cost regions in the following way. If both $D = \{a_1, a_2\}$ and $\beta = 0$, then our objective reduces to finding the optimal path from a given source node a_1 to a known destination point a_2 . The optimal path is the path with minimum weighted length, or equivalently which minimizes construction costs. For a further discussion on the weighted region shortest path problem see for example [27]. Note that weighted region shortest path problem is also obtained when $D = \{a_1, a_2\}$, $\beta = 1$ and $q_e = 1$.

As already mentioned at the beginning of this section, it has been shown that the weighted region shortest path problem cannot be solved analytically [24]. Constructing cost minimizing networks within different cost regions can be seen as a more general form of the weighted region shortest path problem as the cost function is dependent on the flow capacity and multiple destination and Steiner points can be introduced. As a consequence, this problem can also not be solved by analytical means. Therefore, an approximation algorithm will be constructed next. This approximation makes use of local optimization, since global optimization seems infeasible. An attempt is made to minimize $\int_{e_1}^{e_2} g(x, y) dl$ for each $e \in E$ instead of minimizing Equation (6) globally.

4.1.2 An analogy from optics

Most of the know research works on approximation algorithms have made the simplifying assumption that the planar space can be divided into polygons and that the weight within each polygon is uniform [27]. In this way, one can prove that the optimal path of the weighted region shortest path problem consist of straight line segments. The endpoints of each segment are then on the boundary of a polygon [ibid.]. This assumption is also made in this thesis as it turns out to be helpful for constructing an approximation algorithm. In addition to this, it is assumed that the start and destination point of each edge in the network N are known. Use can then be made of local optimization. It has to be emphasized that the Steiner points which are contained in N are also introduced according to a local minimality criterium [15]. In this respect, a network is called locally minimal if no perturbation of the Steiner points reduces the costs of the network [33]. Again, it is not know how to introduce Steiner points in a global minimal way.

Let a particular area comprise of polygons which specify the construction costs in particular regions. The central idea of the approximation algorithm is to first select the boundaries of these polygons alongside which the construction costs between start point e_1 and destination point e_2 is minimized. Once these boundaries are known, costs can be minimized in a continuous like fashion by employing numerical optimization. This procedure can be iterated over all edges in E and yields a (sub-)optimal cost minimizing network. The second step of continuous numerical optimization can be justified by considering an analogy from optics [23].

Let us for the moment assume that a ray of light travels from a source point S to a destination point P . In addition, it travels from a region with index of refraction n_i to a region with a larger index of refraction n_t . A schematic overview of such a situation is given in Figure 5 [7]. According to the *Principle of Least Time*, which was formulated by Fermat, the actual path between points taken by a beam of light is the one that is traversed in the least time [ibid.]. The speed with which light can travel through a region is inversely proportional to index of refraction of that region. Hence, the considered refraction problem from optics can be seen as an instance of the weighted region shortest path problem [23].

By applying the principle of Fermat to the considered refraction case, the fact that light satisfies Snell's law can be derived [7]. The time t required to travel from point S to point P in Figure 5 can be expressed as

$$t = \frac{\sqrt{h^2 + x^2}}{v_i} + \frac{\sqrt{b^2 + (a - x)^2}}{v_t}. \quad (7)$$

As light seeks the path of minimum time, the time t needs to be minimized with respect to x . This problem is equivalent to an instance of the weighted region shortest path problem. A relatively high speed of light in a particular region implies a low index of refraction and therefore low construction cost per unit length in that region. If the index of refraction is interpreted as cost per unit length, then the

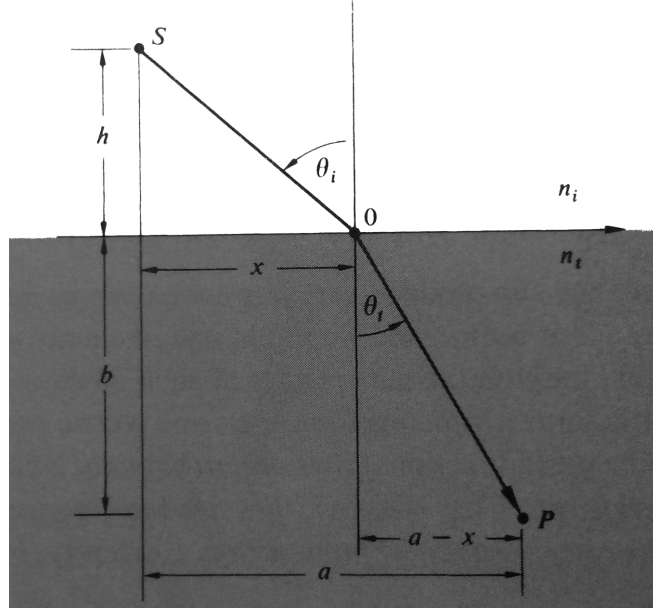


Figure 5: A ray of light traveling from source point S to destination point P while encountering regions with indices of refraction $n_i < n_t$ [7].

construction costs c can be calculated as

$$c = n_i \sqrt{h^2 + x^2} + n_t \sqrt{b^2 + (a - x)^2}. \quad (8)$$

These construction costs also need to be minimized with respect to x and c attains its minimum on the interval $[0, a]$. If the derivative of c with respect to x is calculated and set equal to zero, then the following is obtained:

$$\frac{dc}{dx} = \frac{n_i x}{\sqrt{h^2 + x^2}} + \frac{-n_t(a - x)}{\sqrt{b^2 + (a - x)^2}} = 0. \quad (9)$$

From Figure 5, it can be observed that $\sin \theta_i = \frac{x}{\sqrt{h^2 + x^2}}$ and $\sin \theta_t = \frac{a - x}{\sqrt{b^2 + (a - x)^2}}$. This allows us to rewrite Equation (9) to Snell's law, which is

$$n_i \sin \theta_i = n_t \sin \theta_t. \quad (10)$$

The angle θ_i is defined as the angle between the incoming ray and the normal to the region boundary. Likewise, θ_t is the angle between the transmitted ray and the normal [7].

It was assumed in the preceding analysis that the refraction index of the incident medium was smaller than the refraction index of the transmitting medium. If this assumption is violated, i.e. when $n_i \geq n_t$ then a phenomenon called total reflection can occur. A light ray that hits a boundary with $\theta_i \geq \theta_c = \arcsin(\frac{n_t}{n_i})$, the so-called critical angle, will be totally reflected from the point at which it hits the boundary [23]. The analogy of the weighted region shortest path problem with ray optics breaks down in this case. Once a path hits a boundary at an angle of incidence larger than the critical angle, then there is no longer complete information on where it goes next [ibid.].

Moreover, a ray can encounter multiple boundaries of regions with different indices of refraction. Since the planar space is assumed to be divided into polygons with uniform index of refraction (or weight), the optical path length (OPL) traversed by the ray from S to T can be expressed

$$OPL = n_1 \sqrt{(x_S - x_1)^2 + (y_S - y_1)^2} + \sum_{i=2}^{m-1} n_i \sqrt{(x_{i-1} - x_i)^2 + (y_{i-1} - y_i)^2} + n_m \sqrt{(x_m - x_T)^2 + (y_m - y_T)^2}. \quad (11)$$

Having defined the optimal path length, Fermat's principle can be restated as light traverses the route which has the smallest optical path length [7]. In this way, the weighted region shortest path problem can be seen as equivalent to ray optics refraction.

Recall that the boundaries of the cost differentiation polygons alongside which costs are minimized are assumed to be known. The considered problem therefore reduces to minimizing Equation (11) with respect to the coordinates on the boundaries $\partial\Omega = \{(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)\} := \{(x_i, y_i)_{i=1}^m\}$. Notice that these coordinates are known to vary between certain bounds on a particular boundary and that x_i and y_i cannot vary independently. As a consequence, a path from S to T which is known to have i intermediate points on particular boundaries requires Equation (11) to be minimized over i independent variables. Since these variables are situated at boundaries, the intervals on which these can vary are known. A nonlinear program solver can be used to minimize Equation (11) with respect these variables.

A simple example of constructing a cost minimizing path is given in Figure 6. The source and destination point of this path are numbered with respectively one and two. Two boundaries of polygons are encountered along this path. These boundaries can be specified as $(x_1 = 30, y_1 \in [0, 50])$ and $(x_2 = 60, y_2 \in [0, 50])$. The relevant variables from $\{(x_1, y_1), (x_2, y_2)\}$ are in this case therefore y_1 and y_2 as x_1 and x_2 are fixed. Employing Maples nonlinear program solver yields the result given in Figure 6. As to be expected, it is attempted to shorten the sub-path in the most expensive region. This happens at the cost of lengthening sub-paths in less expensive regions.

Concerning this manner of reducing construction costs, it is not possible to minimize Equation (11) for an arbitrary number of coordinates. Therefore, the weighted path length is iteratively shortened when more than three boundaries are encountered by a path. Let L be a list of some estimated initial points $L = \{(\hat{x}_i, \hat{y}_i)_{i=1}^m\}$. A manner to minimize the cost of a path is to start at the source point S , to consider the adjacent boundary with coordinates (x_1, y_1) and to fix the point (x_2, y_2) on the boundary adjacent to this boundary. The cost function can then be minimized while the point (x_1, y_1) varies on the first encountered boundary. When an optimized point $(\tilde{x}_1, \tilde{y}_1)$ is found, the list L can be updated. In turn, this optimized point is used as a source point for the next iteration. Even for large number of variables, this procedure requires little time in practice. This procedure will result in a locally optimal path however, which is also emphasized by Mitchell and Papadimitriou who coin a similar method [23]. This last observation concludes the considered analogy with optics and enables us to construct an approximation algorithm.

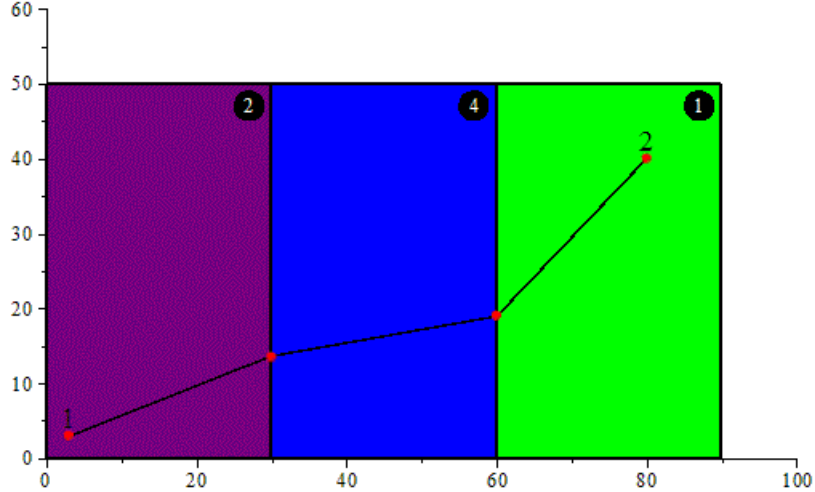


Figure 6: A cost minimizing path between point one and two. The displayed numbers indicate the constructed cost per unit length.

4.1.3 Cost differentiation approximation algorithm

In order to construct an approximation algorithm, let us first recapitulate the various assumptions that have to be made. As mentioned, it is assumed that the planar space can be divided into polygons and that the weight within each polygon is uniform. In this way, one can prove that the optimal path of the weighted region shortest path problem consist of straight line segments. The endpoints of each segment are then on the boundary of a polygon [27]. At a boundary between two polygons P_i and P_j , one can assume without loss of generality that its weight is $\min(n_{P_i}, n_{P_j})$ [23]. In addition to this, it is assumed that the start and destination point of each edge in the network N are known.

Subsequently, Steiner points are introduced according to a local minimality principle, as developed by Heijnen et al. in [15]. These points are only introduced when they lower construction costs. In order to assure this, one must account for the considered different cost regions. The approximation algorithm is applied after the introduction of possible Steiner points because this yields better results in terms of costs than applying the algorithm after the construction of an initial tree. To employ the insights obtained from the analogy with optics, the relevant boundaries for a cost minimizing path have to be determined. These are not known in practice and this is therefore a limitation of the method discussed in [23, pp. 60-61].

A manner to find these boundaries is discussed next. An examination of the costs associated with networks shows that the endpoints of boundaries of different cost regions are often important. These endpoints coincide with the corners of the accompanying polygons. Moreover, Mitchell and Papadimitriou emphasize the importance of the midpoints of these boundaries [ibid.]. Combining these insights allows one to discretize a given area by introducing points at the midpoints and corners of the given polygons. A decision on how to connect these points has to be made. Specifying all possible connections

can be too computationally expensive and we therefore propose to divide the considered area into triangles. Triangulation is also used in [27], although in a slightly different manner. A way to perform the triangulation is to employ Delaunay triangulation [29]. Additionally, the source and destination point are connected with the midpoints and endpoints of the polygon in which they reside. A discretization grid is obtained in this way and an example corresponding to the previously considered example is given in Figure 6.

Having this generic way of constructing a discretization grid, an estimation of the relevant cost minimizing boundaries can be found as follows. The cumulative weights of the introduced connections can be calculated with the use of the specified weight function $g(x, y)$. For instance, the cumulative weight of a connection between points p_1 and p_2 is given by $\int_{p_1}^{p_2} g(x, y) dl$. The cost minimizing boundaries can now be determined by using the well-known algorithm of Dijkstra. This is a manner to determine the relevant boundaries. For the previously considered example, the discretized cost minimizing path runs via points seven and eleven. However, the algorithm is not infallible as it concerns a discretization approach. Introducing more discretization points can reduce the error of not selecting the cost minimizing boundaries. On the other hand, more discretization points causes computational cost to increase. The described approach seems to be a balanced trade-off between these two factors.

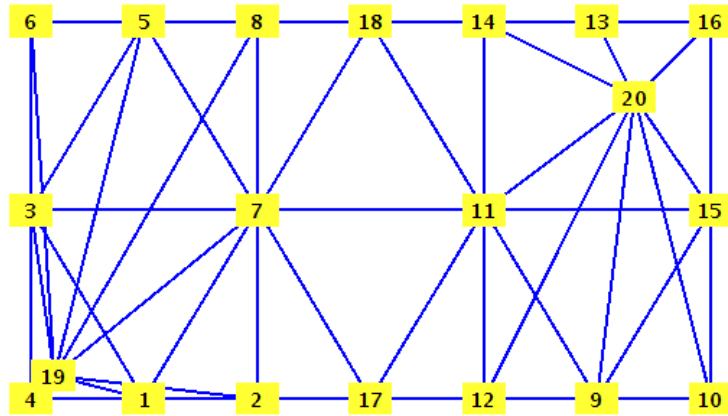


Figure 7: Discretization grid corresponding to Figure 6. Yellow blocks numbered nineteen and twenty are respectively the source and destination point while the other blocks correspond to the midpoints and endpoints of the polygons.

Once an assessment is made of the cost minimizing boundaries, the considered analogy with optics turns out to be of use. The coordinates of the relevant boundaries are now known and therefore intervals for the relevant variables can be specified. Moreover, the discretized cost minimizing path provides one with the estimation $\{(\hat{x}_i, \hat{y}_i)_{i=1}^m\}$. The previously described minimization of Equation (11) can now be employed. Subsequently, parts of the networks that do not satisfy the angle criterion are modified when construction costs can be reduced in this way. This step of the approximation algorithm coincides with

the algorithm constructed by Heijnen et al. [15] with some slight adjustments. An overview of the constructed approximation algorithm is given in Table 2. The intermediate results obtained from this algorithm for a particular example are depicted in Appendix A.1.

Table 2: Approximation algorithm for incorporating costs differentiation in the geometric graph method. It requires the location of the terminals, the required capacities, the weight function and the corresponding polygons as input.

1. An initial network is calculated by constructing an initial tree. This can be a minimal spanning tree (MST) or star network for instance.
2. Steiner points are introduced in particular regions according to a local minimality principle when they lower total construction costs.
3. The resulting network has a set of edges called E .
4. While the difference in construction costs of consecutive networks is larger than a positive quantity ϵ , perform the following steps.
 - (a) Construct a discretization grid by considering the midpoints and corners of the boundaries of the specified polygons.
 - (b) For each edge e in E , which can connect both terminals and Steiner points, do the following:
 - Determine a cost minimizing path from source e_1 to destination point e_2 in the grid with the use of Dijkstra's algorithm
 - Having the cost minimizing boundaries, apply local numerical optimization along the coordinates of these boundaries.
 - (c) Perform local optimization (and possibly introduce Steiner points) for parts of the network that do not satisfy the angle criterion and update the set of edges E .
5. Return the local optimal cost minimizing network.

Before turning to an exemplary result, some final remarks concerning the considered approximation algorithm are given. Firstly, an advantage of the constructed algorithm is that it is relatively tractable. Other algorithms have been constructed, see for example [27] and [23], but implementing these is beyond the scope of this thesis. Secondly, the approximation algorithm is, as to be expected, not infallible. Extensive use is also made of local optimization as global optimization is infeasible. Thirdly and since a continuous weighting function $g(x, y)$ has been specified, there can be numerical issues. Rounding can

cause a solution to slightly deviate from a particular cost minimizing boundary. However, this does not yield a fundamentally different qualitative solution.

Having made these remarks, let us turn to an exemplary result. An example of a cost minimizing network is given in Figures 8. Some further examples are covered in the subsequent section and in Appendix A.2. As indicated, agent-based models are also able to cope with different cost regions and this subject is covered next.

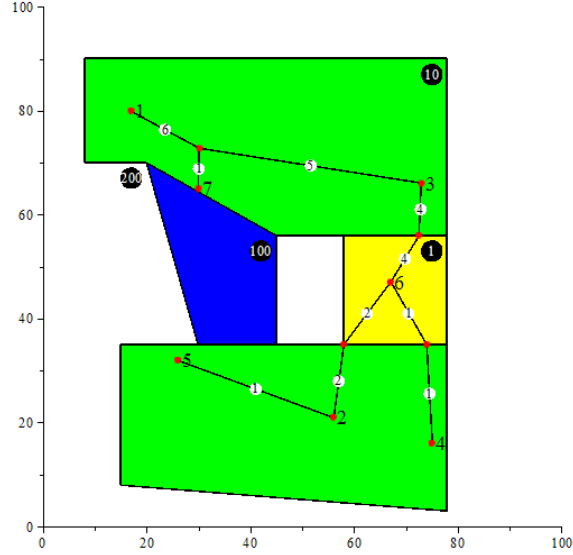


Figure 8: An example result when cost differentiation is incorporated while using the GGM. The displayed numbers concern the cost weighting function.

4.2 Implementing cost differentiation in the ABM

As discussed in Section 3.3, agent-based modeling is fundamentally different from the geometric graph method. Compared to the latter method, it is theoretically straightforward to allow for cost differentiation in agent-based modeling. A no-go area is defined as a relatively costly area and the scent of the nest, which attracts ants to the nest, is set equal to zero. In this way, it is very unlikely that parts of no-go areas are needed for constructing a cost minimizing infrastructure. Currently, go areas are specified as having a particular, relatively low, value. Cost differentiation can be incorporated as follows. The agent-based model imports the considered area as an image consisting of polygons which can have different colors. These different colors can be sensed by the agents and which can act accordingly. These perceived colors can then be used to specify the particular cost of a region.

An exemplary result of allowing for cost differentiation by this procedure in ABM is given in Figure 9. As the proposed infrastructure indicates, it is cheaper to take the longer route from the source to sink number 4. It has to be mentioned, however, that different runs of the agent-based model do not yield the same result. An infrastructure which crosses the expensive region is sometimes proposed, which is

considerable more expensive than the one depicted in Figure 9. The various other involved parameters, as discussed in [14], do not seem to significantly effect the type of networks that are proposed. A manner to improve the solutions proposed by the ABM could be to adjust the speed of the ants in the different regions. An expensive area can be associated with a low speed for instance. This could favor networks for which ants have to cover a larger distance. However, some experimental results showed that the affect of this change on the proposed networks was minimal. Just like the experimental setup in [14], the agent-based model therefore requires a number of different runs in order to assess the value of a proposed network.

Cost differentiation was implemented in both the GGM and ABM in this section. An approximation algorithm was constructed and implemented concerning the former while the latter method required some straightforward additions. Additional examples of cost minimizing networks in practical situations are covered in the subsequent section. A comparison between the results obtained from these two methods when they are challenged with different situations will also be made.

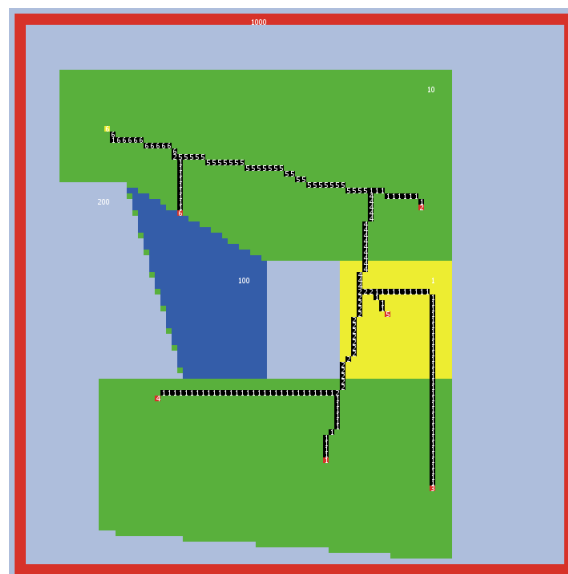


Figure 9: A cost minimizing network when cost differentiation is incorporated in the ABM. The displayed numbers indicate the constructed cost per unit length. This figure can be compared with Figure 8.

5 Results of modeling cost differentiation

After having constructed manners to incorporate cost differentiation in both the geometric graph and agent-based methods, some further examples are considered in this section. Moreover, a quantitative comparison between the results obtained from these methods will be made when they are employed in simplified but different situations.

5.1 Examples of constructing networks in different cost regions

A planner associated with the development of an energy network might be confronted with the following situation. A particular area consists of regions with relatively high and low construction costs. These regions respectively represent a lake and grasslands for instance. Given the location of the terminals, the required capacities and the construction cost associated with these regions, a planner would like to know which network configurations minimize cost. Such a cost minimizing network can be calculated with the extended geometric graph and agent-based methods and is displayed in Figure 10.

In addition to the lake and grasslands, a road might be present. As mentioned in Section 2.2, construction costs are lower on in cultivated regions. It can be less expensive to take a roundabout route than to construct a direct connection in this case. The planner faces therefore a more difficult job. If figures on the construction costs in the three different regions are known, then a cost minimizing network can be calculated with the extended geometric graph and agent-based methods. An example of this is also displayed in Figure 10. This figure demonstrates that it might be cheaper to construct a network which takes a roundabout route due to lower construction costs in particular regions.

An other situation which might be encountered by a planner is the following. A producing terminal, like an energy power plant, might already exist. However, it is located relatively far away from new and to be connected consumers. In addition, different cost regions are present in the considered area. In terms of construction costs, is it beneficial to build a network with relatively long spanning connections or to build a new plant together with connections?

An example of this situation is schematically depicted in Figure 11. In the figure on the left hand side, the producing node is located at the spot numbered one. The producing node in the figure on the right hand side is situated at a different location and relatively closely to the consumers. Let us assume that the construction costs of the cost minimizing network in the former situation is a and in the latter is b . As to be expected from Figure 11, it holds that $a > b$. When we assume that the building costs of the new plant are c , then a planner concerned about costs will do the following. If $a > b + c$, then a new plant and accompanying network will be built. However, long spanning connections will be constructed if $a < b + c$. These two instances exemplify the routing and investment problems faced by a planner.

The foregoing examples give an indication of the wide variety of situations a planner might have to deal with. In these examples, more than two different cost regions could be present and a weight function was needed in order to completely specify the costs of construction in a particular area. In

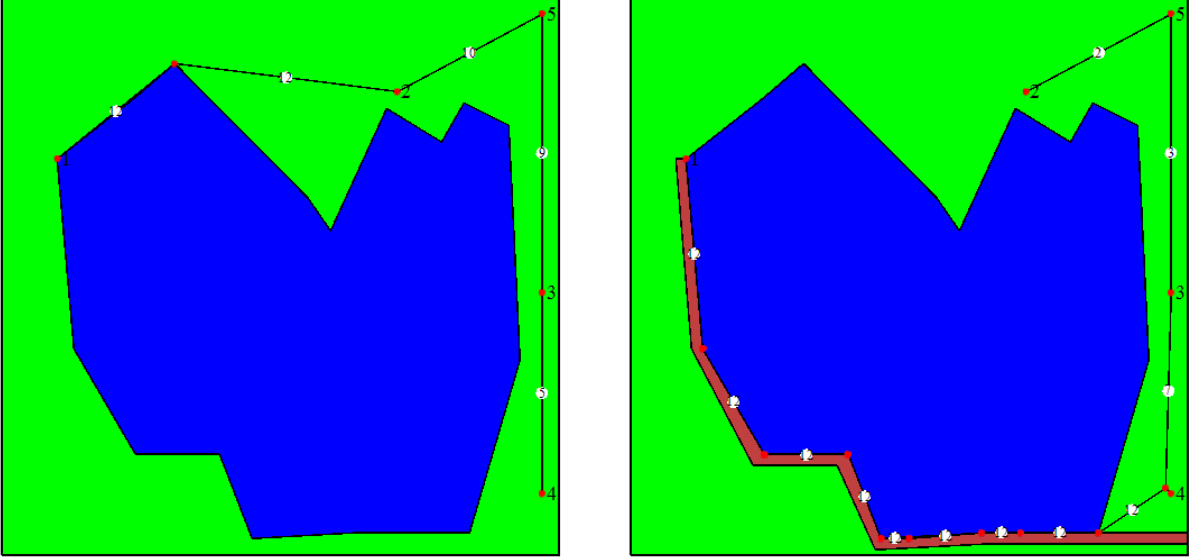


Figure 10: Cost minimizing networks when an area consists of two different cost regions (left) or three different cost regions (right). A weight of 1 is assigned to the green region, 3 to the blue region and 0.3 to the orange region.

order to simplify matters and to compare both methods in a 'fair' way, a topic which is subsequently considered, it is assumed in the following examples that an area comprises of two regions.

The region in which it is more expensive to build is called the offshore region while the other is called the onshore region. Let P_{offshore} denote the polygon of the offshore region. Moreover, let r be the ratio of the offshore over the onshore construction costs. This quantity is also called cost factor. The expression for the construction cost per unit length can now be simplified:

$$h(x, y, q_e) = \begin{cases} rq_e^\beta & \text{for } (x, y) \in P_{\text{offshore}} \\ q_e^\beta & \text{otherwise.} \end{cases} \quad (12)$$

If a connection between two nodes passes the offshore region, then it suffices to calculate the points of intersection with this region. It is not any longer necessary to calculate the integral contained in Equation (6). The offshore regions are assumed to be of different types in the following analysis. Water can form recesses in land and such an area will be called an indented region. An area might also be divided by a river for instance and such a region is named a dividing region. Lastly, an offshore region could be enclosed by land. Examples of these areas are given in Figures 12 and 13.

The issue arises what a realistic value for the ratio r of the offshore over the onshore construction costs is. Although this is one of the factors related to planning uncertainty and not further considered in this thesis, for which we refer to Chapter 2.3, a reasonable value for this ratio seems to be 2.5 in the Middle East and 5 in the United States according to [1]. Some examples of cost minimizing networks in the discussed regions are given in the following figures. These examples demonstrate that it is now possible

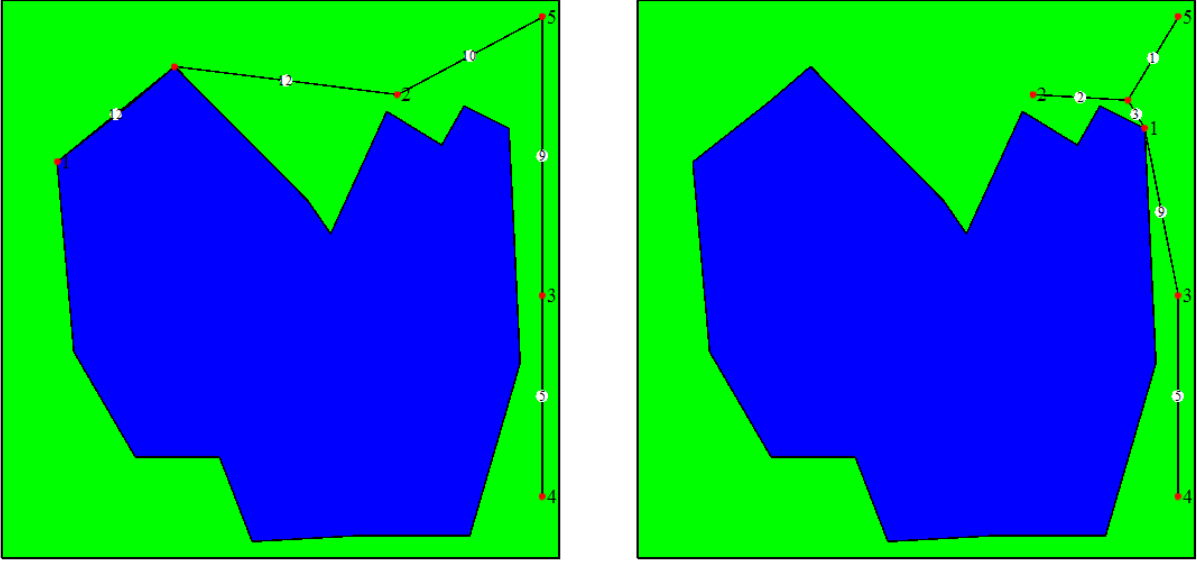


Figure 11: Connecting a relatively distant producing terminal (left hand side) versus constructing a new plant and accompanying network (right hand side). The producing terminal is labeled 1.

to determine whether it is beneficial to construct a network in an expensive region or to circumvent this region. Moreover, the effect of the cost exponent on a cost minimizing network is exemplified in Figure 14. A lower cost exponent ensures that it is advantageous to share a connection. With these simplifications at hand, we can now compare the extended methods in a fair way.

5.2 Comparing cost differentiation in the GGM and ABM

Having examined manners to implement cost differentiation in both the geometric graph and agent-based methods, the results obtained by using these methods can be compared. Let us first start with an example. Figures 8 and 9 display networks proposed by the GGM and ABM. The networks slightly differ in terms of construction costs and the network provided by the GGM is superior. As already indicated, it is difficult to construct random areas with different cost regions as the GGM requires a weight function.

Therefore, examples have been randomly constructed for areas comprising of two regions. Specifying a weight function can be circumvented in this way. For each type of region, fifty random examples have been constructed in the two dimensional plane of size 100x100 units of length. The initial network is chosen to be the minimal spanning tree (MST) as this tree yielded the best results for the considered examples. On the other hand, the star network performed a bit worse in terms of construction costs, but not significantly. The relevant parameters, as listed in Table 3, are chosen randomly from a uniform distribution within the specified range for each example. A polygon, which represents the offshore region, is also randomly constructed and put within the total area. Lastly, the source and consumer nodes are placed randomly in the onshore region.

In order to compare the results, the relative difference in construction costs between the geometric

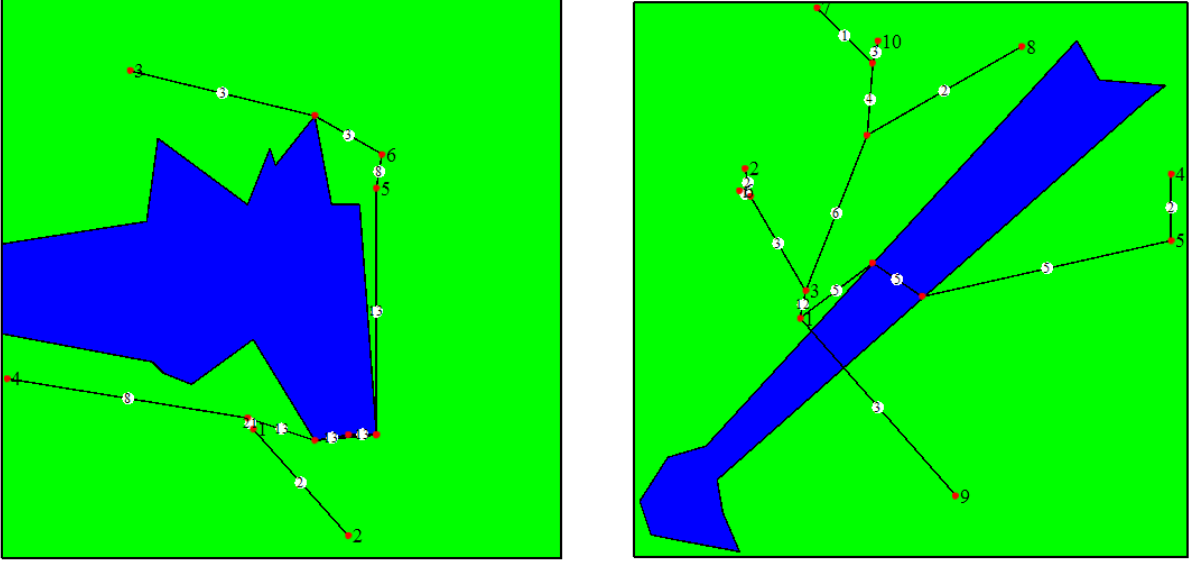


Figure 12: Cost minimizing networks which encounter indented and divided regions when the cost factor is 3.

Table 3: Intervals from which the input parameters for the random examples are drawn.

Parameter	Interval
Number of terminals N	[3,10]
Maximum capacity demand D	[1,9]
Capacity demand of consumer nodes	[1, D]
Cost exponent β	[0,0.9]
Cost factor r	[1,5]

graph and agent-based methods is defined to be

$$\frac{C(N_{\text{GGM}}) - C(N_{\text{ABM}})}{C(N_{\text{ABM}})}. \quad (13)$$

The results of the previously described procedure concerning the three different types of offshore regions are given in Table 4. For each of these type of regions, fifty random examples have been constructed. The third and fourth column of Table 4 show in how many cases which type of initial network yielded a lower cost solution. As explained in Section 3, the geometric graph method can employ both the minimal spanning tree (MST) and star network as an initial network. For the agent-based method, ten runs for each example have been performed.

As these results demonstrate, construction cost according to the GGM are on average ten percent lower than the cost according to the ABM. Comparing the average relative difference in cost and accompanying standard deviation shows that this difference is not significant

In order to assess the significance of various variables on the costs, Table 5 has been constructed.

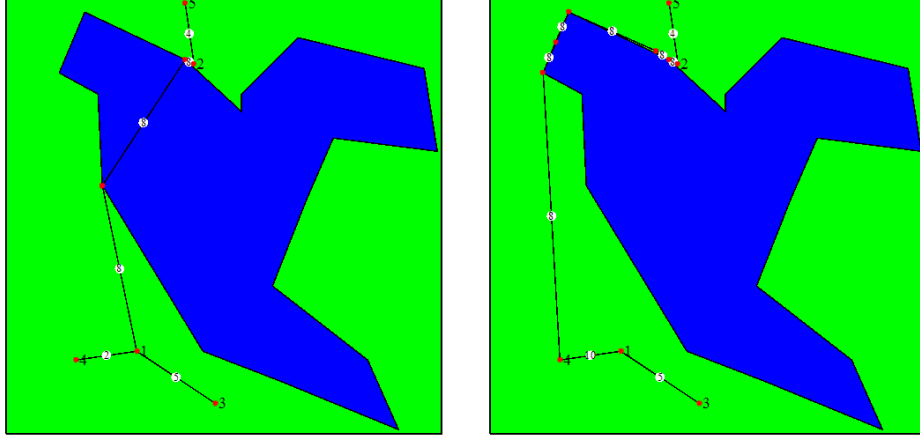


Figure 13: Cost minimizing networks in an enclosed region when the cost factor is 1.75 and 3.

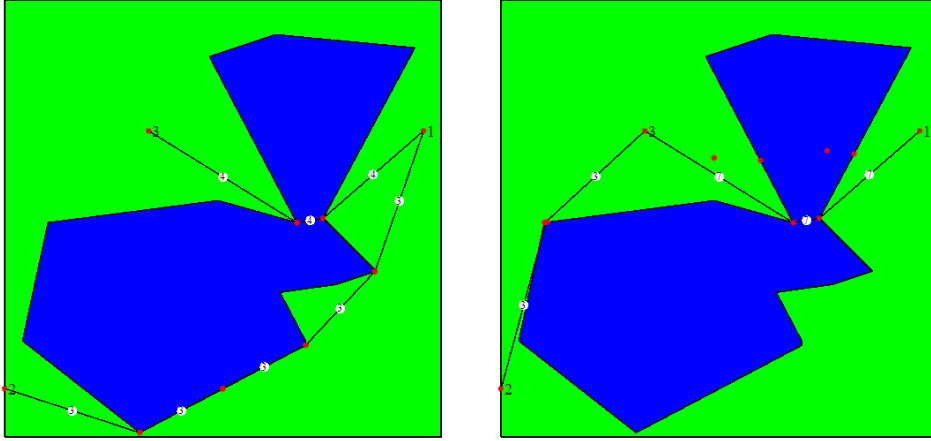


Figure 14: Cost minimizing networks in an enclosed region when the cost exponent is 0.6 and 0.95.

All correlation coefficients in this table are significant at the 0.05 level. This result is in agreement with expectations as the costs of building a specific network is to a high extent dependent on the considered variables. Total costs are determined by the total number of nodes, the costs of construction a larger capacity connection (represented by cost exponent β), the capacity demanded by the consumers and by the mutual weighted distance between the nodes. The latter is summarized in the total weighted length of the minimal spanning tree wl_{MST} . Furthermore, the quantity total variance of costs explained indicates that the building costs of a cost minimizing network are mainly influenced by the considered variables. About seventy percent is determined by these variables while thirty percent is determined by variability in the starting tree All these results are in good agreement with the results of the go and no-go area experiments performed in [15].

Lastly, some comments on the results obtained from the different cost and the go/no-go regions approach are appropriate. The former is namely an extension of the latter. The former approach consists

Table 4: Average relative difference in cost μ between the cost minimizing network proposed by the GGM and ABM for fifty randomly constructed networks. Also displayed is the accompanying standard deviation σ . The initial network for the GGM was the minimal spanning tree (MST) and ten runs have been performed for each example while using the ABM.

Type of region	μ	σ
Enclosed	-0.11	0.08
Indented	-0.12	0.10
Dividing	-0.10	0.06

of using the methods as described in Chapter 5, while the latter relates to employing the methods known from literature [14]. If an area comprises of two regions and the construction costs between these regions do not differ considerably, then the results obtained from these two approaches are not be comparable. As can be seen from Figures 12 to 14, different results are obtained. The considered areas consist of two regions in these instances, but a less expensive network is constructed by partially crossing the expensive region. If different cost regions are present, then the go/no-go regions approach cannot be employed. The latter can only be used when an area comprises of regions which allow or bar construction. Figures 8 to 10 serve as an illustration this observation. In this case, the different cost region approach allows a planner to lower expected building costs than by employing a go/no-go area approach. As covered hereinafter, these results enable us to formulate recommendations that might assist in developing new energy networks in different cost regions.

Table 5: Results of different cost region experiments: relevant correlations and total variance of cost explained.

Correlation between network costs and ...	Parameters	GGM	ABM
Total number of nodes	N	0.46	0.45
Cost exponent	β	0.56	0.57
Maximum capacity demanded by consumers	q_{\max}	0.40	0.38
Mean capacity demanded by consumers	q_{mean}	0.38	0.36
Weighted length of MST, connecting all nodes	wl_{MST}	0.47	0.50
Total variance of costs explained	adjusted R^2	0.67	0.68

6 Conclusions

This thesis is concluded with a summary and some recommendations. Before turning to the recommendations that might assist in developing new energy networks, let us summarize the preceding findings.

6.1 Summary

Energy networks are relevant to society as they enable the distribution of essential energy carriers. In an energy network, suppliers and consumers are involved. It consists of links and nodes from a technical point of view. The connections between suppliers and consumers enable the distribution of energy. The evolution of a network consists of different stages and the development, growth and mature phase can be distinguished. Due to decreasing energy reserves and pollution, new energy networks might have to be developed in the future as these networks have beneficial properties [8], [11]. This constitutes an important reason for focusing on the development stage of energy networks.

Various sources indicate that investment costs play an important role when new energy networks are to be rolled-out [11],[10],[18]. Due to liberalization and internationalization, the construction costs of a project have witnessed increased attention. In this respect, energy networks may need to be developed in an area comprising of different cost regions. Synergy between the construction of different infrastructures in cultivated regions and geographical location give rise to such regions. The introduction and employment of location cost-adjustment factors also hints at this development. Given these observations on the relevance of construction costs, more specifically was looked into the factors that determine the investment costs of new energy networks.

A distinction is made between investment determining factors related to planning uncertainty and factors related to routing problems. The various kinds of relevant factors, as displayed in Table 1, are different in the respect that some of them might be resolved over time through collaboration. These are indirectly related to investment costs while other factors are directly related to costs. The factors related to planning uncertainty have not been further considered in this thesis while the factors related to routing have been. The reason for this is that it is not feasible to examine the entire space of plans as the number of possible network configurations can grow very large. Therefore, the choice is made to focus on the directing of networks along specified courses in different cost regions. Planning uncertainty has to be resolved in the initial stages of decision making before the two considered modeling approaches can be used by planners.

Two fundamentally different methods, which are known from literature, are able to cope with routing problems [15]. The geometric graph method (GGM) is a top-down approach via which it is attempted to deterministically construct a new network with minimal building costs. Agent based modeling (ABM), on the other hand, is a bottom-up, stochastic approach. These modeling approaches are able to cope with go/no-go regions. Given the relevance of construction costs, manners to incorporate different cost regions have been examined.

Cost differentiation was implemented in the GGM by implementing an approximation algorithm. This algorithm needs a weight function which specifies construction costs at a particular location. The ABM required relatively straightforward additions as it employs digital ants which directly interact with the encountered environment. It suffices to construct a map with colors representing construction costs in a particular region. In these ways, it is able to extend both the GGM and ABM such that they can cope with different cost regions. It is wise to keep in mind that both methods are approximation approaches and therefore not infallible. After some further simplifications, random examples have been constructed comprising of two different cost regions. These examples allow one to compare the results from the GGM and ABM in a fair manner. Generally speaking, the results obtained via the GGM and ABM are comparable in terms of construction costs. The GGM yields a bit better results than the ABM, but not significantly.

Lastly, the results obtained from the different cost and the go/no-go regions approach have been compared. If an area comprise of two regions and the construction costs between these regions do not differ significantly, then the results obtained from these two approaches are not comparable. The considered examples illustrate that a less expensive network can be constructed by partially crossing an expensive region. The extended methods enable a planner to substantiate whether it is beneficial to construct a roundabout connection or to partially pass the expensive region. If more than two different cost regions are present, then the go/no-go regions approach cannot be employed. In this case, the different cost region approach allows a planner to lower assessed building costs than by employing a go/no-go area approach.

6.2 Recommendations and future research

Based on these findings, the following recommendations are formulated that might advance the construction of new energy networks. When multiple consumers are involved and different cost regions are encountered, the number of possibilities to construct new energy networks is considerable. Given the required input on the participators in the project, their locations, required capacities and different cost regions, the considered approaches reduce the number of routing possibilities. Based on the actual routing problem at hand, one or multiple cost minimizing networks can be calculated. As the various examples illustrate, relevant alternative network configurations can be sometimes constructed. This allows the participators to focus their attention on these solutions. Therefore, planners faced with routing problems are recommended to employ the considered approaches. Cost minimizing networks in different cost region can provide them with information for streamlining collaboration. Some additional thoughts on alternative networks are contained in Appendix A.3.

Moreover, the considered methods allow the stakeholders to determine whether it is beneficial to construct part of a network in an expensive region or to circumvent this region. The considered examples illustrate that this fact can result in non-obvious solutions. Therefore, the different cost regions present in a particular area should be carefully examined as networks with non-straightforward configurations

might yield a substantial reduction of building costs. As covered in the introduction, these new networks might decrease the rate of decline in energy reserves and reduce the adverse effects from pollution. In these ways, rolling-out new energy networks could bring us one step closer to a more sustainable society.

Future research on the considered topic can deal with different aspects. Concerning the modeling, new networks might also be fed by multiple sources instead of one. Research is currently being performed to incorporate this aspect. Additionally, a further refinement of construction costs might be made by including specific costs for splitting points and corners. Also a practical study of actual construction costs could be valuable in rendering these methods fit for use in reality. Moreover, it could be interesting for planner to compare the costs between constructing a new network and extending an existing one. Some exemplary results obtained from the agent-based method can be found in Appendix A.4. Lastly, the question arises how and in which cases these might be used in practice. Why would planners employ tools based upon these approaches and which aspects might cause them to be reluctant? As the modeling approaches are already quite complex, the latter aspect deserves priority.

A Appendix

Some further details and brief remarks on topics touched upon in the main text are covered in this Appendix. Firstly, the intermediate results of the approximation algorithms for a particular example are displayed. Some additional examples are given next. Lastly, two issues that can be addressed by the agent-based method are considered: the comparison of alternative cost minimizing networks and the extension of existing networks.

A.1 Intermediate results of approximation algorithms

Some intermediate results of the approaches for incorporating differentiation of construction costs are covered next. The approximation algorithm that is employed while using the geometric graph method (GGM) has been discussed in Section 4.1.3. This algorithm is exemplified below in Figures 15 to 18. In this example, an area consists of two regions. Construction costs in the blue region are assumed to be twice as large as in the green region. Moreover, the consecutive steps of the agent-based method (ABM) are depicted in Figure 19. The difference in construction costs between the resulting networks obtained from these methods,

$$\frac{C(N_{\text{GGM}}) - C(N_{\text{ABM}})}{C(N_{\text{ABM}})},$$

is in this case about -0.19.

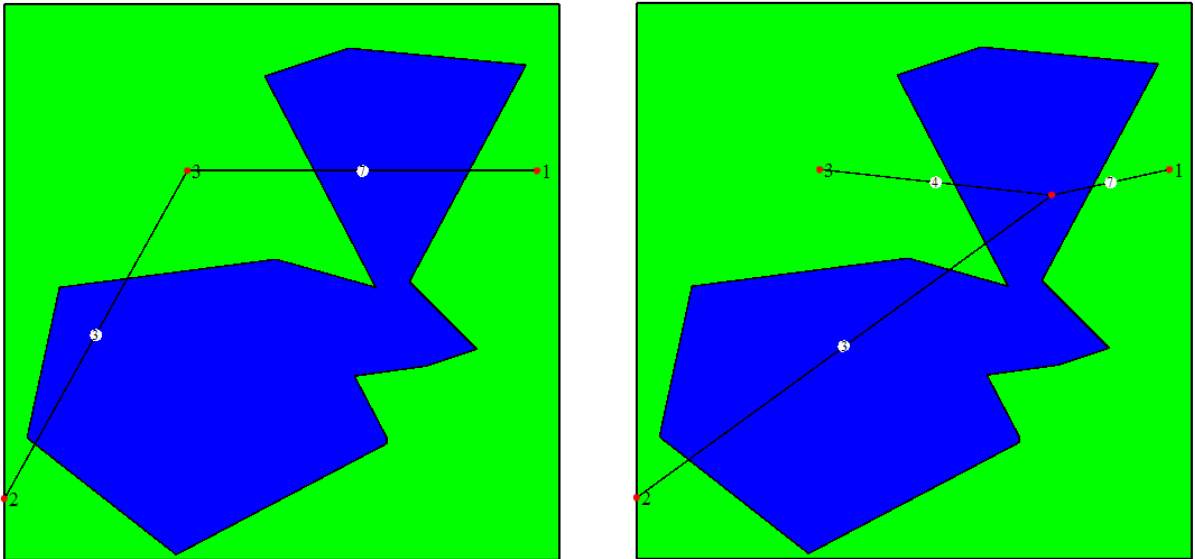


Figure 15: Minimal spanning tree (left), which connects the complete set of terminals, and a network which comprises of a Steiner point (right). The latter network has lower building costs than the former. The edge labels indicate the capacity of the connections.

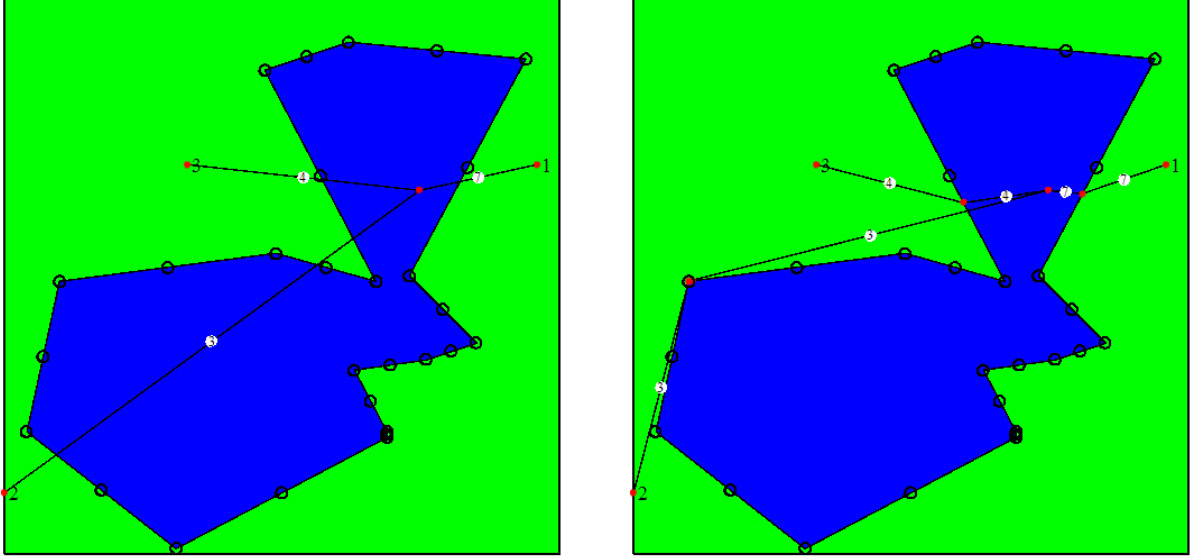


Figure 16: Points of the discretization grid (left), which are introduced at the midpoints and endpoints of the boundaries of the relevant region. Subsequently, the connections between the nodes are redirected by selecting the relevant boundaries and employing numerical optimization. The figure on the right contains the resulting network.

A.2 Additional examples

Cost minimizing networks might also be helpful in constructing energy networks in a harbor region for instance. These can be calculated by using the geometric graph and agent-based methods. The accompanying results are displayed in Figures 20 and 21. Moreover, some additional results of employing the extended geometric graph method in areas comprising of more than two different cost regions are given in Figures 22 and 23.

A.3 Alternative cost minimizing networks

As mentioned in Section 6.2, the ABM can be employed to construct alternative cost minimizing networks. The question arises how different these alternatives are. It has to be mentioned that these alternatives might differ in terms of construction costs. Therefore, they are not alternatives in the strict sense of the word. A measure to determine the 'degree of differenceness' between networks can be to calculate the relative amount of costs determined by a particular region. If a network has total construction costs C and an amount a of these costs can be attributed to a particular region, then a/C is the relative amount of costs which can be attributed to this region. Some exemplary alternative networks are given in Figure 24. These networks might provide planners associated with development of a new network with insights into the relevant alternatives.

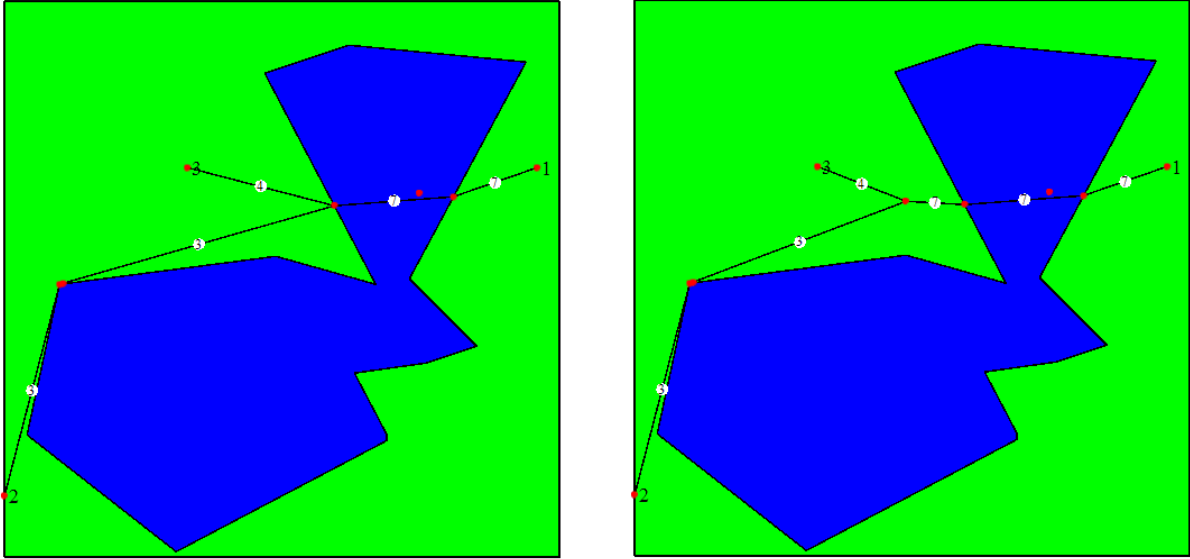


Figure 17: Local optimization (left and right) for parts of the network that are not yet locally optimal. In the figure on the left, the part of the network situated in the expensive region is altered. The resulting network can be improved in terms of construction costs by merging a particular connection, see the figure on the right.

A.4 Expanding existing networks

Next to or instead of the completely new development of a network, the question might arise how to expand an existing network cost efficiently. As mentioned in Section 3.3, the agent-based method already makes use of an extension approach. This method can therefore be straightforwardly adapted. In this way, the ABM can provide planner with cost minimizing extensions of existing networks. An example of some resulting networks in an area comprising of go/no-go regions is given in Figure 25. Such information could enable a planner to make a more substantiated trade-off between the development of a completely new network and the extension of an existing network.

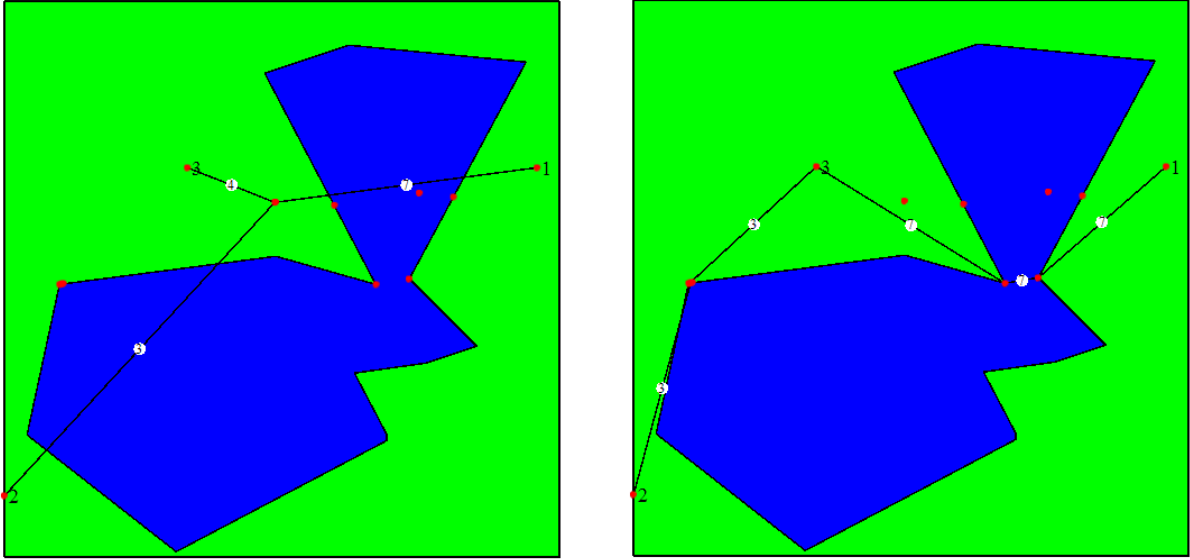


Figure 18: The connections between the nodes are reset (left) and another iteration step is performed. This renders the Steiner point redundant, see the figure on the right.

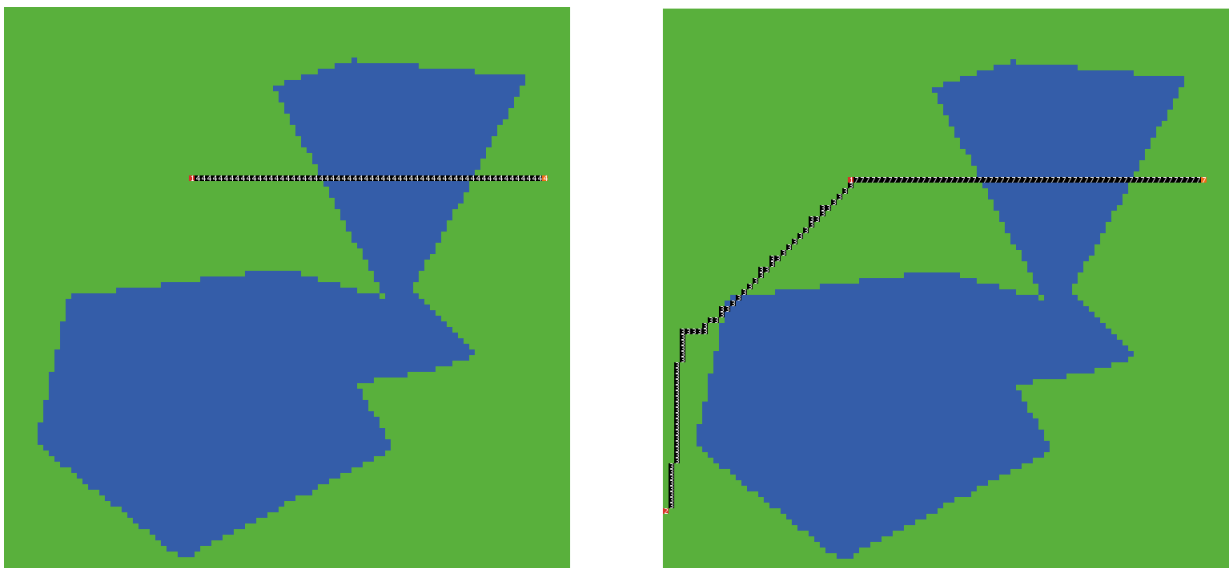


Figure 19: The consecutive steps of the agent-based method, which also connects the three considered terminals. Construction costs in the blue region are twice as large as in the green region.

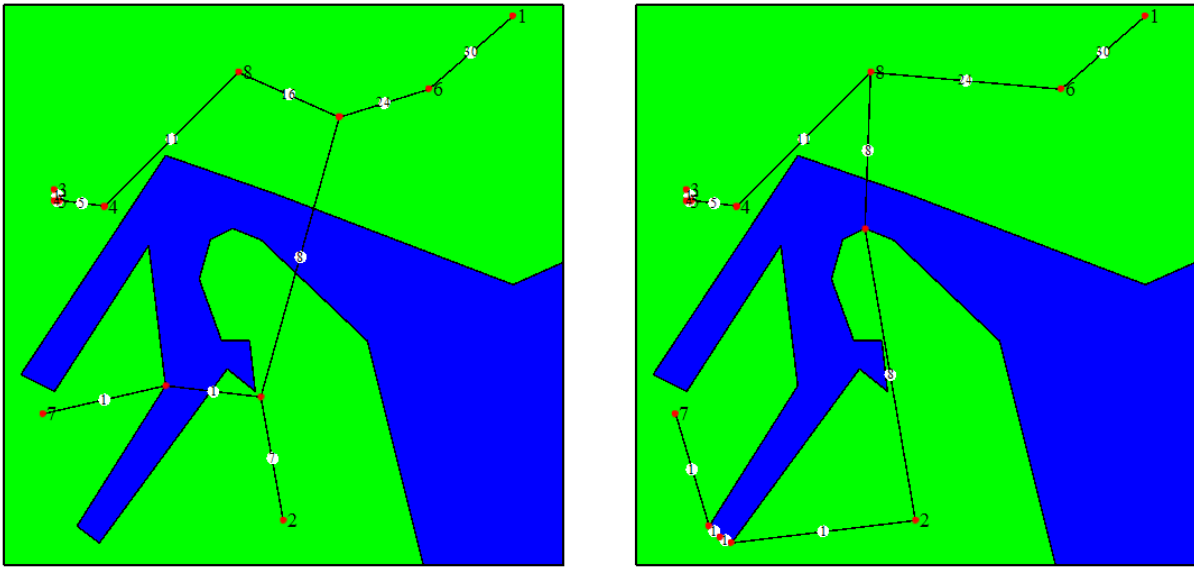


Figure 20: Cost minimizing networks constructed by the GGM when the cost factor is 2.5 and 5.

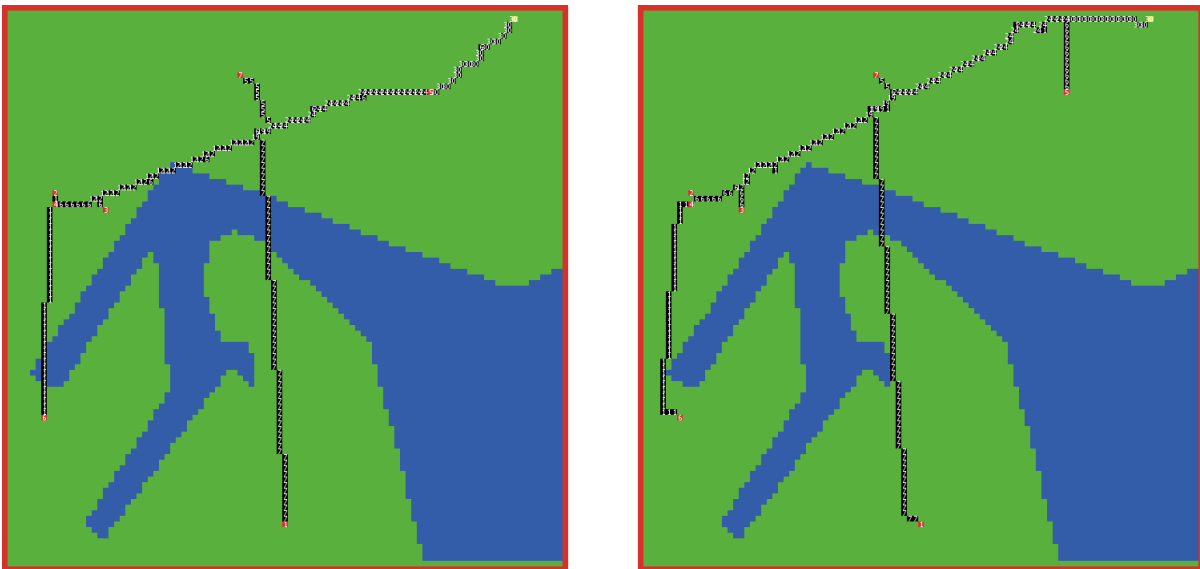


Figure 21: Cost minimizing networks constructed by the ABM when the cost factor is 2.5 and 5.

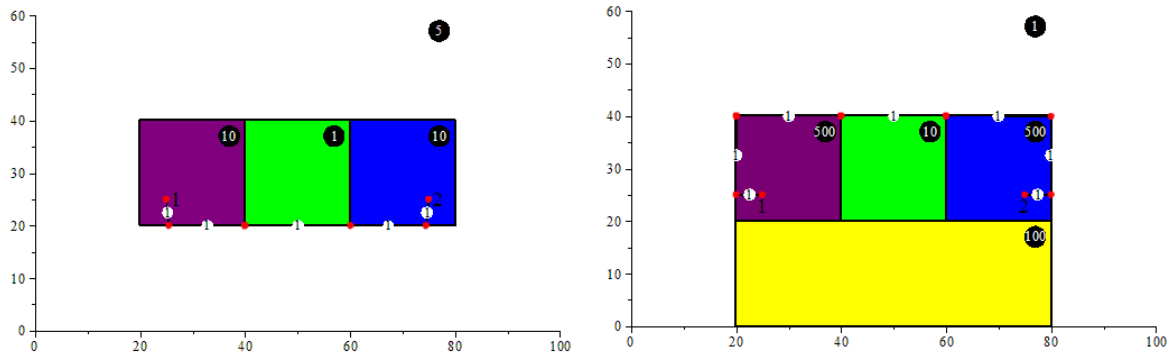


Figure 22: Example result when cost differentiation is embedded in the GGM for an area comprising of more than two different cost regions. The displayed numbers indicate the costs per unit length.

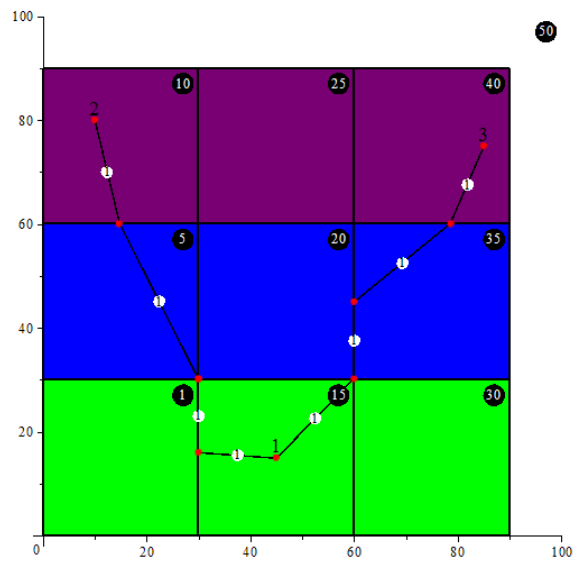


Figure 23: Example result when cost differentiation is incorporated in the GGM.

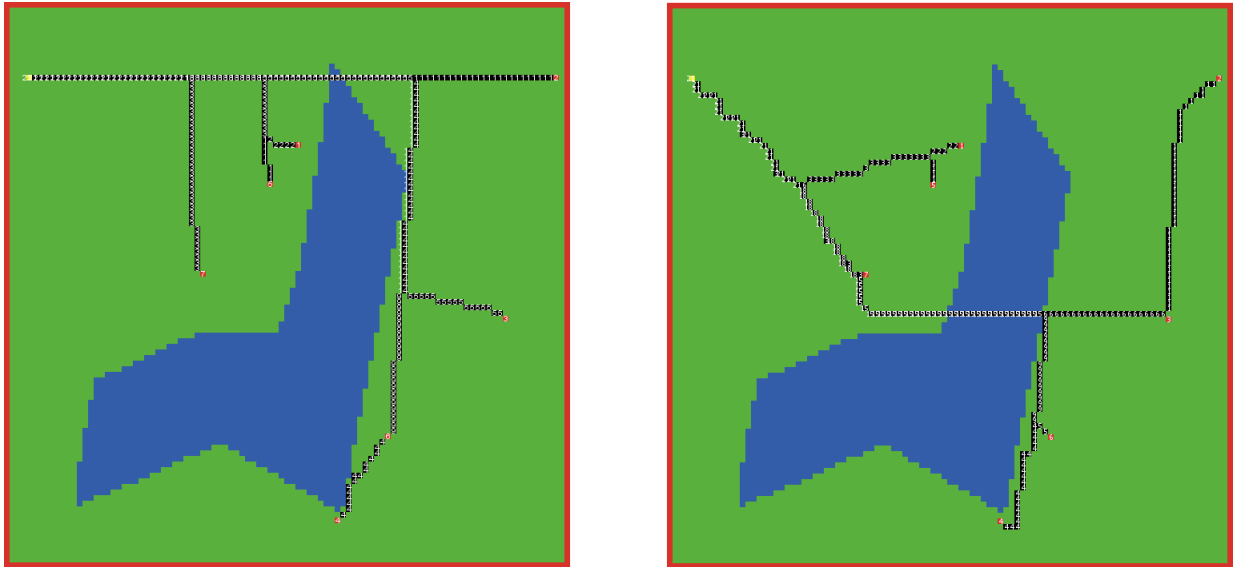


Figure 24: Alternative networks calculated by using the agent-based method. Construction costs in the blue region are three times as large as in the green region. The cost difference between the networks is 0.07. In the figure on the left, 0.05 of the total construction costs is determined by the expensive region while in the figure on the right this is 0.25.

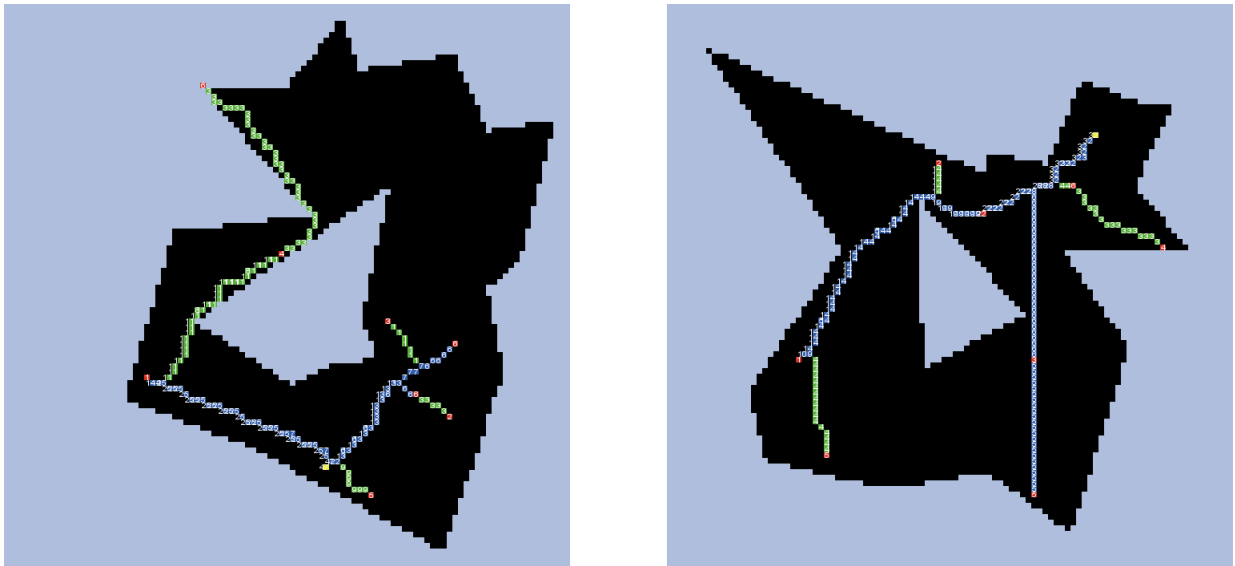


Figure 25: Extending existing networks in areas comprising of go (black) and no-go (light) regions. The network colored blue might already be present and the green parts represent cost efficient extensions of this network.

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