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THREE-DIMENSIONAL EFFECTS IN SONIC BOOM THEORY

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ABSTRACT

Examining the validity and applicability in sonic boom calculations of:

a) the Whitham-Walkden theory and

b) the first order approximation of a perturbation theory, based on the method of characteristics,

it was found by Oswatitsch and Sun, that the two methods yield different pressure signatures in the far field of a supersonic delta wing.
The present paper discusses the reason for the differences; by means of a simple procedure the reliability of both theories is compared in case they give different results.
From these considerations it becomes evident that in the near field of planar systems the method under b) should be used, that in the mid-field a second order approximation is sometimes necessary, and that in the far field the Whitham-Walkden method is reliable and simple.

1. INTRODUCTION

The generally used method for sonic boom calculations for slender configurations was developed by Whitham |1| and Walkden |2|.
As a first step the configuration, generating the sonic boom, is replaced by an equivalent body of revolution, having the same far field disturbances as the original configuration. The conversion makes use of the supersonic area rule. As its fundamental starting point the forward Mach cones Γo (Fig. 1) from the far field points are, near the configuration, approximated by planes Γw tangent to these Mach cones (Fig. 2). This means that the dependence domains for the far field points are slightly enlarged ($D_w = D_o + \Delta D$), but this represents generally a negligible effect in the far field disturbances. After the conversion to the equivalent body of revolution, the far field pressure signatures of this body are calculated with the asymptotic method outlined by Whitham |1| (see section 2).
Examining the validity and applicability of this Whitham-Walkden theory, Oswatitsch and Sun |3| found that in the case of a supersonic delta wing the Whitham-Walkden result for the far field disturbances differs essentially from the disturbances predicted by the general linearized theory with correction of the bicharacteristics. In this method, which will be called corrected linearized theory in the present work, the flow field is first calculated with linearized theory. As a second step the straight linear bicharacteristics are corrected by integrating their local first order direction from the body into the flow field.
The flow properties, originally calculated on the lineair bicharacteristics, are now interpreted as the flow properties on the curved first order bicharacteristics.

The reason for the differences in the results of the two methods for the far field disturbances of the supersonic delta wing is, as pointed out in sections 4 and 5, the slight difference in the dependence domains. From the very near vicinity of the wing, Whitham's theory takes account of the tip effect in the expansion fan from the trailing edge of the

wing, whereas in the corrected linearized theory this tip effect is clearly restricted to the flow behind the expansion.
Comparison of the dependence domains D_o and D_w with the first order domains for points in the expansion-fan, shows that in this special case the Whitham-Walkden theory is preferable to the corrected linearized theory in describing the far field flow, while the near field is better predicted by the corrected linearized theory.
Before discussing the flow field of the supersonic delta wing, the perturbations in the vicinity of the wave front of a trapezoidal leading edge wing with infinite chord (Fig. 3) will be considered in section 4. Here the essential differences between both theories will be shown. The reason for the differences between the discussed theories in the expansion fan from the leading edge shows up more clearly than in the case of the delta wing trailing edge expansion.

2. WHITHAM-WALKDEN THEORY

This generally accepted sonic boom theory is based on the original theory of Whitham concerning the far field disturbance of slender bodies of revolution parallel to the free stream direction x |1|.
The fundamental point of this theory is the following hypothesis:
Linearized theory gives a correct first order approximation everywhere, provided that the value, which it predicts for any physical quantity at a given distance r from the axis on the approximate characteristic $x - \beta r = \text{const.}$, pointing downstream from a given point on the body surface, is interpreted as the value at that distance from the axis on the exact characteristic, which points downstream from the said point.
A good approximation of the exact characteristics is achieved by integrating the local direction along the linear characteristics. The local direction is calculated from the linear perturbation velocities u_1 and v_1 (in x and r-direction respectively) and is given by:

$$\frac{dx}{dr} = \beta - \frac{(\gamma + 1)\, M^4}{2\beta} u_1 - M^2 (v_1 + \beta u_1) \qquad (1)$$

Here $\beta^2 = M^2 - 1$ and γ is the ratio of the specific heats.
To get a satisfactory description of the far field disturbances, however, it is sufficient to calculate the local direction with the asymptotic far field approximations u_a and v_a.
This enables Whitham to give an analytical description of the far field signature of the slender body of revolution.

$$u_1 \simeq u_a = \frac{F(\xi)}{\sqrt{2\beta r}} \qquad v_1 \simeq v_a = -\beta u_a \tag{2}$$

$F(\xi)$ is the so-called Whitham F-function.
Integration of (1) leads for the characteristic ξ = const. to:

$$x = \beta r + \xi - \frac{(\gamma + 1)\, M^4}{\beta\sqrt{2\beta}} F(\xi)\, \sqrt{r} \tag{3}$$

where:

$$F(\xi) = \frac{1}{2\pi} \int_0^{\xi} \frac{S''\, t}{\sqrt{\xi - t}}\, dt \tag{4}$$

$S(x)$ is the area distribution of the body of revolution and t is the integration variable along the x-axis ($\xi = x$ at $r = 0$).

An extension of this theory to more general slender configurations, including configurations generating lift, is given by Walkden |2|. This implies the replacement of the configuration by a body of revolution with the property that its far field behaviour corresponds to the far field behaviour of the original configuration.
The flow disturbances of this equivalent body are calculated according to Whitham's method. The substitution of the equivalent body for the original configuration is based on the "supersonic area rule". This rule states that in $x = x_1$ one part $A(x_1)$ of the cross-section of the equivalent body is given by the cross-section of the configuration with a plane Γw, having the Mach angle with the x-axis and intersecting this x-axis in $x = x_1$. Furthermore this plane should be taken normal to the meridional plane in which the far field disturbances are to be calculated. Besides this equivalent area distribution $A(x)$ due to

volume, an equivalent area distribution B(x) due to lift is to be determined. This is given by

$$B(x) = \frac{B}{2q} \int_0^x \ell(t)\, dt$$

where $\ell(x)$ is the lift distribution found by integration of the pressure perturbations along the contours of the intersection of the configuration and the various planes Γw.

Superposition of A(x) and B(x) yields the Whitham F-function for a general lift producing slender configuration.

$$F(\xi) = \frac{1}{2\pi} \int_0^\xi \frac{A''(t) + B''(t)}{\sqrt{\xi - t}}\, dt \tag{5}$$

The far field disturbances of the complete configuration are then given by (2), (3) and (5).

In fact, the intersection of the configuration and the plane Γw for a certain far field point P is an approximation of the intersection of the configuration and the linear Mach cone Γo from P (Fig. 2a). This means that the domain of dependence D_w in the Whitham theory for P is slightly larger than the linear domain D_o.

For planar systems, the intersection with Γo from a point in the vertical meridional plane is given by a hyperbola. The intersection with Γw is then a straight line, tangent to the apex of the hyperbola, as shown in Fig. 2b. In the far field, the effect of this substitution is negligible as the difference between D_o and D_w tends to zero for large distances r. However, the effect may not be neglected when considering the near and mid field. This matter will be discussed in the following sections.

3. PERTURBATION THEORY BASED ON THE METHOD OF CHARACTERISTICS

This method was originally given by Oswatitsch |4| as an extension of three-dimensional flow problems of the theory of Lin |5|. It consists of a perturbation theory in terms of the characteristic variables ξ, η and ζ, not only for the flow properties but also for the physical coordinates x, y and z.

$$
\begin{aligned}
u(\xi,\eta,\zeta) &= 1 + u_1 + u_2 \ldots. \\
v(\xi,\eta,\zeta) &= \quad\;\; v_1 + v_2 \ldots. \\
w(\xi,\eta,\zeta) &= \quad\;\; w_1 + w_2 \ldots. \\
x(\xi,\eta,\zeta) &= x_o + x_1 + x_2 \ldots. \\
y(\xi,\eta,\zeta) &= y_o + y_1 + y_2 \ldots. \\
z(\xi,\eta,\zeta) &= z_o + z_1 + z_2 \ldots.
\end{aligned}
$$

The flow field to first order is given by the linear non-dimensional perturbation velocities u_1, v_1 and w_1, while the perturbations x_1, y_1 and z_1 are obtained by integrating the local bicharacteristic directions (using u_1, v_1 and w_1) along the original straight linear bicharacteristics As already stated in the introduction, this first order approximation is called the corrected linearized theory.
Essentially this is the same as done in the axisymmetric case by Whitham |1|, who calculates the corrected characteristics with respect to the cylindrical coordinate system.
The only relevant difference with Whitham's far field theory is, that in Oswatitsch's method the corrections are made with the linearized perturbation velocities, valid throughout the whole flow field, while Whitham uses the asymptotic far field perturbations for the corrections.

The advantage of the corrected linearized theory with respect to the Whitham-Walkden procedure is that in the near field the flow calculations are correct to the first order, whereas Whitham-Walkden gives the far field approximation, which is only correct if $\xi/y \ll 1$, i.e. close to the front shock of the configuration.

Another advantage is the fact that this perturbation theory has the possibility to give second order approximations. However, for this second order approximation the effect of the first order perturbations x_1, y_1 and z_1, on the shape of the configuration with respect to ξ, η and ζ has to be calculated. Usually this proves to be a very lengthy procedure.

As shown in sections 4, 5 and 6 however, it is not always necessary to carry out the entire procedure. In some cases it may be possible to calculate in physical coordinates the first order domain D_1 for a field point P, and to subtract from these calculations the essential features of the second order terms.

As shown in the following sections, these may have a relevant influence on the far field perturbations.

4. FLOW PERTURBATIONS IN THE PLANE OF SYMMETRY OF A SEMI-INFINITE WING WITHOUT THICKNESS, HAVING A SUPERSONIC TRAPEZOIDAL LEADING EDGE

4.1. Compression side

An example of a flow, in which small differences in the dependence domains D have no relevant influence on the far field perturbations, is given by the flow in the vicinity of the wave front at the compression side of a wing, having a trapezoidal leading edge shape and infinite chord (Fig. 3).

For this wing, the flow in the vertical plane of symmetry $z = 0$ will be calculated, first with the corrected linearized theory mentioned in section 3, then using the Whitham-Walkden theory of section 2.

4.1.1. Corrected linearized theory

In linearized theory the perturbation velocity potential ϕ at a point P(x,y,0) due to a source distribution I(x,z) in the plane $y = 0$ is given by:

$$\phi = -\frac{1}{2\pi}\iint_{\Sigma_o}\frac{I\,(x_o,z_o)\,dx_o\,dz_o}{\sqrt{(x-x_o)^2-\beta^2y^2-\beta^2z_o^{\,2}}} \tag{7}$$

The integration area Σ_o is the linear domain D_o, bounded by Γo (see Fig. 4). The source distribution can be given by:

$$I\ (x,z) = v_1\ (x,0,z) = \frac{d\phi}{dy}\ (x,0,z) \qquad (8)$$

For the flat wing to be considered v (x,0,z) is constant and equal to the angle of incidence α.
For the perturbation velocities in the plane z = 0 (7) yields:

$$\begin{aligned} u_1\ (x,y,0) &= \frac{d\phi}{dx}\ (x,y,0) \\ v_1\ (x,y,0) &= \frac{d\phi}{dy}\ (x,y,0) \\ w_1\ (x,y,0) &= 0 \end{aligned} \qquad (9)$$

Using these perturbations the first order characteristic direction is calculated by (1). This relation is integrated step by step into the flow field to construct the first order bicharacteristics. This yields the pressure signatures at some distances from the wing as given in Fig. 5.

4.1.2. Whitham-Walkden theory

In this theory the area distribution of the equivalent body of revolution due to lift is given by:

$$B(x) = \frac{\beta}{2q} \int_0^x \ell(t)\ dt$$

where $\ell(x)$ is the lift distribution in x-direction, calculated by integration of the pressure perturbations along Γw (Fig. 4).
As the equivalent body due to volume is zero for the flat wing, the Whitham F-function is given by:

$$F(\xi) = \frac{1}{2\pi} \int_0^{\xi} \frac{B''(t)\ dt}{\sqrt{\xi - t}} = + \frac{1}{2\pi}\,\frac{\beta}{2q} \int_0 \frac{\ell'(t)\ dt}{\sqrt{\xi - t}}$$

Together with (2.3) this yields the pressure signatures shown in Fig. 5.

In the very near vicinity of the shock, including the shock itself, these pressure signatures agree reasonably well with the signatures predicted by the corrected linearized theory. The differences between the signatures grow with increasing distance to the shock. This is not unreasonable, as the Whitham theory can only be expected to give reliable results if $\xi/y \ll 1$, and this condition is only satisfied in the very near vicinity of the shock.
The pressure signatures obtained with the corrected linear theory are not restricted to such conditions and can be thought valid in the whole region.

The very small angle of incidence $\alpha = 1^{\circ}$ in Fig. 5 has been chosen because for larger angles the condition $\xi/y \ll 1$ is only satisfied at large distances from the wing. As our interest is also directed to the near field effects, it seemed not unreasonable to restrict the angle of incidence to $\alpha = 1^{\circ}$.

4.2. Expansion side

At the expansion side of the flat wing, the difference between D_w and D_o appears to result in essential deviations between the pressure signatures in the expansion zone, as calculated by the corrected linearized theory and the Whitham-Walkden method.
As shown in Fig. 6 the linear domain D_o for the point P on the last bicharacteristic of the expansion fan in z = 0 is given by the hyperbola Γo. In z = 0 this hyperbola is tangent to the rear of the leading edge; elsewhere it lies ahead of this edge. As a consequence it does not contain the leading edge tips.
The domain D_w for point P, as used in Whitham-Walkden's theory is bounded by the straight line Γ_w, lying immediately behind the straight part of the leading edge and enclosing the leading edge tips.
This results in strongly different pressure signatures in and close

behind the expansion fan, where both methods are supposed to give reliable results.
In the region further away from the expansion the Whitham-Walkden method cannot be applied as the condition $\xi/y << 1$ is not satisfied. In that region the corrected linearized theory gives reliable results.

The pressure signatures according to both the corrected linearized theory and the Whitham-Walkden method are shown in Fig. 7.

4.3. Discussion of the results

In 4.1. the flow perturbations in the plane of symmetry of a flat wing, having a trapezoidal leading edge shape and infinite chord, are calculated by both the Whitham-Walkden theory and the corrected linearized theory. In the vicinity of the wave front at the compression side the results agree very well. This is not surprising considering the domains of dependence D_o and D_w, enclosed by Γ_o and Γ_w, for a point P at some distance from the wing on the resulting shock (Fig. 4).
At the expansion side of the wing however, the methods mentioned above give essentially different results for the expansion fan. Whitham-Walkden' theory predicts an expansion strength decreasing with distance to the wing, whereas in the corrected linearized theory this strength remains constant (this cannot be seen easily from fig. 7, because the expansion is affected by the formation of a shock). The difference is introduced by the dissimilarity of D_o and D_w. In such a case, when small differences in these domains cause large effects in the flow field, it seems necessary to calculate the first order approximation D_1 of the dependence domain D, and to compare it to D_o and D_w; this can be done using the bicharacteristic method outlined in Appendix A.
In this method the flow field is first calculated according to the corrected linearized theory. Then from the point of interest (e.g. on the last characteristic surface of the expansion fan) a family of bicharacteristics is constructed, forming the envelope of the domain of dependence for the point said. The bicharacteristics are calculated stepwise by simultaneous integration of their local directions.

The most remarkable feature of this first order dependence domain is the coincidence of a part of its enveloping surface with the originating line of the expansion.
Although these calculations have been performed in two-dimensional flow, their results are still useful for the three-dimensional flow at the expansion side of the trapezoidal leading edge wing.
In (corrected) linearized theory the expansion from the trapezoidal wing is quasi-two-dimensional over the width of the central part of the leading edge. Between the tips R - R' in Fig. 8) the first order dependence domain for the trapezoidal wing will therefore coincide with the corresponding two-dimensional domain, and the influence of the non-two-dimensionality is restricted to the part of the domains beyond the leading edge tips.
From Fig. 8 it can be seen that for points at some distance of the expansion's origin, Γ_1 encloses the tips of the central part of the leading edge. Then the influence of these tips is felt in the expansion in the plane of symmetry.
It weakens the expansion and makes its strength tend to zero at infinity, which is not predicted by linearized theory.
This shows that in treating an expansion, the first order corrections in D, although they cause second order corrections in the perturbation velocities only, may not be ignored.

In the paragraphs above arguments are given for the non-two-dimensionality of the expansion in the plane of symmetry of a trapezoidal leading edge wing. These arguments are based on first order dependence domains as calculated in two-dimensional flow. Indeed they show that two-dimensionality (present in corrected linearized theory) is not possible, but the conclusion that the Whitham-Walkden methode is more reliable in the far field may need some further explanation. As shown by stretching the x-coordinate in Fig. 9, Γ_w agrees of course rather well with Γ_1 for larger values of y (y > 8). However, Γ_1 is calculated assuming an expansion of constant strength, whilst it is shown that this assumption is dubious.

To allow the conclusion that the Whitham-Walkden method yields reliable results in the far field, Γ_w has to agree reasonably well with the exact dependence domain in the leading edge expansion of variable strength. From the calculations of Γ_1 it is seen that its particular shape is formed mainly in the part of the expansion fan very close to its origin. This part of the expansion will be also quasi-two-dimensional in exact calculations, so that Γ_1 will be a fairly good approximation of the exact bicharacteristic surface Γ. Consequently, Γ_w will be in reasonable agreement with Γ and Whitham-Walkden's method may be used for the calculation of the far field disturbances in the expansion fan from the leading edge.

In summary it can be stated that the first order flow calculations in the symmetry plane of the trapezoidal leading edge wing, according to the corrected linearized theory are correct in that part of the expansion where the tip effect does not influence the flow. In Fig. 10 the ultimate distance for using this theory is indicated by A.
In the far field Whitham-Walkden theory may give valid results. Approaching the wing, its validity decreases as can be inferred from Fig. 9.
The minimum distance up to which Whitham-Walkden's theory may be used for the calculations in the expansion fan, cannot be defined sharply but will be in the vicinity of B in Fig. 10. In the transition interval A-B a great deal of effort should be given to the calculation of the second order approximation of the velocities u and v, especially to the calculation of the first order dependence domains D_1, which causes the most important second order terms in u and v.
In the region behind the expansion fan Whitham-Walkden's theory may not be applied unless the condition $\xi/y << 1$ is satisfied. Where this condition is not met, the predictions of the corrected linearized theory will be rather reliable, because further downstream D_o approximates D_1 quite well.

5. WAVE FORMATION DUE TO A SUPERSONIC DELTA WING

In this section the influence is discussed of the first order terms in

the dependence domains for points in the flow field of a delta wing with supersonic leading edges (see Fig. 11).
The flow in the vertical plane of symmetry of this delta wing is calculated up to the first order by Oswatitsch and Sun |3| using the corrected linear theory.
The correction of the bicharacteristics in the compression zone A (Fig. 11) leads to intersections with bicharacteristics of the free stream. This results in the formation of a shock, having a constant strength up to point Q, where the last bicharacteristic of zone A meets the shock.
Beyond Q the shock is weakened by the interference with the bicharacteristics of the two-dimensional expansion zone B. This results ultimately in an elimination of the front shock. A fraction of the expansion remains, which at the rearward side is followed by a new shock. The pressure signatures calculated by this method for various distances from the wing are shown in Fig. 12.
Comparing these signatures to the results of the Whitham-Walkden method severe differences are found, as Fig. 12 shows. These are a consequence of the fact that in this Whitham-Walkden theory the strength of the expansion decreases with growing distance to the wing. Therefore the shock expansion interaction cannot be sufficient intensive to cancel the shock.

As in the previous section these differences in the expansion fan are primarily caused by the differences in the dependence domains D_o and D_W. Comparison with the first order domain D_1 shows that in the vicinity of the wing ($y < y_R$, Fig. 11) the corrected linearized theory gives reliable results (Fig. 13).
At larger distances, beyond, y_R, its results are correct in the whole flow field, except in and immediately behind the last part of the expansion fan. In the latter the first order terms in the expression describing the shape of the dependence domain may not be neglected, as can be seen from Fig. 13. At large distances ($y > 8$) the first order domain D_1 shows strong similarity to D_W and large deviation from D_o. This means that in this

region the Whitham results are rather reliable for the whole flow field, including the expansion, while the corrected linearized theory is only correct outside the expansion fan.

The effect of this being incorrect of the corrected linearized theory within the expansion fan on the shock-expansion interaction will now be discussed.
One might consider the possibility that the major part of this interaction occurs at relatively small distances from the wing, where the first order correction of D is not yet relevant and therefore the corrected linearized theory can still be applied. Then the shock cancellation would occur in that part of the flow field where the description of the flow field by Whitham would not be valid. In this hypothetical case the result of the Whitham-Walkden theory would be incorrect in the far field as well.
Some arguments relative to this questions can be taken from Fig. 12 and 13.
Fig. 13 shows that for points on the shock at distances $y > 8$ the approximation of the first order dependence domain by a straight line, as assumed in the Whitham-Walkden theory, will be acceptable. For points behind the shock the approximation is even better. Therefore it seems not unreasonable to expect the Whitham-Walkden theory to be valid for the prediction of the value of the flow perturbations behind the shock.
Furthermore, at $y = 8$ the shock predicted by Whitham-Walkden shows reasonable similarity with the shock calculated by the corrected linearized theory (Fig. 12). Thus Whitham's results for the pressure signature in the far field seem essentially correct. This would mean that beyond $y = 8$ the shock-expansion interaction is reasonably well described by Whitham-Walkden's method and that the shock is not eliminated as was expected from corrected linearized theory.

These considerations are in agreement with, and contain a certain quantification of the suggestions of Seebass and George |6|, who state that in linear theory the distance from the intersection hyperbolas (of the tipcones and the symmetry plane) to the trailing edge expansion becomes very soon negligible. Although the hyperbolas do not penetrate the expansion fan, Seebass and George expect some influence in the two-dimensionality of the expansion, which would yield a Whitham-Walkden behaviour in the far field. The present work shows that these tip cones do not only approach but actually penetrate into the expansion which is necessary for influencing the flow perturbation within it. Another effect that has been criticized in |6| is the shock-expansion interaction in the plane of symmetry. It was argued that this might be incorrect in the corrected linearized theory, because the disturbances in the expansion zone B (Fig. 11) are calculated as a superposition of the compression zone A and a two-dimensional expansion. After correction of the bicharacteristics, this zone A no longer exists beyond Q, and it might be incorrect to keep up the superposition. It can be commented that this effect would also be included in the second order terms of the characteristic perturbation theory of section 2, and might be essential in the calculations for the midfield (roughly between $y = y_R$ and $y = 8$).

6. CONCLUDING REMARKS

In some special cases the predictions of the far field disturbances by the Whitham-Walkden theory deviate essentially from the predictions by the corrected linearized theory. These deviations are introduced by the differences of the dependence domains as used in both theories. Formally, these differences are expected to have negligible influence. In the cases discussed in sections 4.2. and 5 they prove to be essential. In compressing flow fields the differences in the dependence domains generally do not influence the far field perturbations, because a great deal of the detailed information about the shape of the configuration is lost in the formation of shock waves.
In expanding flow fields, however, individual disturbances are not absorbed by shock waves and persist to large distances, while the space in which they are felt is enlarged, because of the diverging bicharacter-

istics.

It is a striking fact that in the expanding fields the predictions of the far field disturbances by the Whitham-Walkden theory, which contains some rather rigorous approximations, seem more reliable than the perturbations calculated by the corrected linearized theory. Applying the perturbation theory, based on the method of characteristics in the far field, it seems necessary to take into account the second order approximation for the perturbation velocities. Comparing Γ_w and Γ_1 however, it seems not unreasonable to expect that the second order method yields the same results as the Whitham-Walkden theory.

In the near field the corrected linearized theory is of course very useful, as it does not contain any further approximations besides the conditions for linearization, while the linear dependence domain D_o does not yet diverge essentially from D_1.

In the mid field, that is to say at a few characteristic lengths from the configuration, both the Whitham-Walkden theory and the corrected linearized theory may yield incorrect results. In this area the second order approximation of the characteristic perturbation theory is expected to yield useful and reliable predictions for the flow perturbations.

As a final remark, it may be noted that a fairly simple procedure can be used to compare the applicability of the sonic boom methods discussed in sections 2 and 3 for a given case. It consists of comparing the dependence domains D_o and D_w, as used in these theories, to the first order dependence domain D_1. From this comparison it can be learned whether a second order approximation of the flow perturbations is necessary, or that it is sufficient to use either the corrected linearized theory (generally in the near field) or the Whitham-Walkden theory (generally in the far field).

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APPENDIX A

The first order approximation of the dependence domain D_1 for a point on a characteristic surface of a two-dimensional expansion fan is calculated using the bicharacteristic method as given by Oswatitsch |4|.
In cartesian coordinates Oswatitsch gives the following relation for the bicharacteristic direction in a given point on a characteristic surface, emanating from a given initial line,

$$x_\lambda : y_\lambda : z_\lambda = (u - c \sin \omega):(v - c \cos \omega \sin \phi):(w + c \cos \omega \cos \phi). \tag{A.1}$$

The index λ refers to the partial derivatives with respect to the bicharacteristic parameter λ, while u, v and w are the velocities in x, y and z-directions and c is the local speed of sound. The angles ϕ and ω couple the local normal vector of the characteristic surface to the cartesian coordinates as given in Fig. A1. Since a characteristic surface can be thought to be generated by bicharacteristics, it is sufficient for the determination of this surface to follow the bicharacteristics into the flow field.
In the scope of the characteristic perturbation theory this procedure, which is given in more detail in |7| can be outlined as follows:
First the flow field is calculated using the linearized supersonic theory. Then the linear bicharacteristics, pointing downstream from a certain point of the configuration, are corrected for the local bicharacteristic direction, and the values of the flow perturbations, calculated on the original straight bicharacteristics are interpreted as the values on the corrected bicharacteristics.
By this procedure the discontinuity, present in the linear representation (Fig. A2a), is spread out in an expansion fan (Fig. A2b).
In this up to first order correct flow field the upstream bicharacteristics, constructing the characteristic surfaces to be determined (the cones A_1, B_1) are calculated stepwise using (A.1), again using the linear perturbation velocities.

By this procedure the first order approximation of the boundaries A_1 and B_1 of the dependence domains for points A and B on the first and last characteristic surface of a two-dimensional expansion, is determined. Its shape is given in Fig. A2.[a,b,c].
This shows the essential feature that the first order domain for B is enlarged with respect to the zeroth order domain. Its boundary Γ_1 coincides partly with the expansion's originating line.
The various bicharacteristics, constructing S - S' (Fig. A2[e]), meet the originating line under various angles ν between ν_1 and ν_2 (as illustrated in Fig. A3). Here ν_1 and ν_2 are the angles of the first and last characteristic of the expansion fan. Therefore Γ_1 essentially traverses through the originating line. This is shown in Fig. A4, where the x-coordinate is stretched at the line, and the location of the points where the bicharacteristics meet the "expansion-zone" is coupled to ν according:

$$\Delta x = \frac{\nu - \nu_1}{\nu_2 - \nu_1}$$

This means that in the stretched "expansion zone" a linearly varying normal velocity v is put in, and that the various bicharacteristics meet the "expansion zone" at a correct intermediate stream angle.
From Fig. A4 it is seen that Γ_1 is tangent to the trailing edge of the expansion zone and traverses through the zone to S and S'. There it leaves the expansion and runs into the flow field ahead of the expansion.
In Fig. A5 the first order approximation of the influence domains for points at the origin of the expansion is given. Two essentially different cases may be distinguished.
In the first case the origin P" of the influence cone is situated immediately behind the origin of the expansion. Then the whole domain is restricted to the space behind the expansion and enveloping the characteristic surface is the Mach cone from the origin, tangent to the last characteristic surface of the expansion fan.
In the second case the origin P' of the influence cone is situated immediately in front of the origin of the expansion. Then the character-

istic cone is tangent to the first characteristic surface of the expansion fan and traverses through the expansion into the space behind the fan.

Here it is tangent to the influence cone from P".

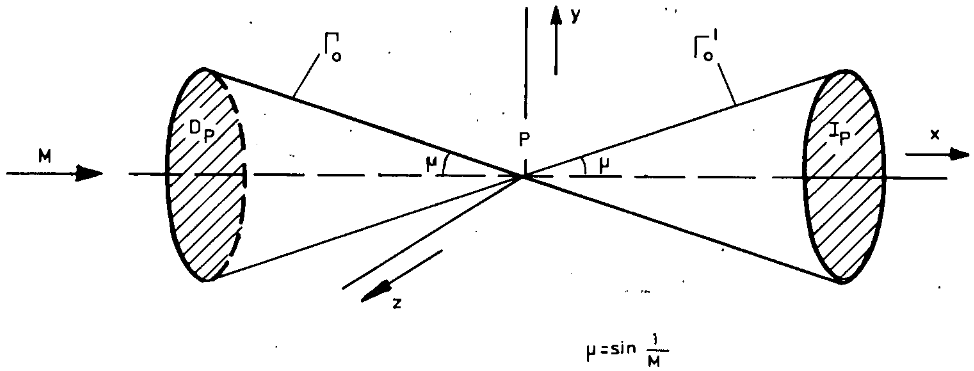


Fig. 1. Dependence and influence domain for the point P.

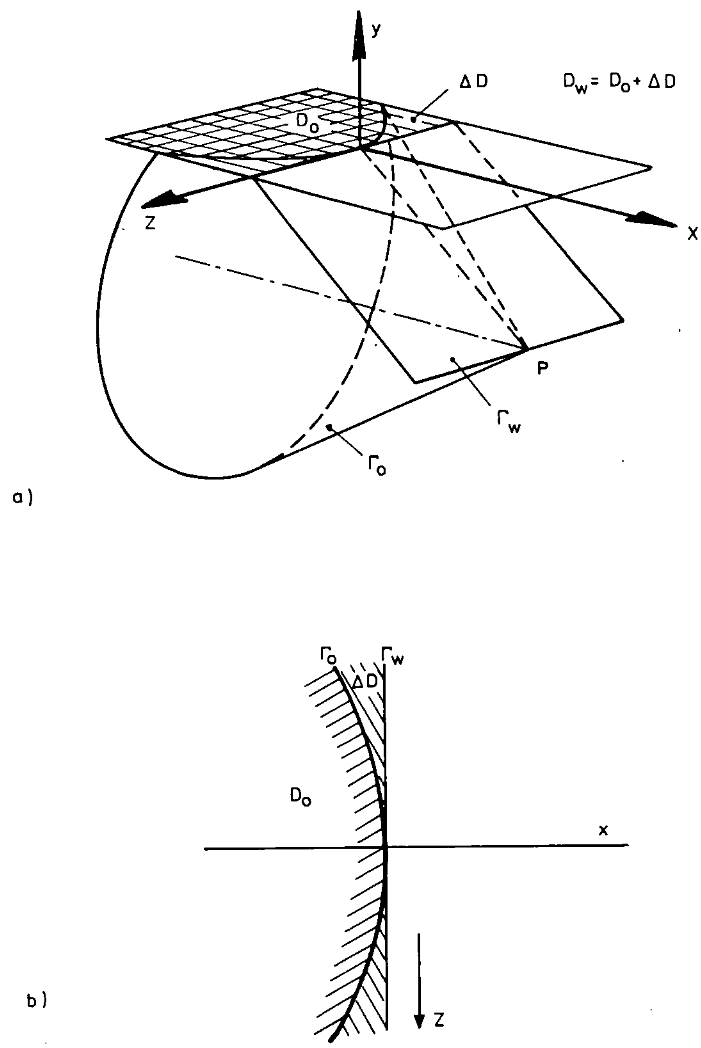


Fig. 2. The approximation of the linear Mach cone Γ_o by the plane Γ_w

a) in spatial representation

b) in planar view

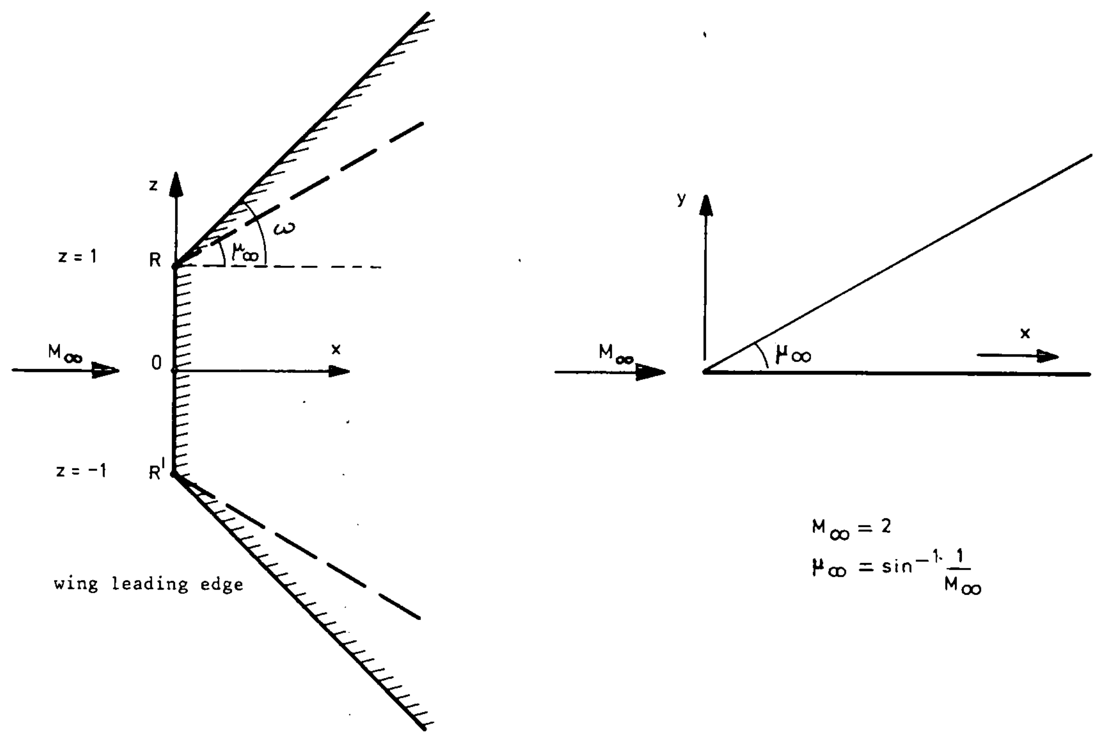


Fig. 3. Trapezoidal leading edge wing without thickness.

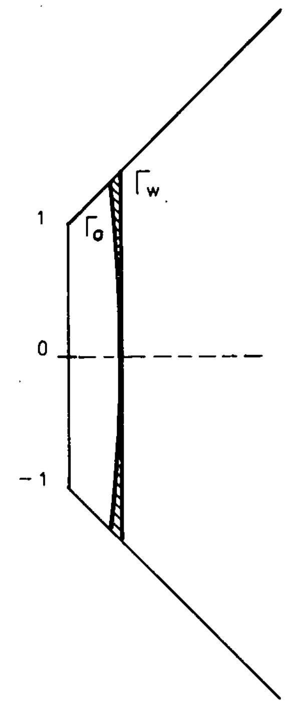


Fig. 4. The approximation of the linear hyperbola Γ_o by the straight line Γ_w for a point on the bicharacteristic $\xi = .4$ at the distance $y = 20$.

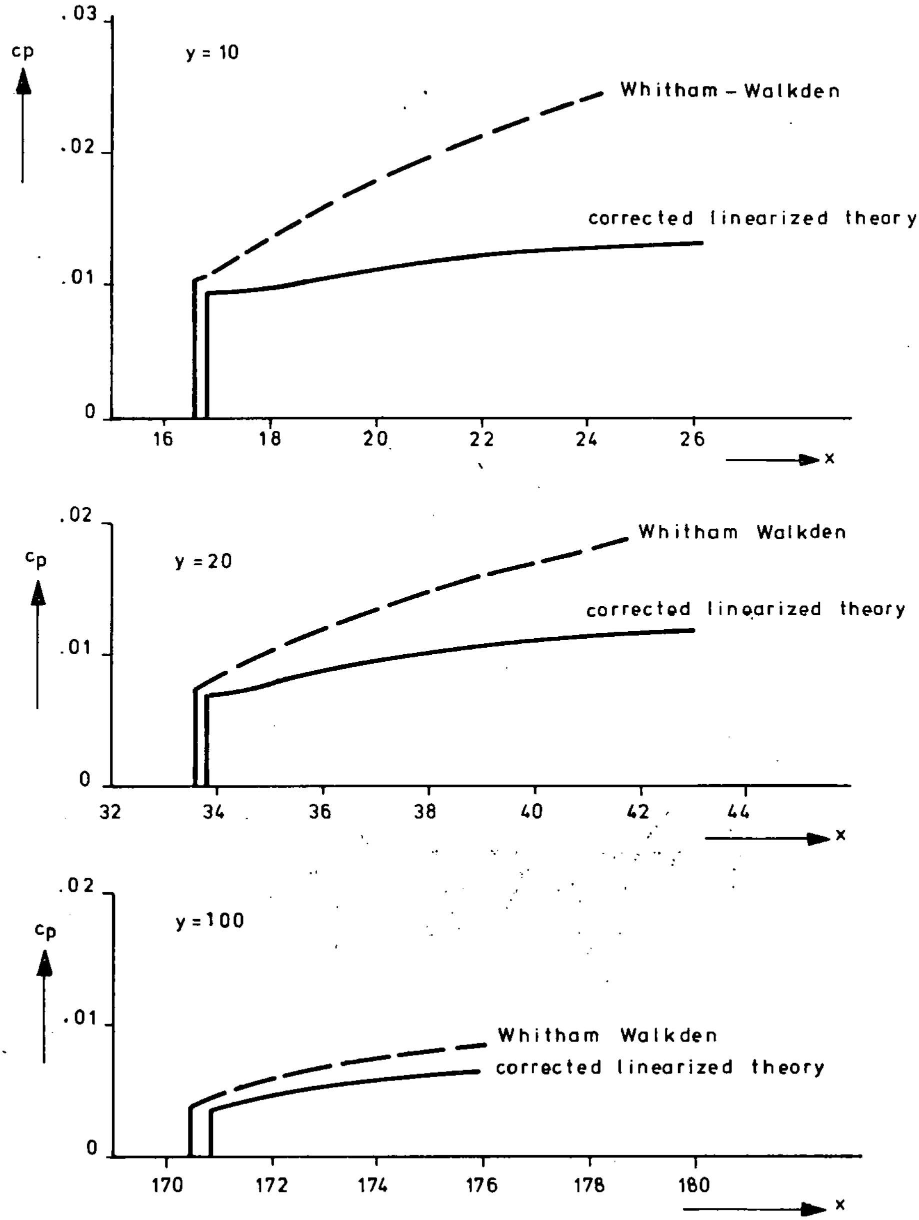


Fig. 5. Pressure signatures for the trapezoidal leading edge wing at vertical distances y=10,20,100 ($\omega=45^{\circ}$, $M=2$, $\alpha=1^{\circ}$)

Compression side.

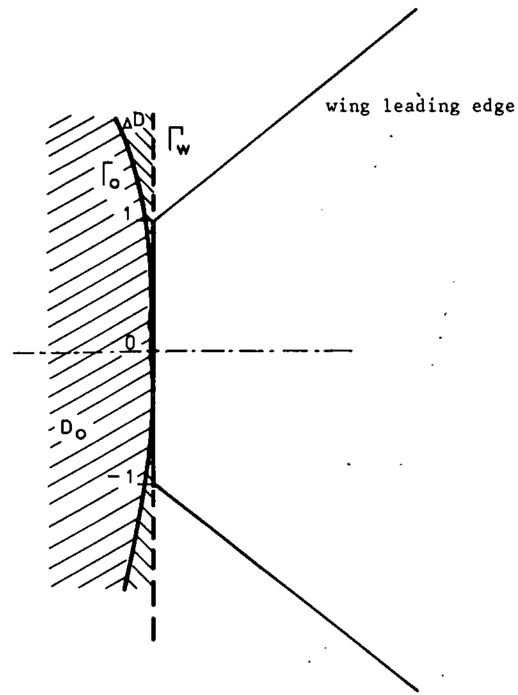


Fig. 6. The approximation of the hyperbola Γ_o by the straight line Γ_w for a point on ξ=+0 at the vertical distance y=20
Dw=Do+ΔD.

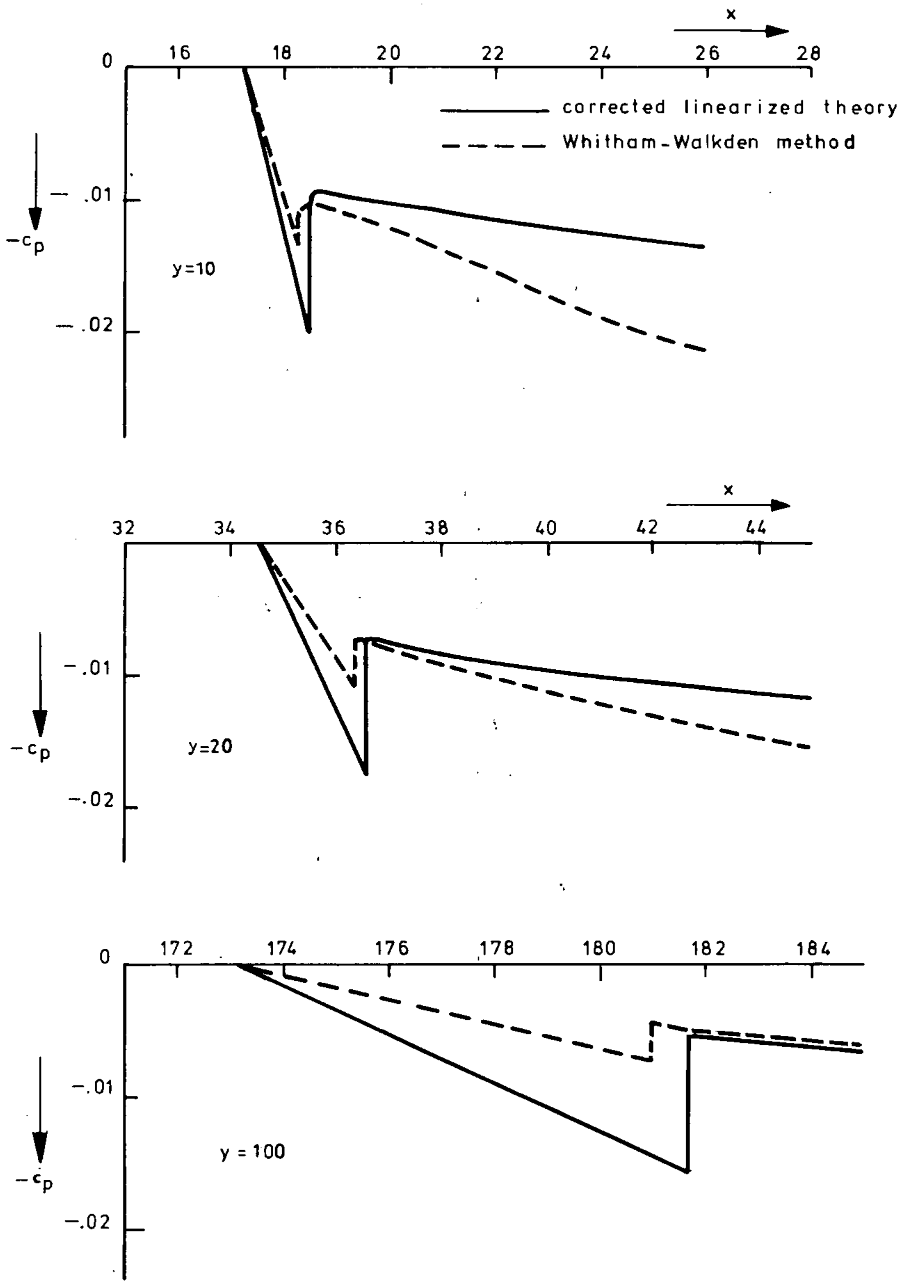


Fig 7. Pressure signatures for the trapezoidal leading edge wing at vertical distances y=10,20,100. ($\omega=55^0$, M=2, $\alpha=1^0$) Expansion side.

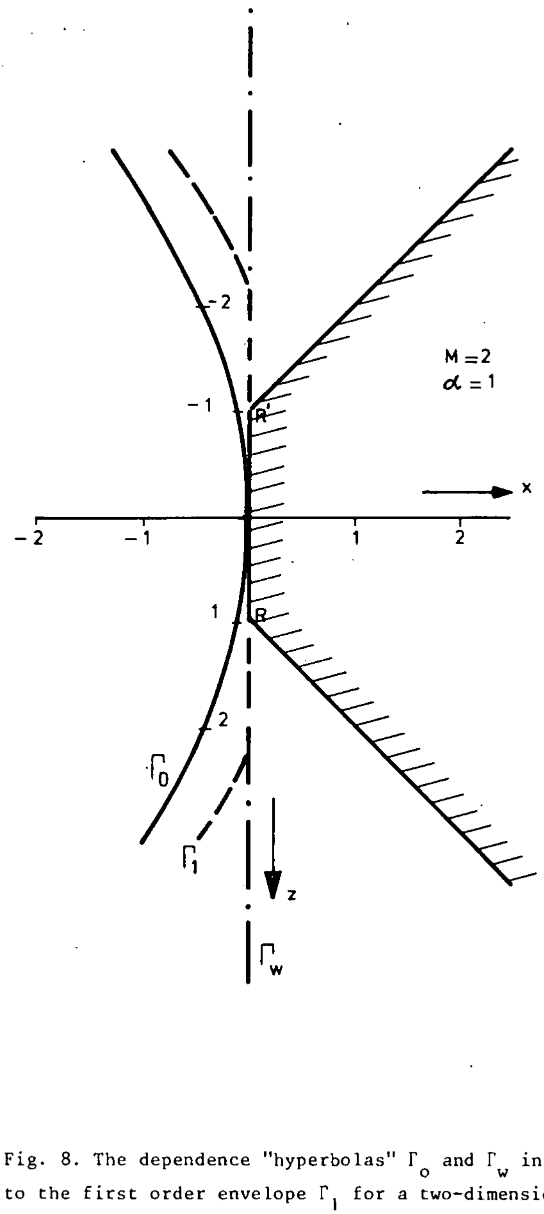


Fig. 8. The dependence "hyperbolas" Γ_o and Γ_w in comparison to the first order envelope Γ_1 for a two-dimensional expansion (for a point at y=8 on the last characteristic surface).

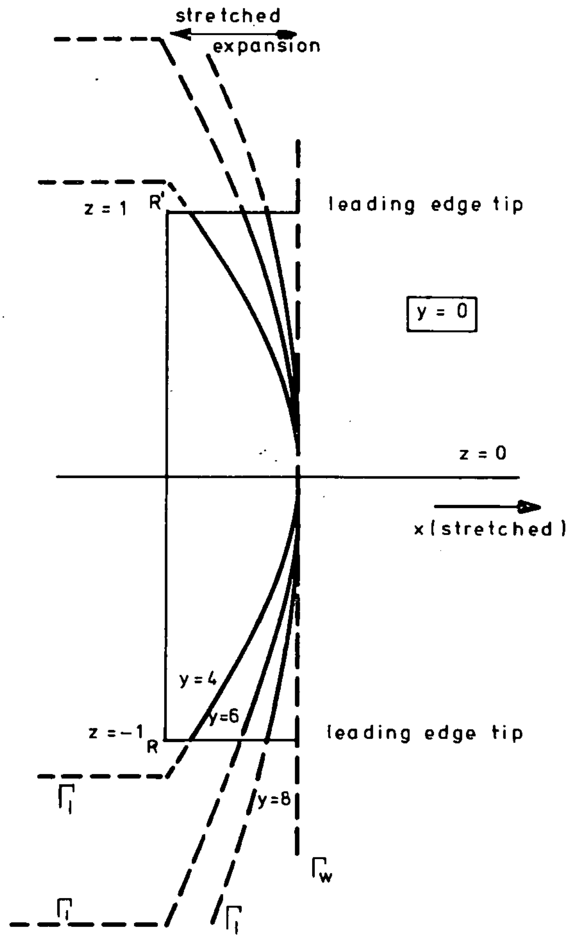


Fig. 9. The boundaries Γ_1 for the first order dependence domains for points on the last bicharacteristic of the trailing edge expansion at various distances from the wing (x-coordinate stretched).

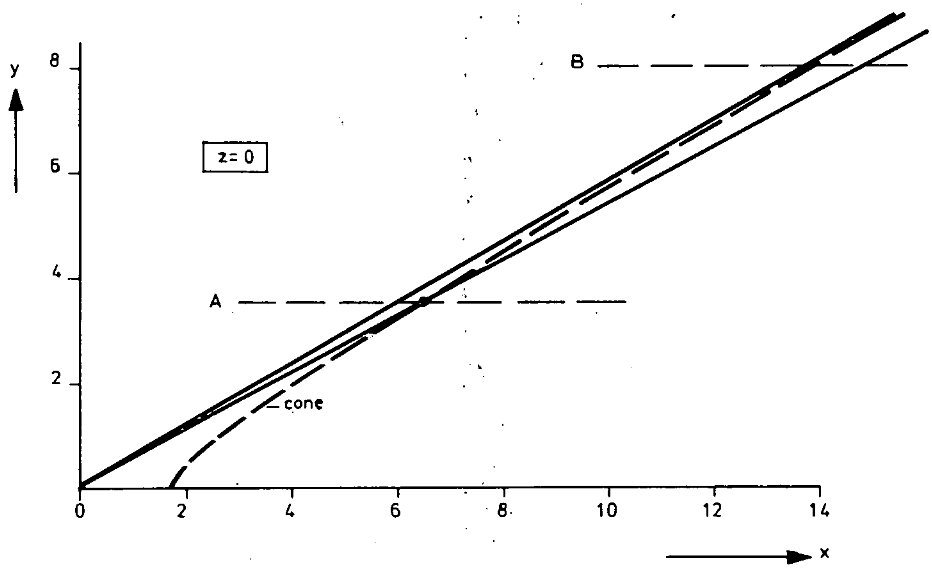


Fig. 10. The intersection of the machcones from R' and R with the symmetry plane of the trapezoidal leading edge wing ($\omega=45^{\circ}$, M=2, $\alpha=1^{\circ}$).

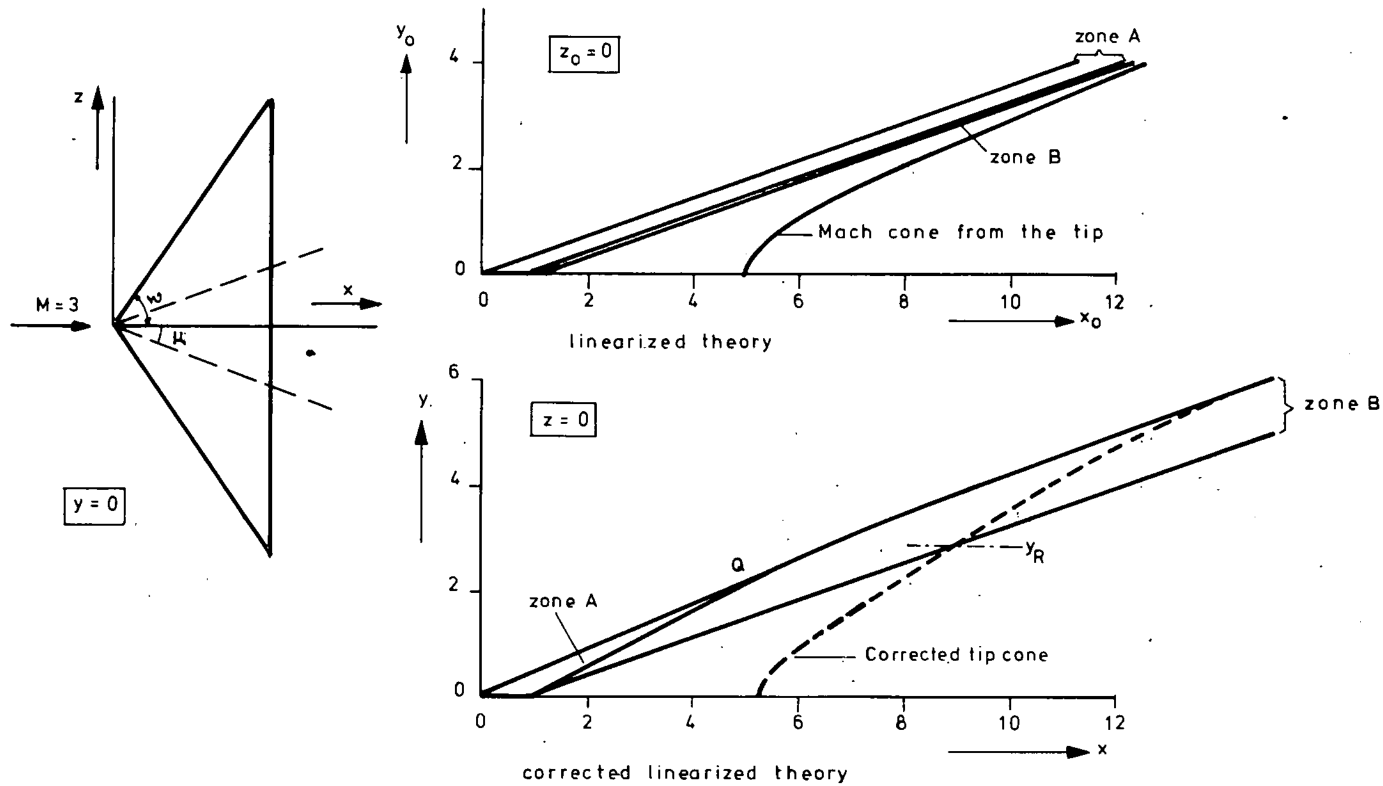


Fig. 11. The supersonic delta wing and its pattern of bicharacteristics (zeroth and first order) in the plane z-o.

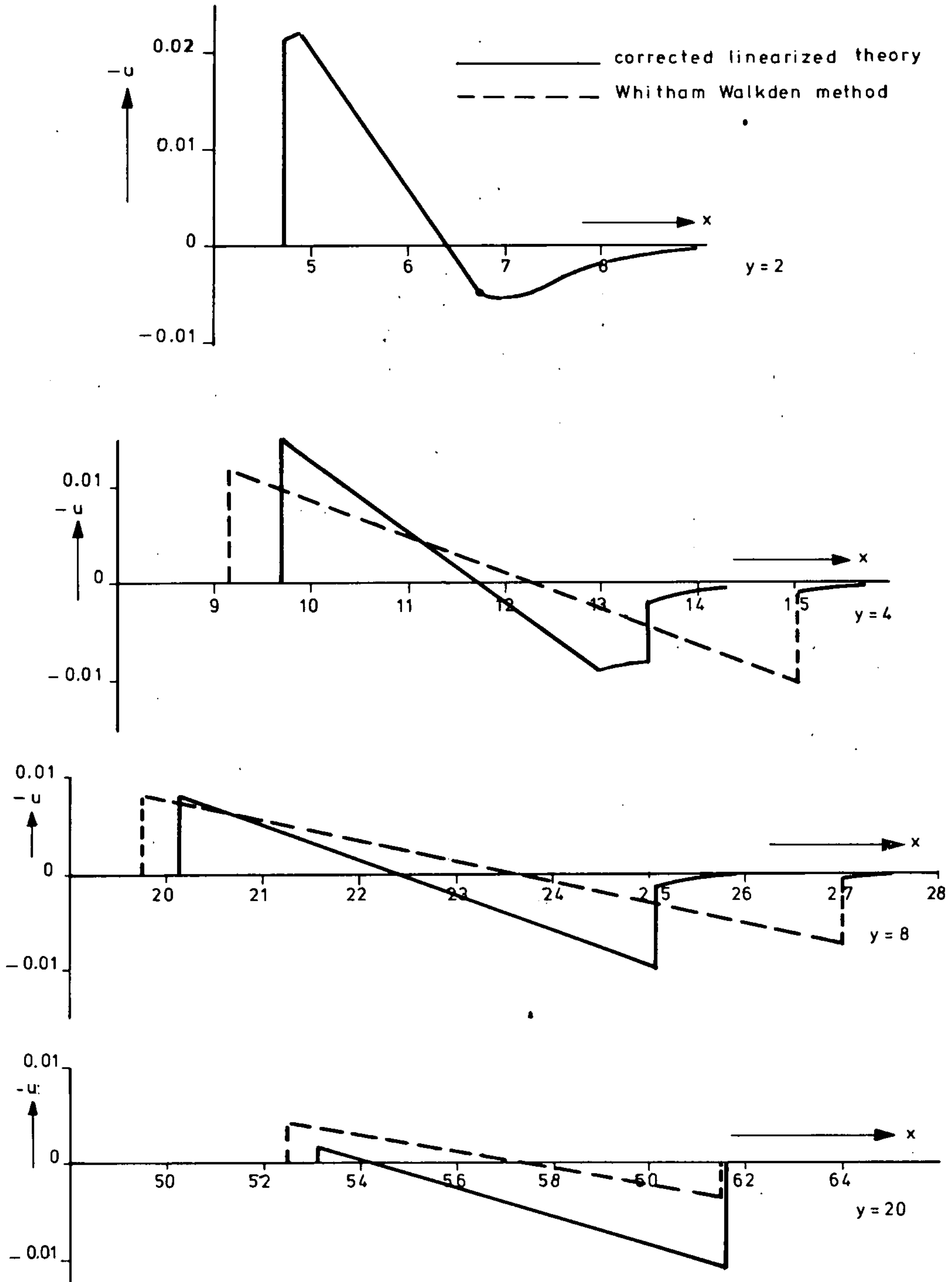


Fig. 12. Pressure signatures for the supersonic delta wing as calculated by t
Whitham -Walkden method and by the corrected linearized theory ($\omega=55^{\circ}$, $M=3$, $\alpha=$

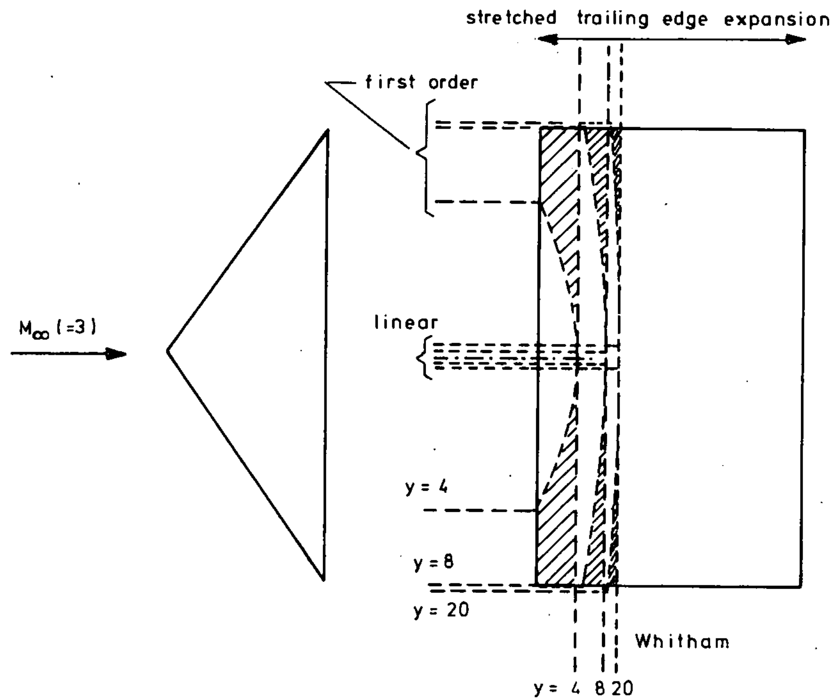


Fig.13. The dependence domains in Whitham-Walkden's theory, compared to the corresponding first order and linear dependence domains for points on the front shock of the delta wing(α=5)

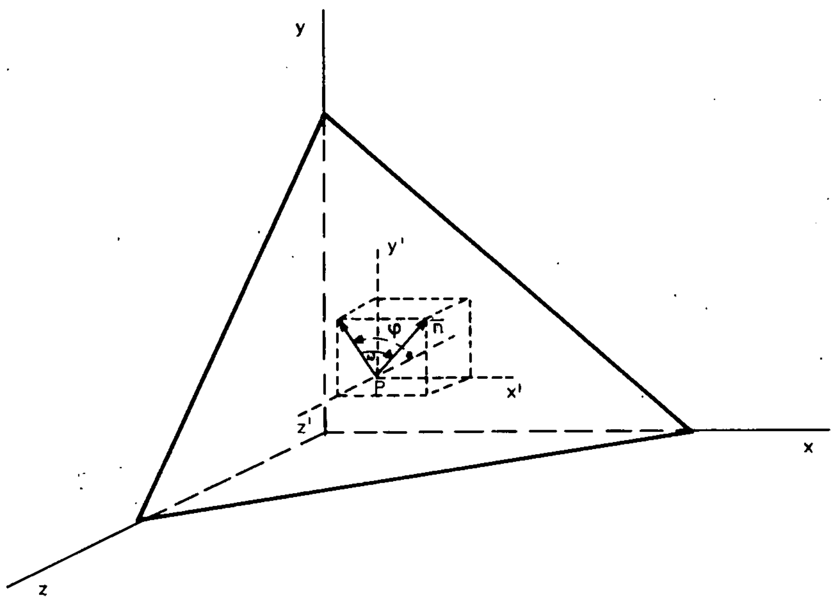
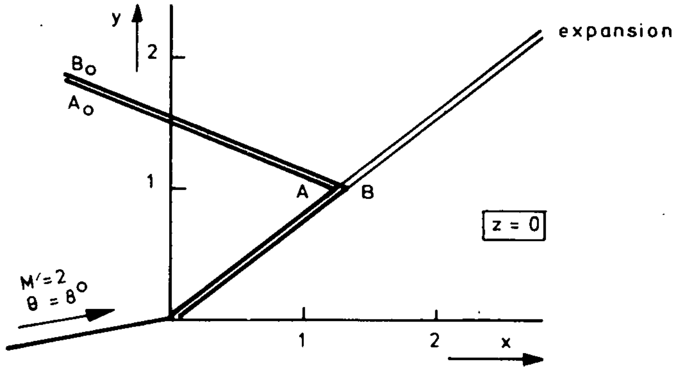
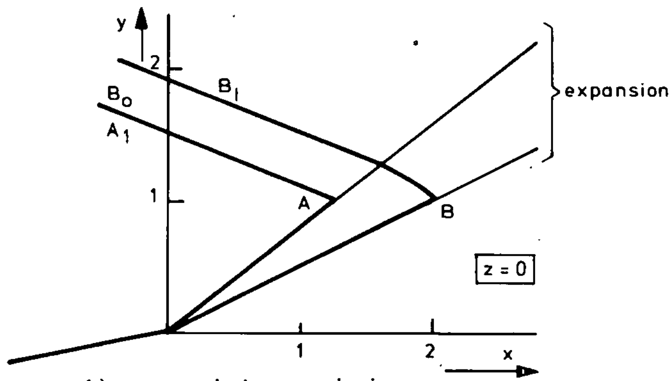


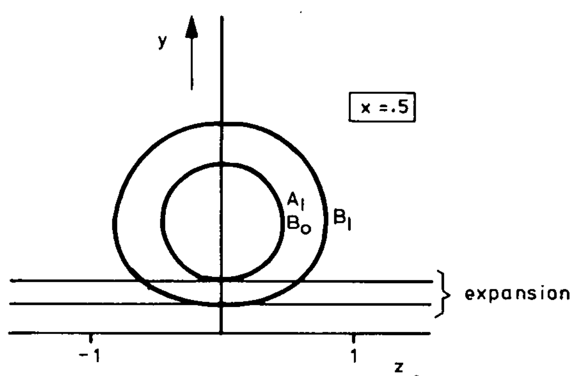
Fig. A1. The coupling of the angles ω and φ to the normal $\bar{n}$ on the characteristic surface at the point P.



a) linear representation



b) corrected characteristics



c) intersection with the plane x=.5

Fig. A2. The first order dependence domains in an expanding flow (for points A and B in the plane y=1).

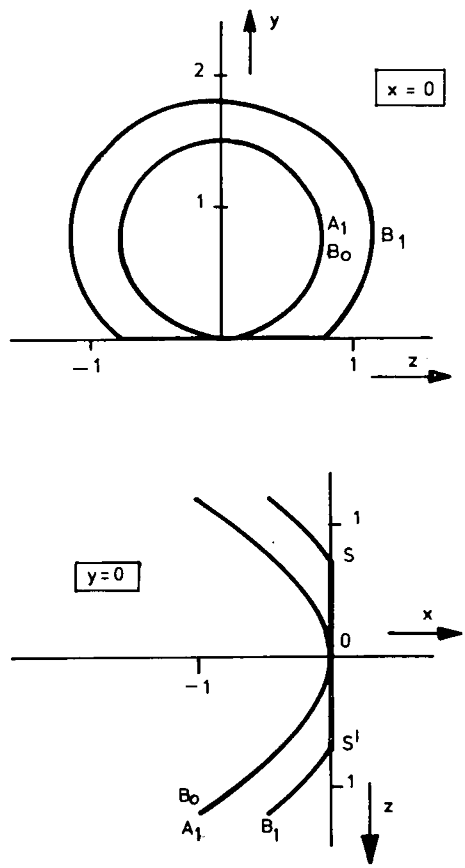


Fig. A2. - concluded -

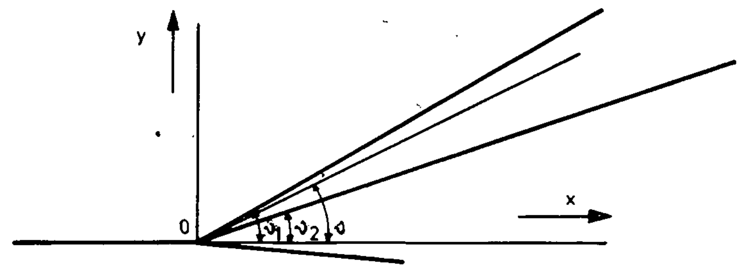


Fig.A3. The angle ϑ with which a specific bicharacteristic meets the expansion's origin.

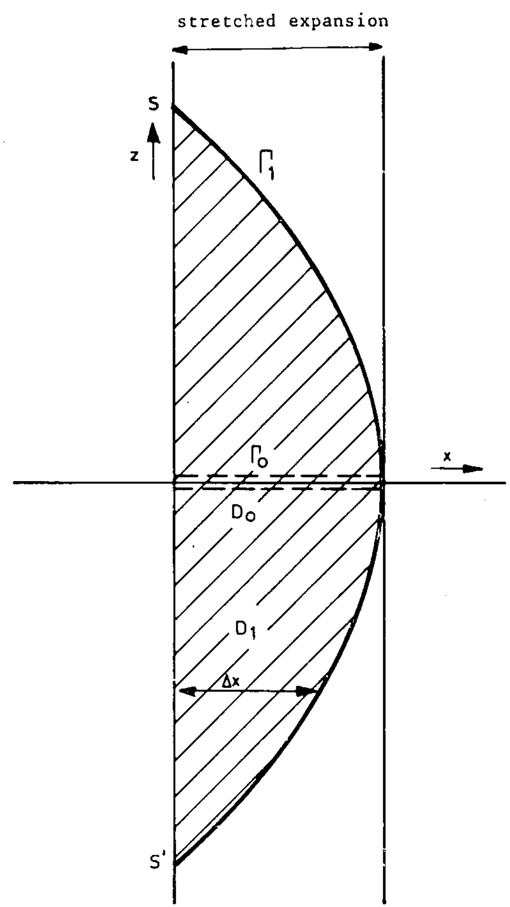


Fig. A4. The comparison of the first order domain D_1 and the zeroth order domain D_o within the stretched expansion ($M = 2$, $\theta = 7.9^o$).

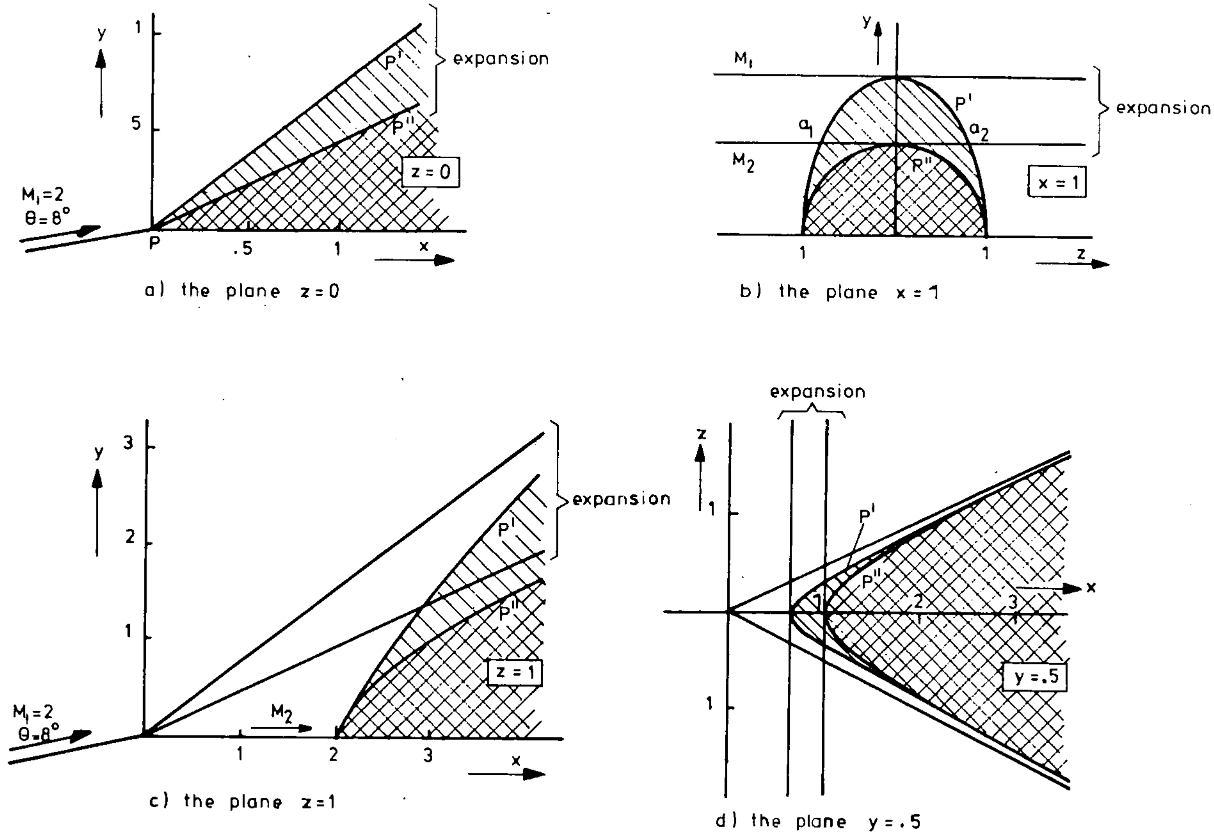


Fig. A5. The first order domains of dependence for a point at the origin of the expansion.