APPLICATION TO AN HYDRAULIC PROBLEM

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Synopsis

This paper discusses the general conditions of the problem of flow distribution in a network of estuarine channels to which an analog computer model was applied. After developing the analog requirements, the model is described, with emphasis on the electronic circuit that provides the required square-law resistance. The equations correlating electrical and hydraulic quantities are developed from the basic electrical and hydraulic relationships. Finally, the methods by which the required boundary conditions were duplicated are discussed.

Introduction

The Delta area of California is a roughly triangular tract of land lying just to the east of Suisun Bay. This area, extending for a distance of about 50 miles north and south with a maximum width of about 25 miles, was originally a marsh with a network of channels threading through it. At the present time this area is agricultural land which has been reclaimed from the marshes by constructing dikes along the old channels to inclose areas that can be pumped out and farmed.

The Delta is traversed by the Sacramento River, entering from the north, by the San Joaquin River, entering from the south, and by the north and south forks of the Mokelumne River that come in from the east. The old network of channels, effectively preserved and stabilized by the process of reclamation, still carries the flow of these streams through the Delta.

Tides coming into San Francisco Bay from the Pacific Ocean propagate themselves through Suisun Bay and enter the Delta channels. Since the tidal currents generally exceed the currents resulting from stream flow, the direction of flow in the channels is periodically reversed and the resulting movement and mixing of fresh and saline waters provides a mechanism capable of propagating ocean salinity into the Delta channels. The salinity encroachment is held in

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check by stream flow that tends to flush the salinity out of the channels. In times of flood the salinity is driven back but in times of low stream flow the tidal ebb and flow succeeds in carrying some salt water into the channels.

Construction of Shasta Dam on the upper Sacramento River has made available a water supply intended for use on some of the lands in the San Joaquin Valley. To supply this demand, the Tracy pumping plant will lift water out of the channels at the south end of the Delta. This water must be brought across the Delta through its channels.

The problem to be solved is how to bring the Sacramento River water across the Delta to the San Joaquin side while maintaining a pattern of flow in the channels which will hold the intrusion of ocean salinity in check and thereby permit the transfer to be made without danger of contamination.

Reasons for Use of an Analog

With the Tracy pumps in operation, it will be necessary to increase the natural transfer of water from the Sacramento channel to the San Joaquin channel in order to replenish the water supply of the southern part of the Delta and thereby maintain a proper balance of flow. A tidal phase difference exists at one of the sites where a channel could be cut through to increase the transfer, and since gates would be necessary in any case for protection during floods, it would be possible to open the gates when the tidal currents were favorable and to close them when the currents were adverse to increase the net flow. Good

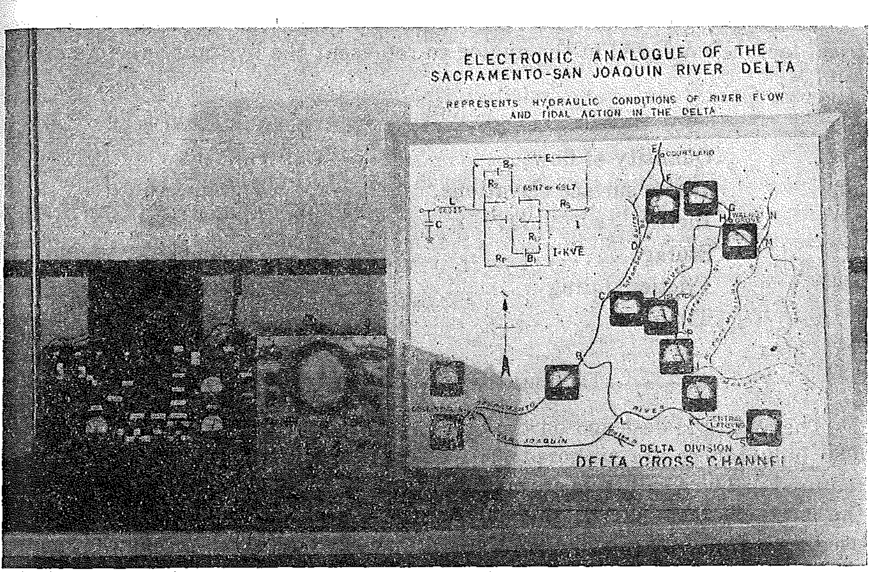

Fig. 1.—External Appearance of Electronic Analog

progress had previously been made for estimating flow patterns by model testing and by use of the procedure of Hardy Cross,[4] Hon. M. ASCE, but the

[4] "Analysis of Flow in Networks of Conduits or Conductors," by Hardy Cross, *Bulletin No. 286*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., November, 1936.

complexities introduced by the tidal factor made it desirable to seek some new method of solution.

The electronic analog computer, built to expedite these computations, was successful for this purpose. The appearance of the completed analog is shown in Fig. 1.

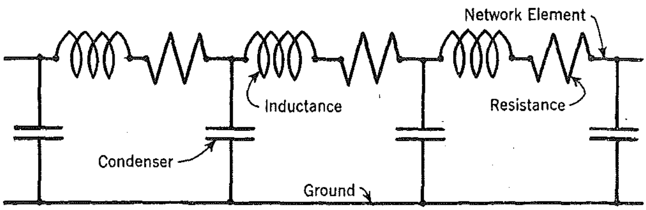

FIG. 2.—BASIC ANALOG CIRCUIT

DESIGN REQUIREMENTS

In order to solve the Delta problem, it was required that the analog be able to reproduce the square-law relation between friction and velocity that is characteristic of fluid flow. In addition, it was required to represent the wave motion associated with the tides. To do this, the factors of inertia and of storage resulting from water level changes had to be accounted for. The electrical factors employed in the analog to represent the hydraulic factors are as follows:

Hydraulic	Electrical
Quantity of flow	Current
Water surface elevations	Voltage
Inertia	Inductance
Storage	Capacity
Frictional drag	Resistance
Time	Time

DESCRIPTION OF THE ANALOG

The analog is designed on the basis of circuits of the type shown in Fig. 2. The inductances are air-cored coils of commercial types or were wound as required. The condensers are commercial units of the paper or mica type. In the large channels having very low frictional resistance, linear resistors were used with appropriate average values for the currents flowing. In the smaller channels, however, it was necessary to use a type of square-law resistor, obtained by taking advantage of certain vacuum tube characteristics that have approximately the required form of variation. These tubes were used with resistors in parallel and in series to obtain the desired characteristic. A biasing voltage was also required in this adjustment. The circuit used in such cases is shown in Fig. 3.

A dual triode tube with sections connected in parallel, opposing, is used to permit current to flow in either direction. This type of resistor is not wholly satisfactory since the tubes show variations that make it necessary to adjust each section separately. The current carrying capacity is restricted within narrow limits, and it is necessary, therefore, to design the analog around these elements. Net current flows were read on direct current milliammeters and tidal amplitudes and phase differences were read on a cathode-ray oscilloscope. The gate keeper was represented by a rectifier circuit that was also found to have some shortcomings near the zero point, introducing an effect analogous to gate leakage. In spite of these minor difficulties, the analog operates in a very satisfactory manner. Some idea of the speed with which the device works may be obtained from the fact that the analog runs through about 500 days of actual tidal changes in each second of operating time.

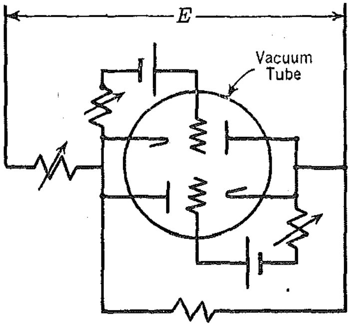

FIG. 3.—SQUARE-LAW RESISTOR

BASIC EQUATIONS

In setting up the correlation equations, the electrical circuits were assumed to have their resistance, inductance, and capacity uniformly distributed along their length. In practice, these elements and the square-law resistances were lumped. The inertia and storage factors were considered together, and the resistances were considered separately.

A longitudinal section of a stream channel is shown in Fig. 4. The shaded element in Fig. 4 represents a lamina of width b_w, depth H, and length dx. For analytical purposes the actual channel is assimilated to a uniform rectangular channel that has the same top width and cross-sectional area as the actual channel. As stated previously, frictional forces are not introduced into the dynamical equations, but are treated separately. Since x represents a distance measured along the stream from some fixed point on the bank, the planes

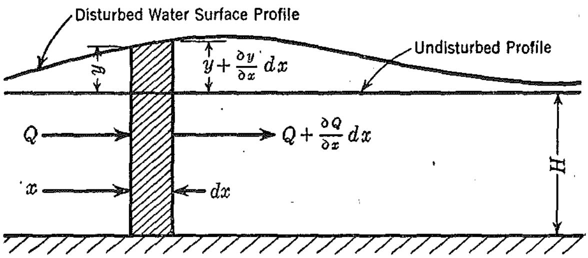

FIG. 4.—LONGITUDINAL SECTION OF A CHANNEL

defined by x and $x + dx$ do not change position with time. It is assumed that y, the surface elevation above sea level, is small compared to H, the depth of the stream.

The continuity condition requires that, if the quantities of water flowing through the planes x and $x + dx$ differ, the surface elevation must rise or fall as required to accommodate the changes of volume. If small quantities are neglected, this requirement is expressed by the equation:

$$b_w\, dx \frac{\partial y}{\partial t} = + Q - \left(Q + \frac{\partial Q}{\partial x}\, dx\right) \quad (1)$$

in which t represents time and Q represents flow. If a surface gradient $\partial y/\partial x$ is present, the water depth on one side of the lamina will be greater than on the other side by the amount $(\partial y/\partial x)\, dx$ and the additional pressure resulting from this head differential will cause the water within the lamina to be accelerated. Thus, the requirements of Newton's law are expressed to a first order of approximation by

$$\frac{\gamma\, b_w H}{g}\, dx \frac{\partial}{\partial t}\left(\frac{Q}{b_w H}\right) = -\gamma\, b_w H \frac{\partial y}{\partial x}\, dx \quad (2)$$

in which γ represents the weight of water per unit volume and g represents the acceleration of gravity. Eqs. 1 and 2 can be simplified by canceling common terms and collecting. Then the equation of continuity becomes

$$\frac{\partial Q}{\partial x} + \frac{b_w\, \partial y}{\partial t} = 0 \quad (3)$$

and Newton's law takes the form:

$$\frac{\partial y}{\partial x} + \frac{1}{g\, H\, b_w} \frac{\partial Q}{\partial t} = 0 \quad (4)$$

It is of interest to note that, if Q is eliminated from Eqs. 3 and 4, the wave equation is obtained:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{g\, H} \frac{\partial^2 y}{\partial t^2} \quad (5)$$

The relation between flow and gradient for the hydraulic channel can be expressed in the form:

$$Q = M \sqrt{\frac{\partial y}{\partial x}} \quad (6)$$

in which M is a constant of the channel specifying its flow resistance. Eq. 6 may be recognized as a form of the Chezy formula. In the electrical circuits let C represent the capacity per unit length of circuit; E the potential with respect to ground; I the current; K a constant applying to a circuit; r the resistance per unit length of circuit; η the time in the analog; λ the inductance per unit length of circuit; and ξ the distance along a circuit.

Then, the equations for the idealized electrical circuits[5] that correspond to Eqs. 3 and 4 for the hydraulic channels are

$$\frac{\partial I}{\partial \xi} + C\,\frac{\partial E}{\partial \eta} = 0 \qquad (7)$$

$$\frac{\partial E}{\partial \xi} + \lambda\,\frac{\partial I}{\partial \eta} = 0 \qquad (8)$$

From Eqs. 7 and 8, there is obtained, on elimination of I,

$$\frac{\partial^2 E}{\partial \xi^2} = \lambda\,C\,\frac{\partial^2 E}{\partial \eta^2} \qquad (9)$$

For the circuits provided with an electronic resistor to represent hydraulic resistances of the type expressed by Eq. 6

$$I = K\sqrt{\frac{\partial E}{\partial \xi}} \qquad (10)$$

or, if the circuit has a linear resistance,

$$I = \frac{1}{r}\,\frac{\partial E}{\partial \xi} \qquad (11)$$

Correlation Equations

The electronic analog operates at a frequency of 1,000 cycles per sec. The sinusoidal variations imposed on the analog approximately represent tidal oscillations having a frequency of about 2 cycles per day. The correlation equations that were found suitable for use with the available electrical components are as follows:

$$y = 0.1\,E \qquad (12a)$$

$$Q = 10{,}000{,}000\,I \qquad (12b)$$

$$x = 10{,}000\,\xi \qquad (12c)$$

$$t = 45{,}000{,}000\,\eta \qquad (12d)$$

Other applications would, of course, require other constants. An analogous electrical quantity is obtained by substituting the foregoing relations into the hydraulic equations. For example, Eq. 6, on substitution becomes

$$10{,}000{,}000\,I = M\sqrt{\frac{0.1\,\partial E}{10{,}000\,\partial \xi}} \qquad (13a)$$

or

$$I = \frac{M}{3.2 \times 10^9}\sqrt{\frac{\partial E}{\partial \xi}} \qquad (13b)$$

[5] "The Theory of Sound," by Lord Rayleigh, Dover Publications, London, England, 1945, Vol. 1, p. 467, paragraph 235.

Then, the quantity $\frac{M}{3.2 \times 10^9}$ is the K-value in Eq. 10. By this choice of constants the electrical circuit is given resistance characteristics that are analogous to the friction in the corresponding hydraulic channel. The other relations are treated in a similar way.

Boundary Conditions

To account for the stream flow it was necessary to introduce direct currents of specified amounts at certain points in the analog and to take them out at certain other points. In general, the currents fed into the network represent river flows entering the Delta area, while currents leaving the network represent the draft of the Tracy pumps and the flow from the Delta area into Suisun Bay. To simulate these currents, voltages of controllable magnitude were introduced between the network and the ground wire (see Fig. 2). Control of the currents was obtained by variable resistors located at the points at which the currents enter and leave the network.

The tides were represented by alternating voltages of specified magnitude applied between the network and the ground wire at the point on the analog representing the entrance to Suisun Bay. A blocking condenser was used here to prevent the flow of direct current through the transformer windings. The actual tides occurring at this point vary somewhat from day to day because of varying phase relations between the lunar and solar components. In the analog, these tidal variations were replaced by a single sinusoidal variation of average amplitude. The connections arranged for introducing the direct currents representing stream flow would permit the alternating currents representing the tides to pass into the ground wire at other points than that representing the entrance to Suisun Bay. Since this would introduce errors, inductive blocking impedances were placed in the direct current circuit wherever necessary to confine the alternating currents to the proper network circuits. At points where stream channels continued beyond the area represented by the analog, lumped impedances were introuced in the circuit to represent those portions beyond the analog area. In most cases these impedances were determined by trial, so that known tidal behavior would be properly represented.

In order to protect the direct current meters from loss of field caused by the alternating current components, these meters were shunted by condensers having impedances to alternating current that were low compared to the resistance of the meter.

Conclusions

An analog of the type described in this paper is an effective means of expediting the work of finding flow distribution patterns in a network of channels. It is particularly effective when tidal effects must also be included in addition to gravity flows. The results obtained have checked well with those obtained by other means.

DISCUSSION

J. VAN VEEN[6].—Tidal flow in estuaries, with which this paper is concerned, is a problem in which the Dutch engineers have been much interested for a long time. It may be worthwhile to mention some Dutch practices.

For deltaic schemes, such as for the making of new tidal channels or improving them by dredging (that is, narrowing or widening them), making new open harbors, closing inlets, and other similar projects, Dutch engineers use three different methods to check one another and to enhance results. They are: (1) Mathematical methods, (2) hydraulic laboratory methods, and (3) electrical methods. These methods give nearly the same results when handled well, but each has a point of advancement over the other. (1) Mathematical research is very satisfactory except in the amount of work that has to be put into the calculations. (2) Hydraulic laboratory tests give quick results, but they may be somewhat lacking in exactness, mainly because of the difficulties arising in reproducing the right amount of friction; baffles must be used, which (produced on the prototype scale) would be enormous structures in the bed of the river, taking perhaps 10% or 20% of the width. (3) Electrical imitation of the tides is essentially the same as the mathematical method because this imitation is a calculating machine, or electronic computer, based on the mathematical formulas. Although the results can be obtained very quickly and accurately, the computer needs much supervision, and none of the many electronic tubes should show deterioration.

The three methods can be coordinated very well. It became possible by taking the average of 2 or 3 runnings of the model to reach an accuracy of about plus and minus 3% in the normal vertical tide on an hydraulic model of the estuary of the Rhine-Maas (the vertical scale was 1:64 and the horizontal scale was 1:2400) when the results of that model were compared with those of accurate measurement data. The accuracy of the data given by the electrical model and by the mathematical computations may be as great. The accuracy of the data for the currents, obtained by electrical models, may be much greater than that obtained by an hydraulic model.

Electrical imitation can do much work alone—namely, the work of determining the water levels and water currents in the channels of an estuary, but not the determination of the sand movements, scouring, and siltations. Those problems are for hydraulic models and mathematics. The literature shows that it took engineers a long time to solve the electrical imitation problems wholly, or at least to great perfection, partly because the intricate details of mathematics had to be solved first. There resulted three different mathematical methods to solve the quadratic tidal formulas: (*a*) Harmonic method, using sinusoids; (*b*) Taylor series development; and (*c*) the method of charac-

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teristic wave components. Without the knowledge of these methods, good results cannot be obtained by electrical computers.

Although giving practically the same results, all three mathematical methods are extremely unwieldy to handle—so much so that for a certain proposed project only a few calculations could be made per year (using about ten calculators), although the results of scores of calculations were needed. Often fifty equations had to be solved each time—hence the need of hydraulic or electrical models. H. A. Lorentz, of Leiden University, initiated the modern method of tidal calculations in 1918 when calculating the Zuider-Zee enclosure scheme. He used formulas of the linear type that (after a slight modification) electrical engineers are accustomed to call the telegraph equation. It would have been easy to imitate electrically the tides in the Zuider-Zee channels according to these linear formulas, and the solution would have been simple. It is the quadratic law of resistance of water movement that necessitates the use of highly complex mathematics and electronics. It increases difficulties a hundredfold. Of course, neither the electrical computers nor the hydraulic models of greatly reduced scale are able to deal with sand movements and salt problems.

Thus, the principle of the computer described by the authors offers the advantage of speed and accuracy, and there are no limitations as to its ability to imitate purely hydraulic phenomena except the great practical and theoretical difficulties of construction of the computer. Some difficulties that were encountered by Dutch engineers in their 30 years of practice may also have been experienced by the authors and are described here.

The accuracy of the electrical method depends mainly on the accuracy, flexibility, and reliability of the elements of self-inductance, capacity, and resistance; on the number of sinusoids used for the boundary conditions; and also on the length of a river section. Engineers of the Dutch Government take these sections no longer than 5 km. The accuracy of the electrical measurements is about 1%. In Holland engineers use a period of $\frac{1}{1,000}$ sec for a tidal period. Cathode-ray tubes are used as indicators only, not for measurements.

Of course, the elements of self-inductance, capacity, and resistance must change with the tidal depths and tidal widths of each section. This caused great difficulties in the construction of the Dutch electrical computer. The authors have not mentioned that problem, therefore, their computer will be fit for small tidal ranges only, say, 1 ft or 2 ft. Another difficulty was the inability of the factories to make special electronic tubes having the desired variable quadratic accuracy. The electrical computers for engineering purposes do not use exact "quadratical"-tubes in great quantities so that the manufacture of these special tubes is costly. When these "quadratical law"-computers become numerous, manufacture of these special electronic tubes may become economically feasible. However, some other electrical design has been evolved which gives good results, although it is not so elegant as the use of special electronic tubes would be.

All sections of the Dutch computer are constructed for universal use; they may be used for channels of any width, depth, and tide within wide ranges. One hundred sections, or elements, or more are needed to imitate the main networks of the Dutch tidal channels. A rather large room is needed to house this computer although each element is contained in a box 1 ft $\times 1\frac{1}{2}$ ft $\times 1\frac{1}{2}$ ft. There are auxiliary instruments for the boundary conditions and for measurements. The universal characteristics of each section imply that any existing or future network and any tide can be imitated with those sections. The exact imitation of tides by electrical gadgets is difficult. Although, under the heading, "Conclusions," the authors assert that they have found their computer particularly effective for their purpose; great difficulties arise when more detail is wanted. The imitation of currents in a network of pipes is not so involved. Water currents in soils, or in dams, can be imitated easily because they are linear. Also, the air currents in an underground railroad system or in a mine can be imitated easily by means of electricity. It has become a general practice in the past few years in Holland.

If the engineers would unite in their efforts to induce electronic tube factories to produce the required kind of electronic tube (especially a tube with a variable quadratic characteristic), this would mean a simplifying and refining of electrical computers. The Dutch engineers would welcome any suggestion as to how to obtain the special variable quadratic valve-tubes.

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W. Douglas Baines,[7] J. M. ASCE.—The representation of a physical phenomenon by another physical phenomenon, its analog, can be divided into two distinct steps. The first step is the accurate mathematical description of the first phenomenon step and the second is the representation of this mathematical description by the second phenomenon. In general, it can be stated that, if the two phenomena are described by the same mathematical equations (with corresponding boundary conditions), the second phenomenon is a perfect analog of the first. In the authors' attempt to make an electrical analog of unsteady flow in a natural river system, they have written sets of mathematical equations to describe the flow in the river channels and the electrical properties of the analog circuit. The mathematical equations are identical—hence, the electrical analog can be expected to solve the equations which are set up for the flow. However, the authors have not clearly explained how the mathematical equations describe the physical aspects of the flow. They have asserted that they are using the first approximation to the complete flow equations, but they have not shown to what extent this approximation describes the flow as it occurs in the natural river channels. The writer's criticism is limited to a discussion of this approximation.

The equations of motion for unsteady flow in an open channel are[8] the continuity equation:

$$\frac{\partial Q}{\partial x} + b_w \frac{\partial Y}{\partial t} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and Newton's second law:

$$S_o - S_f = \left(1 - \frac{Q^2}{b^2{}_w\, g\, Y^3}\right)\frac{\partial Y}{\partial x} + \frac{2\,Q}{b_w\, g\, Y^2}\frac{\partial Q}{\partial x} + \frac{1}{b_w\, g\, Y}\frac{\partial Q}{\partial t} \quad \ldots\ldots (15)$$

in which S_o equals the river bed slope, S_f equals the friction slope, and Y equals $y + H$, thus representing the instantaneous water stage. The equation of continuity (Eq. 3), used by the authors, can be seen to be the exact equation,

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[8] "Flood Routing," by B. R. Gilcrest, in "Engineering Hydraulics" (edited by H. Rouse), John Wiley & Sons, Inc., New York, N. Y., 1950, p. 640.

but several terms have been dropped from Newton's second law as can be observed by comparing Eq. 15 with Eq. 4. In obtaining their first approximation, the authors must, therefore, have made the following assumptions:

1. The tide height disturbance, y, is small as compared to the mean depth, H. It is not clear how small y must be in comparison to H for this condition to be true, but it seems that, if y/H had a maximum value of about 0.1, such an assumption would be justified.

2. The depth H is not a function of x. This is tantamount to assuming that the natural river channel has a uniform cross section and is straight. This assumption may, or may not, be correct and can be checked only by a close examination of the river. Judging that these two assumptions are satisfied, Eq. 15 reduces to

$$S_o - S_f = \left(1 - \frac{Q^2}{b_w\, g\, H^3}\right) \frac{\partial y}{\partial x} + \frac{2\, Q}{b_w\, g\, H^2} \frac{\partial Q}{\partial x} + \frac{1}{b_w\, g\, H} \frac{\partial Q}{\partial t} \quad \text{.....(16)}$$

3. It is assumed that $\frac{Q^2}{b_w\, g\, H^3} \ll 1$. Examination of the small amount of field data at the writer's disposal[9] shows that this assumption is justified. For August 23, 1929, the flow in the San Joaquin River, at Antioch, Calif., has the maximum value for $\frac{Q^2}{b_w\, g\, H^3}$ of $\frac{Q^2}{b_w\, g\, H^3} = F^2 = \frac{(2.5)^2}{32.2 \times 40} = 0.00475 \ll 1$, in which F equals the Froude number of the flow.

4. The term $\frac{2\, Q}{b_w\, g\, H^2} \frac{\partial Q}{\partial x}$ is much smaller than the terms $\frac{\partial y}{\partial x}$ and $\frac{1}{b_w\, g\, H} \frac{\partial Q}{\partial t}$. Again, the only way these terms can be checked is by an examination of field data from the river. The writer does not have enough data available to verify this assumption, and so is unable to decide whether or not it is justified. However, in the case of the Fraser River (in British Columbia, Canada) and other streams which the writer has examined in detail, it has been found that $\frac{2\, Q}{b_w\, g\, H^2} \frac{\partial Q}{\partial x}$ is of the same order as the other terms in the equation. Accepting the contention that the term is negligible for the San Joaquin River, then Eq. 16 is simply

$$S_o - S_f = \frac{\partial y}{\partial x} + \frac{1}{b_w\, g\, H} \frac{\partial Q}{\partial t} \quad \text{....................(17)}$$

5. It is assumed that $S_o - S_f = 0$. It is most common in the analysis of unsteady flow to use Chezy's form for the expression of friction slope, S_f, so that this assumption gives the following equation, which must be satisfied by the discharge:

$$Q = M \sqrt{S_o} \quad \text{........................(18)}$$

It is easily seen that Eq. 18 is not physically correct because of the fact that S_o

[9] "On the Nature of Estuarine Circulation," by H. Stommel and H. G. Farmer, Woods Hole Oceanographic Inst., Woods Hole, Mass., August, 1952.

is a constant, requiring Q to be constant. The authors have side-stepped this difference by replacing S_o with $\frac{\partial y}{\partial x}$, the instantaneous water surface slope, in obtaining Eq. 6. The reason for the choice of $\frac{\partial y}{\partial x}$ is not clear because it produces a condition that is not physically correct. For example, near local high or low tide—$\left(\text{when } \frac{\partial y}{\partial x} = 0\right)$ $Q = 0$ by Eq. 6. It is demonstrated subsequently that this fact is contrary both to the general solution of Eqs. 3 and 4 and to the observed facts. The exact relation between flow and friction is found by solving Eq. 17 for Q, which yields

$$Q = M \sqrt{S_o - \frac{\partial y}{\partial x} - \frac{1}{b_w\, g\, H} \frac{\partial Q}{\partial t}} \qquad (19)$$

It may be true that the expression under the square root sign in Eq. 19 is a good approximation of $\frac{\partial y}{\partial x}$ in this particular case, thereby justifying the authors' assumption. This would be fortuitous indeed.

It becomes clear that there is an inconsistency in the authors' assumptions. If it is accepted that they are using Eqs. 3 and 4 to describe the flow, the complete solution to the problem is defined, and this solution cannot satisfy any other condition except boundary or initial conditions. The general solutions of Eqs. 3 and 4 are[10]

$$\frac{y}{H} = F_1 (x - t \sqrt{g\, H}) + F_2 (x + t \sqrt{g\, H}) \qquad (20)$$

and

$$\frac{Q}{b_w\, H \sqrt{g\, H}} = F_1 (x - t \sqrt{g\, H}) - F_2 (x + t \sqrt{g\, H}) \qquad (21)$$

in which F_1 and F_2 are arbitrary functions. These two equations describe a pair of waves, one moving in the positive x-direction, and the other moving in the negative direction. Each element of the profile for these waves moves with the same celerity, and, consequently, the profile is unchanged in shape and amplitude. If it is assumed that $x = 0$ on the ocean end of the tidal river, $F_2 = 0$ for the region of interest, and the following simple relationship between y and Q must hold:

$$\frac{Q}{b_w\, H \sqrt{g\, H}} = \frac{y}{H} \qquad (22)$$

The form of the function F_1 is determined by boundary or initial conditions which, for tidal-influenced flow, are of the form:

$$y\,(a,\, t) = \sum_{k=1}^{n} A_k \sin \frac{k\, \pi\, t}{T} \qquad (23)$$

[10] "Wave Motion," by G. H. Kuelegan, in "Engineering Hydraulics" (edited by H. Rouse), John Wiley & Sons, Inc., New York, N. Y., 1950, p. 718.

in which a equals a fixed value of x, A_k is a series of constants, and T is the tidal cycle period. If the differential equation for friction (Eq. 6) is combined with Eq. 22, having boundary conditions of the form of Eq. 23, a redundant condition is obtained which cannot be satisfied.

It is easy to demonstrate that Eq. 6 imposes a condition on Eqs. 3 and 4 which it is impossible to satisfy for the sinusoidal type of boundary condition. Eliminating $\frac{\partial y}{\partial x}$ from Eqs. 4 and 6 produces a differential equation:

$$M^2 \, g \, H \, b_w \, Q^2 = - \frac{\partial Q}{\partial t} \quad (24)$$

which has the general solution:

$$Q = [M^2 \, g \, H \, b_w \, t + c_1 (x)]^{-1} \quad (25)$$

in which $c_1(x)$ equals a function of x. This solution, when inserted in Eq. 3, yields the following differential equation:

$$\frac{c'_1(x)}{[M^2 \, g \, H \, b_w \, t + c_1 (x)]^2} = b_w \frac{\partial y}{\partial t} \quad (26)$$

which has the general solution:

$$b_w \, y = - \frac{c'_1(x)}{M^2 \, g \, H \, b_w \, [M^2 \, g \, H \, b_w \, t + c_1 (x)]} + c_2(x) \quad (27)$$

Eq. 27 is a hyperbola expressed in terms of the variable, t, and, as a result, it is impossible to satisfy boundary conditions of the type illustrated by Eqs. 23 and 27. The writer is thus forced to conclude that the authors have not described the flow by Eqs. 3, 4, and 6, but have used other unstated equations. The writer is at a loss to explain how these other equations were obtained because of the meager amount of information contained in the paper.

In spite of the uncertainties in the foregoing assumptions, the electrical analog may be a good representation of the flow in a tidal river; and, if so, it would prove to be valuable to engineers who must plan structures that disturb the natural regime of the river.

It would be interesting to know whether or not the peculiarities of the tidal effects in rivers are demonstrated by the electrical analog. In particular, the change of the shape of the curve relating stage and time at any station should be produced. At the river mouth, this curve resembles a sine curve with rounded peaks and a constantly changing slope on both ebb and flood tides. In the upstream reaches, it is usually observed that the curve relating the stage and the time approaches a saw-toothed shape, with sharp peaks and a linear rise and fall.

Another factor in evaluating the electrical analog is whether or not it produces the correct phase difference between the stage and the velocity. In most rivers, the maximum inflow or outflow velocity precedes the maximum or minimum stage, respectively, by about 3 hours. This factor is very important when sediment transport must be considered because the rate of bed movement depends on both the depth and velocity of flow. The wave equation predicts no phase difference between the stage and the current, and Eq. 6 predicts approximately a 6-hour difference. Thus, neither of these solutions describes the actual physical conditions.

Other investigators have studied the possibility of the electrical analog in tides, particularly A. T. Doodson,[11] but none has been able to produce an analog of practical value. If the authors can show that their analog is a good representation of the actual flow in natural rivers, they are to be commended for a very valuable contribution to engineering science.

T. Blench,[12] M. ASCE.—An analog of the type described by the authors was designed by Mr. van Veen[11,13] who shunted the resistance across the capacitance instead of placing it in series with the inductance. The consequence was that the electric current represented the tidal displacement and the voltage represented the fluid discharge. The technical difficulties of representing branching channels with tidal displacement at the joint may be

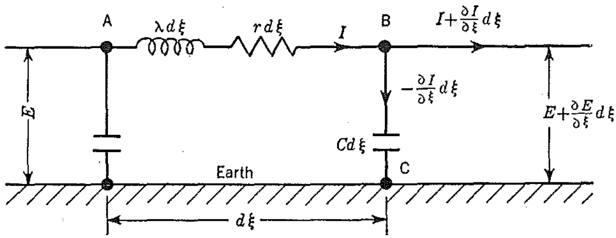

Fig. 5.—Basic Analog Circuit

imagined, and the authors are to be commended for their ingenuity in removing the difficulty; however, the derivations of both the hydraulic and the electric equations seem to be incorrect. Fig. 5 is Fig. 2 with the relevant electrical data inserted. In Fig. 5, from A to B—

$$-\frac{\partial E}{\partial \xi}\,d\xi = r\,d\xi\,I + \lambda\,d\xi\,\frac{\partial I}{\partial \eta}$$

and from B to C—

$$-\frac{\partial I}{\partial \xi}\,d\xi = C\,d\xi\,\frac{\partial E}{\partial \eta}$$

from which

$$\frac{\partial I}{\partial \xi} + C\,\frac{\partial E}{\partial \eta} = 0 \qquad (7)$$

and

$$\frac{\partial E}{\partial \xi} + \lambda\,\frac{\partial I}{\partial \eta} + r\,I = 0 \qquad (28)$$

of which Eq. 28 contradicts Eq. 8.

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[11] "Tide Models," by A. T. Doodson, *Dock and Harbour Authority No. 339*, Vol. 29, January, 1949.

[12] Cons. Eng., Associate Prof. of Civ. Eng., Univ. of Alberta, Edmonton, Alt., Canada.

[13] "Coasts, Estuaries and Tidal Hydraulics," by J. van Veen, from "Civil Engineering Reference Book," Butterworths, London, England, 1951.

The dynamical error seems to be in writing Eq. 4 without friction and then applying Eq. 6 separately, as if it were related to the total unsteady nonuniform flow. The writer feels that the method of Mr. van Veen is correct; that is, to write Eq. 4 as

$$\frac{\partial y}{\partial x} + \frac{1}{g\, H\, b_w} \frac{\partial Q}{\partial t + F} = 0 \qquad (29)$$

in which F is a friction term whose expression depends on the individual's preference in flow formulas. It is worthwhile to note that Eq. 4 omits convective acceleration and, therefore, does not apply to tides of high amplitude; that is, y/H greater than 0.25. The form of F used by Mr. Doodson, who accepted the Chezy formula, is tantamount to

$$F = \frac{K\, Q\,|Q|}{g\, b^2{}_w\, H^3} \qquad (30)$$

in which k is a coefficient, $|Q|$ means "absolute value of Q," and there is zero flow at half tide. The writer had occasion to try to extend the theory to the case of a half-tide flow of velocity U, and to replace the Chezy formula by a more realistic expression. This substitution yielded

$$F = \frac{Q\,|Q|}{c\, b^2{}_w\, H^{3.5}} = \frac{U\,|U|}{c\, H^{1.5}} \qquad (31)$$

in which c is a friction coefficient. The continuity equation (Eq. 3) remains unchanged.

The comparison of Eq. 7 and Eq. 28 with Eq. 3 and Eq. 29 shows that the authors' analog still holds, but a question arises as to whether the conversion factors may not need some numerical modification, and a comparison of predicted results with prototype results would be interesting.

R. E. Glover,[14] M.ASCE, D. J. Hebert,[15] and C. R. Daum[16].—Mr. van Veen describes the experiences of Dutch engineers who have had to deal with hydraulic problems where tidal influences are a factor. His remarks are of interest because they are based on these experiences, and also because they make the results of Dutch research available to American engineers. It is regrettable that the statement that cathode-ray tubes are not used for measurement in Holland was not amplified. It has been the writers' experience that the high speed at which electronic analogs work usually makes oscillograph recording difficult unless the working speed is reduced by use of iron-core inductances. Such expedients usually result in a loss of accuracy.

The difficulties introduced by changes in water-surface areas and hydraulic resistance brought about by changes in depth are real ones. The presence of levees along the channels and the limited variations of depth in the Delta minimized these troubles in the case described by the writers.

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Mr. Baines and Mr. Blench subjected the analytical aspects of the work to a searching scrutiny. The questions they ask are of a type which should be considered whenever an electronic analog is to be used for solving any specific hydraulic problem. The Delta channels are of nearly constant cross section for great distances. Furthermore, the presence of levees holds the changes of width of the water surface to values that are small compared to the original top widths and, since the changes of level to be accounted for were small compared to the original water depths, the difficulties produced by variations of top width and cross section were small enough to be ignored in the construction of the Delta analog. In the case of the problem of routing a flood down a river, cross-sectional areas and top widths would ordinarily be subject to large variations, and an analog of the type described would not be suitable.

Although it is certainly desirable to obtain a highly accurate representation of the hydraulic conditions, electrical difficulties often make compromises necessary. The development of a tube having adjustable characteristics would greatly facilitate overcoming some of these difficulties. Mr. Blench comments on the neglect of the convective acceleration term. Although his purpose was to emphasize the limitations of the analog, it may be added that electrical devices to represent the convective acceleration term in an analog would be difficult to devise. The writers would prefer to replace the linear resistance term in Eq. 28, as presented by Mr. Blench, with a term of the form I^2/K^2 obtained from the relations expressed by Eq. 10. Such a replacement would lead to an effective procedure for selecting the square-law resistor to represent hydraulic friction in the analog. Suppose the friction term in Eq. 17 of Mr. Baines' discussion had been expressed explicitly in the form Q^2/M^2 obtained by squaring Eq. 18, after replacing the quantity S_o by S_f. Since, in the writers' application S_o could be set equal to zero, Mr. Blench's electrical equation and Mr. Baines' hydraulic equation would become analogous and the constants for the square-law resistor could be selected with the use of the correlation equations. Such a procedure should also answer Mr. Baines' question in connection with Eq. 19 because it is a modified form of Eq. 17 which would then be exactly satisfied. This procedure would lead to the same choice of friction constants as were obtained by the writers. The choice of constants for square-law resistors is more readily understood when the friction term is included in the original equations as suggested by Mr. Baines and Mr. Blench. The limitations expressed in assumption No. 2, as listed by Mr. Baines, may be present when lumped electrical components are used to represent a channel, as in the Delta analog. This limitation can be removed by using several electrical components to represent a channel. As many changes of section can then be represented as there are components in the circuit.

Some changes in wave shape are produced in the Delta analog. These probably result from the action of the square-law resistors. The types of wave profile changes occurring in hydraulic channels as the result of the increase of the celerity of wave propagation with depth will not be produced by an analog of the type described, because the analog contains no electrical counterpart of the hydraulic factors that will produce changes of wave propagation speeds with changes of voltage.

Field data are available that make possible evaluation of the success attained by the analog in spite of the difficulties mentioned. The Walnut Grove channel has now been constructed to increase the flow of water in the lower Mokelumne channel and thereby compensate for the flow changes which result from operation of the Tracy pumping plant. Analog readings indicated that, with 10,000 cu ft per sec flowing in the Sacramento River at Sacramento, a total of 5,200 sec-ft should be transferred to the lower Mokelumne channel by the Walnut Grove cut and the Georgina slough. Field measurements indicate that with a flow of 10,600 cu ft per sec at Sacramento, the total transfer is 5,470 sec-ft.

In closing, the writers wish to express their appreciation to those who have contributed discussions to this paper.