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Performance Analysis of the Cooperative ZP-OFDM: Diversity, Capacity and Complexity

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Abstract In this paper, we investigate the diversity, capacity and complexity issues of cooperative Zero-Padding (ZP)-Orthogonal Frequency Division Multiplexing (OFDM) communication. We consider cooperative ZP-OFDM communication over a multipath Rayleigh channel and with multiple Carrier Frequency Offsets (CFOs) existing at different relays. We use a cooperative tall Toeplitz scheme to achieve full cooperative and multipath diversity, while simultaneously combat the CFOs. Importantly, this full diversity scheme only requires Linear Equalizers (LEs), such as Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) equalizers, an issue which reduces the system complexity when compared to a Maximum-Likelihood Equalizer (MLE) or other near-MLEs. Theoretical analysis of the proposed cooperative tall Toeplitz scheme is provided on the basis of the analytical upper bound of the channel orthogonality deficiency derived in this paper. Utilizing only low-complexity linear equalizers, theoretical analysis and simulation results show that the proposed Toeplitz scheme achieves the full cooperative, multipath and outage diversity.

Keywords Cooperative communication · OFDM · Zero-padding · Diversity · Capacity · Complexity · Carrier frequency offsets · Tall Toeplitz · Linear equalizers · Orthogonality deficiency

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Abbreviations

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
CDFs	Cumulative Density Functions
CF	Compress-and-Forward
CFOs	Carrier Frequency Offsets
CP	Cyclic Prefix
CR	Cognitive Radio
DF	Decode-and-Forward
DPS	Digital Phase Sweeping
ECMA	European Computer Manufacturers Association
FFT	Fast Fourier Transform
IFFT	Inverse Fast Fourier Transform
ISI	Inter-Symbol-Interference
LEs	Linear Equalizers
MB	Multi-Band
MLE	Maximum-Likelihood Equalizer
MMSE	Minimum Mean Square Error
od	orthogonality deficiency
OFDM	Orthogonal Frequency Division Multiplexing
OLA	Overlap and Add
PSD	Power Spectral Density
SD	Sphere Decoding
SNR	Signal-to-Noise Ratio
STC	Space Time Coding
STFC	Space-Time-Frequency Coding
SFC	Space Frequency Coding
UWB	Ultra Wide Band
ZF	Zero-Forcing
ZP	Zero-Padding

Notations

$(\cdot)^T$	Transpose of $(\cdot)$
$(\cdot)^*$	Conjugate of $(\cdot)$
$(\cdot)^H$	Hermitian of $(\cdot)$
$(\cdot)^{-1}$	Inverse of $(\cdot)$
$(\cdot)^\dagger$	Pseudo inverse of $(\cdot)$
$\forall$	For all
$\lvert\cdot\rvert$	Absolute value of a scalar or cardinality of a set
$\lVert\cdot\rVert$	2-Norm of a vector/matrix
argue min $(\cdot)$	Argument of minimum of $(\cdot)$
diag $(\cdot)$	Diagonal matrix with main diagonal $(\cdot)$
det $(\cdot)$	Determinant of $(\cdot)$
lim $(\cdot)$	Limit of $(\cdot)$
log $(\cdot)$	Logarithm with base 10
$\log_2 (\cdot)$	Logarithm with base 2

max $(\cdot)$	Maximum of $(\cdot)$
od $(\cdot)$	Orthogonality deficiency of matrix $(\cdot)$
O $(\cdot)$	Landau notation
Prob $(\cdot)$	Probability of $(\cdot)$

1 Introduction

The development of wireless communication applications in the last few years has been unprecedented. High data rate mobile communication, wireless broadband Internet, ubiquitous localization and many other services have recently emerged. Modern mobile communication with high speed and reliable transmission requires higher diversity gains from spatial, temporal and frequency domains. Meanwhile, it also requires a lower computational complexity; this result into saving energy, an issue which is becoming more and more crucial in the modern mobile communication.

For the spatial diversity, high data rates and reliable wireless transmissions can, however, only be achieved for full-rank[1] Multiple-Input-Multiple-Output (MIMO) users [1]. To overcome the limitations of achieving MIMO gains in future wireless networks, we must think of a new technology beyond the traditional point-to-point communications. This brought us to what is known as cooperative communication and networking, which allows different users or nodes in a wireless network to share resources and to create collaboration by means of distributed transmission/processing, in which each user's information is sent out not only by the user but also by the collaborating users [2]. Cooperative communication and networking is a new communication paradigm that promises significant capacity and multiplexing gain increases in wireless networks [3,4]. It realizes a new form of space diversity to combat the detrimental effects of severe fading by mimicking the MIMO, while getting rid of the drawbacks of MIMO such as size limitation and correlated channels [5–7]. There are mainly three relaying protocols: Amplify-and-Forward (AF), Decode-and-Forward (DF) and Compress-and-Forward (CF). In AF, the received signal is amplified and retransmitted to the destination. The advantage of this protocol is its simplicity and low-cost implementation. However, the noise is also amplified at the relay. In DF, the relay attempts to decode the received signals. If successful, it re-encodes the information and retransmits it. If some relays cannot fully decode the signal, they will be discarded. Lastly, CF attempts to generate an estimate of the received signal. This is then compressed, encoded, and transmitted with the hope that the estimated value may assist in decoding the original code word at the destination [1]. In this paper, we limit ourselves to DF protocol, which will be explained later in Sect. 3.

Cooperative techniques have already been considered for wireless and mobile broadband radio and Cognitive Radio (CR) [8]; they have also been under investigation in various IEEE 802 standards. A recent evolution of IEEE 802.11 using mesh networking, i.e., 802.11s considers the update of 802.11 MAC layer operations to self-configuration and multihop topologies. As an amendment to the 802.16 networks, IEEE 802.16j is concerned with multihop relay to enhance coverage, throughput, and system capacity [9].

OFDM technology in modern wireless communication has been widely used. Utilizing the cooperative OFDM communication, and transmitting the data in parallel, reliable high speed transmission can be achieved. For the conventional OFDM technology, a Cyclic Prefix (CP) is exploited to eliminate the Inter-Symbol-Interference (ISI) due to multipath. With CP

[1] If the channel matrix of the MIMO users is an m by n matrix H, Full rank means that the minimum number of independent rows and column of H, i.e., rank (H) $=$ min (m, n).

adding and removing, the linear convolution channel is transformed into a circular convolution channel, and the ISI can be easily resolved. Meanwhile, the channel equalization is also simplified, due to the channel matrix diagonalization. However, the cyclic prefix is not the only way to combat the multipath. ZP has already been proposed as an alternative to the CP in OFDM transmissions [10] and particularly for Cognitive Radio [11]. One of the advantages of using a ZP over CP is its lower spikes in the Power Spectral Density (PSD), this is because, unlike CP, the ZP signal has no circular structure (is completely random). A Multi-Band (MB) ZP-OFDM-based approach to design Ultra Wide Band (UWB) transceivers has been recently proposed in [12] and [13] for the IEEE Standard. In Dec. 2008, the European Computer Manufacturers Association (ECMA) adopted ZP-OFDM for the latest version of the High-rate UWB Standard [14]. Because of its advantage in the low power transmission, ZP-OFDM will have the potential to be used in other low power wireless communications systems.

Average Bit Error Rate (BER) and capacity are two important criteria for quantifying the performance of different communication systems. The BER performance of wireless transmissions over fading channels is usually quantified by two parameters: diversity order and coding gain. The diversity order is defined as the asymptotic slope of the BER curve versus Signal-to-Noise Ratio (SNR). It describes how fast the error probability diminishes with SNR, while the coding gain measures the performance gap between different schemes when they have the same diversity. The higher the diversity, the smaller the error probability is at high-SNR regimes. Most of the existing diversity-enabled schemes adopt MLEs or near-MLEs at the receiver to collect full diversity [15]. Although MLE enjoys the maximum diversity, its exponentially increased decoding complexity makes it unsuitable for certain practical systems. In order to reduce the system complexity, one may apply LEs, such as ZF and MMSE equalizers. With the proper design of the transceivers, LEs can achieve the full diversity.

In order to combine the advantages of both MIMO systems and OFDM, by concatenating a linear pre-coder with a layered space-time mapper, a full-diversity and full-rate Space Time Coding (STC) has been proposed for MIMO-OFDM system [16]. Space Frequency Coding (SFC) MIMO-OFDM system, where two-dimensional coding is applied to distribute channel symbols across space (transmit antennas) and frequency (OFDM tones) within one OFDM block, has been developed to exploit the available spatial, time and frequency diversity [17]. Recently, several research activities on Space Time Coding (STC) and Space Frequency Coding (SFC) have addressed the full spatial and multipath diversity issues for MIMO-OFDM system [18,19]. The Digital Phase Sweeping (DPS) technique based on left multiplying a permutation matrix with the time-domain transmitted symbol has been proposed to obtain the tall Toeplitz channel in order to guarantee the maximum possible spatial and multipath diversity in MIMO-OFDM system [20,21].

Unlike the MIMO system, multiple relays transmissions in the cooperative system may not be either time or frequency synchronized, i.e., signals transmitted from different relays arrive at the receiver at different time instances, and multiple CFOs exist due to the oscillator mismatching. Multiple CFOs introduce time selectivity into the wireless channel, this is similar to high-mobility terminals and scatterers inducing Doppler shifts and so introducing the time selectivity. This similarity can be explained by the resemblance between the multiple CFOs channel matrix and multiple Doppler shifts channel matrix. The time selective channel together with the frequency selective channel caused by the multipath transmission give a so called doubly time-frequency selective channel. Unlike the conventional MIMO system, the existence of multiple CFOs in cooperative systems makes direct CFOs compensation hard if not impossible. To the best knowledge of authors, the cooperative ZP-OFDM system affected by a multipath channel and CFOs is a subject that has not yet been addressed in literature. The channel orthogonality deficiency (*od*) [22], which will be defined in Sect. 4, determines the fundamental condition when LEs collect the same diversity as the MLE, i.e., meaning that the full diversity can be achieved. To collect the same spatial and multipath as MLE does, and to improve the system capacity only with LEs, the equivalent channel matrix needs some "modification" to upper bound the*od*by a constant less than 1. In this paper, based on some new results proposed in [22] and [23], we will illustrate how to simultaneously achieve the full cooperative and multipath diversity, to combat CFOs and to enable low system complexity only with LEs. We also show that, on the basis of the proposed cooperative tall Toeplitz scheme, the same outage diversity as that of MLE is attained by LEs.

The rest of the paper is organized as follows. Section 2 reviews important features of the ZP-OFDM. In Sect. 3, we first give the system model of the DF protocol based cooperative ZP-OFDM communication system with a multipath channel and multiple CFOs. Then, we provide a cooperative tall Toeplitz scheme to illustrate the full diversity design. Different equalization schemes and the concept of channel orthogonality deficiency are shown in Sect. 4. In Sect. 5, we justify the full cooperative and multipath diversity with CFOs and LEs by using the presented cooperative tall Toeplitz scheme. The upper bound of the channel orthogonality deficiency of the cooperative tall Toeplitz scheme is derived to elucidate the parameter's effect. In Sects. 6 and 7, we analyze and discuss the capacity and decoding complexity of different equalizers. Simulation results are illustrated in Sect. 8 to corroborate the theoretical claims, and finally Sect. 9 concludes the paper.

2 ZP-OFDM Basics

OFDM signals usually employ a cyclic prefix (CP-OFDM) or zero padding (ZP-OFDM) as time guard interval. A number of benefits that ZP-OFDM brings to cooperative relay systems originate from the basic features that ZP-OFDM possesses. To appreciate those, we first outline ZP-OFDM's operation using the discrete-time baseband equivalent block model of a single-transceiver system depicted in Fig. 1. The Fig. 1a depicts the transmitter and channel of a ZP-OFDM system, the Fig. 1b and c illustrate the commonly used Overlap and Add (OLA) receiver and FAST[2] receiver [10], respectively.

Different from a serial transmission, OFDM is a multi-carrier block transmission where, as the name suggests, information-bearing symbols are processed in blocks at both the transmitter and the receiver. The vector $\mathbf{x}_f = \left[x_0, \cdots, x_{N-1}\right]^T$ is the so-called frequency signal at one OFDM time symbol duration. Then it will be transferred to $\mathbf{x}_t$ in the time-domain by the N-point Inverse Fast Fourier Transform (IFFT) matrix $\mathbf{F}_N^{-1} = \mathbf{F}_N^H$ with (n, k)-th entry $\exp(j2\pi nk/N)/\sqrt{N}$, i.e., $\mathbf{x}_t = \mathbf{F}_N^H \mathbf{x}_f$, where $\mathbf{F}_N$ is the N-point Fast Fourier Transform (FFT) matrix, and n, $k$denote the index in frequency and time-domain, respectively. Throughout this paper, we use subscript f to indicate the signal vector in frequency domain, and use subscript t to indicate the signal vector in time domain. Then a zero vector with length $L_Z$is appended at the end of the time symbol. If we define

$$\mathbf{T}_{ZP} = \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0} \end{bmatrix}_{Z \times N}, \tag{1}$$

[2] This is so-called FAST because it is a fast version of the corresponding linear or nonlinear equalizer based on the channel matrix diagonalizaiton.

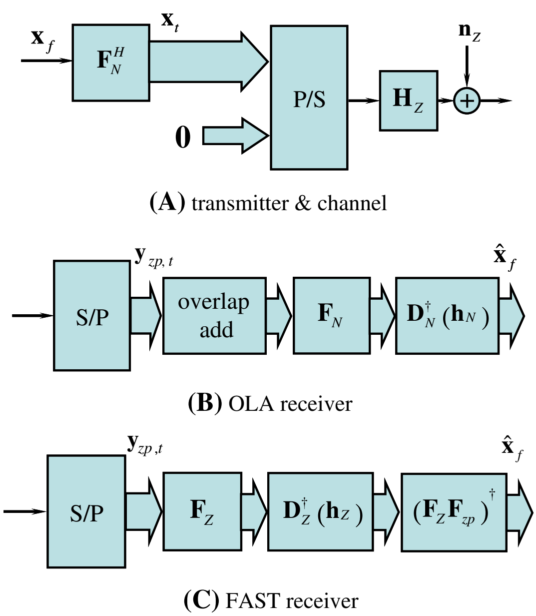

Fig. 1 Discrete-time block equivalent model of ZP-OFDM. **a** Transmitter & channel, **b** OLA receiver, **c** FAST receiver

where $\mathbf{I}_N$ is an $N \times N$ identity matrix and $Z = N + L_Z$, the transmitted OFDM symbol can be denoted as $\mathbf{x}_{zp,t} = \mathbf{T}_{ZP}\mathbf{F}_N^H\mathbf{x}_f$. The received symbol (i.e., $\mathbf{y}_{zp,t}$) is now expressed as:

$$\mathbf{y}_{zp,t} = \mathbf{H}_Z\mathbf{T}_{ZP}\mathbf{F}_N^H\mathbf{x}_f + \mathbf{H}_{ISI}\mathbf{T}_{ZP}\mathbf{F}_N^H\mathbf{x}_{p,f} + \mathbf{n}_{Z,t}, \tag{2}$$

where $\mathbf{H}_{\ Z}$ is the $Z \times Z$ lower triangular Toepliz filtering matrix with first column $[h_1 \cdots h_L \quad 0 \cdots 0]^T$,where L is the channel order (i.e., $h_l = 0, \forall\, l > L$, $h_l$denotes the l-th path gain) and $\mathbf{H}_{ISI}$ is the $Z \times Z$ upper triangular Toeplitz filtering matrix with first row $[0 \cdots 0 \quad h_L \cdots h_2]$, which captures ISI from the previous symbol $\mathbf{x}_{p,f}$. In Eq. (2), $\mathbf{n}_{Z,t}$ denotes the Additive White Gaussian Noise (AWGN) vector with zero mean, variance$N_o = 1$ and length Z. The blocks P/S and S/P in Fig. 1 denote the parallel to serial and serial to parallel operations respectively.

To avoid ISI, we should have $L \leq L_Z$. In this paper, we assume $L = L_Z$. Then, $\mathbf{H}_{ISI}\mathbf{T}_{ZP} = \mathbf{0}$, and Eq. (2) can be rewritten as:

$$\mathbf{y}_{zp,t} = \mathbf{H}_Z\mathbf{T}_{ZP}\mathbf{F}_N^H\mathbf{x}_f + \mathbf{n}_{Z,t}. \tag{3}$$

The OLA receiver and FAST receiver, as shown in Fig. 1b and c, respectively, are elaborated in [10], for estimating $\hat{\mathbf{x}}_f$ from the observation $\mathbf{y}_{zp}$.$\mathbf{D}_N\left(\widetilde{\mathbf{h}}_N\right)$ stands for the $N \times N$ diagonal matrix with vector $\widetilde{\mathbf{h}}_N$ on its diagonal, while $\mathbf{D}_Z\left(\widetilde{\mathbf{h}}_Z\right)$ denotes the $Z \times Z$ diagonal matrix with vector $\widetilde{\mathbf{h}}_Z$ on its diagonal; $\widetilde{\mathbf{h}}_N$ and $\widetilde{\mathbf{h}}_Z$ are the N-point and Z-point frequency response of the channel's impulse response, respectively. $\mathbf{F}_Z$ stands for the Z-point FFT matrix, $\mathbf{F}_{zp} = \mathbf{T}_{ZP}\mathbf{F}_N^H$. The OLA receiver is used to recast the ZP-OFDM as a CP-OFDM. Similar to the circular convolution property in CP-OFDM, the OLA receiver diagonalizes the channel, transfers the broadband frequency-selective channel to a multi-frequency-flat channel, and enables the simple equalization of the ZP-OFDM channel. However, since the multipath channel is transformed to the flat-fading channel, the OLA receiver loses the merit of multipath diversity accordingly. As shown at the Fig. 1c, and by comparing to the OLA receiver we learn that although the extra two FFT matrices slightly increase the equalization complexity, the FAST receiver always holds the linear structure or the tall Toeplitz structure of the ZP-OFDM channel, i.e., $\mathbf{H}_Z\mathbf{T}_{ZP}$. The tall Toeplitz structure can be illustrated by the $(L+M-1)$-row and M-column matrix $\mathcal{T}(\mathbf{v}, L, M)$ as follows:

$$\mathcal{T}(\mathbf{v}, L, M) = \begin{bmatrix} v_1 & 0 & \cdots & 0 \\ v_2 & v_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ v_L & v_{L-1} & \cdots & 0 \\ 0 & v_L & \cdots & v_1 \\ \vdots & 0 & \cdots & v_2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & v_L \end{bmatrix}, \tag{4}$$

where $\mathbf{v} = [v_1, v_2, \cdots, v_L]^T$ is a non-zero column vector of length L. Both the OLA receiver and FAST receiver have their own application fields. Generally speaking, the OLA receiver with ZP-OFDM can mimic the conventional CP-OFDM to obtain a simple equalization, while the FAST receiver keeps the inherent merits of ZP-OFDM, and provides a relatively faster equalization.

In the ZP-OFDM, the tall Toeplitz structure of equivalent channel matrix always guarantees its full rank (it only becomes rank deficient when the channel impulse response is identically zero, which is impossible in practice). In other words, the full rank property guarantees the detection of transmitted symbols. Nevertheless, the zero-padding and linear structure of ZP-OFDM outperforms CP-OFDM in the lower frequency spikes [11,12], as zero-padding replaces cyclic prefix in OFDM symbols, and so significantly reduces the ripples in the PSD. Compared to CP, tailing zeros will save transmit power. Furthermore, by adopting proper filters, the ZP-OFDM will not give rise to out-of-band spectral leakage, either. In the blind channel estimation and blind symbol synchronization area, ZP-OFDM also has its advantage over CP-OFDM in reducing the system complexity, again due to its linear structure [24,25]. In the following sections, we investigate the diversity issue of cooperative ZP-OFDM communications with the unique nature of a tall Toeplitz structure, where we show how the system takes advantage of this nature to achieve the full cooperative and multipath diversity, and to combat the multiple CFOs from different cooperative relays, only with linear equalizers (such as the ZF or MMSE equalizer).

3 Cooperative Tall Toeplitz Scheme

In this section, we consider a DF cooperative ZP-OFDM system as shown in the Fig. 2. It is because in case the relay can fully decode the signal, DF always outperforms AF in the transmission performance. Fully decoding relay can be guaranteed by employing an error detection code, such as cyclic redundancy check, or easily pick up the relay with a SNR larger than the threshold.[3] Therefore, we assume the relays shown in Fig. 2 can fully decode the information, participate in the cooperation, and occupy different frequency bands to forward the data to the destination. We also assume that each relay-destination link undergoes uncorrelated multipath Rayleigh fading. According to the Eq. (3), for the relay r, $r \in [1, 2, \cdots, R]$,

[3] The threshold is $(2^B - 1)/h_{S,Q_r}$; where B is the target rate and h_{S,Q_r} denotes the power gain from source to relay Q_r.

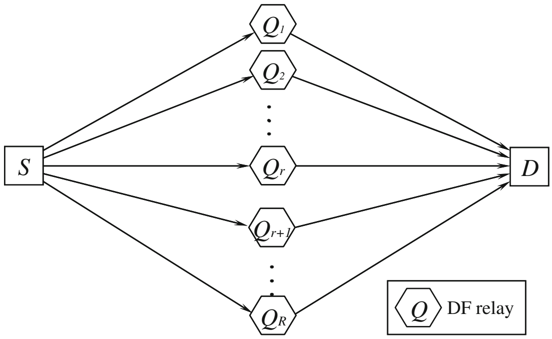

Fig. 2 DF cooperative ZP-OFDM system architecture, (S source, D destination, Q_r r-th Relay)

R is the number of relays, according to the Eq. (3), the received signal of r-th relay can be formulated as

$$\mathbf{y}_{r,f} = \mathbf{F}_Z \mathbf{D}_{Z,r} \mathbf{H}_r \mathbf{T}_{ZP} \mathbf{F}_N^H \mathbf{x}_f + \mathbf{n}_{Z,f}. \tag{5}$$

The subscript r here indicates the index of the $r-$th relay. The matrix $\mathbf{H}_r$ is a $Z \times Z$ lower triangular matrix with first column vector $[h_{1,r}, \cdots, h_{L,r}, 0 \cdots 0]^T$, and first row vector $[h_{1,r}, 0 \cdots 0]$, $h_{L,r}$ denotes the L-th path gain over the r-th relay and destination link. Without loss of generality, we assume that the channel lengths of different relay-destination links are all L. The matrix $\mathbf{D}_{Z,r}$ is a diagonal matrix representing the residual carrier frequency error over the r-th relay and destination link and is defined in terms of its diagonal elements as $\mathbf{D}_{Z,r} = \text{diag}\left(1,\alpha_r, \cdots, \alpha_r^{Z-1}\right)$, with $\alpha_r = \exp\left(j2\pi q_r/N\right)$; q_r is the normalized carrier frequency offset of r-th relay with the symbol duration of ZP-OFDM. $\mathbf{n}_{Z,f}$ is the FFT processed noise, which remains an additive white Gaussian term since $\mathbf{F}_Z$ is a unitary matrix [1,16]. Here, we define $\mathbf{H}_{T,r} = \mathbf{H}_r \mathbf{T}_{ZP}$, which is a full column rank tall Toeplitz matrix, and whose correlation matrix is always guaranteed to be invertible. Consequently, Eq. (5) can be rewritten as:

$$\mathbf{y}_{r,f} = \mathbf{F}_Z \mathbf{D}_{Z,r} \mathbf{H}_{T,r} \mathbf{F}_N^H \mathbf{x}_f + \mathbf{n}_{Z,f}. \tag{6}$$

The DPS technique based on insertion of a permutation matrix

$$\mathbf{P}_r = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{(r-1)L} \\ \mathbf{I}_{Z-(r-1)L} & \mathbf{0} \end{bmatrix} \tag{7}$$

between the channel matrix $\mathbf{H}_r$ and tailing zero matrix$\mathbf{T}_{ZP}$ can be used to form a tight tall Toeplitz channel matrix, which will be illustrated later (Fig. 5). This procedure is regarded as applying a DPS coding on the time-domain signal. It does not change the original data rate, but guarantees the maximum possible spatial and multipath diversity in the MIMO system, due to characteristics of the tight tall Toeplitz channel matrix [20,21]. However, in the cooperative relay system, a CFOs problem due to the oscillator mismatching between different relays is inevitable. In this situation, DPS cannot obtain the cooperative and multipath diversity with linear receivers. We will verify this claim by the theoretical analysis and simulation results later. In order to cope with the CFOs problem and achieve the full cooperative and multipath diversity with only linear equalizers, we design a cooperative tall Toeplitz scheme; we arrange transmitted symbols in different frequency bands according to the corresponding relay, as shown in the Fig. 3.

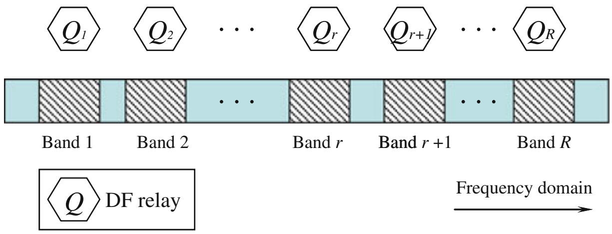

Fig. 3 Cooperative tall Toeplitz design for cooperative ZP-OFDM relays

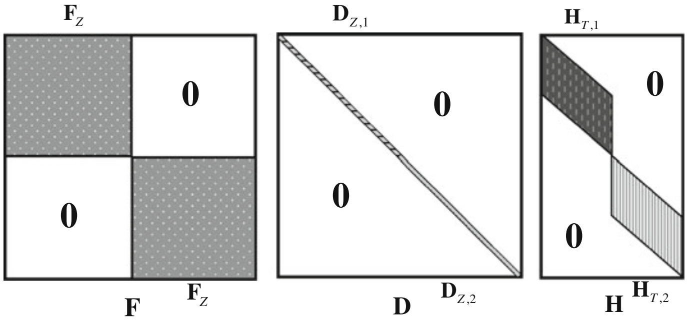

Fig. 4 Structures of the FFT matrix, CFOs matrix and channel matrix for a 2-relay cooperative system; *left*: FFT matrix**F**, *middle*: CFOs matrix**D**, *right*: channel matrix**H**. Blank parts are all 0's, the shaded parts correspond to non-zero entries

We take $\mathbf{x}_f$ as the information symbols correctly received at the r-th relay nodes involved in the DF-cooperative scheme. After full decoding, $\mathbf{x}_f$is assigned to the corresponding r-th frequency band as shown in the Fig. 3, and forwarded to the destination. This design is also suitable for a cognitive radio system when several spectrum holes are available for the cooperative communication. The above design is equivalent to multiplying a matrix $\mathbf{G} = [\mathbf{I}_1, \mathbf{I}_2, \cdots, \mathbf{I}_R]^T$ with $\mathbf{F}_N^H \mathbf{x}_f$, where $\mathbf{I}_r$ is an $N \times N$ identity matrix, $r \in [1, 2, \cdots, R]$; the received signal at the destination from all R relay nodes yields

$$\mathbf{y}_f = \mathbf{FDHGF}_N^H \mathbf{x}_f + \mathbf{n}_{RZ,f}, \tag{8}$$

where $\mathbf{F} = \text{diag}\left(\overbrace{\mathbf{F}_Z, \mathbf{F}_Z, \cdots, \mathbf{F}_Z}^{R \text{ times}}\right)$, $\mathbf{D} = \text{diag}\left(\mathbf{D}_{Z,1}, \mathbf{D}_{Z,2}, \cdots, \mathbf{D}_{Z,R}\right)$, $\mathbf{H} = \text{diag}\left(\mathbf{H}_{T,1}, \mathbf{H}_{T,2}, \cdots, \mathbf{H}_{T,R}\right)$, are all diagonal matrices with R relay's components on their diagonals. $\mathbf{n}_{RZ,f}$ denotes the AWGN vector with zero mean, variance $N_o = 1$ and length RZ. For instance, when we consider a 2-relay cooperation system, i.e., $R = 2$, then the structures of **F**, **D**, and **H** can be illustrated as shown in Fig. 4.

If we denote $\mathbf{HG} = \hat{\mathbf{H}}_T$, then, $\hat{\mathbf{H}}_T = \left[\mathbf{H}_{T,1}^T, \mathbf{H}_{T,2}^T, \cdots, \mathbf{H}_{T,R}^T\right]^T$ will be a linear Toeplitz matrix, or tall Toeplitz matrix, with $\hat{\mathbf{h}}_1 = \left[\widehat{h}_{1,1}, \cdots, \widehat{h}_{L,1}, \mathbf{0}, \widehat{h}_{1,2}, \cdots, \widehat{h}_{L,2}, \mathbf{0}, \cdots, \widehat{h}_{1,R}, \cdots, \widehat{h}_{L,R}, \mathbf{0}\right]^T$ being $\hat{\mathbf{H}}_T$'s first column. $\hat{\mathbf{H}}_T$ can be regarded as a tall Toeplitz channel matrix, with the channel length $L_T = Z \times (R-1) + L$ as well. For the case$R = 2$, $\hat{\mathbf{H}}_T$ is shown at the left hand side of Fig. 5. The DPS technique proposed in [20] is used to convert the R transmit-antenna system, where

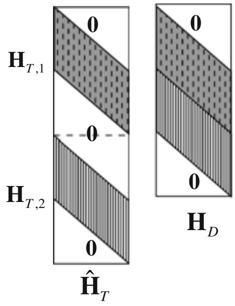

Fig. 5 Structures of the proposed tall Toeplitz channel matrix $\hat{\mathbf{H}}_T$ and channel matrix based on DPS$\mathbf{H}_D$. Blank parts are all 0's, the shaded parts correspond to non-zero entries

each frequency-selective channel has L taps into a single transmits antenna system, where the equivalent channel has RL taps. $\mathbf{H}_D$, shown at the right-hand-side of Fig. 5, is the channel matrix adopting the DPS technique. Comparing $\hat{\mathbf{H}}_T$ with learns that the tight tall Toeplitz structure of $\mathbf{H}_D$ enables the system to have a high bandwidth efficiency. However, when the CFOs exist at different relays, the DPS technique cannot remove this deleterious effect and subsequently degrades the diversity gains. We will show this drawback later in our theoretical analysis and simulation results as well.

We notice that since the relays perform the forwarding in different bands, matrix $\mathbf{G}$ spreads the R copies of the time-domain signal $\mathbf{x}_t = \mathbf{F}_N^H \mathbf{x}_f$, according to the corresponding R cooperative relays. Therefore, matrix $\mathbf{G}$ can be regarded as a coding on the time-domain signal, for different relays and different bands, and is so called the Space-Time-Frequency Coding (STFC) [26,27]. Then, Eq. (8) becomes

$$\mathbf{y}_f = \mathbf{F}\mathbf{D}\hat{\mathbf{H}}_T \mathbf{F}_N^H \mathbf{x}_f + \mathbf{n}_{RZ,f}. \tag{9}$$

If we denote $\mathbb{H} = \mathbf{F}\mathbf{D}\hat{\mathbf{H}}_T \mathbf{F}_N^H$ as the RZ-row times N-column equivalent channel matrix, we get

$$\mathbf{y}_f = \mathbb{H}\mathbf{x}_f + \mathbf{n}_{RZ,f}. \tag{10}$$

$\mathbb{H}$ in Eq. (10) is called the overall equivalent channel. In the Sect. 5, we will exploit $\mathbf{H}$ to show that our cooperative tall Toeplitz scheme can achieve the full cooperative and multipath diversity and combat the CFOs, with only LEs. Beforehand, we review the two concepts: equalization and channel orthogonality deficiency.

4 Equalization and Channel Orthogonality Deficiency

Given the equivalent channel model in Eq. (10), there are various ways to decode $\mathbf{x}$ from the observation $\mathbf{y}$. We first provide the definitions of the equalizers that we consider in this paper. On the one hand, an often used method, which is also optimal, if there is no prior information on the symbols or when symbols are treated as deterministic parameters, is the MLE. The output of the MLE $\mathbf{x}_{ml}$ is then given as

$$\mathbf{x}_{ml} = \underset{\tilde{\mathbf{x}} \in S^N}{\arg\min} \|\mathbf{y} - \mathbb{H}\tilde{\mathbf{x}}\|, \tag{11}$$

where $\tilde{\mathbf{x}}$ is the transmitted symbol and S is the finite alphabet of the transmitted symbols.

On the other hand, LEs, such as the ZF equalizer and MMSE equalizer are favored for their low decoding complexity. The output of the ZF equalizer $\mathbf{x}_{zf}$ is defined as

$$\mathbf{x}_{zf} = \mathbb{H}^{\dagger}\mathbf{y}, \tag{12}$$

where $\mathbb{H}^{\dagger} = (\mathbb{H}^{H}\mathbb{H})^{-1}\mathbb{H}^{H}$ denotes the pseudo-inverse of the channel matrix H.

The output of the MMSE equalizer $\mathbf{x}_{mmse}$ is defined as

$$\mathbf{x}_{mmse} = \left(\mathbb{H}^{H}\mathbb{H} + N_0\mathbf{I}_N\right)^{-1}\mathbb{H}^{H}\mathbf{y}, \tag{13}$$

we note that, with the definition of an extended system

$$\widehat{\mathbb{H}} = \begin{bmatrix} \mathbb{H} \\ \sqrt{N_0}\mathbf{I}_N \end{bmatrix} \text{and } \widehat{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N\times 1} \end{bmatrix}, \tag{14}$$

the MMSE equalizer in Eq. (13) can be rewritten as $\mathbf{x}_{mmse} = \widehat{\mathbb{H}}^{\dagger}\widehat{\mathbf{y}}$; this indicates that the ZF equalizer and MMSE equalizer are both LEs, and share the linear properties. Therefore, some analysis based on the ZF equalizer can be extended to the MMSE equalizer, and vice versa.

The important reason that hinders LEs from getting more attention in theory and practice is that their performance loss, relative to MLEs, is not quantified in general. In the following, to critically quantify the performance gap between LEs and MLE, we adopt the parameter, orthogonality deficiency (*od*), of the channel matrix Has in [22].

Definition 1 (Orthogonality Deficiency): For an equivalent channel matrix $\underline{\mathbb{H}} = \left[\underline{\mathbf{h}}_1, \underline{\mathbf{h}}_2, \cdots, \underline{\mathbf{h}}_N\right]$, with $\underline{\mathbf{h}}_n$ being the $\underline{\mathbb{H}}$'s n-th column, its orthogonality deficiency $od\left(\underline{\mathbb{H}}\right)$ is defined as

$$od\left(\underline{\mathbb{H}}\right) = 1 - \frac{\det\left(\underline{\mathbb{H}}^{H}\underline{\mathbb{H}}\right)}{\prod_{n=1}^{N}\left\|\underline{\mathbf{h}}_n\right\|^2}. \tag{15}$$

If $\underline{\mathbb{H}}$ is singular, $od\left(\underline{\mathbb{H}}\right) = 1$. The closer $od\left(\underline{\mathbb{H}}\right)$ to zero, the more orthogonal the $\underline{\mathbb{H}}$. Given the model in Eq. (10), if $od\left(\underline{\mathbb{H}}\right) = 0$, and thus $\underline{\mathbb{H}}^{H}\underline{\mathbb{H}}$ is diagonal, then LEs have the same performance as that of MLE.

5 Diversity Analysis of the Proposed Cooperative ZP-OFDM Scheme

5.1 Full Cooperative and Multipath Diversity with CFOs and LEs

In the following, we verify that for the cooperative tall Toeplitz scheme, when CFOs appear, the LEs are the only requirement of equalizer to achieve full cooperative and multipath diversity order of *RL*.

Proof We first cite the following theorem from [22]: □

Theorem 1 *Consider the linear system as in* Eq. (10). *A LE achieves the full diversity and collects the same diversity as MLE does, if there exists a constant* $\varepsilon \in (0, 1)$ *such that* $\forall\,\mathbb{H}$, $od\,(\mathbb{H}) \leq \varepsilon$.

In the cooperative tall Toeplitz scheme, and quivalent channel matrix H $= [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_N]$, with $\mathbf{h}_n$ being H's $n-$th column, we note that $\mathbf{F}_Z$,$\mathbf{D}_{Z,r}$,$\mathbf{F}_N^H$, $\mathbf{F}$ and $\mathbf{D}$ are all unitary matrices. Therefore, we have

$$\det\left(\mathbb{H}^H\mathbb{H}\right) = \det\left(\mathbf{F}_N\hat{\mathbf{H}}_T^H\hat{\mathbf{H}}_T\mathbf{F}_N^H\right)$$
$$= \det\left(\mathbf{F}_N\right)\det\left(\mathbf{F}_N^H\right)\det\left(\hat{\mathbf{H}}_T^H\hat{\mathbf{H}}_T\right) = \det\left(\hat{\mathbf{H}}_T^H\hat{\mathbf{H}}_T\right), \tag{16}$$

where $\det\left(\mathbf{F}_N\right)\det\left(\mathbf{F}_N^H\right) = 1$. Since $\hat{\mathbf{H}}_T$ is a tall Toeplitz matrix, then $\det\left(\hat{\mathbf{H}}_T^H\hat{\mathbf{H}}_T\right) > 0$ for any nonzero channel response, i.e., when $\widehat{h}_{l,r}$'s are not equal to zero simultaneously, where $l \in [1, 2, \cdots, L]$, $r \in [1, 2, \cdots, R]$, [28–30]. Consequently, we have $\det\left(\mathbb{H}^H\mathbb{H}\right) > 0$.

Meanwhile, for any practical channel, since the components of vector $\mathbf{h}_n$ cannot be equal to zero simultaneously, $\prod_{n=1}^N \|\mathbf{h}_n\|^2 > 0$ is always satisfied. Therefore, $od\left(\mathbb{H}\right)$ is always smaller than 1, i.e., there exists a constant $\varepsilon \in (0, 1)$ such that $\forall\, \mathbb{H}$, $od(\mathbb{H}) \leq \varepsilon$.

According to *Theorem* 1, we can verify that the proposed cooperative tall Toeplitz scheme can achieve full cooperative and multipath diversity, only with LEs, and can combat the CFOs at the different relays, simultaneously.

Now, we return to the conventional DPS technique. It provides a compact tall Toeplitz structure channel matrix, as shown in the Fig. 5, which results in the CFOs matrices overlapping each other accordingly. The overlapped CFOs matrices causes that the unitary property is lost, and consequently the channel matrix of DPS with CFOs is not a tall Toeplitz any more, and loses the frequency orthogonality, which means thatdet $\left(\mathbb{H}^H\mathbb{H}\right)$ of the DPS case cannot always guarantee to be larger than zero, and $od\left(\mathbb{H}\right)$ may equal to 1. Therefore, according to Theorem 1, when CFOs from different relays appear, the DPS technique with LEs adopted cannot achieve full diversity gains. We will verify this theoretical claim by the simulation result as shown in Sect. 8.

5.2 Upper Bound of the Channel Orthogonality Deficiency of Proposed Scheme

In order to provide a further insight into the channel factors that affect the cooperative transmission performance, we consider the orthogonality deficiency of a pure channel, and denote$\overline{\text{H}} = \mathbf{D}\hat{\mathbf{H}}_T$. The orthogonality deficiency of the pure channel can be represented as

$$od\left(\overline{\mathbb{H}}\right) = 1 - \frac{\det\left(\overline{\mathbb{H}}^H\overline{\mathbb{H}}\right)}{\prod_{n=1}^N\left\|\overline{\mathbf{h}}_n\right\|^2} = 1 - \frac{\det\left(\hat{\mathbb{H}}_T^H\hat{\mathbb{H}}_T\right)}{\prod_{n=1}^N\left\|\overline{\mathbf{h}}_n\right\|^2}, \tag{17}$$

where $\overline{\mathbf{h}}_n$ is $\overline{\mathbb{H}}$'s n-th column. For the RZ-row times N-column tall Toeplitz channel matrix $\hat{\mathbf{H}}_T$, suppose $m = \arg\max_{l_{rz}\in[1,RZ]}\left|\widehat{h}_{l_{rz}}\right|^2$, and $\left|\widehat{h}_m\right|^2 > 0$, the tall Toeplitz channel matrix $\hat{\mathbf{H}}_T$ can be split into three submatrices as $\hat{\mathbf{H}}_T = \left[\hat{\mathbf{H}}_{T,o1}^T, \hat{\mathbf{H}}_{T,m}^T, \hat{\mathbf{H}}_{T,o2}^T\right]^T$, where matrix $\hat{\mathbf{H}}_{T,o1}$ consists of the first $(m-1)$ rows of $\hat{\mathbf{H}}_T$, $\hat{\mathbf{H}}_{T,o2}$ has the last $(RZ-2m)$ rows of $\hat{\mathbf{H}}_T$, and $\hat{\mathbf{H}}_{T,m}$ is of size $N \times N$ with$\widehat{h}_m$ on the diagonal entries. Therefore, we have $\hat{\mathbf{H}}_T^H\hat{\mathbf{H}}_T = \hat{\mathbf{H}}_{T,o1}^H\hat{\mathbf{H}}_{T,o1}^+\hat{\mathbf{H}}_{T,m}^H\hat{\mathbf{H}}_{T,m}^+\hat{\mathbf{H}}_{T,o2}^H\hat{\mathbf{H}}_{T,o2}$. It is easy to show that $\det\left(\hat{\mathbf{H}}_{T,m}^H\hat{\mathbf{H}}_{T,m}\right) = \left(\left|\widehat{h}_m\right|^2\right)^N$ when $N > RZ$. Thus, we bound $\det\left(\overline{\mathbb{H}}^H\overline{\mathbb{H}}\right)$ as

$$\det\left(\overline{\mathbb{H}}^H\overline{\mathbb{H}}\right) \geq \det\left(\hat{\mathbf{H}}_{T,m}^H\hat{\mathbf{H}}_{T,m}\right) = \left(\max_{l_{rz}\in[1,RZ]}\left(\left|\widehat{h}_{l_{rz}}\right|^2\right)\right)^N. \tag{18}$$

We note that, for the unitary CFOs matrix, $\left|\alpha_r^z\right|^2 = 1, z \in [0, 1, \cdots, Z-1]$, and

$$\prod_{n=1}^{N} \left\|\bar{\mathbf{h}}_n\right\|^2 = \left(\sum_{l_{rz}=1}^{RZ} \left|\widehat{h}_{l_{rz}}\right|^2\right)^N , \tag{19}$$

We find for the upper bound of Eq. (17) as

$$od\left(\bar{\mathbb{H}}\right) \leq 1 - \frac{\left(\max_{l_{rz}\in[1,RZ]}\left(\left|\widehat{h}_{l_{rz}}\right|^2\right)\right)^N}{\left(\sum_{l_{rz}=1}^{RZ}\left|\widehat{h}_{l_{rz}}\right|^2\right)^N}. \tag{20}$$

Each column vector of the tall Toeplitz channel matrix $\hat{\mathbf{H}}_T$, includes at mostRL non-zero values. Thus, we obtain

$$\left(\sum_{l_{rz}=1}^{RZ} \left|\widehat{h}_{l_{rz}}\right|^2\right)^N \leq \left(RL\left(\max_{l_{rz}\in[1,\ RZ]}\left(\left|\widehat{h}_{l_{rz}}\right|^2\right)\right)\right)^N . \tag{21}$$

Consequently, we can further rewrite the upper bound of the $od\left(\bar{\mathbb{H}}\right)$as

$$od\left(\bar{\mathbb{H}}\right) \leq 1 - \frac{\left(\max_{l_{rz}\in[1,\ RZ]}\left(\left|\widehat{h}_{l_{rz}}\right|^2\right)\right)^N}{\left(RL\left(\max_{l_{rz}\in[1,\ RZ]}\left(\left|\widehat{h}_{l_{rz}}\right|^2\right)\right)\right)^N} = 1 - \frac{1}{(RL)^N}. \tag{22}$$

Note that RL is the full diversity order. If we keepRLas a constant, and reduce the upper bound of $od\left(\bar{\mathbb{H}}\right)$by decreasing N, i.e., the channel becomes more orthogonal, the upper bound of BER also becomes smaller; this indicates that LEs may achieve a better BER performance with the full diversity order. Later, we will verify this theoretical claim by simulation *Test Case 2* in Sect. 8.

6 Capacity Analysis of the Proposed Cooperative ZP-OFDM Scheme

Besides BER, mutual information is another important criterion when comparing the performance of different systems, since it measures how efficiently the transceivers utilize the channels. The concept "capacity" here denotes the maximum mutual information when a certain transceiver is adopted. Given a random channel, the instantaneous capacity is also random. In this case, to depict the capacity, one not only needs the capacity, but also the outage capacity, i.e., C^{th}, a capacity threshold which indicates the outage behavior [31]. In this section, we compare the outage capacity of the ZF equalizer with that of the MLE. The results can be easily extended to other LEs. We first consider the capacity when no channel state information is available at the transmitter, and the MLE is adopted at the receiver. Given the linear equivalent channel model in Eq. (10), the capacity achieved by MLE, i.e., C_{ml} is given as

$$C_{ml}\left(\mathbb{H}\right) = \log_2\left[\det\left(\mathbf{I}_N + (1/N_o)\,\mathbb{H}^H\mathbb{H}\right)\right]. \tag{23}$$

When a ZF equalizer is adopted at the receiver, the capacity of ZF equalizer given $\mathbb{H}$ can be expressed as [32]

$$C_{zf}\left(\mathbb{H}\right) = \log_2\left[\det\left(\mathbf{I}_N + (1/N_o)\,\mathbb{N}^{-1}\right)\right], \tag{24}$$

where $\sigma_n^2\mathbb{N}$ is called the covariance matrix of the equivalent noise vector with $\mathbb{N} = \text{diag}\left[k_{1,1}, k_{2,2}, \cdots, k_{N,N}\right]$, and $k_{i,i}$being the (i, i)-th entry of matrix $\daleth = \left(\mathbb{H}^H\mathbb{H}\right)^{-1}$. It is well known that $C_{zf}(\mathbb{H}) \leq C_{ml}(\mathbb{H})$ is always satisfied, and the difference between $C_{zf}(\mathbb{H})$ and $C_{ml}(\mathbb{H})$ for each realization of $\mathbb{H}$ can be as approximated by

$$C_{ml}(\mathbb{H}) - C_{zf}(\mathbb{H}) \approx -\log_2\left(1 - od\left(\left(\mathrm{H}^{\dagger}\right)^H\right)\right). \tag{25}$$

This expression shows that the capacity difference between the ZF equalizer and MLE is also related to the *od* of the channel matrix. Similar to the discussion in the previous Section, we also consider the pure channel effect $\overline{\mathbb{H}}$ here. We observe that as $od\left((\overline{\mathbb{H}}^{\dagger})^H\right)$ decreases, i.e., the inverse of the channel matrix is more orthogonal, the capacity gap between the MLE and ZF equalizer decreases.

Next, we show that, with the ZF equalizer, the proposed cooperative ZP-OFDM scheme collects the same outage diversity as that of the MLE. The outage diversity order$G_o$is defined as

$$G_o = \lim_{\text{SNR}\to\infty} -\frac{\log\left(\text{Prob}\left(C < C^{th}\right)\right)}{\log(\text{SNR})}. \tag{26}$$

If two Cumulative Density Functions (CDFs) of channel capacities are in parallel, it can be shown that they have the same outage diversity [22]. In order to prove that the proposed cooperative ZP-OFDM scheme in this paper employing the ZF equalizer achieves the same outage diversity as the MLE, we cite the results from [22] in the following theorem:

Theorem 2 *Given the system model of* Eq. (10) *with channel state information at the receiver but not at the transmitter, and if* $od\left(\overline{\mathbb{H}}\right) \leq \varepsilon, \forall \overline{\mathbb{H}}$, *and* $\varepsilon \in (0, 1)$*, then at high-SNR regime, the ZF equalizer collects the same outage diversity as that of the MLE.*

Note that the condition in *Theorem* 2 is the same as the condition in *Theorem* 1. Similar to the verification for the full cooperative diversity, by taking the advantage of the linear tall Toeplitz structure of the proposed cooperative ZP-OFDM scheme, it means that by utilizing the proposed cooperative ZP-OFDM scheme with the tall Toeplitz equivalent channel matrix, the ZF equalizer has the same outage diversity as that of the MLE.

In summarizing this section, we showed that the mutual information loss between the ZF equalizer and MLE also depends on the *od* of the channel matrix. When the*od* of the channel matrix has an upper bound which is strictly less than one, for example, via the proposed cooperative tall Toeplitz scheme, the performance diversity i.e., cooperative and multipath diversity, and the outage diversity in Eq. (26) of the ZF equalizer are the same as those of MLE.

7 Complexity Comparison Between LEs and MLE

In modern wireless communication systems, the decoding complexity is usually given a significant concern, because a more complex decoding scheme always means a higher computational burden and consequently a more energy consumption. Thus, the decoding complexity is an important measure for the comparison of different equalizers. In this section, we discuss the complexity of the commonly used equalizers, and then show the importance of the LEs.

To quantify the complexity of different equalizers, we count the average number of arithmetic operations in terms of numbers of real multiplications and real additions, needed to estimate Eq. (10). Using the ZF equalizer in Eq. (12) as an example, the complexity results from computing $\mathbb{H}^{\dagger} = \left(\mathbb{H}^{H}\mathbb{H}\right)^{-1}\mathbb{H}^{H}$ using the QR decomposition[4] and calculating $\mathbb{H}^{\dagger}\mathbf{y}$. As shown in [33], if we consider Has an $M \times N$ matrix, $M = R \times (N + L)$, the number of real multiplications for ZF equalizer equals $O\left(N^3\right) + O\left(N^2 M\right) + O\left(NM^2\right)$ and the number of real additions is also $O\left(N^3\right) + O\left(N^2 M\right) + O\left(NM^2\right)$, where $O\left(\cdot\right)$ denotes the Landau notation[5]. The optimum equalizer, MLE in Eq. (11) enjoys the best performance; However, it requires the highest complexity as well. As shown in [33], the number of arithmetic operations for the MLE in Eq. (11) is $O\left(|\mathbf{x}|^N\right) MN$. We learn from the comparison that the major advantage of LEs is their low decoding complexity.

Although the MLE enjoys the maximum diversity performance, its exponential decoding complexity makes it infeasible for certain practical systems. Some near-ML schemes (e.g., Sphere Decoding (SD)) can be used to reduce the decoding complexity. However, at low SNR or when large decoding blocks are sent/or high signal constellations are employed, the complexity of near-ML schemes is still high. As shown in [34], the SD method generally requires an exponential worst case complexity, whereas the heuristic search methods require only $O\left(N^3\right)$computations on the average. This complexity does even not include the complexity from any pre-processing (e.g., decomposition) and it is an average. Simulation results in [35] show that the SD method still has a high complexity compared with conventional LEs, since the SD method adopts linear equalizers as pre-processing steps. To further reduce the complexity, when the system model is linear, one may apply LEs.

8 Simulation Results

In this section, we use the simulation results to show the effect of the proposed cooperative ZP-OFDM scheme on the performance, and to verify our theoretical claims on the diversity and capacity issues. We consider the N sub-carriers ZP-OFDM system with ZP accounts for 25% of the OFDM symbol duration which undergoes the Rayleigh channel fading. We consider the 1-relay and 2-relay cases, the normalized CFOs of relay 1 and relay 2 are $q_1 = 0.3$, and $q_2 = 0.5$, respectively. The details of simulation parameters are shown in the Table 1.

Test Case 1 (Cooperative tall Toeplitz scheme for a full diversity design): In this example, we present simulation results to test the performance of the proposed cooperative tall Toeplitz scheme on ZP-OFDM system with 32 sub-carriers, i.e., $N = 32$, and compare the results with to the conventional DPS technique. Fig. 6 shows the BER performance versus E_b/N_o with different cooperative and multipath diversity orders, i.e., cooperative diversity order $R = 1, 2$, and multipath diversity order $L = 2, 4$. Since the MMSE equalizer can be transformed into the ZF equalizer, in the following two cases, we adopted the MMSE equalizer to show the performance of the LEs. The diversity order can be shown as the asymptotic slope of BER versus E_b/N_0curve. It describes how fast the error probability decays with SNR. We can see from Fig. 6 that, when CFOs appear at the different relays, the proposed cooperative tall Toeplitz scheme can achieve the full cooperative and multipath diversity only with the linear equalization, as the asymptotic slope of the curve increases with the increase of the number of relays and multipath length. However, with CFOs and LEs, the DPS technique loses diversity gains and shows a poor BER performance, which agrees with our theoretical

[4] A QR decomposition (also called a QR factorization) of a matrix is a decomposition of a matrix $\mathbf{A}$ into a product $\mathbf{A} = \mathbf{QR}$ of an orthogonal matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$.

[5] $O\left(\cdot\right)$, the Landau notation describes the limiting behavior of a function when the argument tends towards a particular value or infinity, usually in terms of simpler functions.

Table 1 Simulation parameters for cooperative ZP-OFDM

Modulation scheme	BPSK
Multicarrier scheme	CP-OFDM, ZP-OFDM
Number of OFDM subcarriers	8, 16, 32, 64
Length of guard interval	25% of OFDM symbol duration
Number of multipath	1, 2, 3, 4
Average channel gain of 1–4 path	1, 0.663, 0.487, 0.4255
Transmission bandwidth	500 MHz
Number of random trial symbols	1000
Number of relays	1, 2
Normalized CFOs	$q_1 = 0.3$, $q_2 = 0.5$

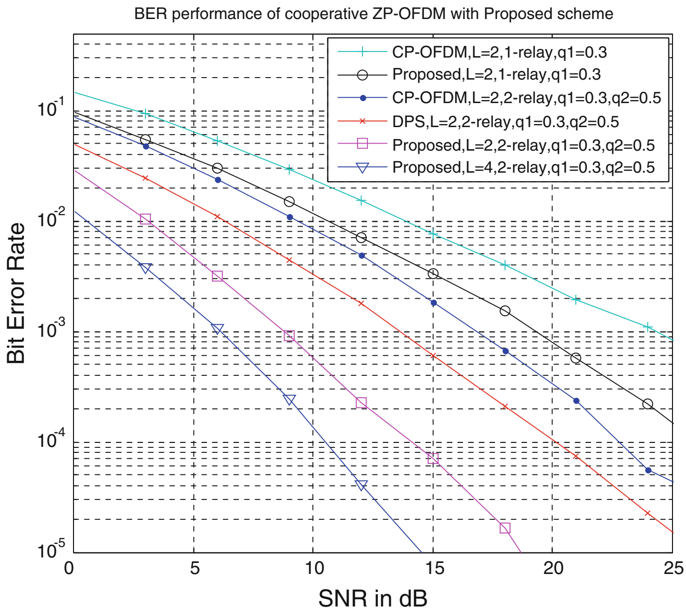

Fig. 6 Comparison of the proposed scheme to other conventional schemes for full diversity with LEs and CFOs

approach in sub-section 5.1. Without DPS technique, the conventional CP-OFDM takes the advantage of easy equalization but loss in multipath diversity gain. Adopting DPS technique, CP-OFDM and ZP-OFDM will achieve the same diversity gain, but still shows a worse performance than the proposed scheme.

Test Case 2 (Bounded channel orthogonality deficiency): In this example, we focus on the upper bound of channel orthogonality deficiency as derived in Eq. (22), and show how a change in N affects the channel orthogonality deficiency and BER performance. The frequency-selective channel order L is fixed to be 2, i.e., the multipath diversity orders are the same. As shown in Fig. 7, after adopting the cooperative tall Toeplitz scheme, $od\left(\overline{\mathbb{H}}\right) \leq \varepsilon < 1$, which means that the full cooperative diversity is achieved with the linear MMSE equalizer. We also notice that whenεgets smaller as N decreases, the BER performance gets

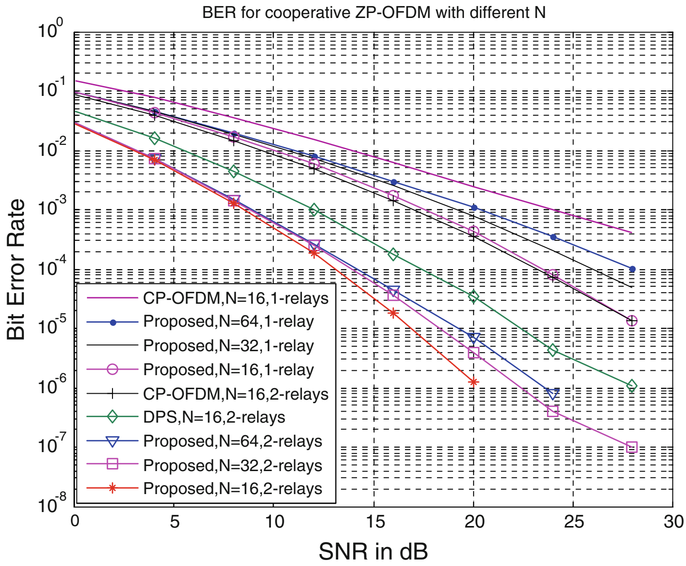

Fig. 7 Comparison of the proposed scheme with different numbers of sub-carriers and relays

better. This is consistent with the analysis, as shown in Eq. (22), i.e., $od\left(\mathbb{H}\right)$ decreases with decreasing N. When ε is smaller, the channel is more orthogonal, and the upper bound of the BER performance also becomes smaller. In general, for LEs, a smaller $od\left(\mathbb{H}\right)$ bound indicates a higher coding gain while the diversity gain is the same. Again, because the DPS technique is unable to cope with the CFOs effect, it shows a worse BER performance than the proposed cooperative tall Toeplitz scheme. Conventional CP-OFDM cannot gain from the multipath diversity, and shows the worst BER performance.

Test Case 3 (Capacity of proposed cooperative tall Toeplitz scheme): Fig. 8 shows the average capacity of a Rayleigh channel with the proposed cooperative tall Toeplitz scheme for the case of two relays cooperation, and without the proposed scheme, i.e., by direct combining of the 2-relays signals in the same frequency band at the destination, the 2-relay system only yields 3 dB power gain. For the low SNR region, average capacity curves are close to each other, and difficult to exhibit the comparison, so we chose to show the SNR region above 0 dB. As shown in the figure, the proposed cooperative tall Toeplitz scheme slightly improves the system capacity, because of exploiting the linear structure and frequency orthogonality of the channel. We notice that the $od\left(\mathbb{H}\right)$ gets smaller as the channel length decreases, and thus the capacity gaps between the ZF and ML equalizer shrink. We also show the average capacity of the CP-OFDM case, which achieves the smallest gap between the ZF and ML equalizer, since CP-OFDM has the pure orthogonal channel matrix. This confirms the observation in Eq. (25) that, the capacity gap between the ZF and ML equalizer not only depends on SNR but also on channel orthogonality. The CDFs of the capacity $\text{Prob}\left(C < C^{th}\right)$ with ZF and ML equalizer are depicted in Fig. 9, with $\text{SNR} = 25$ dB. We notice that, for the ZF equalizer (ZFE) case without the proposed scheme, the curve is not in parallel with the one of the MLE case, which means a loss of outage diversity. By adopting the proposed cooperative tall Toeplitz scheme, the curve of the ZFE becomes parallel with that of MLE, which indicates that the proposed cooperative tall Toeplitz scheme achieves the same outage diversity as MLE. This is consistent with Theorem 2 and our analysis in Sect. 6.

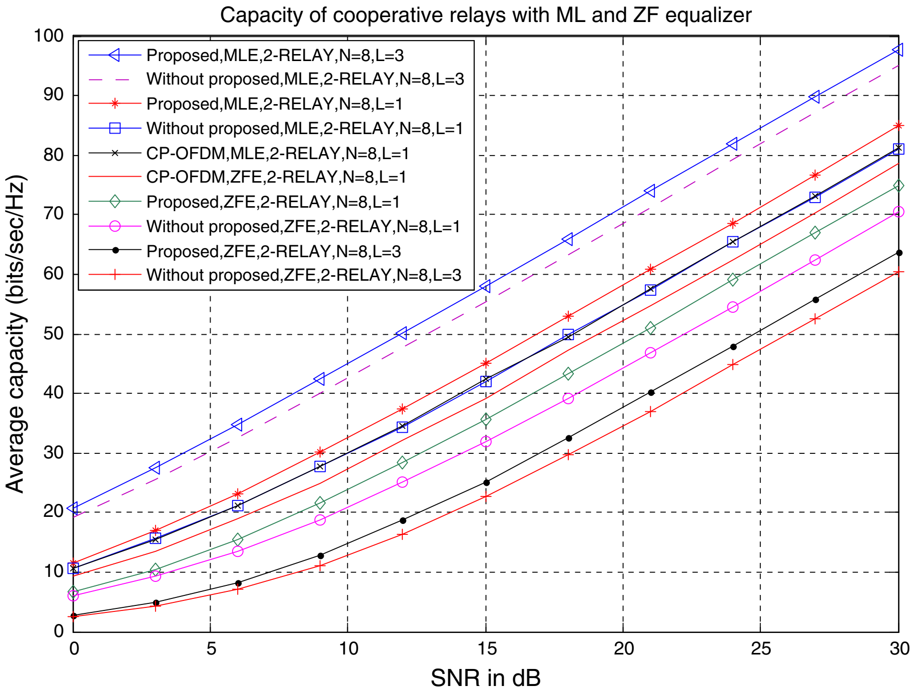

Fig. 8 Average capacity of cooperative ZP-OFDM with ML and ZF equalizer

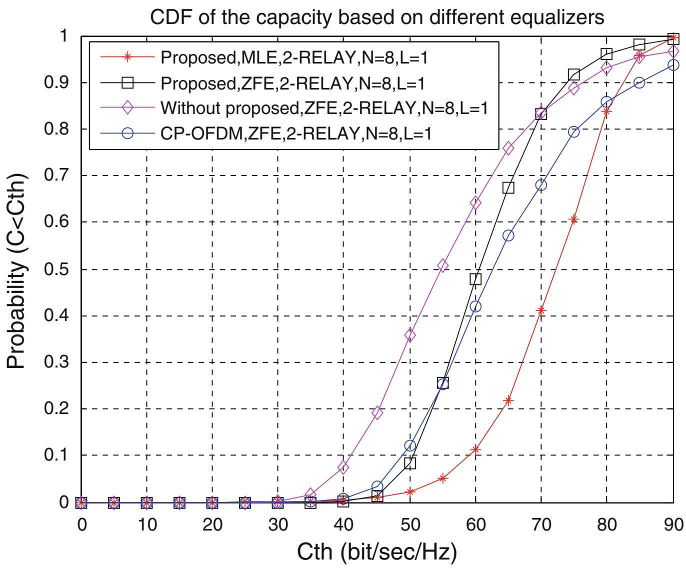

Fig. 9 CDF of the capacity of cooperative ZP-OFDM with ML and ZF equalizer

9 Conclusions

In this paper, we investigated diversity, capacity and complexity issues in cooperative ZP-OFDM communications. We first designed a cooperative tall Toeplitz scheme for the cooperative ZF-OFDM communication system, with different CFOs at different relays and over a multipath Rayleigh channel, i.e., a doubly time-frequency selective channel. In the proposed cooperative tall Toeplitz scheme, the tall Toeplitz structure together with the frequency orthogonality of channel matrix has a unique feature, which guarantees the full cooperative and multipath diversity, and easily combats the CFOs, only with the LEs. We derived the upper bound of the channel orthogonality deficiency, which provides an insight into how the change of channel factors affects the system performance in terms of BER performance and capacity. According to the theoretical analysis and simulation results, only with linear equalizers, the cooperative tall Toeplitz scheme achieves the same cooperative, multipath and outage diversity as those of MLEs, while the system complexity is reduced significantly.

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