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MIMO AMBIGUITY FUNCTIONS OF DIFFERENT CODES WITH APPLICATION TO PHASE-CODED FMCW RADARS

Utku Kumbul[1] , Nikita Petrov[2,1], Sen Yuan[1], Cicero S. Vaucher[2,1], Alexander Yarovoy[1]*

[1]*Department of Microelectronics, Delft University of Technology, Delft, The Netherlands*
[2]*NXP Semiconductors, Eindhoven, The Netherlands*
**E-mail: u.kumbul@tudelft.nl*

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Abstract

The MIMO ambiguity functions of the binary phase codes as applied to phase-coded frequency modulated continuous waveform (PC-FMCW) are studied. The range-angle performance of the PC-FMCW with different code families is investigated and compared with the phase modulated continuous waveform (PMCW). An advantage of the PC-FMCW ambiguity function over the PMCW one is demonstrated in terms of the range resolution and sidelobe level for the same types of codes.

1 Introduction

Autonomous driving has become a new emerging technology that will improve road safety. Automotive radars play a critical role in achieving autonomous driving for detection, tracking and classification in traffic environments as they can operate under diverse weather conditions, and are used in modern cars to enable different levels of advanced driver assistance systems (ADAS) [1]. To achieve fully autonomous driving, the automotive radar needs to provide comprehensive and accurate information about the environment.

The multiple-input multiple-output (MIMO) systems are widely used for automotive radars to improve angular resolution [2]. The MIMO radars exploit the spatial diversity between transmitting and receiving antenna elements to create virtual arrays with a large aperture while using a relatively small number of antenna elements. However, the utilization of such spatial diversity requires high mutual orthogonality between transmitting channels. Various transmitting schemes and radar waveforms with different pros and cons have been proposed to realize MIMO systems in automotive radars [2, 3]. Since there is no room for sensing failures in fully autonomous driving, the radar waveform that gives high mutual orthogonality with good sensing performance is still a focus of research [4].

The automotive radars widely use linear frequency modulated continuous waveforms (LFMCW). LFMCW offer high range resolution, low sidelobes, good Doppler tolerance through simple hardware and low sampling requirements from analog-to-digital converter (ADC). On the other hand, multiple LFMCW in the same operational band lack the mutual orthogonality required for MIMO and thus the time-division multiple access (TDMA) scheme is often used to secure orthogonality between transmission channels. However, this transmission scheme increases the time duration between consecutive chirps and thus loses maximum unambiguous Doppler velocity [3]. An alternative proposed is the usage of LFMCW in adjacent frequency bands [5]. To achieve orthogonality without degrading the unambiguous Doppler velocity and enable simultaneous transmission via the code-division multiple access (CDMA) scheme, phase modulated continuous waveforms (PMCW) have been studied for automotive radars [6]. However, PMCW is vulnerable to Doppler frequency shifts due to target motion, and its poor Doppler tolerance needs to be compensated. Moreover, the utilization of the PMCW waveform results in a requirement of a high sampling rate of the beat-frequency signals, which increases the complexity and costs of the receiver in case of high range resolution [7].

Recently, phase-coded frequency modulated continuous waveforms (PC-FMCW) have been proposed to achieve high mutual orthogonality and realize simultaneous transmission for MIMO with good sensing performance [8–10]. In addition, PC-FMCW can be used to enable other emerging technologies such as joint sensing and communication systems [11–13]. The key advantage of PC-FMCW over PMCW is using linear frequency modulation of the carrier, which shears the ambiguity function of the phase-coded signal. Consequently, PC-FMCW can provide high mutual orthogonality while having high range resolution and Doppler tolerance similar to FMCW. In literature, various code families with different correlation properties have been studied for the phase-coded signal [14]. However, employing a chirp signal as a carrier for these codes changes their correlation properties. Thus, searching for appropriate code families to utilize with the PC-FMCW MIMO radar needs to be studied.

In this study, we study the MIMO ambiguity function to set the boundaries for the separation capability of the PC-FMCW with different code families and compare their resulting range-angle performance. We start with examining the ambiguity function for a single transmitting case and recall the shearing effect of the chirp signal in Section 2. Then in Section 3, we present the signal model for the PC-FMCW MIMO radar with a simultaneous transmission scheme. Afterwards, we give the range-angle profiles achieved with different code families in Section 4. Finally, the conclusions are drawn in Section 5.

2 Preliminaries

The transmitted signal for phase modulated continuous waveform can be written as:

$$x_{\text{pmcw}}(t) = s(t)\exp\left(j\left(2\pi f_c t\right)\right), \quad (1)$$

where $s(t) = \exp\left(j\phi(t)\right)$ is phase-coded signal that controls the phase changes and f_c is the carrier frequency of the radar. As explained in the introduction, PMCW is sensitive to Doppler frequency shifts due to target motion. The chirp signal can be used as a carrier to shear its ambiguity function and improve its Doppler tolerance. The transmitted phase-coded frequency modulated continuous waveform can be represented as:

$$x_{\text{t}}(t) = s(t)\exp\left(j\left(2\pi f_c t + \pi\beta t^2\right)\right). \quad (2)$$

where $\beta = B/T$ represents the chirp rate as B is the chirp bandwidth and T is the chirp duration, respectively. In this study, we consider a binary phase shift keying (BPSK) as a phase modulation scheme where the phase changes between $\{0, \pi\}$ according to the phase sequence. Then the phase-coded signal can be represented as:

$$s(t) = \sum_{n=1}^{N_c} e^{j\phi_n}\text{rect}\left(\frac{t-(n-1/2)T_c}{T_c}\right), \quad (3)$$

where $\text{rect}(t)$ is the rectangle function. The duration of the chip (code) is defined by the number of chips per chirp as $T_c = T/N_c$, where N_c denotes the number of chips within one chirp. Consequently, increasing the N_c raises the bandwidth of the code as $B_c = N_c/T$.

The ambiguity function is widely used for studying radar waveforms and determines the range-Doppler resolution of the transmitted signal for a chosen system parameters [15]. The narrow-band ambiguity function of signal $x(t)$ can be written as a linear convolution of a signal with its time-delayed and frequency-shifted replica:

$$|\,\chi(x(t);\tau,f_d)\,| = \left|\int_{-\infty}^{\infty} x(t)\,x^*(t-\tau)e^{j2\pi f_d t}\;dt\right|, \quad (4)$$

where $(\cdot)^*$ denotes the complex conjugate, τ is the delay and f_d is the Doppler frequency shift. One of the properties of the ambiguity function shows that adding linear frequency modulating shears the resulting ambiguity function as [15]:

$$|\,\chi(s(t)\exp\left(j\pi\beta t^2\right);\tau,f_d)\,| \Longleftrightarrow |\,\chi(s(t);\tau,f_d-\beta\tau)\,|\,. \quad (5)$$

The aforementioned shearing effect of the chirp signal can be seen in Fig. 1, where the ambiguity functions of PMCW and PC-FMCW are demonstrated. Herein, we use $B_c = 0.62$ MHz for the phase-coded signal and $T = 25.6\ \mu$s with $B = 10$ MHz for the chirp signal. For this study, we choose system parameters that are different from those found in standard automotive radars ($B = 300$ MHz) to illustrate the shearing effect clearly. As can be seen from Fig. 1, the ambiguity function of PMCW with $B_c = 0.625$ MHz has poor Doppler tolerance, which makes it vulnerable to target motion. On the other hand, the ambiguity function of PC-FMCW has range-Doppler coupling similar to the chirp signal and thus has good Doppler tolerance. In automotive radars, the Doppler shift can be estimated over multiple sequentially transmitted waveforms; hence

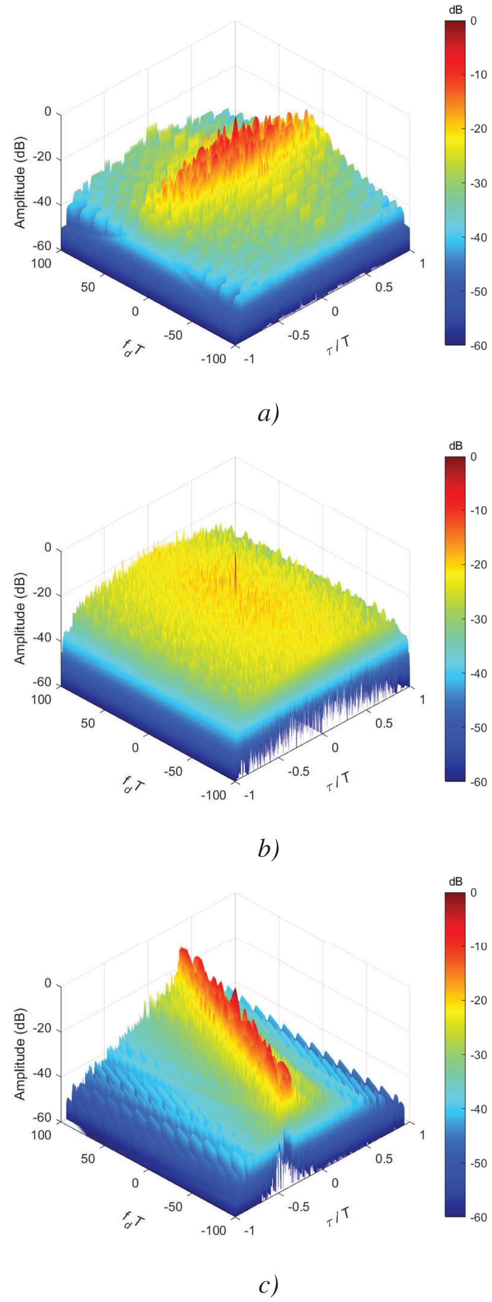
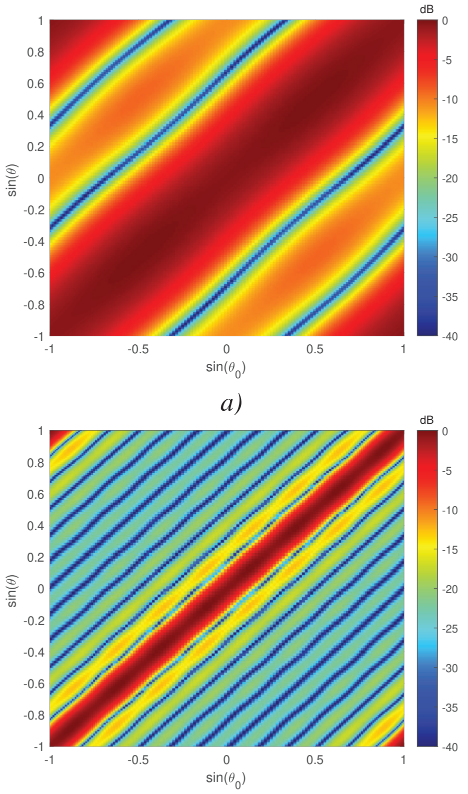


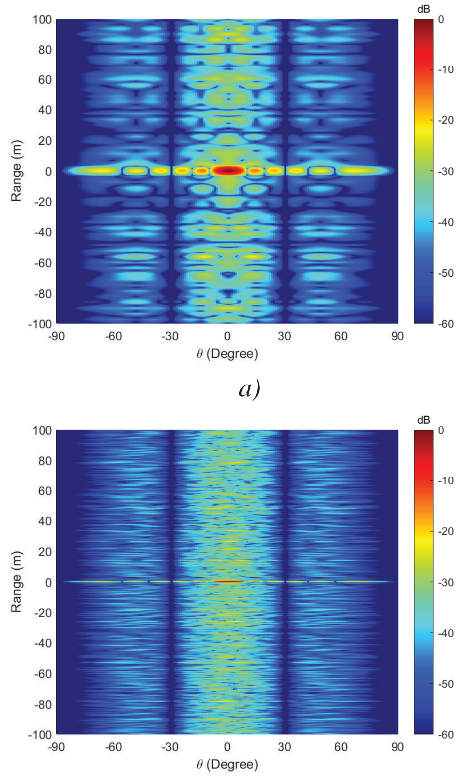
Fig. 1 Range-Doppler ambiguity function of normalized delay versus normalized Doppler a) PMCW with $B_c = 0.62$ MHz b) PMCW with $B_c = 10$ MHz c) PC-FMCW with $B_c = 0.62$ MHz and $B = 10$ MHz



a)

b)

Fig. 2 Hypothesis angle about the target position θ_0 versus the angle of the target θ for the PC-FMCW with the random code a) Transmit ambiguity function b) MIMO ambiguity function



a)

b)

Fig. 3 Range-angle profile of the MIMO ambiguity function ($\theta_0 = 0, f_d = 0$) with the random code a) PMCW b) PC-FMCW

the coupling between range and Doppler can be resolved. In addition, the ambiguity function corresponds to the traditional full-band match filter receiver. However, the acquisition of the signals with its full-band demands high sampling rate from ADC. Thus, the traditional matched filtering is not favorable for automotive radars due to its high demands on ADC performance and processing power. To reduce the sampling demands from ADC, the conventional matched filter for PC-FMCW can be approximated by using the group delay filter receiver, which performs dechirping and decoding strategy, or it can be realized via filter bank after dechirping. The sensing performance of the group delay filter receiver is almost equal to the full-band matched filter if the code bandwidth is much smaller than the ADC sampling frequency [10]. For this reason, we compare the full bandwidth of PC-FMCW with the code bandwidth of PMCW. Consequently, PC-FMCW with $B_c = 0.62$ MHz provides better range resolution than PMCW with $B_c = 0.62$ MHz. Note that in case of PMCW code bandwidth needs to be increased up to $B_c = 10$ MHz, which raises further the sampling requirement of ADC in the receiver, to achieve the same range resolution. Therefore, PC-FMCW has the advantages of mutual orthogonality while keeping the advantages of the LFM signal such as good Doppler tolerance and high range resolution by using the typical chirp bandwidth values. These facts favour the usage of PC-FMCW over PMCW.

3 MIMO Signal Model

Assume the MIMO system simultaneously transmits PC-FMCW with different codes as:

$$x_{\mathrm{t}_i}(t) = s_i(t)\exp\left(j\left(2\pi f_c t + \pi\beta t^2\right)\right) \tag{6}$$

where $1 \leq i \leq P$ is the index of the transmitter, P is the number of transmitters, and $s_i(t)$ is the phase-coded signal for identification of different transmitters, which needs to be orthogonal with each other. Assume there is a target at a range R_0 and angle θ moving with a constant radial velocity v_0. Then the round trip delay between radar and target can be obtained as:

$$\tau_0(t) = \frac{2\left(R_0 + v_0 t\right)}{c} = \tau_0 + \frac{2v_0}{c}t, \tag{7}$$

where c is the speed of light. Consider the MIMO system has L number of receiving antenna elements, and the index of the receiver is represented with $1 \leq j \leq L$. The received signal at j^{th} receiver will be the round trip delayed version of the transmitted signal at i^{th} transmitter. Accounting propagation and back-scattering effects by complex coefficient α_0, the received signals can be represented as:

$$x_{\mathrm{r}_j}(t,\theta) = \alpha_0\, a_{\mathrm{r}_j}(\theta) \sum_{i=1}^{P} a_{\mathrm{t}_i}(\theta) x_{\mathrm{t}_i}(t - \tau_0(t)) + n_j(t), \tag{8}$$

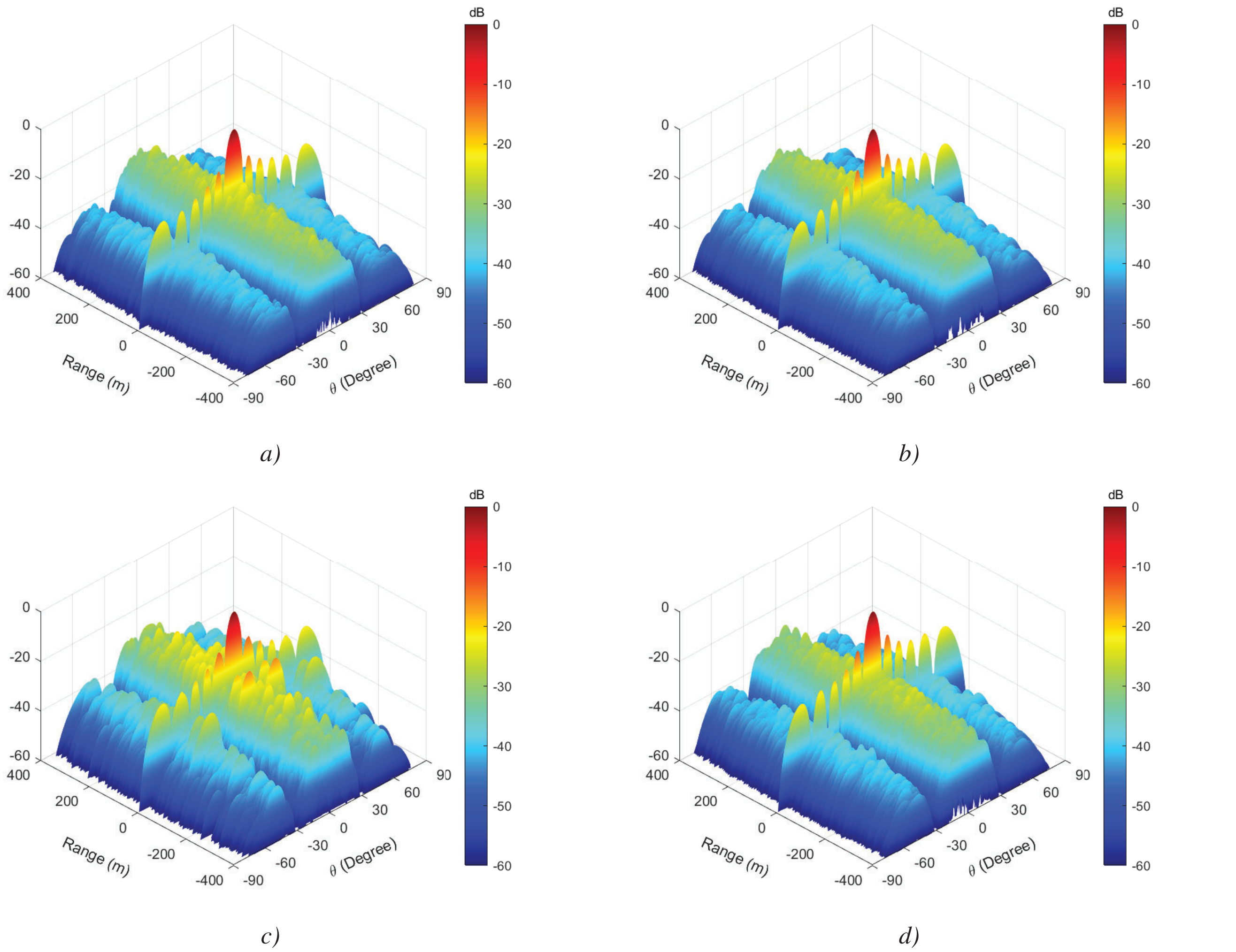


Fig. 4 Range-angle profile of the MIMO ambiguity function ($\theta_0 = 0, f_d = 0$) for the PC-FMCW with different codes a) Random (1024) b) Gold (1023) c) ZCZ (1024) d) Kasami (1023)

where $n_j(t)$ represents the noise signal at j^{th} receiver. The terms $a_{\mathrm{t}_i}(\theta)$ and $a_{\mathrm{r}_j}(\theta)$ are obtained from the steering vectors of the transmitter and receiver antennas, respectively and can be written as:

$$a_{\mathrm{t}_i}(\theta) = e^{j2\pi d_t (i-1)\frac{sin(\theta)}{\lambda}}, \tag{9}$$

and

$$a_{\mathrm{r}_j}(\theta) = e^{j2\pi d_r (j-1)\frac{sin(\theta)}{\lambda}}, \tag{10}$$

where d_t and d_r are the spacing between transmitters and receivers, respectively, and λ is the wavelength.

The optimum receiver that maximizes the signal-to-noise ratio (SNR) in the white noise scenario is the matched filter [15]. After down-conversion to base-band, the matched filter convolves the received signal with the complex conjugate of the transmitted signal as:

$$x_{\mathrm{MF}_{j,i}}(t, \theta) = \int_{-\infty}^{\infty} x_{\mathrm{r}_j}(\zeta, \theta)\, x^*_{\mathrm{t}_i}(t - \zeta)\, d\zeta. \tag{11}$$

The information about the target can be extracted from the output of the matched filter.

The transmit ambiguity function in the absence of Doppler effect can be calculated from the matched filter output, i.e., a convolution of the transmitted signal with its delayed replica [16]. Similarly, the MIMO ambiguity function is equal to the matched filter output and can be written as:

$$|\,\chi(\tau, \theta, \theta_0)\,| = \left| \sum_{j=1}^{L} \sum_{i=1}^{P} \int_{-\infty}^{\infty} x_{\mathrm{r}_j}(t, \theta)\, x^*_{\mathrm{t}_{i,j}}(t - \tau, \theta_0)\ dt \right|, \tag{12}$$

where θ is the actual angle of the target and θ_0 is the hypothesis angle about the target direction, respectively. Note the hypothesis signal also contains the angle information of the virtual array and can be represented as:

$$x_{\mathrm{t}_{i,j}}(t, \theta_0) = a_{\mathrm{r}_j}(\theta_0) a_{\mathrm{t}_i}(\theta_0) x_{\mathrm{t}_i}(t). \tag{13}$$

4 Simulations

In this section, we examine the MIMO ambiguity functions of the PC-FMCW radar obtained with different code families. Assume an automotive radar transmits PC-FMCW with

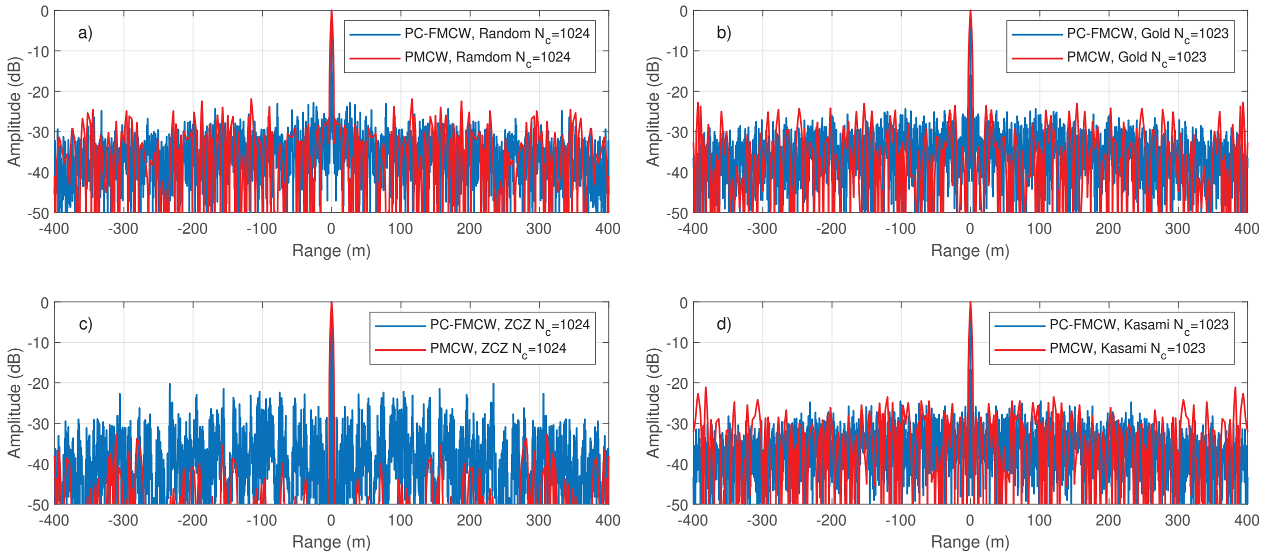


Fig. 5 Range profile comparison of the MIMO ambiguity function ($\theta = 0$,$\theta_0 = 0$,$f_d = 0$) for the PMCW and PC-FMCW with different codes a) Random b) Gold c) ZCZ d) Kasami

a carrier frequency $f_c = 77$ GHz, chirp duration $T = 25.6$ μs and chirp bandwidth $B = 300$ MHz. We use the BPSK sequence as a phase-coded signal, and choose $N_c = 1024$ number of chips per chirp. Thus, the bandwidth of the code signal is $B_c = N_c/T = 40$ MHz. We consider 3 transmitters and 4 receivers for the MIMO configuration, with $d_t = 2\lambda$ and $d_r = \lambda/2$, respectively. Consequently, the virtual array of the MIMO system has 12 elements with $\lambda/2$ spacing.

First, we simulate the transmit ambiguity and the MIMO ambiguity functions for PC-FMCW by using (12). Each phase-coded signal is selected as random codes, which are orthogonal to each other. The angular coverages of both ambiguity functions are shown in Fig. 2 where the hypothesis angle about the target position θ_0 versus the actual target angle θ is computed for $\tau = 0$. Herein, the strong line along the diagonal axis indicates that beam-forming can be achieved without ambiguity. Then, we compare the MIMO ambiguity functions of PMCW and PC-FMCW for a zero Doppler frequency shift $f_d = 0$ and $\theta_0 = 0$ in Fig. 3. For both waveforms, it is observed that the receiver can separate the simultaneously transmitted signals with the price of increased range sidelobes. However, PMCW has poor range resolution and needs to raise the bandwidth of the code, which increases the sampling demands on ADC, to achieve the same range resolution. Thus, the range resolution is improved substantially without increasing receiver's analogue bandwidth by using PC-FMCW.

Next, the MIMO ambiguity functions of PC-FMCW are simulated by using different code families for ($\theta_0 = 0$,$f_d = 0$). We choose four code families to compare with; Gold, Kasami, zero correlation zone (ZCZ), and Random codes. The comparison of the range-angle profiles is illustrated in Fig. 4. It is observed that the Gold, Kasami, and random codes provide similar range-angle performance while the ZCZ code has much higher sidelobes. Afterwards, we compare the range profiles of the MIMO ambiguity functions ($\theta = 0$,$\theta_0 = 0$,$f_d = 0$) of both waveforms with different codes in Fig. 5. We observe that the range profiles of PMCW and PC-FMCW are quite different as the code property is changed after modulating with the chirp signal. It can be seen that PC-FMCW achieves better range resolution with the same code bandwidth. Moreover, PC-FMCW provides a lower sidelobe level, especially in the far range as shown in the random, Gold and Kasami codes. However, PC-FMCW with the ZCZ code has notably higher sidelobe levels compared to PMCW. This is because the ZCZ code property that searches a zero correlation zone is destroyed by modulating with the chirp signal, and hence sidelobes are increased. Therefore, the code families optimized for PMCW might not be suitable for PC-FMCW, and proper code design for PC-FMCW is subject to be considered in future.

Finally, we investigate the influence of Doppler shift on the range-angle performance of both waveforms in Fig. 6. For this purpose, the received signal is considered with a Doppler frequency shift $f_d = 30$ kHz (corresponds to the relative velocity $v \approx 60$ m/s). It is observed that the Doppler frequency shift raises the sidelobe levels and degrades the range-angle performance of PMCW. On the other hand, the range-angle profile of PC-FMCW is not affected significantly due to its good Doppler tolerance and provides sidelobe levels similar to zero Doppler shift.

5 Conclusion

The MIMO ambiguity functions of different code families with application to PC-FMCW are studied. The range-Doppler ambiguity function for the single transmitting case is revisited, and the shearing effect of the chirp signal is analysed. It is shown that PC-FMCW has good Doppler tolerance and high range resolution similar to LFMCW. The range-angle profiles of the MIMO ambiguity functions are demonstrated for the random, Gold, ZCZ, and Kasami codes. It has been observed that the correlation property of the code alters with the frequency

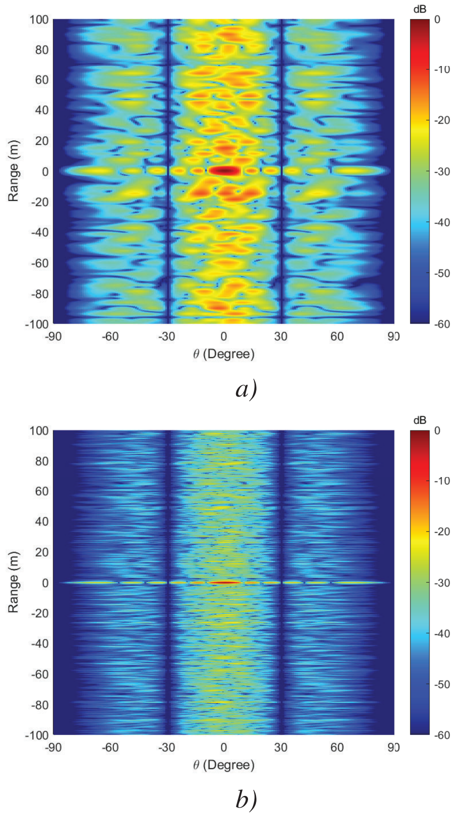


Fig. 6 Range-angle profile of the MIMO ambiguity function ($\theta_0 = 0, f_d = 30$ kHz) with the random code a) PMCW b) PC-FMCW

modulation, and the range profile of PC-FMCW can be substantially different from the code property. Particularly, it is demonstrated that the ZCZ code, which is optimized to find the zero correlation zone, is not suitable to utilize with PC-FMCW. Any practical implementation of the receiver design can use the illustrated performance as a benchmark.

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