Delft University of Technology

Hierarchical model predictive control and moving horizon estimation for open-channel systems with multiple time delays

Guekam, P. ; Segovia , P.; Etienne, L.; Duviella, E.

DOI
10.23919/ECC54610.2021.9655058

Publication date
2021

Document Version
Accepted author manuscript

Published in
Proceedings of the European Control Conference (ECC 2021)

Citation (APA)
Guekam, P., Segovia , P., Etienne, L., & Duviella, E. (2021). Hierarchical model predictive control and moving horizon estimation for open-channel systems with multiple time delays. In *Proceedings of the European Control Conference (ECC 2021)* (pp. 198-203). IEEE. https://doi.org/10.23919/ECC54610.2021.9655058

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright
Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy
Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.
For technical reasons the number of authors shown on this cover page is limited to a maximum of 10.

Hierarchical model predictive control and moving horizon estimation for open-channel systems with multiple time delays

P. Guekam, P. Segovia, L. Etienne and E. Duviella

***Abstract*— This work presents the design of a hierarchical control and state estimation approach for the optimal water level management of open-channel systems using gates and pumping stations as actuators. Each reach may be characterized by a different time delay and a different prioritization of objectives. The design is divided in three layers: the upper layer determines the current operating mode. The intermediate layer is concerned with the design of appropriate controllers and observers to compute the references. Finally, the lower layer solves a scheduling problem to minimize the error between the references and the applied controls by discrete actuators. Simulation of a realistic case study based on part of the inland waterways in the north of France is used to demonstrate the effectiveness of the proposed approach.**

I. INTRODUCTION

Modeling and control of open-channel systems have been extensively studied in the last several decades, motivated by many engineering applications, e.g. control of irrigation [1] and drainage canals [2], inland waterways management [3], hydro-power turbine control [4] and regulation of sewage systems [5]. Open-channel systems are characterized by complex dynamics, and are best described by the Saint-Venant equations, a set of partial differential equations [6]. However, these are not well suited for control, which has motivated the derivation of simplified modeling approaches, e.g., the Integrator Delay (ID) model [7], the Integrator Delay Zero (IDZ) model [8], the Integrator Resonance (IR) model [9], and grey-box [10] and black-box [11] models.

The automated control of these systems aims at reducing labor requirements as well as energy and maintenance costs, providing easy water level regulation and predicting effects of uncontrollable phenomena by properly dispatching water resources. Moreover, these objectives are customized and prioritized for each application. This work focuses particularly on inland waterways, which are large-scale systems, composed of natural rivers and artificial canals, and used mainly for transportation. Their management is usually achieved through the definition of a set of general objectives. However, it is worth noting that their prioritization might be different for each reach, and therefore it might be challenging to attain simultaneously the objectives of each reach.

P. Guekam, L. Etienne and E. Duviella are with CERI Systèmes Numériques, IMT Lille Douai, Lille, France (e-mails: pythagore.guekam@etu.imt-lille-douai.fr, {lucien.etienne, eric.duviella} @imt-lille-douai.fr).

P. Segovia is with the Department of Maritime and Transport Technology, Faculty of Mechanical, Maritime and Materials Engineering, Delft University of Technology, Delft, the Netherlands (e-mail: P.SegoviaCastillo@tudelft.nl).

Inland waterways are generally connected to the sea to dispose of the excess of water in the system, which can be achieved by using the gates and pumping stations (PS) located at the outlet of the last reach. However, it is important to note that sea tides limit the use of such gates, as it is forbidden to operate them in lowlands during high tide periods for safety reasons. This situation gives rise to a hybrid operating mode, and two control scenarios must be considered, one for low tide and another for high tide.

This paper continues the work initiated in [12] by generalizing the approach to any open-channel system that can be described by the Saint-Venant equations. The consideration of multiple-reach systems requires to extend the control design to the multiple time-delay case, and also allows for a different objective prioritization for each reach.

The rest of the paper is organized as follows: Section II states the problem. The proposed approach is described in Section III. A realistic case study based on the inland waterways in the north of France is considered in Section IV. The results allow to draw conclusions in Section V.

II. PROBLEM STATEMENT

Linearized models of open-channel canals can be formulated using the state-space representation [3]

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{B}_n\mathbf{u}_{k-n}, \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{D}_n\mathbf{u}_{k-n}, \end{aligned} \quad (1)$$

where $\mathbf{x}_k$, $\mathbf{u}_k$ and $\mathbf{y}_k$ denote the state, input and output at time instant k, respectively, while n denotes the time delay of the canal (in samples). Then, $\mathbf{u}_{k-n}$ represents the delayed effect of the control actions. Furthermore, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_n$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{D}_n$, are the system matrices of appropriate dimensions.

The control objectives mentioned before can be achieved by optimizing a certain criterion, commonly known as objective function, which is usually built as the weighted sum of several terms, each of them related to a specific objective. Based on the stated operational goals in the case of inland waterways systems, the following terms can be considered:

- Maintain water levels close to the set-points: $J_k^1 = (\mathbf{y}_k - \mathbf{y}_{ref})^\intercal (\mathbf{y}_k - \mathbf{y}_{ref})$, with $\mathbf{y}_{ref}$ the vector of set-point values.
- Minimize control effort: $J_k^2 = \mathbf{u}_k^\intercal \mathbf{u}_k$.
- Minimize fluctuations of the control signals: $J_k^3 = \delta\mathbf{u}_k^\intercal \delta\mathbf{u}_k$, with $\delta\mathbf{u}_k = \mathbf{u}_k - \mathbf{u}_{k-1}$.
- Penalize relaxation of navigability condition: $J_k^4 = \boldsymbol{\alpha}_k^\intercal \boldsymbol{\alpha}_k$, with $\boldsymbol{\alpha}_k$ the relaxation variable.

The formulation of the multi-objective function becomes more challenging when the simultaneous achievement of

© 2021 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

objectives for several canals is considered. To this end, the multi-objective function J can be defined as:

$$J = \sum_{k=1}^{H_p} \sum_{m=1}^{4} \sum_{r=1}^{N_r} \beta^{m,r} J_k^{m,r}, \tag{2}$$

where $\beta^{m,r}$ is the weighting coefficient associated to the m-th objective for the r-th reach, $J_k^{m,r}$ is the value of the m-th objective for the r-th reach at time instant k, H_p is the prediction horizon and N_r is the total number of reaches in the system. Therefore, the operational goals may be achieved for the N_r reaches through the optimization of (2).

On the other hand, the actuators used to apply the control actions might be of both continuous and discrete nature. Therefore, it needs to be ensured that the actions applied to the system are equal or as close as possible to those computed by the controllers. This task might be taken care of by another controller. However, while it might be somewhat realistic to assume that gates are able to supply the exact required flow, the same cannot be said for PS. Indeed, they usually consist in a bank of ON/OFF pumps, which usually prevents the equipment from supplying the exact reference in periods when the pumps are used.

With all this in mind, an approach that is able to handle a multivariable system while coping with constraints is required. Model predictive control (MPC), in addition to meeting the aforementioned requirements, is able to perform online optimization, and its design framework is simple yet powerful. More specifically, MPC computes the control variable trajectories that optimize the future behaviour of the plant output within a time window known as prediction horizon [13]. However, MPC needs to know the system states at the start of the time window, which are seldom measurable and hence need to be estimated. To this end, an observer is coupled to the MPC. The use of a moving horizon estimator (MHE) is proposed, as it is also formulated as an online optimization problem that is able to handle constraints [14].

In view of the above, a centralized MPC-MHE approach structured in three layers is proposed to solve this problem:

- The upper layer determines the tidal period and makes the corresponding settings.
- The intermediate layer solves the MPC problem corresponding to the current tide and provides the local controllers with optimal references. An MHE problem is also solved to estimate the next state of the system.
- The lower layer solves another optimization problem, minimizing the error between the optimal computed references and the pumping actions applied to the plant.

III. Proposed Approach

A. Upper layer

This layer must determine the current tide based on the comparison between the sea and the canal levels, and then make the corresponding settings. As gates cannot be employed during high tide for safety reasons, it is necessary to define two operating modes and design a controller for each tidal period. These can be determined based on historical data of the sea levels or the geographical location of the canal.

B. Intermediate layer

This layer is concerned with the design and resolution of the two MPC (one for each mode) and the MHE (employed in both modes). Their design is adapted from [3] to consider the case of N_r reaches with different delays $\{n_1, n_2, \ldots, n_{N_r}\}$, where n_r is the delay (in samples) corresponding to the r-th reach and $n \triangleq \max(n_r)$, $r \in \{1, ..., N_r\}$. Moreover, the real inland waterways management policy is considered, which allows to know the approximate time of the occurrence of the disturbances ahead of schedule, and thus they do not need to be estimated.

The low tide MPC is formulated as follows:

$$\min_{\substack{\{\mathbf{u}^g_{i|k}\}_{i=k}^{k+H_p-1}, \{\mathbf{u}^p_{i|k}\}_{i=k}^{k+H_p-1}, \\ \{\mathbf{y}_{i|k}\}_{i=k}^{k+H_p-1}, \{\boldsymbol{\alpha}_{i|k}\}_{i=k}^{k+H_p-1}}} J\left(\mathbf{u}^g_{i|k}, \mathbf{u}^p_{i|k}, \mathbf{y}_{i|k}, \boldsymbol{\alpha}_{i|k}\right) \tag{3a}$$

subject to

$$\mathbf{x}_{i+1|k} = \mathbf{A}\mathbf{x}_{i|k} + \mathbf{B}^g_u \mathbf{u}^g_{i|k} + \mathbf{B}^p_u \mathbf{u}^p_{i|k} + \mathbf{B}_d \mathbf{d}_{i|k} + \sum_{r=1}^{N_r} \mathbf{S}_r \left(\mathbf{B}^g_{un} \mathbf{u}^g_{i-n_r|k} + \mathbf{B}^p_{un} \mathbf{u}^p_{i-n_r|k} + \mathbf{B}_{dn} \mathbf{d}_{i-n_r|k}\right), \quad i \in \{k, \ldots, k+H_p-1\}, \tag{3b}$$

$$\mathbf{y}_{i|k} = \mathbf{C}\mathbf{x}_{i|k} + \mathbf{D}^g_u \mathbf{u}^g_{i|k} + \mathbf{D}^p_u \mathbf{u}^p_{i|k} + \mathbf{D}_d \mathbf{d}_{i|k} + \sum_{r=1}^{N_r} \mathbf{S}_r \left(\mathbf{D}^g_{un} \mathbf{u}^g_{i-n_r|k} + \mathbf{D}^p_{un} \mathbf{u}^p_{i-n_r|k} + \mathbf{D}_{dn} \mathbf{d}_{i-n_r|k}\right), \quad i \in \{k, \ldots, k+H_p-1\}, \tag{3c}$$

$$\mathbf{0} = \mathbf{E}^g_u \mathbf{u}^g_{i|k} + \mathbf{E}^p_u \mathbf{u}^p_{i|k} + \mathbf{E}_d \mathbf{d}_{i|k} + \sum_{r=1}^{N_r} \mathbf{S}_r \left(\mathbf{E}^g_{un} \mathbf{u}^g_{i-n_r|k} + \mathbf{E}^p_{un} \mathbf{u}^p_{i-n_r|k} + \mathbf{E}_{dn} \mathbf{d}_{i-n_r|k}\right), \quad i \in \{k, ..., k+H_p-1\}, \tag{3d}$$

$$\underline{\mathbf{u}}^g \leq \mathbf{u}^g_{i|k} \leq \overline{\mathbf{u}}^g, \ i \in \{k, \ldots, k+H_p-1\}, \tag{3e}$$

$$\underline{\mathbf{u}}^p \leq \mathbf{u}^p_{i|k} \leq \overline{\mathbf{u}}^p, \ i \in \{k, \ldots, k+H_p-1\}, \tag{3f}$$

$$\underline{\mathbf{y}} - \boldsymbol{\alpha}_{i|k} \leq \mathbf{y}_{i|k} \leq \overline{\mathbf{y}} + \boldsymbol{\alpha}_{i|k}, \quad i \in \{k, \ldots, k+H_p-1\}, \tag{3g}$$

$$\boldsymbol{\alpha}_{i|k} \geq 0, \ i \in \{k, \ldots, k+H_p-1\}, \tag{3h}$$

$$\mathbf{x}_{i|k} = \hat{\mathbf{x}}^{MHE}_{i|k}, \ i \in \{k-n, \ldots, k\}, \tag{3i}$$

$$\mathbf{u}^g_{i|k} = \mathbf{u}^{MPC(g)}_{i|k}, \ i \in \{k-n, \ldots, k-1\}, \tag{3j}$$

$$\mathbf{u}^p_{i|k} = \mathbf{u}^{MPC(p)}_{i|k}, \ i \in \{k-n, \ldots, k-1\}, \tag{3k}$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$ are the states, $\mathbf{y}_k \in \mathbb{R}^{n_y}$ are the water levels, $\mathbf{u}^g_k \in \mathbb{R}^{n_{u_g}}$ and $\mathbf{u}^p_k \in \mathbb{R}^{n_{u_p}}$ are the total gate and pumping control actions, respectively, $\mathbf{d}_k \in \mathbb{R}^{n_d}$ are the disturbances, and $\boldsymbol{\alpha}_k \in \mathbb{R}^{n_y}$ is a relaxation variable introduced to relax the navigability condition constraint that allows to take into account scenarios in which the levels might be temporarily outside the navigation interval. Moreover, H_p is the prediction horizon, $k \in \mathbb{Z}$ is the current time instant,

$i \in \mathbb{Z}$ is the time instant along the prediction horizon, $\underline{\mathbf{y}}$, $\overline{\mathbf{y}}$, $\underline{\mathbf{u}}^g$, $\overline{\mathbf{u}}^g$, $\underline{\mathbf{u}}^p$ and $\overline{\mathbf{u}}^p$, are the lower and higher navigation levels (LNL and HNL) and control bounds, respectively. Furthermore, (3d) defines the mass balance relations at the junctions, and $\mathbf{S}_r$ is a selector matrix that allows to choose the appropriate output subject to the delayed control. Finally, the superscripts MHE and MPC in constraints (3i)–(3k) denote the use of information computed in previous MPC and MHE iterations, and J is formulated as in (2).

Then, the optimal solution of (3) is given by the sequences $\{\mathbf{u}^g_{i|k}\}_{i=k-n}^{k+H_p-1}$, $\{\mathbf{u}^p_{i|k}\}_{i=k-n}^{k+H_p-1}$, $\{\mathbf{y}_{i|k}\}_{i=k-n}^{k+H_p-1}$ and $\{\boldsymbol{\alpha}_{i|k}\}_{i=k-n}^{k+H_p-1}$†. However, only the controls at time instant k are retained according to the receding horizon philosophy

$$\mathbf{u}_k^{MPC(g)} \triangleq \mathbf{u}^g_{k|k}, \tag{4a}$$
$$\mathbf{u}_k^{MPC(p)} \triangleq \mathbf{u}^p_{k|k}. \tag{4b}$$

As a side comment, the high tide MPC can be derived from (3) by forcing the gate action to be equal to zero.

On the other hand, the MHE is formulated as follows:

$$\min_{\{\hat{\mathbf{x}}_{i|k}\}_{i=k-H_e+1}^{k+1}} \quad \mathbf{w}^{\intercal}_{k-H_e+1|k}\mathbf{P}^{-1}\mathbf{w}_{k-H_e+1|k} + \tag{5a}$$
$$\sum_{i=k-H_e+1}^{k}\left(\mathbf{w}^{\intercal}_{i|k}\mathbf{Q}^{-1}\mathbf{w}_{i|k} + \mathbf{v}^{\intercal}_{i|k}\mathbf{R}^{-1}\mathbf{v}_{i|k}\right) \tag{5b}$$

subject to

$$\mathbf{w}_{k-H_e+1|k} = \hat{\mathbf{x}}_{k-H_e+1|k} - \mathbf{x}_{k-H_e+1}, \tag{5c}$$
$$\mathbf{w}_{i|k} = \hat{\mathbf{x}}_{i+1|k} - \Bigg(\mathbf{A}\hat{\mathbf{x}}_{i|k} + \mathbf{B}^g_u\mathbf{u}^g_{i|k} + \mathbf{B}^p_u\mathbf{u}^p_{i|k} + \mathbf{B}_d\mathbf{d}_{i|k} + \sum_{r=1}^{N_r}\mathbf{S}_r\Big(\mathbf{B}^g_u\mathbf{u}^g_{i-n_r|k} + \mathbf{B}^p_u\mathbf{u}^p_{i-n_r|k} + \mathbf{B}_{dn}\mathbf{d}_{i-n_r|k}\Big)\Bigg),\ i \in \{k-H_e+1,\dots,k\}, \tag{5d}$$
$$\mathbf{v}_{i|k} = \mathbf{y}_{i|k} - \Bigg(\mathbf{C}\hat{\mathbf{x}}_{i|k} + \mathbf{D}^g_u\mathbf{u}^g_{i|k} + \mathbf{D}^p_u\mathbf{u}^p_{i|k} + \mathbf{D}_d\mathbf{d}_{i|k} + \sum_{r=1}^{N_r}\mathbf{S}_r\Big(\mathbf{D}^g_u\mathbf{u}^g_{i-n_r|k} + \mathbf{D}^p_u\mathbf{u}^p_{i-n_r|k} + \mathbf{D}_{dn}\mathbf{d}_{i-n_r|k}\Big)\Bigg),\ i \in \{k-H_e+1,\dots,k\}, \tag{5e}$$
$$\mathbf{0} = \mathbf{E}^g_u\mathbf{u}^g_{i|k} + \mathbf{E}^p_u\mathbf{u}^p_{i|k} + \mathbf{E}_d\mathbf{d}_{i|k} + \sum_{r=1}^{N_r}\mathbf{S}_r\Big(\mathbf{E}^g_{un}\mathbf{u}^g_{i-n_r|k} + \mathbf{E}^p_{un}\mathbf{u}^p_{i-n_r|k} + \mathbf{E}_{dn}\mathbf{d}_{i-n_r|k}\Big),\ i \in \{k-H_e+1,...,k\}, \tag{5f}$$
$$\mathbf{y}_{i|k} = \mathbf{y}_i,\ i \in \{k-H_e+1,\dots,k\}, \tag{5g}$$
$$\underline{\mathbf{x}} \leq \hat{\mathbf{x}}_{i|k} \leq \overline{\mathbf{x}},\ i \in \{k-H_e+1,\dots,k+1\}, \tag{5h}$$
$$\hat{\mathbf{x}}_{i|k} = \hat{\mathbf{x}}_i^{MHE},\ i \in \{k-H_e-n+1,\dots,k-H_e\}, \tag{5i}$$

† $\{\mathbf{u}^g_{i|k}\}_{i=k}^{k+H_p-1} \triangleq \{\mathbf{u}^g_{k|k}, \mathbf{u}^g_{k+1|k}, \cdots, \mathbf{u}^g_{k+H_p-1|k}\}$; $\mathbf{u}^p_{i|k}, \mathbf{y}_{i|k}$ and $\boldsymbol{\alpha}_{i|k}$ are defined in the same manner

$$\mathbf{u}^g_{i|k} = \mathbf{u}_i^{MPC(g)},\ i \in \{k-H_e-n+1,\dots,k-H_e\}, \tag{5j}$$
$$\mathbf{u}^p_{i|k} = \mathbf{u}_i^{MPC(p)},\ i \in \{k-H_e-n+1,\dots,k-H_e\}, \tag{5k}$$

with H_e the length of the estimation window; $\mathbf{P}^{-1}$, $\mathbf{Q}^{-1}$, $\mathbf{R}^{-1}$ the weighting matrices; $\mathbf{x}_{k-H_e+1}$ the best guess for the initial state; and $\mathbf{y}_i$ the measured water levels.

The optimal solution is given by the sequence $\{\hat{\mathbf{x}}_{i|k}\}_{i=k-He+1}^{k+1}$, but according to the MHE philosophy, only the last component is retained, which is expressed as

$$\hat{\mathbf{x}}_k^{MHE} = \hat{\mathbf{x}}_{k+1|k}. \tag{6}$$

C. Lower layer

This layer is in charge of the low-level control, which aims at ensuring that the optimal actions computed at the intermediate layer are applied to the system. To this end, several PS consisting of ON/OFF pumps are installed at the downstream end of the last reach, which has an outlet to the sea. Recalling that gates were assumed to be able to supply the optimal value, the low-level control problem boils down to applying the pumping control reference computed by the MPC using ON/OFF pumps. More precisely, the goal is to determine, for each PS, the subset of available pumps that should be activated at each time instant so that the error between the reference and the applied control is minimized. For each PS, the scheduling problem is formulated as in [12].

D. Simulation of the proposed approach

The approach is simulated by assuming that the upper, intermediate and lower layers work with sampling times T_{s_1}, T_{s_2} and T_{s_3}, respectively, with $T_{s_1} \geq T_{s_2} \geq T_{s_3}$. Moreover, these are chosen such that $\bmod(T_{s_1}, T_{s_2}) = \bmod(T_{s_2}, T_{s_3}) = 0$. The simulation loop is executed using the smallest sampling time. This allows to define $N_1 = T_{s_1}/T_{s_2}$ as the number of times that the MPC and MHE are solved within two upper layer executions, and $N_2 = T_{s_2}/T_{s_3}$ as the number of pumping instants within two consecutive solutions computed by the MPC. Then, one simulation step in the upper layer corresponds to N_1 and $N_1 \cdot N_2$ execution steps in the intermediate and lower layers, respectively.

To clarify the proposed approach, imagine that the upper layer is executed at time instant k_1 to determine the current tidal period. Then, the corresponding MPC problem is solved for every instant in the interval $[k_1, k_1+1, ..., k_1+N_1-1]$, providing the set of optimal references to be applied to the system at each instant. Likewise, the MHE problem is solved N_1 times, but once the N_2 activation states of the pumps have been applied to the system to account for the total pumping effect. Then, the upper layer problem will be executed again at time instant k_1+N_1. On the other hand, for each MPC solution at time instant $k_2 \neq k_1$, the scheduling problem is solved immediately after, yielding the following N_2 activation states of the pumps, which are considered sequentially. Then, at time instant k_2+N_2, the intermediate layer is executed again using updated information.

Algorithm 1 Outline of the simulation

Require: Parameters in problems (3) and (5)

Estimate the initial state $\hat{x_0}$
for k = 0: t_{sim} **do**
  if $\text{mod}(k, N_1 \cdot N_2) = 0$ **then**
    Determine tide and make the corresponding settings
  end if
  if $\text{mod}(k, N_2) = 0$ **then**
    Solve the MPC corresponding to the current tide
    Extract $u^{MPC(g)}$ and $u^{MPC(p)}$ from the solution
    Solve scheduling problem for each PS using $u^{MPC(p)}$ and obtain activation states $\{s_i^j(l)\}_{i=1}^{N_2}$ (state of the l-th pump at the PS j for the next N_2 time instants)
  end if
  Apply $u_k^{MPC(g)}$ and $\sum_{j=1}^{n_j} \sum_{l=1}^{n_{p_j}} u_d^j(l) s^j_{\text{mod}(k,N_2)}(l)$, with n_j the number of PS and n_{p_j} the number of pumps at the j-th PS
  if $\text{mod}(k, N_2) = N_2 - 1$ **then**
    Measure the water levels in the system
    Set $u^{real(p)} \triangleq \sum_{j=1}^{n_j} \sum_{l=1}^{n_{p_j}} \sum_{i=1}^{N_2} u_d^j(l) s_i^j(l)$
    Send the water levels, $u^{MPC(g)}$ and $u^{real(p)}$ to the MHE to estimate the state at the next time instant
  end if
end for

The described scheme is illustrated in Algorithm 1. Note that $u_d^j(l)$, n_{p_j} and n_j denote the design flow of the l-th pump at the j-th PS, the number of available pumps at the j-th PS and the total number of PS, respectively.

IV. Case Study

A. Description of the system

The methodology is tested by considering a realistic system composed of two navigation reaches NR_1 and NR_2 connected by a lock. This system is based on part of the inland waterways in the north of France; the physical data of each reach are given in Table I. NR_1 is bounded at the upstream end by the lock L_1 and at the downstream end by lock L_2, which is also the upstream end of NR_2. The downstream end of NR_2 is bounded by P_3, a PS equipped with four pumps, and the sea outlet gate G_3. Moreover, two additional gates G_1 and G_2 are installed to regulate the levels together with G_3 and P_3. However, the use of G_3 is restricted depending on the condition of the tide, while G_1, G_2 and the pumps may be operated regardless of the tidal period. Sensors are also installed at the end of each canal, and provide the controllers with level measurements. A schematic representation of the system is depicted in Figure 1.

The prioritization of the management objectives is tailored for each canal. In the case of NR_1, the control effort is put into maintaining the level as close to the normal navigation level (NNL) as possible. Conversely, in NR_2 the attention is focused on minimizing the use of pumps for economic

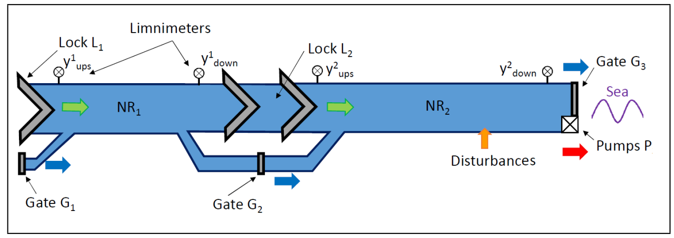

Fig. 1. Schematic representation of the case study

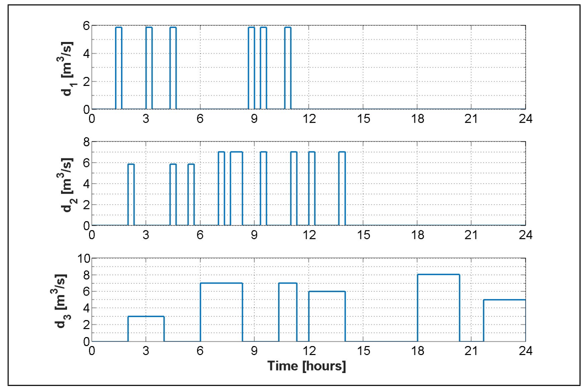

Fig. 2. Considered disturbances at each bounding node

reasons. Moreover, changes in the activation states of the pumps in P_3 are also penalized, aiming at extending their useful life. To this end, the foreseeable excess of water in the canal during high tide will be anticipated, lowering the canal level as much as possible during low tide. Then, water will be accumulated as much as possible during high tide, so that its excess can be released to the sea during the next low tide using G_3. Note that pumps might still need to be employed in low tide, but their use is restricted to the periods in which G_3 has reached its maximum capacity and there still exists the need of additional effort to dispose of water in NR_2.

B. Experimental design

The disturbances, which are assumed to be known in this work, are a consequence of the operation of L_1 and L_2 (d_1 and d_2, respectively) and the pumping actions carried out by farmers (d_3), and are depicted in Figure 2.

The model used to design both MPC and the MHE is described by (3b)–(3d), yielding a different time delay for each reach: two and a half hours for NR_1 and two hours for NR_2. Then, both problems are implemented in MATLAB using CVX, a tool that allows to write convex programs in a straightforward manner and solve them [15]. On the other hand, H_p and H_e are both chosen equal to four hours, whereas the sampling times of the layers are $T_{s_1} = T_{s_2} = 20$ min and $T_{s_3} = 5$ min. Finally, it is considered that two of the pumps in P_3 have a design flow of 2 m^3/s, whereas this value is equal to 4 m^3/s for the other two pumps. Then, the simulation is performed as stated in Algorithm 1.

C. Results

The simulation is divided into a navigation and a non-navigation period. Moreover, the following notation is em-

	Length $[m]$	Width $[m]$	LNL $[m]$	NNL $[m]$	HNL $[m]$	Bottom slope	Manning coeff. $[s/m^{1/3}]$	Average flow $[m^3/s]$
NR_1	42000	50	3.65	3.8	3.95	0	0.035	0.6
NR_2	26720	20	2.05	2.2	2.35	0	0.035	1

TABLE I
PHYSICAL PARAMETERS OF THE CASE STUDY

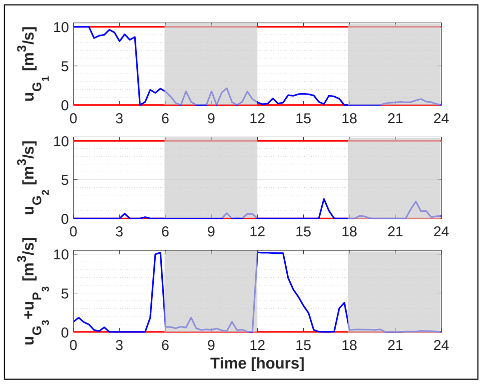

Fig. 3. Control references computed by the MPC and bounds

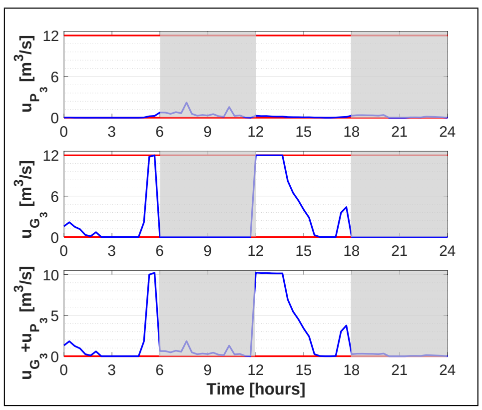

Fig. 4. Pumping, gate and total references at the downstream end of NR_2

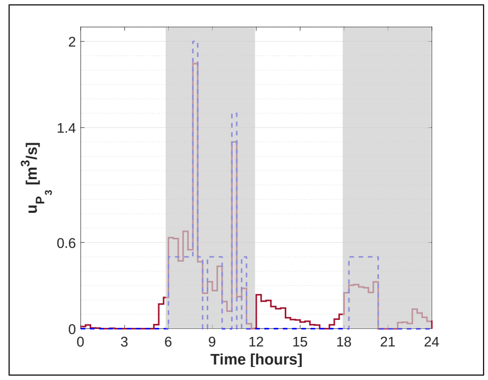

Fig. 5. Optimal reference (solid red) and scheduled pumping (dashed blue)

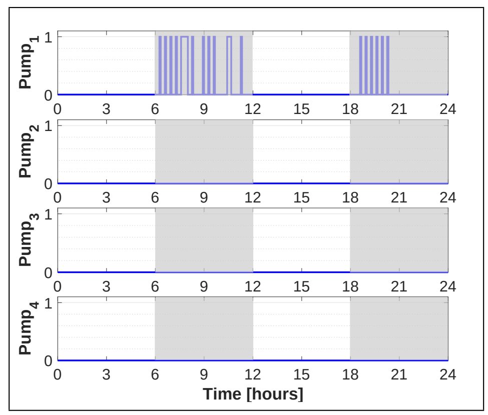

Fig. 6. Activation state of each pump

ployed in the figures: 1 stands for NR_1, 2 for NR_2, *ups* for upstream, and *down* for downstream.

On the other hand, shaded areas are employed to denote high tide periods in the subsequent figures. Note also that the historical record of the sea in this case study allows to consider a semi-diurnal pattern, which consists in two high tides and two low tides per day, each of them lasting about six hours. The heights of the two low tides are similar; the same happens for the two highs.

The optimal control references computed by the MPC are depicted in Figure 3. Note that G_2 is the least operated gate, which is due to the initial condition: the initial levels are below the NNL, and therefore G_1 should open to fill the canal, whereas G_2 should remain closed so that the level can reach the NNL. The most interesting control result concerns G_3 and is shown in more detail in Figure 4. Pumps are barely needed to regulate the levels during low tide, while gates are not employed during high tide for safety reasons, and the use of pumps is kept to a minimum for economic reasons.

The lower-layer results are presented in Figure 5. The shaded areas coincide with the high tide, when the use of pumps is required. Each pump might be activated for $\{0, 5, 10, 15, 20\}$ minutes between two MPC solutions. Their activation states are depicted in Figure 6, showing that their use is minimized despite the disturbances.

The water levels in the two canals resulting from applying the aforementioned control signals are depicted in Figure 7. Note that the level in NR_2 does not stay close to the NNL; in fact, it is allowed to oscillate around it to better anticipate the use of pumps during the day. On the other hand, the level in NR_1 does stay close to the NNL. This behavior is due to the prioritization of objectives described in Section IV.A, thus complying with the described management priority. Moreover, the controller performance is quantified as [16]:

$$TP = 1 - \frac{1}{H_p}\sqrt{\sum_{k=1}^{H_p}\left(\frac{\mathbf{y}_k - \mathbf{y}_r}{\frac{1}{2}\left(\overline{\mathbf{y}}_r - \underline{\mathbf{y}}_r\right)}\right)^2} \qquad (7)$$

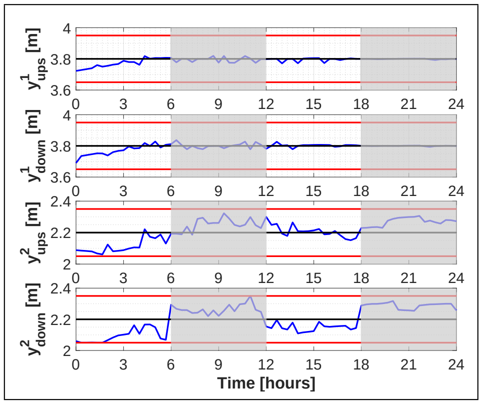

Fig. 7. Water levels (blue), NNL (black), and LNL and HNL (red)

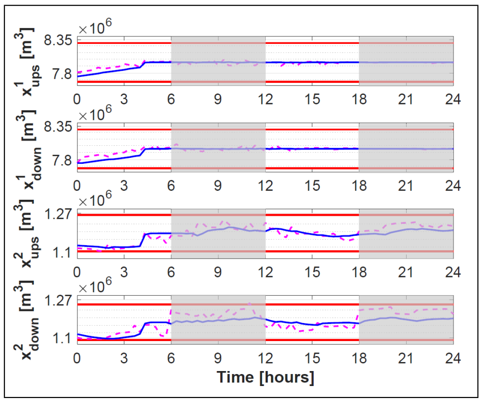

Fig. 8. Computed (in dashed magenta) and estimated (in solid blue) states

which equals 0.9823 for NR_1, and 0.9326 for NR_2, thus allowing to highlight the satisfactory performance of the approach. Note that the index of NR_1 is better than that of NR_2 because of the different prioritization of objectives.

Finally, the controls applied to the system and the levels are sent to the MHE to estimate the states at the next time instant. The comparison between state estimates and computed states (using the model) are depicted in Figure 8.

V. CONCLUSIONS

This paper continues the work initiated in [12] on the design of a hierarchical control approach to regulate the inland waterways levels. New challenges were considered, namely a case study that comprises several interconnected canals and the prioritization of management objectives for each canal. On the one hand, dealing with several canals, each with a different time delay, required to extend the original formulation to the case of multiple time delays. On the other hand, the original cost function was reformulated to allow for individual objective customization. The final formulation is suitable to describe any open-channel system.

A realistic case study built upon the inland waterways in the north of France served to test the methodology and prove its effectiveness. However, as the number of reaches increase, the system becomes difficult to handle using a centralized approach. In this regard, future works could consider non-centralized approaches, given their flexibility and scalability [17]. On the other hand, a large amount of potential energy is lost during lock operations. It might be interesting to consider the use of turbines in inland waterways. Then, this energy, which is generated by the water dispatched during lock operations, could be transformed into electricity, aiming at reducing the dependency on non-renewable energy sources.

REFERENCES

[1] P. O. Malaterre and J. P. Baume, "Modeling and regulation of irrigation canals: existing applications and ongoing researches," in *Proc. of the IEEE Int. Conf. on Systems, Man and Cybernetics*, vol. 4, 1998, pp. 3850 – 3855.

[2] A. Zafra-Cabeza, J. M. Maestre, M. A. Ridao, E. F. Camacho, and L. Sánchez, "A hierarchical distributed model predictive control approach to irrigation canals: a risk mitigation perspective," *Journal of Process Control*, vol. 21, no. 5, pp. 787–799, 2011.

[3] P. Segovia, L. Rajaoarisoa, F. Nejjari, E. Duviella, and V. Puig, "Model predictive control and moving horizon estimation for water level regulation in inland waterways," *Journal of Process Control*, vol. 76, pp. 1–14, 2019.

[4] J. Zhang, V. Leontidis, A. Dazin, A. Tounzi, P. Delarue, G. Caignaert, F. Piriou, and A. Libaux, "Canal lock variable speed hydropower turbine design and control," *IET Renewable Power Generation*, vol. 12, no. 14, pp. 1698–1707, 2018.

[5] B. Joseph-Duran, C. Ocampo-Martínez, and G. Cembrano, "Output-feedback control of combined sewer networks through receding horizon control with moving horizon estimation," *Water Resources Research*, vol. 51, no. 10, pp. 8129–8145, 2014.

[6] A. J. C. B. de Saint-Venant, "Théorie du mouvement non permanent des eaux, avec application aux crues des rivières et à l'introduction des marées dans leurs lits," *Comptes Rendus des séances de l'Académie des Sciences*, vol. 73, pp. 237–240, 1871.

[7] J. Schuurmans, A. J. Clemmens, S. Dijkstra, A. Hof, and R. Brouwer, "Modeling of irrigation and drainage canals for controller design," *Journal of irrigation and drainage engineering*, vol. 125, no. 6, pp. 338–344, 1999.

[8] X. Litrico and V. Fromion, *Modeling and Control of Hydrosystems*. Springer, 2009.

[9] P. J. van Overloop, I. J. Miltenburg, X. Bombois, A. J. Clemmens, R. Strand, and N. van de Giesen, "Identification of resonance waves in open water channels," *Control Engineering Practice*, vol. 18, no. 8, pp. 863–872, 2010.

[10] K. Horváth, E. Duviella, J. Blesa, L. Rajaoarisoa, Y. Bolea, V. Puig, and K. Chuquet, "Gray-box model of inland navigation channel: application to the Cuinchy-Fontinettes reach," *Journal of Intelligent Systems*, vol. 23, no. 2, pp. 183–199, 2014.

[11] E. Weyer, "System identification of an open water channel," *Control Engineering Practice*, vol. 9, no. 12, pp. 1289 – 1299, 2001.

[12] P. Segovia, E. Duviella, and V. Puig, "Multi-layer model predictive control of inland waterways with continuous and discrete actuators," in *21st IFAC World Congress*, 2020, accepted.

[13] E. F. Camacho and C. Bordons, *Model Predictive Control*. London: Springer, 1998.

[14] C. V. Rao, J. B. Rawlings, and J. H. Lee, "Constrained linear state estimation – a moving horizon approach," *Automatica*, vol. 37, no. 10, pp. 1619–1628, 2001.

[15] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.

[16] P. Segovia, L. Rajaoarisoa, F. Nejjari, E. Duviella, and V. Puig, "Input-delay model predictive control of inland waterways considering the backwater effect," in *2018 IEEE Conference on Control Technology and Applications (CCTA)*. IEEE, 2018, pp. 589–594.

[17] P. Segovia, V. Puig, E. Duviella, and L. Etienne, "Distributed model predictive control using optimality condition decomposition and community detection," *Journal of Process Control*, vol. 99, pp. 54–68, 2021.