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The Influence of Directional Spreading of Waves on Mooring Forces

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ABSTRACT

In this paper the basic expressions for the computation of mean and low frequency second order wave drift forces on floating structures in directionally spread seas as can be derived from potential theory are discussed. The resultant expressions for the mean and slowly varying drift forces are applied to a specific form of the directional seas, i.e. cross seas which consist of two irregular long-crested wave trains from different directions. The analysis shows under which conditions the drift forces in irregular cross seas may be calculated based on the superposition of the drift forces from the component long-crested irregular wave trains. Model tests were carried out with the model of a 200,000 DWT tanker moored in regular and irregular cross seas. The results of the model tests confirm theoretical predictions regarding the superposition of mean drift forces and the interaction effects present in slowly varying components of the drift forces in directional seas.

INTRODUCTION

The analysis of the behaviour of vessels moored at sea is generally based on measurements from model tests in irregular uni-directional waves carried out in suitable model facilities.

Data obtained from such model tests has in the past proved to be indispensable in the design of offshore floating structures. Although, at all times, the conditions of the model tests represent a simplified reality, the on-site performance of full scale structures bears witness to the general validity of such model test data as a sound basis for judging the performance of a particular design with regard to both the motion behaviour and the loads on the structures.

References and illustrations at end of paper.

Notwithstanding, however, the generally accepted validity of model testing under such simplified conditions, there is a need to investigate, in more detail, the effect of directional spreading of irregular waves, as it occurs in reality, on the loads and motions of floating offshore structures. See, for instance, ref. [1].

One method of obtaining such information is to conduct model tests in basins fitted with wave generators which have the capability to generate irregular directional waves. Such model tests will produce quantitative data, based on a physical reality, of the effects of directional waves. [1], [2] and [3].

Another method of obtaining data on the effects of multi-directional seas on the behaviour of floating structures is based on theoretical computations; [4]. It will be clear that efforts should be made to compare results of such computations with model test results. This is necessary since significant physical effects peculiar to directional seas may be present which are not accounted for in the theoretical approach.

For irregular uni-directional seas, computation methods exist which can, with reasonable accuracy, predict the behaviour of a moored vessel in both the frequency and the time domain; [5] and [6].

This type of computer program can be used to assess certain aspects associated with multi-directional seas provided a realistic formulation can be given for the wave loads, both oscillatory, first order wave loads and mean and low frequency second order drift forces in irregular directional waves, [7].

In this paper attention is paid to such formulations for the mean and slowly varying drift forces. Expressions are given for the wave drift forces for the case that the undisturbed incident directional wave field is described by a Fourier series with random phase angles.

Based on such descriptions for the second order wave loads, frequency domain results are obtained in terms of force spectra in irregular directional seas. From these expressions further insight can be obtained regarding the effects of directional seas.

Numerical results on the wave drift forces in a special type of directional sea, namely, regular cross waves, obtained using three-dimensional diffraction calculations and direct integration of second order pressure are compared with results of model tests.

Results are also given of tests in irregular unidirectional waves and of tests in irregular cross seas on the basis of which the validity of the superposition principle with respect to mean drift forces could be checked. For these cases computed results are also presented.

Finally some model test results which show the effect of irregular cross seas on low frequency mooring forces are given.

THE INCIDENT WAVES

In order to derive expressions for the drift forces in directional seas we assume that the wave elevation in a point can be described by a double Fourier series, which characterizes the surface elevation as a sum of regular long-crested waves from various directions:

$$\zeta(t) = \sum_{i=1}^{N} \sum_{k=1}^{M} \zeta_{ik} \cos(\omega_i t + \varepsilon_{ik}) \quad \ldots\ldots (1)$$

In a multi-directional sea the amplitudes of the wave components are found from:

$$\zeta_{ik} = \sqrt{2\, S_\zeta(\omega_i,\psi_k)\, \Delta\omega\, \Delta\psi} \quad \ldots\ldots (2)$$

where:

$S_\zeta(\omega_i,\psi_k)$ = directional wave spectrum
$\Delta\omega$ = frequency interval
$\Delta\psi$ = direction interval.

For the directional spectrum $S_\zeta(\omega,\psi)$ various formulations can be chosen, [8] and [9].

After chosing the random phase angles, a time record of the surface elevation can be computed from equation (1).

SECOND ORDER WAVE DRIFT FORCES

Second order wave forces acting on vessels or structures in waves can be computed based on direct integration of second order pressures and forces, [10].

In vector notation the general expression for the second order wave forces is as follows:

$$\bar{F}^{(2)}(t) =$$

$$= -\int_{WL} \tfrac{1}{2}\rho g\, \zeta_r^{(1)2}\, \bar{n}\, dl - \iint_{S_0} \{-\tfrac{1}{2}\rho\, \bar{\nabla}\phi^{(1)2}\}\, \bar{n}\, dS +$$

$$- \iint_{S_0} \{-\rho(\bar{X}^{(1)} \cdot \bar{\nabla}\phi_t^{(1)})\}\, \bar{n}\, dS + \bar{\alpha}^{(1)} \times (M \cdot \ddot{\bar{X}}_g^{(1)}) +$$

$$- \iint_{S_0} \{-\rho(\phi_{w_t}^{(2)} + \phi_{d_t}^{(2)})\}\, \bar{n}\, dS \quad \ldots\ldots (3)$$

Equation (3) shows that the second order force consists of terms involving products of first order quantities or integrals of products of first order quantities.

Using the discrete formulation of equation (1) for the waves it follows that the second order force in irregular directional seas can be written as:

$$F^{(2)}(t) =$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \sum_{\ell=1}^{M} \zeta_{ik}\, \zeta_{j\ell}\, P_{ijk\ell} \cos\{(\omega_i - \omega_j)t +$$

$$+ (\varepsilon_{ik} - \varepsilon_{j\ell})\} +$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \sum_{\ell=1}^{M} \zeta_{ik}\, \zeta_{j\ell}\, Q_{ijk\ell} \sin\{(\omega_i - \omega_j)t +$$

$$+ (\varepsilon_{ik} - \varepsilon_{j\ell})\} \quad \ldots\ldots (4)$$

in which $P_{ijk\ell}$ and $Q_{ijk\ell}$ are the in-phase and quadrature quadratic transfer functions derivable based on equation (3).

For instance, for the quadratic transfer function due to the first contribution for equation (3) we may write:

$$\bar{P}_{ijk\ell} = -\int_{WL} \tfrac{1}{4}\rho g\, \zeta^{*}_{r_{ik}}\, \zeta'_{r_{j\ell}} \cos(\varepsilon_{r_{ik}} - \varepsilon_{r_{j\ell}})\, \bar{n}\, dl \quad \ldots\ldots (5)$$

$$\bar{Q}_{ijk\ell} = \int_{WL} \tfrac{1}{4}\rho g\, \zeta^{*}_{r_{ik}}\, \zeta'_{r_{j\ell}} \sin(\varepsilon_{r_{ik}} - \varepsilon_{r_{j\ell}})\, \bar{n}\, dl \quad \ldots\ldots (6)$$

Each of the components of equation (3) contribute to the total in-phase and quadrature quadratic transfer functions. This will not be treated further here. It should be mentioned that evaluation of these transfer functions can be made through the application of 3-D diffraction computer programs, [10].

The contribution due to the non-linear potentials $\phi_w^{(2)}$ and $\phi_d^{(2)}$ are approximated using the method given in [10] making use of the non-linear second order potential for the undisturbed directional waves given in [11]. For irregular long-crested waves the validity of this approach was recently demonstrated by comparison of results obtained using this approximation and results based on a more exact solution [12] and [13].

We will assume hereafter that the quadratic transfer functions include contributions from all components of equation (3). It should be noted that through regrouping of terms such as given in equations (5) and (6) certain symmetry relationships will apply to $P_{ijk\ell}$ and $Q_{ijk\ell}$. These will be given in a later section.

Mean drift forces

The time average drift force is found from equation (4) for i=j:

$$\overline{F^{(2)}(t)} = \sum_{i=1}^{N} \sum_{k=1}^{M} \sum_{\ell=1}^{M} \zeta_{ik} \zeta_{i\ell} \{P_{iik\ell} \cos(\varepsilon_{ik}-\varepsilon_{i\ell}) + Q_{iik\ell} \sin(\varepsilon_{ik}-\varepsilon_{i\ell})\} \quad \ldots\ldots\ (7)$$

It can be shown that the ensemble average of the drift force is:

$$E[F^{(2)}] = \overline{F} = \sum_{i=1}^{N} \sum_{k=1}^{M} \zeta_{ik}^{2} P_{iikk} \quad \ldots\ (8)$$

which, in continuous form becomes:

$$\overline{F} = 2 \int_0^{\infty} \int_0^{2\pi} S_\zeta(\omega,\alpha)\, P(\omega,\omega,\alpha,\alpha)\, d\alpha\, d\omega \quad \ldots\ (9)$$

in which $P(\omega,\omega,\alpha,\alpha)$ is the quadratic transfer function of the mean drift force in regular waves of frequency ω from wave direction α.

Drift force spectrum

It is of interest to obtain information on the low frequency drift forces in irregular waves in the frequency domain. To this end we use equation (4) as starting point to obtain the spectral density of the drift forces. The following result is obtained:

$$S_F(\mu) = 8 \int_0^{\infty} \int_0^{2\pi} \int_0^{2\pi} S_\zeta(\omega+\mu,\alpha)\, S_\zeta(\omega,\beta) \cdot |T(\omega+\mu,\omega,\alpha,\beta)|^2\, d\alpha\, d\beta\, d\omega \quad \ldots\ (10)$$

in which:

$S_\zeta(\omega+\mu,\alpha)$ = wave spectral density associated with frequency $\omega+\mu$ and direction α.

$$T(\omega+\mu,\omega,\alpha,\beta) = P(\omega+\mu,\omega,\alpha,\beta) + iQ(\omega+\mu,\omega,\alpha,\beta) \quad \ldots\ldots\ldots\ldots\ (11)$$

For the quadratic transfer functions the following symmetry relationships apply:

$$P(\omega+\mu,\omega,\alpha,\beta) = P(\omega,\omega+\mu,\beta,\alpha) \quad \ldots\ldots\ (12)$$

$$Q(\omega+\mu,\omega,\alpha,\beta) = -Q(\omega,\omega+\mu,\beta,\alpha) \quad \ldots\ldots\ (13)$$

which is the same as:

$$T(\omega+\mu,\omega,\alpha,\beta) = T^*(\omega,\omega+\mu,\beta,\alpha) \quad \ldots\ldots\ (14)$$

in which * denotes the complex conjugate. Similar symmetry equations apply to the discrete quadratic transfer functions $P_{ijk\ell}$, $Q_{ijk\ell}$ and the amplitude $T_{ijk\ell}$.

Drift forces in irregular cross seas

We will apply equation (10) to a simple case to show the effect of wave directionality on the wave drift force. To this end we assume that the irregular directional sea consists of a superposition of two irregular long-crested uni-directional seas approaching from directions α_0 and α_1 respectively. This type of sea condition is known as a cross-sea condition. The relevance of the effect of such sea conditions on the behaviour of moored vessels has been demonstrated in [1].

The directional wave spectrum of irregular long-crested waves from the direction α_0 is defined as follows:

$$S_{\zeta_{\alpha_0}}(\omega,\alpha) = \delta(\alpha-\alpha_0)\, S_{\zeta_{\alpha_0}}(\omega) \quad \ldots\ldots\ (15)$$

in which the DIRAC delta function is defined as follows:

$$\delta(\alpha-\alpha_0) = 0 \quad \text{for} \quad \alpha \neq \alpha_0 \quad \ldots\ldots\ (16)$$

and:

$$\int_0^{2\pi} \delta(\alpha-\alpha_0)\, d\alpha = 1 \quad \ldots\ldots\ (17)$$

The directional wave spectrum of two long-crested irregular waves from directions α_0 and α_1 is:

$$S_\zeta(\omega,\alpha) = \delta(\alpha-\alpha_0)\, S_{\zeta_{\alpha_0}}(\omega) + \delta(\alpha-\alpha_1)\, S_{\zeta_{\alpha_1}}(\omega) \quad \ldots\ldots\ (18)$$

in which $S_{\zeta_{\alpha_0}}(\omega)$ and $S_{\zeta_{\alpha_1}}(\omega)$ are ordinary point spectra for long-crested waves.

The mean drift force in an irregular directional sea is as follows:

$$\overline{F} = 2 \int_0^{\infty} \int_0^{2\pi} S_{\zeta}(\omega,\alpha)\; P(\omega,\omega,\alpha,\alpha)\; d\alpha\; d\omega \qquad (19)$$

Substitution of equation (18) and taking into account equations (16) and (17) gives:

$$\begin{aligned}\overline{F} &= \overline{F}_{\alpha_0} + \overline{F}_{\alpha_1} \\ &= 2 \int_0^{\infty} S_{\zeta_{\alpha_0}}(\omega)\; P(\omega,\omega,\alpha_0,\alpha_0)\; d\omega + \\ &\quad + 2 \int_0^{\infty} S_{\zeta_{\alpha_1}}(\omega)\; P(\omega,\omega,\alpha_1,\alpha_1)\; d\omega\end{aligned} \qquad (20)$$

This shows that the mean drift force of two sea states is the sum of the mean forces from the individual wave trains.

The spectrum of the wave drift force in an irregular sea is given by equation (10):

Substitution of (18) in (10) gives:

$$\begin{aligned}S_F(\mu) &= \\ &= 8 \int_0^{\infty} \int_0^{2\pi} \int_0^{2\pi} \left[\delta(\alpha-\alpha_0)\; S_{\zeta_{\alpha_0}}(\omega+\mu) + \right. \\ &\quad \left. + \delta(\alpha-\alpha_1)\; S_{\zeta_{\alpha_1}}(\omega+\mu)\right] \cdot \\ &\quad \cdot \left[\delta(\beta-\alpha_0)\; S_{\zeta_{\alpha_0}}(\omega) + \delta(\beta-\alpha_1)\; S_{\zeta_{\alpha_1}}(\omega)\right] \cdot \\ &\quad \cdot \left|T(\omega+\mu,\omega,\alpha,\beta)\right|^2 d\alpha\; d\beta\; d\omega\end{aligned} \qquad (21)$$

which results in:

$$\begin{aligned}S_F(\mu) &= \\ &= 8 \int_0^{\infty} S_{\zeta_{\alpha_0}}(\omega+\mu)\; S_{\zeta_{\alpha_0}}(\omega)\; \left|T(\omega+\mu,\omega,\alpha_0,\alpha_0)\right|^2 d\omega + \\ &\quad + 8 \int_0^{\infty} S_{\zeta_{\alpha_0}}(\omega+\mu)\; S_{\zeta_{\alpha_1}}(\omega)\; \left|T(\omega+\mu,\omega,\alpha_0,\alpha_1)\right|^2 d\omega + \\ &\quad + 8 \int_0^{\infty} S_{\zeta_{\alpha_1}}(\omega+\mu)\; S_{\zeta_{\alpha_0}}(\omega)\; \left|T(\omega+\mu,\omega,\alpha_1,\alpha_0)\right|^2 d\omega + \\ &\quad + 8 \int_0^{\infty} S_{\zeta_{\alpha_1}}(\omega+\mu)\; S_{\zeta_{\alpha_1}}(\omega)\; \left|T(\omega+\mu,\omega,\alpha_1,\alpha_1)\right|^2 d\omega\end{aligned} \qquad (22)$$

The first and last terms of equation (22) correspond respectively with the drift force spectrum from each of the two sea conditions separately. This shows that the low frequency second order excitation in a sea state consisting of two long-crested irregular seas is larger than the sum of the excitation from each sea independently. The second and third terms of equation (22) show the excitation arising from interaction of both long-crested seas. The nature of these terms also shows under which conditions such interaction terms may be neglected.

Consider, for instance, that waves from direction α_0 are relatively short, wind driven seas while those from direction α_1 are relatively long-period swells. Both spectra are shown in Figure 1.

If the spectra have little or no overlap on the frequency axis, then the products of the spectral densities in the second and third parts of equation (22) are small, even for relatively large μ values. This can have important consequences for simulation computations since such combinations of wind driven and swell seas frequently occur. The impact of this effect will be apparent when it is realized that the major part of the low frequency response of moored vessels is at or near the natural frequency. These frequencies are generally very low. The consequence of this effect is that the system only reacts to very low frequency components in the excitation. If the above-mentioned frequency separation is present then equation (22) shows that the excitation may be assumed to be simply the sum of the excitation arising from each sea independently.

MODEL TESTS

In the previous sections a general outline of the theory regarding mean and low frequency second order wave drift forces has been given. It was found that, given a description of the irregular directional wave field in terms of either a set of spectra of long-crested irregular waves from discrete directions or a continuous directional wave spectrum, the second order drift forces can be determined if the frequency and wave direction dependent quadratic transfer functions $P_{ijk\ell}$ and $Q_{ijk\ell}$ are known. These transfer functions can be obtained from 3-D diffraction calculations. In this section results of such calculations will be compared with results of model tests for some simple, fundamental cases.

The model tests and calculations have been carried out for a 200,000 DWT fully loaded tanker moored in a water depth of 30.24 m. The main particulars of the vessel are given in Table 1. A body plan is shown in Figure 2.

The model tests were carried out at a scale of 1:82.5 in the Wave and Current Laboratory of MARIN. This facility measures 60 m by 40 m with a variable water depth of up to 1 m. The basin is equipped with snake-type wave generators on two sides of the basin. The wave generators on each side can be driven independently to produce regular and irregular cross-sea conditions.

Model tests were carried out in regular cross-wave conditions and irregular uni-directional and cross-sea conditions. In the following a brief description will be given with respect to the choice of the test conditions, the results of tests and the comparison with results of computations.

Tests in regular cross waves

According to equation (7) the mean force in irregular directional seas is given by:

$$\overline{F^{(2)}(t)} = \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{\ell=1}^{M} \zeta_{ik}\, \zeta_{i\ell} \{P_{ik\ell} \cos(\varepsilon_{ik}-\varepsilon_{i\ell}) + Q_{iik\ell} \sin(\varepsilon_{ik}-\varepsilon_{i\ell})\} \quad \ldots\ldots (23)$$

Each of the terms in this equation reflects a contribution due to interaction of a regular wave with frequency ω_i from direction k with a regular wave with frequency ω_i from direction ℓ.

Consider the case that we have one regular wave with frequency ω_1 from direction 1 and one regular wave with the same frequency from direction 2.

The mean drift force would then be:

$$\overline{F^{(2)}(t)} = \zeta_{11}^{2} P_{1111} + \zeta_{12}^{2} P_{1122} + \zeta_{11}\zeta_{12}\{P_{1112}\cos(\varepsilon_{11}-\varepsilon_{12}) + Q_{1112}\sin(\varepsilon_{11}-\varepsilon_{12})\} + \zeta_{12}\zeta_{11}\{P_{1121}\cos(\varepsilon_{12}-\varepsilon_{11}) + Q_{1121}\sin(\varepsilon_{12}-\varepsilon_{11})\} \quad \ldots\ldots\ldots\ldots (24)$$

Taking into account the symmetry relationship of equations (12) through (14) this results in:

$$\overline{F^{(2)}(t)} = \zeta_{11}^{2} P_{1111} + \zeta_{12}^{2} P_{1122} + 2\,\zeta_{11}\zeta_{12}\{P_{1112}\cos(\varepsilon_{11}-\varepsilon_{12}) + Q_{1112}\sin(\varepsilon_{11}-\varepsilon_{12})\} \quad \ldots\ldots (25)$$

The first two components of equation (25) are the mean forces due to each regular wave independently. The third contribution is due to interaction effects of the two regular waves in the mean drift force. It is seen that this component is, besides being a function of the quadratic transfer functions P_{1112} and Q_{1112}, a function of the phase angles ε_{11} and ε_{12} of the two regular waves.

The quadratic transfer functions P_{1111} and P_{1122} can be found from tests or computations for regular waves from one direction. For this particular tanker, these uni-directional mean drift force transfer functions have been given in [10].

The purpose of the tests in regular cross waves is specifically to identify the quadratic transfer functions P_{1112} and Q_{1112} which represent the interaction effects due to the simultaneous presence of two regular waves.

Equation (25) shows that, in order to identify these effects, tests in regular cross waves should be carried out whereby in each test two regular waves with the same frequency and amplitude are generated. The phase angle difference of the components $(\varepsilon_{11} - \varepsilon_{12})$ must be different for each test however.

From equation (25) it is seen that the measured mean drift forces on the vessel will then contain a mean part and a part which is a harmonic function of the phase angle difference $(\varepsilon_{11} - \varepsilon_{12})$.

We have chosen to carry out tests in regular cross waves with the component waves at right angles to each other and at 45 degrees to the port and starboard bow of the vessel respectively. The set-up is shown in Figure 3.

Regular waves were adjusted separately from both sides of the basin. The frequencies were the same and the wave amplitudes (first harmonic component of the wave elevation record) measured at the mean position of the centre of gravity of the vessel were almost the same. A review of the regular waves adjusted from each side of the basin is given in Table 2.

During the tests in regular cross waves, the drive shafts of the wave generators on both sides of the basin were locked to each other mechanically, thus allowing good adjustment of the phase angle difference $(\varepsilon_{11} - \varepsilon_{12})$ from one test to the next and at the same time assuring that both wave generators were running at the same frequency.

The model of the tanker was moored in a soft spring mooring system. The longitudinal and transverse forces exerted by the mooring system on the vessel were measured by means of force transducers situated at the fore and aft connection points of the mooring system. The set-up is shown in Figure 3.

Series of tests were carried out in regular cross waves of a given frequency and amplitude whereby the phase angle difference $(\varepsilon_{11} - \varepsilon_{12})$ between the component regular waves was changed by 60 degrees for each consecutive test. In each test the mooring forces were measured. The time average of the mooring forces yielded the mean longitudinal and transverse drift forces and mean yaw moment as a function of the phase difference.

The results of measurements are shown in Figures 4 through 7 in terms of the mean forces and yaw moment to a base of the phase angle between the wave generators. Also included is the amplitude of the first harmonic of the resultant undisturbed wave elevation measured at the centre of gravity of the vessel.

The results shown in these figures confirm that the mean forces contain a constant part and a part which varies periodically with the phase angle difference of the wave generators.

Due to the symmetry of the test set-up, i.e. regular waves of the same frequency and approximately the same amplitude approaching from 135° and 225°, the mean values of the transverse force F_y and the yaw moment M_ψ should be equal to zero leaving only the periodic component of the mean force and moment. Due to slight differences in wave amplitudes and possibly due to slight error in model alignment this is not quite the case. The periodic parts of these mean forces are, however, dominant.

The mean longitudinal force F_x contains a mean component corresponding to the sum of the mean forces due to the component regular waves and the additional component which is periodic with the phase difference between the wave generators.

When comparing the mean forces and yaw moment as a function of the wave generator phase angle difference, it is seen that except for tests at wave frequency 0.267 rad/s, the mean transverse force is in phase with the mean yaw moment. At 0.267 rad/s, a phase difference of about 180° is seen between these quantities. In general, the mean longitudinal force F_x is about 90° out-of-phase with F_y and M_ψ. This is in agreement with equation (25) when the symmetry of the test set-up and wave conditions are taken into account.

In a fully symmetrical case in regular cross waves, i.e. when the phase angle difference $(\varepsilon_{11} - \varepsilon_{12})$ is equal to zero, the wave elevation pattern consists of a square pattern of three-dimensional peaks and troughs travelling in a direction parallel to the longitudinal axis of the vessel with the maximum peaks and troughs moving in a line exactly on the centre line of the vessel.

In this case the mean transverse force F_y and yaw moment M_ψ are equal to zero. Assuming that the component wave amplitudes are equal, this implies that for the transverse force and yaw moment, the value of P_{1112} in equation (25) is equal to zero. Based on similar reasoning it can be shown that for the mean longitudinal force F_x, the value of Q_{1112} is equal to zero.

Consequently, according to equation (25) the varying part of F_x will be 90° out-of-phase with respect to the varying parts of F_y and M_ψ. The periodic part of the variations in the mean value of the forces to a base of the wave generator phase difference are governed by the third component and fourth component for F_x and for F_y, M_ψ respectively in equation (25).

In order to compare the results of model tests in regular cross waves with the results of computations, the following analysis was applied to the measured data:

From the results given in Figures 4 through 7, the amplitude of the first harmonic of the mean forces and moment and their respective phase angles relative to the origin were obtained by standard harmonic analysis. The amplitudes thus obtained were divided by $2\,\zeta_1\,\zeta_2$, ζ_1 and ζ_2 being the amplitudes of the component irregular waves given in Table 2 for the various frequencies.

For the surge force F_x, the computed values of the quadratic transfer functions $P_{ijk\ell}$ and $Q_{ijk\ell}$ are given in Table 3 and in Table 4 for the case that the direction index k corresponds to the waves from 135° and the index ℓ corresponds to waves from 225°.

All combinations have been given. In this paper use is only made of results for the case i=j. The cases when i≠j correspond to regular cross waves with non-equal frequencies.

For comparison purposes, the quadratic transfer functions for the surge force are also given for the case that both waves come from the same direction, i.e. for the case that k is equal to ℓ. These results are given in Tables 5 and 6.

The transfer functions were then compared with the computed values of the amplitude T_{1112} for F_x, F_y and M_ψ. The results of the comparisons are shown in Figures 8 through 10.

The phase angles, $\varepsilon_{F\zeta}$, of the forces and moment relative to the point at which the amplitude of the undisturbed regular cross waves is at a maximum are also compared with the theoretical values in these figures.

In general, the amplitudes of the quadratic transfer functions T_{1112} are reasonably well predicted by the computations as are the phase angles relative to the undisturbed regular cross waves. On the whole, the agreement is considered to be satisfactory.

Tests in irregular cross seas

In the previous section model tests in regular cross waves were described. It was seen that, dependent on the relative phase angles of the component regular waves, mean drift forces could be higher or lower than the total mean drift force due to the sum of the drift forces from each wave component independently.

In irregular seas all relative phase angles between wave components from different directions are equally probable. The result of this is that the total mean drift forces in irregular seas is simply the sum of contributions from all wave components. This is expressed in equation (9) for the general case of directionally spread seas and in equation (20) for the case of irregular cross seas consisting of a superposition of two long-crested irregular wave trains.

On the other hand, if we consider that in an irregular directional sea the relative phase angles between wave components from different directions are continually changing quantities, the results shown in Figures 4 through 7 also reflect the additional low frequency force components due to interaction effects predicted by, for instance, equation (22).

Tests in irregular cross seas were carried out to check whether the theoretical prediction is borne out by experimental findings with respect to the mean forces and the low frequency horizontal motions.

The model set-up was the same as used for tests in regular cross waves (see Figure 3), i.e. the model was moored in a soft linear spring system and irregular long-crested seas were approaching the vessel from 135° or 225° or both.

In order to check the validity of the superposition principle with respect to the mean drift forces, tests were carried out in three phases, i.e. (1) one test to measure mean drift forces in irregular seas from 135°, (2) one test in waves from 225° and (3) one test in irregular cross seas consisting of a superposition of both irregular wave trains.

Before treating the test results, it is of interest to check the superposition principle with respect to the undisturbed wave elevation at the centre of gravity of the vessel.

The spectra of the long-crested, uni-directional wave trains generated from either of the two basin sides are shown in Figure 11. In Figure 12 the calculated sum of the individual spectra are compared with the spectra obtained from the wave elevation records measured in irregular cross seas consisting of two wave trains generated from both sides using independent random wave generator control signals. The agreement shows that the superposition principle holds very well for the undisturbed wave trains.

The test duration for tests in irregular seas corresponded to 90 minutes full scale.

The mean wave drift forces on the tanker measured in the various irregular long-crested, uni-directional seas along with calculated data are given in Table 7.

In general the measured and calculated mean surge and sway forces $\bar{F}_x$ and $\bar{F}_y$ are in good agreement. For spectrum 1, the calculated mean surge force is some 30% below the measured value. The reason for this difference is not clear at this time. In general differences in the forces are less than 10%.

The difference between the calculated and measured mean yaw moments appear to be larger. It should be remembered, however, that the mean yaw moment is derived from the difference in the mean transverse forces measured fore and aft. The moment arm amounted to about 320 m for the full scale vessel. The mean yaw moments are therefore rather small in terms of transverse forces applied at the fore and aft ends.

In Table 8 the measured mean forces in irregular cross seas are presented together with calculated data obtained by adding the mean values measured in irregular uni-directional seas, and calculated data obtained by adding calculated data for irregular uni-directional seas.

The calculated mean forces for irregular uni-directional seas were obtained based on equation (19), using the measured uni-directionl wave spectra presented in Figure 11. The transfer function $P(\omega,\omega,\alpha,\alpha)$ was given again, based on 3-D diffraction calculations and on equation (3). See, for example, Table 5 for surge force F_x.

The results shown in Table 8, in general, show that the superposition principle holds very well for the mean wave drift forces in irregular cross seas.

In general, the calculated data based on 3-D diffraction theory also are in good agreement except for those tests involving spectrum 1. In those cases, diffraction calculations underestimate the mean surge force $\bar{F}_x$ to some extent. This is in agreement with the results given in Table 7 for the uni-directional seas.

Finally, we will look at the effect of irregular cross-seas on the low frequency horizontal mooring forces. In Table 9 the standard deviation of the surge, sway and yaw forces of the tanker are given for the four mentioned wave spectra.

For the test in irregular cross seas the results are given in Table 10. In this table results are also given of the forces obtained based on the tests in irregular long-crested seas under the assumption that no interaction effects are present. In this case the cross sea results are obtained by taking the square root of the sum of the squares of the standard deviations from the two relevant tests in long-crested seas.

Comparison of the results reveals that the interaction effects in the sway and yaw mode are small. In the surge mode interaction effects are significant for the case of wave spectrum 2 combined with wave spectrum 3.

CONCLUSIONS

In this paper, some aspects of the general theory regarding mean and low frequency second order drift forces in irregular directional seas were discussed. It was shown that, given the bi-directional and bi-frequency dependent quadratic transfer function for the wave drift forces, the mean forces and spectral density of the slowly varying part of the forces could be computed using the directional wave spectrum as description for the sea condition.

This frequency domain representation, although very useful, is not entirely complete however, since strictly speaking information should also be given on the distribution of low frequency forces. The general problem of the distribution of the second order drift forces in irregular long-crested seas has been treated among others in ref. [14] and [15], to which the reader is referred.

Time domain representations of drift forces in directional seas through direct summation of Fourier components or through the application of the second order term of a Functional Polynomial have been discussed. In both cases it was assumed that the wave field consists of a sum of long-crested irregular wave trains from a number of discrete directions. The Functional Polynomial allows a deterministic comparison to be made between measured and computed drift force records in irregular cross seas. This comparison has not been treated in this paper however. This aspect will be addressed in more detail in future research.

The results of model tests with a 200,000 DWT tanker in regular cross waves and irregular cross seas have confirmed theoretical predictions with respect to the applicability of the superposition principle for the mean wave drift forces in irregular seas while it has also been shown that additional drift forces occur through the simultaneous presence of two regular wave fields. The latter effect can be used to clarify the theoretically predicted increase in low frequency excitation due to the interaction of waves approaching a vessel from different directions.

Results have been presented of the comparisons between drift forces obtained from experiments in regular cross waves and in irregular cross seas and obtained from 3-dimensional diffraction theory calculations. In general, the computed and measured data are in good agreement. The comparison has, for the present, been restricted to the mean drift forces and the low frequency mooring force components. In a future phase it is envisaged to carry out experiments and calculations to check the accuracy of predictions with respect to the slowly varying part of the drift forces also.

For such comparisons use will be made of a general method to produce time domain records of drift forces based on measured wave elevation records and the application of a Volterra series expension for the force, see for instance [16].

NOMENCLATURE

i, j	wave frequency indices
k, ℓ	wave direction indices
M	number of directional components
N	number of discrete frequency components
$\overline{n}$	outward pointing normal vector to the hull
S_o	mean wetted hull surface
$\overline{X}^{(1)}$	first order oscillatory linear motion vector
$\overline{\alpha}^{(1)}$	first order oscillatory angular motion vector of the body
ε_{ik}	random phase angle
$\varepsilon_{r_{ik}}$	phase angle of relative wave elevation
ζ_{ik}	wave amplitude
$\zeta_r^{(1)}$	first order relative wave elevation around the waterline WL
$\zeta'_{r_{ik}}$	first order transfer function for the relative wave elevation for wave frequency ω_i and wave direction ψ_k
$\phi^{(1)}$	first order velocity potential including effects of incoming waves, diffracted waves and waves generated by the body motions
$\phi_d^{(2)}$	second order "diffraction" potential
$\phi_w^{(2)}$	second order "incoming wave" potential
ω_i	wave frequency

REFERENCES

1. Grancini, G., Iovenitti, L.M. and Pastore, P.: "Moored tanker behaviour in crossed sea. Field experiences and model tests". Symposium on Description and Modelling of Directional Seas, Technical University of Denmark, Copenhagen, 1984.
2. Marol, P., Römeling, J.U. and Sand, S.E.: "Bi-articulated mooring tower tested in directional waves". Symposium on Description and Modelling of Directional Seas, Technical University of Denmark, Copenhagen, 1984.
3. Teigen, P.S.: "The response of a TLP in short-crested waves". Paper No. OTC 4642, Offshore Technology Conference, Houston, 1983.
4. Marthinsen, T.: "The effect of short-crested seas on second order forces and motions". International Workshop on Ship and Platform Motions, Berkeley, 1983.
5. Van Oortmerssen, G., Pinkster, J.A. and Van den Boom, H.J.J.: "Computer simulations as an aid for offshore operations". WEMT, Paris, 1984.
6. Wichers, J.E.W.: "Wave-current interaction effects on moored tankers in high seas". Paper No. OTC 5631, Offshore Technology Conference, Houston, 1983.
7. Molin, B. and Fauveau, V.: "Effect of wave-directionality on second-order loads induced by set-down". Applied Ocean Research, Vol. 6, No. 2, 1984.
8. Mitsuyasu, H.: "Directional spectra of ocean waves in generation areas". Conference on Directional Wave Spectra Applications, Berkeley, 1981.
9. Hasselman, K., Dunckel, M. and Ewing, J.A.: "Directional wave spectra observed during JONSWAP 1973". Journal of Physical Oceanography, 8, 1264-1280, 1980.
10. Pinkster, J.A.: "Low frequency second order wave exciting forces on floating structures". N.S.M.B. Publication No. 650, Wageningen, 1980.
11. Bowers, E.C.: "Long period oscillation of moored ships subject to long waves". R.I.N.A., 1975.
12. Benschop, A.: "The contribution of the second order potential to low frequency second order wave exciting forces on vessels". Department of Mathematics, Technical University of Delft, Delft, 1985.
13. Benschop, A., Hermans, A.J., Huijsmans, R.H.M., "Second order diffraction forces on a ship in irregular waves". Applied Ocean Research, Vol. 6, No. 2, 1987.
14. Vinje, T.: "On the statistical distribution of second order forces and motions". International Shipbuilding Progress, March, 1983.
15. Langley, R.S.: "The statistics of second order wave forces". Applied Ocean Research, Vol. 6, No. 4, 1984.
16. Pinkster. J.A.: "Drift forces in directional seas". Marintec '85. Shanghai 1985.

TABLE 1—MAIN PARTICULARS AND STABILITY DATA OF LOADED 200,000-DWT TANKER

Designation	Symbol	Unit	Magnitude
Length between perpendiculars	L_{PP}	m	310.00
Breadth	B	m	47.17
Depth	H	m	29.60
Draft fore	T_F	m	18.90
Draft mean	T_M	m	18.90
Draft aft	T_A	m	18.90
Displacement weight	Δ	tf	240,697
Block coefficient	C_B	-	0.850
Midship section ceofficient	C_M	-	0.995
Waterplane coefficient	C_W	-	0.868
Centre of buoyancy forward of section 10	$\overline{FB}$	m	6.61
Centre of gravity above keel	$\overline{KG}$	m	13.32
Metacentric height	$\overline{GM}$	m	5.78
Radius of gyration in air:			
- transverse	k_{xx}	m	14.77
- longitudinal	k_{yy}	m	77.47
- vertical	k_{zz}	m	79.30

TABLE 2—AMPLITUDE OF FIRST HARMONIC OF ADJUSTED REGULAR UNI-DIRECTIONAL WAVE COMPONENTS

Wave frequency in rad/s	Amplitude of first harmonic in m	
	Wave from east side (225°)	Wave from south side (135°)
0.267	1.90	1.94
0.443	1.88	1.90
0.713	1.99	2.02
0.887	1.82	1.87

TABLE 3—QUADRATIC TRANSFER FUNCTION P_{ijkl} FOR THE LONGITUDINAL FORCE IN CROSS SEAS

$\omega_j \rightarrow$ / $\omega_i \downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	0	-7	-11	-9	0	0	0	0	0
0.3	-7	1	-4	-8	-5	0	0	0	0
0.4	-11	-4	-5	-10	-6	5	0	0	0
0.5	-9	-8	-10	-11	-7	6	8	0	0
0.6	0	-5	-6	-7	-9	-2	8	5	0
0.7	0	0	5	6	-2	-10	-1	10	12
0.8	0	0	0	8	8	-1	-7	4	15
0.9	0	0	0	0	5	10	4	-6	-4
1.0	0	0	0	0	0	12	15	-4	-25

ψ_k = 135°

ψ_ℓ = 225°

$P_{ijk\ell}$ in tf/m^2

TABLE 4—QUADRATIC TRANSFER FUNCTION Q_{ijkl} FOR THE LONGITUDINAL FORCE IN CROSS SEAS

$\omega_j \rightarrow$ / $\omega_i \downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	0	-31	-38	-29	0	0	0	0	0
0.3	30	0	-17	-26	-21	0	0	0	0
0.4	38	17	0	-15	-21	-19	0	0	0
0.5	29	26	15	0	-12	-20	-3	0	0
0.6	0	21	21	12	0	-12	-6	10	0
0.7	0	0	19	20	12	0	-9	0	10
0.8	0	0	0	3	6	9	0	-7	-5
0.9	0	0	0	0	-10	0	7	0	-10
1.0	0	0	0	0	0	-10	5	10	0

ψ_k = 135°

ψ_ℓ = 225°

$Q_{ijk\ell}$ in tf/m^2

TABLE 5—QUADRATIC TRANSFER FUNCTION P_{ijkk} FOR THE LONGITUDINAL FORCE IN LONG-CRESTED SEAS

$\omega_j \rightarrow$ / $\omega_i \downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	0	0	-23	-51	0	0	0	0	0
0.3	0	0	-4	-21	-26	0	0	0	0
0.4	-23	-4	-10	-14	-17	-7	0	0	0
0.5	-51	-21	-14	-20	-12	-6	2	0	0
0.6	0	-26	-17	-12	-13	-6	-4	7	0
0.7	0	0	-7	-6	-6	-12	-8	0	20
0.8	0	0	0	2	-4	-8	-10	-3	12
0.9	0	0	0	0	7	0	-3	-10	-11
1.0	0	0	0	0	0	20	12	-11	-31

ψ_k = 135°

ψ_ℓ = 135°

P_{ijkk} in tf/m^2

TABLE 6—QUADRATIC TRANSFER FUNCTION Q_{ijkk} FOR THE LONGITUDINAL FORCE IN LONG-CRESTED SEAS

$\omega_j \rightarrow$ / $\omega_i \downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	0	-146	-166	-116	0	0	0	0	0
0.3	146	0	-81	-90	-45	0	0	0	0
0.4	166	81	0	-50	-50	-17	0	0	0
0.5	116	90	50	0	-35	-37	4	0	0
0.6	0	45	50	35	0	-32	-19	23	0
0.7	0	0	17	37	32	0	-29	-4	28
0.8	0	0	0	-4	19	29	0	-27	1
0.9	0	0	0	0	-23	4	27	0	-30
1.0	0	0	0	0	0	-28	-1	30	0

ψ_k = 135°

ψ_ℓ = 135°

Q_{ijkk} in tf/m^2

TABLE 7—MEASURED AND CALCULATED MEAN DRIFT FORCES AND MOMENT IN IRREGULAR UNI-DIRECTIONAL SEAS

Mean forces and moment	Unit	Spectrum 1 (225°)		Spectrum 2 (225°)		Spectrum 3 (135°)		Spectrum 4 (135°)	
		Measured	Calculated	Measured	Calculated	Measured	Calculated	Measured	Calculated
$\overline{F}_x$	tf	-75.7	-53.0	-16.9	-16.6	-16.9	-18.1	-4.9	-3.3
$\overline{F}_y$	tf	-286.5	-293.2	-98.2	-107.5	93.5	112.6	15.4	17.9
$\overline{M}_\psi$	tfm	-4685.0	-3489.0	-582.0	-471.0	1608.0	600.0	1014.0	255.0

TABLE 8—MEASURED AND CALCULATED MEAN DRIFT FORCES AND MOMENT IN IRREGULAR CROSS SEAS

Mean forces and moment	Unit	Spectrum 1 (225°) + + Spectrum 3 (135°)			Spectrum 1 (225°) + + Spectrum 4 (135°)		
		Measured	Calculated (1)	Calculated (2)	Measured	Calculated (1)	Calculated (2)
$\bar{F}_x$	tf	-98.6	-71.1	-92.6	-84.4	-56.3	-80.6
$\bar{F}_y$	tf	-195.8	-180.6	-193.0	-277.6	-275.3	-271.1
$\bar{M}_\psi$	tfm	-3242.0	-2889.0	-3077.0	-4745.0	-3234.0	-3671.0
		Spectrum 2 (225°) + + Spectrum 3 (135°)			Spectrum 2 (225°) + + Spectrum 4 (135°)		
		Measured	Calculated (1)	Calculated (2)	Measured	Calculated (1)	Calculated (2)
$\bar{F}_x$	tf	-32.8	-34.7	-33.8	-18.4	-19.9	-21.8
$\bar{F}_y$	tf	0.6	5.1	-4.7	-88.1	-89.6	-82.8
$\bar{M}_\psi$	tfm	708.0	129.0	1026.0	-294.0	-216.0	432.0

Calculated (1): Sum of calculated values given in Table 7

Calculated (2): Sum of measured values given in Table 7

TABLE 9—STANDARD DEVIATION OF MOORING FORCES MEASURED IN IRREGULAR LONG-CRESTED SEAS

Wave spectrum No.	Wave direction in degrees	Standard deviation of mooring force		
		σ_{F_x} in tf	σ_{F_y} in tf	σ_{M_ψ} in tfm
1	225	275.9	357.0	26643.7
2	225	30.8	187.6	8260.2
3	135	38.1	107.8	8635.1
4	135	14.0	65.6	6198.4

TABLE 10—COMPARISON OF THE STANDARD DEVIATION OF MOORING FORCES MEASURED IN IRREGULAR CROSS-SEAS AND COMPUTED FROM RESULTS OF TESTS IN IRREGULAR LONG-CRESTED SEAS

Sea condition	Standard deviation of mooring force					
	σ_{F_x} in tf		σ_{F_y} in tf		σ_{M_ψ} in tfm	
	Measured	Computed	Measured	Computed	Measured	Computed
Spectrum 1 + 3	311.3	278.5	340.0	372.9	27674.8	28009.3
Spectrum 1 + 4	266.7	276.2	364.8	363.0	26873.6	27355.2
Spectrum 2 + 3	88.2	49.0	219.2	216.4	12919.8	11952.6
Spectrum 2 + 4	36.6	33.8	198.5	198.7	9549.2	10327.2

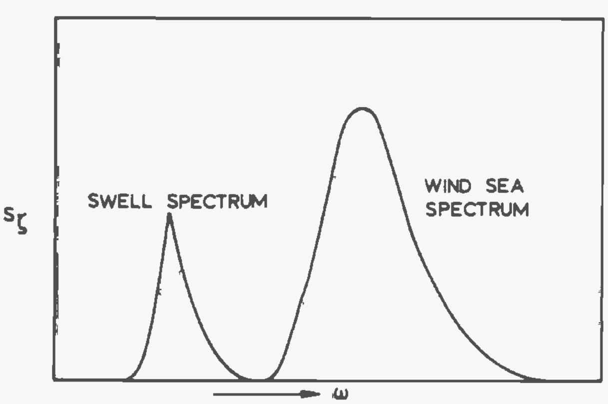

Fig. 1—Schematic representation of wind sea and swell spectrum.

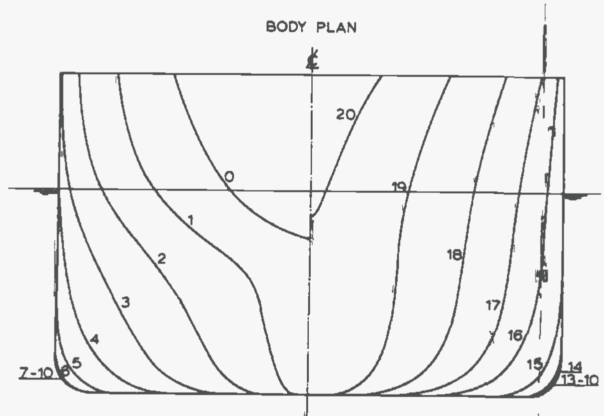

Fig. 2—Body plan of 200,000-DWT tanker.

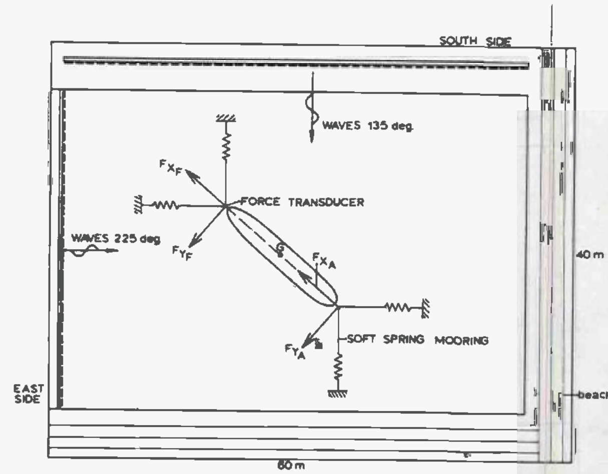

Fig. 3—Schematic representation of setup of tanker model in wave and current laboratory.

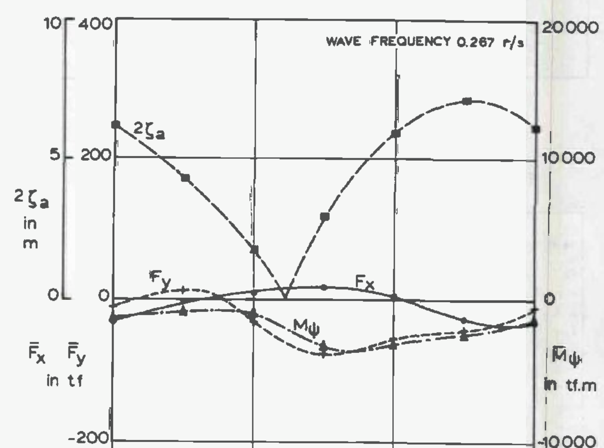

Fig. 4—Measured mean drift forces and yaw moment and undisturbed wave height in regular cross waves—wave frequency 0.267 rad/s.

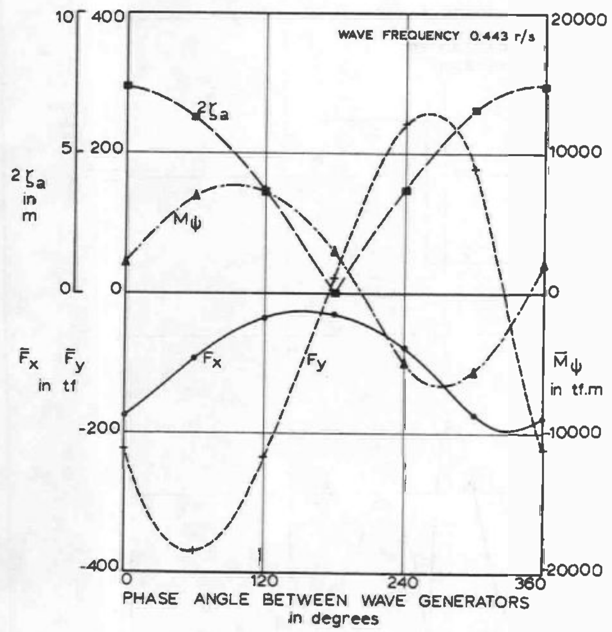

Fig. 5—Measured mean drift forces and yaw moment and undisturbed wave height in regular cross waves—wave frequency 0.443 rad/s.

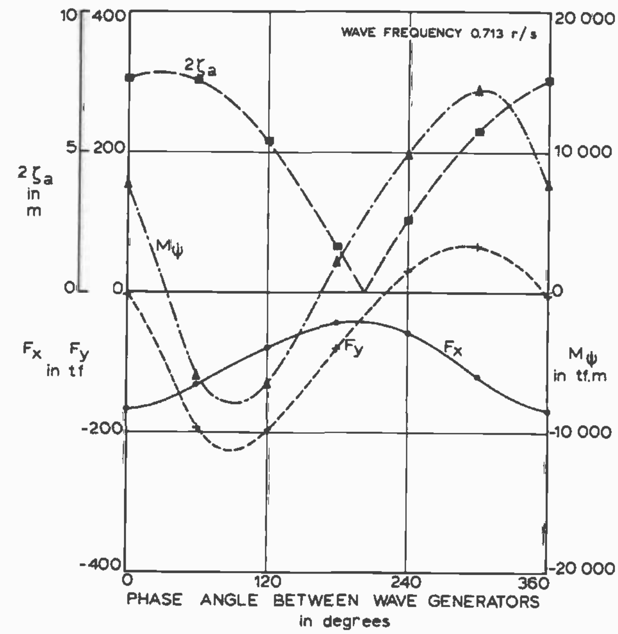

Fig. 6—Measured mean drift forces and yaw moment and undisturbed wave height in regular cross waves—wave frequency 0.713 rad/s.

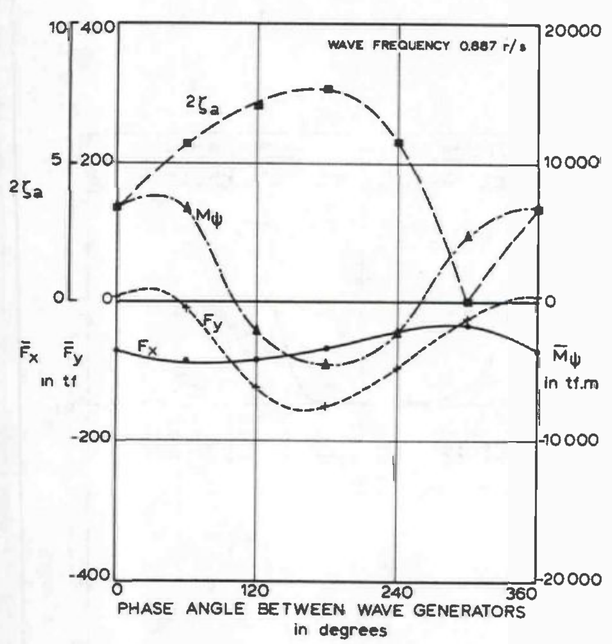

Fig. 7—Measured mean drift forces and yaw moment and undisturbed wave height in regular cross waves—wave frequency 0.887 rad/s.

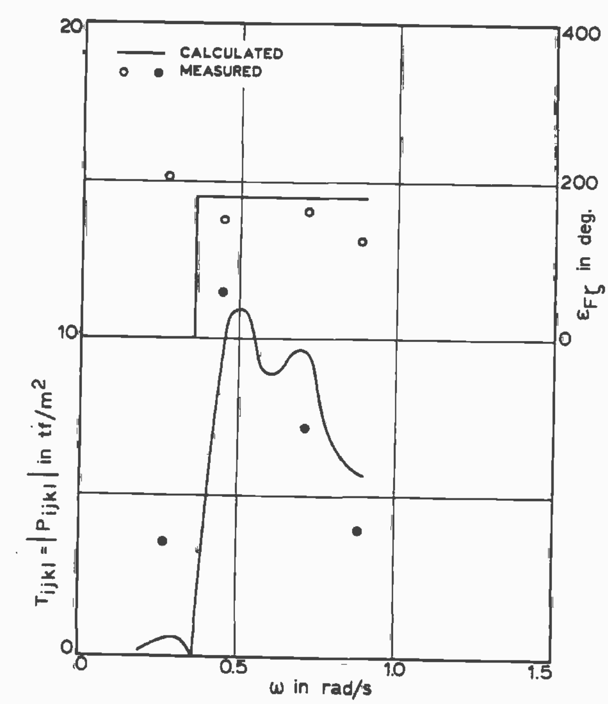

Fig. 8—Amplitude and phase angle of mean longitudinal drift force due to interaction effects in regular cross waves.

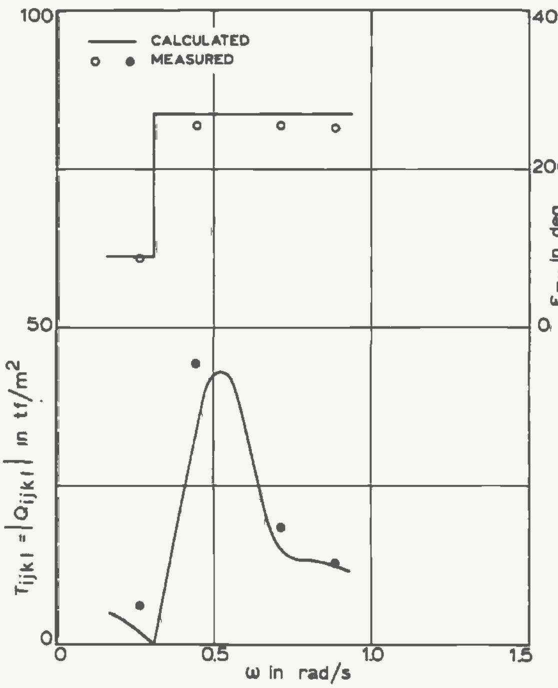

Fig. 9—Amplitude and phase angle of mean transverse drift force due to interaction effects in regular cross waves.

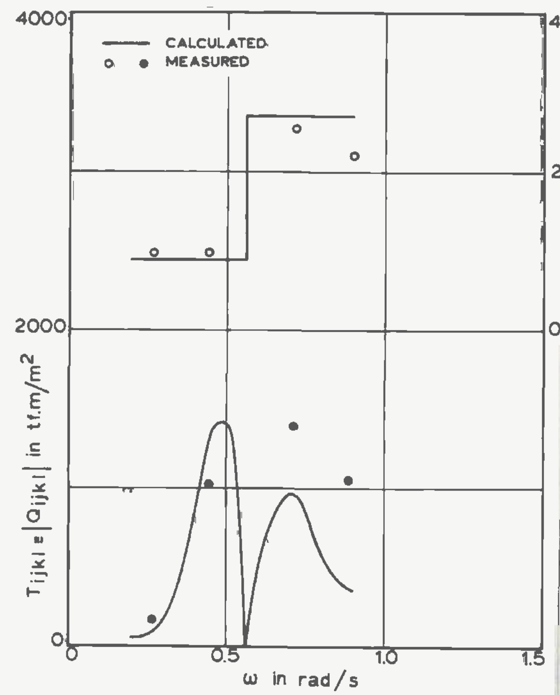

Fig. 10—Amplitude phase angle of mean yaw drift moment due to interaction effects in regular cross waves.

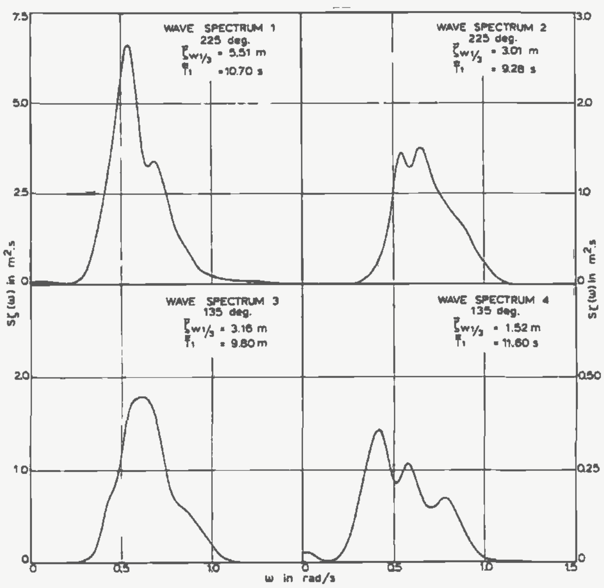

Fig. 11—Spectra of adjusted uni-directional irregular seas.

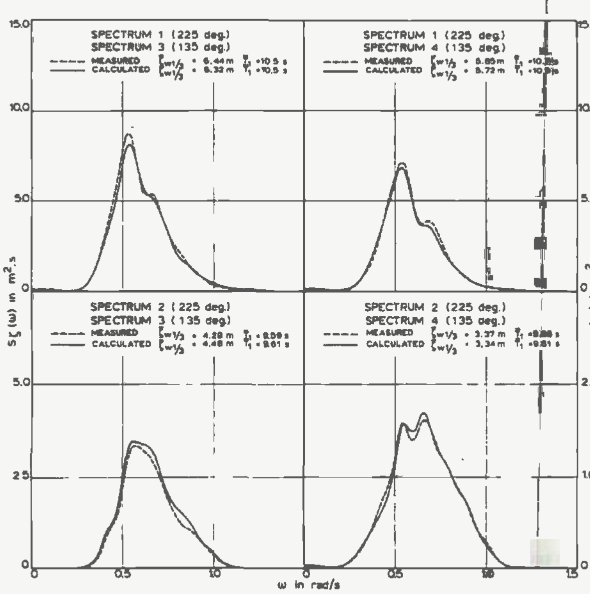

Fig. 12—Spectra of irregular cross seas: comparison of measured results with results calculated from superposition of uni-directional irregular seas.