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Target Selection for Tracking in Multifunction Radar Networks: Nash and Correlated Equilibria

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We consider a target selection problem for multitarget tracking in a multifunction radar network from a game-theoretic perspective. The problem is formulated as a noncooperative game. The radars are considered to be players in this game with utilities modeled using a proper tracking accuracy criterion and their strategies are the observed targets whose number is known. Initially, for the problem of coordination, the Nash equilibria are characterized and, in order to find equilibria points, a distributed algorithm based on the best response dynamics is proposed. Afterward, the analysis is extended to the case of partial target observability and radar connectivity and heterogeneous interests among radars. The solution concept of correlated equilibria is employed and a distributed algorithm based on the regret matching is proposed. The proposed algorithms are shown to perform well compared to the centralized approach of significantly higher complexity.

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I. INTRODUCTION

Radar networks that employ multiple distributed stations have attracted a lot of attention due to the improvements in tracking and detection performance they may offer over conventional standalone radars. Furthermore, recent advances in sensor technologies enabled a large number of controllable degrees of freedom in modern radars. One such system is the multifunction radar (MFR), and it typically employs phased array antennas that allow the radar beam to be controlled almost instantaneously [1]–[3]. Thus, the MFR is much more flexible than conventional dedicated radars by being capable of performing multiple functions simultaneously—Volume surveillance, fire control, and multiple target tracking, to name a few. In this paper, we focus on the latter function [4]–[8]; specifically, each MFR performs the track filtering of several targets.

The aforementioned flexibility introduces a need to effectively manage available radar resources to achieve specified objectives while conforming to operational and technical constraints [9], [10]. Even for a standalone MFR, the radar resource management plays a crucial role. Most of the existing approaches to MFR resource management roughly fit into the following two categories [11]–[13]. The first category consists of the rule-based techniques [14]–[16], which control the resource allocation parameters indirectly, under low computational burden. The main drawback of these techniques is that they are hard to say what performance can be achieved since it highly depends on the application scenario and on the sensors being deployed. The other category is related to the methods that formulate the problem as an optimization one; and thus, they may achieve the optimal performance, see [1], [8], [17]–[19], and the references therein. In the network setting, which is the focus of this paper, the first category of approaches is difficult to be extended, whereas the second one may involve excessive complexity due to the network dimension [20]–[21]. Thus, to reduce such complexity, one may aim to find, in either centralized or distributed way, a close-optimal solution to the radar management problem that is considered, see e.g., [21]–[23]. In this paper, we propose a distributed approach based on game theory (GT), so as to model target selection for multitarget track filtering in an MFR network.

GT is the mathematical study of conflict and cooperation between intelligent rational decision-makers [24]. Apart from economics and political sciences, over the last decade, GT is being applied to control, signal processing and wireless communications, mainly due to the issues dealing with networking [25]–[32]. More recently, GT has been applied to solve certain radar problems, mostly related to the multiple-input multiple-output radar networks. For instance, the problem of waveform design has been investigated [33]–[36]; in [33], by formulating a two-player zero-sum (TPZS) game between the radar design engineer and an opponent, in [35], by a potential game in which the radars choose among the prefixed transmit codes, and a proof of the uniqueness of the Nash equilibrium

(NE) of a potential game waveform design problem was presented in [36]. Next, the interaction between a jammer and a radar has also been modeled as a TPZS game [37], [38]. Furthermore, the problem of transmission power control was addressed by using noncooperative GT in [39] and by employing a coalitional game-theoretic solution concept, called the Shapley value, in [42]. Although not dealing with radar management, a useful work in [43] related to the multitarget tracking application uses correlated equilibrium (CE) to solve the data association problem at a single radar, and by introducing an exponential forgetting factor, it has been extended in [44] to deal with a varying number of targets. Finally, the works in [11], [45], and [46] utilize a market mechanism, called the continuous double auction, in order to choose the global optimum parameters for each individual task given the global (finite) resource constraint. The method provided a superior performance over its competing heuristic-based algorithms; however, its main drawback is in the implementation complexity [21].

In this paper, we apply game theory to multitarget track filtering in an MFR network and extend the initial results from [47]. The main contributions of this article are the following ones.

- A new formulation of the track selection problem for a multitarget tracking scenario in a resource-limited MFR network using the noncooperative games is proposed.
- The track selection problem is analyzed using the NE of the underlying coordination game for the setting with full target observability and radar connectivity as well as the homogeneous interests (target priorities) of radars. Also, to solve the problem in a distributed manner, a low-complexity algorithm based on the best response dynamics (BRD) is proposed.
- The track selection problem is extended to the case of partial target observability and radar connectivity and heterogeneous interests among radars. Due to the particularities of this case, the solution concept of correlated equilibria is employed and a distributed algorithm based on the regret matching (RM) is proposed.

The structure of this paper is as follows. Section II provides some background on game theory and the solution concepts employed in this paper. The problem formulation is given in Section III. Next, Sections IV and V deal with the analyses of the scenarios where the observability and connectivity conditions as well as radar interests are being equal and heterogeneous, respectively. Specifically, in Section IV, the former scenario is modeled as a coordination game, its NE are characterized in terms of their existence conditions and efficiency, and a distributed algorithm based on the BRD is proposed. On the other hand, Section V provides a distributed algorithm that tracks the set of correlated equilibria points. In Section VI, the effectiveness of the proposed algorithms is demonstrated via computer simulations. Finally, Section VII summarizes this paper.

II. BRIEF PRELIMINARIES ON GAME THEORY

In this section, we provide notation and recall some formal definitions and solution concepts related to game theory that will be used throughout this paper. The focus is put on noncooperative game theory, the dominant branch of game theory, and specifically on so-called normal-form games [24].

DEFINITION 1 A finite, N -person normal-form game is a tuple $\Gamma = (\mathcal{N}, \mathcal{S}, u)$, where the following statements hold.

- $\mathcal{N}$ is a finite set of N players.
- $\mathcal{S} = \mathcal{S}_1 \times \dots \times \mathcal{S}_N$, where $\mathcal{S}_i$ is a finite set of actions (strategies) available to player i , $\forall i \in \mathcal{N}$. Each vector $s = (s_1, \dots, s_N) \in \mathcal{S}$ is called an action (strategy) profile.
- $u = (u_1, \dots, u_N)$ where $u_i : \mathcal{S} \rightarrow \mathbb{R}$ is a real-valued utility (or payoff) function for player i , $\forall i \in \mathcal{N}$.

To reason about multiplayer games, one can rely on solution concepts, i.e., principles according to which interesting outcomes of a game can be identified. Some fundamental concepts, which will be used throughout this paper, are described in the sequel. A basic and the most widely accepted one is the celebrated NE. Formally, in case where players make deterministic choices (pure strategies), the NE is defined as follows [24].

DEFINITION 2 A strategy profile $s = (s_1, \dots, s_N)$ is a pure-strategy NE if, for all players i and for all strategies $s'_i \neq s_i$, it holds that

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$$

where $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N)$ is defined as a strategy profile s without player i 's strategy.

Otherwise stated, an NE is a state of a noncooperative game where no player can unilaterally improve its utility by taking a different strategy, if the other players remain constant in their strategies.

Next, we define the concepts of Pareto domination and Pareto optimality.

DEFINITION 3 Strategy profile s *Pareto dominates* strategy profile s' if $\forall i \in \mathcal{N}$, $u_i(s) \geq u_i(s')$, and there exist some $j \in \mathcal{N}$ for which $u_j(s) > u_j(s')$. Also, strategy profile s is *Pareto optimal* if there does not exist another strategy profile $s' \in \mathcal{S}$ that Pareto dominates s .

To evaluate the (in)efficiency of NE, there is a notion called the price of anarchy (PoA), which is defined as the ratio of a centralized solution to the worst case equilibrium in terms of the utility sum that is in economics literature known as “social welfare.”

DEFINITION 4 The PoA is given as

$$\text{PoA} = \frac{\max_{s \in \mathcal{S}} \sum_{i \in \mathcal{N}} u_i(s)}{\min_{s \in \mathcal{S}^{\text{NE}}} \sum_{i \in \mathcal{N}} u_i(s)}$$

where $\mathcal{S}^{\text{NE}}$ is the set of NE of the game.

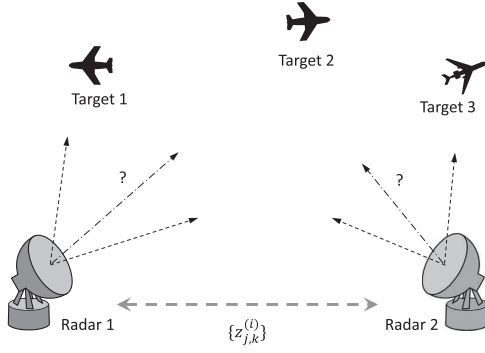


Fig. 1. Track selection problem in multitarget tracking.

Note that in case where the equilibria are fully efficient, the PoA is equal to 1.

Finally, we define the notion of CE, which is a generalization of NE [48], [49].

DEFINITION 5 A CE consists of a probability vector¹ π on $\mathcal{S}$ such that the following is satisfied, $\forall i \in \mathcal{N}$ and $\forall s_i, s'_i \in \mathcal{S}_i$:

$$\sum_{s_{-i} \in \mathcal{S}_{-i}} \pi(s_i, s_{-i}) [u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i})] \geq 0.$$

To interpret the above-mentioned inequality, let us first divide it by the marginal probability $\pi(s_i)$, which yields

$$\sum_{s_{-i} \in \mathcal{S}_{-i}} \pi(s_{-i} | s_i) [u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i})] \geq 0.$$

Thus, an intuitive interpretation of CE is as follows. Suppose that a strategy profile $s \in \mathcal{S}$ is chosen at random, e.g., by some virtual referee, according to the joint distribution π . Each player i is then given, by the “referee,” its own recommendation s_i . The above-mentioned inequality means that player i cannot obtain a higher expected utility by selecting strategy s'_i instead of the “recommended” one, i.e., s_i . Also, in every finite game, the set of correlated equilibria is nonempty, closed, and convex.

III. PROBLEM FORMULATION SECTION

Let us consider a network of MFRs that aims at tracking several targets, e.g., see Fig. 1. Let $\mathcal{N}$ denote the set of N radars and $\mathcal{T}$ denote the set of T targets. We consider that the position of each radar node² $i \in \mathcal{N}$ is known. Although there are works in the tracking literature that consider unknown number of targets, e.g., [6] and [7], in this paper, we focus on the case where the number of targets at each time instant is known. The current positions of targets are assumed to be known approximately. Also, the targets are assumed to be well separated; thus, the data association problem is trivial and different transmission beams are required so as to illuminate distinct targets. Furthermore, assume that there is no central processing node to perform

track filtering; in other words, fusion is done at each radar node.

Next, the dynamics of each target $j \in \mathcal{T}$, at each discrete time k , are represented using the so-called white noise constant velocity model [4], [17] given by

$$\mathbf{x}_{j,k} = F \cdot \mathbf{x}_{j,k-1} + \mathbf{w}_{j,k-1} \quad (1)$$

$$\mathbf{z}_{j,k}^{(i)} = h_j^{(i)}(\mathbf{x}_{j,k}) + \mathbf{v}_{j,k}^{(i)} \quad (2)$$

where the following conditions hold.

- The state vector $\mathbf{x}$ for each target j is comprised of the two-dimensional (2-D) coordinates (x_j, y_j) and velocity³ $(v_{j,x}, v_{j,y})$, i.e., $\mathbf{x}_j = [x_j, y_j, v_{j,x}, v_{j,y}]^\top$ where $[\cdot]^\top$ stands for the transposition of the argument.
- F is a 4×4 matrix corresponding to the deterministic target dynamics given as follows:

$$F = \begin{bmatrix} 1 & t_u \\ 0 & 1 \end{bmatrix} \otimes I_2 \quad (3)$$

with $\otimes$ being the Kronecker product, I_2 stands for a 2×2 identity matrix and t_u is the update time that is fixed.

- The process noise $\mathbf{w}$ is Gaussian with zero mean and covariance

$$Q = \sigma_w^2 \cdot \begin{bmatrix} t_u^3/3 & t_u^2/2 \\ t_u^2/2 & t_u \end{bmatrix} \otimes I_2 \quad (4)$$

where σ_w^2 models maneuverability.

- The measurement vector $\mathbf{z}_{j,k}^{(i)}$, at each radar $i \in \mathcal{N}$, consists of range and azimuth, i.e., $\mathbf{z}_{j,k}^{(i)} = [r_{j,k}^{(i)}, a_{j,k}^{(i)}]^\top$.
- The nonlinear transformation $h_j^{(i)}(\mathbf{x}_j)$ is given by

$$h_j^{(i)}(\mathbf{x}_j) = \begin{bmatrix} \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \\ \arctan((y_j - y_i)/(x_j - x_i)) \end{bmatrix}. \quad (5)$$

- The measurement noise $\mathbf{v}_j^{(i)}$ is zero-mean Gaussian with covariance $R_{j,i} = \text{diag} \{ [\sigma_{r_j}^{(i)}]^2, [\sigma_{a_j}^{(i)}]^2 \}$, where $\sigma_{r_j}^{(i)}$ and $\sigma_{a_j}^{(i)}$ stand for the standard deviation in range and azimuth, respectively.

The radars have limited time budget in the sense that they cannot take measurements of all targets during the same time slot. Thus, the number of measurements per scan that each radar can make is given by $m < |\mathcal{T}|$. Since there is no central entity that may coordinate actions of the radars, a distributed approach is needed. Therefore, the main aim of this paper is to propose distributed solutions to the problem of target selection in order to perform multitarget track filtering.

Furthermore, radars may experience different target observability conditions; thus, the set of the targets that are observable at each radar i is denoted by $\mathcal{T}_i$, and it satisfies

¹A probability vector is a vector whose coordinates are all nonnegative and sum up to 1.

²In this paper, we use the terms radar and node interchangeably.

³Although here we assume a 2-D case, the extension to a 3-D case is straightforward.

$\mathcal{T}_i \subseteq \mathcal{T}$. The interaction among the radars is existing but limited to sharing the measurements $\{z_{j,k}^{(i)}\}$ related to the selected targets. The communication neighborhood of any particular radar i , together with radar i , is denoted as $\mathcal{N}_i$, where $\mathcal{N}_i \subseteq \mathcal{N}$. The total number of transmissions each radar i collects from its neighborhood $\mathcal{N}_i$, and which are related to some target $j \in \bigcup_{i \in \mathcal{N}_i} \mathcal{T}_i$, is denoted as $m_j^t(i)$. For notational simplicity, in the rest of this section, we drop the index j for targets where no confusion is possible.

At each radar i and for each target j , the tracking process is performed by an extended-Kalman filter. First, the prediction step occurs, i.e.,

$$\mathbf{x}_{k|k-1} = F \cdot \mathbf{x}_{k-1|k-1} \quad (6)$$

$$P_{k|k-1} = F P_{k-1|k-1} F^\top + Q \quad (7)$$

where $\mathbf{x}_{k|k-1}$ and $P_{k|k-1}$ are the state estimate and the error covariance matrix for time step k given all measurements till time step $k-1$. Then, the updating step takes place where each available measurement for target j of some radar $n \in \mathcal{N}$ is used in a cyclic manner. In particular, for each $p \in \{1, \dots, m_j^t(n)\}$

$$K_k^{(p)} = P_{k|k}^{(p-1)} \left[H_{k,n}^{(p)} \right]^\top \left(H_{k,n}^{(p)} P_{k|k}^{(p-1)} \left[H_{k,n}^{(p)} \right]^\top + R_n \right)^{-1} \quad (8)$$

$$\mathbf{x}_{k|k}^{(p)} = \mathbf{x}_{k|k}^{(p-1)} + K_k^{(p)} \left(z_k^{(n)} - h^{(n)} \left(\mathbf{x}_{k|k}^{(p-1)} \right) \right) \quad (9)$$

$$P_{k|k}^{(p)} = \left(I - K_k^{(p)} H_{k,n}^{(p)} \right) P_{k|k}^{(p-1)} \quad (10)$$

where $P_{k|k}^{(p)}$ denotes the error covariance matrix after p incremental updates at the same time step k , with $P_{k|k}^{(0)} = P_{k|k-1}$ and $\mathbf{x}_{k|k}^{(0)} = \mathbf{x}_{k|k-1}$. The linearized measurement matrix of radar n at time k is $H_{k,n}^{(p)} = \partial h^{(n)} / \partial \mathbf{x}$ evaluated at $\mathbf{x}_{k|k}^{(p-1)}$. Note that, due to the fact that the coordinates (x_n, y_n) of each radar $n \in \mathcal{N}$ are known, the radars do not need to exchange $\{H_{k,n}\}$ matrices in order to implement the above-mentioned algorithm.

In the following, we study a natural game-theoretic variant of this problem. Specifically, we assume that the radars are autonomous decision-makers interested in optimizing their own tracking performance. In other words, the selections of each radar are autonomous in the sense that there is no entity to tell radars what to do in a hierarchical type of structure, nor is there any negotiation among radars. We analyze two indicative scenarios with respect to the observability conditions, communication topology as well as the radars' interests.

- i) *Scenario 1*: A scenario where each radar i observes all targets, i.e., $\bigcap_{i \in \mathcal{N}} \mathcal{T}_i = \mathcal{T}$, communicates with all neighbors (all radars communicate through the full graph), i.e., $\mathcal{N}_i = \mathcal{N}$, and is interested in tracking all targets in $\mathcal{T}$ (all targets have the same importance).
- ii) *Scenario 2*: A more general scenario where the radars do not necessarily have the same target interests and where the observability and communication equalities above

(full observability and full connectivity conditions) do not need to hold, i.e., $\exists i \in \mathcal{N} \mid \mathcal{N}_i \subset \mathcal{N} \vee \mathcal{T}_i \subset \mathcal{T}$.

For both scenarios, the fact that each radar (or the radar operator) autonomously and rationally decides to track the targets that increase its utility can be modeled as a one-stage noncooperative game in normal form, which is the most fundamental representation type in game theory [24]. In following two sections, we analyze the track selection problem in each scenario separately.

IV. SCENARIO 1: THE PROBLEM OF COORDINATION

First, note that there are many classes of normal-form games; however, due to the particularities of the scenario considered, in this section, we focus on coordination games, which do not rest solely upon conflict among players. Instead, as their name suggests, more emphasis is put on the coordination issue where players may have an incentive to conform with or to differ from what others do. In the latter case, this kind of games are usually called *anticoordination games* [24], [50]–[51].

A. Game-Theoretic Model

We assume that the players are rational and their objective is to maximize their payoff, i.e., the tracking accuracy of all targets. Formally, the track selection game $\Gamma^{(1)} = (\mathcal{N}, \mathcal{S}, u)$ has the subsequent components.

- The players are the radars represented by the set $\mathcal{N}$.
- The strategy of each radar i is represented by a T -tuple $s_i = (s_{i,1}, s_{i,2}, \dots, s_{i,T})$, where $s_{i,j} = a$ if radar i devotes a transmission beams to a target j with $a \leq m$. Each strategy tuple has at most m transmissions, i.e., $\sum_{j=1}^T s_{i,j} \leq m$. Also, note that

$$m_j^t(i) = m_j^t = \sum_{i=1}^N s_{i,j}. \quad (11)$$

- The utility for each radar i is given by

$$u_i(s_i, s_{-i}) = \sum_{j=1}^T \text{gain}_j(m_j^t) \quad (12)$$

where the term $\text{gain}_j(m_j^t)$ represents the tracking accuracy gain for target $j \in \mathcal{T}$ and it is defined by

$$\text{gain}_j(m_j^t) = \text{Tr} \left\{ P_{j,k|k-1} - P_{j,k|k}^{(m_j^t)} \right\} \quad (13)$$

where $\text{Tr}\{\cdot\}$ stands for the trace operator and all radars are assumed to have the same initial guesses $\mathbf{x}_{j,0|0}$ and $P_{j,0|0}$.

In other words, the strategy of radar i defines the number of transmissions per target, at a given time slot, see Fig. 2. Due to the fact that radars share their measurements, their tracking accuracy gains for a specific target are dependent on all radars' measurements related to that target.

radars					
r3	0	1	0	1	1
r2	0	3	0	0	0
r1	1	0	2	0	0
	a	b	c	d	e
	targets				

Fig. 2. Example strategy profile displayed as a matrix for $\mathcal{T} = \{a, \dots, e\}$ and $|\mathcal{N}| = m = 3$.

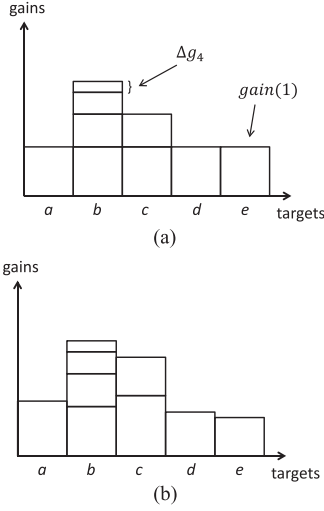


Fig. 3. Example of a track allocation in terms of gains per target where $\mathcal{T} = \{a, \dots, e\}$ and $|\mathcal{N}| = m = 3$. Each box represents a gain increment due to a measurement, and the number of measurements per target m_j^t varies between 1 and 4 across targets in $\mathcal{T}$. In case (a), the gains are equal for the same number of measurements, whereas in case (b) they differ.

Note that the gain in (13) can be expressed as follows:

$$gain_j(m_j^t) = \begin{cases} \sum_{p=1}^{m_j^t} \Delta g_p^{(j)}, & \text{if } m_j^t \geq 1 \\ 0, & \text{if } m_j^t = 0 \end{cases} \quad (14)$$

where

$$\Delta g_p^{(j)} = \text{Tr}\{P_{j,k|k}^{(p-1)} - P_{j,k|k}^{(p)}\} \quad (15)$$

and $\Delta g_1^{(j)} = gain_j(1)$. To analyze the proposed game, we proceed by adopting the following practical assumptions, for all $j \in \mathcal{T}$ and $p \in \{1, \dots, m_j^t\}$ with $m_j^t \geq 1$.

ASSUMPTION 1 The gain function in (13) is increasing with the number of measurements m_j^t , i.e., $\Delta g_p^{(j)} > 0$.

ASSUMPTION 2 Estimation accuracy gain increment $\Delta g_p^{(j)}$ decreases as the order of measurements p grows, i.e., $\Delta g_p^{(j)} > \Delta g_{p+1}^{(j)}$.

Finally, the following two cases are analyzed:

- a) $\Delta g_p^{(j)} = \Delta g_p$, for all $j \in \mathcal{T}$ and $p \in \{1, \dots, m_j^t\}$; and
- b) $\Delta g_p^{(j)} \neq \Delta g_p^{(\ell)}$, for $j \neq \ell$, and $\min_{j \in \mathcal{T}} \Delta g_p^{(j)} > \max_{j \in \mathcal{T}} \Delta g_{p+1}^{(j)}$.

Case a) represents an idealistic case where all nodes would have very similar measurements among themselves and are related to all targets, see Fig. 3(a). A more realistic scenario, corresponding to case b), is illustrated in

Fig. 3(b). In Section IV-B, we characterize the NE of the aforementioned cases.

B. Nash Equilibria

Generally, in a coordination game, there are multiple NE. If the players have the same payoffs, and the equilibria are equal, the game is a *pure* coordination one. In fact, in such a game, all NE are Pareto optimal. On the other hand, in a *ranked* one, the NE differ and usually there is only one Pareto optimal equilibrium [52].

Now, the main findings related to the NE for cases a) and b) are provided.

PROPOSITION 1 The game for case a) has $PoA = 1$, and any track assignment is an NE, if $\sum_{j=1}^T s_{i,j} = m$, and if the following statements hold:

- $m_j^t \leq 1, \forall j \in \mathcal{T}$, for a scenario where $N \cdot m \leq T$; and
- $\max_{j, \ell \in \mathcal{T}} \{|m_j^t - m_\ell^t|\} \leq 1, \forall j, \ell \in \mathcal{T}$, for a scenario where $N \cdot m > T$.

PROOF First, let us assume that there is a radar i such that $\sum_{j=1}^T s_{i,j} < m$ and that the corresponding s^* is an NE. Then, radar i can change its strategy by taking an additional measurement. Due to the fact that, for $m_j^t \geq 1$, the radar's gain function in (13) is increasing with the number of measurements, its utility will be increased. But that contradicts our initial assumption that s^* is an NE; thus, as per our intuition, each radar should make all possible transmissions toward the target(s) at each time instant. Next, note that if the total number of measurements is less than or equal to the number of targets, the radars are worse off if more than one measurement in total are devoted to the same target. Also, due to the structure of gain function, NE are precisely $\frac{T!}{(T-N \cdot m)!}$ outcomes in which each measurement is devoted to a distinct target. On the other hand, if $N \cdot m > T$, the corresponding condition states that all targets should be covered as equally as possible. Here, each NE corresponds to a balanced allocation. For instance, the allocation in Fig. 3(a) is not an NE since the payoffs can be increased if some players move its measurement from target b to any other target. Finally, since the gain of any target is the same for the same number of measurements, the game appears to be a pure anticoordination one. Thus, every NE is also Pareto optimal, which finally implies that $PoA=1$. ■

PROPOSITION 2 The game for case b) has $PoA > 1$, and any track assignment is an NE, if $\sum_{j=1}^T s_{i,j} = m$, and if the following statements hold:

- for a scenario with $N \cdot m \leq T$, each radar chooses its most accurate target that has not been selected; and
- for $N \cdot m > T$, the first $\lceil \frac{N \cdot m}{T} \rceil - 1$ levels are filled in, i.e., $m_j^t \geq \lceil \frac{N \cdot m}{T} \rceil - 1, \forall j \in \mathcal{T}$, and for the $\lceil \frac{N \cdot m}{T} \rceil$ -th level, each radar chooses its most accurate target that has not been selected by others, where $\lceil \cdot \rceil$ is the ceiling function.

PROOF Similar arguments hold as for Proposition 1. Yet, the above-mentioned game seems to be a ranked anticoordination game. Note that here are still multiple NE, but

not all NE are necessarily equal, and hence, not every NE is Pareto optimal (only one is). So, the above-mentioned conditions are not sufficient to have also a Pareto optimal NE, and consequently, PoA is strictly greater than 1. ■

C. BRD-Based Distributed Track Selection Algorithm

In the sequel, we present a simple low-complexity distributed algorithm based on the BRD [24], [26], [27] literature, that looks for the NE of the analyzed game. Toward this goal, let us first define the notion of radar i 's best response to the vector of strategies s_{-i} , denoted by $BR_i(s_{-i})$, as the set-valued function

$$BR_i(s_{-i}) = \arg \max_{s_i \in \mathcal{S}_i} u_i(s_i, s_{-i}).$$

Note that there are two versions of BRD that can be used; namely, the sequential version

$$\begin{aligned} s_i(k+1) &\in BR_i(s_1(k+1), \dots \\ &\dots, s_{i-1}(k+1), s_{i+1}(k), \dots, s_N(k)) \end{aligned}$$

where $s_i(k+1)$ is the action selected by radar i at time step $(k+1)$ and the simultaneous one where all players update their actions synchronously

$$s_i(k+1) \in BR_i(s_{-i}(k)).$$

Although the former one is more frequently used [27], it requires the definition of a cyclic path that covers all nodes, which is an NP-hard problem [53], [54], and furthermore, it has limited applicability in large and delay-intolerant networks if the whole cycle has to be performed at each time instant. Thus, we focus on the simultaneous BRD implementation which, on the other hand, may experience the problem of a coordination failure due to strategic uncertainty (see Fig. 4). Nevertheless, this problem can be alleviated if radars select their best responses with some probability $\alpha < 1$. For instance, α can be set to be 0.5 and can be kept fixed.

In the above-mentioned games where $N \cdot m > T$, in general, two types of NE may arise: one where a radar illuminates only different targets; and the other where it chooses the same target more than once. In practice, it is of interest to exploit the radars' diversity; thus, we focus on the former type. Let $\mathcal{T}_{\text{sel}}^{(i)}$ denote the set of targets selected by radar i .

Then, a summary of the proposed algorithm is provided in the following.

In the context of general BRD algorithms, players need to observe the actions played by the others; however, in our algorithm, it can be verified that the knowledge of the numbers of transmissions per target j , i.e., $\{m_j^t\}_{j=1}^T$, is sufficient. Specifically, note that $\{m_j^t\}_{j=1}^T$ are aggregate functions of the radars' actions and, due to $u_i(s_i, s_{-i}) = u_i(m_1^t, \dots, m_T^t)$, observing the actions themselves is not necessary.

Note that there are no convergence results for general games using BRD, i.e., a BRD-based algorithm may miss an NE [24], [27]. Fortunately, for some special classes of games, there exist sufficient conditions under which the

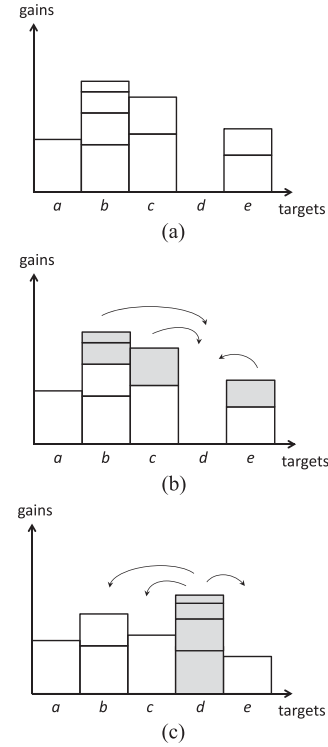


Fig. 4. Coordination failure example: (a) initial track allocation, (b) four radars decide to change their current track choices (gray boxes) and illuminate target d , and (c) track allocation in the following time instant and possible radar choices (denoted by the arrows) that may result into the cyclic behavior.

Algorithm 1: Low-complexity BRD (LC-BRD) Based Distributed Scheme for Track Selection.

- Start with any strategy profile $s(0)$.
 - At each time instant $k = 1, 2, \dots$, each radar $i \in \mathcal{N}$ performs the following steps:
 - s1) Count $m_j^t, \forall j \in \mathcal{T}$, and reallocate the measurements for $\forall j \in \mathcal{T}_{\text{sel}}^{(i)}$ satisfying $s_{i,j} > 1$ to a target $\arg \min_{\ell \in \{\mathcal{T} \setminus \mathcal{T}_{\text{sel}}^{(i)}\}} m_\ell^t$.
 - s2) With some fixed probability α , reallocate the measurement from target j to ℓ until
 - * $\exists j \in \mathcal{T}_{\text{sel}}^{(i)}$ such that $m_j^t > \lceil \frac{N \cdot m}{T} \rceil$ and the measurement for ℓ is the most accurate one of those satisfying $\arg \min_{q \in \{\mathcal{T} \setminus \mathcal{T}_{\text{sel}}^{(i)}\}} m_q^t$, or
 - * $m_j^t - m_\ell^t = 1$, where $m_j^t = \max_{q \in \mathcal{T}_{\text{sel}}^{(i)}} m_q^t$ and $m_\ell^t = \min_{q \in \{\mathcal{T} \setminus \mathcal{T}_{\text{sel}}^{(i)}\}} m_q^t$, and if measurement for ℓ is more accurate than the one for j .
 - s3) Transmit/receive measurements, and $\forall j \in \mathcal{T}$, execute (6) and (7) and employ all available measurements in (8)–(10).
 - s4) (optional) if $u_i(s_i(k), s_{-i}(k)) < u_i(s_i(k-1), s_{-i}(k-1))$ revert back to the strategy from $k-1$ and skip the first 2 steps, i.e., $s_i(k+1) = s_i(k-1)$.
-

convergence of the sequential BRD to a pure NE is always guaranteed. For instance, one such class is related to the so-called potential games [55], which we will define further.

DEFINITION 6 A finite, N -person normal-form game $\Gamma = (\mathcal{N}, \mathcal{S}, u)$ is called a *potential game*⁴ if there exists a function $\Phi : \mathcal{S} \rightarrow \mathbb{R}$ such that $\forall i \in \mathcal{N}$ and for all $(s_i, s_{-i}), (s'_i, s_{-i}) \in \mathcal{S}$

$$u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i}) = \Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i})$$

and such a function Φ is called potential function of the game.

In every finite potential game, every improvement path is finite. Since a finite game has a finite strategy space, the potential function takes on finitely many values and the above-mentioned sequence must terminate in finitely many steps in an equilibrium point.

Unlike the sequential BRD, there does not seem to exist general convergence results for the simultaneous BRD, yet only a few application-specific proofs [27]. Nevertheless, the proposed Algorithm 1 does converge to a pure NE.

THEOREM 1 The proposed Algorithm 1, with the s4) step, does converge to a pure NE of the proposed one-shot track selection game $\Gamma^{(1)} = (\mathcal{N}, \mathcal{S}, u)$, as defined in Section IV-A.

PROOF Let us first analyze a hypothetical sequential version of the proposed algorithm. Note that one may construct a potential function Φ for the analyzed game, i.e., by setting $\Phi = u_i(s), \forall i \in \mathcal{N}$. Thus, a sequential BRD-based strategy for the analyzed game would converge. Now, for the proposed (simultaneous) algorithm, note that in general case, Φ is not nondecreasing as time progresses; however, due to the s4) step, only the states where Φ is not smaller than the best previous Φ value are actually kept. Specifically, in case where the players at time k select a coordination failure profile that may result in $\Phi(k) < \Phi(k-1)$ (such as one given in Fig. 4), this step ensures that $\Phi(k+1) = \Phi(k-1)$. Then, due to $\alpha < 1$, there is a nonnegligible probability that only one player will update (as in the asynchronous version) and Φ will increase; thus, the algorithm will eventually converge. ■

REMARK 1 Strictly speaking, the proposed algorithm with the s4) step is not a traditional simultaneous BRD, since it requires that each player also stores in memory the action and the utility value from the previous time step. This additional yet small memory requirement is sufficient but not necessary for the algorithm to converge. For properly set α , our simulations have shown that the proposed algorithm,

⁴Strictly speaking, the game defined in the above-mentioned definition is formally known as *exact potential game*. There are other variants of potential games, where probably the most general one is the so-called *ordinal potential game* in which the condition $u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i}) > 0$ iff $\Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i}) > 0$ holds. Most importantly, both types of potential games are still guaranteed to have pure-strategy NE.

even without the s4) step, actually converges and performs well (see also [47]).

Dynamic scenario: Note that the tracking accuracy gain in (13), which constitutes the utility of each radar in (12), generally depends on measurement noise covariance $R_{j,i}$, deterministic target dynamics F , and process noise covariance Q . To account for time-varying accuracy measures, i.e., range and azimuth variances, and to deal with possibly high target dynamics, the proposed algorithm can be modified in one of the following ways:

- the LC-BRD algorithm can be repeated every K time instants, where K is an integer number that can be set by the radar operator(s), so as to search for other NE during the tracking process; or
- each radar running LC-BRD may randomly change its strategy in step s2) (regardless of the conditions in this step) with a small ϵ probability. In other words, step s2) in LC-BRD is run with probability $1 - \epsilon$.

The above-mentioned modifications achieve similar performance, as it will be shown in Section VI.

V. SCENARIO 2 COORDINATION AND CONFLICT

In Section IV, we have analyzed the scenario where all radars share the same interests; thus, the main challenge has been to tackle the problem of coordination among radars. In practice, not all targets are necessary of the same priorities, so proper weights should be introduced in the radar utilities. To determine target priorities, one may use the so-called situation assessment or threat assessment function [4], which is the highest level of abstraction in the tracking process. In a setting where there is a single MFR, it is clear that the radar (actually, the radar operator) may have different priorities over different targets. On the other hand, in a network setting, there are two cases: first, that all radars have the same priorities per target; and second, that their priorities may differ for specific target(s). On the one hand, the former case is suitable for modeling situation where there is a homogeneous radar network system, or simply a single network operator, where the radars are part of the same mission. Therefore, the target weights are the same for each radar, which can be seen in scenario 1, which was considered in Section IV. On the other hand, the latter case, in which radar priorities may differ for specific target(s), may model situations where there are several radar operators controlling different parts (different radars) of the radar network. For instance, radar operators can be interested only in a specific region and/or in a specific type of targets. Also, radar operators may have different but overlapped areas of responsibility, so that tracking an object leaving some area and entering another one can be of different importance to the corresponding operators. These situations may arise in military and safety missions, air-traffic control, space debris tracking, vehicle-to-vehicle networks, etc. Yet, in such situations, it is still important to exploit the network cooperation, as in the scenario analyzed in Section IV. Thus, here we focus on a more demanding scenario where radars

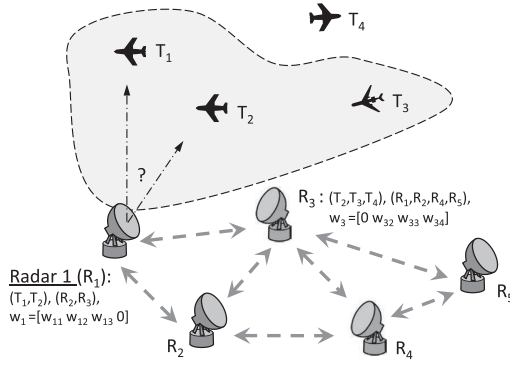


Fig. 5. Track selection problem in a scenario with partial observability, limited communication, and different interests among radars.

may have different interests and where issues of conflict may also arise. Specifically, we assume the following statements about radars that they:

- i) do not necessarily have the same target interests;
- ii) are limited to partial target observability; and
- iii) do not communicate with all other radars.

An example of such a scenario is depicted in Fig. 5. For instance, radar 1 in Fig. 5, denoted as R_1 , communicates with only two neighboring radars (R_2 and R_3), observes only two targets (T_1 and T_2) while being interested in tracking three targets (T_1 , T_2 , and T_3), i.e., there are three nonzero weights, which correspond to T_1 , T_2 , and T_3 , in its weight vector w_1 . On the other hand, R_3 has different yet overlapped interests and different neighbors and observability conditions.

A. Game-Theoretic Model

Here, we redefine the track selection game $\Gamma^{(2)} = (\mathcal{N}, \mathcal{S}, u)$ as follows.

- The players are the radars represented by the set $\mathcal{N}$.
- The strategy of each radar i is represented by a T -tuple $s_i = (s_{i,1}, s_{i,2}, \dots, s_{i,T})$ where

$$s_{i,j} = \begin{cases} a, & \text{if } j \in \mathcal{T}_i \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

where a is the number of transmission beams that radar i devotes to target $j \in \mathcal{T}_i$ and it holds that $a \leq m$. Each strategy tuple has at most m transmissions, i.e., $\sum_{j \in \mathcal{T}_i} s_{i,j} \leq m$. Now, the number of transmissions each radar i collects from $\mathcal{N}_i$ and related to some target $j \in \bigcup_{i \in \mathcal{N}_i} \mathcal{T}_i$ is given as $m_j^t(i) = \sum_{i \in \mathcal{N}_i} s_{i,j}$.

- The utility for each radar i is given by

$$u_i(s_i, s_{-i}) = \sum_{j=1}^T w_{i,j} \cdot \text{gain}_j(m_j^t(i)) \quad (17)$$

with $\text{gain}_j(m_j^t(i)) = \text{Tr} \{ P_{j,k|k-1} - P_{j,k|k}^{(m_j^t(i))} \}$ and $w_{i,j}$ being the weight that a radar i gives to some target j . In fact, $w_{i,j}$ can be seen as (i, j) th element of $N \times T$ matrix W that defines the target interests across all radars. Also, note that $u_i(s_i, s_{-i}) = u_i(\{s_l\}_{l \in \mathcal{N}_i})$.

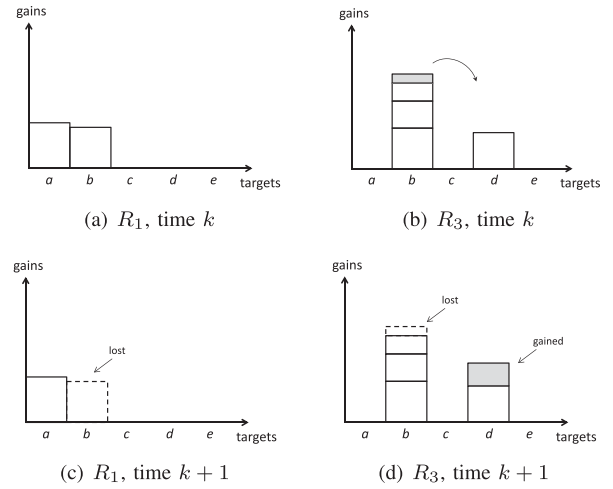


Fig. 6. Assume a track allocation across the network as in Fig. 3(b). Due to limited connectivity and different interests, at time k , radar 1 (R_1) experiences the gains given in (a), and those in (b) are for radar 3 (R_3) that decides to change its selection. At time $k+1$, (c) R_1 has a (great) loss while (d) R_3 has a (relatively small) gain.

B. Correlated Equilibria and RM

The scenario considered in the previous example resembles the well-known *Battle of the Sexes* game [24] where players have a common interest to coordinate (or in our case to anticorrelate), but they have different preferences regarding the (anti)coordinated states of the game (which are NE). However, for a general setting of our above-defined game, it is not easy to characterize possible NE neither in terms of their efficiency nor even their existence. Furthermore, we cannot ensure that the game is potential. This is due to the fact that, in general case, it is difficult to construct a potential function Φ since an action profile change can influence different players in an arbitrarily different way (see an example in Fig. 6).

Remark 2: In the extreme case where radars have totally different interests (with no overlap w.r.t. the interests and communication topology), there it would be easy to define a potential function (just the sum of all utilities). However, the solution (NE) is trivial since the problem is totally decoupled (there is no interdependence); each radar's utility depends only on its own strategy selection $u_i(s) = u_i(s_i), \forall i \in \mathcal{N}$.

For the above-mentioned reasons, here we focus on the solution concept of CE. Note first that, as mentioned in Section II, a CE always exists in a finite game [49]. Actually, every NE is a CE and NE correspond to the special case of a CE for which the joint distribution over the strategy profiles $\pi(s_i, s_{-i})$ factorizes as the product of its marginals, i.e., the play of different players is independent [48], [49]. Furthermore, in certain settings, the set of CE may include even the distribution that is not in the convex hull of the NE distributions.

Next, we will exploit a class of simple adaptive algorithms, called RM, in order to reach a CE of the analyzed track selection game. It does not entail any sophisticated updating, prediction, or fully rational behavior [56]. The

approach can be summarized as follows: At each time instant, a radar may either continue playing the same target strategy as in the previous time instant, or switch to other strategies, with probabilities that are proportional to how much higher his accumulated accuracy gain would have been had he always made that change in the past. Specifically, at each time instant k and for any two distinct strategies $s'_i \neq s_i$, the regret that radar i experiences at time k for not playing s'_i is given by

$$R_{i,k}(s_i, s'_i) = \max \{D_{i,k}(s_i, s'_i), 0\} \quad (18)$$

where the term $D_{i,k}(s_i, s'_i)$ represents the average payoff at time k for not having played, every time that s_i was played in the past $k_p \leq k$, the different strategy s'_i

$$\begin{aligned} D_{i,k}(s_i, s'_i) &= \frac{1}{k} \sum_{k_p \leq k} [u_i(s'_i(k_p), s_{-i}(k_p)) - u_i(s_i(k_p), s_{-i}(k_p))] \end{aligned} \quad (19)$$

Next, the probability at time $k + 1$ for radar i to play some strategy $s'_i \in \mathcal{S}_i$ is a linear function of its regret vector, i.e.,

$$\begin{cases} \pi_i^{k+1}(s'_i) = \frac{1}{\mu} R_{i,k}(s_i, s'_i), & \text{for all } s'_i \neq s_i \\ \pi_i^{k+1}(s_i) = 1 - \sum_{s'_i \neq s_i} \pi_i^{k+1}(s'_i), & \text{otherwise} \end{cases} \quad (20)$$

where the fixed constant $\mu > 0$ is selected to be large enough such that $\pi_i^{k+1}(s_i) > 0$. Finally, for every k , we define the empirical distribution η_k of the strategy profiles played up to time k , i.e., for each $s \in \mathcal{S}$

$$\eta_k(s) = \frac{1}{k} \#\{k_p \leq k : s(k_p) = s\} \quad (21)$$

with $\#(\cdot)$ stands for the number of times the event inside the brackets occurs while $s(k_p)$ is the action profile played at time k_p .

THEOREM 2 If every radar i selects targets according to the probability distribution in (20), then the empirical distributions η_k converge almost surely as $k \rightarrow \infty$ to the set of CE distributions of the game $\Gamma^{(2)}$.

PROOF For the proof, see [56]. ■

Dynamic scenario: Due to possibly time-varying accuracy measures and high target dynamics, as explained at the end of Section IV-C, as well as time-varying radar interests, the suggested approach has to be modified so as to take into account the aforementioned effects. In fact, by incorporating an adaptive mechanism in the calculation of the average regret, it can be shown that the resulting algorithm can track the changes if they are sufficiently small.

First, note that the average regret in (19) can be computed recursively, i.e.,

$$\begin{aligned} D_{i,k}(s_i, s'_i) &= \frac{k-1}{k} D_{i,k-1}(s_i, s'_i) \\ &\quad + \frac{1}{k} [u_i(s'_i(k), s_{-i}(k)) - u_i(s_i(k), s_{-i}(k))] \end{aligned} \quad (22)$$

Also, the average regret in (19) and (22) exploits the history of all past selections. This is not desirable due to the fact that the tracking accuracy gains slight change in time due to the aforementioned effects. Thus, to compute the average regret, each radar should exponentially discount the influence of its past selections. Specifically, similarly to [57], we rewrite the average regret recursion as follows:

$$\begin{aligned} D_{i,k}(s_i, s'_i) &= D_{i,k-1}(s_i, s'_i) \\ &\quad + \theta_k [u_i(s'_i(k), s_{-i}(k)) - u_i(s_i(k), s_{-i}(k)) \\ &\quad - D_{i,k-1}(s_i, s'_i)] \end{aligned} \quad (23)$$

where θ_k is a positive step size. In case where the step size θ_k is decreasing with time, the algorithm will converge with probability 1 to the correlated equilibria of a static game. In fact, if $\theta_k = \frac{1}{k}$, then the recursions in (22) and (23) are identical; thus, the convergence arguments from [56] directly apply. However, for the decreasing step size, the algorithm may not adapt to the changes caused by the target dynamics. On the other hand, with the fixed step size $\theta_k = \theta$, the algorithm is able to adapt to the changes and can be proved to converge to the set of CE by using the arguments from stochastic averaging theory [58]. For a more detailed discussion, see [57].

Finally, we provide the algorithm based on RM.

Algorithm 2: RM-distributed Scheme for Track Selection.

- Start with some initial probability vector $\pi^1(s)$
 - At each time instant $k = 1, 2, \dots$, each radar $i \in \mathcal{N}$ performs the following steps:
 - s1) Select target(s) according to probabilities $\pi_i^k(s'_i)$ and $\pi_i^k(s_i) \forall s'_i \neq s_i$, and denote the selection by s_i .
 - s2) Calculate $D_{i,k}(s_i, s'_i)$ using (23).
 - s3) Calculate regret $R_{i,k}(s_i, s'_i)$ using (18).
 - s4) Find probabilities for the following time instant, i.e., $\pi_i^{k+1}(s'_i)$ and $\pi_i^{k+1}(s_i)$ using (20).
-

Computational complexity: It is of interest to comment on the computational complexity of the distributed algorithms proposed in this paper. For illustration only, let us consider that the number of observed targets is the same for all radars, i.e., $|\mathcal{T}_i| = |\mathcal{T}_n|, \forall i, n \in \mathcal{N}$, and that the radars are interested in all targets that are observable to them. Then, the RM-based algorithm has the complexity that is linear in the number of radars but exponential in m , i.e., $\mathcal{O}(N \cdot |\mathcal{T}_i|^m)$. This is in contrast to the centralized approach that can be realized by an exhaustive search and that has the exponential complexity also in the number of radars, i.e., $\mathcal{O}(|\mathcal{T}_i|^{N \cdot m})$. On the other hand, note that the LC-BRD proposed in Section IV is the most efficient from the computational perspective; its complexity is in the order of $\mathcal{O}(N \cdot m \cdot |\mathcal{T}_i|)$.

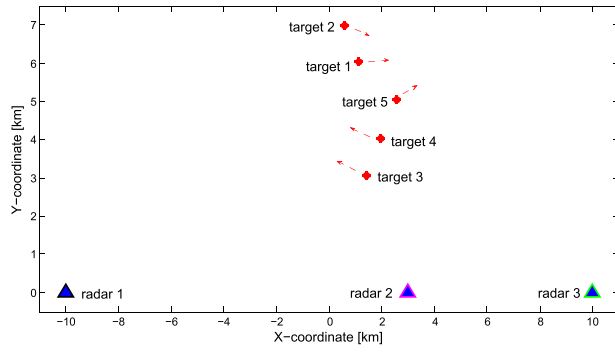


Fig. 7. Coordinates of radars and targets.

TABLE I
Radar Positions

	x [km]	y [km]
radar 1	-10	0
radar 2	3	0
radar 3	10	0

TABLE II
Target Parameters

	x [km]	y [km]	v_x [km/s]	v_y [km/s]
Target 1	1	6	0.5	0.1
Target 2	0.5	7	0.35	-0.1
Target 3	1.5	3	-0.3	0
Target 4	2	4	-0.2	0.1
Target 5	2.5	5	0.3	0.2

VI. SIMULATIONS

In this section, we provide some computer simulations that verify the main findings and demonstrate the effectiveness of the proposed algorithms.

First, we consider an MFR network of $N = 3$ radars, each of them making $m = 2$ measurements per scan and aiming at tracking $T = 5$ targets, see Fig. 7. Specifically, the coordinates of radars are given in Table I. The targets follow white noise constant velocity trajectories with initial x , y coordinates and velocities provided in Table II. Initial guesses $\mathbf{x}_{j,0|0}$ are noisy versions of the initial states $\mathbf{x}_{j,0}$ and initial covariances are equal to $\mathbf{P}_{j,0|0} = \mathbf{P}_{0|0} = \text{diag}\{(0.1 \text{ km})^2, (0.1 \text{ km})^2, (0.1 \text{ km/s})^2, (0.1 \text{ km/s})^2\}$. The update time is $t_u = 0.25$ s, and in order to model moderate maneuverability, σ_w^2 is set to $2.5 \times 10^{-5} \text{ km}^2/\text{s}^3$. Also, the standard deviation in azimuth is $\sigma_{a_j}^{(i)} = \sigma_a = 2 \text{ mrad}$, whereas the range accuracy varies among the radars and targets as $\sigma_{r_j}^{(i)} = b_{i,j} \cdot \sigma_r$, where $\sigma_r = 15 \text{ m}$ and coefficient $b_{i,j}$ is taken from the interval $[1, 4.5]$.

Most figures present the weighted sum of $\text{Tr}\{\mathbf{P}_{j,k|k}^{(m_j^{(i)})}\}$ over all targets and over all radars, i.e.,

$$\sum_{i=1}^N \sum_{j=1}^T w_{i,j} \cdot \text{Tr}\{\mathbf{P}_{j,k|k}^{(m_j^{(i)})}\} \quad (24)$$

as a function of time k . Initially, we focus on the case analyzed in Section IV and compare the following strategies.

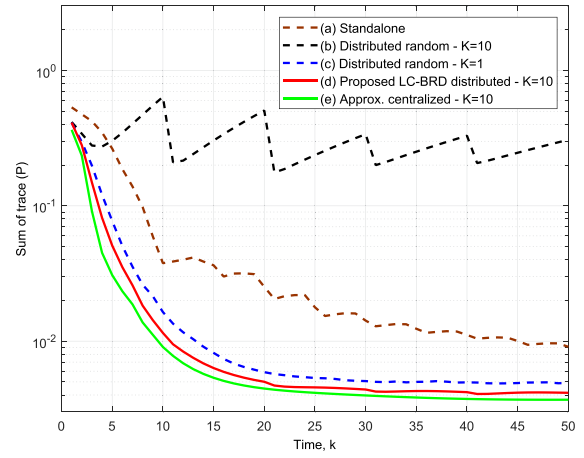


Fig. 8. Sum of traces of error covariance matrices for all targets during time in the setting with $T = 5$ targets, $N = 3$ radars, $m = 2$ measurements per scan, and update time $t_u = 0.25$ s.

- Standalone*—The standalone radar that does not send/receive measurements. It sequentially chooses $m = 2$ different targets at each time instant.
- Distributed random with $K = 10$* —Distributed strategy where the radars exchange the measurements while each of them randomly selects targets each $K = 10$ time instants.
- Distributed random with $K = 1$* —Same as in (b), except that targets are being randomly chosen at each time instant, i.e., $K = 1$.
- Proposed LC-BRD distributed with $K = 10$* —The proposed low-complexity BRD-based distributed algorithm seeking NE while being reinitialized every $K = 10$ time instants. The probability α is set to the value of 0.5.
- Approximated centralized with $K = 10$* —The approximated centralized approach based on analytically resolved measurements-to-target allocation every $K = 10$ time instants. Due to its exponential search complexity in the total number of measurements, i.e., $\mathcal{O}(T^{Nm})$, the centralized exhaustive search is computationally challenging even for the considered scenario of $T = 5$, $N = 3$, and $m = 2$. For this reason, the coefficients in noise variances $\sigma_{r_j}^{(i)}$, which are in the interval $[1, 4.5]$, are set in such a way that the best centralized measurements-to-target allocations can be easily analytically determined and changed every $K = 10$ time instants.⁵

Fig. 8 compares the above-mentioned strategies. The results are averaged over 100 realizations. Not surprisingly, due to the high process' dynamics, a standalone noncooperative radar experiences weak performance since it utilizes only its own measurements that are not sufficient to cover all targets. Although approach in (b) uses $N \cdot m = 6$ measurements, due to the lack of coordination it performs poorly. However, the distributed random strategy can be significantly improved if strategies are constantly being

⁵This is only done for the purpose of a comparison. In the scenarios with limited observability, and thus less computational complexity, we will provide the exhaustive search results as a benchmark.

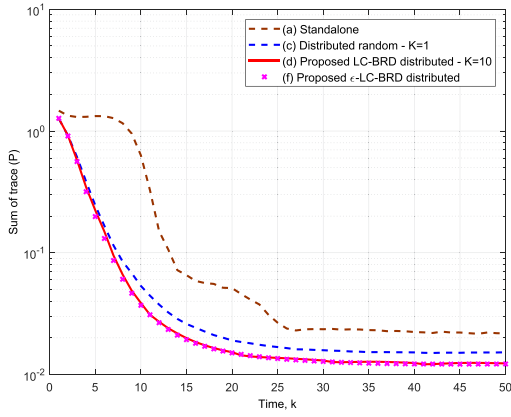


Fig. 9. Setting with $T = 15$ targets, $N = 3$ radars, $m = 6$ measurements per scan, and update time $t_u = 0.25$ s.

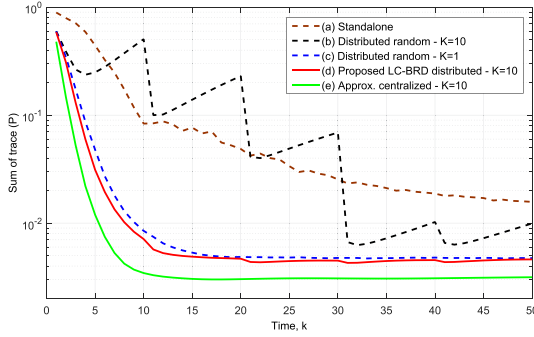


Fig. 10. Setting with $T = 5$ targets, $N = 5$ radars, $m = 2$, and update time $t_u = 0.25$ s.

changed, given that there are no track migration costs involved. Note that the proposed LC-BRD distributed algorithm, which learns underlying NE allocations, outperforms the aforementioned strategies. On the other hand, it closely approaches the performance of the approximated centralized one while mitigating its inherent complexity.

The results related to a setting with more targets and measurements per scan than in the previous setting are plotted in Fig. 9. Here, we also include the following.

- f) ϵ -LC-BRD distributed—The proposed low-complexity BRD-based distributed algorithm seeking NE and where each radar may change its strategy (even if in an NE) with a small ϵ probability. The probabilities α and ϵ are chosen to be 0.5 and 0.02, respectively.

Although without the curve for approximated centralized solution as a benchmark, the results that Fig. 9 provides are similar to those in Fig. 8. Also, note that the two versions of the proposed LC-BRD algorithm exhibit pretty similar performance.

For the plots in Figs. 10 and 11, two additional radars in the network are considered w.r.t. the setting in Fig. 7, i.e., $(x_4, y_4) = [-4 \text{ km}, 0 \text{ km}]$, $(x_5, y_5) = [7 \text{ km}, 0 \text{ km}]$. This is probably the least favorable scenario for LC-BRD w.r.t. the distributed totally random ($K = 1$) algorithm due to the fact that now $\text{mod}(N \cdot m, T) = 0$, i.e., all targets can be selected with the same number of measurements. In Figs. 10 and 11, the update time is set to $t_u = 0.25$ s and $t_u = 0.025$ s,

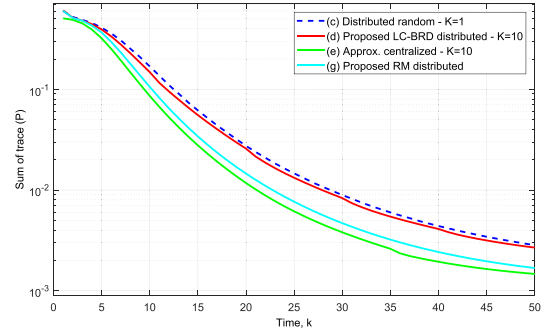


Fig. 11. Scenario is the same as in Fig. 10 except that $t_u = 0.025$ s and the comparison is made with the proposed distributed algorithm based on RM.

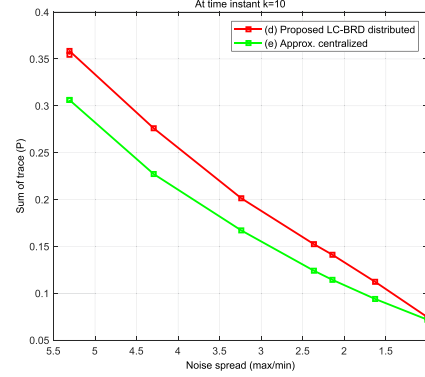


Fig. 12. Performance comparison as a function of the measurements diversity in terms of the noise variance spread, for the same setting as in Fig. 11.

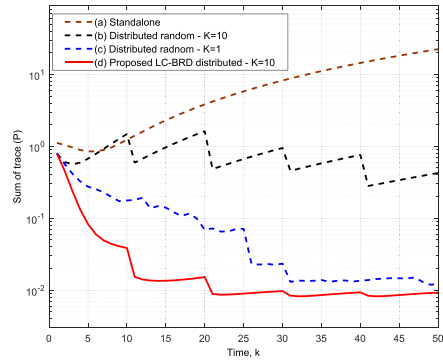


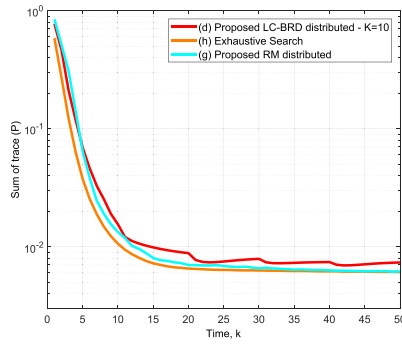
Fig. 13. Scenario with $3 \leq |T_i| \leq 5$, $5 \leq |N_i| \leq 6$ and $W = 1_N \cdot 1_{|T|}^T$, where it holds $\bigcup_{i \in N_i} T_i = T$, $\forall i \in N$.

respectively. Also, in Fig. 11, the following strategy is used in the comparison.

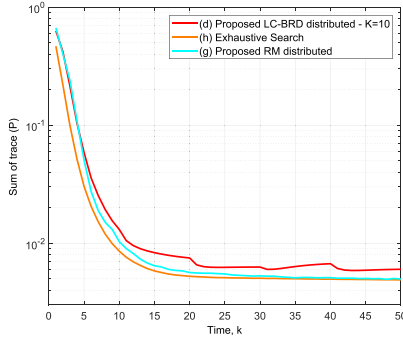
- g) RM distributed—The proposed distributed algorithm based on RM that tracks CE, with $\theta_k = 0.5$.

It can be noticed that the RM distributed algorithm clearly outperforms other distributed strategies. This is due to its more sophisticated learning mechanism, which comes at the expense of somewhat higher computational complexity than the other distributed strategies.

Regarding the LC-BRD algorithm, it should be mentioned that its performance difference w.r.t. the centralized approach mainly depends on the measurement diversity, as suggested by Propositions 1 and 2 in Section IV.



(a) Interest: All targets



(b) Interest: 4 targets

Fig. 14. Scenario with $|\mathcal{T}_i| = 4$ and $\mathcal{N}_i = \mathcal{N}$, with $T = 5$, $N = 6$, $m = 1$, and $t_u = 0.25$ s. The case where all radars have the same interests (all targets) is in (a), and the case where their interests differ in general is in (b).

Specifically, the more similar measurements' quality is, the gap w.r.t. the centralized approach is smaller ($PoA \rightarrow 1$). This is illustrated in Fig. 12 by simulating the performance of the LC-BRD and the approximated centralized solution as a function of the noise variance spread in the network over all targets, i.e., $\max_{i,j}(\sigma_{r_j}^{(i)})/\min_{i,j}(\sigma_{r_j}^{(i)})$.

So far, the focus has been on the scenarios where all radars had the same interests, full observability, and the radar network was fully connected. Now, let us remove these restrictions by first analyzing the scenario where the observability of each radar varies between 3 and 5 and the radar connectivity for some radars is not full while all radars are still interested in all targets. We set $T = 5$ targets, $N = 6$ radars, $m = 1$ measurements per scan, and update time $t_u = 0.25$ s. Note that the compared algorithms were modified accordingly in order to take into account the above-mentioned constraints. As it can be seen in Fig. 13, the LC-BRD algorithm clearly outperforms the distributed random one ($K = 1$) due to the fact that the equally balanced target allocations (from a single radar perspective) are not necessary reasonable, in contrast to the scenario in Figs. 10 and 11.

Finally, we compare the proposed strategies with the centralized solution based on exhaustive search, i.e.,

h) *Exhaustive search*—The centralized search is implemented with full knowledge of all radars' interests, observability, and connectivity conditions, and at each time

instant, the best allocation optimizing the sum of all radars' utilities is selected.

Fig. 14 shows that the LC-BRD performs well given its complexity. Note also that for the case where the interests of the radars are not necessary the same four targets, there are no theoretical guarantees that the NE exist(s) nor that a BRD-based algorithm may achieve an NE point; however, the LC-BRD still performs relatively well. On the other hand, the RM-based algorithm, which is designed for more general scenarios, performs better than the LC-BRD and it closely approaches the centralized exhaustive search solution.

VII. CONCLUSION

In this article, we have proposed a new formulation of the track selection problem for a multitarget tracking scenario in an MFR network using the noncooperative games. The target selections of each radar are considered to be autonomous; there is no central entity to tell radars what to do nor is there any negotiation process among the radars. We have analyzed two indicative scenarios with equal and heterogeneous conditions of observability and connectivity as well as radar interests. In the former scenario, the NE of the underlying anticoordination games have been analyzed and a simple yet effective distributed algorithm that introduces a balancing effect in track selections has been proposed. Afterward, for a more demanding scenario, the solution concept of correlated equilibria has been employed and a more sophisticated distributed algorithm based on the RM has been proposed. Finally, computer simulations have verified that both proposed algorithms closely approximate the centralized solution while mitigating its inherent complexity.

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