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A DSC method for strict-feedback nonlinear systems with possibly unbounded control gain functions

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Abstract

In dynamic surface control (DSC) methods, the control gain functions of systems are always assumed to be bounded, which is a restrictive assumption. This work proposes a novel DSC approach for an extended class of strict-feedback nonlinear systems whose control gain functions are continuous and possibly unbounded. Appropriate compact sets are constructed in such a way that the trajectories of the closed-loop system do not leave these sets, therefore, in these sets, maximums and minimums values of the continuous control gain functions are well defined even if the control gain functions are possibly unbounded. By using Lyapunov theory and invariant set theory, semi-globally uniformly ultimately boundedness is analytically proved: all the signals of closed-loop system will always stay in these compact sets, while the tracking error is shown to converge to a residual set that can be made as small as desired by adjusting design parameters appropriately. Finally, the effectiveness of the designed method is demonstrated via two examples.

Keywords: Dynamic surface control, Adaptive neural control, Robust control, Invariant set theory

1. Introduction

Recent years have witnessed a great amount of research in approximation-based adaptive control for nonlinear uncertain systems due to both the practical need and theoretical challenges [1-15]. In these works, neural networks (NNs) or fuzzy-logic systems (FLS) are typically used to approximate nonlinear functions with little knowledge of system to be controlled, which has effectively removed the restrictive matching conditions for system uncertainties. In addition, as a breakthrough in the nonlinear control area, the adaptive backstepping approach has been extensively employed to obtain global stability for many classes of nonlinear systems with the help of neural networks or fuzzy systems [16-21]. However, because of the employment of the backstepping technique, the aforementioned approaches suffer from the problem of "explosion of complexity", which is caused by repeated differentiations of the virtual control law designed at each step and seriously limits the application of conventional backstepping schemes. As a consequence, more recently, the dynamic surface control (DSC) technique has been creatively proposed to avoid this problem effectively by introducing a first-order low-pass filter at each step of the conventional backstepping design procedure. With the help of the DSC technique, NN-based adaptive backstepping control approaches have been successfully constructed for a large class of nonlinear systems, with excellent control performance [22-26]. Therefore, the DSC technique has become an established powerful tool in the field of adaptive control for nonlinear systems [25, 27-33]. For example, by combining DSC and minimal learning parameter (MLP) techniques, in [25], a RBF NN-based robust adaptive tracking control algorithm is proposed for a class of strict-feedback SISO nonlinear systems. Moreover, In [34], an adaptive neural control scheme is presented for a wide class of perturbed strict-feedback nonlinear time-delay systems with unknown virtual control coefficients using Lyapunov-Krasovskii functions to overcome the possible controller singularity problem and so on.

However, it should be pointed out that, for the DSC technique to work, the control gain functions have always been assumed to be bounded, some efforts have been made in order to remove this restrictive assumption such as in [31, 35] which use alternative techniques than DSC, however, to the best of the authors' knowledge, the restrictive assumption of bounded control gain functions is required in all existing DSC-based schemes. The restriction arises from the fact that the upper and lower constants bounds of control gain functions may be difficult to acquire in some practical systems, or even nonexistent [36], which motivates us to explore new methods to overcome this limitation of DSC technique.

Overcoming this limitation is challenging, due to the fact that neural networks approximation errors will inevitably occur while using NNs to approximate unknown continuous functions on a compact set, this, combined with external disturbances, may degrade system performance, or even lead to instability of closed-loop system [37, 38]. We need to develop a design technique that guarantees that the trajectories of closed-loop systems do not leave appropriate compact sets.

The main contributions of this work are as follows.

1) Compared with previous literatures on DSC technique, we allow the control gain functions to be possibly unbounded. Only the sign of the control gain functions is assumed to be known.

2) To handle this larger class of nonlinear systems, appropriate compact sets are constructed. By using Lyapunov theory and invariant set theory, the closed-loop trajectories are guaranteed not to leave the compact sets so that we can handle continuous control gain functions which are bounded on compact sets. Consequently, the restrictive assumption has been removed, and the application range of the DSC technique is drastically enlarged.

3) Finally, it is analytically proved that all the closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB) while the tracking error is shown to converge to a residual set that can be made as small as desired by adjusting design parameters appropriately.

The rest of this paper is organized as follows. Section 2 gives the problem formulation and preliminaries. The adaptive DSC scheme design and stability analysis are presented in Section 3 and Section 4, respectively. In Section 5, simulation studies are performed to show the effectiveness of the proposed scheme. Finally, the Section 6 concludes the work.

2. Problem statement and preliminaries

2.1. Problem formulation

Consider a class of uncertain SISO strict-feedback nonlinear dynamic systems of the following form[25]:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \Delta_i(x,t), \quad 1 \le i \le n-1 \\ \dot{x}_n = f_n(x) + g_n(x)u + \Delta_n(x,t) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, ..., x_i]^T \in R^i$ and $x = [x_1, x_2, ..., x_n]^T \in R^n$ are the system state variables; $u, y \in R$ are the system input and output, respectively. The functions $f_i(\cdot)$ are unknown continuous functions with $f_i(0) = 0$. $g_i(\cdot)$ are unknown continuous control gain functions and $\Delta_i(x,t)$ denote unknown external disturbances with $i = 1, 2, \ldots, n$.

The control objective of this study is to construct a DSC-based robust adaptive NN tracking controller u such that the system output y follows the desired trajectory y_d, the output tracking error can be rendered arbitrary small and all the signals in the closed-loop systems (1) are semi-globally uniformly ultimately bounded (SGUUB) in the presence of external disturbances.

Before proceeding to the adaptive neural control design of system (1), we make the following Assumptions and Lemmas.

Assumption 1: The signs of unknown control-gain functions $g_i(\cdot)$ are known. Without losing generality, it is further assumed that $g_i(\bar{x}_i) > 0$ for $i = 1, 2, \ldots, n$.

Remark 1: It should be pointed out that in all the existing schemes based on DSC technique, e.g. [25,27-33], the control-gain functions $g_i(\bar{x}_i), i = 1, 2, \ldots, n$ are assumed to satisfy $0 < g_{i,m} \le |g_i(\bar{x}_i)| \le \bar{g}_{i,M}$ with $\bar{g}_{i,M} > 0$ and $g_{i,m} > 0$ being known constants. This assumption is sufficient for controllability of system (1) [25,31,33]. However, in practice, the assumption $0 < g_{i,m} \le |g_i(\bar{x}_i)| \le \bar{g}_{i,M}$ is too restrictive since it may be difficult or even impossible to have prior knowledge of $g_i(\bar{x}_i)$. Moreover, the upper bound $\bar{g}_{i,M}$ and lower bound $g_{i,m}$ of $g_i(\bar{x}_i)$ may be nonexistent: for example, suppose the control-gain functions $g_i(\bar{x}_i), i = 1, 2, \ldots, n$ have the following form

$$g_i(\bar{x}_i) = e^{x_i} \tag{2}$$

The assumption $0 < g_{i,m} \leq |g_i(\bar{x}_i)| \leq \bar{g}_{i,M}$ does not hold since $\bar{g}_{i,M}$ and $g_{i,m}$ do not exist: nevertheless, the system is controllable and Assumption 1 holds since $g_i(\bar{x}_i) > 0$.

Assumption 2 [4]: The reference signal $y_d(t)$ is a sufficiently smooth function of t, and there exists a positive constant B_0 such that $\Pi_0 := \left\{[y_d, \dot{y}_d, \ddot{y}_d]^T \left|(y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \leq B_0\right.\right\}$, where Π_0 is a compact set.

Assumption 3 [25]: For $\forall t > 0$, $\Delta_i(x, t)$ are bounded, that is, there exist unknown positive constants Δ_i^* such that $|\Delta_i(x, t)| \leq \Delta_i^*, i = 1, 2, \ldots, n$.

The following Lemmas will be functional to the DSC design

Lemma 1 [39]: Consider the dynamic system of the following form

$$\dot{Z}(t) = -aZ(t) + bv(t), \quad Z(t) \in R \tag{3}$$

where a and b are positive constants and $v(t)$ is a positive function. Then, for any given bounded initial condition $Z(t_0) \geq 0$, then $Z(t) \geq 0$ for all $\forall t \geq 0$.

Lemma 2 [40]: For any $q \in R$ and $\varsigma > 0$,the hyperbolic tangent function fulfills

$$0 \leq |q| - q \tanh(q/\varsigma) \leq 0.2785\varsigma \tag{4}$$

Lemma 3 [41]: (Young's inequality with ε) For any $(x, y) \in R^2$, the following inequality holds

$$xy \leq \frac{\varepsilon^2}{p}|x|^2 + \frac{1}{q\varepsilon^2}|y|^2 \tag{5}$$

where $\varepsilon > 0$, $q > 1$, $p > 1$ and $(p-1)(q-1) = 1$.

2.2. RBF neural network approximation

In this note, the radial basis function neural network (RBF NN) is considered to be applied to approximate the unknown continuous function $h(Z) : R^n \to R$ as follows

$$h(Z) = \Theta^T \psi(Z) \tag{6}$$

where $Z \in \Pi_Z \subset R^n$ is the input vector to the basis function; $\Theta = [\Theta_1, \Theta_2, ..., \Theta_l]^T \in R^l$ is the weight vector with l being the number of basic functions. The basic functions $\psi(Z) = [\psi_1(Z), ..., \psi_n(Z)]^T \in R^l$ are chosen to be Gaussian functions of the following form

$$\psi_i(Z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{\sigma_i^2}\right], \quad i = 1, 2, ..., l \tag{7}$$

where $\mu_i \in R^l$ and $\sigma_i \in R$ are the center and the width of the Gaussian function, respectively.

It has been proved that any continuous function can be approximated by a neural network as in (7) with any desired accuracy over a compact set $\Pi_Z \subset R^n$ provided that we select enough neural network nodes. The best approximation is denoted with

$$h_{nn}(Z) = \Theta^{*T} \psi(Z) + \varepsilon(Z), \quad \forall Z \in \Pi_Z \subset R^n \tag{8}$$

where Θ^* is the ideal constant weight vector, and $\varepsilon(Z)$ is the approximation error which satisfies $\|\varepsilon(Z)\| \leq \varepsilon^*$ with $\varepsilon^* > 0$ being an unknown constant. It should be noticed that the optimal weight vector Θ^* is an 'artificial' vector required only for analysis purpose which satisfies

$$\Theta^* := \arg\min_{\Theta \in R^l}\left\{\sup_{Z \in \Pi_Z}|h_{nn}(Z) - \Theta^T \psi(Z)|\right\} \tag{9}$$

For compactness, let ε denote $\varepsilon(Z)$, $\|x\|$ and $\|A\|$ denote the 2-norm of vector x and matrix A, respectively.

3. Adaptive neural controller methodology

In this section, we will develop a RBF NN-based robust adaptive tracking control scheme for system (1) using the DSC technique and invariant set theory. The specific procedure of the controller design is given as follows:

Step 1: To begin the design, define the output tracking error $z_1 = x_1 - r$. From (1), the time derivative of z_1 is

$$\dot{z}_1 = f_1(x_1) + g_1(x_1)x_2 + \Delta_1(x, t) - \dot{r} \tag{10}$$

Since the continuous function $f_1(x_1)$ is unknown, a RBF NN is used to approximate $f_1(x_1)$ as follows

$$f_1(x_1) = \Theta_1^{*T} \psi_1(x_1) + \varepsilon_1, \quad x_1 \in \Pi_x \tag{11}$$

where ε_1 is the approximation error satisfying $|\varepsilon_1| \leq \varepsilon_1^*$ with $\varepsilon_1^* > 0$ being an unknown constant.

Substituting (11) into (10), one has

$$\dot{z}_1 = \Theta_1^{*T} \psi_1(x_1) + g_1(x_1)x_2 + \Delta_1(x, t) + \varepsilon_1 - \dot{r} \tag{12}$$

To consider the stabilization of subsystem (10), choose the following quadratic function as

$$V_{z_1} = \frac{1}{2}z_1^2 \tag{13}$$

Using (12), the time derivative of V_{z_1} is

$$\dot{V}_{z_1} = z_1\left(\Theta_1^{*T} \psi_1(x_1) + \varepsilon_1 + g_1(x_1)x_2 + \Delta_1(x, t) - \dot{r}\right) \tag{14}$$

Define a compact set $\Pi_{z_1} := \left\{z_1 \left|V_{z_1} \leq p\right.\right\}$, with p being a positive design constant. In the compact set Π_{z_1}, the following Lemma 4 holds for control-gain function $g_1(x_1)$ of subsystem (10).

Lemma 4 : The unknown continuous function $g_1(x_1)$ has a maximum and a minimum over compact set $\Pi_{z_1} \times \Pi_0$, namely, there exist positive constants $g_{1,m}$ and $\bar{g}_{1,M}$ such that $g_{1,m} = \min_{\Pi_{z_1}\times\Pi_0} g_1(x_1)$ and $\bar{g}_{1,M} = \max_{\Pi_{z_1}\times\Pi_0} g_1(x_1)$.

Proof: Note that $z_1 = x_1 - r$ and $x_1 = z_1 + r$, so that the continuous function $g_1(x_1)$ can be rewritten as

$$g_1(x_1) = \mu_1(z_1, r) \tag{15}$$

where $\mu_1(z_1, r)$ is a continuous function of z_1 and r, and $\Pi_{z_1} \times \Pi_0$ is a compact set since Π_{z_1} and Π_0 are also compact sets. Furthermore, it is seen from (15) that all the variables of $\mu_1(z_1, r)$ lie in the compact set $\Pi_{z_1} \times \Pi_0$. Therefore, $\mu_1(z_1, r)$ has a maximum $\bar{g}_{1,M}$ and a minimum $g_{1,m}$ in the compact set $\Pi_{z_1} \times \Pi_0$. Consequently, we have

$$0 < g_{1,m} \le g_1(x_1) \le \bar{g}_{1,M}, \quad x_1 \in \Pi_{z_1} \times \Pi_0 \tag{16}$$

Choose a virtual controller α_1 for x_2 in (12) and parameters adaptation laws as follows

$$\begin{aligned}\alpha_1 = &-c_1 z_1 - \frac{\hat{\vartheta}_1 z_1}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) \\ &-\hat{\delta}_1 \tanh\left(\frac{z_1}{\upsilon_1}\right) - \xi_1 \dot{r} \tanh\left(\frac{z_1 \dot{r}}{\upsilon_1}\right)\end{aligned} \tag{17}$$

$$\dot{\hat{\vartheta}}_1 = \frac{\rho_1 z_1^2}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) - \sigma_1\rho_1\hat{\vartheta}_1 \tag{18}$$

$$\dot{\hat{\delta}}_1 = \beta_1 z_1 \tanh\left(\frac{z_1}{\upsilon_1}\right) - \sigma_1\beta_1\hat{\delta}_1 \tag{19}$$

where $c_1 > 0$, $b_1 > 0$, $\upsilon_1 > 0$, $\xi_1 \ge g_{1,m}^{-1}$, $\rho_1 > 0$, $\sigma_1 > 0$ and $\beta_1 > 0$ are design parameters. $\hat{\vartheta}_1$ and $\hat{\delta}_1$ are estimates of the unknown constants $\vartheta_1 = g_{1,m}^{-1}\left\|\Theta_1^*\right\|^2$ and $\delta_1 = g_{1,m}^{-1}\left(\varepsilon_1^* + \Delta_1^*\right)$, respectively.

Remark 2: Note that (18) and (19) satisfy the conditions of Lemma 1, therefore, by choosing $\hat{\vartheta}_1(0) \ge 0$ and $\hat{\delta}_1(0) \ge 0$, we have $\hat{\vartheta}_1(t) \ge 0$ and $\hat{\delta}_1(t) \ge 0$ for $\forall t \ge 0$. Furthermore, since the initial conditions of $\hat{\vartheta}_1(0)$ and $\hat{\delta}_1(0)$ are design parameters, we choose the initial conditions $\hat{\delta}_1(0) = \hat{\vartheta}_1(0) = 0$ in this paper.

To avoid repeatedly differentiating α_1, which leads to the so called "explosion of complexity", in the next steps, the DSC technique originally presented in [26] is exploited here. Introduce a first-order filter and let α_1 pass through it with time constant τ_2 so as to obtain $\alpha_{2,f}$ as

$$\tau_2\dot{\alpha}_{2,f} + \alpha_{2,f} = \alpha_1, \quad \alpha_{2,f}(0) = \alpha_1(0) \tag{20}$$

Define the output error of this filter as $y_2 = \alpha_{2,f} - \alpha_1$, which yields $\dot{\alpha}_{2,f} = -y_2/\tau_2$ and

$$\begin{aligned}\dot{y}_2 &= -\frac{y_2}{\tau_2} + \left[-\frac{\partial\alpha_1}{\partial z_1}\dot{z}_1 - \frac{\partial\alpha_1}{\partial\hat{\vartheta}_1}\dot{\hat{\vartheta}}_1 - \frac{\partial\alpha_1}{\partial\hat{\delta}_1}\dot{\hat{\delta}}_1 - \frac{\partial\alpha_1}{\partial\dot{r}}\ddot{r}\right] \\ &= -\frac{y_2}{\tau_2} + \chi_2\left(z_1, z_2, y_2, \hat{\vartheta}_1, \hat{\delta}_1, r, \dot{r}, \ddot{r}\right)\end{aligned} \tag{21}$$

where $\chi_2(\cdot)$ is a continuous function and it will be employed in the stability analysis later.

By noting that $x_2 = z_2 + \alpha_{2,f}$ and $y_2 = \alpha_{2,f} - \alpha_1$, one has

$$x_2 = z_2 + y_2 + \alpha_1 \tag{22}$$

Substituting (22) into (14), we have

$$\begin{aligned}\dot{V}_{z_1} = &z_1(\Theta_1^{*T}\psi_1(x_1) + \varepsilon_1 + g_1(x_1)(z_2 + y_2 + \alpha_1) \\ &+\Delta_1(x, t) - \dot{r})\end{aligned} \tag{23}$$

Applying Lemma 3 yields

$$z_1\Theta_1^{*T}\psi_1(x_1) \le \frac{z_1^2\left\|\Theta_1^*\right\|^2}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) + \frac{b_1^2}{2} \tag{24}$$

According to Assumption 3, $|\varepsilon_1| \le \varepsilon_1^*$ and using (24), we can rewrite (23) as

$$\begin{aligned}\dot{V}_{z_1} \le &z_1 z_2 g_1(x_1) + z_1 g_1(x_1) y_2 + z_1 g_1(x_1)\alpha_1 - z_1\dot{r} \\ &+\frac{z_1^2\left\|\Theta_1^*\right\|^2}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) + |z_1|\left(\varepsilon_1^* + \Delta_1^*\right) + \frac{b_1^2}{2}\end{aligned} \tag{25}$$

Substituting the virtual controller α_1 into (25) and utilizing (16) and $\xi_1 g_{1,m} \ge 1$, we have

$$\begin{aligned}\dot{V}_{z_1} \le &z_1 z_2 g_1(x_1) + z_1 g_1(x_1) y_2 - c_1 g_{1,m} z_1^2 \\ &-\frac{\hat{\vartheta}_1 z_1^2 g_{1,m}}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) - z_1 g_{1,m}\hat{\delta}_1 \tanh\left(\frac{z_1}{\upsilon_1}\right) \\ &-z_1\dot{r}\tanh\left(\frac{z_1\dot{r}}{\upsilon_1}\right) + \frac{z_1^2\left\|\Theta_1^*\right\|^2}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) \\ &+|z_1|\left(\varepsilon_1^* + \Delta_1^*\right) - z_1\dot{r} + \frac{b_1^2}{2}\end{aligned} \tag{26}$$

Noting $\vartheta_1 = g_{1,m}^{-1}\left\|\Theta_1^*\right\|^2$ and $\delta_1 = g_{1,m}^{-1}\left(\varepsilon_1^* + \Delta_1^*\right)$, we can obtain

$$\begin{aligned}-\frac{\hat{\vartheta}_1 z_1^2 g_{1,m}}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) = &\frac{\tilde{\vartheta}_1 z_1^2 g_{1,m}}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) \\ &-\frac{z_1^2\left\|\Theta_1^*\right\|^2}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1)\end{aligned} \tag{27}$$

$$-z_1 g_{1,m}\hat{\delta}_1 \tanh\left(\frac{z_1}{\upsilon_1}\right) = -z_1(\varepsilon_1^* + \Delta_1^*)\tanh\left(\frac{z_1}{\upsilon_1}\right) + z_1 g_{1,m}\tilde{\delta}_1 \tanh\left(\frac{z_1}{\upsilon_1}\right) \tag{28}$$

According to (27), (28) and Lemma 2, we can rewrite (26) as follows

$$\begin{aligned}\dot{V}_{z_1} \le & -c_1 g_{1,m} z_1^2 + z_1 z_2 g_1(x_1) + z_1 g_1(x_1) y_2 \\ & + \frac{\tilde{\vartheta}_1 z_1^2 g_{1,m}}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1) + a_1 \\ & + z_1 g_{1,m}\tilde{\delta}_1 \tanh\left(\frac{z_1}{\upsilon_1}\right)\end{aligned} \tag{29}$$

where $a_1 = 0.2785\upsilon_1(\varepsilon_1^* + \Delta_1^* + 1) + \frac{b_1^2}{2}$

Define the Lyapunov function candidate

$$V_1 = V_{z_1} + \frac{g_{1,m}\tilde{\delta}_1^2}{2\beta_1} + \frac{g_{1,m}\tilde{\vartheta}_1^2}{2\rho_1} + \frac{1}{2}y_2^2 \tag{30}$$

where $\tilde{\delta}_1 = \delta_1 - \hat{\delta}_1$ and $\tilde{\vartheta}_1 = \vartheta_1 - \hat{\vartheta}_1$ are the estimation errors of δ_1 and ϑ_1, respectively.

It follows from (21) and (29) that the time derivative of V_1 is

$$\begin{aligned}\dot{V}_1 \le & -c_1 g_{1,m} z_1^2 + z_1 z_2 g_1(x_1) + z_1 g_1(x_1) y_2 - \frac{y_2^2}{\tau_2} \\ & - \frac{g_{1,m}\tilde{\delta}_1}{\beta_1}\left[\dot{\hat{\delta}}_1 - \beta_1 z_1 \tanh\left(\frac{z_1}{\upsilon_1}\right)\right] + |y_2|\,|\chi_2(\cdot)| \\ & - \frac{g_{1,m}\tilde{\vartheta}_1}{\rho_1}\left[\dot{\hat{\vartheta}}_1 - \frac{\rho_1 z_1^2}{2b_1^2}\psi_1^T(x_1)\psi_1(x_1)\right] + a_1\end{aligned} \tag{31}$$

Substituting parameters adaptation laws (18) and (19) into (31), we obtain

$$\begin{aligned}\dot{V}_1 \le & -c_1 g_{1,m} z_1^2 + z_1 z_2 g_1(x_1) + z_1 g_1(x_1) y_2 \\ & + \sigma_1 g_{1,m}\tilde{\delta}_1\hat{\delta}_1 - \frac{y_2^2}{\tau_2} + \sigma_1 g_{1,m}\tilde{\delta}_1\hat{\delta}_1 \\ & + |y_2|\,|\chi_2(\cdot)| + a_1\end{aligned} \tag{32}$$

Step i ($2 \le i \le n-1$): A similar procedure is recursively employed for each step i, $i = 2, 3, \ldots, n-1$. Define $z_i = x_i - \alpha_{i,f}$. The time derivative of z_i is

$$\dot{z}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \Delta_i(x,t) - \dot{\alpha}_{i,f} \tag{33}$$

Similar to Step 1, a RBF NN is used to approximate the unknown continuous functions $f_i(\bar{x}_i)$ as follows

$$f_i(\bar{x}_i) = \Theta_i^{*T}\psi_i(\bar{x}_i) + \varepsilon_i, \qquad x_i \in \Pi_x \tag{34}$$

where ε_i is the approximation error satisfying $|\varepsilon_i| \le \varepsilon_i^*$ with $\varepsilon_i^* > 0$ being an unknown constant.

Consider the i-th subsystem quadratic function

$$V_{z_i} = \frac{1}{2}z_i^2 \tag{35}$$

It follows from (33) and (34) that the time derivative of V_{z_i} is

$$\dot{V}_{z_i} = z_i\left(\Theta_i^{*T}\psi_i(\bar{x}_i) + \varepsilon_i + g_i(\bar{x}_i)x_{i+1} + \Delta_i(x,t) - \dot{\alpha}_{i,f}\right) \tag{36}$$

Design the virtual controller α_i for x_{i+1} in (36) and the adaptations laws of the i-th subsystem as follows

$$\begin{aligned}\alpha_i = & -c_i z_i - \frac{\hat{\vartheta}_i z_i}{2b_i}\psi_i^T(\bar{x}_i)\psi_i(\bar{x}_i) \\ & - \hat{\delta}_i \tanh\left(\frac{z_i}{\upsilon_i}\right) - \xi_i \frac{y_i}{\tau_i}\tanh\left(\frac{z_i y_i}{\tau_i \upsilon_i}\right)\end{aligned} \tag{37}$$

$$\dot{\hat{\vartheta}}_i = \frac{\rho_i z_i^2}{2b_i^2}\psi_i^T(\bar{x}_i)\psi_i(\bar{x}_i) - \sigma_i\rho_i\hat{\vartheta}_i \tag{38}$$

$$\dot{\hat{\delta}}_i = \beta_i z_i \tanh\left(\frac{z_i}{\upsilon_i}\right) - \sigma_i\beta_i\hat{\delta}_i \tag{39}$$

where $c_i > 0$, $b_i > 0$, $\upsilon_i > 0$, $\xi_i \ge g_{i,m}^{-1}$, $\rho_i > 0$, $\sigma_i > 0$ and $\beta_i > 0$ $(i = 2, 3, \ldots, n)$ are design parameters. $\hat{\vartheta}_i$ and $\hat{\delta}_i$ are estimates of the unknown constants $\vartheta_i = g_{i,m}^{-1}\left\|\Theta_i^*\right\|^2$ and $\delta_i = g_{i,m}^{-1}\left(\varepsilon_i^* + \Delta_i^*\right)$, respectively. Similar to Remark 2, it can be noticed that we have $\hat{\vartheta}_i(t) \ge 0$ and $\hat{\delta}_i(t) \ge 0$ for $\forall t \ge 0$ by appropriately choosing the initial values $\hat{\delta}_i(0) = \hat{\vartheta}_i(0) = 0$

Let the virtual controller α_i pass through a first-order filter to obtain the output $\alpha_{i+1,f}$ as follows

$$\tau_{i+1}\dot{\alpha}_{i+1,f} + \alpha_{i+1,f} = \alpha_i, \quad \alpha_{i+1,f}(0) = \alpha_i(0) \tag{40}$$

Define $y_{i+1} = \alpha_{i+1,f} - \alpha_i$, which yields $\dot{\alpha}_{i+1,f} = -y_{i+1}/\tau_{i+1}$ and

$$\dot{y}_{i+1} = -\frac{y_{i+1}}{\tau_{i+1}} + \chi_{i+1}\left(\bar{z}_{i+1}, \bar{y}_{i+1}, \bar{\hat{\vartheta}}_i, \bar{\hat{\delta}}_i, r, \dot{r}, \ddot{r}\right) \tag{41}$$

where $\bar{z}_{i+1} = [z_1, z_2, \cdots, z_{i+1}]^T$, $\bar{y}_{i+1} = [y_2, \cdots, y_{i+1}]^T$, $\bar{\hat{\vartheta}}_i = \left[\hat{\vartheta}_1, \hat{\vartheta}_2, \cdots, \hat{\vartheta}_i\right]^T$, $\bar{\hat{\delta}}_i = \left[\hat{\delta}_1, \hat{\delta}_2, \cdots, \hat{\delta}_i\right]^T$ and $\chi_{i+1}(\cdot)$ is a continuous function.

From $x_i = z_i + \alpha_{i,f}$ and $y_i = \alpha_{i,f} - \alpha_{i-1}$, we have

$$x_i = z_i + y_i + \alpha_{i-1} \tag{42}$$

In view of (37), we know that the virtual control α_{i-1} is a continuous function with respect to z_{i-1}, $\hat{\delta}_{i-1}$, y_{i-1}, and $\hat{\vartheta}_{i-1}$. Therefore, x_i is a continuous function of z_i, y_i, $\hat{\delta}_{i-1}$ and $\hat{\vartheta}_{i-1}$. From $x_1 = z_1 + r$ and (42), it follows that the control gain functions $g_i(\bar{x}_i)$ can be expressed in the following form

$$g_i(\bar{x}_i) = \mu_i\left(\bar{z}_i, \bar{y}_i, \bar{\hat{\delta}}_{i-1}, \bar{\hat{\vartheta}}_{i-1}, r\right) \tag{43}$$

where $\mu_i(\cdot)$ is a continuous function.

Define the following compact sets

$$\begin{aligned}\Pi_i :=&\{[\bar{z}_i^T, \bar{y}_i^T, \bar{\hat{\vartheta}}_{i-1}^T, \bar{\hat{\delta}}_{i-1}^T]^T | z_i^2 \\ &+ \sum_{j=1}^{i-1} (z_j^2 + y_{j+1}^2 + \frac{g_{j,m}\tilde{\delta}_j^2}{\beta_j} + \frac{g_{j,m}\hat{\vartheta}_j^2}{\rho_j} \le 2p)\}\end{aligned} \tag{44}$$

where p is the same positive design constant after (14).

Lemma 5: The unknown continuous functions $g_i(\bar{x}_i)$ have a maximum and a minimum in compact set $\Pi_0 \times \Pi_i$, namely, there exist positive constants $\bar{g}_{i,M}$ and $g_{i,m}$ such that $\bar{g}_{i,M} = \max_{\Pi_0\times\Pi_i} g_i(\bar{x}_i)$ and $g_{i,m} = \min_{\Pi_0\times\Pi_i} g_i(\bar{x}_i)$

Proof: It follows from (43) and (44) that all the variables of $\mu_i(\cdot)$ are included in the compact set $\Pi_i \times \Pi_0$, thus, the continuous functions $\mu_i(\cdot)$ have a maximum and a minimum in the compact set $\Pi_0 \times \Pi_i$. Meanwhile, in view of (43), we know that $g_i(\bar{x}_i)$ have a maximum and a minimum on the compact set $\Pi_0 \times \Pi_i$, namely, Lemma 5 holds, and the following inequality holds on $\Pi_0 \times \Pi_i$

$$0 < g_{i,m} \le g_i(\bar{x}_i) \le \bar{g}_{i,M}, \quad x_i \in \Pi_i \times \Pi_0 \tag{45}$$

Choose the Lyapunov function candidate

$$V_i = V_{z_i} + \frac{g_{i,m}\tilde{\delta}_i^2}{2\beta_i} + \frac{g_{i,m}\tilde{\vartheta}_i^2}{2\rho_i} + \frac{1}{2}y_{i+1}^2 \tag{46}$$

where $\tilde{\delta}_i = \delta_i - \hat{\delta}_i$ and $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$

According to (36) and (41), and note $x_{i+1} = z_{i+1} + y_{i+1} + \alpha_i$, we obtain the time derivative of V_i as

$$\begin{aligned}\dot{V}_i \le& z_i z_{i+1} g_i(\bar{x}_i) + z_i g_i(\bar{x}_i) y_{i+1} + z_i g_i(\bar{x}_i)\alpha_i \\ &+ |z_i|(\varepsilon_i + \Delta_i) + \frac{z_i^2 \|\Theta_i\|^2}{2b_i}\psi_i^T(\bar{x}_i)\psi_i(\bar{x}_i) \\ &- z_i\dot{\alpha}_{i,f} - \frac{g_{i,m}\tilde{\delta}_i}{\beta_i}\dot{\hat{\delta}}_i - \frac{g_{i,m}\tilde{\vartheta}_i}{\rho_i}\dot{\hat{\vartheta}}_i \\ &- \frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}||\chi_{i+1}(\cdot)| + \frac{b_i^2}{2}\end{aligned} \tag{47}$$

Substituting (37), (38) and (39) into (47), and following the same way as step 1, one has

$$\begin{aligned}\dot{V}_i \le& - c_i g_{i,m} z_i^2 + z_i z_{i+1} g_i(\bar{x}_i) + z_i g_i(\bar{x}_i) y_{i+1} - \frac{y_{i+1}^2}{\tau_{i+1}} \\ &+ g_{i,m}\sigma_i\tilde{\vartheta}_i\hat{\vartheta}_i + g_{i,m}\sigma_i\tilde{\delta}_i\hat{\delta}_i + |y_{i+1}||\chi_{i+1}(\cdot)| + a_2\end{aligned} \tag{48}$$

where $a_2 = 0.2785\upsilon_i\left(\varepsilon_i^* + \Delta_i^* + 1\right) + \frac{b_i^2}{2}$.

Step n: Define $z_n = x_n - \alpha_{n,f}$, whose time derivative along (1) is

$$\dot{z}_n = f_n(x) + g_n(x)u + \Delta_n(x,t) - \dot{\alpha}_{n,f} \tag{49}$$

To consider the stabilization of the n-th subsystem, choose the following quadratic function

$$V_{z_n} = \frac{1}{2}z_n^2 \tag{50}$$

Then time derivative of (50) along (49) is

$$\dot{V}_{z_n} = z_n\left[f_n(x) + g_n(x)u + \Delta_n(x,t) - \dot{\alpha}_{n,f}\right] \tag{51}$$

Similarly to the previous steps, we apply a RBF NN to approximate the unknown continuous function $f_n(\bar{x}_n)$ as follows

$$f_n(x) = \Theta_n^{*T}\psi_n(x) + \varepsilon_n, \quad \forall x \in \Pi_x \tag{52}$$

where $|\varepsilon_n| \le \varepsilon_n^*$ with $\varepsilon_n^* > 0$ being a constant.

From (52), we can rewrite (51) as follows

$$\dot{V}_{z_n} = z_n\left[\Theta_n^{*T}\psi_n(x) + \varepsilon_n + g_n(x)u + \Delta_n(x,t) - \dot{\alpha}_{n,f}\right] \tag{53}$$

Similar to step i, the continuous function $g_n(x)$ can be expressed in the following form

$$g_n(x) = \mu_n\left(\bar{z}_n, \bar{y}_n, \bar{\hat{\vartheta}}_{n-1}, \bar{\hat{\delta}}_{n-1}, r\right) \tag{54}$$

where $\mu_n(\cdot)$ is a continuous function.

Define the following compact set

$$\begin{aligned}\Pi_n :=&\{[\bar{z}_n^T, \bar{y}_n^T, \bar{\hat{\vartheta}}_{n-1}^T, \bar{\hat{\delta}}_{n-1}^T]^T | z_n^2 \\ &+ \sum_{j=1}^{n-1} (z_j^2 + y_{j+1}^2 + \frac{g_{j,m}\tilde{\delta}_j^2}{\beta_j} + \frac{g_{j,m}\tilde{\vartheta}_j^2}{\rho_j}) \le 2p\}\end{aligned} \tag{55}$$

It should be noted that all the variables of $\mu_n(\cdot)$ are included in the compact set $\Pi_n \times \Pi_0$, that is, the continuous function $\mu_n(\cdot)$ has a maximum $\bar{g}_{n,M} = \max_{\Pi_n\times\Pi_0} g_n(x)$ and a minimum $g_{n,m} = \min_{\Pi_n\times\Pi_0} g_n(x)$ such that

$$0 < g_{n,m} \le g_n(x) \le \bar{g}_{n,M} \tag{56}$$

Design the actual control law u as

$$\begin{aligned}u =& -c_n z_n - \frac{\hat{\vartheta}_n z_n}{2b_n^2}\psi_n^T(x)\psi_n(x) - \hat{\delta}_n \tanh\left(\frac{z_n}{\upsilon_n}\right) \\ &- \xi_n\frac{y_n}{\tau_n}\tanh\left(\frac{z_n y_n}{\tau_n\upsilon_n}\right)\end{aligned} \tag{57}$$

with the corresponding adaptation laws determined as

$$\dot{\hat{\vartheta}}_n = \frac{\rho_n z_n^2}{2b_n^2}\psi_n^T(x)\psi_n(x) - \sigma_n\rho_n\hat{\vartheta}_n \tag{58}$$

$$\dot{\hat{\delta}}_n = \beta_n z_n \tanh\left(\frac{z_n}{\upsilon_n}\right) - \sigma_n\beta_n\hat{\delta}_n \tag{59}$$

where $c_n > 0$, $b_n > 0$, $\upsilon_n > 0$, $\xi_n \geq g_{n,m}^{-1}$, $\rho_n > 0$, $\sigma_n > 0$ and $\beta_n > 0$ are design parameters. $\hat{\vartheta}_n$ and $\hat{\delta}_n$ are the estimates of the unknown constants $\vartheta_n = g_{n,m}^{-1}\left\|\Theta_n^*\right\|^2$ and $\delta_n = g_{n,m}^{-1}(\varepsilon_n^* + \Delta_n^*)$, respectively. By recalling Lemma 1, we have $\hat{\delta}_n(t) \geq 0$ and $\hat{\vartheta}_n(t) \geq 0$ after choosing $\delta_n(t) = 0$ and $\vartheta_n(t) = 0$

Design the Lyapunov function candidate of the following form

$$V_n = V_{z_n} + \frac{g_{n,m}\tilde{\delta}_n^2}{2\beta_n} + \frac{g_{n,m}\tilde{\vartheta}_n^2}{2\rho_n} \tag{60}$$

where $\tilde{\delta}_n = \delta_n - \hat{\delta}_n$ and $\tilde{\vartheta}_n = \vartheta_n - \hat{\vartheta}_n$

Then, with the help of (53), the time derivative of V_n can be expressed as

$$\begin{aligned}\dot{V}_n = & z_n\left(\Theta_n^{*T}\psi_n(x) + \varepsilon_n + g_n(x)u + \Delta_n(x,t) - \dot{\alpha}_{n,f}\right)\\ & -\frac{g_{n,m}\tilde{\delta}_n}{\beta_n}\dot{\hat{\delta}}_n - \frac{g_{n,m}\tilde{\vartheta}_n}{\rho_n}\dot{\hat{\vartheta}}_n\end{aligned} \tag{61}$$

Substituting the actual control law (57) into (61) and following the same way as the former steps, we have

$$\begin{aligned}\dot{V}_n \leq & -c_n g_{n,m} z_n^2 - \frac{g_{n,m}\tilde{\delta}_n}{\beta_n}\left[\dot{\hat{\delta}}_n - \beta_n z_n \tanh\left(\frac{z_n}{\upsilon_n}\right)\right]\\ & -\frac{g_{n,m}\tilde{\vartheta}_n}{\rho_n}\left[\dot{\hat{\vartheta}}_n - \frac{\rho_n z_n^2}{2b_n^2}\psi_n^T(x)\psi_n(x)\right] + a_3\end{aligned} \tag{62}$$

where $a_3 = 0.2785\upsilon_n(\varepsilon_n^* + \Delta_n^* + 1) + \frac{b_n^2}{2}$

The substitution of adaptation laws (58) and (59) into (62) finally yields

$$\dot{V}_n \leq -c_n g_{n,m} z_n^2 + g_{n,m}\sigma_n\tilde{\vartheta}_n\hat{\vartheta}_n + g_{n,m}\sigma_n\tilde{\delta}_n\hat{\delta}_n + a_3 \tag{63}$$

4. Stability analysis

The main stability result of the proposed approach is summarized in the following theorem 1.

Choose the Lyapunov function as follows

$$V = V_1 + V_2 \cdots V_n = \sum_{i=1}^{n} V_i \tag{64}$$

with

$$\begin{aligned}V_i &= \frac{z_i^2}{2} + \frac{g_{i,m}\tilde{\delta}_i^2}{2\beta_i} + \frac{g_{i,m}\tilde{\vartheta}_i^2}{2\rho_i} + \frac{y_{i+1}^2}{2}, (i = 1, \ldots, n-1)\\ V_n &= \frac{1}{2}z_n^2 + \frac{g_{n,m}\tilde{\delta}_n^2}{2\beta_n} + \frac{g_{n,m}\tilde{\vartheta}_n^2}{2\rho_n}\end{aligned} \tag{65}$$

Theorem 1: Consider the strict feedback nonlinear system (1), and let Assumptions 1∼3 hold. Consider the control design given by the virtual control laws (17) and (37), filters (20) and (40), actual control law (57) and adaptation laws (18), (19), (38), (39) and (58), (59). For any $p > 0$, and bounded initial conditions satisfying $\hat{\delta}(t) \geq 0$, $\hat{\vartheta}(t) \geq 0$, and $V(0) \leq p$, there exist design parameters c_i, b_i, υ_i, ξ_i, ρ_i, σ_i, τ_i and β_i such that

1) The compact set $\Pi_n \times \Pi_0$ is an invariant set, namely, $V(t) \leq p$ for $\forall t > 0$, and hence all the closed-loop signals are semi-globally uniformly ultimately bounded;

2) The output tracking error z_1 satisfies $\lim_{t\to\infty}|z_1| \leq \sqrt{2\Lambda}$, where $\Lambda > 0$ is a constant depending on the design parameters.

Proof: In view of (32), (48) and (63), the time derivative of V is

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n} c_i g_{i,m} z_i^2 + \sum_{i=1}^{n-1}\bar{g}_{i,M}\left(|z_{i+1}| + |y_{i+1}|\right)|z_i|\\ & -\frac{1}{2}\sum_{i=1}^{n}\sigma_i g_{i,m}\left(\tilde{\delta}_i^2 + \tilde{\vartheta}_i^2\right) + \frac{1}{2}\sum_{i=1}^{n}\sigma_i g_{i,m}\left(\delta_i^2 + \vartheta_i^2\right)\\ & + \sum_{i=1}^{n}\left(\frac{b_i^2}{2} + 0.2785\upsilon_i\left(\varepsilon_i^* + \Delta_i^* + 1\right)\right)\\ & + \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}\chi_{i+1}(\cdot)|\right)\end{aligned} \tag{66}$$

By completion of squares, we have

$$|y_{i+1}\chi_{i+1}(\cdot)| \leq \frac{y_{i+1}^2\chi_{i+1}^2(\cdot)}{2d_1} + \frac{d_1}{2}$$

$$\bar{g}_{i,M}|z_i||y_{i+1}| \leq \frac{\bar{g}_{i,M}^2 y_{i+1}^2 d_2}{2} + \frac{z_i^2}{2d_2}$$

$$\bar{g}_{i,M}|z_i||z_{i+1}| \leq \frac{\bar{g}_{i,M}z_i^2}{2} + \frac{\bar{g}_{i,M}z_{i+1}^2}{2}$$

where $d_1 > 0$ and $d_2 > 0$ are unknown constants. Thus, we can rewrite (66) as

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n} c_i g_{i,m} z_i^2 - \frac{1}{2}\sum_{i=1}^{n} \sigma_i g_{i,m}\left(\tilde{\delta}_i^2 + \tilde{\vartheta}_i^2\right) \\ & + \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\tau_{i+1}} + \frac{y_{i+1}^2 \chi_{i+1}^2(\cdot)}{2d_1} + \frac{\bar{g}_{i,M}^2 y_{i+1}^2 d_2}{2}\right) \\ & + \frac{1}{2}\sum_{i=1}^{n-1} \bar{g}_{i,M}\left(z_i^2 + z_{i+1}^2\right) + \sum_{i=1}^{n}\frac{z_i^2}{2d_2} + a_4\end{aligned} \tag{67}$$

where $a_4 = \sum_{i=1}^{n}\left(\frac{b_i^2}{2} + 0.2785\upsilon_i\left(\varepsilon_i^* + \Delta_i^* + 1\right)\right) + \frac{(n-1)d_1}{2} + \frac{1}{2}\sum_{i=1}^{n}\sigma_i g_{i,m}\left(\vartheta_i^2 + \delta_i^2\right)$

Then, let us briefly investigate the characteristics of the continuous functions $\chi_{i+1}(\cdot), i = 1, 2, \ldots, n-1$, defined in (21) and (41). According to the expression of the compact set $\Pi_i, i = 2, ..., n$, it can be noticed that all the variables of the continuous functions $\chi_{i+1}(\cdot)$ are included in the compact set $\Pi_{i+1} \times \Pi_0$. Consequently, $\chi_{i+1}(\cdot)$ has a maximum D_{i+1} over $\Pi_{i+1} \times \Pi_0$, $i = 1, 2, \ldots, n-1$

Then, let $1/\tau_{i+1} \geq D_{i+1}^2\big/(2d_1) + \bar{g}_{i,M}^2 d_2\big/2 + \lambda_1$ with λ_1 being a positive constant. Hence we have

$$-\frac{y_{i+1}^2}{\tau_{i+1}} + \frac{y_{i+1}^2\chi_{i+1}^2(\cdot)}{2d_1} + \frac{\bar{g}_{i,M}^2 y_{i+1}^2 d_2}{2} \leq -\lambda_1 y_{i+1}^2 \tag{68}$$

Substituting (68) into (67) and letting $c_i \geq g_{i,m}^{-1}\left(\bar{g}_{Max} + 1/(2d_2) + \lambda_1\right)$ with $\bar{g}_{Max} = \max\{\bar{g}_{1,M}, \bar{g}_{2,M}, \ldots, \bar{g}_{n,M}\}$. Then, we can further rewrite (67) as

$$\dot{V} \leq -\omega V + a_4 \tag{69}$$

where

$$\omega = \min\{2\lambda_1, \beta_i\sigma_i, \rho_i\sigma_i\} \tag{70}$$

It can be seen from (70) that a_4/ω can be made arbitrarily small by reducing σ_i, υ_i and b_i , and meanwhile increasing λ_1, β_i and ρ_i. It is always possible to make $a_4/\omega \leq p$ by choosing the design parameters appropriately. Then, in view of (69), we have that $\dot{V} \leq 0$ holds for $V = p$: consequently, the compact set $\Pi_n \times \Pi_0$ is an invariant set and all signals of closed-loop system are SGUUB. Therefore, property (1) of Theorem 1 is proved.

Multiplying (69) by $e^{\omega t}$ and integrating over [0, t] yields

$$V(t) \leq [V(0) - \Lambda]\, e^{-\omega t} + \Lambda \tag{71}$$

with $\Lambda = a_4/\omega$ being a positive constant.

Thus, we can further have

$$\lim_{t\to\infty} |z_1| \leq \lim_{t\to\infty} \sqrt{2V} \leq \sqrt{2\Lambda} \tag{72}$$

This completes the proof of Theorem 1.

Remark 3: It is worth mentioning that (45) is only satisfied on $\Pi_i \times \Pi_0$, and we do not assume $g_i(\bar{x}_i)$ to be bounded: we relax this assumption by making use of the fact that the continuous functions $g_i(\bar{x}_i)$ are bounded on the compact set $\Pi_i \times \Pi_0$. In addition, $g_{i,m}$ and $\bar{g}_{i,M}$ may be unknown and are only used in the stability analysis.

Remark 4: It should be noted that all of the above stability analysis are achieved based on (16), (45) and (56). Specifically speaking, (16) only holds for $\Pi_{z_1} \times \Pi_0$, (45) only holds for $\Pi_i \times \Pi_0, (i = 2, ..., n-1)$, and (56) only holds for $\Pi_n \times \Pi_0$. It is also worth noting that $\Pi_n \subset \Pi_{n-1} \times R^4 \subset \cdots \subset \Pi_3 \times R^{4(n-3)} \subset \Pi_2 \times R^{4(n-2)} \subset \Pi_{z_1} \times R^{4(n-1)}$, as a result, (16), (45) and (56) are satisfied on the compact set $\Pi_n \times \Pi_0$, and all the state variables stay inside of the compact set $\Pi_n \times \Pi_0$ all the time since $\Pi_n \times \Pi_0$ is an invariant set which has been proved in the former steps.

5. Simulation results

In this section, a practical example and a numerical example are given to illustrate the effectiveness of the proposed method in this paper.

Example 1: Consider the dynamics of a one-link manipulator actuated by a brush dc (BDC) motor described as follows[25]:

$$\begin{cases} D\ddot{q} + B\dot{q} + N\sin(q) = I + \Delta_I \\ M\dot{I} = -HI - K_m\dot{q} + V \end{cases} \tag{73}$$

where q, $\dot{q}$ and $\ddot{q}$ are the link angular position, velocity, and acceleration, respectively. I denotes the motor current; Δ_I is the current disturbance; V represents the input control voltage. The parameters values with appropriate units are given in [42] by $D = 1$, $B = 1$, $M = 0.05$, $H = 0.5$, $N = 10$, and $K_m = 10$. Let the torque disturbance to be $\Delta_I = 0.2x_1\sin(x_2x_3)$ with $x_1 = q$, $x_2 = \dot{q}$ and $x_3 = I$. Define the desired reference signal $y_d = (\pi/2)\sin(t)\left(1 - e^{-0.1t^2}\right)$

Therefore, system (73) can be expressed in the following form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (-10\sin(x_1) - x_2) + x_3 + 0.2x_1\sin(x_2x_3) \\ \dot{x}_3 = -10x_2 - 10x_3 + 20u \\ y = x_1 \end{cases}$$

According to Theorem 1, we choose virtual controllers and actual control law as follows

$$\alpha_1 = -3z_1 - \frac{\hat{\vartheta}_1 z_1}{2\times 0.2^2}\psi_1^T(x_1)\psi_1(x_1) - \hat{\delta}_1 \tanh(z_1) - 0.5\dot{q}_d \tanh(z_1\dot{q}_d)$$

$$\alpha_2 = -z_2 - \frac{\hat{\vartheta}_2 z_2}{2\times 0.2^2}\psi_2^T(\bar{x}_2)\psi_2(\bar{x}_2) - \hat{\delta}_2 \tanh(z_2) - 5\frac{y_2}{0.01}\tanh\left(\frac{z_2 y_2}{0.01}\right)$$

$$u = -2z_3 - \frac{\hat{\vartheta}_3 z_3}{2\times 0.2^2}\psi_3^T(\bar{x}_3)\psi_3(\bar{x}_3) - \hat{\delta}_3 \tanh(z_3) - 0.05\frac{y_3}{0.01}\tanh\left(\frac{z_3 y_3}{0.01}\right)$$

where $z_1 = x_1 - r$, $z_2 = x_2 - \alpha_{2,f}$ and $z_3 = x_3 - \alpha_{3,f}$, and the remaining design parameters are set as: $\rho_1 = \rho_2 = 1$ $\rho_3 = 1.5$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.5$ $\beta_1 = 5$ and $\beta_2 = \beta_3 = 1$. Let the initial conditions for $[x_1(0), x_2(0), x_3(0)]^T = [0, 0, 0]^T$, $\hat{\vartheta}_1(0) = \hat{\vartheta}_2(0) = \hat{\vartheta}_3(0) = \hat{\delta}_1(0) = \hat{\delta}_2(0) = \hat{\delta}_3(0) = 0$. Accordingly, the centers and widths are chosen on a regular lattice in the respective compact set. Specifically, neural network $\Theta_2^T\ \psi_2(\bar{x}_2)$ contains 9 nodes with centers evenly spaced in the interval $[-4, 4] \times [-4, 4]$, width equals to 2, and the RBF NN for $f_3(\bar{x}_3)$ contains 11 nodes with centers evenly spaced in $[-10, 10] \times [-10, 10] \times [-10, 10]$ and width is 2 as well. The simulation results are shown as Figs.1∼5.

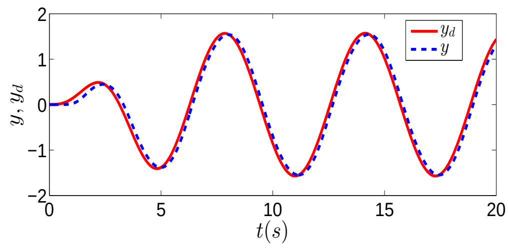

Figure 1: Angular position y and reference signal y_d

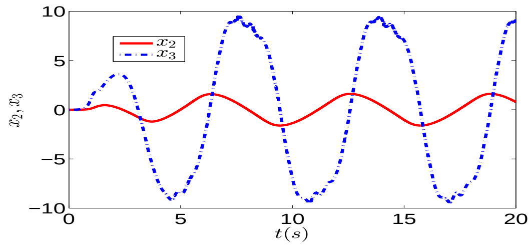

Figure 2: Angular velocity x_2 and current x_3

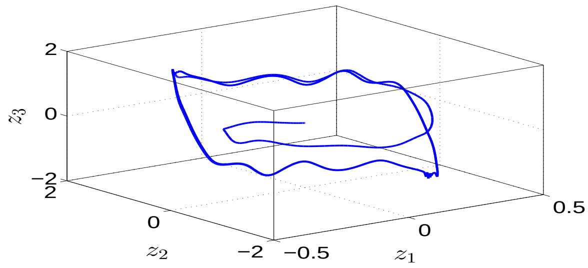

Figure 3: The phase portrait of z_1, z_2 and z_3

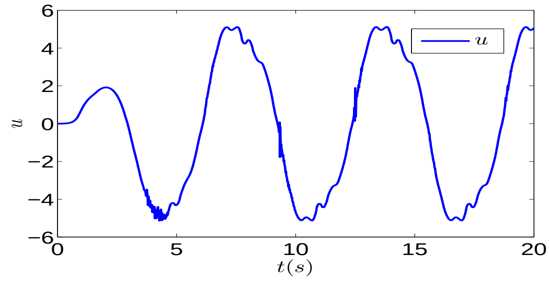

Figure 4: The curve of control input u

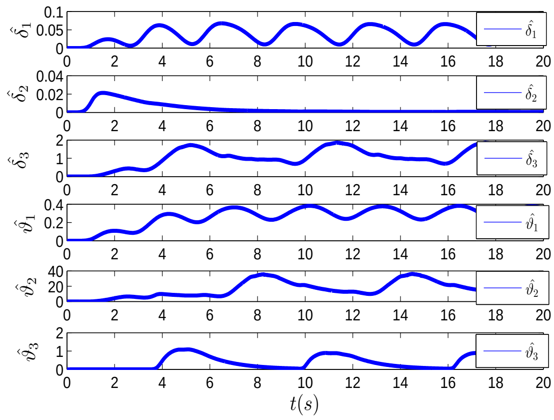

Figure 5: Curves of adaptation parameters $\hat{\delta}_1$, $\hat{\delta}_2$, $\hat{\delta}_3$, $\hat{\vartheta}_1$, $\hat{\vartheta}_2$ and $\hat{\vartheta}_3$

It can be easily known from Fig.1 that the one-link robot angular position y can follow the reference signal y_d very well and fairly good tracking performance has been achieved using the proposed scheme. The motor control input current $I(t)$ and angular velocity $\dot{q}$ are described in Fig.2. Fig.3 is given to explain phase portrait of z_1, z_2 and z_3, and control input voltage is presented in Fig.4. In addition, the response curves of adaptive parameters $\hat{\delta}_1$, $\hat{\delta}_2$ and $\hat{\delta}_3$, and $\hat{\vartheta}_1$, $\hat{\vartheta}_2$ and $\hat{\vartheta}_3$ are depicted in Fig. 5. It should be noted that only 2 adaptive parameters are needed for each subsystem in our paper by utilizing MLP technique, which reduces the number of adaptive parameters drastically.

Example 2: To further validate the applicability of the proposed method, Consider the following second-order uncertain nonlinear system[43]:

$$\begin{cases} \dot{x}_1 = x_1 e^{-0.5x_1} + \left(1 + e^{-0.1x_1^2}\right) x_2 + d_1(x,t) \\ \dot{x}_2 = x_1 x_2^2 + (3 + \cos(x_1 x_2))\, u + d_2(x,t) \\ y = x_1 \end{cases} \tag{74}$$

where $d_1(x,t) = 0.5\sin(x_2)\cos(t)$ and $d_2(x,t) = 0.5\left(x_1^2 + x_2^2\right)\sin(x_2)$. We assume the reference signal $y_d = 0.5(\sin(t) + \sin(0.5t))$.

Based on Theorem 1, the virtual control law and actual control law are designed as follows:

$$\alpha_1 = -10z_1 - \frac{\hat{\vartheta}_1 z_1}{2\times 0.25^2}\psi_1^T(x_1)\psi_1(x_1) - \hat{\delta}_1\tanh(z_1) - 2\dot{y}_d\tanh(z_1\dot{y}_d)$$

$$u = -4z_2 - \frac{\hat{\vartheta}_2 z_2}{2\times 0.25^2}\psi_2^T(\bar{x}_2)\psi_2(\bar{x}_2) - \hat{\delta}_2\tanh(z_2) - \frac{2y_2}{0.01}\tanh\left(\frac{z_2 y_2}{0.01}\right)$$

where $z_1 = x_1 - y_d$ and $z_2 = x_2 - \alpha_{2,f}$. The remaining design parameters of the simulation are taken as: $\rho_1 = \rho_2 = 1$, $\sigma_1 = \sigma_2 = 0.5$, $\beta_1 = 1$ and $\beta_2 = 2$. Let the initial conditions for $[x_1(0), x_2(0)]^T = [0.5, 0]^T$ and $\hat{\vartheta}_1(0) = \hat{\vartheta}_2(0) = \hat{\delta}_1(0) = \hat{\delta}_2(0) = 0$, respectively. The parameters of RBF NNs are the same as Example 1. It is worth mentioning that, for comparison later, the system output tracking responses of the proposed approach (scheme 1) and the robust adaptive fuzzy controller of [43] (scheme 2) are both investigated to further validate the effectiveness of the proposed method. The simulation results are shown in the following Figs. 6∼10.

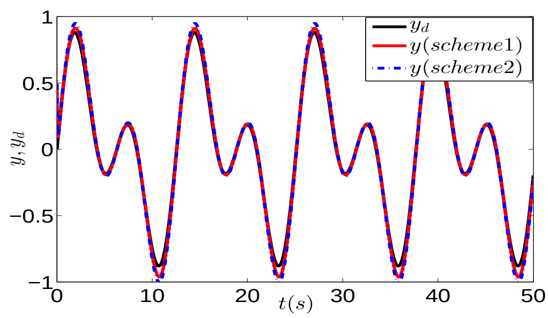

Figure 6: System output y of two schemes

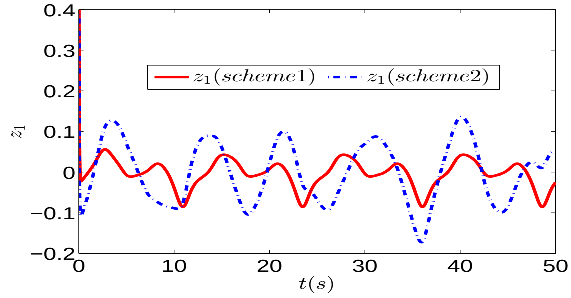

Figure 7: Tracking errors of two schemes

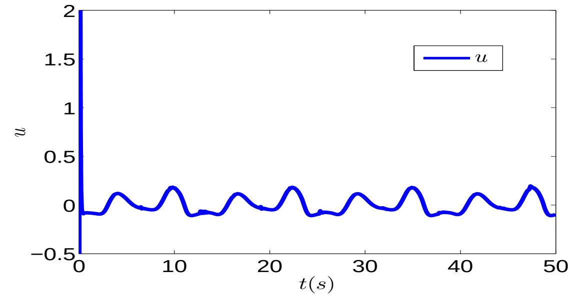

Figure 8: Control input u

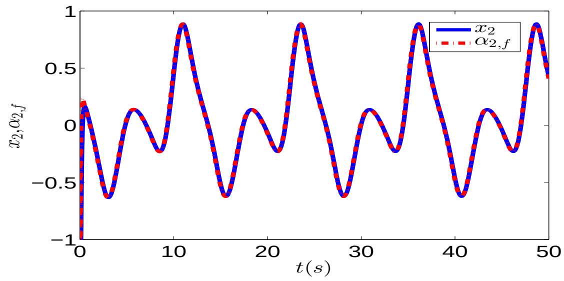

Figure 9: State x_2 and variable $\alpha_{2,f}$

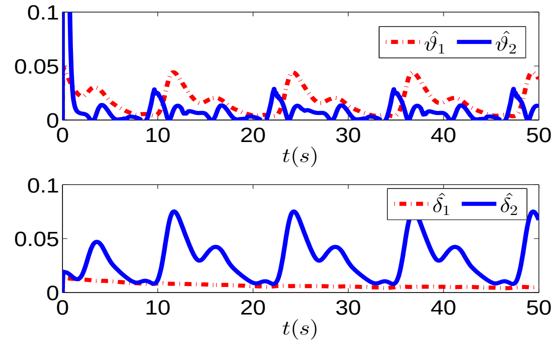

Figure 10: Adaptation parameters $\hat{\delta}_1$, $\hat{\delta}_2$, $\hat{\vartheta}_1$, and $\hat{\vartheta}_2$

The output responses under two methods are depicted in Figs. 6 and 7. Moreover, the bounds of control input u and variables x_2 and $\alpha_{2,f}$ are presented in Figs. 8 and 9, respectively. In addition, the responses curves of adaptive parameters $\hat{\delta}_1$, $\hat{\delta}_2$, $\hat{\vartheta}_1$ and $\hat{\vartheta}_2$ are depicted in Fig.10. It can be obviously observed from Figs. 6 and 7 that the proposed controller can track the reference signal as well as the robust adaptive fuzzy controller (scheme 2), and the output y and state x_2 converge rapidly to the reference signals y_d and $\alpha_{2,f}$, respectively. What's more, the tracking error of scheme 1 is smaller than scheme 2, which implies that our scheme has a better tracking performance in the presence of system nonlinear functions and external disturbances.

To further investigate the influence of different design parameters in the dynamic response of the system, three cases with three sets of design parameters are considered.

Case 1: $\sigma_1 = \sigma_2 = 0.5$, $b_1 = b_2 = 0.35$, $\upsilon_1 = \upsilon_2 = 1$; $c_1 = 10$, $c_2 = 1$, $\beta_1 = \beta_2 = 1$, $\rho_1 = \rho_2 = 2$, $\tau_2 = 0.01$, $\xi_1 = \xi_2 = 2$.

Case 2: $\sigma_1 = \sigma_2 = 0.35$, $b_1 = b_2 = 0.2$, $\upsilon_1 = \upsilon_2 = 0.75$; $c_1 = 10$, $c_2 = 2.5$, $\beta_1 = \beta_2 = 2$, $\rho_1 = \rho_2 = 3$, $\tau_2 = 0.01$, $\xi_1 = \xi_2 = 2$.

Case 3: $\sigma_1 = \sigma_2 = 0.2$, $b_1 = b_2 = 0.05$, $\upsilon_1 = \upsilon_2 = 0.5$; $c_1 = 10$, $c_2 = 3.5$, $\beta_1 = \beta_2 = 3$, $\rho_1 = \rho_2 = 4$, $\tau_2 = 0.01$, $\xi_1 = \xi_2 = 2$.

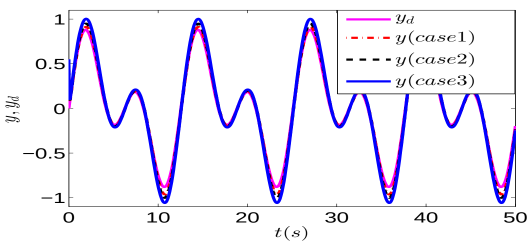

Figure 11: Output y under 3 cases

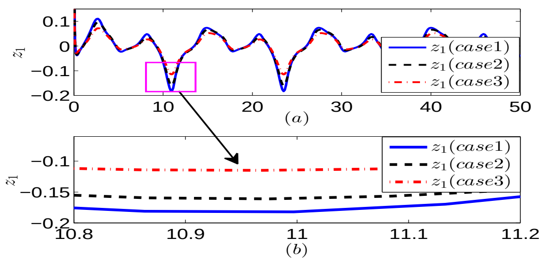

Figure 12: Tracking errors under 3 cases

Based on the closed-loop control system of Example 2, we simulate under the above three cases. And the remaining parameters remain unchanged. The system output responses of three sets of different design parameters are illustrated in Figs.11 and 12, and the simulation results imply that the tracking errors can be made small by reducing σ_i, b_i and υ_i, and meanwhile increasing c_i, β_i and ρ_i, which validates the effectiveness of the method of adjusting parameters.

Remark 5: The design parameters have various influences on the performance of the proposed scheme. In particular, the adaptation gains ρ_i and β_i in (38) and (39) are employed to tune the convergence rate of the adaptation process, and higher adaptation gains can make the convergence rate faster. In addition, the design parameters b_i and υ_i can also affect the convergence rate of the adaptation process and smaller b_i and υ_i can contribute to a faster convergence rate as well. Besides, the small positive constant σ_i is a σ-modification factor that can enhance the stability of (18), (19), (38), (39), (58) and (59) in the presence of disturbances and approximation errors. The design parameter $\xi_i \geq g_{i,m}^{-1}$, does not affect the size of tracking error z_1, and we can tune its value from trial simulations since the positive constant $g_{i,m}$ is unknown.

6. Conclusion

A novel adaptive neural control scheme based on DSC has been proposed for a more general class of uncertain strict-feedback nonlinear system. In particular, the assumption that the control gain functions must be bounded has been relaxed by assuming continuous (possibly unbounded) control gain functions, which are bounded on compact set. This significantly relaxes a severe limitation of DSC technique. Fundamental to achieving this relaxation was the construction of an invariant set that guarantees that the closed-loop trajectories do not leave appropriate compact sets. The stability of the closed-loop system has been rigorously proved by Lyapunov analysis and invariant set theory, while the tracking error has been shown to converge to a residual set that can be made as small as desired by adjusting design parameters appropriately. Finally, the performance of the proposed approach has been verified through two simulation examples.

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