INSTITUTE OF AEROPHYSICS

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ATTITUDE STABILITY OF ARTICULATED GRAVITY-ORIENTED SATELLITES

Part II - Lateral Motion

by

H. Maeda

JUNE, 1963

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SUMMARY

By a procedure similar in principle to that for the longitudinal equations of motion, the lateral equations of a specific compound satellite system were derived. The system is substantially identical with that of the previous report (Part I).

As a result of linearization for small perturbations, the effect of orbit ellipticity vanishes in the lateral motion. Both the general case, i. e. with hinged yaw-stabilizers, and a simpler case, i. e. with fixed yaw-stabilizers, are discussed. The latter is considered to be better from the practical standpoint.

After calculating numerical examples, the configuration was found to provide damping of the lateral motion to $\frac{1}{2}$ amplitude in about 0.28 orbits, which is a little better than was previously found for the longitudinal modes.

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SYMBOLS

A	total moment of inertia of a satellite (including stabilizers) about ξ_b-axis
A_1, A_2, A_3	constant coefficients (Eq. 2.15)
A_1', A_2'	constant coefficients (Eq. 2.19)
a, a'	satellite body dimensions (Fig. 1)
$a_o \ldots a_5$	constant coefficients (App.)
b, b'	stabilizer dimensions (Fig. 1)
C	total moment of inertia of a satellite (including stabilizers) about ζ_b-axis.
C'	constant coefficients (Eq. 2.19)
C_1, C_2, C_3	constant coefficients (Eq. 2.15)
$\bar{c}_1, \bar{c}_2$	damping coefficients of hinges
d	differential operator $d/d\gamma$
D	constant coefficients (Eq. 2.15)
E	constant coefficients (Eq. 2.19)
F	constant coefficients (Eq. 2.19)
I	moment of inertia by dumbbell mass
n_j	real part of the roots of characteristic equation (App.)
$O_{\frac{1}{2}}$	orbits to $\frac{1}{2}$ amplitude
T_i	constant coefficients of characteristic equations
α_k	weighting numbers (App.)
β_k	control variables (App.)
δ, ϵ	angular displacement of yaw stabilizer rods (Fig. 1)
κ, λ	angular displacements of roll stabilizer rods (Fig. 1)

μ	Lagrange multiplier (App.)
ω_j	imaginary part of the roots of characteristic eq. (App.)
Γ_1, Γ_2	$\bar{c}_1/\omega_o$, $\bar{c}_2/\omega_o$
ϕ, ψ	Euler angles giving orientation of satellite body.

I. INTRODUCTION

This report presents an analysis of the lateral motion of a compound satellite system.

The first part of the analysis (Sec. II) is the derivation of the lateral equations of motion of the system, applying the general formulae for the forces and moments given in the previous report (Part I) (Ref. 1). The system analyized is substantially the same as that of Ref. 1.

The second part (Sec. III) gives numerical solutions of the equations of motion. The following two cases are calculated separately.

(1) Case with fixed yaw-stabilizers

(2) General case, i. e. case with hinged yaw-stabilizers

In the lateral motion, no steady state oscillation occurs, so that the numerical results are only concerned with the transient motion.

The damping of lateral motion obtained in the initial series of calculations was for both cases unsatisfactory compared with that of the longitudinal motion (Part I), hence further parameter variations were made. A dumbbell mass on the Y-axis of the system was found effective to improve the lateral stability for the case with fixed yaw-stabilizers.

Finally, the so-called "steepest-descent method" (Ref. 3) is applied to optimize the solution. The actual procedure of this method is presented in Appendix 1.

II. DERIVATION OF THE LATERAL EQUATIONS OF MOTION

2.1 Lateral Equations of Motion for the Particular System

In this analysis, it is assumed that the system to be studied is the same as the particular system which is suggested in the previous report (See Ref. 1) from the standpoint of passive attitude stabilization. It consists of the satellite body, two roll stabilizer rods and two yaw stabilizers. The roll stabilizers are identical with the pitch stabilizers of the longitudinal motion, and are universally-hinged at the top and bottom of the satellite body. The yaw stabilizers are hinged at the front and back of the body and can rotate only in yaw. Subscripts δ, ϵ, κ and λ are used to denote the four stabilizers respectively, which is shown in Fig. 1 (a) and (b). Subscript b is used to denote the satellite body only. (Note: subscript b is used to express the satellite body plus two yaw stabilizers in the longitudinal case - see Ref. 1.) The damping coefficients in the yaw and roll stabilizer hinges are $\bar{c}_1$ and $\bar{c}_2$ respectively, so that the rods are acted on by couples $-\bar{c}_1\dot{\delta}$, $-\bar{c}_1\dot{\epsilon}$, $-\bar{c}_2\dot{\kappa}$ and $-\bar{c}_2\dot{\lambda}$.

As the generalized coordinates we take 6 angular displacements, i.e. ϕ, ψ, δ, ϵ, κ and λ, defined as shown in Fig. 1. ϕ, ψ correspond to the conventional Euler angles used in airplane dynamics, see Ref. 2. In this figure, C_b, C_δ, C_ϵ, C_κ and C_λ are the mass-centres of constituent bodies, and O is the mass-centre of the whole system. The mass-centre coordinates are given in terms of the generalized coordinates by

$$
\begin{aligned}
\bar{x}_\delta &= \bar{x}_b + (a' + b'\cos\delta)\cos\psi - b'\sin\delta\cos\phi\sin\psi \\
\bar{y}_\delta &= \bar{y}_b + (a' + b'\cos\delta)\sin\psi + b'\sin\delta\cos\phi\cos\psi \\
\bar{z}_\delta &= \bar{z}_b + b'\sin\delta\cdot\sin\phi \\
\bar{x}_\epsilon &= \bar{x}_b - (a' + b'\cos\epsilon)\cos\psi + b'\sin\epsilon\cos\phi\sin\psi \\
\bar{y}_\epsilon &= \bar{y}_b - (a' + b'\cos\epsilon)\sin\psi - b'\sin\epsilon\cos\phi\cos\psi \\
\bar{z}_\epsilon &= \bar{z}_b - b'\sin\epsilon\sin\phi \\
\bar{x}_\kappa &= \bar{x}_b + b\sin\kappa\cos\phi\sin\psi + (a + b\cos\kappa)\sin\phi\sin\psi \\
\bar{y}_\kappa &= \bar{y}_b - b\sin\kappa\cos\phi\cos\psi - (a + b\cos\kappa)\sin\phi\cos\psi \\
\bar{z}_\kappa &= \bar{z}_b - b\sin\kappa\sin\phi + (a + b\cos\kappa)\cos\phi \\
\bar{x}_\lambda &= \bar{x}_b - b\sin\lambda\cos\phi\sin\psi - (a + b\cos\lambda)\sin\phi\sin\psi \\
\bar{y}_\lambda &= \bar{y}_b + b\sin\lambda\cos\phi\cos\psi + (a + b\cos\lambda)\sin\phi\cos\psi \\
\bar{z}_\lambda &= \bar{z}_b + b\sin\lambda\sin\phi - (a + b\cos\lambda)\cos\phi
\end{aligned}
\tag{2.1}
$$

These positions are also connected by the following relations, which express the fact that O is the mass centre.

$$m_b \bar{x}_b + m_\delta \cdot \bar{x}_\delta + m_\epsilon \bar{x}_\epsilon + m_\kappa \bar{x}_\kappa + m_\lambda \bar{x}_\lambda = 0$$

$$m_b \bar{y}_b + m_\delta \bar{y}_\delta + m_\epsilon \bar{y}_\epsilon + m_\kappa \bar{y}_\kappa + m_\lambda \bar{y}_\lambda = 0 \qquad (2.2)$$

$$m_b \bar{z}_b + m_\delta \cdot \bar{z}_\delta + m_\epsilon \bar{z}_\epsilon + m_\kappa \cdot \bar{z}_\kappa + m_\lambda \bar{z}_\lambda = 0$$

where $\quad m_\delta = m_\epsilon , \qquad m_\kappa = m_\lambda \ (= m_\alpha)$

In Eq. (2. 1), ϕ, ψ, δ, ϵ, κ and λ are the first order small quantities, so that, using the relation of Eq. (2. 2), we find approximately,

$$\bar{x}_b = 0$$

$$\bar{y}_b = -\frac{m_\delta}{m} \cdot b'(\delta - \epsilon) + \frac{m_\kappa}{m} \cdot b(\kappa - \lambda) \qquad (2.3)$$

$$\bar{z}_b = 0$$

$$\bar{x}_\delta = a' + b'$$

$$\bar{y}_\delta = (a'+b')\psi + \left(1 - \frac{m_\delta}{m}\right) b'\delta + \frac{m_\delta}{m} b'\epsilon + \frac{m_\kappa}{m} b(\kappa - \lambda)$$

$$\bar{z}_\delta = 0$$

$$\bar{x}_\epsilon = -(a'+b')$$

$$\bar{y}_\epsilon = -(a'+b')\psi - \frac{m_\delta}{m} b'\delta - \left(1 - \frac{m_\delta}{m}\right) b'\epsilon + \frac{m_\kappa}{m} b(\kappa - \lambda)$$

$$\bar{z}_\epsilon = 0$$

$$\bar{x}_\kappa = 0$$

$$\bar{y}_\kappa = -(a+b)\phi - (1 - \frac{m_\kappa}{m}) b\kappa - \frac{m_\kappa}{m} b\lambda - \frac{m_\delta}{m} b'(\delta - \epsilon) \tag{2.4}$$

$$\bar{z}_\kappa = a + b$$

$$\bar{x}_\lambda = 0$$

$$\bar{y}_\lambda = (a+b)\phi + \frac{m_\kappa}{m} b\kappa + (1 - \frac{m_\kappa}{m}) b\lambda - \frac{m_\delta}{m} b'(\delta - \epsilon)$$

$$\bar{z}_\lambda = -(a+b)$$

where $m = m_b + 2\, m_\delta + 2\, m_\kappa$ = total mass of the satellite.

After differentiating Eq. (2.1) and (2.2) with respect to time t we neglect the higher order small quantities, so that we find approximately,

$$\dot{\bar{x}}_b = 0$$

$$\dot{\bar{y}}_b = -\frac{m_\delta}{m} b'(\dot{\delta} - \dot{\epsilon}) + \frac{m_\kappa}{m} b(\dot{\kappa} - \dot{\lambda})$$

$$\dot{\bar{z}}_b = 0$$

$$\dot{\bar{x}}_\delta = 0$$

$$\dot{\bar{y}}_\delta = (a'+b')\dot{\psi} + (1 - \frac{m_\delta}{m}) b'\dot{\delta} + \frac{m_\delta}{m} b'\dot{\epsilon} + \frac{m_\kappa}{m} b\dot{\kappa} - \frac{m_\kappa}{m} b\dot{\lambda}$$

$$\dot{\bar{z}}_\delta = 0$$

$$\dot{\bar{x}}_\epsilon = 0$$

$$\dot{\bar{y}}_\epsilon = -(a'+b')\dot{\psi} - \frac{m_\delta}{m} b'\dot{\delta} - (1 - \frac{m_\delta}{m}) b'\dot{\epsilon} + \frac{m_\kappa}{m} b\dot{\kappa} - \frac{m_\kappa}{m} b\dot{\lambda}$$

$$\dot{z}_\epsilon = 0$$

$$\dot{x}_\kappa = 0$$

$$\dot{y}_\kappa = -(a+b)\dot{\phi} - \frac{m_\delta}{m} b'\dot{\delta} + \frac{m_\delta}{m} b'\dot{\epsilon} - (1 - \frac{m_\kappa}{m}) b\dot{\kappa} - \frac{m_\kappa}{m} b\dot{\lambda}$$

$$\dot{z}_\kappa = 0$$

$$\dot{x}_\lambda = 0 \tag{2.5}$$

$$\dot{y}_\lambda = (a+b)\dot{\phi} - \frac{m_\delta}{m} b'\dot{\delta} + \frac{m_\delta}{m} b'\dot{\epsilon} + \frac{m_\kappa}{m} b\dot{\kappa} + (1 - \frac{m_\kappa}{m}) b\dot{\lambda}$$

$$\dot{z}_\lambda = 0$$

2.2 Kinetic Energy

The kinetic energies of the five constituent bodies are given by

$$T_b = \frac{1}{2} m_b (\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2) + \frac{1}{2} A_b \dot{\phi}^2 + \frac{1}{2} C_b \dot{\psi}^2 \quad *$$

$$T_\delta = \frac{1}{2} m_\delta (\dot{x}_\delta^2 + \dot{y}_\delta^2 + \dot{z}_\delta^2) + \frac{1}{2} C_\delta (\dot{\psi} + \dot{\delta})^2$$

$$T_\epsilon = \frac{1}{2} m_\epsilon (\dot{x}_\epsilon^2 + \dot{y}_\epsilon^2 + \dot{z}_\epsilon^2) + \frac{1}{2} C_\epsilon (\dot{\psi} + \dot{\epsilon})^2 \tag{2.6}$$

$$T_\kappa = \frac{1}{2} m_\kappa (\dot{x}_\kappa^2 + \dot{y}_\kappa^2 + \dot{z}_\kappa^2) + \frac{1}{2} A_\kappa (\dot{\phi} + \dot{\kappa})^2$$

$$T_\lambda = \frac{1}{2} m_\lambda (\dot{x}_\lambda^2 + \dot{y}_\lambda^2 + \dot{z}_\lambda^2) + \frac{1}{2} A_\lambda (\dot{\phi} + \dot{\lambda})^2$$

where $m_\delta = m_\epsilon$, $m_\kappa = m_\lambda$, $C_\delta = C_\epsilon$, $A_\kappa = A_\lambda$

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* The exact expression for the kinetic energies should be

$$T_b = \frac{1}{2} m_b (\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2) + \frac{1}{2} A_b \cdot P_b^2 + \frac{1}{2} C_b \cdot R_b^2$$

$$T_\delta = \frac{1}{2} m_\delta (\dot{x}_\delta^2 + \dot{y}_\delta^2 + \dot{z}_\delta^2) + \frac{1}{2} C_\delta \cdot R_\delta^2$$

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However, ϕ, ψ, δ, ϵ .... are assumed to be small quantities, so that approximately

$$P_b = \dot{\phi}, \quad R_b = \dot{\psi}, \quad R_\delta = \dot{\psi} + \dot{\delta}, \quad etc.$$

The total kinetic energy T is given by

$$T = T_b + T_\delta + T_\epsilon + T_\kappa + T_\lambda \tag{2.7}$$

After substitution of the values of Eq. (2.5) into Eq. (2.6), the partial derivatives of T required for the Lagranges equation of motion are presented as follows:

$$\frac{\partial T}{\partial \dot\phi} = m_b \dot{\bar y}_b \frac{\partial \dot{\bar y}_b}{\partial \dot\phi} + A_b \dot\phi + m_\delta \dot{\bar y}_\delta \frac{\partial \dot{\bar y}_\delta}{\partial \dot\phi} + m_\delta \dot{\bar y}_\epsilon \frac{\partial \dot{\bar y}_\epsilon}{\partial \dot\phi}$$

$$+ m_\kappa \dot{\bar y}_\kappa \frac{\partial \dot{\bar y}_\kappa}{\partial \dot\phi} + A_\kappa(\dot\phi + \dot\kappa) + m_\kappa \dot{\bar y}_\lambda \frac{\partial \dot{\bar y}_\lambda}{\partial \dot\phi} + A_\kappa(\dot\phi + \dot\lambda)$$

$$= A_b\dot\phi + A_\kappa(\dot\phi + \dot\kappa) + A_\kappa(\dot\phi + \dot\lambda) \tag{2.8}$$

$$- m_\kappa(a+b)\left[-(a+b)\dot\phi - \frac{m_\delta}{m} b'\dot\delta + \frac{m_\delta}{m} b'\dot\epsilon - \left(1 - \frac{m_\kappa}{m}\right) b\dot\kappa - \frac{m_\kappa}{m} b\dot\lambda\right]$$

$$+ m_\kappa(a+b)\left[(a+b)\dot\phi - \frac{m_\delta}{m} b'\dot\delta + \frac{m_\delta}{m} b'\dot\epsilon + \frac{m_\kappa}{m} b\dot\kappa + \left(1 - \frac{m_\kappa}{m}\right) b\dot\lambda\right]$$

$$= \dot\phi\left[A_b + 2A_\kappa + 2m_\kappa(a+b)^2\right] + \dot\kappa\left[A_\kappa + m_\kappa b(a+b)\right]$$

$$+ \dot\lambda\left[A_\kappa + m_\kappa b(a+b)\right]$$

Similarly,

$$\frac{\partial T}{\partial \dot\psi} = \dot\psi\left[C_b + 2C_\delta + 2m_\delta(a'+b')^2\right] + \dot\delta\left[C_\delta + m_\delta b'(a'+b')\right]$$

$$+ \dot\epsilon\left[C_\delta + m_\delta b'(a'+b')\right] \tag{2.9}$$

$$\frac{\partial T}{\partial \dot\delta} = \dot\psi\left[C_\delta + m_\delta b'(a'+b')\right] + \dot\delta\left[C_\delta + m_\delta\left(1 - \frac{m_\delta}{m}\right) b'^2\right]$$

$$+ \dot\epsilon \cdot \frac{m_\delta^2}{m} \cdot b'^2 + \dot\kappa \frac{m_\delta \cdot m_\kappa}{m} bb' - \dot\lambda \frac{m_\delta m_\kappa}{m} bb' \tag{2.10}$$

$$\frac{\partial T}{\partial \dot\epsilon} = \dot\psi\left[C_\delta + m_\delta b'(a'+b')\right] + \dot\delta \frac{m_\delta^2}{m} b'^2$$

$$+ \dot\epsilon\left[C_\delta + m_\delta\left(1 - \frac{m_\delta}{m}\right) b'^2\right] - \dot\kappa \frac{m_\delta m_\kappa}{m} bb' + \dot\lambda \frac{m_\delta m_\kappa}{m} bb' \tag{2.11}$$

$$\frac{\partial T}{\partial \dot{\kappa}} = \dot{\phi}\left[A_K + m_K b(a+b)\right] + \dot{\delta}\,\frac{m_\delta \cdot m_K}{m} b b' - \dot{\epsilon}\,\frac{m_\delta \cdot m_K}{m} b b'$$

$$+ \dot{\kappa}\left[A_K + m_K\left(1 - \frac{m_K}{m}\right) b^2\right] + \dot{\lambda}\,\frac{m_K^2}{m} b^2 \qquad (2.12)$$

$$\frac{\partial T}{\partial \dot{\lambda}} = \dot{\phi}\left[A_K + m_K b(a+b)\right] - \dot{\delta}\,\frac{m_\delta m_K}{m} b b' + \dot{\epsilon}\,\frac{m_\delta m_K}{m} b b'$$

$$+ \dot{\kappa}\,\frac{m_K^2}{m} b^2 + \dot{\lambda}\left[A_K + m_K\left(1 - \frac{m_K}{m}\right) b^2\right] \qquad (2.13)$$

And

$$\frac{\partial T}{\partial \phi} = \frac{\partial T}{\partial \psi} = \frac{\partial T}{\partial \delta} = \frac{\partial T}{\partial \epsilon} = \frac{\partial T}{\partial \kappa} = \frac{\partial T}{\partial \lambda} = 0 \qquad (2.14)$$

The equations of motion therefore become

$$A\ddot{\phi} + A_1\ddot{\kappa} + A_1\ddot{\lambda} = \frac{\partial W}{\partial \phi}$$

$$C\ddot{\psi} + C_1\ddot{\delta} + C_1\ddot{\epsilon} = \frac{\partial W}{\partial \psi}$$

$$C_1\ddot{\psi} + C_2\ddot{\delta} + C_3\ddot{\epsilon} + D\ddot{\kappa} - D\ddot{\lambda} = \frac{\partial W}{\partial \delta}$$

$$C_1\ddot{\psi} + C_3\ddot{\delta} + C_2\ddot{\epsilon} - D\ddot{\kappa} + D\ddot{\lambda} = \frac{\partial W}{\partial \epsilon}$$

$$A_1\ddot{\phi} + D\ddot{\delta} - D\ddot{\epsilon} + A_2\ddot{\kappa} + A_3\ddot{\lambda} = \frac{\partial W}{\partial \kappa}$$

$$A_1\ddot{\phi} - D\ddot{\delta} + D\ddot{\epsilon} + A_3\ddot{\kappa} + A_2\ddot{\lambda} = \frac{\partial W}{\partial \lambda} \qquad (2.15)$$

where

$$A = A_b + 2A_K + 2m_K(a+b)^2$$

: total moment of inertia about ξ_b-axis

$$A_1 = A_K + m_K b(a+b)$$

$$A_2 = A_K + m_K\left(1 - \frac{m_K}{m}\right) b^2$$

$$A_3 = \frac{m_K^2}{m} b^2$$

$$C = C_b + 2C_\delta + 2m_\delta(a'+b')^2$$

: total moment of inertia about ζ_b -axis

$$C_1 = C_\delta + m_\delta b'(a'+b')$$

$$C_2 = C_\delta + m_\delta\left(1-\frac{m_\delta}{m}\right)b'^2$$

$$C_3 = \frac{m_\delta^2}{m}b'^2$$

$$D = \frac{m_\delta \cdot m_\kappa}{m}bb'$$

2.3 Generalized Forces

Since we deal with the lateral motion, the total work done is given by

$$\begin{aligned}\delta\overline{W} = {} & F_{xb}\cdot\delta\bar{x}_b + F_{yb}\cdot\delta\bar{y}_b + F_{zb}\cdot\delta\bar{z}_b + L_b\cdot\delta\phi + N_b\cdot\delta\psi \\ & + F_{x\delta}\cdot\delta\bar{x}_\delta + F_{y\delta}\cdot\delta\bar{y}_\delta + F_{z\delta}\cdot\delta\bar{z}_\delta + N_\delta\cdot\delta\psi_\delta - \bar{c}_1\dot{\delta}\cdot\delta(\delta) \\ & + F_{x\epsilon}\cdot\delta\bar{x}_\epsilon + F_{y\epsilon}\cdot\delta\bar{y}_\epsilon + F_{z\epsilon}\cdot\delta\bar{z}_\epsilon + N_\epsilon\cdot\delta\psi_\epsilon - \bar{c}_1\dot{\epsilon}\cdot\delta\epsilon \\ & + F_{x\kappa}\cdot\delta\bar{x}_\kappa + F_{y\kappa}\cdot\delta\bar{y}_\kappa + F_{z\kappa}\cdot\delta\bar{z}_\kappa + L_\kappa\cdot\delta\phi_\kappa - \bar{c}_2\dot{\kappa}\cdot\delta\kappa \\ & + F_{x\lambda}\cdot\delta\bar{x}_\lambda + F_{y\lambda}\cdot\delta\bar{y}_\lambda + F_{z\lambda}\cdot\delta\bar{z}_\lambda + L_\lambda\cdot\delta\phi_\lambda - \bar{c}_2\dot{\lambda}\cdot\delta\lambda \qquad (2.16)\end{aligned}$$

note: $\psi_\delta = \psi + \delta,\quad \psi_\epsilon = \psi + \epsilon$

$\phi_\kappa = \phi + \kappa,\quad \phi_\lambda = \phi + \lambda$

The generalized forces are obtained from the virtual work $\delta\overline{W}$ as follows:

$$\begin{aligned}\mathcal{F}_\phi = {} & \frac{\partial W}{\partial\phi} \\ = {} & (L_b + L_\kappa + L_\lambda) + F_{xb}\frac{\partial\bar{x}_b}{\partial\phi} + F_{yb}\frac{\partial\bar{y}_b}{\partial\phi} + F_{zb}\frac{\partial\bar{z}_b}{\partial\phi} \\ & + F_{x\delta}\frac{\partial\bar{x}_\delta}{\partial\phi} + F_{y\delta}\frac{\partial\bar{y}_\delta}{\partial\phi} + F_{z\delta}\frac{\partial\bar{z}_\delta}{\partial\phi} + F_{x\epsilon}\frac{\partial\bar{x}_\epsilon}{\partial\phi} + F_{y\epsilon}\frac{\partial\bar{y}_\epsilon}{\partial\phi} + F_{z\epsilon}\frac{\partial\bar{z}_\epsilon}{\partial\phi} \\ & + F_{x\kappa}\frac{\partial\bar{x}_\kappa}{\partial\phi} + F_{y\kappa}\frac{\partial\bar{y}_\kappa}{\partial\phi} + F_{z\kappa}\frac{\partial\bar{z}_\kappa}{\partial\phi} + F_{x\lambda}\frac{\partial\bar{x}_\lambda}{\partial\phi} + F_{y\lambda}\frac{\partial\bar{y}_\lambda}{\partial\phi} + F_{z\lambda}\frac{\partial\bar{z}_\lambda}{\partial\phi}\end{aligned}$$

Similarly, other forces are

$$\mathcal{F}_\psi = \frac{\partial W}{\partial \psi}$$

$$\mathcal{F}_\delta = \frac{\partial W}{\partial \delta}$$

$$\mathcal{F}_\epsilon = \frac{\partial W}{\partial \epsilon}$$

$$\mathcal{F}_\kappa = \frac{\partial W}{\partial \kappa}$$

$$\mathcal{F}_\lambda = \frac{\partial W}{\partial \lambda} \qquad (2.17)$$

In Eq. (2.17), the forces and moments are given by Eq. (2.26) and Eq. (2.29) of Ref. 1, i.e.

$$F_x = m\omega_0^2\left[(e\cos\gamma)\bar{x} - (2e\sin\gamma)\bar{z} + 2\frac{d\bar{z}}{d\gamma}\right]$$

$$F_y = -m\omega_0^2(1+3e\cos\gamma)\bar{y}$$

$$F_z = m\omega_0^2\left[(2e\sin\gamma)\bar{x} + (3+10e\cos\gamma)\bar{z} - 2\frac{d\bar{x}}{d\gamma}\right]$$

$$L = \omega_0^2\left[4(C-B)\phi + (A+C-B)\frac{d\psi}{d\gamma}\right]$$

$$N = -\omega_0^2\left[(B-A)\psi + (A+C-B)\frac{d\phi}{d\gamma}\right] \qquad (2.18)$$

Furthermore, using Eq. (2.1) and (2.2), all derivatives involved in the generalized forces are given in the following table. With Eq. (2.18) and the values of the table, therefore, the generalized forces in Eq. (2.17) can be reduced, after some calculation, to equations (2.19).

	$\frac{\partial}{\partial\phi}$	$\frac{\partial}{\partial\psi}$	$\frac{\partial}{\partial\delta}$	$\frac{\partial}{\partial\epsilon}$	$\frac{\partial}{\partial\kappa}$	$\frac{\partial}{\partial\lambda}$
$\bar{x}_b$	0	$\frac{m_\delta}{m}b'(\delta-\epsilon) - \frac{m_\kappa}{m}b(\kappa-\lambda)$	$\frac{m_\delta}{m}b'(\delta+\psi)$	$-\frac{m_\delta}{m}b'(\epsilon+\psi)$	$-\frac{m_\kappa}{m}b\psi$	$\frac{m_\kappa}{m}b\psi$
$\bar{x}_\delta$	0	$\frac{m_\delta}{m}b'(\delta-\epsilon) - \frac{m_\kappa}{m}b(\kappa-\lambda) - (a'+b')\psi - b'\delta$	$\frac{m_\delta}{m}b'(\delta+\psi) - b'(\delta+\psi)$	$-\frac{m_\delta}{m}b'(\epsilon+\psi)$	$-\frac{m_\kappa}{m}b\psi$	$\frac{m_\kappa}{m}b\psi$
$\bar{x}_\epsilon$	0	$\frac{m_\delta}{m}b'(\delta-\epsilon) - \frac{m_\kappa}{m}b(\kappa-\lambda) + (a'+b')\psi + b'\epsilon$	$\frac{m_\delta}{m}b'(\delta+\psi)$	$-\frac{m_\delta}{m}b'(\epsilon+\psi) + b'(\epsilon+\psi)$	$-\frac{m_\kappa}{m}b\psi$	$\frac{m_\kappa}{m}b\psi$
$\bar{x}_\kappa$	$(a+b)\psi$	$\frac{m_\delta}{m}b'(\delta-\epsilon) - \frac{m_\kappa}{m}b(\kappa-\lambda) + b\kappa + (a+b)\phi$	$\frac{m_\delta}{m}b'(\delta+\psi)$	$-\frac{m_\delta}{m}b'(\epsilon+\psi)$	$-\frac{m_\kappa}{m}b\psi + b\psi$	$\frac{m_\kappa}{m}b\psi$
$\bar{x}_\lambda$	$-(a+b)\psi$	$\frac{m_\delta}{m}b'(\delta-\epsilon) - \frac{m_\kappa}{m}b(\kappa-\lambda) - b\lambda - (a+b)\phi$	$\frac{m_\delta}{m}b'(\delta+\psi)$	$-\frac{m_\delta}{m}b'(\epsilon+\psi)$	$-\frac{m_\kappa}{m}b\psi$	$\frac{m_\kappa}{m}b\psi - b\psi$
$\bar{y}_b$	0	0	$-\frac{m_\delta}{m}b'$	$\frac{m_\delta}{m}b'$	$\frac{m_\kappa}{m}b$	$-\frac{m_\kappa}{m}b$
$\bar{y}_\delta$	0	$a'+b'$	$(1-\frac{m_\delta}{m})b'$	$\frac{m_\delta}{m}b'$	$\frac{m_\kappa}{m}b$	$-\frac{m_\kappa}{m}b$
$\bar{y}_\epsilon$	0	$-(a'+b')$	$-\frac{m_\delta}{m}b'$	$-(1-\frac{m_\delta}{m})b'$	$\frac{m_\kappa}{m}b$	$-\frac{m_\kappa}{m}b$
$\bar{y}_\kappa$	$-(a+b)$	0	$-\frac{m_\delta}{m}b'$	$\frac{m_\delta}{m}b'$	$-(1-\frac{m_\kappa}{m})b$	$-\frac{m_\kappa}{m}b$
$\bar{y}_\lambda$	$a+b$	0	$-\frac{m_\delta}{m}b'$	$\frac{m_\delta}{m}b'$	$\frac{m_\kappa}{m}b$	$(1-\frac{m_\kappa}{m})b$
$\bar{z}_b$	$-\frac{m_\delta}{m}b'(\delta-\epsilon) + \frac{m_\kappa}{m}b(\kappa-\lambda)$	0	$-\frac{m_\delta}{m}b'\phi$	$\frac{m_\delta}{m}b'\phi$	$\frac{m_\kappa}{m}b(\phi+\kappa)$	$-\frac{m_\kappa}{m}b(\phi+\lambda)$
$\bar{z}_\delta$	$-\frac{m_\delta}{m}b'(\delta-\epsilon) + \frac{m_\kappa}{m}b(\kappa-\lambda) + b'\delta$	0	$-\frac{m_\delta}{m}b'\phi + b'\phi$	$\frac{m_\delta}{m}b'\phi$	$\frac{m_\kappa}{m}b(\phi+\kappa)$	$-\frac{m_\kappa}{m}b(\phi+\lambda)$
$\bar{z}_\epsilon$	$-\frac{m_\delta}{m}b'(\delta-\epsilon) + \frac{m_\kappa}{m}b(\kappa-\lambda) - b'\epsilon$	0	$-\frac{m_\delta}{m}b'\phi$	$\frac{m_\delta}{m}b'\phi - b'\phi$	$\frac{m_\kappa}{m}b(\phi+\kappa)$	$-\frac{m_\kappa}{m}b(\phi+\lambda)$
$\bar{z}_\kappa$	$-\frac{m_\delta}{m}b'(\delta-\epsilon) + \frac{m_\kappa}{m}b(\kappa-\lambda) - b\kappa - (a+b)\phi$	0	$-\frac{m_\delta}{m}b'\phi$	$\frac{m_\delta}{m}b'\phi$	$\frac{m_\kappa}{m}b(\phi+\kappa) - b(\phi+\kappa)$	$-\frac{m_\kappa}{m}b(\phi+\lambda)$
$\bar{z}_\lambda$	$-\frac{m_\delta}{m}b'(\delta-\epsilon) + \frac{m_\kappa}{m}b(\kappa-\lambda) + b\lambda + (a+b)\phi$	0	$-\frac{m_\delta}{m}b'\phi$	$\frac{m_\delta}{m}b'\phi$	$\frac{m_\kappa}{m}b(\phi+\kappa)$	$-\frac{m_\kappa}{m}b(\phi+\lambda) + b(\phi+\lambda)$

$$\mathfrak{F}_\phi = -\omega_0^2\left\{\phi\cdot 4[A-(A_b+C_b-B_b)] - \frac{d\psi}{d\gamma}(A_b+C_b-B_b) + \kappa\cdot 4A_1 + \lambda\cdot 4A_1\right\}$$

$$= -\omega_0^2\left\{F\cdot\phi - E\frac{d\psi}{d\gamma} + A_1'\kappa + A_1'\lambda\right\}$$

$$\mathfrak{F}_\psi = -\omega_0^2\left\{\frac{d\phi}{d\gamma}(A_b+C_b-B_b) + \psi[C-(A_b+C_b-B_b)] + \delta\cdot C_1 + \epsilon\cdot C_1\right\}$$

$$= -\omega_0^2\left\{E\frac{d\phi}{d\gamma} + C'\psi + C_1\delta + C_1\epsilon\right\}$$

$$\mathfrak{F}_\delta = -\omega_0^2\left\{\psi\cdot C_1 + \frac{\bar{C}_1}{\omega_0}\cdot\frac{d\delta}{d\gamma} + \delta\cdot C_2 + \epsilon\cdot C_3 + \kappa D - \lambda D\right\}$$

$$= -\omega_0^2\left\{C_1\psi + T_1\frac{d\delta}{d\gamma} + C_2\delta + C_3\epsilon + D\kappa - D\lambda\right\}$$

$$\mathfrak{F}_\epsilon = -\omega_0^2\left\{\psi C_1 + \frac{\bar{C}_1}{\omega_0}\cdot\frac{d\epsilon}{d\gamma} + \delta\cdot C_3 + \epsilon\cdot C_2 - \kappa D + \lambda D\right\}$$

$$= -\omega_0^2\left\{C_1\psi + C_3\delta + T_1\frac{d\epsilon}{d\gamma} + C_2\epsilon - D\kappa + D\lambda\right\}$$

$$\mathfrak{F}_\kappa = -\omega_0^2\left\{\phi\cdot 4A_1 + \frac{\bar{C}_2}{\omega_0}\cdot\frac{d\kappa}{d\gamma} + \delta\cdot D - \epsilon\cdot D + \kappa(3A_1+A_2) + \lambda A_3\right\}$$

$$= -\omega_0^2\left\{A_1'\phi + D\delta - D\epsilon + T_2\frac{d\kappa}{d\gamma} + A_2'\kappa + A_3\lambda\right\}$$

$$\mathfrak{F}_\lambda = -\omega_0^2\left\{\phi\cdot 4A_1 + \frac{\bar{C}_2}{\omega_0}\cdot\frac{d\lambda}{d\gamma} - \delta D + \epsilon D + \kappa A_3 + \lambda(3A_1+A_2)\right\}$$

$$= -\omega_0^2\left\{A_1'\phi - D\delta + D\epsilon + A_3\kappa + T_2\frac{d\lambda}{d\gamma} + A_2'\lambda\right\} \qquad (2.19)$$

where additional simple notations are as follows:

$$E = A_b + C_b - B_b\ , \qquad F = 4(A-E)$$

$$A_1' = 4A_1\ , \qquad A_2' = 3A_1 + A_2$$

$$C' = C - E$$

$$T_1 = \frac{\bar{C}_1}{\omega_0}\ , \qquad T_2 = \frac{\bar{C}_2}{\omega_0}$$

By combining Eq. (2. 15) and (2. 19) and noting $\frac{d^2}{dt^2} = \omega_0^2 \cdot \frac{d^2}{d\gamma}$, the equations of motion are, finally,

$$
\begin{bmatrix}
Ad^2+F & -Ed & 0 & 0 & A_1d^2+A_1' & A_1d^2+A_1' \\
Ed & Cd^2+C' & C_1d^2+C_1 & C_1d^2+C_1 & 0 & 0 \\
0 & C_1d^2+C_1 & C_2d^2+\Gamma_1 d+C_2 & C_3d^2+C_3 & Dd^2+D & -(Dd^2+D) \\
0 & C_1d^2+C_1 & C_3d^2+C_3 & C_2d^2+\Gamma_1 d+C_2 & -(Dd^2+D) & Dd^2+D \\
A_1d^2+A_1' & 0 & Dd^2+D & -(Dd^2+D) & A_2d^2+\Gamma_2 d+A_2' & A_3d^2+A_3 \\
A_1d^2+A_1' & 0 & -(Dd^2+D) & Dd^2+D & A_3d^2+A_3 & A_2d^2+\Gamma_2 d+A_2'
\end{bmatrix}
\begin{bmatrix} \phi \\ \psi \\ \delta \\ \epsilon \\ K \\ \lambda \end{bmatrix} = 0 \tag{2. 20}
$$

where $d = d/d\gamma$.

As a result of assumption $e \ll 1$, no effect of the ellipticity of the orbit appears in the equations of motion. Furthermore, since Eq. (2. 20) is the homogeneous equations, the disturbed motion is only the transient motion and no forced motion occurs.

III. SOLUTION OF THE EQUATIONS OF MOTION

3. 1 Characteristic Equation (i) with fixed Yaw-Stabilizers

Since the characteristic equation of the lateral motion, derived from Eq. (2. 20), is the equation of the 12th degree, it is too complicated and inconvenient to discuss. Hence, to begin with, we assume a simpler case, i. e. with fixed yaw-stabilizers, in which case

$$\delta = \epsilon = 0 \tag{3. 1}$$

and from Eq. (2. 20), the equations of motion become

$$
\begin{bmatrix}
Ad^2+F & -Ed & A_1d^2+A_1' & A_1d^2+A_1' \\
Ed & Cd^2+C' & 0 & 0 \\
A_1d^2+A_1' & 0 & A_2d^2+\Gamma_2 d+A_2' & A_3d^2+A_3 \\
A_1d^2+A_1' & 0 & A_3d^2+A_3 & A_2d^2+\Gamma_2 d+A_2'
\end{bmatrix}
\begin{bmatrix} \phi \\ \psi \\ K \\ \lambda \end{bmatrix} = 0 \tag{3. 2}
$$

where

$$C = C_b + 2C_\delta + 2m_\delta (a' + b')^2$$

$$B = B_b + 2B_\delta + 2m_\delta (a' + b')^2$$

and assumed

$$a' = a \qquad \text{(spherical body)}$$

$$C_\delta = B_\delta = \frac{1}{3} m_\delta b'^2 \qquad \text{(slender rods)}$$

The characteristic equation of Eq. (3.2) is

$$\begin{vmatrix} A\lambda^2 + F & -E\lambda & A_1\lambda^2 + A_1' & A_1\lambda^2 + A_1' \\ E\lambda & C\lambda^2 + C' & 0 & 0 \\ A_1\lambda^2 + A_1' & 0 & A_2\lambda^2 + T_2\lambda + A_2' & A_3\lambda^2 + A_3 \\ A_1\lambda^2 + A_1' & 0 & A_3\lambda^2 + A_3 & A_2\lambda^2 + T_2\lambda + A_2' \end{vmatrix} = 0 \tag{3.3}$$

It has the expansion

$$\left[(A_2 - A_3)\lambda^2 + T_2\lambda + (A_2' - A_3) \right] \times$$

$$\begin{vmatrix} A\lambda^2 + F & -E\lambda & 2(A_1\lambda^2 + A_1') \\ E\lambda & C\lambda^2 + C' & 0 \\ A_1\lambda^2 + A_1' & 0 & (A_2 + A_3)\lambda^2 + T_2\lambda + (A_2' + A_3) \end{vmatrix} = 0 \tag{3.4}$$

The characteristic equation can, therefore, be factored into two equations, i.e. the quadratic

$$(A_2 - A_3)\lambda^2 + T_2\lambda + (A_2' - A_3) = 0 \tag{3.5}$$

and the sextic

$$T_6\lambda^6 + T_5\lambda^5 + T_4\lambda^4 + T_3\lambda^3 + T_2\lambda^2 + T_1\lambda + T_0 = 0 \tag{3.6}$$

where

$$T_6 = AC(A_2+A_3) - 2A_1^2 C$$

$$T_5 = T_2 AC$$

$$T_4 = (AC'+FC+E^2)(A_2+A_3) + AC(A_2'+A_3) - 2(2A_1A_1'C + A_1^2C')$$

$$T_3 = T_2(AC'+FC+E^2)$$

$$T_2 = FC'(A_2+A_3) + (AC'+FC+E^2)(A_2'+A_3) - 2(A_1'^2C + 2A_1A_1'C')$$

$$T_1 = T_2 FC'$$

$$T_0 = FC'(A_2'+A_3) - 2A_1'^2C'$$

The quadratic equation (3.5) corresponds to the symmetric or "staggering" mode, because if $\phi = \psi = 0$, $\lambda = -K$ are substituted into Eq. (3.2), the first and second equations are identically satisfied, and either of the remaining two equations will become

$$(A_2 - A_3)d^2K + T_2 dK + (A_2' - A_3)K = 0 \tag{3.7}$$

Hence, the characteristic equation of this mode is identical with Eq. (3.5). This mode of motion is illustrated in Fig. 2(a). The sextic equation (3.6) can likewise be identified as the characteristic equation associated with the antisymmetric modes, for which $K = \lambda$ (Fig. 2(b)).

For example, if the satellite body is an uniform sphere and it has no yaw-stabilizer, by definition

$$A_b = B_b = C_b \quad , \qquad C = C_b$$

$$\therefore \quad C' = 0$$

hence in the sectic equation (3.6)

$$T_1 = T_0 = 0 \tag{3. 8}$$

i.e. the characteristic equation has two zero roots. The mode of motion corresponding to those zero roots will naturally be considered the yawing motion, and it means the satellite has no directional sense.

However, in the case with fixed yaw-stabilizers, C' is not zero but positive by definition, and therefore the yawing motion will be oscillatory, and by the coupling effect between rolling and yawing motion, we can expect the possibility to damp out the transient yawing motion, or in other words, to stabilize the whole system.

3.2 Characteristic Equation (ii) General Case

The characteristic equation of the general case, i.e. with hinged yaw-stabilizers, becomes from Eq. (2.20)

$$\begin{vmatrix} A\lambda^2+F & -E\lambda & 0 & 0 & A_1\lambda^2+A_1' & A_1\lambda^2+A_1' \\ E\lambda & C\lambda^2+C' & C_1\lambda^2+C_1 & C_1\lambda^2+C_1 & 0 & 0 \\ 0 & C_1\lambda^2+C_1 & C_2\lambda^2+T_1\lambda+C_2 & C_3\lambda^2+C_3 & D\lambda^2+D & -(D\lambda^2+D) \\ 0 & C_1\lambda^2+C_1 & C_3\lambda^2+C_3 & C_2\lambda^2+T_1\lambda+C_2 & -(D\lambda^2+D) & D\lambda^2+D \\ A_1\lambda^2+A_1' & 0 & D\lambda^2+D & -(D\lambda^2+D) & A_2\lambda^2+T_2\lambda+A_2' & A_3\lambda^2+A_3 \\ A_1\lambda^2+A_1' & 0 & -(D\lambda^2+D) & D\lambda^2+D & A_3\lambda^2+A_3 & A_2\lambda^2+T_2\lambda+A_2' \end{vmatrix} = 0 \qquad (3.9)$$

After some manipulation of the determinant, it becomes

$$\begin{vmatrix} A\lambda^2+F & -E\lambda & 0 & 2(A_1\lambda^2+A_1') & 0 & A_1\lambda^2+A_1' \\ E\lambda & C\lambda^2+C' & 2(C_1\lambda^2+C_1) & 0 & C_1\lambda^2+C_1 & 0 \\ 0 & C_1\lambda^2+C_1 & [(C_2+C_3)\lambda^2+T_1\lambda+(C_2+C_3)] & 0 & C_3\lambda^2+C_3 & -(D\lambda^2+D) \\ A_1\lambda^2+A_1' & 0 & 0 & [(A_2+A_3)\lambda^2+T_2\lambda+(A_2'+A_3)] & -(D\lambda^2+D) & A_3\lambda^2+A_3 \\ 0 & 0 & 0 & 0 & [(C_2-C_3)\lambda^2+T_1\lambda+(C_2-C_3)] & 2(D\lambda^2+D) \\ 0 & 0 & 0 & 0 & 2(D\lambda^2+D) & [(A_2-A_3)\lambda^2+T_2\lambda+(A_2'-A_3')] \end{vmatrix} = 0 \qquad (3.10)$$

This equation has the expansion

$$\begin{vmatrix} (C_2-C_3)\lambda^2+T_1\lambda+(C_2-C_3) & 2(D\lambda^2+D) \\ 2(D\lambda^2+D) & (A_2-A_3)\lambda^2+T_2\lambda+(A_2'-A_3) \end{vmatrix} \times$$

$$\begin{vmatrix} A\lambda^2+F & -E\lambda & 0 & 2(A_1\lambda^2+A_1') \\ E\lambda & C\lambda^2+C' & 2(C_1\lambda^2+C_1) & 0 \\ 0 & C_1\lambda^2+C_1 & [(C_2+C_3)\lambda^2+T_1\lambda+(C_2+C_3)] & 0 \\ A_1\lambda^2+A_1' & 0 & 0 & [(A_2+A_3)\lambda^2+T_2\lambda+(A_2'+A_3)] \end{vmatrix} = 0 \qquad (3.11)$$

The characteristic equation can, therefore, be separated into two equations, i.e. the quartic equation and the equation of the 8th degree. In a similar way as the previous case, the former corresponds to the symmetric modes, for which

$$\phi = \psi = 0, \quad \delta = -\epsilon, \quad \kappa = -\lambda$$

and the latter corresponds to the antisymmetric modes, for which

$$\delta = \epsilon, \quad \kappa = \lambda$$

These motions are illustrated in Fig. 3 (a) and (b).

3.3 Numerical Examples

Numerical examples are divided into two groups, i.e. the case with fixed yaw-stabilizers and the general case. The numerical data which have been used for calculation are as follows:*

$$\frac{m_\kappa}{m_b} = .005\frac{b}{a}, \qquad \frac{m_\delta}{m_b} = .005\frac{b'}{a}$$

$$1 \leq \frac{b}{a} \leq 11, \qquad 0 \leq \frac{b'}{a} \leq 5$$

$$0 \leq \hat{T}_1 = \frac{T_1}{A_b} \leq 1.0, \qquad 0 \leq \hat{T}_2 = \frac{T_2}{A_b} \leq 1.0$$

$$A_b = B_b = C_b = 0.4\, m_b a^2 \qquad \text{(uniform sphere)}$$

$$A_\kappa = \frac{1}{3} m_\kappa b^2, \qquad C_\delta = \frac{1}{3} m_\delta b'^2$$

---

* The numerical values given here are almost the same as those given in the previous report (see Ref. 1). However, owing to some basic assumptions, e.g. the satellite body is assumed to be a sphere with uniform mass distribution, in this report a few values have been changed slightly.

Since it is convenient to make the characteristic equations non-dimensional during the actual solution, the formulae for the various coefficients which occur in the characteristic equations are expressed as follows:

$$\hat{A} = \frac{A}{A_b} = 1 + \frac{5}{3}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)^2 + 5\left(\frac{m_K}{m_b}\right)\left(1+\frac{b}{a}\right)^2$$

$$\hat{A}_1 = \frac{A_1}{A_b} = \frac{5}{6}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)^2 + \frac{5}{2}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)\left(1+\frac{b}{a}\right)$$

$$\hat{A}_2 = \frac{A_2}{A_b} = \frac{5}{6}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)^2 + \frac{5}{2}\left(\frac{m_K}{m_b}\right)\left(1-\frac{m_K}{m}\right)\left(\frac{b}{a}\right)^2$$

$$\hat{A}_3 = \frac{A_3}{A_b} = \frac{5}{2}\left(\frac{m_K}{m_b}\right)\left(\frac{m_K}{m}\right)\left(\frac{b}{a}\right)^2$$

$$\hat{A}_1' = \frac{A_1'}{A_b} = \frac{10}{3}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)^2 + 10\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)\left(1+\frac{b}{a}\right)$$

$$\hat{A}_2' = \frac{A_2'}{A_b} = \frac{10}{3}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)^2 + \frac{5}{2}\left(\frac{m_K}{m_b}\right)\left(1-\frac{m_K}{m}\right)\left(\frac{b}{a}\right)^2 + \frac{15}{2}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)\left(1+\frac{b}{a}\right)$$

$$\hat{C} = \frac{C}{A_b} = 1 + \frac{5}{3}\left(\frac{m_\delta}{m_b}\right)\left(\frac{b'}{a}\right)^2 + 5\left(\frac{m_\delta}{m_b}\right)\left(1+\frac{b'}{a}\right)^2$$

$$\hat{C}' = \frac{C'}{A_b} = \hat{C} - 1$$

$$\hat{C}_1 = \frac{C_1}{A_b} = \frac{5}{6}\left(\frac{m_\delta}{m_b}\right)\left(\frac{b'}{a}\right)^2 + \frac{5}{2}\left(\frac{m_\delta}{m_b}\right)\left(\frac{b'}{a}\right)\left(1+\frac{b'}{a}\right)$$

$$\hat{C}_2 = \frac{C_2}{A_b} = \frac{5}{6}\left(\frac{m_\delta}{m_b}\right)\left(\frac{b'}{a}\right)^2 + \frac{5}{2}\left(\frac{m_\delta}{m_b}\right)\left(1-\frac{m_\delta}{m}\right)\left(\frac{b'}{a}\right)^2$$

$$\hat{C}_3 = \frac{C_3}{A_b} = \frac{5}{2}\left(\frac{m_\delta}{m_b}\right)\left(\frac{m_\delta}{m}\right)\left(\frac{b'}{a}\right)^2$$

$$\hat{D} = \frac{D}{A_b} = \frac{5}{2}\left(\frac{m_\delta}{m_b}\right)\left(\frac{m_K}{m}\right)\left(\frac{b}{a}\right)\left(\frac{b'}{a}\right)$$

$$\hat{E} = \frac{E}{A_b} = 1$$

$$\hat{F} = \frac{F}{A_b} = \frac{20}{3}\left(\frac{m_K}{m_b}\right)\left(\frac{b}{a}\right)^2 + 20\left(\frac{m_K}{m_b}\right)\left(1+\frac{b}{a}\right)^2 \qquad (3.12)$$

where

$$m = m_b + 2m_\delta + 2m_K$$

The characteristic equations for each case are, therefore, given as follows:

(1) with fixed yaw-stabilizers

$$(\hat{A}_2 - \hat{A}_3)\lambda^2 + \hat{T}_2\lambda + (\hat{A}_2' - \hat{A}_3) = 0 \qquad (3.13)$$

and

$$\begin{vmatrix} \hat{A}\lambda^2 + \hat{F} & -\hat{E}\lambda & 2(\hat{A}_1\lambda^2 + \hat{A}_1') \\ \hat{E}\lambda & \hat{C}\lambda^2 + \hat{C}' & 0 \\ \hat{A}_1\lambda^2 + \hat{A}_1' & 0 & (\hat{A}_2 + \hat{A}_3)\lambda^2 + \hat{T}_2\lambda + (\hat{A}_2' + \hat{A}_3) \end{vmatrix} = 0 \qquad (3.14)$$

(2) with hinged yaw-stabilizers (general case)

$$\begin{vmatrix} (\hat{C}_2 - \hat{C}_3)\lambda^2 + \hat{T}_1\lambda + (\hat{C}_2 - \hat{C}_3) & 2(\hat{D}\lambda^2 + \hat{D}) \\ 2(\hat{D}\lambda^2 + \hat{D}) & (\hat{A}_2 - \hat{A}_3)\lambda^2 + \hat{T}_2\lambda + (\hat{A}_2' - \hat{A}_3) \end{vmatrix} = 0 \qquad (3.15)$$

and

$$\begin{vmatrix} \hat{A}\lambda^2 + \hat{F} & -\hat{E}\lambda & 0 & 2(\hat{A}_1\lambda^2 + \hat{A}_1') \\ \hat{E}\lambda & \hat{C}\lambda^2 + \hat{C}' & 2(\hat{C}_1\lambda^2 + \hat{C}_1) & 0 \\ 0 & \hat{C}_1\lambda^2 + \hat{C}_1 & (\hat{C}_2 + \hat{C}_3)\lambda^2 + \hat{T}_1\lambda + (\hat{C}_2 + \hat{C}_3) & 0 \\ \hat{A}_1\lambda^2 + \hat{A}_1' & 0 & 0 & (\hat{A}_2 + \hat{A}_3)\lambda^2 + \hat{T}_2\lambda + (\hat{A}_2' + \hat{A}_3) \end{vmatrix} = 0 \qquad (3.16)$$

The above characteristic equations were solved on the IBM 7090 at the U. of T. Institute of Computer Science. For each root of the equation, the characteristic decay time (time to $\frac{1}{2}$ amplitude) and the period were calculated. For antisymmetric modes, the mode shapes, i.e. ψ_0/ϕ_0, κ_0/ϕ_0 etc., were also calculated.

(i) Some sample results for the case with fixed yaw-stabilizers are shown in Table 1 (a) and (b) and in Fig. 4. The principal variables are b/a, b'/a and $\hat{T}_2$ in this case, and depending on those values, both oscillatory and non-oscillatory modes were obtained. Figure 4 shows plots of the least-damped modes for two combinations of b/a and b'/a. The best performance, from the standpoint of the number of orbits to $\frac{1}{2}$ amplitude for the particular cases shown in Fig. 4 was obtained for the combination b/a = 3.0, b'/a = 2.5, $\hat{T}_2$ = 0.7 for the antisymmetric mode. The best value is seen to be nearly 1.35 orbits. The damping and period of the symmetric (or staggering) mode are also shown in Fig. 4 for b/a = 3.0 and 4.0, but this mode is less important, since it does not involve angular motion of the satellite body.

(ii) The principal results of the general case are shown in Table 2 (a) and (b) and in Fig. 5. The variables of this case are b/a, b'/a, $\hat{T}_1$ and $\hat{T}_2$. Figure 5 shows plots of the least-damped modes for two sets of combination of b/a, b'/a and $\hat{T}_2$, and the best performance for the antisymmetric mode is nearly 1.2 orbits. It is clearly seen that when $\hat{T}_1$ is large, weak damping and long period (sometimes aperiodic) mode occurs in each case of the antisymmetric mode.

However, the above-stated damping of the antisymmetric mode of lateral motion, i.e.

$O_{\frac{1}{2}} = 1.35$ with rigid yaw-stabilizers

$O_{\frac{1}{2}} = 1.2$ with hinged yaw-stabilizers

are both unsatisfactory compared with that of the longitudinal motion. Hence, further parameter variations were made in a search for better performance. The equations of motion of the general case are so complicated that it is inconvenient to use them for such a purpose. Furthermore, from the practical standpoint, the equipment of fixed yaw-stabilizers is much simpler than that of hinged yaw-stabilizers. Hence, the following discussions are only concerned with the case of fixed yaw-stabilizers.

(iii) The least damped mode of the case of fixed yaw stabilizers is mostly connected to the yawing motion and, as already discussed, the damping of this mode depends strongly on the coupling between yawing and rolling motion. This in turn is seen to be entirely governed by the two terms containing E in Eq. 3.2. In other words, by changing the value of E , we can expect to obtain the better results. From the practical point of view, the value of E can be controlled by adding additional mass along the Y-axis. Namely, by definition,

$$E = A_b + C_b - B_b \tag{3.17}$$

When a dumbbell mass for example is attached along Y-axis, as shown in Fig. 6,

$$\begin{aligned} A_b &= A_{b_o} + I \\ C_b &= C_{b_o} + I \\ B_b &\doteq B_{b_o} \end{aligned} \tag{3.18}$$

where A_{b_o} , B_{b_o}, C_{b_o} are the original moments of inertia (without dumbbell mass), and I is the additional moment of inertia about either the X or Z axes by virtue of dumbbell mass.

From Eq. (3.17) and (3.18), therefore

$$E = E_o + 2I \tag{3.19}$$

where $E_o = A_{b_o} + C_{b_o} - B_{b_o}$

or in nondimensional form,

$$\hat{E} = \hat{E}_o + 2\hat{I} \tag{3.19'}$$

where $\hat{I} = \dfrac{I}{A_b}$

Furthermore, several other coefficients of the characteristic equations are affected by the dumbbell mass, i.e.

$$\begin{aligned} \hat{A} &= \hat{A}_o + \hat{I} \\ \hat{C} &= \hat{C}_o + \hat{I} \\ \hat{C}' &= \hat{C}'_o - \hat{I} \\ \hat{F} &= \hat{F}_o - 4\hat{I} \end{aligned} \tag{3.20}$$

where subscript o means the original values without dumbbell mass.

After substituting Eq. (3.19') and (3.20) into the characteristic Eq. (3.16), it was solved on the IBM 7090 for two sets of variables b/a, b'/a and $\hat{T}_2$. The principal results are shown in Table 3 and Fig. 7.

Figure 7 shows clearly, as expected, that the dumbbell mass is effective to improve the stability of lateral motion. The best performance or the minimum number of orbits to $\frac{1}{2}$ amplitude is about 0.38 orbits at b/a = 4.0, b'/a = 3.0, $\hat{T}_2$ = 0.8 and $\hat{I}$ = 0.3. This value is of the same order as the best damping of longitudinal motion obtained in Ref. 1. Figure 7 presents kinks in the plot of orbits to half amplitude and jumps in the plot of period. This is because the least-damped mode changes at these points from one mode to another.

(iv) By the above-mentioned numerical computation, the best stability was obtained for combination of variables b/a = 4.0, b'/a = 3.0, $\hat{T}_2$ = 0.8 and I = 0.3 and this value ($O_{\frac{1}{2}}$ = 0.38) appears to be very good from the practical standpoint.

However, since these numerical values were chosen more-or-less arbitrarily, the better performance will be expected for another combination of variables around these values.

The so-called 'steepest-descent method' (see Ref. 3) is conveniently applied for solving the optimization problem like this. The actual procedure of our problem is described in Appendix 1. The numerical values which were used for calculation are as follows:

Starting conditions: (1) $(b/a)^* = 3.0$, $(b'/a)^* = 2.5$

$$\hat{\Gamma}_2^* = 0.70 \qquad \hat{I}^* = 0.15$$

(2) $(b/a)^* = 4.0$, $(b'/a)^* = 3.0$

$$\hat{\Gamma}_2^* = 0.80 \qquad \hat{I}^* = 0.30$$

Small perturbations: $\Delta \frac{b}{a} = 0.01$ $\quad \Delta \frac{b'}{a} = 0.01$

$$\Delta \hat{\Gamma}_2 = 0.001 \qquad \Delta \hat{I} = 0.001$$

Weighting numbers: $\alpha_{\frac{b}{a}} = 100$ $\quad \alpha_{\frac{b'}{a}} = 100$

$$\alpha_{\hat{\Gamma}_2} = 1 \qquad \alpha_{\hat{I}} = 1$$

The results are shown in Table 4 and Fig. 8. Figure 8 shows clearly that the least-damped mode is improved remarkably by this method. Namely, as shown in Fig. 8, the damping or orbits to half amplitude of the starting point is nearly 0.5 orbits in this example, but it is about 0.28 orbits after 12 times of interation of the computation. The optimum combination of variables corresponding to this optimum damping mode is as follows:

$$b/a = 3.3231 \qquad b'/a = 3.0872$$

$$\hat{\Gamma}_2 = 0.6368 \qquad \hat{I} = 0.2184$$

Since these variables except $\hat{I}$ affect the longitudinal stability as well, then the longitudinal stability must be considered simultaneously to obtain the best overall performance of attitude stabilization of a satellite. It means some compromise between longitudinal and lateral stability is probably necessary and the best combination of those principal variables should be chosen from this point of view. No attempt is made here to demonstrate such a compromise solution, since it becomes essentially a design problem very much dependent on the particular configuration.

IV. CONCLUDING REMARKS

The lateral equations of motion which are derived for a particular compound satellite system are the homogeneous equations and hence the disturbed motion is only the transient motion and no forced motion occurs, unlike the longitudinal motion.

The numerical calculations were separated into two cases, i.e. the general case and the case with fixed yaw-stabilizers. Since the latter is more convenient to deal with and also considered better from the practical point of view, it was mostly discussed by the numerical examples. The results show that the best performance of lateral motion or the decay time to $\frac{1}{2}$ amplitude is roughly 0.28 orbits for the following combination of variables:

$$b/a = 3.3231 \qquad b'/a = 3.0872$$

$$\hat{\Gamma}_2 = 0.6368 \qquad \hat{I} = 0.2183$$

However, to obtain the best overall performance of attitude stabilization of a satellite, some compromises or in other words some changes of the value of variables from this optimum combination are probably necessary for its design.

The principal objective of this analysis (both Parts I and II) has been to show that the basic concept presented for passive attitude stabilization can lead to acceptably short damping times. This is seen to have been successfully accomplished.

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2.	Etkin, B.	Dyanmics of Flight, John Wiley and Sons, 1959.
3.	Bryson, A. E. Denham, W. F.	A Steepest-Ascent Method for Solving Optimum Programming Problems, Journal of Applied Mechanics, Vol. 29, No. 2, June, 1962.

APPENDIX 1

Application of the Steepest-Descent Method to Optimize the Stability of Perturbed Motion of the Satellite (see Ref. 3)

For the optimization problem stated in Section 3. 3 (iv), we have to solve the characteristic equations and find the real part of the roots. However, since the mode of motion to be optimized is the antisymmetric mode of the lateral motion, the characteristic equation is expressed by the sextic equation as follows:

$$\lambda^6 + a_5 \lambda^5 + \cdots + a_1 \lambda + a_0 = 0 \tag{1}$$

where

$$a_i = f(\beta_k) \qquad \begin{matrix} (i = 0, \cdots, 5) \\ (k = 1, \cdots, 4) \end{matrix} \tag{2}$$

and β_k are the 'control' variables, i. e.

$\beta_1 = \hat{I}$: dumbbell mass inertia

$\beta_2 = b/a$: roll-stabilizer length

$\beta_3 = b'/a$: yaw-stabilizer length

$\beta_4 = \hat{\Gamma}_2$: damping coefficient of roll-stabilizers

Roots of the characteristic equation are given, in general, by

$$\lambda_j = n_j \pm i\omega_j \tag{3}$$

if all the roots are complex, j = 1, 2, 3

if the roots are real, $\omega_j = 0$ and j = 1, 2, 3 . . .

The stability criterion is the number of orbits to $\frac{1}{2}$ amplitude $O_{\frac{1}{2}}$ and

$$O_{1/2} = \frac{0.110}{|n_j|} \quad \text{orbits} \tag{4}$$

Therefore, $(O_{\frac{1}{2}})_{max}$ corresponds to $|n_j|$ min or n_j max because $n_j < 0$ for the stable motion.

At the starting point, the control variables are

$$\hat{I} = \hat{I}^*, \quad \left(\frac{b}{a}\right) = \left(\frac{b}{a}\right)^*, \quad \left(\frac{b'}{a}\right) = \left(\frac{b'}{a}\right)^*, \quad \hat{\Gamma}_2 = \hat{\Gamma}_2^*$$

and the roots of the equation are

$$\lambda_j^{\,*} = n_j^{\,*} \pm i\,\omega_j^{\,*}$$

Consider small perturbations of the control variables about the starting point, i. e.

$$\hat{I} = \hat{I}^{*} + \Delta\hat{I}\,, \qquad \frac{b}{a} = \left(\frac{b}{a}\right)^{*} + \Delta\frac{b}{a}$$

$$\frac{b'}{a} = \left(\frac{b'}{a}\right)^{*} + \Delta\frac{b'}{a}\,, \qquad \hat{T}_2 = \hat{T}_2^{\,*} + \Delta\hat{T}_2$$

These perturbations cause small changes of the roots,

$$\lambda_j^{\,*} + \Delta\lambda_j = (n_j^{\,*} + \Delta n_j) \pm i\,(\omega_j^{\,*} + \Delta\omega_j)$$

Since λ or n is a function of coefficients a_i, then

$$n_j = F(\beta_k)$$

$$\therefore\; dn_j = \sum_{k=1}^{4}\left(\frac{\partial n_j}{\partial \beta_k}\right)^{*} \cdot d\beta_k \tag{5}$$

When $n_1 > n_2 > n_3$ at the starting point, n_1 should be chosen as the value to be optimized (i. e. minimized) from Eq. (4).

In order to apply the steepest-descent method, we define

$$(dP)^2 = \sum_{k=1}^{4} \alpha_k (d\beta_k)^2 = const. \tag{6}$$

where α_k are the positive weighting numbers. To maximize dn for a small perturbation $d\beta$ under a constraint condition given by Eq. (6), consider the quantity

$$dn = \sum_k \left(\frac{\partial n}{\partial \beta_k}\right)^{*} \cdot d\beta_k + \mu\left[(dP)^2 - \sum_k \alpha_k (d\beta_k)^2\right] \tag{7}$$

where μ is a Lagrange multiplier. The maximum of dn occurs when

$$\left(\frac{\partial n}{\partial \beta_k}\right)^{*} - 2\mu\alpha_k(d\beta_k) = 0 \qquad (k = 1, \cdots, 4) \tag{8}$$

$$\therefore\; d\beta_k = \frac{1}{2\mu\alpha_k}\left(\frac{\partial n}{\partial \beta_k}\right)^{*} \tag{8'}$$

Substituting Eq. (8') into Eq. (6)

$$(dP)^2 = \left(\frac{1}{2\mu}\right)^2 \sum_k \frac{1}{\alpha_k}\left(\frac{\partial n}{\partial \beta_k}\right)^{*2} \tag{9}$$

or

$$\frac{1}{2\mu} = \frac{dP}{\left\{ \sum_k \frac{1}{\alpha_k} \left(\frac{\partial n}{\partial \beta_k} \right)^{*2} \right\}^{\frac{1}{2}}} \tag{9'}$$

Substituting Eq. (9') into Eq. (8')

$$d\beta_k = dP \cdot \frac{1}{\alpha_k} \cdot \frac{\left(\frac{\partial n}{\partial \beta_k} \right)^{*}}{\left\{ \sum_k \frac{1}{\alpha_k} \left(\frac{\partial n}{\partial \beta_k} \right)^{*2} \right\}^{\frac{1}{2}}} \tag{10}$$

Since dn should be negative, $d\beta_k$ must be chosen so that $\left(\frac{\partial n}{\partial \beta_k} \right)^{*} \cdot d\beta_k$ is negative from Eq. (5), i.e. when

$$\left(\frac{\partial n}{\partial \beta_k} \right)^{*} > 0 \quad : \quad d\beta_k < 0$$

$$\left(\frac{\partial n}{\partial \beta_k} \right)^{*} < 0 \quad : \quad d\beta_k > 0$$

or

$$d\beta_k = -|dP| \frac{\frac{1}{\alpha_k} \left(\frac{\partial n}{\partial \beta_k} \right)^{*}}{\left\{ \sum_k \frac{1}{\alpha_k} \left(\frac{\partial n}{\partial \beta_k} \right)^{*2} \right\}^{\frac{1}{2}}} \tag{11}$$

For the next step, $\beta_k = \beta_k^* + d\beta_k$ (k = 1 . . . 4) are the starting points and the same procedure is repeated. This process should be repeated several times until the gradient dn/dP or

$$\frac{dn}{dP} = - \left\{ \sum_k \frac{1}{\alpha_k} \left(\frac{\partial n}{\partial \beta_k} \right)^{*2} \right\}^{\frac{1}{2}} \tag{12}$$

is nearly zero. The optimum value of n is then obtained.

TABLE 1

(a) Antisymmetric Modes (with rigid yaw-stabilizers)

Hinge Damping $\hat{\Gamma}_z$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$	Hinge Damping $\hat{\Gamma}_z$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
b/a = 3.0 b'/a = 2.0			b/a = 3.0 b'/a = 2.5		
0.2	0.4715	0.3093	0.2	0.4713	0.3143
	0.7553	0.7312		0.7758	0.7503
	2.3201	5.1603		1.9188	3.5618
0.3	0.5116	0.2093	0.3	0.5099	0.2150
	0.7232	0.4705		0.7483	0.4703
	2.3018	3.4716		1.8953	2.4129
0.4	0.6043	0.1476	0.4	0.6056	0.1591
	0.6580	0.4106		0.6783	0.3613
	2.2759	2.6436		1.8621	1.8633
0.5	0.7413	0.1030	0.5	0.7677	0.1074
	0.6299	0.5496		0.6371	0.4907
	2.2426	2.1667		1.8205	1.5674
0.6	1.0384	0.0806	0.6	1.1325	0.0827
	0.6226	0.6946		0.6286	0.6293
	2.2028	1.8734		1.7739	1.4143
0.7	--	0.0573	0.7	--	0.0524
	--	0.0797			0.0965
	0.6193	0.8342		0.6251	0.7607
	2.1585	1.6941		1.7283	1.3565
0.8	--	0.0364	0.8	--	0.0359
	--	0.1317		--	0.1479
	0.6175	0.9706		0.6232	0.8880
	2.1128	1.5940		1.6889	1.3641
1.0	--	0.0248	1.0	--	0.0247
	--	0.2099		--	0.2296
	0.6157	1.2377		0.6212	1.1363
	2.0309	1.5531		1.6339	1.4911
1.2	--	0.0194	1.2	--	0.0194
	--	0.2842		--	0.3037
	0.6148	1.5009		0.6202	1.3802
	1.9718	1.6365		1.6023	1.6867

TABLE 1 (a)
(continued)

Hinge Damping $\hat{\Gamma}_2$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$	Hinge Damping $\hat{\Gamma}_2$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
b/a = 4.0 b'/a = 2.5			b/a = 4.0 b'/a = 3.0		
0.2	0.4638	0.5124	0.2	0.4641	0.5243
	0.6884	0.7993		0.7043	0.7916
	1.7331	7.0810		1.5183	5.7644
0.3	0.4832	0.3533	0.3	0.4827	0.3636
	0.6723	0.5059		0.6902	0.4971
	1.7254	4.7951		1.5094	3.9414
0.4	0.5224	0.2865	0.4	0.5178	0.2996
	0.6378	0.3418		0.6617	0.3306
	1.7149	3.6802		1.4974	3.0700
0.5	0.6186	0.1709	0.5	0.6395	0.1807
	0.5667	0.4587		0.5638	0.4100
	1.7019	3.0386		1.4829	2.5877
0.6	0.6761	0.1239	0.6	0.6983	0.1272
	0.5648	0.6313		0.5655	0.5760
	1.6869	2.6388		1.4668	2.3084
0.7	0.7665	0.1000	0.7	0.7982	0.1016
	0.5640	0.7811		0.5655	0.7176
	1.6703	2.3820		1.4503	2.1525
0.8	0.9317	0.0845	0.8	0.9891	0.0853
	0.5636	0.9224		0.5653	0.8503
	1.6531	2.2192		1.4344	2.0788
1.0	--	0.0531	1.0	--	0.0500
	--	0.0836		--	0.0937
	0.5631	1.1937		0.5652	1.1039
	1.6196	2.0742		1.4073	2.0870
1.2	--	0.0326	1.2	--	0.0322
	--	0.1411		--	0.1494
	0.5629	1.4575		0.5651	1.3499
	1.5913	2.0778		1.3876	2.2148

TABLE 1

(b) Symmetric Mode (with rigid yaw-stabilizers)

$\hat{\Gamma}_z$	T	$O_{\frac{1}{2}}$	$\hat{\Gamma}_z$	T	$O_{\frac{1}{2}}$
b/a = 3.0			b/a = 4.0		
0.1	0.4555	0.9716	0.1	0.4631	2.2868
0.2	0.4574	0.4858	0.2	0.4634	1.1434
0.3	0.4605	0.3239	0.3	0.4640	0.7623
0.4	0.4650	0.2429	0.4	0.4648	0.5717
0.6	0.4785	0.1619	0.6	0.4671	0.3811
0.8	0.4996	0.1215	0.8	0.4705	0.2859
1.0	0.5313	0.0972	1.0	0.4749	0.2287

TABLE 2

(a) Antisymmetric Modes (with hinged yaw stabilizers)

Yaw-Hinge Damping $\hat{\Gamma}_1$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$	Yaw-Hinge Damping $\hat{\Gamma}_1$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
b/a = 3.0, b'/a = 2.5 $\hat{\Gamma}_2$ = 0.70			0.30	1.1255	0.0488
				0.6141	0.6923
				1.3534	0.4042
0.03	1.6896	0.0627		--	0.2445
	0.6056	0.9390		--	7.0609
	1.0048	1.9908			
	12.273	4.0895	0.40	1.2127	0.0429
				0.6171	0.6929
0.06	1.5765	0.0618		1.4880	0.4477
	0.6047	0.8725		--	0.1828
	1.0182	1.0400		--	9.5489
	14.571	2.2202			
			0.60	--	0.0290
0.10	1.4435	0.0604		--	0.0410
	0.6050	0.8010		0.6202	0.7041
	1.0487	0.6679		1.6097	0.5674
	36.287	1.3944		--	0.1388
				--	14.463
0.15	1.3078	0.0580			
	0.6070	0.7430	0.80	--	0.0201
	1.1060	0.4953		--	0.0456
	--	0.5617		0.6216	0.7141
	--	3.1268		1.6534	0.6678
				--	0.1239
0.20	1.2090	0.0551		--	19.349
	0.6096	0.7121			
	1.1817	0.4259	1.0	--	0.0158
	--	0.3900		--	0.0474
	--	4.5016		0.6225	0.7215
				1.6743	0.7466
				--	0.1167
				--	24.223

TABLE 2 (a)
(continued)

Yaw-Hinge Damping $\hat{\Gamma}_1$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$	Yaw-Hinge Damping $\hat{\Gamma}_1$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
b/a = 4.0, b'/a = 3.0, $\hat{\Gamma}_2$ = 0.80			0.30	0.7721	0.0664
				0.5550	0.8908
				1.1725	0.5952
0.03	0.7803	0.0789		--	0.1920
	0.5557	1.1765		--	5.3918
	1.0025	3.4077			
	9.8861	3.6481	0.40	0.8294	0.0613
				0.5564	0.8564
0.06	0.7741	0.0779		1.2472	0.6096
	0.5548	1.1314		--	0.1320
	1.0097	1.7670		--	7.3070
	11.393	1.9226			
			0.60	1.8030	0.0617
0.10	0.7667	0.0763		0.5588	0.8301
	0.5541	1.0725		1.3326	0.7279
	1.0256	1.1243		--	0.0560
	21.347	1.1822		--	11.083
0.15	0.7601	0.0741			
	0.5537	1.0081	0.80	1.3035	0.0780
	1.0540	0.8195		0.5603	0.8238
	--	0.4771		1.3690	0.8535
	--	2.3336		--	0.0289
				--	14.834
0.20	0.7577	0.0716			
	0.5539	0.9570	1.00	1.1883	0.0810
	1.0898	0.6841		0.5613	0.8231
	--	0.3213		1.3873	0.9623
	--	3.4126		--	0.0220
				--	18.574

TABLE 2

(b) Symmetric Mode (with hinged yaw-stabilizers)

Yaw-Hinge Damping $\hat{\Gamma}_1$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$	Yaw-Hinge Damping $\hat{\Gamma}_1$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
b/a = 3.0,	b'/a = 2.5,	$\hat{\Gamma}_2$ = 0.70	b/a = 4.0	b'/a = 3.0	$\hat{\Gamma}_2$ = 0.80
0.10	0.04879	0.1387	0.10	0.4704	0.2857
	1.0197	0.5644		1.0065	0.9720
0.20	0.4879	0.1387	0.20	0.4705	0.2857
	1.0864	0.2822		1.0268	0.4860
0.30	0.4879	0.1387	0.30	0.4705	0.2856
	1.2345	0.1881		1.0635	0.3240
0.40	0.4880	0.1387	0.40	0.4705	0.2856
	1.6042	0.1411		1.1223	0.2430
0.60	0.4880	0.1388	0.60	0.4705	0.2856
	--	0.0617		1.3656	0.1620
	--	0.1970			
0.80	0.4880	0.1389	0.80	0.4705	0.2856
	--	0.0399		2.3887	0.1215
	--	0.3052			
1.00	0.4880	0.1388	1.00	0.4706	0.2857
	--	0.0303		--	0.0659
	--	0.4010		--	0.1845

TABLE 3

Dumbbell Mass Effect (Antisymmetric Mode)

Dumbbell Mass $\hat{I}$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
b/a = 3.0,	b'/a = 2.5	$\hat{I}_z$ = 0.70
0	--	0.0524
	--	0.0965
	0.6251	0.7607
	1.7283	1.3565
0.01	--	0.0529
	--	0.0982
	0.6275	0.7370
	1.7509	1.2677
0.02	--	0.0535
	--	0.1000
	0.6298	0.7146
	1.7750	1.1845
0.03	--	0.0540
	--	0.1021
	0.6322	0.6932
	1.8007	1.1061
0.04	--	0.0545
	--	0.1043
	0.6346	0.6729
	1.8282	1.0323
0.06	--	0.0556
	--	0.1097
	0.6393	0.6352
	1.8900	0.8965
0.08	--	0.0566
	--	0.1166
	0.6441	0.6007
	1.9631	0.7738
0.10	--	0.0577
	--	0.1259
	0.6489	0.5692
	2.0520	0.6613
0.15	--	0.0601
	--	0.1846
	0.6610	0.5006
	2.3865	0.3995

Dumbbell Mass $\hat{I}$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
b/a = 4.0	b'/a = 3.0	$\hat{I}_z$ = 0.80
0	0.9891	0.0853
	0.5653	0.8503
	1.4344	2.0788
0.02	1.0032	0.0879
	0.5672	0.7925
	1.4550	1.8416
0.04	1.0182	0.0906
	0.5689	0.7408
	1.4773	1.6336
0.06	1.0342	0.0935
	0.5705	0.6943
	1.5014	1.4503
0.08	1.0513	0.0967
	0.5719	0.6523
	1.5278	1.2879
0.10	1.0697	0.1001
	0.5732	0.6144
	1.5569	1.1434
0.15	1.1226	0.1106
	0.5757	0.5340
	1.6462	0.8440
0.20	1.1871	0.1253
	0.5771	0.4705
	1.7784	0.6089
0.25	1.2544	0.1507
	0.5771	0.4203
	2.0464	0.4162
0.30	1.2200	0.1993
	0.5757	0.3809
	3.5189	0.2879

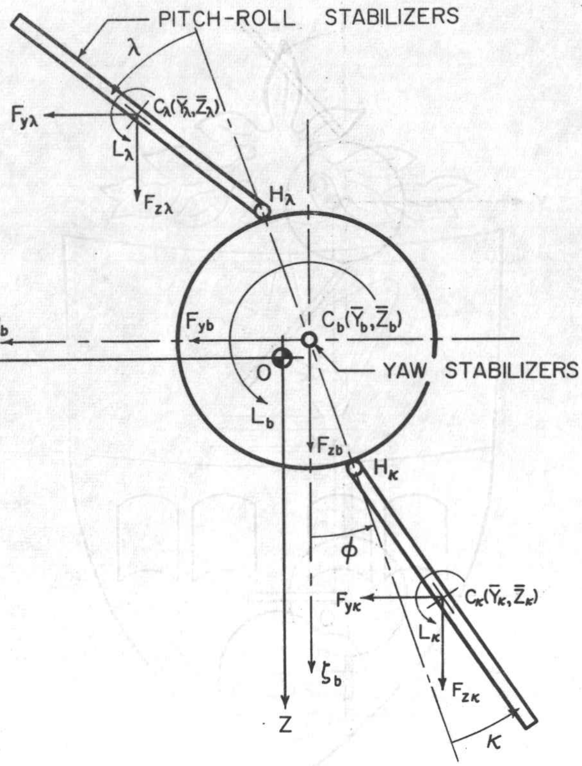
TABLE 4

Steepest-Descent Method

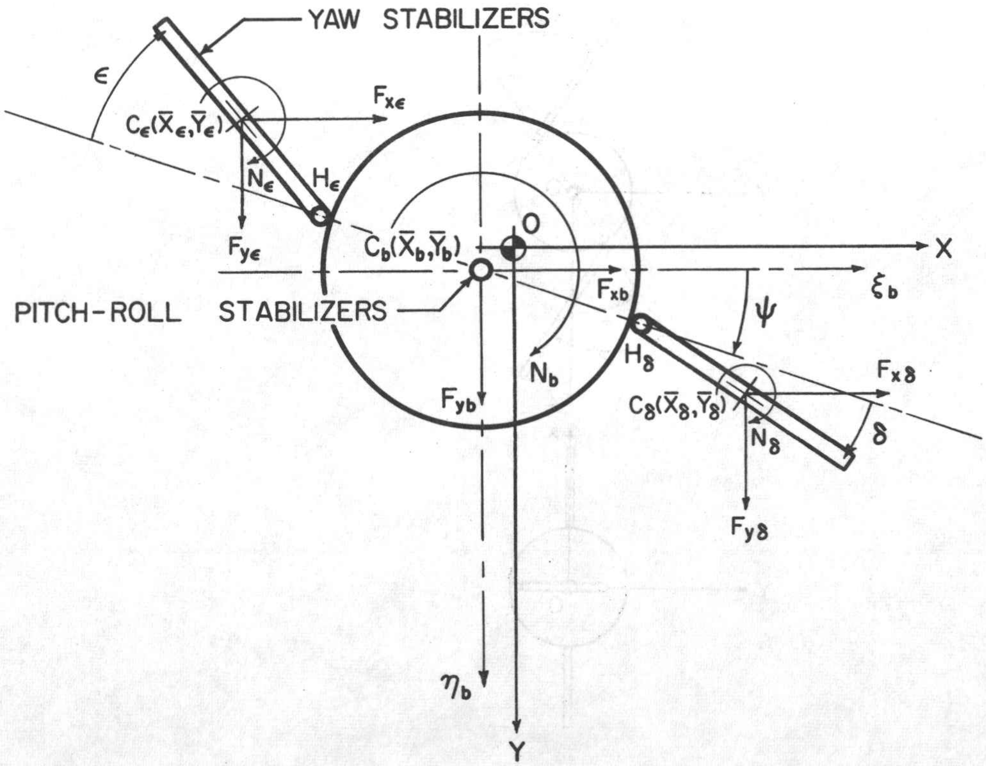
Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$	Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
(1) $b/a = 3.00$	$b'/a = 2.50$	(7) $b/a = 3.2550$	$b'/a = 2.7978$
$\hat{\Gamma}_2 = 0.70$	$\hat{I} = 0.15$	$\hat{\Gamma}_2 = 0.6727$	$\hat{I} = 0.1823$
--	0.0601	1.4910	0.1363
--	0.1846	0.6391	0.3720
0.6610	0.5006	2.5173	0.3386
2.3865	0.3996		
(2) $b/a = 3.0667$	$b'/a = 2.5444$	(8) $b/a = 3.2892$	$b'/a = 2.8489$
$\hat{\Gamma}_2 = 0.6962$	$\hat{I} = 0.1547$	$\hat{\Gamma}_2 = 0.6673$	$\hat{I} = 0.1881$
--	0.0734	1.3517	0.1472
--	0.1422	0.6352	0.3537
0.6547	0.4750	2.5909	0.3306
2.4001	0.4059		
(3) $b/a = 3.1267$	$b'/a = 2.5916$	(9) $b/a = 3.3158$	$b'/a = 2.8997$
$\hat{\Gamma}_2 = 0.6922$	$\hat{I} = 0.1597$	$\hat{\Gamma}_2 = 0.6617$	$\hat{I} = 0.1941$
3.7710	0.1034	1.2621	0.1595
0.6492	0.4513	0.6316	0.3366
2.4024	0.4089	2.7060	0.3205
(4) $b/a = 3.1800$	$b'/a = 2.6408$	(10) $b/a = 3.3344$	$b'/a = 2.9496$
$\hat{\Gamma}_2 = 0.6878$	$\hat{I} = 0.1650$	$\hat{\Gamma}_2 = 0.6558$	$\hat{I} = 0.2001$
2.0745	0.1103	1.1994	0.1736
0.6444	0.4292	0.6284	0.3205
2.4021	0.4070	2.8896	0.3093
(5) $b/a = 3.2266$	$b'/a = 2.6914$	(11) $b/a = 3.3443$	$b'/a = 2.9980$
$\hat{\Gamma}_2 = 0.6878$	$\hat{I} = 0.1650$	$\hat{\Gamma}_2 = 0.6496$	$\hat{I} = 0.2063$
1.6606	0.1179	1.1534	0.1893
0.6402	0.4085	0.6256	0.3055
2.4063	0.4009	3.1932	0.2977
(6) $b/a = 3.2135$	$b'/a = 2.7470$	(12) $b/a = 3.3448$	$b'/a = 3.0443$
$\hat{\Gamma}_2 = 0.6777$	$\hat{I} = 0.1767$	$\hat{\Gamma}_2 = 0.6432$	$\hat{I} = 0.2124$
1.7407	0.1267	1.1187	0.2067
0.6434	0.3914	0.6235	0.2916
2.4677	0.3433	3.7465	0.2863

TABLE 4
(continued)

Period Orbits T	Orbits to $\frac{1}{2}$ Amplitude $O_{\frac{1}{2}}$
(13) $b/a = 3.3231$, $\hat{\Gamma}_2 = 0.6368$	$b'/a = 3.0872$, $\hat{I} = 0.2184$
1.0975 0.6244 5.7041	0.2263 0.2787 0.2696
(14) $b/a = 3.3068$, $\hat{\Gamma}_2 = 0.6301$	$b'/a = 3.1285$, $\hat{I} = 0.2243$
1.0747 0.6237 -- --	0.2462 0.2667 0.2262 0.3085
(15) $b/a = 3.3970$, $\hat{\Gamma}_2 = 0.6289$	$b'/a = 3.1156$, $\hat{I} = 0.2204$
1.0479 0.6090 3.6660	0.2323 0.2726 0.3092



(a) $\psi = \delta = \epsilon = 0$



(b) $\phi = \kappa = \lambda = 0$

FIG. 1 STABILIZER SYSTEM (LATERAL)

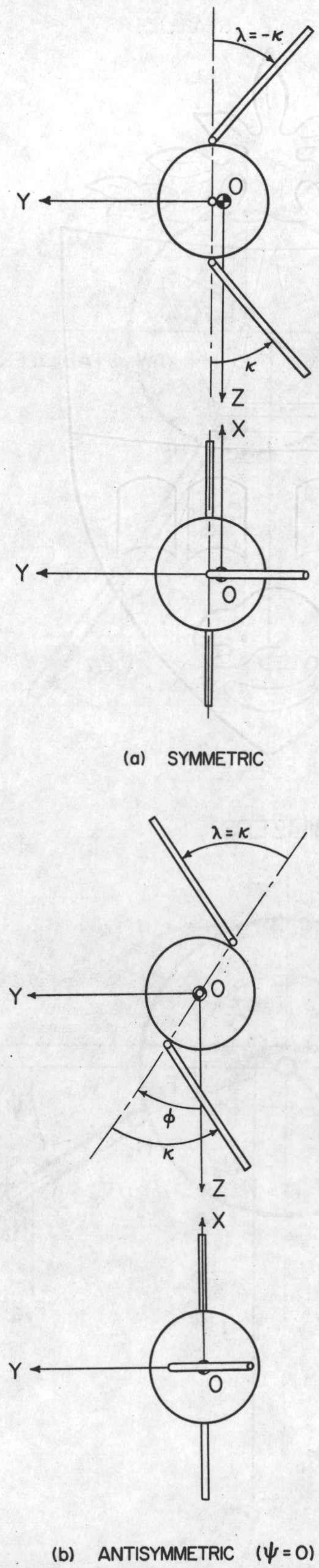


FIG. 2 CHARACTERISTIC MODES (FIXED YAW-STABILIZERS)

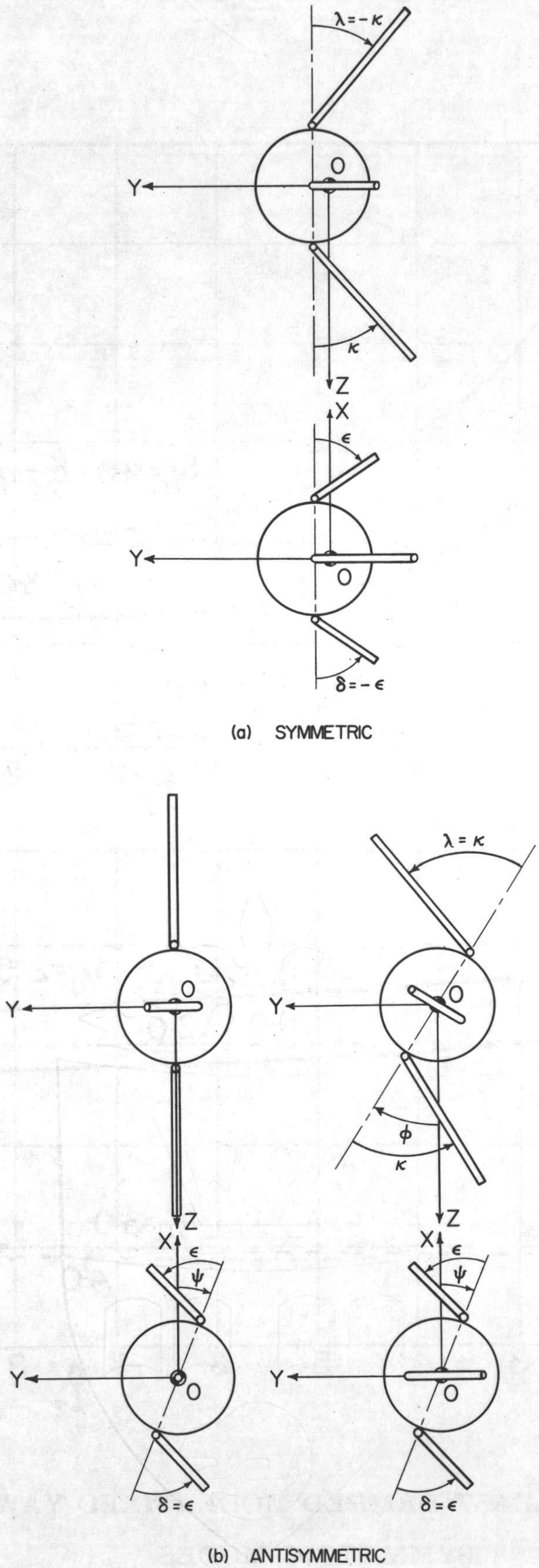


FIG. 3 CHARACTERISTIC MODES (GENERAL CASE)

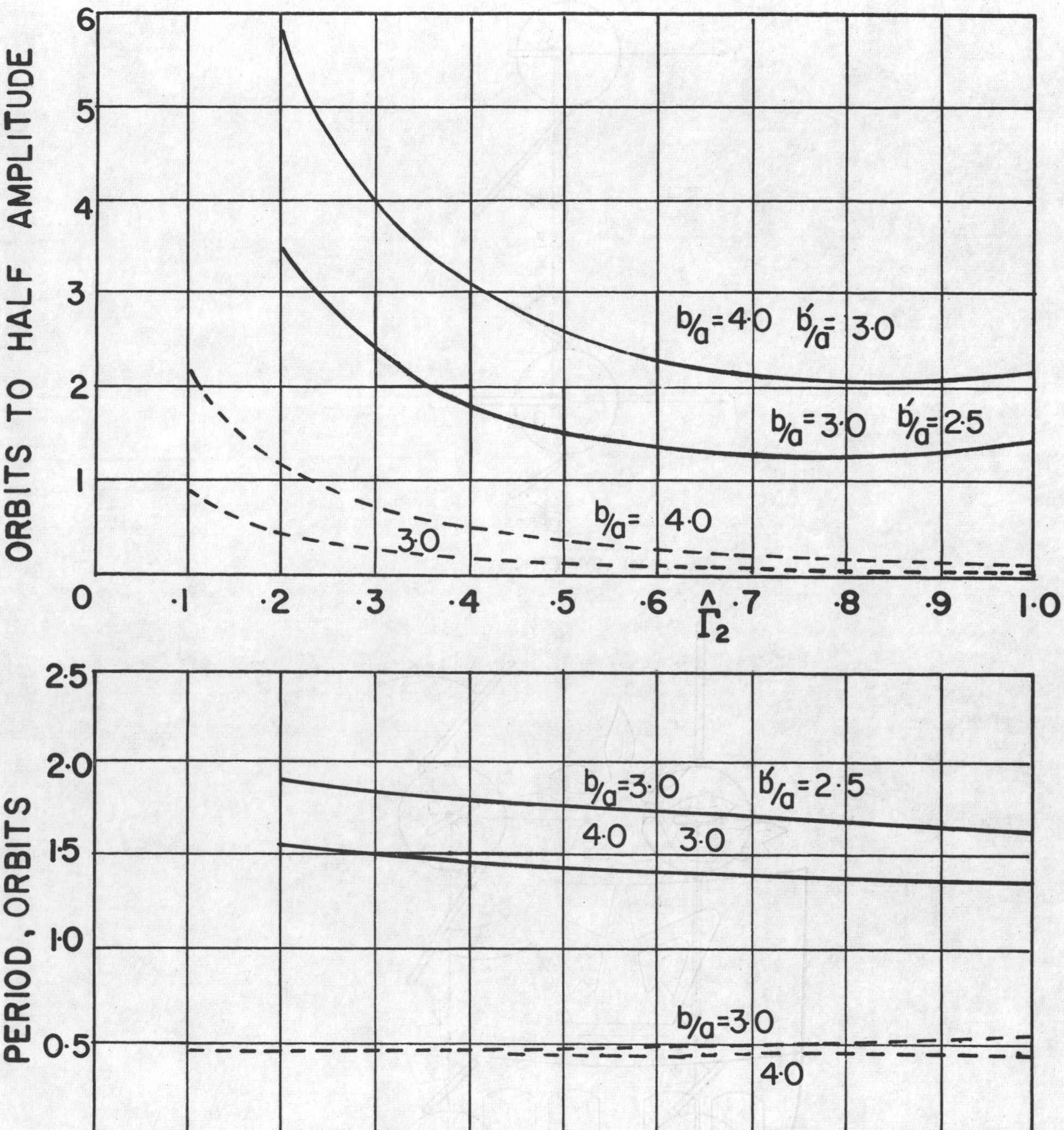


FIG. 4 DAMPING OF LEAST DAMPED MODE (FIXED YAW-STABILIZERS)

SOLID LINE - ANTISYMMETRIC MODES
DOTTED LINE - SYMMETRIC MODES

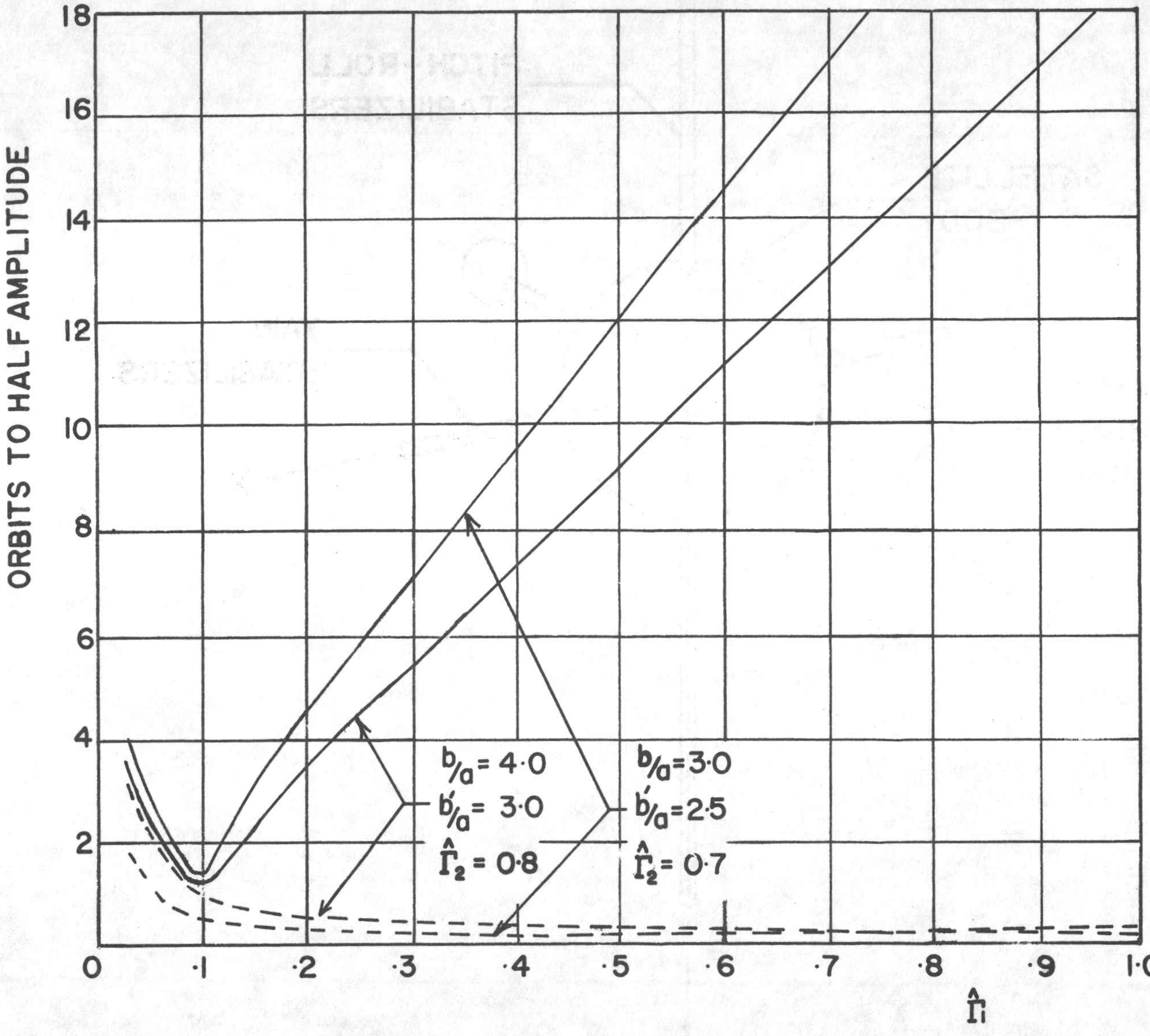


FIG. 5 DAMPING OF LEAST DAMPED MODE (GENERAL CASE)

SOLID LINE - ANTISYMMETRIC MODES

DOTTED LINE - SYMMETRIC MODES

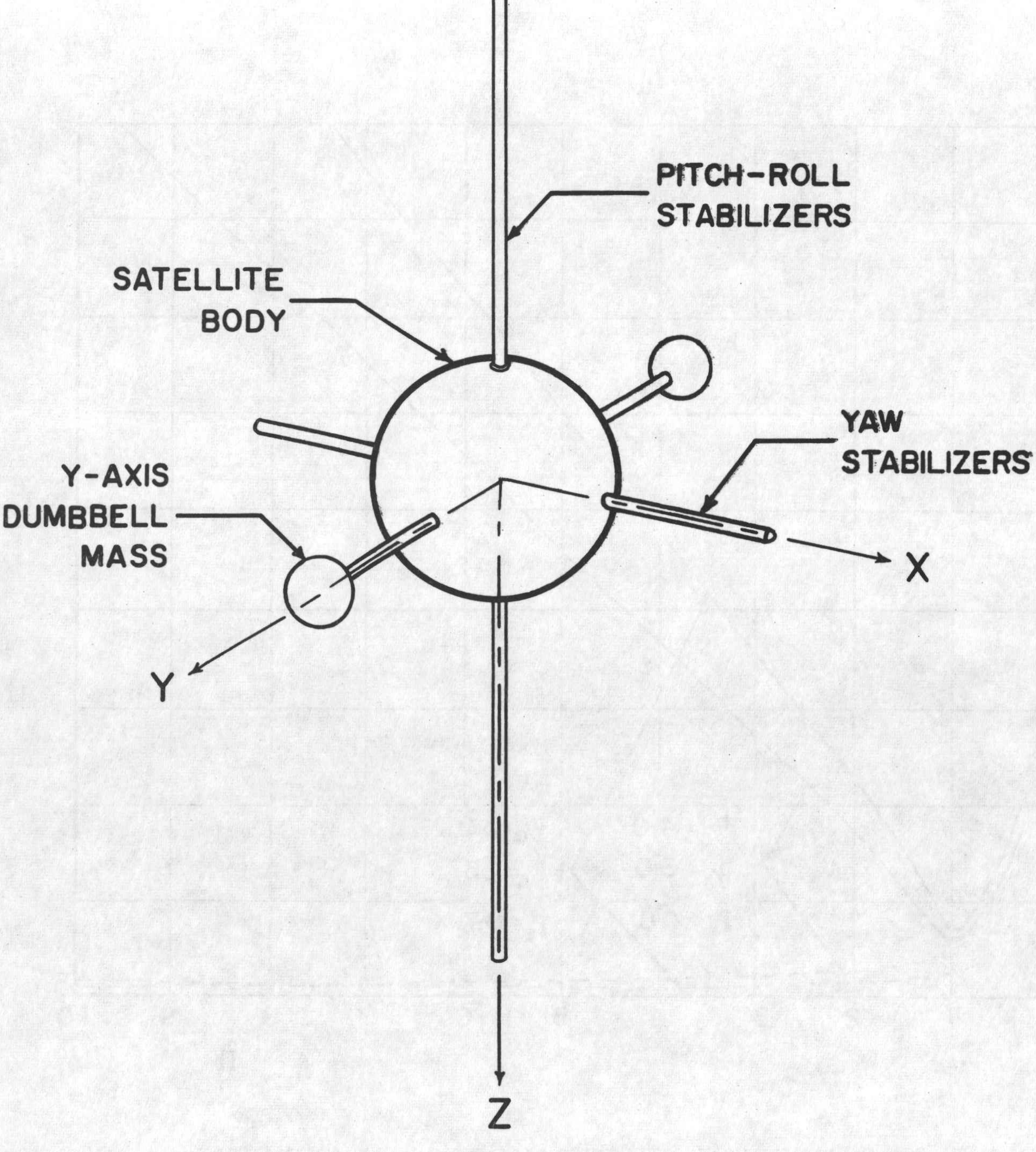


FIG. 6 DUMBBELL MASS

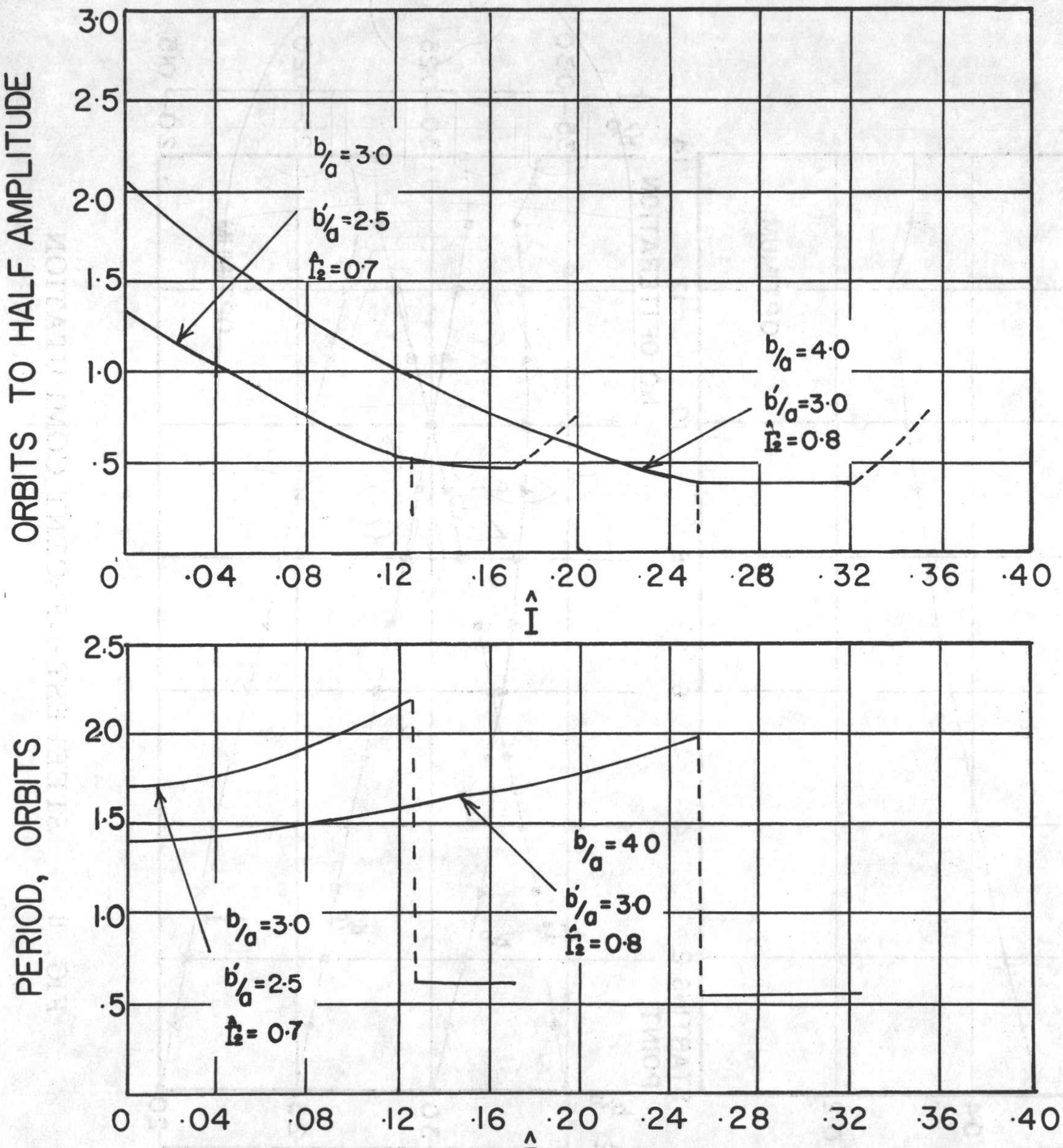


FIG. 7 DUMBBELL MASS EFFECT

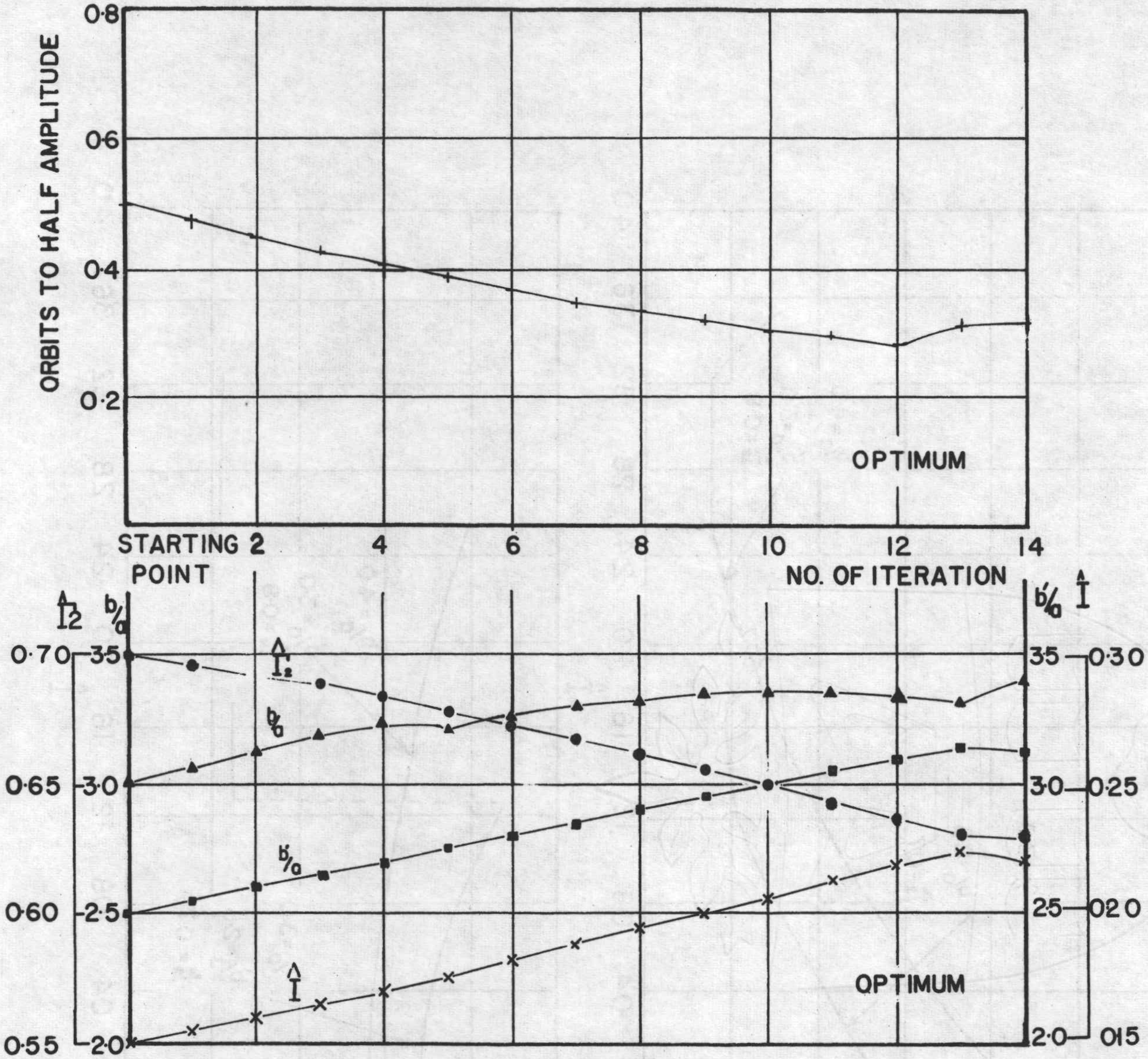


FIG. 8 STEEPEST-DESCENT COMPUTATION