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Optimizing the day-ahead energy market for Central Western Europe subject to flow-based market coupling network constraints

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Abbreviations

ATC	Available Transfer Capacity
ATM	At-the-money
bid	Maximum price for which a buyer wants to purchase
CHP	Convex Hull Pricing
CWE	Central Western Europe
DA	Day Ahead
EUPHEMIA	Pan-European Hybrid Electricity Market Integration Algorithm
FB	Flow Based
FBI	Flow Based Intuitive
IP	Integer Programming
ITM	In-the-money
LOC	Lost Opportunity Costs
LP	Linear Program
MILP	Mixed Integer Linear Program

NTC	Net Transfer capacity
offer	Minimum price for which a seller wants to sell
order	A bid or an offer
OTC	Over-the-counter
OTM	Out-of-the-money
PD	Primal-Dual
PTDF	Power Transfer Distribution Factor
RAM	Remaining Available Margin
TSO	Transmission System Operator

1 Introduction

Europe's day-ahead electricity markets operate on an hourly granularity, where market participants can submit *bids* (maximum price for which to buy) and *offers* (minimum price for which to sell). These market participants are not individual consumers, but energy suppliers, large industrial users and trading companies. As the name suggests, the electricity in the day-ahead market is traded one day before actual delivery. Electricity can be traded in two ways, over-the-counter (OTC) or with the exchange. When electricity is traded OTC, it means that a buyer and a seller jointly agree on a price, usually facilitated by a broker. Besides OTC trading, market participants have the option to bring their electricity to the exchange. Until 12.00 CET the day prior to delivery, every producer can submit the minimum price per hour for which he wants to sell his energy, while every consumer can submit the maximum price for which he wants to purchase energy. All the bids and offers applying to the same physical location are being aggregated to form demand and supply curves, respectively. If these were the only trading products, then the settlement price and total traded volume could be easily determined by simply finding the intersection of the two curves. Furthermore, all the bids below the settlement price as well as all the offers above the settlement price would see their *order* (bid or offer) accepted. There are, however, also orders that extend over multiple consecutive time periods so that the order has to be cleared entirely for all hours, we will refer to these as block orders. The *market clearing problem* amounts to finding a set of hourly and block orders as well as prices and cross-border electricity flows that maximize *social welfare* (consumer surplus + producer surplus + congestion rent) under operational constraints from market participants, as well as network constraints defined by Transmission System Operators (TSOs).

The aim of this thesis is to develop a mathematical model for the market clearing problem for the five countries of the CWE (Central Western European) region, that can be solved within a reasonable time. The model will be subject to the exact same requirements as EUPHEMIA (the algorithm currently in use for solving Europe's market clearing problem). In order to fully understand these requirements and their implications, however, some background information is necessary. We will therefore wait until the introduction of Chapter 5 before specifying them. The problem will be formulated as a Mixed Integer Linear Progam (MILP) and solved using the optimization package CPLEX with real-world data from the first 15 days of July 2019 CPLEX (2020). The goal here is to see if it is computationally feasible to solve instances of realistic size, as the five countries included contain roughly one third of all the orders in Europe. This thesis is organized as follows. Chapter 2 will give an example of a market clearing problem, so that the problem becomes more tangible to the reader. Chapter 3 will review the most important contributions from other authors. Chapter 4 will look into the data — the different parameters, sets and variables used, and explain the market clearing problem in depth. The heavy work is done in Chapter 5 where the model is derived. We finish off with the results of some tests in Chapter 6, and the conclusion can be found in Chapter 7.

2 Background

This chapter will illustrate the trade-off that has to be made in solving the market clearing problem. We start with a discussion of Definition 2.1 and 2.2 followed by an example of a market clearing problem containing block orders that will show why a market equilibrium with uniform prices in a non-convex setting is impossible most of the time.

Definition 2.1 (Uniform prices)**.** *A price system is* uniform *if all transactions between market participants depend only and proportionally on a single commodity price per area and hour.*

Definition 2.2 (Market (Walrasian) equilibrium I)**.** *A solution to the market clearing problem forms a* market equilibrium *if no market participant desires another level of execution.*

If a set of prices and orders form a market equilibrium, then all consumers who want to purchase above the settlement price and all producers who want to sell below the settlement price will see their order accepted. It is a desirable property to have as it allows decision making by market participants to be decentralized. Furthermore, each participant can easily verify why its bids were accepted or rejected, which contributes to the legitimacy and transparency of the market (O'Neill, Sotkiewicz, & Rothkopf, 2007).

A set of prices is uniform if they are *non-discriminatory*, i.e., every participant receives the price at the marginal bid or offer. When the owner of a power plant with a marginal cost of €20/MWh offers his electricity at this price, he does not actually receive this price (unless the market happens to coincidentally clear at €20), instead he receives the settlement price (whenever it settles at or above €20). This has a number of benefits. For starters, uniform prices give incentives to invest, because they allow the investor to make a profit and thereby recover their investment. Conversely, under a pay-as-bid pricing scheme, market participants have an incentive to misrepresent their marginal costs. They will try to bid as closely to the settlement price as possible whenever the settlement price is above the variable cost. The problem here is that market players are not always able to accurately forecast the settlement price. It can for example happen that an expensive gas plant estimates a lower settlement price than a relatively cheaper nuclear plant. If they bid according to their forecast then the gas plant will see its bid accepted instead of the more efficient nuclear plant. This is suboptimal from both an economic and environmental perspective. Under a uniform pricing scheme this would never happen as the primary determinant of a supplier's offer is the marginal cost. In the long run these dispatch inefficiencies raise costs, which are ultimately passed on to consumers Cramton (2007). Finally, there is what is known as the *missing money problem*. In a uniform price setting, the total amount of money collected from consumers is exactly enough to cover the total amount needed to pay producers, as there is one price and supply and demand match. In the non-uniform case this does not have to be true, and it often happens that the amount needed to pay-out in order to compensate certain participants exceeds the amount received. In practice this is not a major issue as the amount remains small compared to the financial transfers derived from trade at market prices, and also compared to the welfare generated (Van Vyve, 2011), nevertheless it is an undesirable property.

We will now introduce *convex* problems. Convex problems are those whose feasible set and objective function are convex. A set S is convex if for all $x, y \in S$ and $\theta \in [0, 1]$ we have that $\theta x + (1 - \theta)y \in S$. Intuitively this means that the line segment between any two points of the set lies within the set. A function f is convex if its domain is convex and for all x, y in its domain and all $\theta \in [0, 1]$ we have that $f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y)$. This means that the line segment between any two points on the graph of the function lies above or on the graph. Convexity is a desirable trait as there are often polynomial time algorithms available to solve such problems. And, equally important, from a convex problem, dual variables can be retrieved that provide uniform prices that form a market equilibrium (Madani & Van Vyve, 2018). Unfortunately, the cost structure of power plants does not allow for a perfectly convex problem. In order to maintain acceptable efficiency rates, plants usually have to reach minimum output levels. In addition, there can be extra costs incurred due to the start-up or shut-down of the plant, which the owner of the power plant wants to recover over the duration of the production period. Finally, plants often have to run for a minimum number of periods in order to not put too much stress on the mechanics of the power plant. These peculiarities are integral to the nature of power and in order to accurately account for them, new financial products in the form of block orders were introduced. They differ from regular orders in two ways. First, they are tied across two or more consecutive hours, and second, they have to be cleared completely or not at all (fill or kill). In this case, strong duality fails and a market equilibrium with uniform prices is mathematically impossible, unless the optimal solution for the relaxed Linear Problem (LP) is coincidentally integral (Madani & Van Vyve, 2015). In order to show this we will first introduce Definition 2.3, which is equivalent to Definition 2.2, but easier to work with. Consider an hourly order consisting of a quantity/price pair (q, p), and let π be the settlement price. As q is counted positive for bids and negative for offers, the following holds for all orders. An order is

- *in the money* (ITM) when $(p - \pi) \cdot q > 0$
- *at the money* (ATM) when $(p - \pi) \cdot q = 0$
- *out of the money* (OTM) when $(p - \pi) \cdot q < 0$.

Let $h \in H$. As blocks apply to multiple hours we extend this to

- $\sum_{h \in H}(p - \pi_h) \cdot q_h > 0$ for OTM blocks
- $\sum_{h \in H}(p - \pi_h) \cdot q_h = 0$ for ATM blocks
- $\sum_{h \in H}(p - \pi_h) \cdot q_h < 0$ for OTM blocks.

This means that a block can be out OTM in some hours and still overall be ITM as long as the sum over the total period is positive. Next, we define a block to be *paradoxically accepted* when it is accepted but OTM and *paradoxically rejected* when it is rejected but ITM.

Table 1: Example input

Orders	Quantity	Price	Decision Var.
A (hourly)	11	50	x_1
B (hourly)	14	10	x_2
C (block)	-10	5	y_1
D (block)	-20	10	y_2

Definition 2.3 (Market equilibrium II)**.** *A solution* (x^*, y^*, π^*) *to the market clearing problem forms a market equilibrium if*

1. *fully executed orders are ITM or ATM*
2. *fractionally executed orders are ATM*
3. *rejected orders are ATM or OTM*

The example in Table 1 consists of two regular bids and two block offers that all apply to the same location and time period. Therefore, the only difference between the hourly orders and the blocks is that the blocks cannot be partially executed. This is enough, however, to illustrate the problem. The MILP formulation corresponding to Table 1 is defined below at 2.1a - 2.1e. Here, the variables x_1 and x_2 denote the ratio of execution of the hourly orders, while y_1 and y_2 do the same for the block orders. The objective function can be found at 2.1a and aims to maximize social welfare. The balance equation is defined at 2.1b and makes sure that total demand and supply match, while the remaining constraints force all variables to be within their respective bounds. First off, it is clear that it is impossible for both blocks to be accepted, as the total supply would be 30 MW, while the total demand can be 25 MW at most. Since the blocks can only be fully accepted or rejected there are two trading options, either block C or block D gets accepted.

$$\max_{x,y} \quad x_1 \cdot 11 \cdot 50 + x_2 \cdot 14 \cdot 10 + y_1 \cdot (-10) \cdot 5 + y_2 \cdot (-20) \cdot 10 \tag{2.1a}$$

$$\text{s.t.} \quad x_1 \cdot 11 + x_2 \cdot 14 + y_1 \cdot (-10) + y_2 \cdot (-20) = 0 \tag{2.1b}$$

$$x_1, x_2 \geq 0 \tag{2.1c}$$

$$x_1, x_2 \leq 1 \tag{2.1d}$$

$$y_1, y_2 \in \{0, 1\} \tag{2.1e}$$

Let us assume a solution in which block C gets accepted. In that case supply is 10MW and hourly order A gets partially accepted, with $x_1 = 10/11$. According to Definition 2.3 the settlement price should be €50, which means that Block D is paradoxically rejected, as it wants to produce for a lower price than €50/MW. The total social welfare in this case would be $10/11 \cdot 11 \cdot 50 + 0 \cdot 14 \cdot 10 + 1 \cdot (-10) \cdot 5 + 0 \cdot (-20) \cdot 10 =$ €450. Now assume a solution wherein block D gets accepted instead, supply would be 20MW and in order to match this order A would be fully accepted and order B would be partially accepted with $x_2 = 9/14$. The market clearing price would be €10/MW as B is ATM, this means block C is in the money and thus paradoxically rejected. Social welfare in this scenario would be €440, Table 2 summarises the results. As the objective of 2.1a - 2.1e is to maximize social welfare, the solution would be to accept block C if solved to optimality. This shows that it is not always possible to have a market equilibrium with uniform prices in the case of indivisible products.

Table 2: Example results

Scenario	Settl. Price	Traded Vol.	Social welfare
accept C	50	10	450
accept D	10	20	440

3 Literature Review

This chapter will review the work of other authors who studied partial market equilibria. There are three main approaches to solving the market clearing problem, the Integer Programming (IP) approach, the Convex Hull (CHP) approach and the Primal-Dual (PD) method. There are other approaches and modifications of the approaches discussed here, see Liberopoulos and Andrianesis (2016) for an overview.

The IP pricing method uses the standard approach of marginal-cost pricing, with the addition that "uplift" values are computed and added to commodity prices in order to make sure that no market participant loses money on the trade. First, the market clearing problem is solved, including integer variables for indivisible products. Subsequently, an identical problem is created, except that now the integer variables are fixed to their optimal values. The result is a convex problem from which dual variables can be extracted. The commodity prices are now simply the dual variables related to the balancing constraints. Under these commodity prices, however, some market participants would be making a loss. Therefore, dual variables can be retrieved from the constraints that control the non-convexities. The dual variables can be interpreted as start-up costs for producers, or additional fees for bulk-purchasers and can be used to calculate so called uplifts, which are then added to the commodity price to form individual prices for all bidders, thereby deviating from a uniform price scheme but maintaining a market equilibrium (O'Neill, Sotkiewicz, Hobbs, Rothkopf, & Stewart, 2005). The start-up costs can be both positive and negative and as such imply zero profits for all market participants. As zero profits do not entice participants to bid in their power to the exchange, it is not used in practice.

An alternative pricing rule, called Convex Hull Pricing (CHP) was developed by Gribik, Hogan, and Pope (2007), drawing from the works of Hogan and Ring (2003) and Ring (1995). CHP introduces the concept of side-payments and aims to minimize these as a best compromise to uniform prices. Side-payments consist of two parts: lost opportunity cost (LOC) payments and excess product payments. LOC payments make sure that each committed resource receives its maximum possible profit given prices and its bidding constraints. We have

$$\text{LOC} = \text{Maximal profit} - \text{profit as dispatched}$$

In the case of an accepted block that is ITM, the maximal profit equals the profit as dispatched, so the LOC is 0. When the block is accepted but OTM (paradoxically accepted block) the maximal profit is 0 and the profit as dispatched is negative so the LOC becomes positive. The LOC in this case is referred to as a *make-whole payment* (the payment needed to make the block break even). It is a common misunderstanding that LOC are simply make-whole payments. When a rejected block is ITM (paradoxically rejected block) the profit as dispatched is zero and the LOC becomes the profit that would be taken if the plant was dispatched. When a block is rejected and OTM both maximal profit and profit as dispatched are 0. Excess product payments ensure that the total payment made by all buyers covers the total payments needed by all sellers, this is necessary because CHP may create positive prices for products that are not short given the market clearing (Schiro, Zheng, Zhao, & Litvinov, 2016).

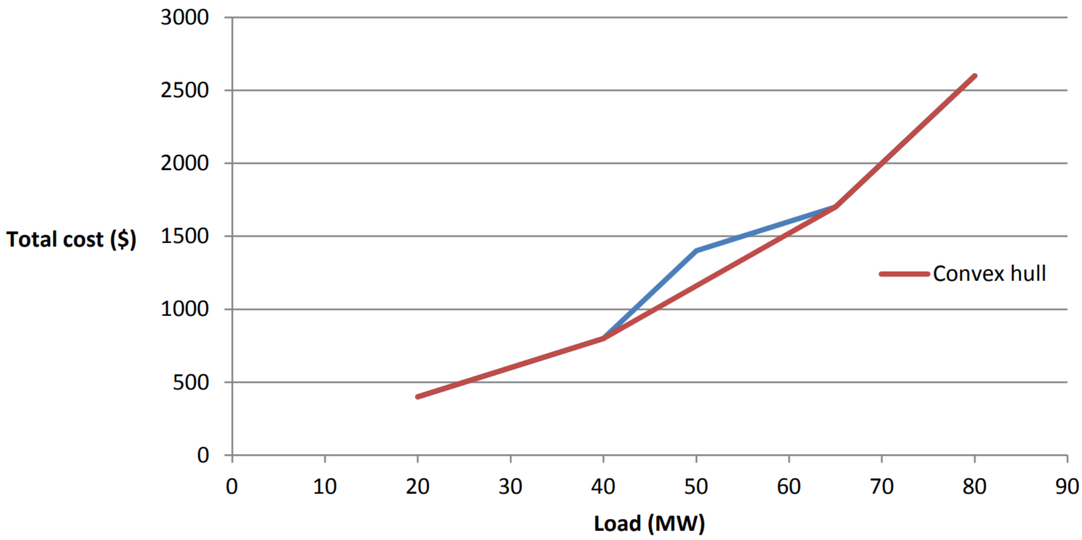


Figure 1: Convex hull of optimal total cost curve

CHP is similar to IP pricing in the sense that both make use of a commodity price and on top of that an uplift that is unique per trade to generate a solution that forms a market equilibrium. The methods differ, however, in the determination of the commodity price. Under IP, the commodity price is the dual variable of the balance constraint, while under CHP it is determined by approximating the cumulative non-convex cost of the original MILP with its convex hull. The cumulative non-convex curve is determined as a function of load based on suppliers' offers. The convex hull of this total cost curve is defined as *"the greatest convex function that is bounded above by the optimal total cost curve"* (Schiro et al., 2016, p. 12). The commodity price is now the slope of the convex hull of the optimal total cost curve at the actual load. Figure 1 shows the optimal total cost curve in blue, and its convex hull in red. It is important and interesting to note that the shape of this convex hull is partially determined by uncommitted resources. Another difference between IP pricing and CHP is that the uplift payment under CHP can never be negative. A rigorous implementation of CHP is not used in practice as it is computationally infeasible for large scale instances, however, some power exchanges in the United States (PJM, New York, New England, MidWest SO, ERCOT) use a variation of CHP pricing.

The third method is the Primal-Dual (PD) method, and it is also the method that we will follow. A detailed derivation can be found in Chapter 5, but a brief discussion can be found below. It is assumed that the optimal set of blocks for the market clearing problem is known a priori. The integrality constraints of the market clearing problem are then replaced with inequalities that force the decision variables of the blocks to their optimal values, resulting in an LP. Next, the dual of this problem is defined, so that now the dual variables related to the constraints forcing the blocks to be accepted or rejected can be interpreted as upper boundaries to the incurred losses or missed opportunity costs, respectively. Finally, a new problem is defined that includes all constraints from the original MILP (including integrality constraints) as well as the dual problem, and in which all dual variables related to the upper bound of incurred losses are set to zero. The resulting MILP now satisfies all requirements from Definition 2.3 except point 3. Ruiz, Conejo, and Gabriel (2012) use non-linear constraints, which are linearized with auxiliary variables. Madani and Van Vyve (2015) found a more efficient formulation that does not require auxiliary variables. The main difference between the Primal-Dual method and the two approaches discussed prior, is that under PD the resulting prices are uniform, i.e., there is one price for all market participants.

The algorithm currently in use in Europe is colloquially known as EUPHEMIA, and is a decomposition-based branch-and-bound algorithm that consists of two steps (Euphemia, 2019). First, the market clearing problem including integrality constraints is solved. The result is an optimal selection of hourly orders and blocks that ensure that the allocation respects network security constraints. Subsequently, a second optimization problem is defined that aims to find prices that meet all the requirements in Definition 2.3 except for point 3. Of course it is not always true that such a set of prices exist for the blocks found in the first step. In that case, cuts are added to the primal problem to exclude blocks that violate this requirement, and the primal problem is solved again. Note that like the PD method, this solution is asymmetric, it allows paradoxically rejected orders but not paradoxically accepted orders. This is justified because the owner of a paradoxically accepted order is actually losing money on the trade while the owner of a paradoxically rejected order only has a theoretical lost opportunity cost. To realise this opportunity cost, he has to find a counter party at market prices, which is costly and not always possible, so the deviation from equilibrium is arguably the least bad one.

4 Welfare optimization

This chapter will discuss the market clearing problem that applies to the CWE area specifically. There are three subsections, each of which will give general information as well as explain the sets, parameters and variables used to model the problem. A complete overview of the notation used, including dual variables, can be found in Section 5.1. We start with the hourly orders. Next, we explain the different kind of block orders and we finalize with a discussion about the transmission system.

4.1 Hourly orders

In this section we will discuss the sets, parameters and variables related to the hourly orders. The sets A and H contain the five areas and the 24 hours of the day. We will use $|A| \times |H| = 5 \cdot 24$ index sets of $I_{a,h}$, which are used to uniquely identify all the orders for a given area and hour. This means that every order can be identified using $a \in A$, $h \in H$, $i \in I_{a,h}$. Every order is associated with two parameters: a quantity $q_{a,h,i}$ and a

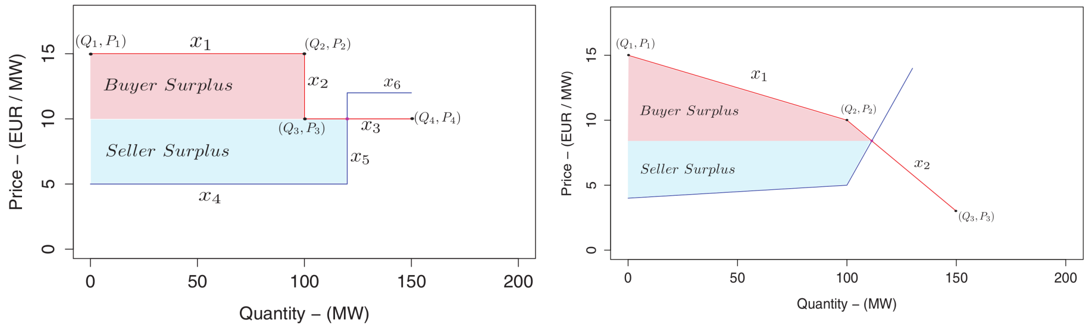


Figure 2: Stepwise curve (left) and linear curve (right) with the red line the aggregate demand (bid) curve and the blue line the aggregate supply (offer) curve (Madani & Van Vyve, 2015). Q stands for quantity while P stands for price. Every order is made up of a finite number of breakpoints $s \in S$

price $p_{a,h,i}$ as well as a continuous variable $0 \leq x_{a,h,i} \leq 1$ that indicates the acceptance ratio. Below we briefly explain how to find the quantities and prices for each order.

As said in the introduction, market participants can submit bids and offers to the power exchange for each area $a \in A$, and hour $h \in H$. They submit them in the form of stepwise or piecewise linear curves, specified by a number of quantity/price combinations $\{(Q_s, P_s)\}_{s \in S}$, S being the set of steps, see Figure 2. The bids and offers of all participants are collected by the power exchange and sorted and aggregated into aggregate supply and demand curves that reflect the demand and supply of the entire market. Supply curves are non-decreasing ($P_s \leq P_{s+1}$), since a higher price means more producers are willing to generate power. Demand curves are non-increasing ($P_{s+1} \leq P_s$), as a lower price means more consumers are willing to purchase power. Stepwise curves have the property that $P_s = P_{s+1}$ if $Q_s \neq Q_{s+1}$ and $P_s \neq P_{s+1}$ if $Q_s = Q_{s+1}$, while for piecewise linear curves it is possible to have $P_s \neq P_{s+1}$ and $Q_s \neq Q_{s+1}$. The hourly orders are now computed from each two consecutive points of the curve. For both the stepwise and the piecewise linear curve the quantity of an hourly order is simply computed by subtracting consecutive points $q_{a,h,i} = Q_{s+1} - Q_s$, where purchase orders are counted positively and sell orders negatively. This will prove to be useful in formulating the balance constraints and the objective function. In the case of the stepwise curve, if $Q_s = Q_{s+1}$ then the quantity will obviously be zero and we can omit the order since it will not contribute anything to the model. This means that only orders where $Q_s \neq Q_{s+1}$ will be included and in this case we have $P_s = P_{s+1}$ is equal to the price $p_{a,h,i}$ of that order. In the piecewise linear case the order starts to get accepted at P_s and ends at P_{s+1}. Therefore the price depends on the level of execution of the order $p_{a,h,i} = P_s + x_{a,h,i}(P_{s+1} - P_s)/2$. Since this would give a quadratic term in the objective function we set $p_{a,h,i} = (P_{s+1} + P_s)/2$. This is justified as for fully executed orders we have that $x_{a,h,i} = 1$ and therefore $P_s + x_{a,h,i}(P_{s+1} - P_s)/2 = (P_{s+1} + P_s)/2$. Only in the case of a partially accepted order is there a difference. However, orders will only be partially accepted when they are equal to the settlement price, which can be at most one bid and one offer for each area and hour.

4.2 Block orders

The sets J_a contain the block IDs, which are used to uniquely identify the block orders for a given area $a \in A$. Blocks are characterised by a single price $pb_{a,j}$ and quantities $qb_{a,j,h}$ for each hour of the day, again purchase blocks have positive volume while sellers have negative volume. The binary variable $y_{a,j}$ determines if the order is rejected or accepted in its entirety. The power exchanges for the five countries we deal with use four different types of block orders, C01, C02, C04 and C88 orders. C04 block orders are also called exclusive orders and come in groups. Market participants have the option to construct up to 24 block orders for a single group, out of which at most one can get accepted, which will be the one that generates the most social welfare. To model this we use sets G_a that contain all the IDs of the different groups. The sets E_g contain all the IDs of the blocks in every group. C02 block orders are also known as linked family block orders, every C02 block order is a child of either another C02 or a standard C01 block order, which is called the parent block order. The defining feature here is that a child can only get accepted if its parent is accepted. Every linked block order has a parameter $lp_{a,j}$ with the ID of its parent. A set of blocks that are linked together are called a family or a tree. Every parent can have up to six children, but a child can only have one parent. Furthermore, the maximum number of generations in a linked block order family is seven and the total amount of block orders in the family cannot exceed 40. The "highest" block in the family, also known as the root, is always a regular (C01) block order, while the rest of the family are linked family (C02) block orders. The set $lc_{a,j}$ contains the IDs of the children blocks. All regular blocks that are at the root of a linked family tree and all linked block orders except the ones at the leaves of the tree contain such a set. C01 orders are standard block orders without any additional constraints attached, except for the fact that they can be parent of a C02 order. In the dual formulation we will need this information so we introduce a parameter $lc_{a,j}$ that refers to the ID of its child. The last type is the C88 block order, or loop family block order. C88s come in pairs, where one is a purchase and the other a sell order. In this group of two, either both blocks get accepted or both get rejected. We deal with them in the same way as the exclusive block orders, U_a contains all the unique IDs for each group, while each O_u contains the two block IDs in that group.

4.3 The electricity network

As a country's power supply is closely related to its national security, electricity markets were historically organized nationally, where each country focused mainly on self-sufficiency. With the gradual integration of the economies of Western Europe, the need for integration of the electricity markets became apparent. Based on these foundations, the model for electricity trading makes use of a zonal approach, building on a number of interconnected markets. Within each zone, electricity can be traded without taking into account network restriction. For cross-border trade, however, network constraints become vitally important. That is why coordination among zones is important, as power flows are not bounded by commercial restrictions but follow the laws of physics.

For example, as Germany exports power to France, part of it will flow through the Netherlands and Belgium, instead of directly going into France, and will therefore also impact the flow capacity on the Dutch and Belgian borders. There are currently two approaches being used for determining this cross-border capacity, the Available Transfer Capacity (ATC) model, and the Flow Based (FB) model.

4.3.1 The available transfer capacity model

The ATC network model was once applied to all of Europe's cross-border trade. Within the CWE region it has been replaced by the more efficient FB model, but it is still very relevant as it is applied in the remainder of Europe's borders and will be briefly explained for the sake of completeness. The TSO of each country determines a Net Transfer Capacity (NTC) for each of its borders. They do this based on historical data, taking into account potential loop flows, seasonal impact and a security margin known as total reliability margin (TRM). This NTC can be interpreted as the maximum allowed flow that pushes the critical points of a network to their maximum load. After this, the TSO coordinates bilaterally with each of the TSO's responsible for their common border, usually selecting the lowest NTC among the two. Subsequently, the ATC is derived by subtracting capacity that has already been auctioned.

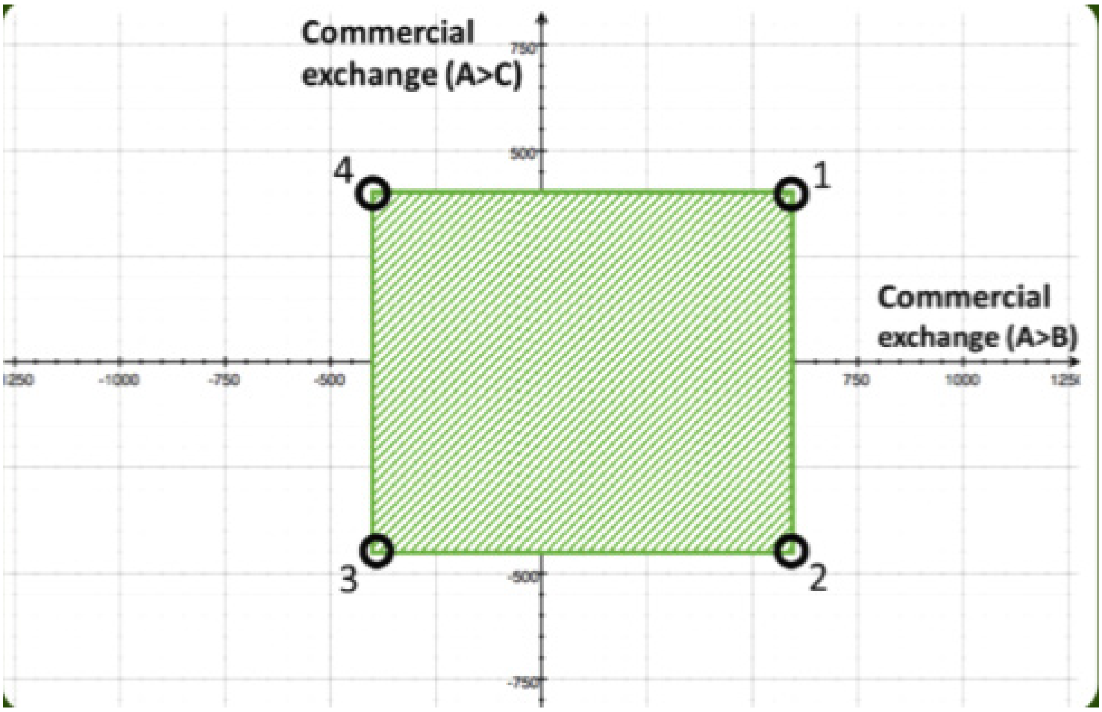


Figure 3: Feasible region ATC model

Figure 3 contains an example from the point of view of area A, which is connected to both area B and C. Each combination of flows falling inside the green rectangle is allowed. Corner 1 represents the case in which area A exports maximally to both B and C, while corner 3 represents maximum import from B and C.

4.3.2 The flow based model

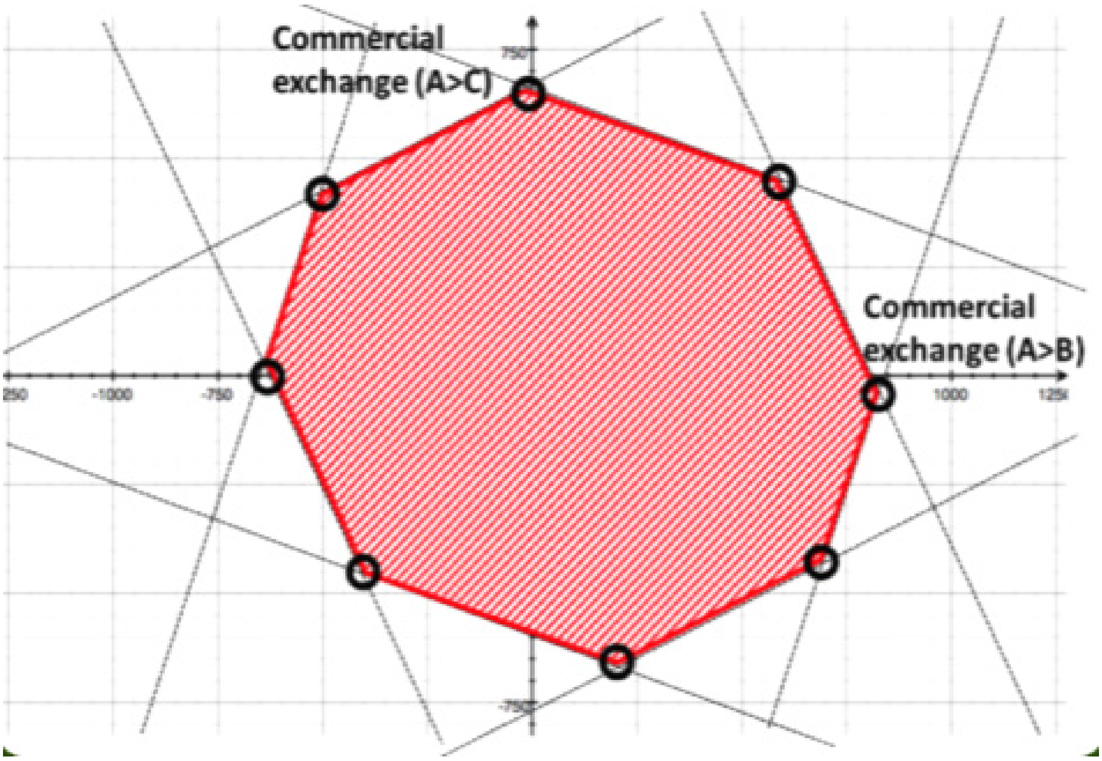


Figure 4: Feasible region FB model

Instead of supplying fixed capacities like the ATC model, the FB approach formulates the constraints that reflect the physical limits of the grid. Each TSO selects a number of physical nodes and lines in the grid based on their vulnerability with respect to internal contingencies and CWE cross-border exchanges. These bottlenecks of the grid are referred to as Critical Branches (CBs). The Remaining Available Margin (RAM) for each CB is determined based on the maximum physical capacity of the line after subtracting a Flow Reliability Margin (FRM) and can be seen as the maximum allowed load. Power Transmission Distribution Factors (PTDFs) denote the physical flow on a transmission line as a result of power being injected at a specific zone. The set K contains pairs of h, k, with $h \in H$ the hour that the constraint applies to and k an ID of the specific CB. $w_{h,k}$ is the Remaining Available Capacity for that branch, and $C_{h,k,a}$ is the PTDF for branch k that applies to hour $h \in H$ for area $a \in A$.

The feasible region will look like Figure 4, the FB domain corresponds with the global Security of Supply domain. Instead of assuming one NTC capacity value per direction on each border, all constraints imposed by the critical branches are considered. Figure 5 includes both the ATC and FB domains, it is clear that the FB domain contains the ATC domain. The reason for this is that when a TSO provides ATC constraints, a choice needs to be made on how to split the capacity among its borders (A to B and A to C), before the bids and offers in each market are known. While under the FB approach, the entire Security of Supply domain is available, and the market itself will

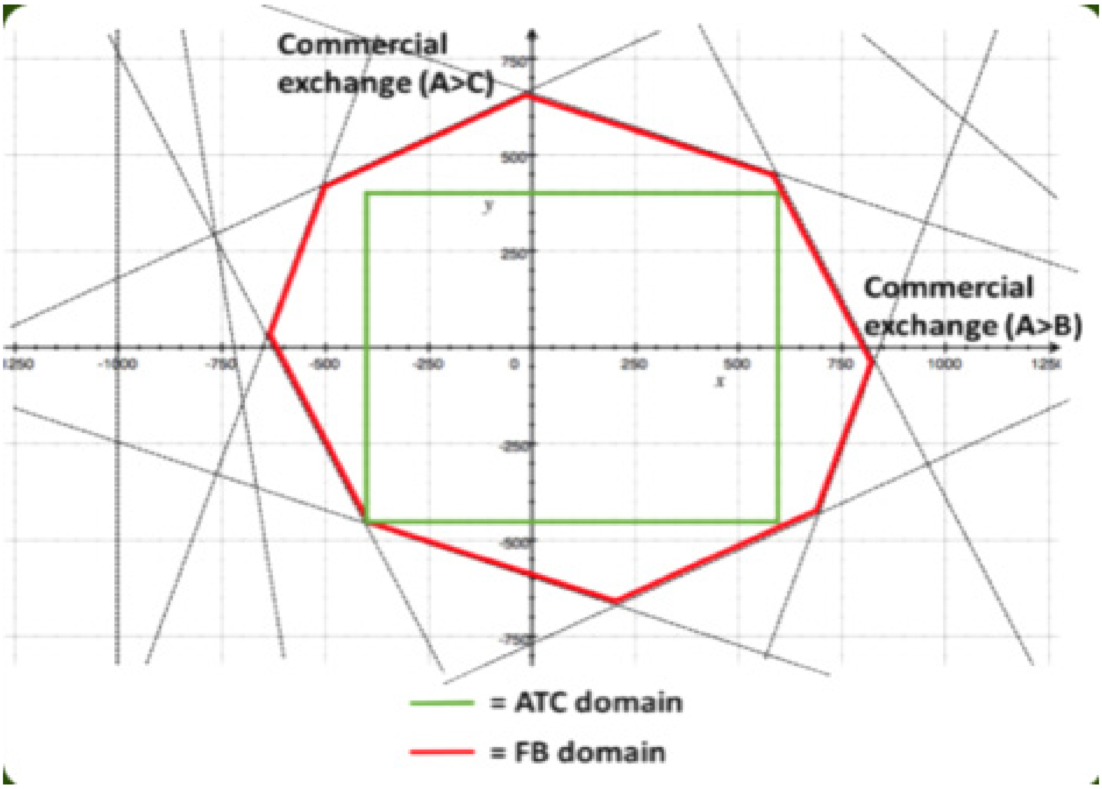


Figure 5: Feasible region of both ATC and FB model

decide on the division of commercial capacity between areas. This is clearly illustrated by the shape of the feasible regions. In Figure 3 the region is squared, which means that the maximum flow from A to B is the same regardless of the flow between A and C. Conversely, in Figure 4 we have that the flow from A to C impacts the capacity between A and C. From Figure 5 it can be seen that the difference is most notable when one of two flows is zero, as the FB model can allocate more capacity to the remaining border. Since, the FB domain encapsulates the ATC domain, it offers more transport options to the market and will therefore result in a better or equally good solution than under an ATC model. The constraint that a CB imposes is the following:

$$\text{PTDF} \cdot nex \leq \text{RAM}$$

Here, nex is the vector of net positions, the net position is the difference in MW sold and bought, as all the power that will be generated but not consumed needs to be exported, while on the other hand all the power that will be consumed but not generated needs to be imported. The plain flow based model has a significant flaw though, as it is possible for power to flow from high to lower priced areas. Market participants can see this as unfair anti-competitive behavior or "price-dumping" and it decreases the transparency of the results, even though it is a natural phenomenon that can be perfectly explained by the physical properties of the system (Vlachos & Biskas, 2016). For this reason an extra requirement is added, namely that only flows from low

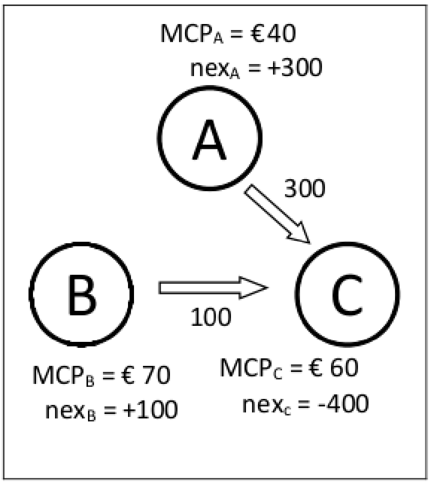


Figure 6: Example of net positions decomposed into flows

to equal or higher prices areas are allowed, referred to as a Flow Based Intuitive (FBI) model. The example in Figure 6 comes from the public documentation of Euphemia (2019) and shows why these non-intuitive situations might happen. It contains three markets and one critical branch so that the imposed constraint looks like the following:

$$0.25 \cdot nex_a - 0.5 \cdot nex_b - 0.25 \cdot nex_c \leq 125$$

In the representation of the result, bilateral exchanges between areas are shown, in addition to the net export positions. Theoretically there are infinitely many potential decompositions of the net export position in bilateral exchanges. Any decomposition will lead to market B exporting to a lower priced area, however, since it is exporting and it is also the highest priced area. The reason for these non-intuitive flows is that some non-intuitive flows might free up capacity on a branch allowing larger flows between other areas. Rewriting the net export positions into the sum of flows will show why.

$$\begin{aligned} PTDF_a \cdot nex_a + PTFDF_b \cdot nex_b + PTDF_c \cdot nex_c \leq 125 \\ PTDF_a \cdot (flow_{a,b} - flow_{b,a} + flow_{a,c} - flowc, a)+ \\ PTDF_b \cdot (flow_{b,a} - flow_{a,b} + flow_{b,c} - flow_{c,b})+ \\ PTDF_c \cdot (flow_{c,a} - flow_{a,c} + flow_{c,b} - flow_{b,c}) \leq 125 \\ flow_{a,b} \cdot (PTDF_a - PTDF_b) + flow_{b,a} \cdot (PTDF_b - PTDF_a)+ \\ flow_{a,c} \cdot (PTDF_a - PTDF_c) + flow_{c,a} \cdot (PTDF_c - PTDF_a)+ \\ flow_{b,c} \cdot (PTDF_b - PTDF_c) + flow_{c,b} \cdot (PTDF_c - PTDFB) \leq 125 \end{aligned} \tag{4.1}$$

Which, after substituting the PTDF values results in:

$$\begin{gathered}
flow_{a,b} \cdot (0.25 - (-0.5)) + flow_{b,a} \cdot ((-0.5) - 0.25) + \\
flow_{a,c} \cdot (0.25 - (-0.25)) + flow_{c,a} \cdot ((-0.25) - 0.25) + \\
flow_{b,c} \cdot (-0.5 - (-0.25)) + flow_{c,b} \cdot ((-0.25) - (-0.5)) \leq 125 \\
flow_{a,b} \cdot 0.75 + flow_{b,a} \cdot (-0.75) + flow_{a,c} \cdot 0.5 + \\
flow_{c,a} \cdot (-0.5) + flow_{b,c} \cdot (-0.25) + flow_{c,b} \cdot 0.25 \leq 125
\end{gathered} \tag{4.2}$$

We see that exporting from area B to area C loads the critical branch with -0.25 MW for each MW exchanged, in other words: the flow relieves the load on the line. This means that flowing from B to C frees up capacity that can be used to transport more between other markets, thereby increasing social welfare. This is an undesirable situation as it decreases transparency for market participants. In extreme cases where several markets end up at maximum price, the PTDF coefficients can lead to unfair distribution of the available energy so that the solution that maximizes the welfare is the one where one market is totally curtailed, while all the available energy is given to another market that is not necessarily at maximum price. A solution is to cancel out the relieving effects of the exchange between area B and C, by replacing coefficient $PTDF_b$ - $PTDF_c$ with max($PTDF_b$ - $PTDF_c$, 0).

5 Model

This chapter will show how the model was derived. In Section 5.1 you can find the notation that has already largely been introduced in Chapter 4. The model we will develop here is very similar to the one in Madani and Van Vyve (2018) and can be classified under the Primal-Dual approach. The reason we picked this method is because the requirement from the European power exchanges is that the prices need to be uniform. The only deviation from a perfect market equilibrium is that paradoxically rejected orders are allowed. Furthermore, the model should correctly handle the different block order types, discussed in Section 4.2. Finally, it should be able to deal with a flow based transmission model, while maintaining a *spatial equilibrium* (no power flows from lower priced to higher priced areas).

The first step is to define an MILP that maximizes social welfare under all relevant constraints, this model will be referred to as the "primal" and discussed in section 5.2. Subsequently, we assume that the optimal selection of blocks is known in advance and replace the integrality constraints for each block by an inequality that forces the decision variable to its a priori "known" value. This turns the problem from an MILP to an LP, with the advantage that strong duality holds. As strong duality holds, dual variables have a straightforward interpretation: the dual variable gives the improvement in the objective function if the constraint is relaxed by one unit. So using this LP we are now able to construct a dual problem and complementary slackness constraints. The dual variables related to the constraints forcing the decision variables of the "accepted" blocks to be greater than or equal to 1 can be interpreted as upper bounds on the loss the blocks make. The dual variables related to the constraints forcing the decision variables of the "rejected" blocks to be less than or equal to 0, on the other hand, can be interpreted as upper bounds on the missed opportunity costs. The next step is to formulate the feasible region that is described by the primal, dual and complementary slackness constraints. This is done by putting together the primal and dual constraints and adding a constraint that forces the objective value of the primal to be less than or equal to the dual. To enforce a partial market equilibrium that does not allow losses for accepted blocks, we set the dual variables related to the "accepted" blocks to 0. Note that we still allow blocks to be rejected but ITM so there is no full market equilibrium, i.e., point 3 from Definition 2.3 is not satisfied. From this we describe a final model whose feasible set is the one described in the previous step and whose objective function is to maximize social welfare.

There are three main differences between our model and the one by Madani and Van Vyve (2018). First, it includes all the different types of block orders that are employed in the CWE region. Second, it enforces a spatial equilibrium, discussed in Subsection 4.3.2. Finally, it omits the flexible minimum acceptance ratio, i.e., the model in Madani and Van Vyve (2018) can handle the constraint that some plants either want to shut down completely or produce more than a certain ratio of the offered volume (e.g. 50%). As this parameter is not made available by the power exchanges of the CWE region, all the blocks in our model can either be completely rejected or accepted, i.e., the minimum acceptance ratio is always one.

5.1 Notation

sets

A	Set of Areas, with $a \in A$
H	Set of Hours, with $h \in H$
$I_{a,h}$	Sets of Indices of hourly orders for a $\in A, h \in H$, with $i \in I_{a,h}$
J_a	Set of Blocks, for $a \in A$ with $j \in J_a$
B_a	Sets of borders, for $a \in A$, with $b \in B_a$
K_h	Sets of Critical Branches, for $h \in H$, with $k \in K$
G_a	Sets of IDs of exclusive (C04) blocks, for $a \in A$ with $g \in G$
E_g	Sets of blocks that are in the same exclusive group, for $g \in G$ with $j \in E_g$
U_a	Sets of IDs of loop family (C88) blocks, for $a \in A$ with $u \in U$
O_u	Sets of blocks that are in the same loop family group, for $u \in U$ with $j \in O_u$
L_a	Sets of all linked family (C02) blocks, for $a \in A$ with $j \in L$
N_a	Sets of all regular (C01) blocks, for $a \in A$ with $j \in N$
J_a^a	Sets of all accepted blocks, for $a \in A$, with $j \in J_a^a$
J_a^r	Sets of all accepted blocks, for $a \in A$, with $j \in J_a^r$

parameters

$p_{a,h,i}$	Price for hourly orders, with $a \in A, h \in H, i \in I_{a,h}$
$q_{a,h,i}$	Quantity for hourly orders, with $a \in A, h \in H, i \in I_{a,h}$
$pb_{a,j}$	Price for block orders, with $a \in A, j \in J_a$
$qb_{a,j,h}$	Quantity for block orders, with $a \in A, j \in J_a, h \in H$
$lp_{a,j}$	Block ID of parent block, with $j \in L$
$lc_{a,j}$	Block IDs of children blocks, if any, with $j \in N \cup L$
$w_{h,k}$	Remaining available margin, with $h, k \in K$
$c_{h,k,a}$	Coefficient for Critical Branch, with, $h, k \in K, a \in A$

primal variables

$0 \leq x_{a,h,i} \leq 1$	Variable associated to execution ratio of hourly orders, with $a \in A, h \in H, i \in I_{a,h}$
$y_{a,j} \in \{0, 1\}$	Variable associated to execution ratio of block orders, with $a \in A, j \in J_a$
$nex_{a,h}$ (free)	Variable denoting the net export position, with $a \in A, h \in H$
$0 \leq f_{a,b,h}$	Flow variable, with $a, \in A, b \in B_a, h \in H$

dual variables

$0 \leq s_{a,h,i}$	Variable representing the surplus associated to the hourly bid $x_{a,h,i}$, with $a \in A, h \in H, i \in I_{a,h}$
$0 \leq z_{a,j}$	Variable representing the surplus associated to the block order $y_{a,j}$, with $a \in A, j \in J_a$
$0 \leq v_{h,k}$	Variable representing the value of critical branch, with $h \in H, k \in K_h$
$-500 \leq \pi_{a,h} \leq 3000$	Variable representing a uniform price, with $a \in A, h \in H$
$0 \leq c02_{a,j}$	Variable that transfers the surplus of a child to its parent $c02$
$0 \leq c04_g$	Variable representing the surplus of the group g
$c88_u$ (free)	Variable representing the surplus of the group u
π_h^{sys}	Variable representing the system price, with $h \in H$
$\delta_{a,h}$	Variable representing the deviation from π_h^{sys}, with $a \in A, h \in H$
$du_{a,j}^a$	Variable representing the upper bound on incurred losses of an accepted block
$du_{a,j}^r$	Variable representing the upper bound on missed opportunity costs of a rejected block

5.2 The primal problem

The primal model in standard form can be found in 5.1, including network and integrality constraints as well as all constraints to handle the different types of blocks. The dual variables are written after each constraint and are given within brackets [ ]. After giving the first description we discuss the various expressions and make some modifications.

Primal

$$\max_{x,y} \sum_{a \in A, h \in H, i \in I_{a,h}} x_{a,h,i} \cdot p_{a,h,i} \cdot q_{a,h,i} + \sum_{a \in A, j \in J_A} y_{a,j} \cdot pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} \tag{5.1a}$$

s.t.

$$\sum_{i \in I_{a,h}} x_{a,h,i} \cdot q_{a,h,i} + \sum_{j \in J_A} y_{a,j} \cdot qb_{a,j,h} + nex_{a,h} = 0 \qquad \forall a \in A, h \in H \quad [\delta_{a,h}] \tag{5.1b}$$

$$\sum_{a \in A} nex_{a,h} = 0 \qquad \forall h \in H \quad [\pi_h^{sys}] \tag{5.1c}$$

$$\sum_{a \in A} nex_{a,h} \cdot c_{h,k,a} \leq w_{h,k} \qquad \forall h \in H, k \in K_h \quad [v_{h,k}] \tag{5.1d}$$

$$\sum_{j \in E_g} y_{a,j} \leq 1 \qquad \forall a \in A, g \in G_a \quad [c04_g] \tag{5.1e}$$

$$y_{a,j} - y_{a,lp_{a,j}} \leq 0 \qquad \forall a, j \in L \quad [c02_{a,j}] \tag{5.1f}$$

$$y_{a,j_1 \in O_u} - y_{a,j_2 \in O_u} = 0 \qquad \forall a \in A, u \in U_a \quad [c88_u] \tag{5.1g}$$

$$-x_{a,h,i} \leq 0 \qquad \forall a \in A, h \in H, i \in I_{a,h} \tag{5.1h}$$

$$x_{a,h,i} \leq 1 \qquad \forall a \in A, h \in H, i \in I_{a,h} \quad [s_{a,h,i}] \tag{5.1i}$$

$$-y_{a,j} \leq 0 \qquad \forall a \in A, j \in J_a \tag{5.1j}$$

$$y_{a,j} \leq 1 \qquad \forall a \in A, j \in J_a \quad [z_{a,j}] \tag{5.1k}$$

$$y_{a,j} \in \mathbb{Z} \qquad \forall a \in A, j \in J_a \tag{5.1l}$$

$$-f_{a,b,h} \leq 0 \qquad \forall h \in H, a \in A, b \in B_a \tag{5.1m}$$

The first summation of the objective 5.1a handles the hourly orders while the second takes care of all the block orders. Inequalities 5.1b, 5.1c and 5.1d enforce the constraints laid out by the network. The first one makes sure that the net export position is equal to the difference between the total volume bought and sold, as the volume for sell orders is negative the net export position becomes positive in case an area is exporting. The second constraint forces the sum of all the net positions to be equal to zero, this is necessary so that the entire system is in balance and the total export matches total import. Finally 5.1d forces the load on each branch to be smaller than the capacity.

$$\sum_{i \in I_{a,h}} x_{a,h,i} \cdot q_{a,h,i} + \sum_{j \in J_A} y_{a,j} \cdot qb_{a,j,h} + \sum_{b \in B_a} (f_{a,b,h} - f_{b,a,h}) = 0 \qquad \forall a \in A, h \in H \quad [\pi_{a,h}] \tag{5.2}$$

$$\sum_{a \in A, b \in B_a} f_{a,b,h} \cdot (c_{h,k,a} - c_{h,k,b}) \leq w_{h,k} \qquad \forall h \in H, k \in K_h \quad [v_{h,k}] \tag{5.3}$$

In order to be able to enforce the intuitiveness requirement, constraints 5.2 and 5.3 should be used instead of 5.1b, 5.1c and 5.1d. Equality 5.2 directly decomposes the net export position into the sum of incoming and outgoing flows, instead of using a variable that represents the net export position. This is necessary so that 5.3 can replace 5.1d, see the system of equations 4.2 for the derivation. In addition, it also makes 5.1c obsolete as the flow variable appears in balance constraint 5.2 of both the sending and receiving area, thereby linking supply and demand in the two areas. Note that the dual variables π_h^{sys} related to 5.1c can be interpreted as the system price and the dual variables $\delta_{a,h}$ related to 5.1d as each country's deviation from this system price. The dual variables $\pi_{a,h}$ related to constraint 5.2, on the other hand, directly give the settlement price so that we have $\pi_{a,h} = \pi_h^{sys} + \delta_{a,h}$.

Constraints 5.1e, 5.1f and 5.1g enforce the special requirement for each type of block order. Given a group of exclusive block orders, inequality 5.1e forces the sum of all the decision variables of the blocks in this group to be smaller or equal than 1. With respect to the linked family blocks, 5.1f forces the decision variable of the parent block to be greater than or equal to the decision variable of the child. Finally, 5.1g forces the decision variables of the two blocks in the loop family group to be equal. The remaining constraints force the variables to be within their respective bounds.

We now follow the reasoning in Madani and Van Vyve (2018) and assume we know the optimal set of blocks for 5.1. For this, consider the partition $J_a = J_a^a \cup J_a^r$, with J_a^a the accepted blocks and J_a^r the rejected blocks for each area. We now add constraints 5.4, and 5.5 and drop 5.1l to obtain a Linear Problem.

$$-y_{a,j_a} \leq -1 \qquad \forall a \in A, j_a \in J_a^a \quad [du_{a,j}^a] \tag{5.4}$$

$$y_{a,j_r} \leq 0 \qquad \forall a \in A, j_r \in J_a^r \quad [du_{a,j}^r] \tag{5.5}$$

5.3 The dual problem and complementary slackness constraints

Problem 5.1 including constraints 5.2, 5.3, 5.4 and 5.5 but without 5.1b, 5.1c, 5.1d and 5.1l yields the dual problem found in 5.6. The derivation is a bit arduous due to the different kind of blocks. We effectively get two constraints for every type of block, one for an accepted block and one for a rejected block, except for the C88's that require four constraints.

Constraints 5.6c and 5.6d apply to linked family (C02) blocks, the variable $c02_{a,j}$ is there because this linked family block is necessarily a child of another block. The sum $-\sum_{j_1 \in lc_{a,j}} c02_{a,j_1}$ exists because this block might also be the parent of some other C02 blocks. Note that this does not have to be the case, $lc_{a,j}$ might be empty. $lc_{a,j}$ also appears in constraints 5.6e and 5.6f for regular (C01) blocks since a C01 block can be the parent of a C02 block. Constraints 5.6g and 5.6h apply to the exclusive (C04) blocks and are rather straightforward. Finally, there are the loop family (C88) blocks that require four constraints. If the block appears as the first block in primal constraint 5.1g, then $c88_u$ should be positive and, depending on execution, 5.6i or 5.6j applies. Conversely, if the block appears last in the primal constraint $c88_u$ should negative so that 5.6k or 5.6l applies. Note that we restricted the range of the settlement prices π to [-500, 3000] as these are the limit prices for orders.

Dual

$$\min_{s,z,c04,v,du^a} \sum_{a \in A, h \in H, i \in I_{a,h}} s_{a,h,i} + \sum_{a \in A, j \in J_A} z_{a,j} + \sum_{g \in G} c04_g + \sum_{h,k \in K} v_{h,k} \cdot w_{h,k} - \sum_{a \in A, j \in J_a^a} du_{a,j}^a \tag{5.6a}$$

s.t.

$$s_{a,h,i} + \pi_{a,h} \cdot q_{a,h,i} \geq p_{a,h,i} \cdot q_{a,h,i} \qquad \forall a \in A, h \in H, i \in I_{a,h} \tag{5.6b}$$

$$z_{a,j_a} + c02_{a,j^a} - \sum_{j_1 \in lc_{a,j^a}} c02_{a,j_1} - du_{a,j^a}^a +$$

$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_a,h}\geq pb_{a,j^a}\cdot\sum_{h\in H}qb_{a,j^a,h}\quad\forall a\in A, j^a\in L_a\cap J_a^a \tag{5.6c}$$

$$z_{a,j^r}+c02_{a,j^r}-\sum_{j_1\in lc_{a,j^r}}c02_{a,j_1}+du^r_{a,j^r}+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^r,h}\geq pb_{a,j^r}\cdot\sum_{h\in H}qb_{a,j^r,h}\quad\forall a\in A, j^r\in L_a\cap J_a^r \tag{5.6d}$$

$$z_{a,j^a}-\sum_{j_1\in lc_{a,j^a}}c02_{a,j_1}-du^a_{a,j^a}++\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^a,h}\geq pb_{a,j^a}\cdot\sum_{h\in H}qb_{a,j^a,h}\quad\forall a\in A, j^a\in N_a\cap J_a^a \tag{5.6e}$$

$$z_{a,j^r}-\sum_{j_1\in lc_{a,j^r}}c02_{a,j_1}+du^r_{a,j^r}++\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^r,h}\geq pb_{a,j^r}\cdot\sum_{h\in H}qb_{a,j^r,h}\quad\forall a\in A, j^r\in N_a\cap J_a^r \tag{5.6f}$$

$$z_{a,j^a}+c04_g-du^a_{a,j^a}+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^a,h}\geq pb_{a,j^a}\cdot\sum_{h\in H}qb_{a,j^a,h}\quad\forall a\in A, g\in G_a, j^a\in E_g\cap J_a^a \tag{5.6g}$$

$$z_{a,j^r}+c04_g+du^r_{a,j^r}+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^r,h}\geq pb_{a,j^r}\cdot\sum_{h\in H}qb_{a,j^r,h}\quad\forall a\in A, g\in G_a, j^r\in E_g\cap J_a^r \tag{5.6h}$$

$$z_{a,j_1^a}+c88_u-du^a_{a,j_1^a}+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_1^a,h}\geq pb_{a,j_1^a}\cdot\sum_{h\in H}qb_{a,j_1^a,h}\quad\forall a\in A, u\in U_a, j_1^a\in O_u\cap J_a^a \tag{5.6i}$$

$$z_{a,j_1^r}+c88_u+du^r_{a,j_1^r}+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_1^r,h}\geq pb_{a,j_1^r}\cdot\sum_{h\in H}qb_{a,j_1^r,h}\quad\forall a\in A, u\in U_a, j_1^r\in O_u\cap J_a^r \tag{5.6j}$$

$$z_{a,j_2^a}-c88_u-du^a_{a,j_2^a}+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_2^a,h}\geq pb_{a,j_2^a}\cdot\sum_{h\in H}qb_{a,j_2^a,h}\quad\forall a\in A, u\in U_a, j_2^a\in O_u\cap\ J_a^a \tag{5.6k}$$

$$z_{a,j_2^r}-c88_u+du^r_{a,j_2^r}+$$

$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_2^r,h} \geq pb_{a,j_2^r}\cdot\sum_{h\in H} qb_{a,j_2^r,h} \quad \forall a\in A, u\in U_a, j_2^r\in O_u\cap\ J_a^r \tag{5.6l}$$

$$\pi_{a,h}-\pi_{b,h}+\sum_{k\in K_h}(c_{h,k,a}-c_{h,k,b})\cdot v_{h,k}\geq 0 \quad \forall h\in H, a\in A, b\in B_a \tag{5.6m}$$

$$s_{a,h,i}\geq 0 \quad \forall a\in A, h\in H, i\in I_{a,h} \tag{5.6n}$$

$$z_{a,j}\geq 0 \quad \forall a\in A, j\in J_a \tag{5.6o}$$

$$v_{h,k}\geq 0 \quad \forall h\in H, k\in K_h \tag{5.6p}$$

$$\pi_{a,h}\geq -500 \quad \forall a\in A, h\in H \tag{5.6q}$$

$$-\pi_{a,h}\geq -3000 \quad \forall a\in A, h\in H \tag{5.6r}$$

$$c02_{a,j}\geq 0 \quad \forall a, j\in L \tag{5.6s}$$

$$c04_g\geq 0 \quad \forall g\in G \tag{5.6t}$$

We continue by writing down the complementary slackness constraints corresponding to the primal and dual, these hold in the case of optimality, see Theorem A.1.

Complementary Slackness Constraints

$$s_{a,h,i}(1-x_{a,h,i})=0 \quad \forall a\in A, h\in H, i\in I_{a,h} \tag{5.7a}$$

$$z_{a,j}(1-y_{a,j})=0 \quad \forall a\in A, j\in J_a \tag{5.7b}$$

$$v_{h,k}(\sum_{a\in A, b\in B_a}(w_{h,k}-f_{a,b,h}\cdot(c_{h,k,a}-c_{h,k,b})))=0 \quad \forall h\in H, k\in K_h \tag{5.7c}$$

$$(1-y_{a,j})du^a_{a,j}=0 \quad \forall a\in A, j\in J_a^a \tag{5.7d}$$

$$y_{a,j}du^r_{a,j}=0 \quad \forall a\in A, j\in J_a^r \tag{5.7e}$$

$$x_{a,h,i}(s_{a,h,i}+q_{a,h,i}\pi_{a,h}-q_{a,h,i}p_{a,h,i})=0 \quad \forall a\in A, h\in H, i\in I_{a,h} \tag{5.7f}$$

$$y_{a,j^a}(z_{a,j^a}+c02_{a,j^a}-\sum_{j_1\in lc_{a,j^a}}c02_{a,j_1}-du^a_{a,j^a}+ \sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^a,h}-pb_{a,j^a}\cdot\sum_{h\in H}qb_{a,j^a,h})=0 \quad \forall a\in A, j^a\in L_a\cap J_a^a \tag{5.7g}$$

$$y_{a,j^r}(z_{a,j^r}+c02_{a,j^r}-\sum_{j_1\in lc_{a,j^r}}c02_{a,j_1}+du^r_{a,j^r}+ \sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^r,h}-pb_{a,j^r}\cdot\sum_{h\in H}qb_{a,j^r,h})=0 \quad \forall a\in A, j^r\in L_a\cap J_a^r \tag{5.7h}$$

$$y_{a,j^a}(z_{a,j^a}-\sum_{j_1\in lc_{a,j^a}}c02_{a,j_1}-du^a_{a,j^a}+$$

$$+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^a,h}-pb_{a,j^a}\cdot\sum_{h\in H}qb_{a,j^a,h})=0 \quad \forall a\in A, j^a\in N_a\cap J_a^a \tag{5.7i}$$

$$y_{a,j^r}(z_{a,j^r}-\sum_{j_1\in lc_{a,j^r}}c02_{a,j_1}+du^r_{a,j^r}+$$
$$+\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^r,h}-pb_{a,j^r}\cdot\sum_{h\in H}qb_{a,j,h})=0 \quad \forall a\in A, j^r\in N_a\cap J_a^r \tag{5.7j}$$

$$y_{a,j^a}(z_{a,j^a}+c04_g-du^a_{a,j^a}+$$
$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^a,h}-pb_{a,j^a}\cdot\sum_{h\in H}qb_{a,j^a,h})=0 \quad \forall a\in A, g\in G_a, j^a\in E_g\cap J_a^a \tag{5.7k}$$

$$y_{a,j^r}(z_{a,j^r}+c04_g+du^r_{a,j^r}+$$
$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j^r,h}-pb_{a,j^r}\cdot\sum_{h\in H}qb_{a,j^r,h})=0 \quad \forall a\in A, g\in G_a, j^r\in E_g\cap J_a^r \tag{5.7l}$$

$$y_{a,j_1^a}(z_{a,j_1^a}+c88_u-du^a_{a,j_1^a}+$$
$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_1^a,h}-pb_{a,j_1^a}\cdot\sum_{h\in H}qb_{a,j_1^a,h})=0 \quad \forall a\in A, u\in U_a, j_1^a\in O_u\cap J_a^a \tag{5.7m}$$

$$y_{a,j_1^r}(z_{a,j_1^r}+c88_u+du^r_{a,j_1^r}+$$
$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_1^r,h}-pb_{a,j_1^r}\cdot\sum_{h\in H}qb_{a,j_1^r,h})=0 \quad \forall a\in A, u\in U_a, j_1^r\in O_u\cap J_a^r \tag{5.7n}$$

$$y_{a,j_2^a}(z_{a,j_2^a}-c88_u-du^a_{a,j_2^a}+$$
$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_2^a,h}-pb_{a,j_2^a}\cdot\sum_{h\in H}qb_{a,j_2^a,h})=0 \quad \forall a\in A, u\in U_a, j_2^a\in O_u\cap J_a^a \tag{5.7o}$$

$$y_{a,j_2^r}(z_{a,j_2^r}-c88_u+du^r_{a,j_2^r}+$$
$$\sum_{h\in H}\pi_{a,h}\cdot qb_{a,j_2^r,h}-pb_{a,j_2^r}\cdot\sum_{h\in H}qb_{a,j_2^r,h})=0 \quad \forall a\in A, u\in U_a, j_2^r\in O_u\cap J_a^r \tag{5.7p}$$

We will now prove some Lemmas regarding the interpretation of the dual variables. Keep in mind that for bids, quantity parameters $q_{a,h,i}, qb_{a,j,h} > 0$, while for offers $q_{a,h,i}, qb_{a,j,h} < 0$.

Lemma 5.1 (Interpretation of s and equilibrium for hourly orders)**.** *Variables $s_{a,h,i}$ correspond to the surplus of order $x_{a,h,i}$, i.e.:*

$$s_{a,h,i} = (q_{a,h,i}p_{a,h,i} - q_{a,h,i}\pi_{a,h})x_{a,h,i} \tag{5.8}$$

In addition, the following equilibrium conditions are valid, so that no agent prefers another level of execution (Definition 2.2):

1. *An hourly order that is fully accepted is ITM or ATM and the surplus is non-negative.*

2. An hourly order that is partially accepted is ATM

3. An hourly order that is fully rejected is OTM or ATM

Proof. We omit subscripts in this proof as we use the same a, h, i for all parameters and variables.

1. If the order is fully executed, $x = 1$, now from 5.7f and 5.6n we have that $s = qp - q\pi \geq 0$, so the bid is ITM or ATM. Multiplying the inequality by $x = 1$ we obtain 5.8.
2. If $0 < x < 1$, then from 5.7a we have $s = 0 = sx$. Now 5.7f gives $s = qp - q\pi = 0$, so the bid is ATM, multiplying by x gives 5.8.
3. If $x = 0$, then $s = 0$ according to 5.7a. 5.6b gives $pq - \pi q < 0$ so the order it OTM or ATM. Since $x = 0 = s$, 5.8 is trivially true.

□

Lemma 5.2 (Interpretation of $z_{a,j}, du^a_{a,j}, du^r_{a,j}$)**.** *Variables $z_{a,j}$ correspond to the potential surplus of a block order. Variables $du^a_{a,j}$ can be interpreted as a limit on the allowed loss of an accepted block order and $du^r_{a,j}$ as an upper bound on missed opportunity costs of a rejected block order. Moreover, for an accepted regular block order without any children blocks equality 5.9 holds.*

$$z_{a,j} - du^a_{a,j} = (\sum_{h \in H} pb_{a,j}qb_{a,j,h} - \pi_{a,h}qb_{a,j,h})y_{a,j} \qquad \forall a \in A, j \in N_a \cap J^a_a \qquad (5.9)$$

Proof. The first equality follows immediately from 5.7i with $y_{a,j} = 1$ and $lc_{a,j}$ empty. As $z_{a,j}, du^a_{a,j} \geq 0$ the interpretation follows straight away.

Now, as a rejected block implies $y_{a,j} = 0$, then from 5.7b we have $z_{a,j} = 0$ as well. Finally 5.6f with $lc_{a,j}$ empty then gives $du^r_{a,j} \geq \sum_{h \in H} pb_{a,j}q_{a,j,h} - \pi_{a,h}q_{a,j,h}$, which implies $du^r_{a,j}$ is an upper bound on the missed profit. □

Theorem 5.1 (Non-negative profits for blocks)**.** *If $du^a_{a,j} = 0$ for an accepted block then it is ITM.*

Proof. Remember that a block is in the money if $\sum_{h \in H} pb_{a,j}qb_{a,j,h} - \pi_{a,h}qb_{a,j,h} \geq 0$, as for a purchase ($q_{a,j,h} > 0$) it means that the buyer wants to pay more than the settlement price, and for a seller ($q_{a,j,h} < 0$) it means he wants to produce for less than the settlement price. If $du^a_{a,j} = 0$, then since $z_{a,j} \geq 0$ the left hand side of 5.9 is non-negative. Since $y_{a,j} = 1$, this means that $\sum_{h \in H} pb_{a,j}qb_{a,j,h} - \pi_{a,h}qb_{a,j,h} \geq 0$. □

Lemma 5.3 (Interpretation of $c02_{a,j}$)**.** *$c02_{a,j}$ transfers the surplus of a child block to its parent.*

Proof. Take $a, j_1 \in N_a \cap J^a_a$ as the parent block, and $a, j_2 \in L_a \cap J^a_a$ as its child block. We set $du^a_{a,j_1} = du^a_{a,j_2} = 0$ to not allow any losses for blocks. We consider two situations: 1 the parent is ITM while the child is OTM and 2 the parent is OTM while the child is ITM.

1. Assume the parent block is ITM by $\delta_1 > 0$, so $\sum_{h \in H} pb_{a,j_1} qb_{a,j_1,h} - \pi_{a,h} qb_{a,j_1,h} = \delta_1$, and the child block is OTM by $\delta_2 < 0$, so $\sum_{h \in H} pb_{a,j_2} qb_{a,j_2,h} - \pi_{a,h} qb_{a,j_2,h} = \delta_2$. Furthermore, assume that $\delta_1 + \delta_2 \geq 0$, i.e. the profit of the child is greater or equal than the loss of the parent. Then the corresponding dual constraints for parent and child are 5.6e and 5.6c respectively and simplify to:

$$z_{a,j_1} - c02_{a,j_2} \geq \delta_1$$
$$z_{a,j_2} + c02_{a,j_2} \geq \delta_2$$

Remember that the objective of the dual is to minimize z. The optimal solution for the first equation is $z_{a,j_1} = \delta_1, c02_{a,j_2} = 0$, while for the second equation any value of z_{a,j_2} suffices (as $z \geq 0$). Its important to note here that changing the value of $co2_{a,j_2}$ does not help.

2. Assume now that the parent is OTM by δ_2 and its child is ITM by δ_1. The dual constraint become:

$$z_{a,j_1} - c02_{a,j_2} \geq \delta_2$$
$$z_{a,j_2} + c02_{a,j_2} \geq \delta_1$$

Now the optimal solution is $c02_{a,j_2} = -\delta_2$, $z_{a,j_1} = 0$ and $z_{a,j_2} = \delta_1 + \delta_2$. These observations show that $c02_{a,j_2}$ can transfer surplus from a child to its parent while the other way around is impossible.

□

Lemma 5.4 (Interpretation of $c88_u$). *$c88_u$ distributes the surplus over the two blocks in the group u.*

Proof. Take $j_1, j_2 \in O_u \cap J_a^a$. We will again consider two situations, in the first one j_1 is ITM by $\delta_1 > 0$ while j_2 is OTM by $\delta_2 < 0$, in the second one the cases are reversed. The corresponding dual constraints are 5.6i and 5.6k for j_1 and j_2 respectively, and we set again $du^a_{a,j_1} = du^a_{a,j_2} = 0$.

1. In the first case these constraints simplify to:

$$z_{a,j_1} + c88_u \geq \delta_1$$
$$z_{a,j_2} - c88_u \geq \delta_2$$

and optimal values are $z_{a,j_1} = \delta_1 + \delta_2, c88_u = \delta_2, z_{a,j_2} = 0$.

2. In the second case the constraints become

$$z_{a,j_1} + c88_u \geq \delta_2$$
$$z_{a,j_2} - c88_u \geq \delta_1$$

and optimal values are $z_{a,j_1} = 0, c88_u = \delta_2, z_{a,j_2} = \delta_1 + \delta_2$.

Therefore $c88_u$ can transfer surplus from j_1 to j_2 and vice versa. □

The difference between $c88_u$ and $c02_{a,j}$ is that $c88_u$ can be negative, unlike $c02_{a,j}$ which can therefore only transfer surplus in one direction. In Section 5.4 below we define an MILP feasible region that exactly matches the region described by the constraints of the primal 5.1e - 5.1m, 5.2 - 5.5, its dual 5.6b - 5.6t and complementary slackness constraints 5.7a - 5.7p.

5.4 The final model

The system in 5.10 describes a region that contains all possible combinations of blocks given by $J_a = J_a^a \cup J_a^r$, $\forall a \in A$. In addition, each combination will satisfy the primal, dual and complementary slackness constraints, this is made precise in Theorem 5.2. Most importantly, it contains inequality 5.10a that enforces all complementary slackness conditions without containing any quadratic terms. Moreover, "Big Ms" are introduced that limit the loss of rejected blocks (5.10p) and missed opportunity costs of accepted blocks (5.10q) to 0.

Feasible Region

$$\sum_{a\in A,h\in H,i\in I_{a,h}} x_{a,h,i} \cdot p_{a,h,i} \cdot q_{a,h,i} + \sum_{a\in A,j\in J_A} y_{a,j} \cdot pb_{a,j} \cdot \sum_{h\in H} qb_{a,j,h} \leq \sum_{a\in A,h\in H,i\in I_{a,h}} s_{a,h,i} + \sum_{a\in A,j\in J_A} z_{a,j} + \sum_{g\in G} c04_g + \sum_{h,k\in K} v_{h,k} \cdot w_{h,k} - \sum_{a\in A,j\in J_a^a} du_{a,j}^a \tag{5.10a}$$

$$\sum_{i\in I_{a,h}} x_{a,h,i} \cdot q_{a,h,i} + \sum_{j\in J_A} y_{a,j} \cdot qb_{a,j,h} + \sum_{b\in B_a} f_{a,b,h} - f_{b,a,h} = 0 \quad \forall a \in A, h \in H \tag{5.10b}$$

$$\sum_{a\in A,b\in B_a} f_{a,b,h} \cdot (c_{h,k,a} - c_{h,k,b}) \leq w_{h,k} \quad \forall h \in H, k \in K_h \tag{5.10c}$$

$$\sum_{j\in E_g} y_{a,j} \leq 1 \quad \forall a \in A, g \in G_a \tag{5.10d}$$

$$y_{a,j} - y_{a,lp_{a,j}} \leq 0 \quad \forall a, j \in L \tag{5.10e}$$

$$y_{a,j_1\in O_u} - y_{a,j_2\in O_u} = 0 \quad \forall a \in A, u \in U_a \tag{5.10f}$$

$$x_{a,h,i}, y_{a,j}, f_{a,b,h} \geq 0 \tag{5.10g}$$

$$x_{a,h,i}, y_{a,j} \leq 1 \tag{5.10h}$$

$$y_{a,j} \in \mathbb{Z} \tag{5.10i}$$

$$s_{a,h,i} + \pi_{a,h} \cdot q_{a,h,i} \geq p_{a,h,i} \cdot q_{a,h,i} \quad \forall a \in A, h \in H, i \in I_{a,h} \tag{5.10j}$$

$$z_{a,j} + c02_{a,j} - \sum_{a,j_1\in lc_{a,j}} c02_{a,j_1} - du_{a,j}^a + du_{a,j}^r + \sum_{h\in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h\in H} qb_{a,j,h} \quad \forall a \in A, j \in L_a \tag{5.10k}$$

$$z_{a,j} - \sum_{j_1 \in lc_{a,j}} c02_{a,j_1} -$$
$$du^a_{a,j} + du^r_{a,j} + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} \quad \forall a \in A, j \in N_a \tag{5.10l}$$

$$z_{a,j} + c04_g - du^a_{a,j} + du^r_{a,j} + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} \quad \forall a \in A, g \in G_a, j \in E_g \tag{5.10m}$$

$$z_{a,j} + c88_u - du^a_{a,j} + du^r_{a,j} + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} \quad \forall a \in A, u \in U_a, j_1 \in O_u \tag{5.10n}$$

$$z_{a,j} - c88_u - du^a_{a,j} + du^r_{a,j} + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} \quad \forall a \in A, u \in U_a, j_2 \in O_u \tag{5.10o}$$

$$du^r_{a,j} \leq M_{a,j} \cdot (1 - y_{a,j}) \quad \forall a \in A, h \in H, i \in I_{a,h} \tag{5.10p}$$

$$du^a_{a,j} \leq M_{a,j} \cdot y_{a,j} \quad \forall a \in A, j \in J_a \tag{5.10q}$$

$$\pi_{a,h} - \pi_{b,h} + \sum_{k \in K_h} (c_{h,k,a} - c_{h,k,b}) \cdot v_{h,k} \geq 0 \quad \forall h \in H, a \in A, b \in B_a \tag{5.10r}$$

$$s_{a,h,i}, z_{a,j}, v_{h,k}, c02_{a,j}, c04_g \geq 0 \quad \forall a \in A, h \in H, i \in I_{a,h} \tag{5.10s}$$

$$\pi_{a,h} \geq -500 \quad \forall a \in A, h \in H \tag{5.10t}$$

$$-\pi_{a,h} \geq -3000 \quad \forall a \in A, h \in H \tag{5.10u}$$

We now need to prove that the two regions are the same, which will be the task of Theorem 5.2.

Theorem 5.2 (Equality of feasible regions)**.** *Let* $(x, y, f, s, z, c02, c04, c88, \pi, v, du^a, du^r)$ *be a point inside the feasible region of 5.10, in addition define for all* $a \in A$ $J^a_a = \{j | y_{a,j} = 1\}$ *and* $J^r_a = \{j | y_{a,j} = 0\}$*. Then:*

1. $(x, y, f, s, z, c02, c04, c88, \pi, v, du^a_{j \in J^a}, du^r_{j \in J^r})$ *of 5.10 satisfies all the the conditions in 5.1e - 5.1m, 5.2 - 5.5, 5.6b - 5.6t and 5.7a - 5.7p.*

2. *Conversely, let* $(x, y, f, s, z, c02, c04, c88, \pi, v, du^a_{j \in J^a}, du^r_{j \in J^r})$ *be any point satisfying the conditions 5.1e - 5.1m, 5.2 - 5.5, 5.6b - 5.6t and 5.7a - 5.7p related to some block selection* $J_a = J^a_a \cup J^r_a$ *for every* $a \in A$*, then this point can be lifted to obtain a point* $(x, y, f, s, z, c02, c04, c88, \pi, v, du^a, du^r)$ *that satisfies all conditions in 5.10.*

Proof. 1. The conditions from the primal problem 5.1b - 5.1m, 5.2 and 5.3 are trivially satisfied as they also appear in 5.10. The partitions $J_a^a \cup J_a^r$ from 5.4 and 5.5 are defined using the values of $y_{a,j}$, so that the constraints 5.10k - 5.10q make sure that 5.6c - 5.6l are satisfied. The remaining constraints from 5.6 can directly be found in 5.10. The complementary constraints in 5.7a - 5.7p are fulfilled since the solution adheres to all primal and dual constraints and inequality 5.10a.

2. For all $a \in A$ define $du^a_{a,j} = 0, \forall j \in J_a^r$ and $du^r_{a,j} = 0, \forall j \in J_a^a$. Then constraints 5.10k - 5.10p are satisfied thanks to 5.6c - 5.6l. 5.10a is satisfied relying on the optimality conditions for the primal subject to dual constraints 5.6b - 5.6t and complementarity constraints 5.7a - 5.7p. The remaining constraints from 5.10 can directly be checked.

□

Final model

Since we want to force all accepted blocks to be ITM we apply Theorem 5.1 and set $du^a_{a,j} = 0$, this means that 5.10q can be dropped. Then 5.10k - 5.10p can be simplified to 5.11k - 5.11o. We can now add condition 5.11p to force all accepted blocks to be ITM. It is important that the Big Ms in 5.11 are large enough to not restrain any values. On the other hand they should not be too big as this can create numerical difficulties for the solver, that is why we compute the Big Ms individually for every block.

$$\sum_{a\in A,h\in H,i\in I_{a,h}} x_{a,h,i} \cdot p_{a,h,i} \cdot q_{a,h,i} + \sum_{a\in A,j\in J_A} y_{a,j} \cdot pb_{a,j} \cdot \sum_{h\in H} qb_{a,j,h} \tag{5.11a}$$

$$\sum_{i\in I_{a,h}} x_{a,h,i} \cdot q_{a,h,i} + \sum_{j\in J_A} y_{a,j} \cdot qb_{a,j,h} + \sum_{b\in B_a} f_{a,b,h} - f_{b,a,h} = 0 \qquad \forall a \in A, h \in H \tag{5.11b}$$

$$\sum_{a\in A,b\in B_a} f_{a,b,h} \cdot (c_{h,k,a} - c_{h,k,b}) \leq w_{h,k} \qquad \forall h \in H, k \in K_h \tag{5.11c}$$

$$\sum_{j\in E_g} y_{a,j} \leq 1 \qquad \forall a \in A, g \in G_a \tag{5.11d}$$

$$y_{a,j} - y_{a,lp_{a,j}} \leq 0 \qquad \forall a, j \in L \tag{5.11e}$$

$$y_{a,j_1\in O_u} - y_{a,j_2\in O_u} = 0 \qquad \forall a \in A, u \in U_a \tag{5.11f}$$

$$x_{a,h,i}, y_{a,j}, f_{a,b,h} \geq 0 \tag{5.11g}$$

$$x_{a,h,i}, y_{a,j} \leq 1 \tag{5.11h}$$

$$y_{a,j} \in \mathbb{Z} \tag{5.11i}$$

$$s_{a,h,i} + \pi_{a,h} \cdot q_{a,h,i} \geq p_{a,h,i} \cdot q_{a,h,i} \qquad \forall a \in A, h \in H, i \in I_{a,h} \tag{5.11j}$$

$$z_{a,j} - y_{a,j} M_{a,j} + c02_{a,j} - \sum_{a,j_1 \in lc_{a,j}} c02_{a,j_1} + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} - M_{a,j} \quad \forall a \in A, j \in L_a \tag{5.11k}$$

$$z_{a,j} - y_{a,j} M_{a,j} - \sum_{j_1 \in lc_{a,j}} c02_{a,j_1} + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} - M_{a,j} \quad \forall a \in A, j \in N_a \tag{5.11l}$$

$$z_{a,j} - y_{a,j} M_{a,j} + c04_g + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} - M_{a,j} \quad \forall a \in A, g \in G_a, j \in E_g \tag{5.11m}$$

$$z_{a,j} - y_{a,j} M_{a,j} + c88_u + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} - M_{a,j} \quad \forall a \in A, u \in U_a, j_1 \in O_u \tag{5.11n}$$

$$z_{a,j} - y_{a,j} M_{a,j} - c88_u + \sum_{h \in H} \pi_{a,h} \cdot qb_{a,j,h} \geq pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} - M_{a,j} \quad \forall a \in A, u \in U_a, j_2 \in O_u \tag{5.11o}$$

$$y_{a,j} M_{a,j} + \sum_{h \in H} \pi_{a,h} qb_{a,j,h} \leq pb_{a,j} \sum_{h \in H} qb_{a,j,h} + M_{a,j} \qquad \forall a \in A, j \in J_a \tag{5.11p}$$

$$\pi_{a,h} - \pi_{b,h} + \sum_{k \in K_h} (c_{h,k,a} - c_{h,k,b}) \cdot v_{h,k} \geq 0 \qquad \forall h \in H, a \in A, b \in B_a \tag{5.11q}$$

$$y_{a,j} M_{a,j} + \sum_{h \in H} \pi_{a,h} qb_{a,j,h} \leq pb_{a,j} \sum_{h \in H} qb_{a,j,h} + M_{a,j} \qquad \forall a \in A, j \in J_a \tag{5.11r}$$

$$\sum_{a \in A, h \in H, i \in I_{a,h}} x_{a,h,i} \cdot p_{a,h,i} \cdot q_{a,h,i} + \tag{5.11s}$$

$$\sum_{a \in A, j \in J_A} y_{a,j} \cdot pb_{a,j} \cdot \sum_{h \in H} qb_{a,j,h} \leq \sum_{a \in A, h \in H, i \in I_{a,h}} s_{a,h,i} + \tag{5.11t}$$

$$\sum_{a\in A,j\in J_A} z_{a,j} + \sum_{g\in G} c04_g + \sum_{h,k\in K} v_{h,k}\cdot w_{h,k} - \sum_{a\in A,j\in J_a^a} du_{a,j}^a \tag{5.11u}$$

$$s_{a,h,i}, z_{a,j}, v_{h,k}, c02_{a,j}, c04_g \geq 0 \qquad \forall a\in A, h\in H, i\in I_{a,h} \tag{5.11v}$$

$$\pi_{a,h} \geq -500 \qquad \forall a\in A, h\in H \tag{5.11w}$$

$$-\pi_{a,h} \geq -3000 \qquad \forall a\in A, h\in H \tag{5.11x}$$

6 Experiment

6.1 Setup

We have used the OR-Tools package in python to implement the MILP from 5.11 and export it to a .lp file. .lp is a common file format used to represent optimization problems. We use CPLEX 12.8.0.0, with default settings as the solver. The hardware being used is a Dell XPS running on Ubuntu 18.04.3. The technical specifications include an Intel Core i7-8565U CPU, with 8 cores @1.80GHz and 15.3 GB of RAM.

6.2 The algorithm

As the aim is to create intuitive prices, we need to implement a flow based intuitive model. In order to do this the model needs to be solved and modified several times until there are no more prices from high to lower priced areas. This is done in the following way. A first instance of the model is created in python, exported to a .lp file, and imported and solved with CPLEX. CPLEX solves the model to optimality and returns the results in the form of a solution file (.sol format). After solving, the prices, flows and congested Critical Branches are extracted from the .sol file and the intuitiveness condition is checked for every flow. If there are any non-intuitive flows, then for every flow the following procedure is performed. We go over all the congested Critical Branches for the hour where the flow is non-intuitive, and the alleviating effect of the flow on the branch is removed. This means that for all the non-intuitive flows, the congested branches in that particular hour are adjusted and 5.11c becomes 6.1 for these.

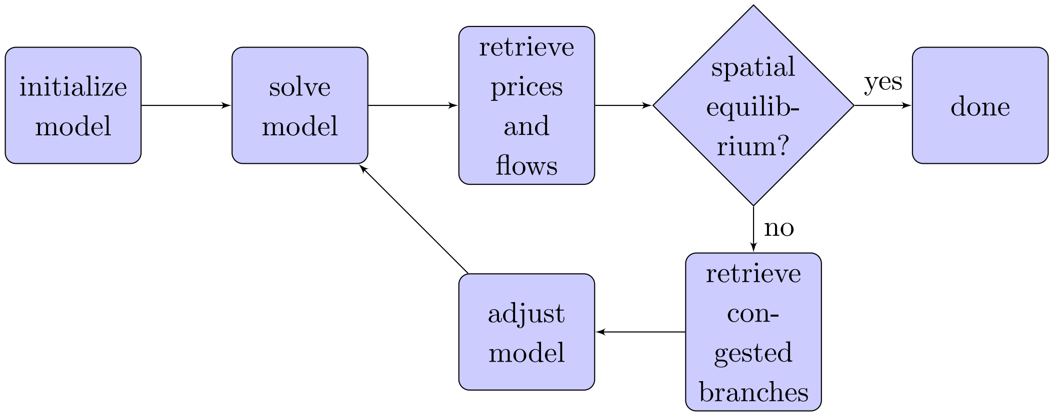
$$\sum_{a\in A, b\in B_a} f_{a,b,h} \cdot \max(c_{h,k,a} - c_{h,k,b}, 0) \leq w_{h,k} \qquad \forall h \in H, k \in K_h \tag{6.1}$$

Now as we change the coefficient in the "primal" constraints of the problem we should also adjust the "dual" part of the problem. This means that 5.11q becomes 6.2.

$$\pi_{a,h} - \pi_{b,h} + \sum_{k\in K_h} \max(c_{h,k,a} - c_{h,k,b}, 0) \cdot v_{h,k} \geq 0 \qquad \forall h \in H, a \in A, b \in B_a \tag{6.2}$$

This cycle is repeated until there are no more non-intuitive flows, figure 7 contains a flowchart of the algorithm. In order to improve the solving time we use the integer variables of each previous iteration as a warm start to the next iteration.

Figure 7: flowchart describing the algorithm to get a spatial equilibrium



6.3 Results

We test the algorithm with real data. The data for the Netherlands and Belgium are supplied by APX, while France's, Germany's and Austria's data are published by EPEX. Table 3 shows the objective value, the total runtime of the algorithm as well as the amount of constraints for each instance. Table 4 shows the amount of block orders per type for each instance, as well as the total amount of continuous variables. Table 5 shows the solving time per iteration. A few comments have to be made here regarding the solving times. First is that the average solving time is greatly influenced by the instance of 2019-07-01. Second is that only the first iteration of the instance of 2019-07-07 was solved. The second iteration was manually aborted after the solving time exceeded 20000 seconds, so we were unfortunately unable to solve all instances. It is difficult to compare results with EUPHEMIA though, as EUPHEMIA solves the market clearing problem for all of Europe and therefore contains many more areas and borders as well as roughly three times as many hourly orders. In addition, EUPHEMIA also deals with some products that we did not include here such as PUN orders from Italy, which are notoriously hard to deal with. Regardless of the larger instances that it deals with, EUPHEMIA is still considerably faster than our model. The average total solving time for our model is around 5300 seconds, whereas it takes EUPHEMIA about 5 minutes. Some improvements can possible be made here by tuning the parameters of CPLEX. An advantage for us is that our model solves to optimality. EUPHEMIA, on the other hand, is a heuristic, so there is no guarantee that the global optimum is reached.

Table 3: Total solving time per instance

date	objective	total runtime (s)	constraints
2019-07-01	3970135151.51	42797	89872
2019-07-02	3786400750.3	3124	91189
2019-07-03	3754094836.94	1263	91942
2019-07-04	3654158566.61	2949	85529
2019-07-05	3490973364.51	7812	87303
2019-07-06	3770348336.4	1386	86589
2019-07-07	3753728439.29	1008	90588
2019-07-08	3446584568.23	1270	89178
2019-07-09	3751396896.44	2299	87652
2019-07-10	3901516125.64	3542	93448
2019-07-11	3712081040.49	1592	88510
2019-07-12	3709250371.52	6855	86061
2019-07-13	3261857473.43	721	77312
2019-07-14	3730265221.4	2172	80122
2019-07-15	3835351849.58	1568	84584

Table 4: Size of each instance

		blocks				
date	continuous variables	total	reg. (C01)	excl. (C04)	linked fam (C02)	loop fam (C88)
2019-07-01	169623	1770	545	1163	62	0
2019-07-02	171994	1669	570	1068	31	0
2019-07-03	173825	1772	669	1069	34	0
2019-07-04	160697	1843	644	1152	47	0
2019-07-05	163761	1860	700	1102	52	6
2019-07-06	163228	1857	614	1158	79	6
2019-07-07	170603	1931	609	1245	77	0
2019-07-08	166760	1845	576	1223	46	0
2019-07-09	163575	1868	612	1234	19	4
2019-07-10	175247	1711	595	1077	38	2
2019-07-11	166106	1877	736	1109	31	2
2019-07-12	161337	1841	666	1125	44	6
2019-07-13	145827	1768	521	1189	52	6
2019-07-14	151390	1678	619	1017	42	0
2019-07-15	159778	1846	637	1176	33	0

Table 5: Solving time (s) per iteration

date	1	2	3	4	5	6
2019-07-01	4529	3076	4939	4238	20925	5090
2019-07-02	809	574	1015	726		
2019-07-03	472	189	301	301		
2019-07-04	1025	523	484	562	163	192
2019-07-05	1472	2539	2777	1024		
2019-07-06	358	157	440	431		
2019-07-07	1008					
2019-07-08	593	440	237			
2019-07-09	860	758	341	340		
2019-07-10	1475	1593	235	239		
2019-07-11	660	226	222	243	241	
2019-07-12	4765	1377	240	237	236	
2019-07-13	92	180	199	124	126	
2019-07-14	644	396	351	255	263	263
2019-07-15	231	298	203	411	425	

7 Conclusion

The primary goal of this work was to develop a model that solved the market clearing problem while abiding by all the special requirements set by the European power exchanges. These are (1) uniform prices, (2) a flow based intuitive network model, and (3) the ability to handle different kind of block orders.

This thesis started with a brief introduction about the European day ahead energy market. Chapter 2 delved into the concepts of *uniform prices* and *market equilibria*, explaining the advantages and disadvantages of each property. Near the end of the Chapter a small market clearing problem illustrated the impossibility of having both properties in the case of a non convex problem. Chapter 3 discussed the most important academic efforts to solve the market clearing problem as well as the algorithm in use by the European power exchanges. As the method by Madani and Van Vyve (2018) is the only one that provides uniform prices, we chose to follow this method and to make modifications to include more types of block orders and a flow-based intuitive network. Chapter 4 gave general information about the European market clearing problem, such as the types of block orders and the transmission models. Chapter 5 saw the derivation of the model, which was done in a similar way as in Madani and Van Vyve (2018). Finally, Chapter 6 provided us with results of the test.

At the end of the day we can say that we successfully derived a model that solves the European day ahead market clearing problem. The secondary goal was to see if it would be practically feasible to solve the day-ahead auction problem using this approach. The answer to that is probably no. The reason is that the solution needs to be published 42 minutes after the market closes, while some of our instances took longer to solve. Besides that, the actual problem of solving Europe's entire day-ahead market clearing problem is even larger than the one tested here.

Appendices

A Formulas and Proofs

Theorem A.1 (Complementary Slackness). *Let x and y be feasible solutions to symmetric form primal and dual linear programs. Then x and y are optimal solutions to the primal and dual respectively if and only if $(b-Ax)^T y = 0$ and $(A^T y - c)^T x = 0$.*

Proof. $\Rightarrow$ Feasibility implies that $(b-Ax)^T y \geq 0$ and $(A^T y - c)^T x \geq 0$. Now, if x and y are optimal solutions to their primal and dual respectively, then strong duality yields:

$$(b-Ax)^T y + (A^T y - c)^T x = b^T y - c^T x = 0$$

Therefore $(b-Ax)^T y = 0$ and $(A^T y - c)^T x = 0$.
$\Leftarrow$ If $(b-Ax)^T y = 0$ and $(A^T y - c)^T x = 0$ then:

$$(b-Ax)^T y + (A^T y - c)^T x = b^T y - c^T x = 0$$

Now the optimality of x and y follows again from strong duality. □

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