

A model predictive framework of Ground Source Heat Pump coupled with Aquifer Thermal Energy Storage System in heating and cooling equipment of a building

Rostampour Samarin, Vahab; Bloemendal, Martin; Keviczky, Tamas

Publication date 2017

Document Version Final published version

Published in

Proceedings 12th IEA Heat Pump Conference

Citation (APA)

Rostampour Samarin, V., Bloemendal, M., & Keviczky, T. (2017). A model predictive framework of Ground Source Heat Pump coupled with Aquifer Thermal Energy Storage System in heating and cooling equipment of a building. In Proceedings 12th IEA Heat Pump Conference Stichting HPC 2017.

Important note

To cite this publication, please use the final published version (if applicable). Please check the document version above.

Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Please contact us and provide details if you believe this document breaches copyrights. We will remove access to the work immediately and investigate your claim.



12th IEA Heat Pump Conference 2017



A Model Predictive Framework of Ground Source Heat Pump coupled with Aquifer Thermal Energy Storage System in Heating and Cooling Equipment of a Building

Vahab Rostampour^{a,*}, Martin Bloemendal^{b,c}, and Tamás Keviczky^a

^aDelft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628 CD, Delft, The Netherlands ^bFaculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands ^cKWR Watercycle Research Institute, Groningenhaven 7, 3433 PE Nieuwegein, The Netherlands

Abstract

This paper presents a complete model of a building heating and cooling equipment and a ground source heat pump (GSHP) coupled with an aquifer thermal energy storage (ATES) system. This model contains detailed mathematical representations of building thermal dynamics, ATES system dynamics, heat exchanger (HE), and GSHP together with heating and cooling circulated water pipelines. In particular, we derive a static (steady-state) model for a countercurrent HE and an electrical GSHP as a function of water flow rates and input/output water temperatures in both sides of HE and GSHP to determine the amount of heat transfer from ATES to HE then HE to GSHP, and vice versa. We then develop a model predictive control framework to keep track of the desired building zone temperature, while meeting operational constraints. In such a framework, a finite-horizon mixed-integer nonlinear optimization problem is solved at each sampling time. A simulation study illustrates the functioning of our proposed predictive model.

© 2017 Stichting HPC 2017.

Selection and/or peer-review under responsibility of the organizers of the 12th IEA Heat Pump Conference 2017.

Keywords: Ground Source Heat Pump (GSHP) Model; Heat Exchanger (HE) Model; Aquifer Thermal Energy Storage System (ATES); Building Thermal Energy Balance; Smart Thermal Grids (STGs); Model Predictive Control (MPC);

This research was supported by the Uncertainty Reduction in Smart Energy Systems (URSES) research program funded by the Dutch organization for scientific research (NWO) and Shell under the project Aquifer Thermal Energy Storage Smart Grids (ATES-SG) with grant number 408-13-030.

^{*} Corresponding author. Tel.: +31-15-27-85056; fax: +31-15-27-86679. E-mail address: v.rostampour@tudelft.nl

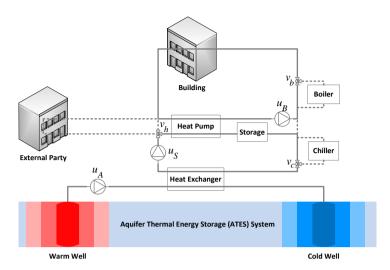


Fig. 1: Conceptual representation of heating and cooling equipment of a building together with an aquifer thermal energy storage (ATES) system.

1. Introduction

Worldwide there is a growing interest in energy saving technologies. Aquifer thermal energy storage (ATES) systems are used to store large quantities of thermal energy in subsurface aquifers enabling the reduction of energy usage and emissions of the heating and cooling networks in buildings [1]. An ATES system is considered as a seasonal storage system that can be a heat source or sink, or as a storage for thermal energy. This is achieved by injection and extraction of water into and from saturated underground aquifers, simultaneously [2]. An ATES system consists of two wells and operates in a seasonal mode [3]. One well is used for the storage of cold water, the second for the storage of heat. In warm seasons, cold water is extracted from the aquifer using the cold storage well and through a heat exchanger to provide cooling to a building. This heats up the water, which is simultaneously injected back into the aquifer via the warm storage well. This procedure is reversed during cold seasons where the flow direction is reversed such that the warmer water is extracted from the warm well to provide heating to a building. ATES systems are suitable for heating and cooling processes of utility buildings such as offices, hospitals, musea, universities and greenhouses. In the heating and cooling processes of such buildings, ATES systems are coupled with ground source heat pump (GSHP). GSHP plays a crucial role in the performance and overall efficiency of ATES systems. Figure 1 depicts the operational modes of an ATES system for a single building [4]. To this end, any attempt to optimize/improve efficiency of operating an ATES system and suboptimal use of subsurface space has to consider building thermal dynamics together with mathematical models of all incorporated equipment. This highlights the necessity of developing a complete model of the heating and cooling equipment of a building that can be used in a predictive control framework to obtain an optimal operating strategy.

In this paper, we develop a complete model of a building heating and cooling equipment and a GSHP coupled with an ATES system. Detailed mathematical representations of building thermal dynamics, ATES system dynamics, heat exchanger (HE), and GSHP are developed together with algebraic equations to determine the internal connections between equipment. In particular, we derive two separate fractional mapping functions from the input water temperatures to the output water temperatures as a function of flow rates of the water in both sides of a HE and a GSHP. Such transfer functions are necessary to determine the amount of heat transfer from ATES to HE then HE to GSHP, and the other way around. Using the developed model, we provide a model predictive control (MPC) framework to keep track of the desired building zone temperature, while meeting operational constraints. In such a framework, a finite-horizon mixed-integer nonlinear optimization problem formulation is solved at each sampling time. We illustrate the functioning of our proposed predictive model by a simulation study.

The structure of this paper is as follows. Section 2 provides a complete mathematical model for each component of the heating and cooling processes of a building, whereas in Section 3 a nonlinear MPC strategy is proposed. Section 4 presents a simulation study to illustrate the functioning of our proposed predictive model. In Section 5 we provide some concluding remarks.

2. Heating and Cooling Equipment of a Building

Consider a building with a single zone and wall layers. Building zone temperature is a dynamical system that evolves over time. The lower surface of a zone (floor) is considered as a heating source via the supplied water temperature that flows into cyclic pipes. Cooling energy is supplied via the upper interior surface (ceiling) of a zone. The conceptual piping of the water circulation of the building is depicted in Figure 1. The heating and cooling equipment of the building consists of: 1) an ATES system 2) a storage tank, a boiler and a chiller 3) a heat exchanger 4) a heat pump. Moreover, we define an external party to capture the possibility of transmitting surplus thermal energy from one building to another via heat pump in a campus of several buildings. We therefore consider a single building thermal dynamics and then, as for the external party we use a similar model with different building parameters.

In what follows, we first develop a mathematical model for each component separately, and then, we present the internal algebraic equations that hold between equipment with respect to the heating or cooling requirement of a building. We refer to passing variable between component as an internal input variable of each component.

2.1. Thermal Dynamics of Building

The building model parameters, e.g. heat transfer coefficients are inspired by [5] and [6]. The model has been formed based on the total heat content of a building. The dynamic behavior of the states are described using the first order differential equations as follows:

$$\frac{dT_z^B}{dt} = \frac{Q_{rdz}(t) + Q_{fcz}(t) + Q_p(t) + Q_e(t) - Q_{ven}(t) - Q_{zs}(t) - Q_{zo}(t)}{(0.75\rho_z c_{p,z} + 0.25\rho_f c_{p,f})V_z},$$
(1a)

$$\frac{dT_{fc}^{B}}{dt} = \frac{Q_{in}^{fc}(t) - Q_{out}^{fc}(t) - Q_{fcz}(t) - Q_{fcs}(t)}{\rho_{w}c_{p,w}V_{fc}},$$
(1b)

$$\frac{d\mathbf{T}_{s}^{B}}{dt} = \frac{\mathbf{Q}_{rds}(t) + \mathbf{Q}_{zs}(t) + \mathbf{Q}_{fcz}(t) - \mathbf{Q}_{so}(t)}{(0.7\rho_{s}c_{p,s} + 0.2\rho_{w}c_{p,w} + 0.1\rho_{z}c_{p,z})\mathbf{V}_{s}},$$
(1c)

where $\frac{dT_c^B}{dt}$, $\frac{dT_{fc}^B}{dt}$, $\frac{dT_c^B}{dt}$ [K s⁻¹] correspond to the rate of change of the building zone temperature, floor or ceiling temperature and solid (wall) temperature, respectively. $\rho_z, \rho_f, \rho_w, \rho_s$ [kg m⁻³] are the density of building zone, furniture, water, solid, and $c_{p,z}, c_{p,f}, c_{p,w}, c_{p,s}$ [J kg⁻¹K⁻¹] are the specific heat capacity of the building zone, furniture, water, solid, and V_z, V_{fc}, V_s [m³] are the volume of building zone, floor or ceiling, solid, respectively. The convective heat transfer from the floor or ceiling to the building zone air $Q_{fcz}(t)$, the building zone air to the solid $Q_{zs}(t)$, and the solid to the outdoor air $Q_{so}(t)$ are defined, respectively, by the following relations:

$$\begin{aligned} \mathbf{Q}_{fcz}(t) &= A_{fc}\alpha_{fcz}(t)(\mathbf{T}_{fc}^{\mathrm{B}}(t) - \mathbf{T}_{z}^{\mathrm{B}}(t)) \quad , \quad \mathbf{Q}_{zo}(t) = A_{zw}\alpha_{zwo}(t)(\mathbf{T}_{z}^{\mathrm{B}}(t) - \mathbf{T}_{o}(t)) \quad , \\ \mathbf{Q}_{zs}(t) &= A_{s}\alpha_{zs}(t)(\mathbf{T}_{z}^{\mathrm{B}}(t) - \mathbf{T}_{s}^{\mathrm{B}}(t)) \quad , \quad \mathbf{Q}_{so}(t) = A_{s}\alpha_{so}(t)(\mathbf{T}_{s}^{\mathrm{B}}(t) - \mathbf{T}_{o}(t)) \quad , \end{aligned}$$

where $T_o(t)$ [K] is the outdoor air temperature, A_{fc} , A_{zw} , A_s are the surface area of floor or ceiling, the building zone windows and solid, respectively. The surface area of the solid is determined by $A_s = n_l h_s l_s w_s$ where n_l , h_s , l_s , w_s are the number, the height, the length and the width of the layers (walls). It is considered that the surface area of the building zone windows is $A_{zw} = 0.4A_s$. It is assumed that the surface areas of pipes in the floor and ceiling are equal and can be determined by $A_{fc} = n_s n_p \pi d_p l_p$ where n_s , n_p , d_p , l_p are the number of spans, the number of the pipes, and the diameter with the length of the pipes, respectively. $\alpha_{fcz}(t)$, $\alpha_{zs}(t)$, $\alpha_{zwo}(t)$, $\alpha_{so}(t)$ are the heat transfer coefficients from the floor or ceiling to the building zone, from the building zone to the solid, from the building zone windows and the solid to the outdoor air, respectively, and are defined by

$$\alpha_{fcz}(t) = \frac{1.28}{(d_p)^{0.25}} |T_{fc}^B(t) - T_z^B(t)|^{0.25} \quad , \quad \alpha_{zs}(t) = \begin{cases} 1.7 |T_s^B(t) - T_z^B(t)|^{0.35} & T_s^B(t) < T_z^B(t) \\ 1.3 |T_s^B(t) - T_z^B(t)|^{0.25} & T_s^B(t) \ge T_z^B(t) \end{cases} \quad ,$$

$$\alpha_{zwo}(t) = \begin{cases} 3.8 + 1.2 v_o(t) & v_o(t) < 4 \\ 2.5 v_o(t)^{0.8} & v_o(t) > 4 \end{cases} \quad , \quad \alpha_{so}(t) = \begin{cases} 2.8 + 1.2 v_o(t) & v_o(t) < 4 \\ 2.5 v_o(t)^{0.8} & v_o(t) > 4 \end{cases} \quad ,$$

where $v_o(t)$ [m s⁻¹] denotes the outdoor wind velocity. $Q_{rdz}(t) = A_{zw}\eta_fI_o(t)$ and $Q_{rds}(t) = A_s\eta_sI_o(t)$ are related to the shortwave radiation absorption by the building zone and the solid. η_f, η_s are the shortwave radiation absorption coefficient by the furniture inside zone and by the solid, respectively. $Q_{fcs}(t) = A_{fc}E_{fc}E_sF_{fcs}\sigma(T_{fc}^{B^4}(t)-T_s^{B^4}(t))$ presents the longwave radiation absorption from floor or ceiling to solid, where E_{fc}, E_s [-] are the emission coefficients, F_{fcs} [-] is the view factor, σ [W m⁻² K⁻⁴] is the Stefan-Boltzmann constant. The heat transfer due to the occupancy of the building zone by the people and the heat gain due to the electrical devices that are used in the building zone are shown by $Q_p(t)$ and $Q_e(t)$, respectively. $Q_p(t)$ and $Q_e(t)$ are modeled as normal and uniform distributed random variables. We consider two operating modes, i.e. working mode $t \in [7, 19]$ and rest mode $t \notin [7, 19]$, for a building zone and the mean, variance of the distributions depend of the building zone operating time. The mean and variance of the distribution in the rest mode are approximately ten times smaller than the working mode. Thus $Q_p(t)$ and $Q_e(t)$ are considered as

$$Q_p(t) \sim \begin{cases} \mathcal{N}_t(np \; \alpha_p, 0.25 \; np \; \alpha_p) & t \in [7, 19] \\ \mathcal{N}_t(0.1 \; np \; \alpha_p, 0.025 \; np \; \alpha_p) & t \notin [7, 19] \end{cases}, \quad Q_e(t) \sim \begin{cases} \mathcal{U}_t(n_{e,\min} \; \eta_e, n_{e,\max} \; \eta_e) & t \in [7, 19] \\ \mathcal{U}_t(0.1 \; n_{e,\min} \; \eta_e, 0.1 \; n_{e,\max} \; \eta_e) & t \notin [7, 19] \end{cases},$$

where np corresponds to the number of people, α_p denotes the heat generated by one person, $n_{e,\min}, n_{e,\max}$ are the minimum, maximum number of the electrical devices, respectively, and η_e is the average heat generated by one device. It is assumed that the heat lost due to the ventilation process $Q_{\text{ven}}(t)$ is $0.1Q_p(t)$. In our control problem formulation (15), we however consider one possible scenario (realization of stochastic process) for each $Q_p(t)$ and $Q_e(t)$ to be given. Thermal energy transport terms due to the water flow into and out of the floor or ceiling of the building zone are defined by $Q_{in}^{fc}(t) = \rho_w c_{p,w} u_B(t) T_{in}^{fc}(t)$, and $Q_{out}^{fc}(t) = \rho_w c_{p,w} u_B(t) T_{out}^{fc}(t)$, respectively. $u_B(t)$ [m³ s⁻¹] is the building pump flow rate, $T_{in}^{fc}(t)$, $T_{out}^{fc}(t)$ [K] are the water temperatures entering and leaving the floor or ceiling of the building zone.

The proposed building system dynamics (1) is a continuous-time system that represents the rate of change of the building zone temperature, the floor or ceiling temperature and the solid (wall) temperature. We use the Forward Euler method to discretize the above continuous-time system. The prediction interval [0, T] is partitioned into N equal sub-intervals of width $\tau = \frac{T}{N} > 0$. We define $x_{B,k} := [T_{z,k}^B, T_{fc,k}^B, T_{s,k}^B] \in \mathbb{R}^3$ to be the state vector and a vector of given external parameters is $w_{B,k} := [T_{o,k}, I_{o,k}, v_{o,k}] \in \mathbb{R}^3$, $v_{B,k} := T_{in,k}^{fc}$ is the internal input variable and the control input variable is $u_{B,k}$. Given an initial condition $x_{B,0}$, consider now the discrete-time building dynamical model with sampling period of τ for each sampling time as follows.

$$x_{B,k+1} = F_B(x_{B,k}, u_{B,k}, v_{B,k}, w_{B,k}, \tau),$$
 (2a)

$$y_{B,k} = G_B(x_{B,k}, u_{B,k}),$$
 (2b)

where $y_{B,k}$ denotes the output performance of the building model dynamics that is the return water temperature of the building $T^B_{\text{ret},k}$. $T^B_{\text{ret},k}$ is equal to the outflow water temperature of the floor or ceiling $T^{fc}_{out,k}$, and following [6] is defined by

$$\mathbf{T}_{\mathrm{ret},k}^{\mathrm{B}} = \mathbf{T}_{out,k}^{fc} = \mathbf{T}_{fc,k} - \frac{\mathbf{Q}_{fcz,k} + \mathbf{Q}_{fcs,k}}{\rho_w c_{p,w} u_{\mathrm{B},k}} \; . \label{eq:teta_fcs_k}$$

The developed model (2) is a nonlinear stochastic hybrid system due to the dependency on uncertain variables such as occupancy, etc., and on switching parameters such as heat transfer coefficients.

2.2. Aquifer Thermal Energy Storage System

An ATES system consists of two wells containing either warm or cold water. Depending on the season, the operation of the ATES system changes. Figure 2b shows a block diagram model of an ATES system. During a cold season, the cold water is stored in the cold-well by circulating water from the warm-well to the cold-well through a heat exchanger to extract a required thermal energy from the water. This procedure is opposite during a warm season, since there is a demand for cooling; the stored cold water is used to cool the building. Following the work in [4],

consider a dynamical model of an ATES system as follows:

$$V_{w,k+1}^{aq} = V_{w,k}^{aq} + (s_{w,k} - s_{c,k}) V_{in,k}^{aq} , \qquad (3a)$$

$$V_{c,k+1}^{\text{aq}} = V_{c,k}^{\text{aq}} + (s_{c,k} - s_{w,k}) V_{in,k}^{\text{aq}},$$
(3b)

$$T_{w,k+1}^{aq} = \frac{V_{w,k}^{aq}}{V_{w,k}^{aq} + s_{w,k}V_{in,k}^{aq}} T_{w,k}^{aq} + \frac{s_{w,k}V_{in,k}^{aq}}{V_{w,k}^{aq} + s_{w,k}V_{in,k}^{aq}} T_{in,k}^{aq} - \frac{\alpha(T_{w,k}^{aq} - T_{amb,k}^{aq})}{V_{w,k}^{aq} + s_{w,k}V_{in,k}^{aq}},$$
(3c)

$$T_{c,k+1}^{\text{aq}} = \frac{V_{c,k}^{\text{aq}}}{V_{c,k}^{\text{aq}} + s_{c,k} V_{in,k}^{\text{aq}}} T_{c,k}^{\text{aq}} + \frac{s_{c,k} V_{in,k}^{\text{aq}}}{V_{c,k} + s_{c,k} V_{in,k}^{\text{aq}}} T_{in,k}^{\text{aq}} - \frac{\alpha (T_{c,k}^{\text{aq}} - T_{\text{amb},k}^{\text{aq}})}{V_{c,k}^{\text{aq}} + s_{c,k} V_{in,k}^{\text{aq}}},$$
(3d)

where $V_{w,k}^{aq}[m^3]$, $V_{c,k}^{aq}[m^3]$, $T_{w,k}^{aq}[K]$, $T_{c,k}^{aq}[K] \in \mathbb{R}$ denote the volume and the temperature of the warm- and the coldwell at each sampling time k, respectively. $V_{in,k}^{aq}[m^3] \in \mathbb{R}$ corresponds to the input volume of water that is injected into the warm- or cold-well and extracted from the counterpart well and it can be determined by $V_{in,k}^{aq} := \tau u_{A,k}$ where $u_{A,k}[m^3s^{-1}]$ is the ATES system pumping flow rate at each sampling time k and τ [s] is the sampling period. $s_{w,k}, s_{c,k} \in \{0,1\}$ denote binary variables that represent the operating modes of an ATES system. $\alpha \in (0,1)$ represents a loss term and we refer the interested reader to [4] for detailed steps on how to tune this parameter of the ATES model. In Figure 2b the solid line represents the heating mode (cold season) and the dashed line represents the cooling mode (warm season). The output performance of the ATES system dynamics $y_{A,k}$ is the water temperature $T_{out,k}^{aq} := s_{c,k} T_{w,k}^{aq} + s_{w,k} T_{c,k}^{aq}$ leaving the ATES system.

 $T^{aq}_{out,k} := s_{c,k} T^{aq}_{w,k} + s_{w,k} T^{aq}_{c,k}$ leaving the ATES system.

Define $x_{A,k} := [V^{aq}_{w,k}, T^{aq}_{w,k}, V^{aq}_{c,k}, T^{aq}_{c,k}] \in \mathbb{R}^4$ to be the state vector. The internal input variable is $v_{A,k} := T^{aq}_{in,k}$, the control input variable is $u_{A,k}$ and the integer variables are $s_{w,k}, s_{c,k}$. Given an initial condition $x_{A,0}$, consider now the discrete-time ATES dynamical model with sampling period of τ for each sampling time k as follows:

$$x_{A,k+1} = F_A(x_{A,k}, u_{A,k}, v_{A,k}, s_{w,k}, s_{c,k}, \tau),$$
(4a)

$$y_{A,k} = G_A(x_{A,k}, s_{w,k}, s_{c,k})$$
 (4b)

The developed model (4) is a nonlinear hybrid system due to the dependency on both continuous and discrete variables.

Remark 1. It is important to notice that the developed model of ATES system in (4) is considered as a control system model that will be used to predict the temperature and volume of stored water in each well at each sampling instance k, and such a model is not meant to be an aquifer groundwater simulator.

2.3. Boiler, Chiller and Storage Tank

We consider a building that is equipped with a boiler and a chiller (see Figure 1). A boiler is a closed vessel in which water is heated and it is used for additional heating if the heat pump cannot supply enough heat. A chiller is a closed vessel in which water is cooled via vapor-compression. It is used to further reduce the water temperature infiltrated into the ATES system in case the water output temperature on the evaporator side of the heat pump is not cold enough. The boiler is used to heat the supply water temperature up to a temperature of about 90°C during cold seasons, whereas the chiller is used to chill the water temperature down to a temperature of about 5°C during cold seasons. We therefore define the boiler outflow water temperature to be $T_{out,k}^{boi} = T_0 + 90$ [K] and the chiller outflow water temperature to be $T_{out,k}^{chi} = T_0 + 5$ [K], where $T_0 = 273.15$ [K] is the absolute zero Kelvin temperature.

Consider a storage tank that holds water for the short term storage of thermal energy. The storage tank is placed between the heat pump and the heat exchanger to keep thermal energy at the desired level for each time period. The dynamics of a storage tank system is described with the following first-order difference equations:

$$V_{s,k+1} = V_{s,k} + V_{in,k} - V_{out,k} , (5a)$$

$$T_{s,k+1} = \frac{V_{s,k}}{V_{s,k} + V_{in,k}} T_{s,k} + \frac{V_{in,k}}{V_{s,k} + V_{in,k}} T_{in,k}^{s} , \qquad (5b)$$

where $V_{s,k}[m^3]$, $T_{s,k}[K] \in \mathbb{R}$ denote the volume and the temperature of the storage tank at each sampling time k, respectively. $V_{in.k}$, $V_{out.k}[m^3] \in \mathbb{R}$ correspond to the input and output volume of water that is injected into and extracted

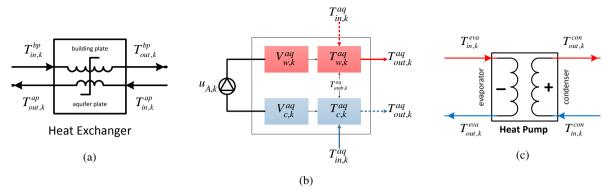


Fig. 2: A symbolic representation of a heat exchanger (a) and a heat pump (c). A block diagram model of an ATES system (b).

from the storage tank, respectively. It is assumed $V_{in,k} = V_{out,k} = \tau u_{S,k}$, where $u_{S,k}$ [m³s⁻¹] is the storage pumping flow rate at each sampling time k and τ [s] is the sampling period. The output performance of the storage tank system dynamics $y_{S,k}$ is the water temperature $T_{out,k}^s$ leaving the storage tank system and can be defined by $T_{out,k}^s := T_{s,k}$.

We now define $x_{S,k} := [V_{s,k}, T_{s,k}] \in \mathbb{R}^2$ to be the state vector of the storage tank. The internal input variable is $v_{S,k} := T_{in,k}^s$, the control input variable is $u_{S,k}$. Given an initial condition $x_{S,0}$, consider now the discrete-time storage tank dynamical model with sampling period of τ for each sampling time k as follows:

$$x_{S,k+1} = F_S(x_{S,k}, u_{S,k}, v_{S,k}, \tau)$$
, (6a)

$$y_{S,k} = G_S(x_{S,k}). ag{6b}$$

The developed model (6) is a nonlinear dynamical system.

2.4. Heat Exchanger

Consider a countercurrent heat exchanger that consists of two plates, as it is shown in Figure 2a, namely the building plate and the aquifer plate. During the warm season the heat exchanger is used to cool water inside pipes with water from cold-well of the ATES system and then, the cold water of ATES system is heated by the heat extracted from building in order to store into the warm-well. The extracted heat energy $Q_{he,k}[W]$ from the building is stored inside the warm-well of ATES system during warm season. By reversing the direction of water, one can pump warm water inside the building using water from the warm-well of ATES system and then, the warm water of ATES system is chilled by the cold extracted from building in order to infiltrate into the cold-well of ATES system. A heat exchanger can be in general considered as an inverse model such that the stored heat energy $Q_{he,k}[W]$ in the ATES system is extracted from the warm-well of ATES system during cold season.

We develop a static model to describe the complete relations between input and output water temperatures for each plate. To this purpose, the energy transport due to the water flow in the aquifer plate and the building plate is given by

$$\mathbf{Q}_{\mathrm{he},k} = \rho_w c_{p,w} u_{\mathrm{A},k} (\mathbf{T}_{out,k}^{\mathrm{ap}} - \mathbf{T}_{in,k}^{\mathrm{ap}}) \quad , \quad \mathbf{Q}_{\mathrm{he},k} = \rho_w c_{p,w} u_{\mathrm{S},k} (\mathbf{T}_{in,k}^{\mathrm{bp}} - \mathbf{T}_{out,k}^{\mathrm{bp}}) \quad , \label{eq:Qhe}$$

where $\rho_w[\text{kg m}^{-3}]$ is the density of water, $c_{p,w}[\text{J kg}^{-1}\text{K}^{-1}]$ is the specific heat capacity of water and $u_{\text{A},k}, u_{\text{S},k}$ are the flow rate of pumps in the aquifer and the building plate, respectively, $T_{in,k}^{\text{ap}}$, $T_{out,k}^{\text{bp}}$, $T_{out,k}^{\text{bp}}$, $T_{out,k}^{\text{bp}}$, denote the input and out water temperatures of the aquifer plate and the building plate of the heat exchanger, respectively. By equating the above equations, one can find the water temperature $T_{out,k}^{\text{bp}}$ as follows:

$${\bf T}^{\rm bp}_{out,k} = {\bf T}^{\rm bp}_{in,k} - u_{{\rm S},k}^{-1} u_{{\rm A},k} ({\bf T}^{\rm ap}_{out,k} - {\bf T}^{\rm ap}_{in,k}) \; . \label{eq:total_pot}$$

The thermal energy of the heat exchanger can be also obtained via: $Q_{he,k} = k_{he}A_{he}\Delta T_m^{he}$, where k_{he} [W m⁻² K⁻¹], A_{he} [m²] correspond to the heat transfer coefficient and the surface for heat transfer of the heat exchanger, respectively. ΔT_m^{he} is the mean temperature difference for the heat transfer [7] and is given by $\Delta T_m^{he} = \left(\Delta T_{max}^{he} - \Delta T_{min}^{he}\right) / \ln \left(\frac{\Delta T_{max}^{he}}{\Delta T_{min}^{he}}\right)$,

where $\Delta T_{\max}^{he} = T_{in,k}^{bp} - T_{out,k}^{ap}$ and $\Delta T_{\min}^{he} = T_{out,k}^{bp} - T_{in,k}^{ap}$. We are now able to determine the water temperature $T_{out,k}^{ap}$ using:

$$T_{out,k}^{ap} = (1 - \delta_{he,k}) \frac{c_{he} T_{in,k}^{ap} (u_{A,k} - u_{S,k}) + T_{in,k}^{bp} u_{S,k} (c_{he} - 1)}{c_{he} u_{A,k} - u_{S,k}} + \delta_{he,k} T_{in,k}^{ap} ,$$

where $c_{\text{he}} = \exp\left(\frac{k_{\text{he}}A_{\text{he}}}{\rho_w c_{p,w}}(u_{\text{S},k}^{-1} - u_{\text{A},k}^{-1})\right)$, $\delta_{\text{he},k} = 1$, if $u_{\text{S},k} = u_{\text{H},k}$ and $\delta_{\text{he},k} = 0$, when $u_{\text{S},k} \neq u_{\text{H},k}$.

Define $v_{\text{he},k} := [T_{in,k}^{\text{ap}}, T_{in,k}^{\text{bp}}] \in \mathbb{R}^2$ and $y_{\text{he},k} := [T_{out,k}^{\text{ap}}, T_{out,k}^{\text{bp}}] \in \mathbb{R}^2$ to be an input variable vector and an output variable vector, respectively. Consider now the heat exchanger model for each sampling time k as follows:

$$y_{\text{he},k} = H_{\text{he}}(v_{\text{he},k}, u_{\text{A},k}, u_{\text{S},k})$$
 (7)

The proposed model is an algebraic fractional mapping from the input water temperatures to the output water temperatures as a function of flow rates of the water in both sides of a heat exchanger.

2.5. Ground Source Heat Pump

Consider an electrical countercurrent heat pump that consists of two sides, i.e. condenser side and evaporator side as it is shown in Figure 2c. The outflow water from the condenser, (the evaporator) is warmer, (colder) than the inflow water into the condenser, (the evaporator), respectively. During cold season, the return cold water from the building enters the condenser side of the heat pump to be heated in the internal process of a heat pump. A heat pump heats the water by using the water that comes from the warm-well of an ATES system after the heat exchanger. During warm season, one can use the evaporator side of a heat pump for a cooling purpose. In this process the return hot water from the building is entering the heat pump and then, the heat is transferred to the water that comes from the cold-well of an ATES system. The warm water obtained in this process is fed to the warm-well of an ATES system after the heat exchanger.

We now determine the relations between the inflow water temperatures $T_{in,k}^{con}$, $T_{in,k}^{eva}$, the outflow water temperatures $T_{out,k}^{con}$, $T_{out,k}^{eva}$ and the flow rate on each side of the heat pump $u_{B,k}$ and $u_{S,k}$. The fluid evaporates on the evaporator side of the heat pump when the thermal energy $Q_{c,k}$ is added from the warm-well of ATES system after the heat exchanger during cold seasons. The fluid condensates on the condenser side of the heat pump when the thermal energy $Q_{h,k}$ is subtracted in order to transfer heat to the return water temperature $T_{\text{ret},k}^B$ from the building during cold season. One can find $Q_{h,k}$ and $Q_{c,k}$ using the water flow on both sides of the heat pump as

$$Q_{h,k} = \rho_w c_{p,w} u_{B,k} (T_{out,k}^{con} - T_{in,k}^{con}) \quad , \quad Q_{c,k} = \rho_w c_{p,w} u_{S,k} (T_{in,k}^{eva} - T_{out,k}^{eva}) \quad ,$$
 (8)

where ρ_w [kg m⁻³] is the density of water, $c_{p,w}$ [J kg⁻¹K⁻¹] is the specific heat capacity of water and $u_{\rm B,k}$, $u_{\rm S,k}$ are the flow rate of pumps in the building side and the heat exchanger side, respectively. The thermal energies $Q_{h,k}$ and $Q_{c,k}$ can be also obtained via $Q_{h,k} = k_{\rm hp} A_{\rm hp} \Delta T_{m,h}^{\rm hp}$, $Q_{c,k} = k_{\rm hp} A_{\rm hp} \Delta T_{m,c}^{\rm hp}$, where $k_{\rm hp}$ [W m⁻² K⁻¹], $A_{\rm hp}$ [m²] corresponds to the heat transfer coefficient and the surface for heat transfer of the heat pump, respectively. $\Delta T_{m,h}^{\rm he}$ and $\Delta T_{m,c}^{\rm he}$ are the mean temperature differences for the heat transfer and given by

$$\Delta T_{m,h}^{\text{hp}} = \frac{\left(\Delta T_{\text{max},h}^{\text{hp}} - \Delta T_{\text{min},h}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{\text{max},h}^{\text{hp}}}{\Delta T_{\text{min},h}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{\text{max},c}^{\text{hp}} - \Delta T_{\text{min},c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{\text{max},c}^{\text{hp}}}{\Delta T_{\text{min},c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{max,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{max,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{max,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{max,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{max,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{max,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{m,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{max,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{min,c}^{\text{hp}} = \frac{\left(\Delta T_{max,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{min,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{min,c}^{\text{hp}} = \frac{\left(\Delta T_{min,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{min,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{min,c}^{\text{hp}} = \frac{\left(\Delta T_{min,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{min,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{min,c}^{\text{hp}} = \frac{\left(\Delta T_{min,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{min,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{min,c}^{\text{hp}} = \frac{\left(\Delta T_{min,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{min,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{min,c}^{\text{hp}} = \frac{\left(\Delta T_{min,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\frac{\Delta T_{min,c}^{\text{hp}}}{\Delta T_{min,c}^{\text{hp}}}\right)} \quad , \quad \Delta T_{min,c}^{\text{hp}} = \frac{\left(\Delta T_{min,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)}{\ln\left(\Delta T_{min,c}^{\text{hp}} - \Delta T_{min,c}^{\text{hp}}\right)} \quad , \quad \Delta T_{min$$

where $\Delta T_{\max,h}^{hp} = T_{hs} - T_{in,k}^{con}$, $\Delta T_{\min,h}^{hp} = T_{hs} - T_{out,k}^{con}$, $\Delta T_{\max,c}^{hp} = T_{in,k}^{eva} - T_{cs}$, and $\Delta T_{\min,c}^{hp} = T_{out,k}^{eva} - T_{cs}$. By equating both equations for $Q_{h,k}$, one can determine T_{hs} , the warm (condensation) side temperature of the heat pump, and by equating the both equations for $Q_{c,k}$, one can determine T_{cs} , the cold (evaporation) side temperature of the heat pump. This leads to

$$T_{hs} = \frac{h_{hp} T_{out,k}^{con} - T_{in,k}^{con}}{h_{hn} - 1} , \quad T_{cs} = \frac{c_{hp} T_{out,k}^{eva} - T_{in,k}^{eva}}{c_{hp} - 1} , \qquad (9)$$

where $h_{hp} = \exp(k_{hp}A_{hp}(\rho_w c_{p,w}u_{B,k})^{-1})$ and $c_{hp} = \exp(k_{hp}A_{hp}(\rho_w c_{p,w}u_{S,k})^{-1})$. The coefficient of performance (COP) of a heat pump is defined as COP = $Q_{h,k}(Q_{h,k} - Q_{c,k})^{-1}$. By substituting (8) into the latter expression, we have

$$COP = \frac{u_{B,k}(T_{out,k}^{con} - T_{in,k}^{con})}{u_{B,k}(T_{out,k}^{con} - T_{in,k}^{con}) - u_{S,k}(T_{in,k}^{eva} - T_{out,k}^{eva})}.$$
(10)

On the other side, assuming that an ideal water to water heat pump works with a Carnot cycle, the COP can be computed via COP = $\eta_{hp}T_{hs}\left(T_{hs}-T_{cs}\right)^{-1}$ where η_{hp} is considered as the efficiency of the heat pump to correct the heat pump operation according to the Carnot cycle. Substituting (9) into the latter equation of COP, we have

$$COP = \frac{\eta_{hp}(c_{hp} - 1)(h_{hp}T_{out,k}^{con} - T_{in,k}^{con})}{(c_{hp} - 1)(h_{hp}T_{out,k}^{con} - T_{in,k}^{con}) + (h_{hp} - 1)(-c_{hp}T_{out,k}^{eva} + T_{in,k}^{eva})}.$$
(11)

By equating (10) with (11), we can derive the water temperature $T_{out\,k}^{eva}$ leaving the heat pump evaporator side:

$$T_{out,k}^{\text{eva}} = \left(u_{\text{B},k}(c_{\text{hp}} - 1)(\eta_{\text{hp}} - 1)(h_{\text{hp}} T_{out,k}^{\text{con}} - T_{in,k}^{\text{con}})(T_{out,k}^{\text{con}} - T_{in,k}^{\text{con}}) - T_{in,k}^{\text{con}} - T_{in,k}^{\text{con}}) - T_{in,k}^{\text{con}} - T_{in,k}^{\text{con}} - T_{in,k}^{\text{con}}) + \eta_{\text{hp}} u_{\text{S},k}(c_{\text{hp}} - 1)(h_{\text{hp}} T_{out,k}^{\text{con}} - T_{in,k}^{\text{con}})\right)$$

$$\left(-c_{\text{hp}} u_{\text{B},k}(h_{\text{hp}} - 1)(T_{out,k}^{\text{con}} - T_{in,k}^{\text{con}}) - \eta_{\text{hp}} u_{\text{S},k}(c_{\text{hp}} - 1)(h_{\text{hp}} T_{out,k}^{\text{con}} - T_{in,k}^{\text{con}})\right)^{-1}.$$

$$(12)$$

To determine the water temperature $T_{out,k}^{con}$ leaving the heat pump during cold season, we need to have another equation for COP in order to equate with (11) and substituting $T_{out,k}^{eva}$ by (12). We therefore consider $T_{out,k}^{con}$ to be the desired set-point for operating the heat pump.

Now define $v_{\text{hp},k} := [T_{out,k}^{\text{con}}, T_{in,k}^{\text{con}}, T_{in,k}^{\text{eva}}] \in \mathbb{R}^3$ to be the input variable vector and $y_{\text{hp},k} := T_{out,k}^{\text{eva}}$ to be the output variable. The control variables are $u_{\text{B},k}, u_{\text{S},k}$, the pump flow rate of building side and the storage tank side, respectively. Now consider the heat pump static model for each sampling time k as follows:

$$y_{hp,k} = H_{hp}(v_{hp,k}, u_{B,k}, u_{S,k})$$
 (13)

The proposed model is an algebraic fractional mapping from the input water temperatures to the output water temperature as a function of flow rates of the water on both sides of a heat pump.

2.6. Relationships between the Internal Variables of each Subsystem

We define conditions for each equipment in a particular mode of operation with respect to the outside air temperature $T_{o,k}$. The desired building zone temperature is equal to $T_{\text{des},k}^B$ and if $T_{z,k}^B$ resides between $T_{\text{des},k}^B \pm 2^{\circ}\text{C}$, there is no need for cold or heat thermal energy and all components are in off mode. If the outside air temperature $T_{o,k}^B \pm 2^{\circ}\text{C}$, there is a request for a light cooling energy meaning that we can use the stored cold energy in the cold-well of ATES system without using the evaporator side of a heat pump. In case the outside air temperature $T_{o,k}^B$ goes beyond $T_{z,ub}^B + 5^{\circ}\text{C}$, we have to turn on the evaporator side of a heat pump. The counterpart of the above conditions are as follows. If the outside air temperature $T_{o,k}$ drops below the lower bound of building zone temperature $T_{z,lb}^B$ but is still above the bound of cold temperature $T_{z,lb}^B - 5^{\circ}\text{C}$, there is a request for a light heating energy meaning that we can use the stored hot energy in the warm-well of ATES system without using the condenser side of a heat pump. In case the outside air temperature $T_{o,k}$ drops below $T_{z,lb}^B - 5^{\circ}\text{C}$, we have to turn on the condenser side of a heat pump. When the outside air temperature $T_{o,k}$ drops further below, it is time to use a boiler as an additional heat source for the building. In all above conditions, there is a possibility to use an external party to contribute to the required amount of thermal energy. However, there is also a limitation for the external party. It is considered to have a possibility of using the heat pump with partial capacity in a parallel mode with direct connection to the heat exchanger by defining a valve to control the input water of the heat pump together with a bypass pipe.

The aforementioned operating modes are defined for all sampling times $k \in \{1, 2, 3, \dots\}$ along with the following control input decision variables:

- Continuous variables: we consider pump flow rates and valve positions with the following definitions.
 - 1. Pump flow rates: define $u_{B,k}, u_{S,k}, u_{A,k} \in \mathbb{R}_+$ to be the pump flow rates in the building, in the storage tank
 - and in the ATES system side, respectively.[-1cm] 2. Valve positions: consider $v_{b,k}, v_{c,k}, v_{h,k} \in [0,1]$ to be the valve positions for the boiler, the chiller and the heat pump, respectively. $v_{b,k}, v_{c,k}, v_{h,k} = 1$ if they are working with full capacity, $0 < v_{b,k}, v_{c,k}, v_{h,k} < 1$ if they are working with partial capacity, and $v_{b,k}$, $v_{c,k}$, $v_{h,k} = 0$ when they are off.
- Discrete (integer) variables: we define $s_{n,k} \in \{0,1\}$ to be an integer variable that corresponds to the operating mode of building. We distinguish between the different season-related operating modes of building by using $s_{c,k}, s_{w,k} \in \{0, 1\}$ to specify the cold season and warm season, respectively:

$$s_{n,k} := \begin{cases} 1 & \text{operating with the HE and ATES} \\ 0 & \text{operating with the external party} \end{cases}, \text{ and } \begin{cases} T_{o,k} > T_{z,ub}^B & s_{w,k} = 1 \text{ , } s_{c,k} = 0 \\ T_{z,lb}^B \le T_{o,k} \le T_{z,ub}^B & s_{w,k} = 0 \text{ , } s_{c,k} = 0 \\ T_{z,lb}^B > T_{o,k} & s_{w,k} = 0 \text{ , } s_{c,k} = 1 \end{cases}$$

All internal input variables $v_{A,k}$, $v_{he,k}$, $v_{S,k}$, $v_{hp,k}$, $v_{B,k}$ and output variables $y_{A,k}$, $y_{he,k}$, $y_{S,k}$, $y_{hp,k}$, $y_{B,k}$ of the aforementioned heating and cooling equipment are connected to each other by the following relations.

The water temperature $T_{in,k}^{aq}$ entering the ATES system is equal to the water temperature $T_{out,k}^{ap}$ leaving the aquifer plate of a heat exchanger, $T_{in,k}^{aq} = T_{out,k}^{ap}$. The water temperature $T_{in,k}^{ap}$ entering the heat exchanger aquifer plate is equal to the water temperature $T_{out,k}^{aq}$ leaving the ATES system, $T_{in,k}^{ap} = T_{out,k}^{aq}$. The water temperature $T_{in,k}^{bp}$ entering the heat exchanger building plate is equal to the water temperature $T_{out,k}^{s}$ leaving the storage tank and the water temperature leaving the chiller $T_{out,k}^{\text{chi}}$ depending on the chiller valve position $v_{c,k}$, $T_{in,k}^{\text{bp}} = v_{c,k}T_{out,k}^{\text{chi}} + (1 - v_{c,k})T_{out,k}^s$. The water temperature $T_{in,k}^s$ entering the storage tank system is equal to the water temperature $T_{out,k}^{\text{con}}$ or $T_{out,k}^{\text{eva}}$ leaving the heat pump condenser or evaporator side, and the return water temperature of the building thermal system depending on the mode of operation. It can be determined by $T_{in,k}^s = v_{h,k}(s_{w,k}T_{out,k}^{\text{con}} + s_{c,k}T_{out,k}^{\text{eva}}) + (1 - v_{h,k})T_{\text{ret},k}^B$. The water temperature $T_{in,k}^{con}$ entering the heat pump condenser side is equal to the return water temperature $T_{ret,k}^{B}$ from the building or the water temperature $T_{out,k}^{bp}$ leaving the building plate of the heat exchanger or the water temperature $T_{out,k}^{ext}$ leaving an external party, depending on the mode of the operation. The water temperature $T_{in,k}^{\text{eva}}$ entering the heat pump evaporator side is equal to the return water temperature $T^{B}_{ret,k}$ from the building or the water temperature $T^{bp}_{out,k}$ leaving the building plate of the heat exchanger or the water temperature $T_{out,k}^{\text{ext}}$ leaving an external party, depending on the working mode. We thus have

$$\begin{split} \mathbf{T}_{in,k}^{\text{con}} &= s_n (\mathbf{T}_{\text{ret},k}^{\text{B}} s_c + \mathbf{T}_{out,k}^{\text{bp}} s_w) + (1 - s_n) (\mathbf{T}_{\text{ret},k}^{\text{B}} s_c + \mathbf{T}_{out,k}^{\text{ext}} s_w) \;, \\ \mathbf{T}_{in,k}^{\text{eva}} &= s_n (\mathbf{T}_{\text{ret},k}^{\text{B}} s_w + \mathbf{T}_{out,k}^{\text{bp}} s_c) + (1 - s_n) (\mathbf{T}_{\text{ret},k}^{\text{B}} s_w + \mathbf{T}_{out,k}^{\text{ext}} s_c) \;. \end{split}$$

The input water temperature of the floor or ceiling $T_{in,k}^{fc}$ is equal to the supply water temperature for the building. The supply water temperature can be determined by

$$T_{\sup,k}^{\rm B} = \nu_{h,k}(s_{w,k}T_{out,k}^{\rm eva} + s_{c,k}((1-\nu_{b,k})T_{out,k}^{\rm con} + \nu_{b,k}T_{out,k}^{\rm boi})) + (1-\nu_{h,k})T_{out,k}^{\rm bp} \;,$$

Using the developments (2), (4), (6), (7), (13) in previous parts and the aforementioned algebraic equations, one can rewrite a building heating and cooling system as a single dynamical model for each sampling time k in a more compact notation

$$x_{k+1} = f(x_k, u_k, v_k, s_k, w_k)$$
, (14)

where $x_k := [x_{B,k}, x_{S,k}, x_{A,k}] \in \mathbb{R}^9$ is the state vector, $u_k := [u_{B,k}, u_{S,k}, u_{A,k}] \in \mathbb{R}^3$ is a vector that contains pump flow rates, $v_k := [v_{b,k}, v_{c,k}, v_{h,k}] \in [0, 1]^3$ corresponds to a vector of valve positions, $s_k := [s_{w,k}, s_{c,k}, s_{h,k}] \in \{0, 1\}^3$ is related to a vector of binary variables, and $w_k := [T_{o,k}, I_{o,k}, v_{o,k}] \in \mathbb{R}^3$ is a vector of outside weather parameters.

3. Model Predictive Control Problem

Consider $k \in \{0, 1, 2, \dots, N_h\}$, where N_h is the length of our prediction horizon. To ensure that the building zone has the required comfort properties such as the desired building zone temperature $T_{\text{des},k}^B$, we consider a set-point tracking

term for the building zone temperature $T_{z,k}^B$ to be our objective. We aim to minimize $\gamma(T_{z,k}^B - T_{set})^2$ as an objective function that reflects our control purpose, where γ is a weighting coefficient. Our goal is to find u_k, v_k, s_k the vector of pump flow rates, valve positions and status of switches as our control variables at each sampling time k.

We now formulate a finite-horizon optimal control problem for each sampling time k:

$$\underset{\{u_{k}, v_{k}, s_{k}\}_{k=0}^{N_{h}}}{\text{minimize}} \qquad \sum_{k=0}^{N_{h}} \gamma (T_{z,k}^{B} - T_{\text{des}}^{B})^{2} \tag{15a}$$

subject to
$$x_{k+1} = f(x_k, u_k, v_k, s_k, w_k)$$
, (15b)

$$x_{\min} \le x_k \le x_{\max} \,, \tag{15c}$$

$$u_{\min} \le u_k \le u_{\max}$$
, (15d)

$$0 \le v_k \le 1 \,, \tag{15e}$$

$$s_k \in \{0, 1\}$$
. (15f)

Let us fix the initial state value x_{ξ} and a given outside weather parameters w_{ξ} at each sampling time ξ , respectively. It is assumed that the entire predicted state vector $[x_1, x_2, \cdots, x_{N_h+1}]$ of the system dynamics (15b) is known at each time instant, given the initial state value $x_0 = x_{\xi}$. The proposed optimization problem (15) is a multistage mixed-integer nonlinear program, whose stages are coupled by the discrete-time dynamical system equation (15b). The proposed nonlinear MPC framework is summarized in Algorithm 1. Figure 3 depicts a block diagram of our developed model of the heating and cooling equipment of a building with a control (optimizer) unit.

Algorithm 1 Nonlinear Model Predictive Control (MPC)

- Initialize the state x₀ = x_{\xi} and the parameter along the prediction horizon {w_k}^{N_h}_{k=0}
 Solve optimization program (15) and determine an optimal solution sequence {u₀^{*}, v₀^{*}, s₀^{*}, ···, u_{N_h}^{*}, v_{N_h}^{*}, s_{N_h}^{*}}
- 3: Apply the first element of optimal solution $u_{\xi} := u_0^{\star}, v_{\xi} := v_0^{\star}, s_{\xi} := s_0^{\star}$ to the system (14)
- 4: Measure the state and update the parameter
- 5: Go to step 2

4. Simulation and Numerical Study

We simulate the proposed nonlinear MPC framework (Algorithm 1), where it is considered to have a day-ahead prediction horizon length with hourly-based sampling time. In the interest of space, the table of parameters of our proposed model in formulation (15) is omitted and we refer the interested reader to [2] and [4] for a detailed table of parameters, symbols and values. During the numerical study, we were faced with computational time issues which may be expected due to the final optimization problem (15) which leads to a mixed-integer nonlinear (signomial) program. We therefore fixed binary variables and chose to focus our simulation study on one month, March 2010, given the outside weather data from the weather station in Delft, The Netherlands [8]. The simulation environment was MATLAB using the solver 'fmincon' together with the YALMIP toolbox [9] as an interface.

Figure 4 depicts the obtained results of our numerical study. Figure 4a presents the pump flow rates at each sampling time, where the 'green' line denotes the pump flow rate of ATES system u_A , the 'blue' line corresponds to the pump flow rate of storage tank u_S , and the 'red' line is related to the flow rate of water inside pipes in the floor or ceiling of building $u_{\rm B}$. Notice that in our building model, we consider to have heating and cooling sources from floor and ceiling in each zone, respectively, by using pipes that are installed inside compositions of ceiling and floor with the surface ratio of 25%. In Figure 4b, we demonstrate the dynamics of temperatures, where the 'red' line corresponds to the outside air temperature which is given to our control problem, the solid' black' line represents the temperature profile of the building walls, the 'blue' line is related to the circulated water temperature in building floor pipes, the dashed black lines show the upper-lower bounds of our desired building zone temperature, and finally, the 'green' line shows the dynamical behavior of building zone temperature over time. As it is clearly presented, the building zone air temperature is kept within the desired bounds throughout the simulation by using our proposed control framework.

Within this specific one month time period of our simulation, the ATES well temperature does not have a great impact on the results. However, this simulation illustrates the functioning of our proposed predictive model. If we apply longer horizons, the well temperature starts playing an important role in finding the optimal control strategy.

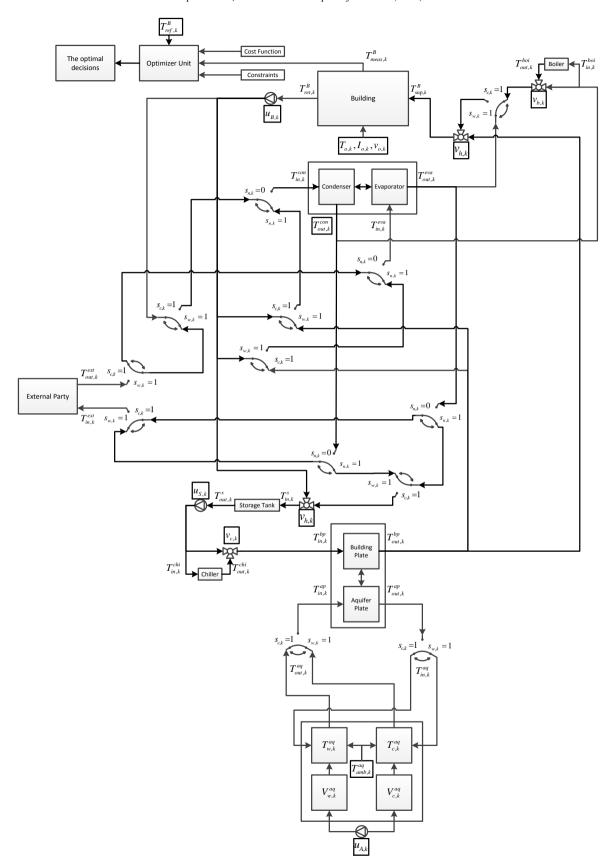


Fig. 3: Control block diagram of building heating and cooling equipment.

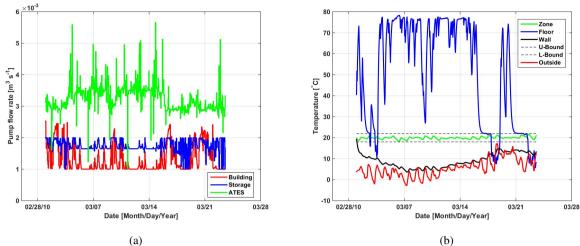


Fig. 4: Pump flow rates of building heating and cooling equipment (a). Dynamics of building temperatures (b).

5. Conclusions and Future Works

In this paper, we built a complete predictive model of building heating and cooling equipment that consists of dynamical models of building zone temperatures, short-term storage tank, an ATES system as a seasonal storage system, and a mathematical description of a ground source heat pump together with a heat exchanger. Heating or cooling requirements of a building was determined with respect to the outside weather conditions. Using our proposed unified model, we developed a nonlinear MPC framework, where our objective was to track the desired building zone temperature, while respecting all limitations, constraints on the operation of each component, separately, and having different operating conditions. We simulated a building heating and cooling system using the proposed nonlinear MPC strategy which showed good performance. The results point towards a general trade-off between complexity of model and simulation time to achieve collective ATES performance.

Our current research direction is to incorporate the developed ATES system dynamics into our proposed thermal grid framework in [10]. Demand for ATES is increasing due to energy saving ambitions and cost. Therefore it is required to intensify and optimize the use of aquifers. However, intensified use of the subsurface may result in unwanted mutual interaction between warm and cold wells. A future research direction is to organize the use of the subsurface efficiently in order to manage/prevent negative interactions [11].

References

- [1] Bloemendal, M., Olsthoorn, T., Boons, F.. How to achieve optimal and sustainable use of the subsurface for aquifer thermal energy storage. Energy Policy 2014;66:104–114.
- [2] Rostampour, V., Jaxa-Rozen, M., Bloemendal, M., Keviczky, T.. Building climate energy management in smart thermal grids via aquifer thermal energy storage systems. Journal of Energy Procedia 2016;.
- [3] Sommer, W., Valstar, J., Leusbrock, I., Grotenhuis, T., Rijnaarts, H.. Optimization and spatial pattern of large-scale aquifer thermal energy storage. Applied energy 2015;137:322–337.
- [4] Rostampour, V., Bloemendal, M., Jaxa-Rozen, M., Keviczky, T.. A control-oriented model for combined building climate comfort and aquifer thermal energy storage system. European Geothermal Congress, September 19-23, 2016;.
- [5] Zwart, d.H.. Analyzing energy-saving options in greenhouse cultivation using a simulation model. Ph.D. thesis; Wageningen University and Research Centre; 1996.
- [6] Ooteghem, C.. Optimal control design for a solar greenhouse. Ph.D. thesis; Wageningen University and Research Centre; 2007.
- [7] Colburn, A.P., de, E.d.P.. Mean temperature difference and heat transfer coefficient in liquid heat exchangers. Industrial & Engineering Chemistry 1933;25(8):873–877.
- [8] KNMI, . Hourly data 1980-2010, climatic projections KNMI W+ scenario 2010-2045 for different weather stations in NL. Tech. Rep.; Royal Dutch Meteorological Institute; 2013.
- [9] Löfberg, J.. YALMIP: A toolbox for modeling and optimization in MATLAB. In: International Symposium on Computer Aided Control Systems Design. IEEE; 2004, p. 284–289.
- [10] Rostampour, V., Keviczky, T.. Robust randomized model predictive control for energy balance in smart thermal grids. European Control Conference, June 29 July 1, 2016;:1201–1208.
- [11] Rostampour, V., Keviczky, T.. Probabilistic energy management for building climate comfort in smart thermal grids with seasonal storage systems. Submitted to the International Federation of Automatic Control (IFAC), July 9-14, 2017;.