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Optimizing the performance of the feedback controller for state-based switching bilinear systems

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Summary

This article is concerned with the design and performance optimization of feedback controllers for state-based switching bilinear systems (SBLSs), where subsystems take the form of bilinear systems in different state space polyhedra. First, by further dividing the subregions into smaller regions and designing region-dependent feedback controllers in the resulting regions, the SBLSs can be transformed into corresponding switching linear systems (SLSs). Then, for these SLSs, by imposing contractility conditions on the Lyapunov functions, an upper bound on the infinite horizon quadratic cost can be obtained. Optimizing this upper bound yields the controller design. The optimization problem is formulated as a linear matrix inequalities optimization problem, which can be solved efficiently. Finally, the stability of the close-loop system under the proposed controller is established step by step through a decreasing overall Lyapunov function.

KEYWORDS

bilinear system with state-based switching, switching bilinear system control, Lyapunov stability, LMIs

1 | INTRODUCTION

Most of the problems found in practice are normally nonlinear problems, which are usually complex. In order to optimize or control these kind of realistic problems, the nonlinear complex systems are usually described by multiple simple models, such as linear models, bilinear models, Markov models, statistic models, and so on. Many research works have been done to identify the individual simple models and their connections that build up the nonlinear systems.[1-9]

A special kind of nonlinear system, that is, the bilinear system, contains the sum of a linear term and a bilinear term. Bilinear systems have been investigated a lot since the 1960s.[10-15] It has been proved bilinear systems have a better performance than linear systems in optimal control,[16] since bilinear systems have a variable structure due to the existence of the bilinear term. In practice, there are systems that naturally have a bilinear term with the states multiplying the control inputs, such as in the field of sociology, biology, power systems, and so on.[10,17] Usually, the reason for the existence of the term is that the influence of the control input on the system depends on the current system state.

In practice, some complex nonlinear systems can be approximated by dividing into multiple state-based bilinear subsystems.[18,19] In each state region, a bilinear subsystem is activated, and the bilinear subsystems switch between each other according to the switching of the state regions. This results in a state-based switching bilinear system (SBLS).[20] Developing the theory on stabilizing controllers for the state-based SBLSs provides a methodology to design controllers for systems with complex nonlinear features in practice. Inspired by this, a stabilizing controller design based on linear matrix inequalities (LMI) has been addressed in Reference 21. It should be pointed out that generally many controllers can be designed to achieve the stabilizability of bilinear systems,[22,23] but maybe more work needs to be done to improve the close-loop performance by utilizing the remaining degrees of freedom. For bilinear systems, optimal control problems have attracted much attention.[10,24-26] However, to the best knowledge of the authors, for state-based SBLSs few results exist focusing on the performance optimization of the controller. Motivated by this, this article is devoted to optimizing the performance of stabilizing controllers for state-based SBLSs.

To deal with the state-based SBLSs, the subregions where subsystems are activated are further divided into some multiple regions, then region-dependent controllers are designed for the resulting subregions, which transforms the bilinear systems into linear ones. For the resulting state-based switching linear systems (SLSs), the infinite horizon quadratic cost is difficult to calculate explicitly. To solve this problem, contractility conditions on the Lyapunov function are used to derive an upper bound on the quadratic cost. Then instead of directly optimizing the infinite horizon quadratic cost, an LMI optimization problem is formulated to optimize this upper bound.

The remainder of the article is organized as follows. In Section 2, the problem statement is given. The main results including the transformation of the bilinear systems into linear ones and the derivation of an upper bound on the quadratic cost are given in Section 3. In the end, a numerical example is given in Section 4 to illustrate the proposed approach. Finally, some conclusions are drawn in Section 5.

2 | PROBLEM STATEMENT

Consider a SBLS

$$\dot{x} = A_i x + \sum_{j=1}^{m_i} (G_{i,j} x + b_{i,j}) u_{i,j}, \text{ if } x \in \Omega_i, i \in \Lambda, \tag{1}$$

where A_i and $G_{i,j}$ are $[n \times n]$ matrices, $b_{i,j}$ is an $[n \times 1]$ vector, Ω_i is the corresponding state space polyhedron with $i \in \Lambda$ the state space partition of $\Omega \subset \mathbb{R}^n$ ($\cup_{i\in\Lambda}\Omega_i = \Omega$, $\Omega_i \neq \emptyset$, $\forall i \in \Lambda$, $\Omega_i \cup \Omega_j = \emptyset$, $\forall i, j \in \Lambda$, $i \neq j$), $j \in M_i = \{1, \ldots, m_i\}$, and $U_i = [u_{i,1}\ u_{i,2} \cdots u_{i,m_i}]^{\mathrm{T}} \in \mathbb{R}^{m_i}$ is an m_i-dimensional control input.

In order to find the relationship between the bilinear term and the linear term, the bilinear system can be further adapted. Since each control input $u_{i,j}$ is a scalar, then $\mathrm{rank}(G_{i,j}) = 1$, so it can be expressed as the inner product of two vectors. Then, we can write (1) as

$$\dot{x} = A_i x + \sum_{j=1}^{m_i} b_{i,j} (c_{i,j}^{\mathrm{T}} x + 1) u_{i,j}, \text{ if } x \in \Omega_i, i \in \Lambda. \tag{2}$$

Due to the similarity between SBLSs and SLSs, we could define the control inputs as

$$u_{i,j} = \frac{k_{i,j} x}{c_{i,j}^{\mathrm{T}} x + 1}, \quad \text{if } x \in \Omega_i, i \in \Lambda. \tag{3}$$

so as to obtain a corresponding SLS for the original SBLS, as

$$\dot{x} = A_i x + \sum_{j=1}^{m_i} b_{i,j} k_{i,j} x, \text{ if } x \in \Omega_i, i \in \Lambda. \tag{4}$$

Herein, the controller can be designed for the derived corresponding SLS.

3 | OPTIMIZED STATE-FEEDBACK CONTROL DESIGN FOR SLSS

Instead of designing a controller for the SBLS directly, we consider designing a state-feedback controller for the corresponding SLS of the original system. Based on the similarity between the two systems, the derived controller can be extended to be used for the SBLS easily.

3.1 | Corresponding SLS

For SBLSs, in order to design stabilizing switching division controllers for each bilinear subsystem $i \in \Lambda$, we need to partition the state space polyhedron Ω_i into more subregions. If for subbilinear system $i \in \Lambda$, the control input is $u_{i,j}(i \in \Lambda, j \in M_i)$, then for each control input $u_{i,j}$ two state-feedback controllers should be designed. The polyhedral partition of $\Omega_i (i \in \Lambda)$ for bilinear subsystem i can be defined as $\{\Omega_{i,l}\}_{i \in \Lambda, l \in \Gamma_i}$, where $\cup_{l \in \Gamma_i} \Omega_{i,l} = \Omega_i, \Omega_{i,l} \neq \emptyset, \forall l \in \Gamma_i, \Omega_{i,l_1} \cap \Omega_{i,l_2} \neq \emptyset, \forall l_1 \neq l_2, l_1, l_2 \in \Gamma_i$.

Based on the polyhedral partition of the state space and defining the equilibrium as the origin, the controller is designed for each polyhedron $\Omega_{i,l}$ as

$$U_{i,l} = [u_{i,l,1} \; u_{i,l,2} \cdots u_{i,l,m_i}]^{\mathrm{T}}, \quad i \in \Lambda, \tag{5}$$

where each control element is designed according to (3). If we substitute (3) into (4), then the bilinear terms are eliminated, and the bilinear system in (4) becomes a SLS, which is the corresponding SLS of the SBLS. In order to control the SBLS, we can first consider to design a stabilizing state-feedback controller for the following corresponding SLS:

$$\dot{x} = (A_i + B_i K_{i,l})x, \quad \text{if } x \in \Omega_{i,l}, l \in \Gamma_i, i \in \Lambda, \tag{6}$$

where

$$\begin{aligned} B_i &= [b_{i,1} \; b_{i,2} \cdots b_{i,m_i}], \\ K_{i,l} &= [k_{i,l,1} \; k_{i,l,2} \cdots k_{i,l,m_i}]^{\mathrm{T}}. \end{aligned} \tag{7}$$

Therefore, by dividing the state space into more subregions, the SBLS can be adapted into the corresponding SLS. The corresponding SLS and the SBLS are actually the same model working on different divisions of state space.

3.2 | Lyapunov functions and boundary constraints

Each polyhedral region $\Omega_{i,l}$ can be described as a system of linear inequalities:

$$\underbrace{[F_{i,l} \quad f_{i,l}]}_{\bar{F}_{i,l}} \begin{bmatrix} x \\ 1 \end{bmatrix} \geq 0, \quad \text{if } x \in \Omega_{i,l}, \tag{8}$$

and the boundary hyperplane for two neighboring regions $\Omega_{i,l}$ and $\Omega_{i',l'}$ is characterized by an equality and inequality as

$$\underbrace{[\Psi_{ii',ll'} \quad \psi_{ii',ll'}]}_{\bar{\Psi}_{ii',ll'}} \begin{bmatrix} x \\ 1 \end{bmatrix} = 0, \text{ and } \underbrace{[\Phi_{ii',ll'} \quad \phi_{i',ll'}]}_{\bar{\Phi}_{ii',ll'}} \begin{bmatrix} x \\ 1 \end{bmatrix} \geq 0, \quad \forall\, x \in \Omega_{i,l} \cap \Omega_{i',l'}. \tag{9}$$

Lyapunov functions are defined for each polyhedral region $\Omega_{i,l}(l \in \Gamma_i, i \in \Lambda)$ with the following format

$$V_{i,l}(x) = \begin{bmatrix} x \\ 1 \end{bmatrix}^{\mathrm{T}} \underbrace{\begin{bmatrix} P_{i,l} & \star \\ s_{i,l}^{\mathrm{T}} & r_{i,l} \end{bmatrix}}_{\bar{P}_{i,l}} \underbrace{\begin{bmatrix} x \\ 1 \end{bmatrix}}_{\bar{x}}, \quad \forall\, l \in \Gamma_i, i \in \Lambda, x \in \Omega_{i,l}, \tag{10}$$

with $\bar{x} = [x \; 1]^{\mathrm{T}}$, $P_{i,l} \in \mathbb{R}^{n \times n}$ a symmetric matrix, $s_{i,l}$ an $[n \times 1]$ dimensional vector, and $r_{i,l} \in \mathbb{R}$. $\star$ stands for the transpose of its symmetrical element.

3.3 | Optimized state-feedback control for SLS

In this part, optimal switching state-feedback control laws will be designed for the SLS in (7), to asymptotically steer any state in the feasible region to the origin, and to guarantee the minimization of a given objective function along the system state trajectory at the same time. More related reference work could be found in References 18,27.

The following theorem gives a sufficient condition to design optimal switched state-feedback control laws for the SLS in (7) that, to asymptotically bring the state to the origin (the equilibrium for at least one of the subsystems), and to optimize the objective function along the system state trajectory. Since the switchings are unknown among the subregions, and the objective function along the system state trajectory is not certain, it is not possible to explicitly optimize the objective function along the state trajectory as

$$J(\infty) = \int_0^\infty \left[x^T Q_J x + u^T R_J u\right] dt. \tag{11}$$

Therefore, instead of optimizing the infinite objective function, we optimize the upper bound of the infinite objective function in a min max format, and prove the realization with LMIs in the following theorems.

In the description below, the augmented system matrices are used to describe the linear affine systems as follows:

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, \; \overline{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}. \tag{12}$$

Theorem 1. *For the optimization problem*

$$\begin{aligned} &\min_u \max_x \quad J(\infty) = \int_0^\infty \left[x^T Q_J x + u^T R_J u\right] dt \\ &\text{s.t.} \quad (16) - (19), \end{aligned} \tag{13}$$

if there exists a solution satisfying all the constraints, with positive definite matrices $\overline{Q}_{i,l}$, $Q_{i,l}$, $R_{i,l}$, and $M_{i,l}$, then taking the state-feedback control laws with gains as

$$\overline{K}_{i,l} = \overline{N}_{i,l}\overline{Q}_{i,l}^{-1} \quad \forall\,(i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \notin \Omega_{i,l}\}, \tag{14}$$

and

$$K_{i,l} = N_{i,l}Q_{i,l}^{-1} \quad \forall\,(i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \in \Omega_{i,l}\}, \tag{15}$$

asymptotically stabilizes the SLS system in (7), and guarantees the minimization of the objective function along the state trajectory.

$$\begin{bmatrix} \overline{Q}_{i,l} & \star \\ \overline{F}_{i,l}\overline{Q}_{i,l} & R_{i,l} \end{bmatrix} > 0, \quad \forall\,(i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \notin \Omega_{i,l}\}, \tag{16}$$

$$\begin{bmatrix} Q_{i,l} & \star \\ F_{i,l}Q_{i,l} & R_{i,l} \end{bmatrix} > 0, \quad \overline{Q}_{i,l} = \begin{bmatrix} Q_{i,l} & \star \\ 0 & q_{i,l} \end{bmatrix} > 0, \quad \forall\,(i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \in \Omega_{i,l}\}, \tag{17}$$

$$\begin{bmatrix} A_iQ_{i,l} + Q_{i,l}A_i^T + B_iN_{i,l} + N_{i,l}^TB_i^T & \star & \star & \star \\ F_{i,l}Q_{i,l} & -M_{i,l} & 0 & 0 \\ Q_{i,l} & 0 & -Q_J^{-1} & 0 \\ K_{i,l}Q_{i,l} & 0 & 0 & -R_J^{-1} \end{bmatrix} < 0, \quad \forall\,(i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i\}, \tag{18}$$

$$\begin{bmatrix} \overline{Q}_{i,l} & \star & \star & \star \\ \overline{Q}_{i,l} & \overline{Q}_{i',l'} & \star & \star \\ \overline{\Psi}_{ii',ll'}\overline{Q}_{i,l} & 0 & -\lambda_{ii',ll'} & \star \\ \overline{\Phi}_{ii',ll'}\overline{Q}_{i,l} & 0 & 0 & -\Theta_{ii',ll'} \end{bmatrix} > 0, \quad \text{if } d_{i,j} > d_{i',j'}, \text{ and } \{\Omega_{i,l} \cap \Omega_{i',l'}\} \neq \emptyset, \ \forall i, i' \in \Lambda, l \in \Gamma_i, l' \in \Gamma_{i'} \tag{19}$$

Proof. First, using the Schur complement for (16), and multiplying the result from the right and left side by $\overline{Q}_{i,l}^{-1} = \overline{P}_{i,l}$, yields

$$\overline{P}_{i,l} - \overline{F}_{i,l}^{\mathrm{T}} R_{i,l}^{-1} \overline{F}_{i,l} > 0, \quad \forall\, (i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \notin \Omega_{i,l}\}, \tag{20}$$

which guarantees that the Lyapunov function on each state polyhedron is positive because of the positiveness of the matrix $R_{i,l}$, that is,

$$V_{i,l} > 0, \quad \text{if } \overline{F}_{i,l}\overline{x}_{i,l} \geq 0 \text{ and } \overline{x}_{i,l} \neq 0, \quad \forall\, (i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \notin \Omega_{i,l}\}. \tag{21}$$

For the case for the subsystem containing the origin, that is, for the polyhedron with $0 \in \Omega_{i,l}$, the LMIs in (17) and (18) are applied to make sure obtain a positive Lyapunov function and a negative derivative of Lyapunov function on the region. The row and column corresponding to the augmented variable are removed here, to guarantee that the derivative of the Lyapunov function $\dot{V}_{i,l}$ would be zero only when the state x is zero.

Second, the Schur complement is applied on (18), and the obtained result is multiplied from left and the right side by $Q_{i,l}^{-1} = P_{i,l}$. With the feedback laws (14), we obtain

$$P_{i,l}(A_i + B_i K_{i,l}) + (A_i + B_i K_{i,l})^{\mathrm{T}} P_{i,l} < -F_{i,l}^{\mathrm{T}} M_{i,l}^{-1} F_{i,l} - Q_J - K_{i,l}^{\mathrm{T}} R_J K_{i,l}, \quad \forall\, (i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \notin \Omega_{i,l}\}, \tag{22}$$

since the parameter matrices in the objective functions (Q_J and R_J) are positive definite, and $M_{i,l}$ is also positive definite; therefore it guarantees that the derivative of the Lyapunov function on each state polyhedron is negative, as

$$\dot{V}_{i,l} < 0, \quad \text{if } F_{i,l}x_{i,l} \geq 0 \text{ and } x_{i,l} \neq 0, \quad \forall\, (i,l) \in \{(i,l) | i \in \Lambda, l \in \Gamma_i, 0 \notin \Omega_{i,l}\}. \tag{23}$$

The LMI that makes sure the derivative of the Lyapunov function is negative is written in the format of (18), because for the linear affine switching subsystems, the derivative of the affine offset is 0.

Then, we perform the Schur complement on (19) 3 times, each time with respect to the last row and column. Similarly, we multiply the result from the right and left side by $\overline{Q}_{i,l}^{-1} = \overline{P}_{i,l}$; and use (14), then we obtain the following inequalities to guarantee the boundary condition:

$$\overline{P}_{i,l} - \overline{P}_{i',l'} + \lambda_{ii',ll'}^{-1} \overline{\Psi}_{ii',ll'}^{\mathrm{T}} \overline{\Psi}_{ii',ll'} + \overline{\Phi}_{ii',ll'}^{\mathrm{T}} \Theta_{ii',ll'}^{-1} \overline{\Phi}_{ii',ll'} > 0, \quad \text{if } d_{i,j} > d_{i',j'}, \text{ and } \Omega_{i,l} \cap \Omega_{i',l'} \neq \emptyset, \quad \forall i, i' \in \Lambda, l \in \Gamma_i, l' \in \Gamma_{i'}, \tag{24}$$

which ensures that $V_{i,l} \geq V_{i',l'}$ for all the states $\overline{x} \in \mathcal{S}_{ii',ll'}$ on the boundary of $\Omega_{i,l}$ and $\Omega_{i',l'}$. $d_{i,j}$ is the shortest distance between the origin and the polyhedron $\Omega_{i,j}$.The augmented $\overline{Q}_{i,l}$ is defined for the polyhedron $\Omega_{i,l}$ containing the origin in (17), to make it comparable on the boundary conditions with other polyhedron without the origin.

Because the Lyapunov functions reduce during the switchings of the regions in the state space, there is a sequence of polyhedra in Ω, whose distances to the origin are reducing, which satisfy

$$d_p \geq d_{p-1} \geq \cdots \geq d_1 \geq 0, \tag{25}$$

with p as the total number of polyhedron $\Omega_{i,l}$, $\forall i \in \Lambda$, $l \in \Gamma_i$ in Ω, which is corresponding to a sequence of decreasing Lyapunov functions for all the polyhedra as

$$V_p(x_p^*) \geq V_{p-1}(x_{p-1}^*) \geq \cdots \geq V_1(x_1^*) \geq 0, \tag{26}$$

that can make the system state asymptotically converge to the origin, from an initial state x_0 within any of the polyhedra in Ω. At the same time, the upper bound of the cost function is minimized along the trajectory, to make sure the objective function of the worst case is minimized under the uncertain switchings. Consequently, by solving the optimization in

Theorem 1, it is possible to design the optimal switched state-feedback control laws for the SLS in (7) that, asymptotically bring the state to the origin (the equilibrium for at least one of the subsystems), and optimize the objective function along the system state trajectory. ■

In order to solve the min max optimization problem, the upper bound of the objective function is derived to further solve the optimization with LMIs.

Since the objective function is

$$J(\infty) = \int_0^{\infty} \left[x^{\mathrm{T}} Q_J x + u^{\mathrm{T}} R_J u\right] dt, \tag{27}$$

and for the derived sequence of polyhedra in Ω, according to (22), we have

$$\dot{V}_p(x) < -x^{\mathrm{T}} F_p^{\mathrm{T}} M_p^{-1} F_p x - x^{\mathrm{T}} Q_J x - x^{\mathrm{T}} K_p^{\mathrm{T}} R_J K_p x, \tag{28}$$

for polyhedron Ω_p. Integrating along the trajectory for Ω_p on both sides of (28), yields

$$\int_{x_{p,s}}^{x_{p,e}} x^{\mathrm{T}} F_p^{\mathrm{T}} M_p^{-1} F_p x \, dx + \int_{x_{p,s}}^{x_{p,e}} \left[x^{\mathrm{T}} Q_J x + x^{\mathrm{T}} K_p^{\mathrm{T}} R_J K_p x\right] dx < \int_{\Omega_p} \dot{V}_p(x) dx, \tag{29}$$

that is

$$C_p + J_p < V_p(x_{p,\mathrm{s}}) - V_p(x_{p,\mathrm{e}}), \tag{30}$$

where $x_{p,\mathrm{s}}$ and $x_{p,\mathrm{e}}$ are the starting and ending states on polyhedron Ω_p. For the sequence of polyhedra in Ω, we have

$$\begin{aligned} C_1 + J_1 &< V_1(x_{1,\mathrm{s}}) - V_1(x_{1,\mathrm{e}}), \\ C_2 + J_2 &< V_1(x_{2,\mathrm{s}}) - V_2(x_{2,\mathrm{e}}), \\ &\vdots \\ C_p + J_p &< V_p(x_{p,\mathrm{s}}) - V_p(x_{p,\mathrm{e}}), \end{aligned} \tag{31}$$

where $x_{1,\mathrm{s}} = x_0$. In addition, according to (19), we have

$$\begin{aligned} V_1(x_{1,\mathrm{e}}) &> V_0(x_{0,\mathrm{s}}), \\ V_0(x_{0,\mathrm{e}}) &> V_3(x_{3,\mathrm{s}}), \\ &\vdots \\ V_{p-1}(x_{p-1,\mathrm{e}}) &> V_p(x_{p,\mathrm{s}}). \end{aligned} \tag{32}$$

If we sum up (31) along the switching sequence for all the polyhedra to the equilibrium, then we have

$$C + J < V_1(x_0) - V_p(x_{p,\mathrm{e}}), \tag{33}$$

where C is the integrating of $x^{\mathrm{T}} F_p^{\mathrm{T}} M_p^{-1} F_p x$ along the state trajectory, which is larger than 0 because the matrices M_p are positive definite. Since $\lim_{t\to\infty} V_p(x_{p,\mathrm{e}}) = 0$, and C is positive, thus $J < V_1(x_0)$, an upper bound of the objective function is $V_1(x_0) = x_0^{\mathrm{T}} P_1 x_0$. Therefore, the min max problem can be solved by minimizing the upper bound of the objective function with the following theorem satisfying the following constraint

$$\begin{bmatrix} \gamma & \star \\ x_0 & Q_1 \end{bmatrix} \geq 0, \tag{34}$$

which guarantees $x_0^{\mathrm{T}} P_1 x_0 \leq \gamma$.

Theorem 2. *For the optimization problem*

$$\min \quad \gamma$$
$$\text{s.t.} \quad (16)-(19) \text{ and } (34), \tag{35}$$

if there exists a solution satisfying all the constraints (16)-(19) and (34), with positive definite matrices $\overline{Q}_{i,l}$, $Q_{i,l}$, $R_{i,l}$, *and* $M_{i,l}$, *then taking the state-feedback control laws with gains as*

$$\overline{K}_{i,l} = \overline{N}_{i,l}\overline{Q}_{i,l}^{-1} \quad \forall\, (i,l) \in \{(i,l)|i \in \Lambda, l \in \Gamma_i, 0 \notin \Omega_{i,l}\}, \tag{36}$$

and

$$K_{i,l} = N_{i,l}Q_{i,l}^{-1} \quad \forall\, (i,l) \in \{(i,l)|i \in \Lambda, l \in \Gamma_i, 0 \in \Omega_{i,l}\}, \tag{37}$$

asymptotically stabilizes the SLS system in (7), and guarantees the minimization of the upper bound of the objective function along the state trajectory with uncertain switchings.

4 | EXAMPLE

In this section, an example is presented to evaluate the performance of the optimal controller designed for a SBLS based on Theorem 2.

In the example, we use the conditions presented in Theorem 2 to design state-feedback control laws optimizing the upper bound of the infinite objective function. We directly use the SBLS model in (4) with the following vectors and matrices:

$$A_1 = \begin{bmatrix} -3 & 1 \\ -5 & -8 \end{bmatrix}, \quad b_{1,1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad c_{1,1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -1 & -3 \\ 2 & -5 \end{bmatrix}, \quad b_{2,1} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad c_{2,1} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$
$$Q_J = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad R_J = 0.1.$$

There are 2 bilinear subsystems separated by $x_1 - x_2 = 0$. According to Sec. 3.1, the state space is partitioned into 4 regions with $\Lambda = \{1, 2\}$ and $\Gamma_1 = \{1, 2\}, \Gamma_2 = \{1, 2\}$. Then, the parameters for the obtained corresponding SLS with the format (7) are:

$$A_1 = \begin{bmatrix} -3 & 1 \\ -5 & -8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$F_{1,1} = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, \quad \overline{F}_{1,2} = \begin{bmatrix} -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix},$$
$$\overline{\Psi}_{11,12} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \quad \overline{\Phi}_{11,12} = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & -3 \\ 2 & -5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$
$$F_{2,1} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \quad \overline{F}_{2,2} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix},$$
$$\overline{\Psi}_{22,12} = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}, \quad \overline{\Phi}_{22,12} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

Using the Yalmip toolbox (with the SeDuMi solver) to solve the optimization problem (ie, the LMIs) (16)-(19) and (34), an decreasing overall Lyapunov function is obtained as given in Figure 1.

As the overall Lyapunov function shows, the Lyapunov function is smooth with each subregion, positive, and decreasing gradually to the origin of the state space. In addition, the Lyapunov function is able to jump and decrease on the boundaries of the state space switchings of the sequence of the switching state polyhedra. As a result, applying the derived Lyapunov function, the controllers are obtained according to (3), (36), and (37), as

$$U_{1,1} = \frac{K_{1,1}x}{x_1+1}, \quad U_{1,2} = \frac{\overline{K}_{1,2}\overline{x}}{x_1+1}, \; U_{1,12} = 0,$$
$$U_{2,1} = \frac{K_{2,1}x}{-x_2+1}, \quad U_{1,2} = \frac{\overline{K}_{2,2}\overline{x}}{-x_2+1}, \; U_{2,12} = 0,$$

where

$$K_{1,1} = [-0.2048 \; 0.0080], \quad \overline{K}_{1,2} = [-0.2980 \; 0.0082 \; 0],$$
$$K_{2,1} = [0.0967 \; -0.1556], \quad \overline{K}_{2,2} = [0.0967 \; -0.1556 \; 0].$$

The simulation shows that the designed controllers are able to steer state to the origin for different initial conditions, as in Figures 2 and 3.

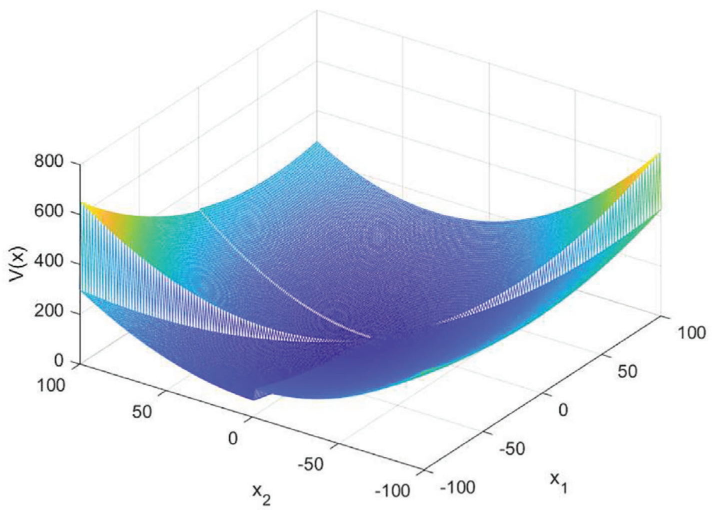

FIGURE 1 Illustration for the overall Lyapunov function [Colour figure can be viewed at wileyonlinelibrary.com]

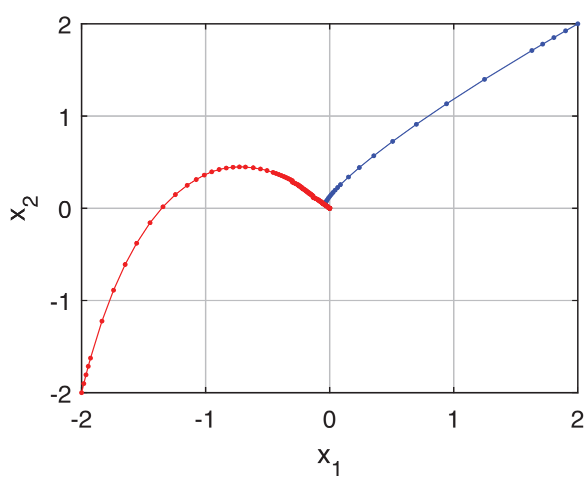

FIGURE 2 The closed-loop trajectories with initial states $[2\ 2]^T$ and $[-2\ -2]^T$ [Colour figure can be viewed at wileyonlinelibrary.com]

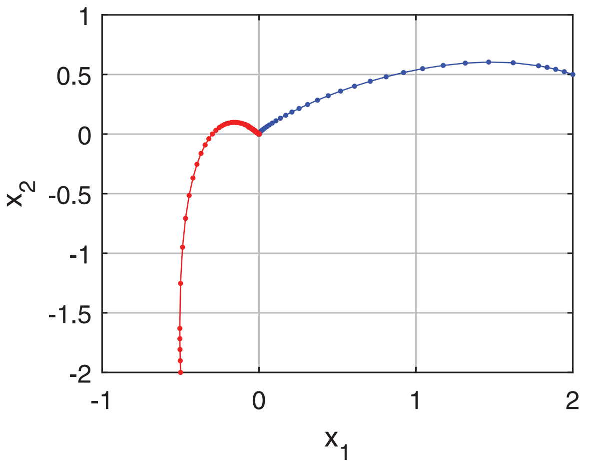

FIGURE 3 The closed-loop trajectories with initial states $[2\ 0.5]^T$ and $[-0.5\ -2]^T$ [Colour figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSIONS

In practice, there are some complex nonlinear systems that can be approximated by SBLSs. Designing stabilizing controller for SBLSs makes it possible to better control these kind of nonlinear systems. To deal with the state-based SBLSs, the subregions where subsystems are activated are further divided into some regions utilizing the special features of bilinear systems. And then, region-dependent controllers are designed in resulting subregions, which transform the bilinear systems into linear ones. Based on the linear property of the system, a state-feedback controller design method is proposed considering the infinite horizon quadratic cost function to minimize the total cost along the state trajectory. By solving the series of derived LMIs, optimized switching state-feedback control laws will be obtained for the SBLS, to asymptotically steer any state in the feasible region to the equilibrium, and can guarantee the minimization of the upper bound of the objective function along the system state trajectory at the same time. The numerical result shows that the designed controller is able to stabilize the system. In the future, we will apply the proposed method in traffic flow control, and try to use it solving real traffic problems.

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SUPPORTING INFORMATION

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