An evaluation of a fatigue crack growth prediction model for variable-amplitude loading (PREFFAS)

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ABSTRACT

Aliaga, Davy and Schaff have proposed a prediction model (PREFFAS) for fatigue crack growth under stationary variable-amplitude load histories with a short recurrence period, e.g. flight-simulation loading. This cycle-by-cycle prediction model is based on crack closure. It is described and analysed in the present paper. Consequences, limitations and possible extensions are discussed. The model is applied to a series of simplified flight-simulation tests.

CONTENTS

1. INTRODUCTION

The prediction of fatigue crack growth under variable-amplitude loading is still a difficult problem. The prediction problem is highly relevant to aircraft structures in view of the so-called damage tolerance requirements. Several prediction models have been proposed in the literature. Recently the PREFFAS model was proposed by Aliaga, Davy and Schaff [1-3] (PREFFAS ≙ PREvision de la Figuration en Fatigue AéroSpatiale). The model is proposed for flight simulation load histories, where a load spectrum is repeated periodically.

The PREFFAS model is essentially based on the Elber crack closure concept. The crack opening stress level and the crack growth increments have to be calculated cycle-by-cycle. The opening level depends on the previous load history. From a computational point of view the model is rather simple. The authors of the model have claimed good predictions for a variety of flight-simulation test results. It was thought to be worthwhile to analyse the characteristic features of the PREFFAS model and to explore computational aspects. The result is presented in this report. Aspects discussed are:
- the cycle-by-cyle variation of K_{op} under variable-amplitude loading;
- rain-flow effect;
- calculation procedures;
- material and thickness effects;
- negative loads.

The model is applied to a series of highly simplified flight-simulation tests [4] and a preliminary evaluation of PREFFAS prediction model is made.

2. K_{op} IN THE PREFFAS MODEL

In the PREFFAS model it is assumed that crack growth occurs cycle-by-cycle. Each cycle, with cycle number (i), is defined by a minimum and a maximum stress intensity factor: $K_{min,i}$ and $K_{max,i}$. The crack growth increment δa_i directly depends on the crack opening stress level in that cycle. The Elber concept is adopted (fig. 1):

$$\delta a_i = \alpha.(K_{max,i} - K_{op,i})^\beta = \alpha.\Delta K_{eff,i}^\beta \tag{1}$$

An essential part of the PREFFAS model is the question how $K_{op,i}$ depends on the previous K-history. To determine $K_{op,i}$ in cycle (i) all previous cycles have to be considered to find the maximum retardation effect. For each of those cycles (cyle number (j), j < i, see fig. 1) its maximum K-value ($K_{max,j}$) has to be combined with the lowest K_{min}-value occurring between $K_{max,j}$ and $K_{max,i}$. That K_{min}-value occurring in cycle (k) ($K_{min,k}$, j < k $\leq$ i, fig. 1) will determine the reduction of the retardation induced by $K_{max,j}$. The two values, $K_{max,j}$ and $K_{min,k}$, are adopted to calculate the K-opening value ($K_{op,i,j}$) in cycle (i), as it is affected by cycle (j). The calculation again follows the Elber conception:

$$K_{max,j} - K_{op,i,j} = U.(K_{max,j} - K_{min,k}) \tag{2}$$

where the crack closure factor U is supposed to be linearly dependent on the stress ratio R.

$$U = A + B.R \tag{3}$$

A and B are material constants, and here

$$R = K_{min,k}/K_{max,j} \tag{4}$$

Rewriting of eq. (2) gives:

$$K_{op,i,j} = K_{max,j} - U.(K_{max,j} - K_{min,k}) \tag{5}$$

Different cycles (j) will lead to different $K_{op,i,j}$-values. The maximum of these values will give the maximum retardation effect, and, according to the PREFFAS

model, that maximum is chosen to be the applicable K_{op} in cycle (i). Thus, the opening level in cycle (i) is defined by:

$$K_{op,i} = \text{maximum of } K_{op,i,j} \qquad \text{for } j = 1 \text{ to } j = i - 1 \qquad (6)$$

This equation is a key feature of the PREFFAS model. Another essential assumption of PREFFAS should help to understand the physical meaning of $K_{op,i}$ in eq. (6). It is assumed that the load spectrum has a relatively short 'recurrence' period (in terms of ref. 1: short spectra), after which it is repeated again and again. The assumption is used to justify the following step. Any effect of crack extension in cycles between cycle (j) and cycle (i) on $K_{op,i}$ is ignored. In other words, the effect of cycle (j) on $S_{op,i}$ is not affected by intermediate crack growths. Obviously this should not be correct, if the recurrence period is long. In such a case the retardation effect (i.e. the K_{op}-increasing effect) of $K_{max,j}$ could have vanished if the crack growth increment between cycle (j) and cycle (i) has exceeded the plastic zone size associated with $K_{max,j}$.

It is noteworthy that equations for the plastic zone size (r_p) do not occur in the PREFFAS model. Crack tip plasticity is accounted for by effects of $K_{max,j}$ and $K_{min,k}$ on $K_{op,i}$ (fig. 1). Because $K_{max,j}$ is the largest K-value occurring before cycle (i), it is supposed here that cycle (j) introduces the largest plastic zone, which is still the dominant monotonic plastic zone for cycle (i). However, its retardation can be reduced by reversed plasticity. The maximum reduction is caused by the lowest K_{min} between cycle (j) and cycle (i), which is $K_{min,k}$ (fig. 1). Those two K-values may be expected to determine K_{op} in cycle (i), and thus justify the basic assumptions for eq. (6). Obviously, this can only be correct if intermediate crack growths increments are small as compared to the plastic zone sizes. Hence it requires variable-amplitude load spectra with a short recurrence period.

According to the definition of $K_{op,i}$ in eq. (6) all previous cycles have to be considered. However, in reality the number of cycles, which will affect $K_{op,i}$, is not large. First, it should be understood that a K_{max}-value, which is larger than all previous K_{max}-values, erases all history effects of those previous cycles. In figure 2 $K_{max,j2}$ is larger than $K_{max,j1}$, and $K_{min,k2}$ is higher than $K_{min,k1}$. The calculation of $K_{op,i,j}$ requires that $K_{max,j1}$ is combined with $K_{min,k1}$, and $K_{max,j2}$ is coupled to $K_{min,k2}$. The last combination will obviously lead to a higher $K_{op,i,j}$ because both the maximum and the minimum K are higher.

Of course a $K_{min,k2}$ lower than $K_{min,k1}$ may also occur. However, then both $K_{max,j1}$ and $K_{max,j2}$ have to be combined with that lower K_{min}-value. Again $K_{max,j1}$ will be a non-relevant K_{max} for $K_{op,i}$. It implies that the largest K_{max}-value of the K-history erases the effects of all previous cycles. Only lower K_{max}-values coming later should be considered.

If $K_{max,j2}$ is lower than $K_{max,j1}$ it does not eliminate the possible significance of that previous value. However, for the same arguments used above, all K_{max}-values occurring between cycle (j1) and cycle (j2), which are lower than $K_{max,j2}$ need not be considered any longer. In other words, the relevant K_{max}-values for calculating $K_{op,i}$ should be a series of decreasing values (fig. 3).

By definition the calculation of $K_{op,i,j}$ requires the selection of the lowest K_{min} between cycles (j) and (i). It is easy to understand that the lowest K_{min} of the K-history makes it unnecessary to consider K_{min}-values of previous cycles. Consequently, the relevant K_{min} values form a series of increasing values. As a result the K-values to be considered for calculating K_{op} in cycle (i) will be an alternating series of K-values, with decreasing K_{max}-values and increasing K_{min} values. Such K-values were labelled by the authors of PREFFAS as history values KH_{max} and KH_{min}, see the illustration in figure 3.

The retardation effect of a KH_{max}-value on a later $K_{op,i}$ is dependent on the reduction of that effect by the subsequent KH_{min}-value. It implies that a KH_{max} is always related with a later KH_{min}. As a result KH_{max} and KH_{min}-values do occur in pairs. Each pair determines a 'historical' K_{op}-level (KH_{op}), which should be calculated according to eqs. (3) to (5). If the history levels have the rank number p the result is:

$$KH_{op,p} = KH_{max,p} - U \cdot (KH_{max,p} - KH_{min,p}) \qquad (7)$$

Each pair $KH_{max,p}$, $KH_{min,p}$ will cause its own opening level $KH_{op,p}$. If a new pair of a potential KH_{max} and KH_{min} value does not increase $K_{op,i}$, their effect will be overruled by a previous pair. It then will not become a new pair of history levels. In figure 3 new KH-levels might be expected between $KH_{min,3}$ and the current cycle (i), because the peak values fit into a decreasing series of K_{max} and an increasing series of K_{min}. However, there is another requirement, which is an increased K_{op}, and that has to be checked for each cycle.

The conditions for the history levels can now be summarized as:

$$KH_{max,p} < KH_{max,p-1} \qquad (8a)$$

$$KH_{min,p} > KH_{min,p-1} \qquad (8b)$$

$$KH_{op,p} > KH_{op,p-1} \qquad (8c)$$

In view of these three requirements and the 'relatively' short load spectrum period the number of history levels is significantly restrained. According to Aliaga et. al [1] the rank number of the KH-levels to be recalled for the calculation of $K_{op,i}$ will not exceed 10 in practical cases of crack growth predictions under flight-simulation loading.

3. THE RAIN-FLOW EFFECT

The authors of PREFFAS realized that small intermediate load variations can lead to unconservative predictions. A simple case is shown in figure 4. The history levels $KH_{max,r-1}$ and $KH_{min,r-1}$ determine the opening level $K_{op,r-1}$. For the small intermediate load cycle between the levels $KH_{max,r}$ and $KH_{min,r}$, the opening level $KH_{op,r}$ is larger than $KH_{op,r-1}$. Consequently, these levels are history levels indeed. According to the model in the previous section the sum of the two crack growth increments is:

$$\delta a = \alpha \, (\Delta K_{eff,r-1})^{\beta} + \alpha \, (\Delta K_{eff,r})^{\beta} \qquad (9)$$

If the small intermediate load variation does not occur, only one ΔK_{eff} has to be considered, which is $\Delta K_{eff,rf}$ in figure 4. The crack extension then is:

$$\delta a = \alpha \, (\Delta K_{eff,rf})^{\beta} \qquad (10)$$

Because of the exponential ΔK effect (β in the order of 3) the last δa will be considerably larger than δa in eq. (9). PREFFAS might give an unconservative prediction. Therefore the authors of the model have extended PREFFAS to account for this aspect by the well known rain-flow approach. Figure 5 illustrates how the two original ΔK_{eff} variations (black bars) are replaced by two other effective variations (open bars). The rain-flow approach in fig. 5 will count

one large load variation AE and one small load cycle BCD. Similarly to this approach the PREFFAS model recognizes one large ΔK_{eff} related to the large load variation and a small damage contribution from the small intermediate cycle. It only occurs if K_{max} exceeds a previous KH_{max} level. Figure 6 illustrates the rain-flow procedure if K_{max} exceeds two previous KH_{max}-levels. As illustrated by figures 5 and 6, if K_{max} exceeds a previous KH_{max}-value, the exceeding part is supposed to be an extension of ΔK_{eff} associated with that previous KH_{max}. It extends the damage done in that previous cycle.

If only one previous KH_{max} is exceeded (fig. 5) the PREFFAS model including the rain-flow effect calculates the crack extension in that cycle to be:

$$\delta a_i = \alpha \, (K_{max,i} - K_{op,i-1})^\beta + \alpha \, (K_{max,i-1} - K_{op,i})^\beta$$

$$- \alpha \, (K_{max,i-1} - K_{op,i-1})^\beta \tag{11}$$

The first term accounts for the rain-flow effect, the second one is the contribution of the small intermediate cycle and the last minus term must be added because otherwise that part of the damage would be added twice. If more previous KH_{max}-values are exceeded (fig. 6), say from $j = p + 1$ until $j = \ell$ (the last one), eq. (11) must be generalized to:

$$\delta a_i = \alpha \, (K_{max,i} - K_{op,p})^\beta + \alpha \sum_{j=p+1}^{\ell} (KH_{max,j} - KH_{op,j})^\beta$$

$$- \alpha \sum_{j=p+1}^{\ell} (KH_{max,j} - KH_{op,j-1})^\beta \tag{12}$$

It should be noted that not all cycles counted by the rain-flow count method will be similarly treated by the PREFFAS model. As an example, the three smaller cycles in figure 7 will all give K_{op}-values lower than $KH_{op,p}$. Only the middle one will give a damage contribution. However, in none of these cycles the last $KH_{max,p}$ is exceeded, and according to the PREFFAS model there is no rain-flow effect, although all these cycles would be counted by the rain-flow counting method. There are more differences. The rain-flow counting method has been justified in the literature by considering closed hysteresis loops during cyclic plastic deformation and the material memory for previous loops. Also these arguments cannot be reconciled with the rain-flow effect of the PREFFAS model. The introduction of the latter effect is based on a 'reasonable' and more

conservative handling of small intermediate load variations when large variations do occur.

4. CALCULATION OF THE PREDICTED CRACK GROWTH

The crack growth prediction according to the PREFFAS model is a relatively simple procedure based on the variation of KH_{op}. It is simple because the crack length (a) for the calculation of the K-history is assumed to have the same constant value during a period of the load spectrum. As a consequence, during one period, the K-values and the S-values (S = stress) are linearly proportional. Hence, all previous considerations on KH-values can be simply translated to SH-values (history stress levels to be considered for the calculation of the crack opening stress level S_{op}). This allows an easy calculation of the average crack growth rate during one period and subsequent periods as shown below.

If the crack growth increment in one period is indicated by Δa, this increment can be written as:

$$\Delta a = \sum_{i=1}^{n} \delta a_i = \alpha \sum_{i=1}^{n} \Delta K_{eff,i}^{\beta} \tag{13}$$

where n is the number of load cycles in one load spectrum period.

$$\Delta K_{eff,i} = f(a) \cdot \Delta S_{eff,i} \sqrt{\pi a} \tag{14}$$

where f(a) is the wellknown geometry factor.
Because it was assumed that the crack length 'a' has a constant value during one period, eq. (13) can be written as:

$$\Delta a = \alpha \left[f(a) \sqrt{\pi a} \right]^{\beta} \sum_{i=1}^{n} \Delta S_{eff,i}^{\beta} \tag{15}$$

In ref. (3) Aliaga has defined the sequence efficiency (symbol EF) as:

$$EF = \sum_{i=1}^{n} \Delta S_{eff,i}^{\beta} \tag{16}$$

and thus:

$$\Delta a = \alpha \left[f(a) \sqrt{\pi a} \right]^{\beta} . EF \qquad (17)$$

The average crack growth rate per cycle (n cycles in one period) is:

$$\left(\frac{da}{dN}\right)_{av} = \frac{\Delta a}{n} = \alpha \left[f(a) \sqrt{\pi a} \right]^{\beta} . \frac{EF}{n} \qquad (18)$$

The sequence efficiency EF has to be calculated only once for one load spectrum period.

(Note: during the first period the SH_{op} development cannot yet be affected by load cycles of a previous period. As a consequence, there are transient initializing effects. The stationary EF value is constant for the second and subsequent period of the load spectrum. That EF-value should be used in crack growth predictions.)

Equation (18) explicitly gives the average crack growth rate as a function of the crack length because of the term $f(a) \sqrt{\pi a}$. The effects of crack length and geometry are fully separated from the effect of the stress history. Such a separation of variables was previously adopted by Gunther and Goranson [5]. The effect of the stress history is characterized by EF. The separation of variables is possible in the PREFFAS model because the variations of KH-levels (and SH-levels) are assumed to be independent of intermediate crack growth during one period of the load spectrum. It implies that the crack length is supposed to remain stationary for some time, while δa_i values are calculated, but not yet added to the crack length a. This emphasizes that $(da/dN)_{av}$ in eq. (18) is an average crack growth rate associated with the crack length to be substituted in $f(a) \sqrt{\pi a}$.

The computer program to calculate EF is described in some more details in the appendix. The calculation of EF requires a cycle-by-cycle calculation of $\Delta S_{eff,i}$ based on history levels SH_{max}, SH_{min} and SH_{op}. These levels have to satisfy a similar set of conditions as given in eq. (8) for the KH-levels.

$$SH_{max,p} < SH_{max,p-1} \qquad (19a)$$

$$SH_{min,p} > SH_{min,p-1} \qquad (19b)$$

$$SH_{op,p} > SH_{op,p-1} \qquad (19c)$$

where p is again the rank number of the SH-levels.

Equation (18) allows another interesting evaluation. For this purpose all stress levels of a load history (or a load spectrum) are compared to a characteristic reference stress of the load spectrum: S_{char}. Each stress level S_i can be written as:

$$S_i = q_i \cdot S_{char} \qquad (20)$$

with $\qquad q_i = S_i/S_{char} \qquad (21)$

If all stress levels are proportionally increased (or decreased), the characteristic stress level will vary in the same proportion. As a consequence the q_i values will remain constant. The shape of the load spectrum does not change. Only the scale of the stress levels is changed. Similarly to eq. (20) $\Delta S_{eff,i}$ can be written as:

$$\Delta S_{eff,i} = \frac{\Delta S_{eff,i}}{S_{char}} \cdot S_{char} = q_{eff,i} \cdot S_{char} \qquad (22)$$

It is easy to understand that rescaling of the stress level will affect S_{char}, S_i and $\Delta S_{eff,i}$ in the same proportion. As a consequence, a variation of S_{char}, which does not affect q_i in eq. (21), will also not affect $q_{eff,i}$ in eq. (22). It implies that EF in eq. (16) can be written as:

$$EF = \sum_{i=1}^{n} \left[q_{eff,i} \cdot S_{char} \right]^{\beta} = S_{char}^{\beta} \cdot \sum_{i=1}^{n} q_{eff,i}^{\beta} \qquad (23)$$

Since $q_{eff,i}$ is independent of S_{char} the $\Sigma q_{eff,i}$ will be constant also. It depends on the shape of the load history, but not on the stress scale.

$$EF = Q \cdot S_{char}^{\beta} \qquad \text{with} \qquad Q = \Sigma q_{eff,i}^{\beta} \qquad (24)$$

Substitution in eq. (18) gives:

$$(da/dN)_{av} = (\alpha/n) \, Q \left[f(a) \, S_{char} \, \sqrt{\pi a} \right]^{\beta} = (\alpha/n) \, Q \, K_{char}^{\beta} \qquad (25)$$

with $\qquad K_{char} = f(a) \, S_{char} \, \sqrt{\pi a} \qquad (26)$

For one particular shape of the load spectrum (including one particular load sequence) some interesting conclusions can now be based on eq. (26):

1. The relation between da/dN and K_{char} if plotted on a double log scale should be linear with the same slope β applicable to the constant-amplitude data (Paris relation).

2. Tests with different characteristic stress levels (but the same load spectrum shape and load history) should comply with the same relation between da/dN and K_{char} (eq. (26)). This allows a simple extrapolation to other nominal stress levels.

3. A third conclusion applies to the crack growth life between a specified initial crack length and a specified final crack length. Because the average growth rate at any crack length is inversely proportional to S_{char}^{β}, the crack growth life will also be inversely proportional to S_{char}^{β}, or:

$$N.S_{char}^{\beta} = \text{constant} \tag{27}$$

Such an S-N relation (Basquin relation) is log-linear with a slope $-1/\beta$.

5. MATERIAL AND THICKNESS EFFECTS

In the prediction equation (18) there are two material constants, α and β, which are the two constants of the Paris relation.

$$da/dN = \alpha.\Delta K_{eff}^{\beta} \tag{1}$$

However, there are two more material-dependent constants in the crack closure function:

$$U = A + B.R \tag{3}$$

According to Aliaga et al [1,2] it is generally found that

$$A + B = 1 \tag{28}$$

leads to good results for aluminium alloys and steels used in aircraft structures. Another relation between A and B is obtained experimentally from constant-amplitude tests (R = 0.1) with periodic overloads every 1000 cycles,

while $K_{overload}/K_{max}$ = 1.7 (fig. 8). The overload factor 1.7 was selected because it was supposed to be 'representative' for civil aircraft wing spectra. Results from Aliaga et al are shown in figure 8. In agreement with the prediction by the model (see previous discussion) two parallel scatterbands were obtained, i.e. the same β is applicable to the constant-amplitude (CA) test results and the tests with periodic overloads (OL). As a consequence, the retardation ratio, defined as:

$$\text{retardation ratio} = \frac{(da/dN)_{CA}}{(da/dN)_{CA+OL}} \quad \text{(at same } \Delta K_{CA})$$

must have a constant value, which can easily be predicted by the model, because in the tests with periodic overloads (SH_{op}) is constant and equal to S_{op} of the OL-cycles. The prediction result is (with A = 1 - B, eq. 28):

$$\text{retardation ratio} = \frac{1001 \left\{ (1 - 0.9\ B)\ 0.9 \right\}^{\beta}}{\left\{ (1 - \frac{16}{17}\ B)\ 1.6 \right\}^{\beta} + 1000 \left\{ (1 - \frac{16}{17}\ B)\ 1.6 - 0.7 \right\}^{\beta}}$$

$$(29)$$

From the experimental ratio and the β value, the value of B can now be obtained from eq. (29). In figure 8 the ratio = 10 and β = 3.2 which leads to B = 0.42 and A = 0.58. Estimated B-values based on test results presented in [2] for different thicknesses are:

Material	B-values (A + B = 1)		
	1 mm --------→ 8 mm --------→ 15 mm		
2024-T3 (and T351)	~0.45	~0.4	~0.35
7075/7010/7050 (T6... and T7...)	~0.35	~0.3	
steel (S_u = 1000 MPa)		~0.25	

The results illustrate that A and B depend on the material and its thickness. For β = 3 the retardation ratio according to eq. (29) is shown in figure 9. A larger B-value implies more crack closure and more retardation. Larger B-values

apply to more ductile materials and lower sheet thicknesses. It corresponds to larger plastic zones at the crack tip.

6. APPLICATION OF PREFFAS TO SIMPLIFIED FLIGHT-SIMULATION TESTS

Results of highly simplified flight-simulation tests on 2024-T3 sheet specimens (t = 2 mm) are presented in [4]. The load sequences adopted I to III are shown in figure 10. All flights in one test are equal. The tests can also be labelled as constant-amplitude tests with periodic overloads and underloads. Crack opening stress levels according to the PREFFAS model (SH_{op}) are shown in figure 11. For comparison the opening stress level in a pure constant-amplitude test (S_{op}) is also shown. The latter values were used for non-interaction (Miner type) predictions. Whenever $SH_{op} > S_{op}$ retardation does occur, whereas accelerations apply if $SH_{op} < S_{op}$.

Crack growth life (N) predictions were obtained by integration of eq. (18) from a = 4 mm to a = 30 mm (W = 100 mm). The integration leads to:

$$N = \frac{\text{Integral}}{\pi \frac{\alpha}{n} W^{\beta/2-1} EF} \qquad (30)$$

with:

$$\text{Integral} = \int \left(\frac{\cos \frac{\pi a}{W}}{\frac{\pi a}{W}} \right)^{-\beta/2} d \frac{\pi a}{w} \qquad (31)$$

and EF according to eq. (16). For constant-amplitude loading, n = 1 and EF = ΔS_{eff}^{β}. Eq. (30) with eq. (3) reduces to:

$$N = \frac{\text{Integral}}{\pi \alpha W^{\beta/2-1} \left[(A + BR) \Delta S \right]^{\beta}} \qquad (32)$$

The material constants are α and β of the Paris relations, and A and B of the crack closure relation (eq. 3). Values for A, B and β were borrowed from the results of Aliaga et al [2] for 2024-T3, which implies U = 0.58 + 0.42 R and β = 3.2. The other constant α was fitted to the constant-amplitude data of the material for which the PREFFAS predictions have to be made [4]. The result is $\alpha = 1.56 * 10^{-10}$ (da/dN in m/c and ΔK in MPa $\sqrt{m}$. In [1] $\alpha = 2.1 * 10^{-10}$ was found).

The PREFFAS predictions are compared to the test results in table 1 and figure 10. Results for tests with tensile stresses were considered only, because the application of PREFFAS is limited to this condition. Six constant-amplitude tests, calculated with eq. (32) are also presented in figure 10. For these results the agreement between prediction and test is very good, as might be expected in view of the data fitting procedure to obtain α. For 10 different flight-simulation tests, the predictions do not always agree with the test results (each test result is the average result of two tests). Quantitatively, the predictions do not deviate more than a factor of 2 from the test results, and in most cases it is significantly better. Qualitatively some more comments can be made, also in comparison to the non-interaction prediction presented in table 1.

Large retardations apparently occurred in the tests of type II and type III with overload cycles and 100 cycles per flight, if compared to the tests of type I (no overload cycles). PREFFAS does indeed predict a large retardation, although the quantitative agreement is not always as good as desirable. The non-interaction predictions do not indicate any retardation, on the contrary a small reduction on life (as should be expected for occasional large cycles and a non-interaction concept).

A remarkable result of reference [4] was the small and non-systematic difference between results for tests of type II and type III. In this case PREFFAS predicts a systematically higher crack growth life for type II. For 100 cycles per flight it is almost twice as high, which is not borne out by the test results. The predicted difference is due to a higher crack opening stress level and thus a lower ΔK_{eff} for tests of type II, see the comparison of ΔS_{eff} in figure 11. The non-interaction prediction indicate negligible differences between II and III, but as said before the quantitative non-interaction predictions are poor anyhow for tests with large retardations.

7. DISCUSSION

The PREFFAS model was introduced by Aliaga et al as an intermediate approach between a rather simple model of the present author [6] and more complex models, which can also handle non-stationary load histories. In my own simple model it was proposed that a constant S_{op} could apply to a stationary flight-simulation

load history. The S_{op}-level should be a function of the maximum stress and the minimum stress of the load spectrum, and later [7] it was proposed to adopt the minimum stress of the ground-to-air cycle as that minimum stress level. It is correct to consider the PREFFAS model as a kind of an intermediate approach, because it includes variations of S_{op} as a function of the load history. Moreover, some rain-flow type influences are also accounted for, and finally the computational effort is very limited by the separation of the effects of crack lengths (a) and geometry on one hand, and the effect of the load history on the other hand.

There are a few apparent limitations in the PREFFAS model.
(1) Negative loads are ignored, which implies that the compressive part of cycles is truncated at S = 0. This should be done because the Elber type equation U = A + BR cannot be reconciled with compressive stresses as shown elsewhere [8]. However, if another U(R) relation is adopted it should not be difficult to extend the PREFFAS model and include negative loads as well. Actually the ground-to-air cycle in the compressive range has a systematic effect on crack growth (reviewed in [9]).
(2) Another limitation is the assumed crack growth relation: $da/dN = \alpha \ \Delta K_{eff}^{\beta}$ (Paris relation). As a result of this assumption, the separation of variables (crack length and load history) was possible. However, any other non-linear relation could also be adopted as an extension of the PRESSAS model. The model is then essentially reduced to the assumptions made to calculate $K_{op,i}$ (eq. (6)) and the procedure to account for some rain-flow effect. For other crack growth relations separation of the variables will become impossible (unless the Paris relation is replaced by a higher order polynomial equation). More computational effort will be required, but that can not really be considered to be a serious disadvantage if an improved prediction reliability can be obtained.
(3) As pointed out by Aliaga et al the model cannot be applicable when crack growth rates are high due to ductile tearing at high K_{max} values. Crack growth rates should not be too high anyway, because as indicated by Aliaga [3], Δa in one load spectrum period must be significantly smaller than the plastic zone size associated with the maximum load of the spectrum. Otherwise, the ignorance of crack growth to calculate KH_{op} values can no longer be acceptable.
(4) Another question is, whether the model can still be applicable to very small cracks (threshold regime), where a unique correlation between ΔK_{eff} and da/dN may become questionable for several reasons.

(5) In various prediction models plastic zone size calculations are an essential part of the model. In the CORPUS model [10] and the ONERA model [11] the transition from plane strain to plane stress is also incorporated. These aspects are not explicitly addressed the the PREFFAS model. However, plastic zone size considerations are essential to justify the history effects included into the model. It may be stated that the determination of the A and B value by a test with periodic overloads implies a kind of a calibration for both plasticity and the transition from the state of stress at the crack tip.

Two types of criteria for prediction models were indicated in [12]:
(i) Empirical trends in variable-amplitude tests must be quantitatively predicted with a sufficient quantitative accuracy.
(ii) A prediction model involves a mechanistic conception. It should be a physically sound conception, in agreement with visual and microscopic observations on fatigue crack growth, including striation measurements.
Several important empirical trends of variable-amplitude tests are summarized in [12]. It is not the purpose of this paper to analyse the prediction potential of the PREFFAS model for all these trends. However, an exception is made for the so-called delayed retardation phenomenon, see figure 12. The PREFFAS model does not predict delayed retardation, and the same is true for the CORPUS model [10] and the ONERA model [11]. An interesting aspect now is that delayed retardation most probably occurred in the constant-amplitude tests with periodic overloads, used for the 'calibration' of the A and B value. With this observation in mind it is a valid question whether another type of a variable-amplitude testing might be adopted to assess unknown constants of the prediction model. An obvious choice would be a judiciously selected flight-simulation test. Experimentally, such a test can hardly be considered to be more complex than any other type of variable-amplitude tests. The proposal to use a flight-simulation test as a calibration test, implies that the PREFFAS model then is used to extrapolate from that test result to other flight-simulation conditions. According to Aliaga et al [1,2] the PREFFAS model predicts load spectrum variations with a reasonable accuracy. Further checks and analysis should be recommended. It is interesting to note that Habibie's prediction model [13] also started from available results of one flight-simulation test.

Another empirical trend mentioned in [12] is the occurrence of a transient initial crack growth retardation occurring during several periods of the load spectrum. The observation was especially made when fatigue cracks started from

artificial notches, such as a saw cut. The three models mentioned cannot predict this trend, but it was argued in [12] that this need not be necessary for a practically useful model. However, a model then should not be checked with test results which are still affected with the transient behaviour. Such a transient behaviour can only be revealed when crack growth rates are considered, rather than crack growth lives.

8. CONCLUSIONS

The basic assumptions and consequences of the PREFFAS fatigue crack growth prediction model are discussed in the present paper. The model was developed by Aliaga, Davy and Schaff for stationary variable-amplitude load histories with a short recurrence period. The model is essentially based on crack closure to account for load interaction effects. The following comments and conclusions can be given.

1. The PREFFAS model predicts a cycle-by-cycle variation of the crack opening stress level in a relatively simple way. The computational efforts involved in predictions for flight-simulation tests are very limited, which is a result of separating in the calculations the variables of the load history on one side and the crack length and geometry effects on the other side. The model accounts for the rain-flow effect of small load variations, which interrupt large load variations.

2. The applicability of K_{char} to stationary flight-simulation tests at different stress level (same shape of load spectrum and load history) is a natural consequence of the PREFFAS model.

3. Retardation trends observed in simplified flight-simulation tests, were predicted by the PREFFAS model, whereas the non-interaction approach predicts small accelerations.

4. The PREFFAS model ignores compressive loads, and it is restricted to a log-linear crack growth behaviour. It is suggested that the model can easily be extended beyond those limitations.

5. The model does not include plastic zone size calculations. However, material thickness and ductily do affect a fitting factor to be obtained from a constant-amplitude test with periodic overloads on the relevant material.

6. The constant-amplitude test with periodic overloads can be considered as a kind of a calibration test to adjust the single free constant of the model. It

is suggested that a flight-simulation test may be more appropriate for this purpose.

7. The model and possible extensions, should be further verified against experimentally obtained crack growth rates, rather than by comparison crack growth lives.

9. REFERENCES

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type of loading (fig.10)	S_{OL} (MPa)	cycles per flight	life in kilocycles				
			test result	predictions		$\dfrac{prediction}{test\ result}$	
				PREFFAS	non-int.	PREFFAS	non-int.
I	-	5 100	47.9 60.6	58.4 67.5	58.4 67.5	1.22 1.11	1.22 1.11
II	160	5 100	45.1 194.8	56.0 165.4	39.6 65.7	1.24 0.85	0.88 0.34
	200	5 100	21.3 270.1	35.0 391.7	25.4 62.8	1.64 1.45	1.19 0.23
III	160	5 100	44.2 175.6	44.6 85.6	42.8 66.2	1.01 0.49	0.97 0.38
	200	5 100	21.1 321.3	33.1 216.9	28.6 63.7	1.57 0.68	1.36 0.20

Table 1: Test results and predictions for the simplified flight-simulation tests
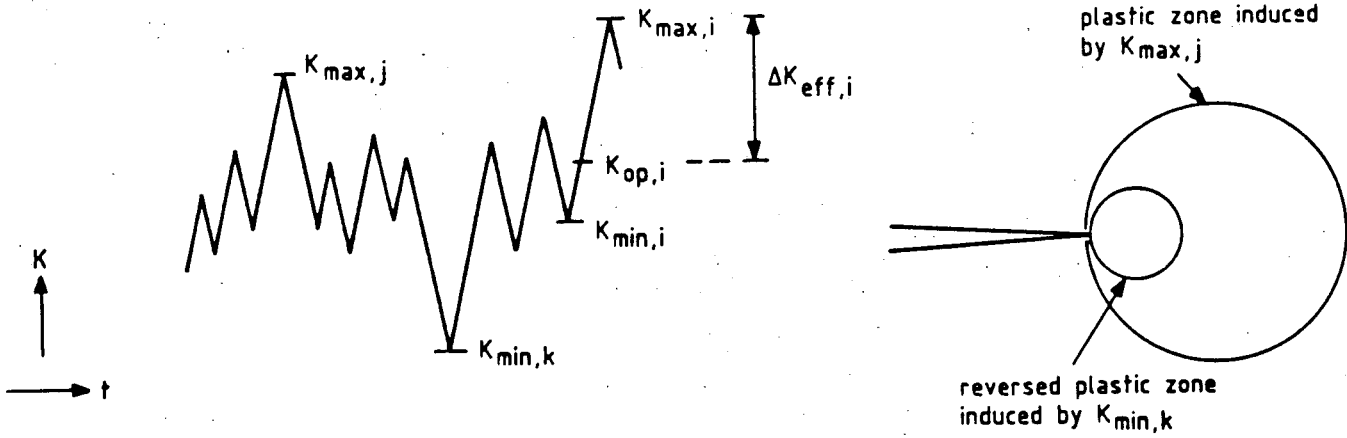
Figure 1: K-history.
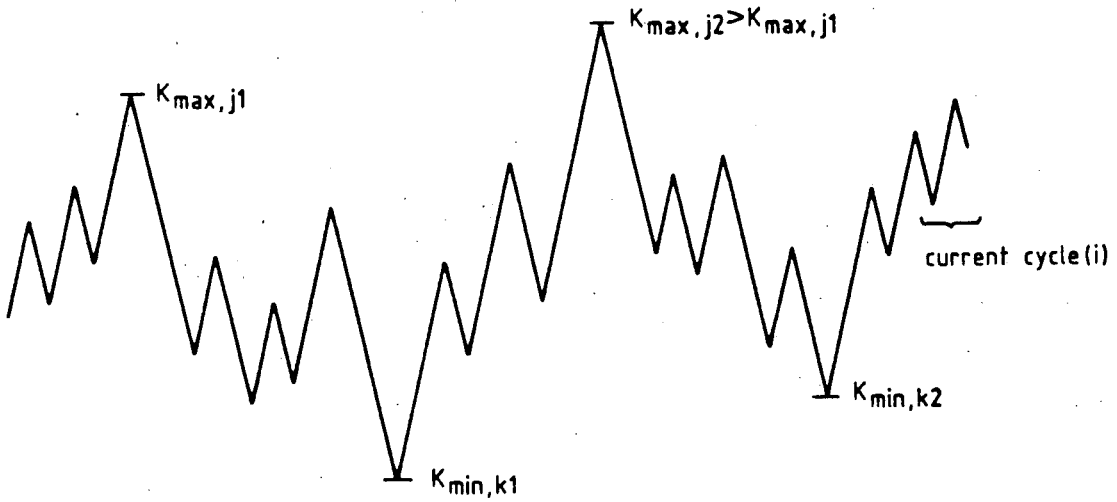
Figure 2: A K_{max} larger than maximum values of previous cycles erases all history effects of those previous cycles.
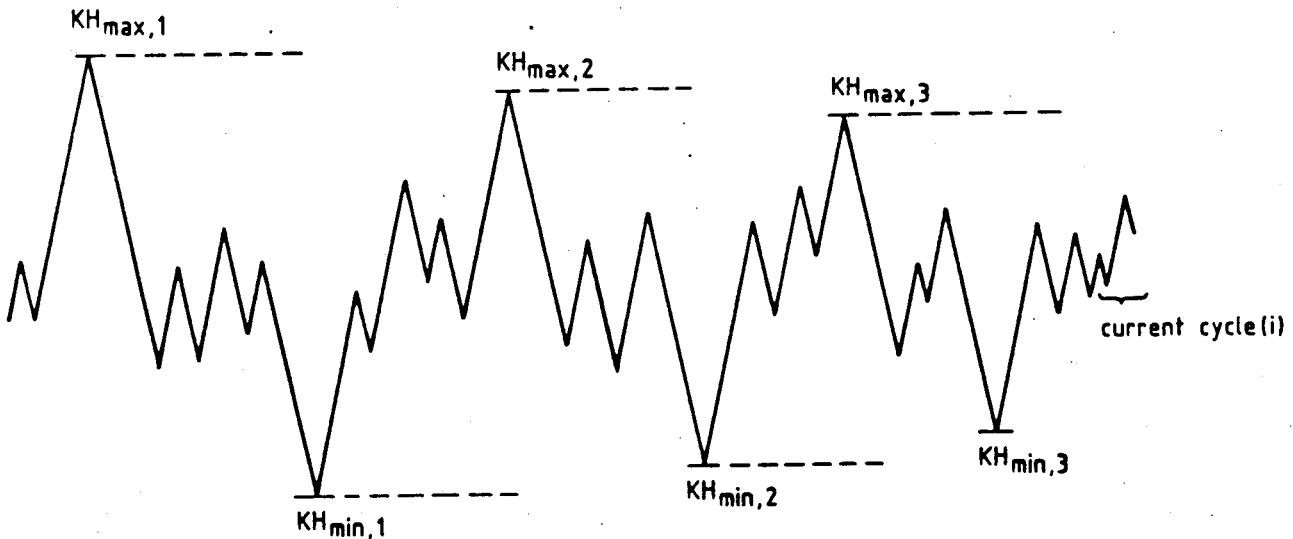
Figure 3: Decreasing KH_{max}-values, increasing KH_{min}-values.
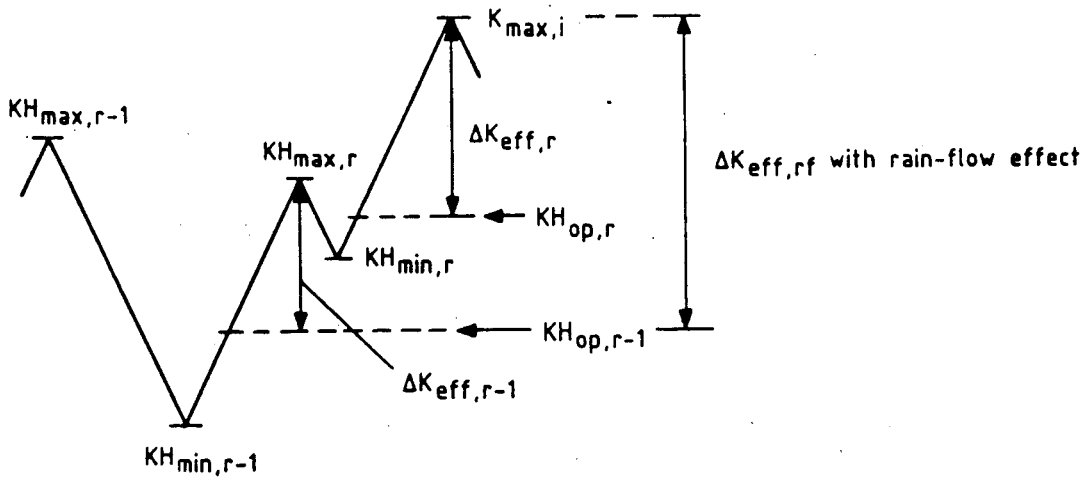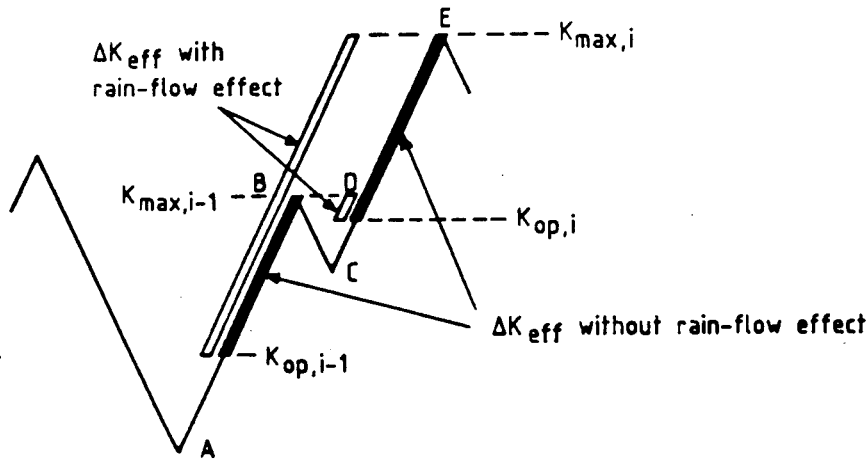
Figure 4: Small intermediate load variation.

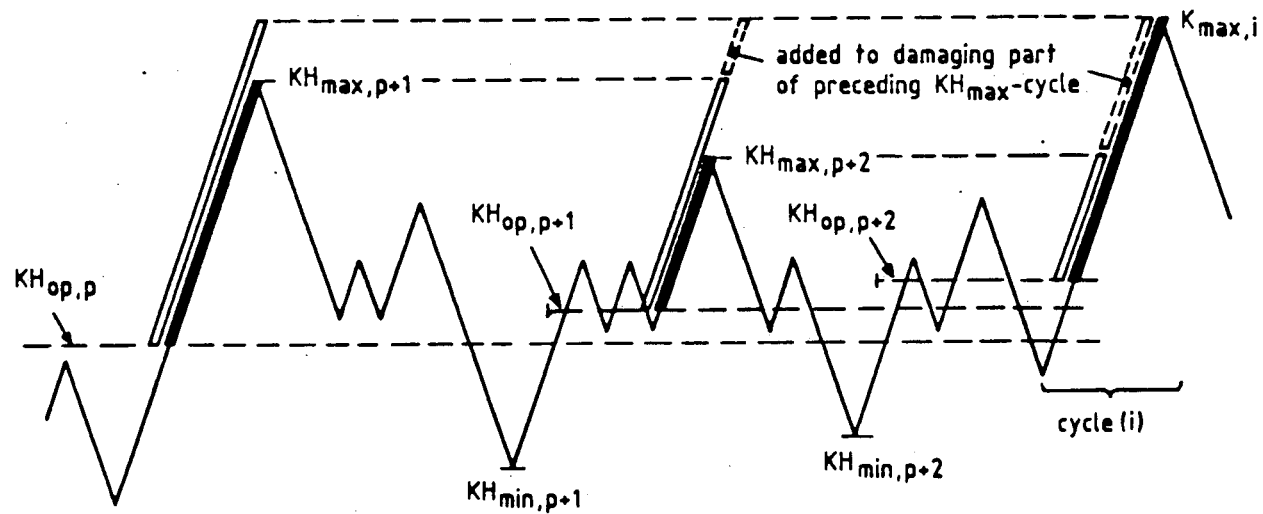
Figure 5: Same K-history as in fig. 4. Rain-flow effect.

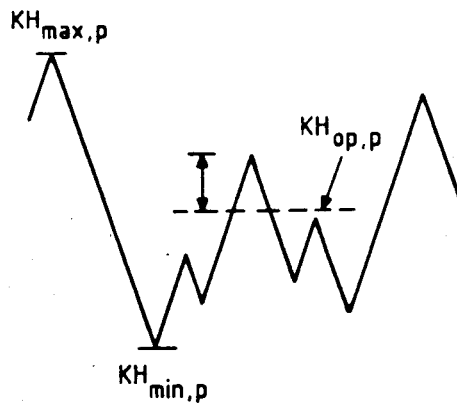Figure 6: Damaging parts of KH_{max}-cycles without rain-flow effect ($\blacksquare$) and with rain-flow effect ($\square$).

Figure 7: Three smaller cycles counted by the rain-flow counting method, but without rain-flow effect in the PREFFAS model.
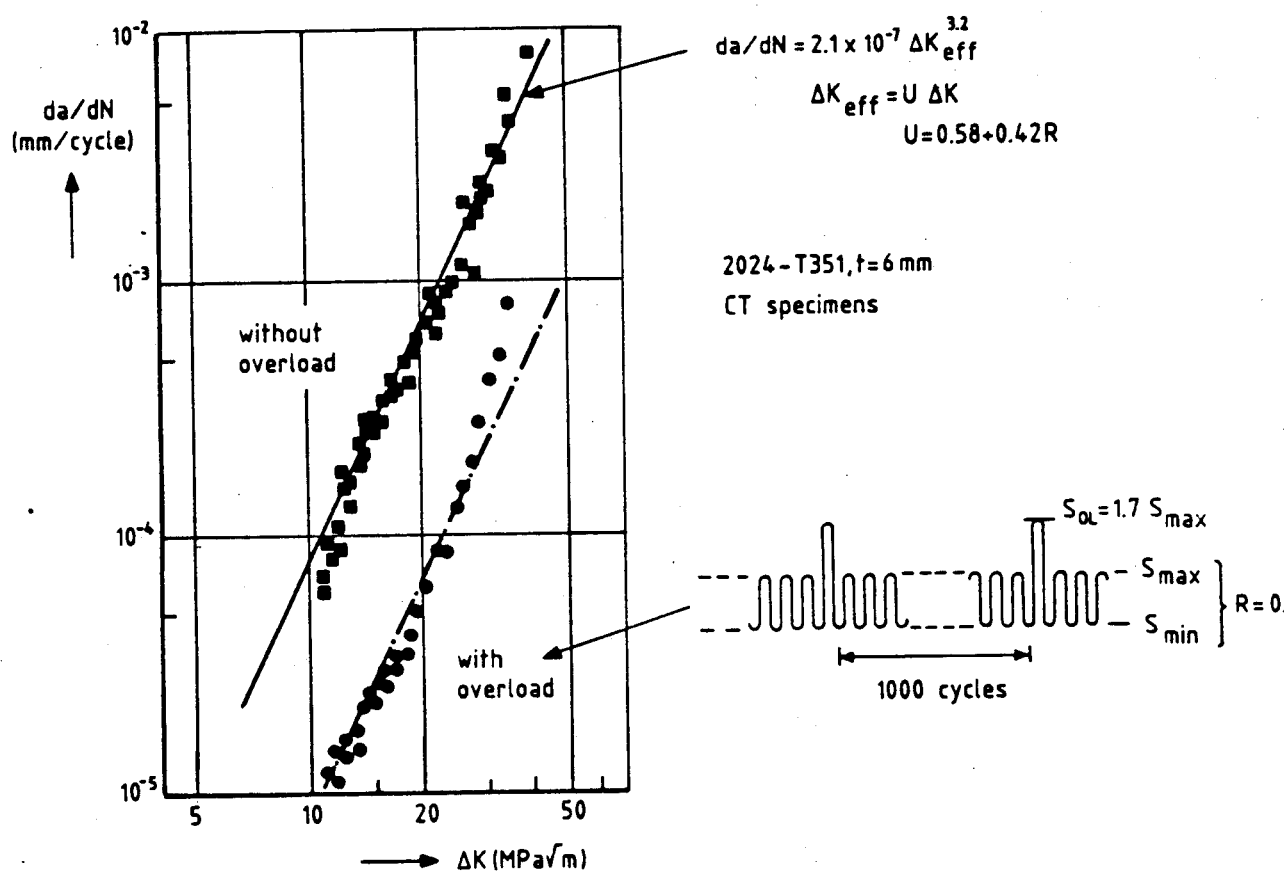
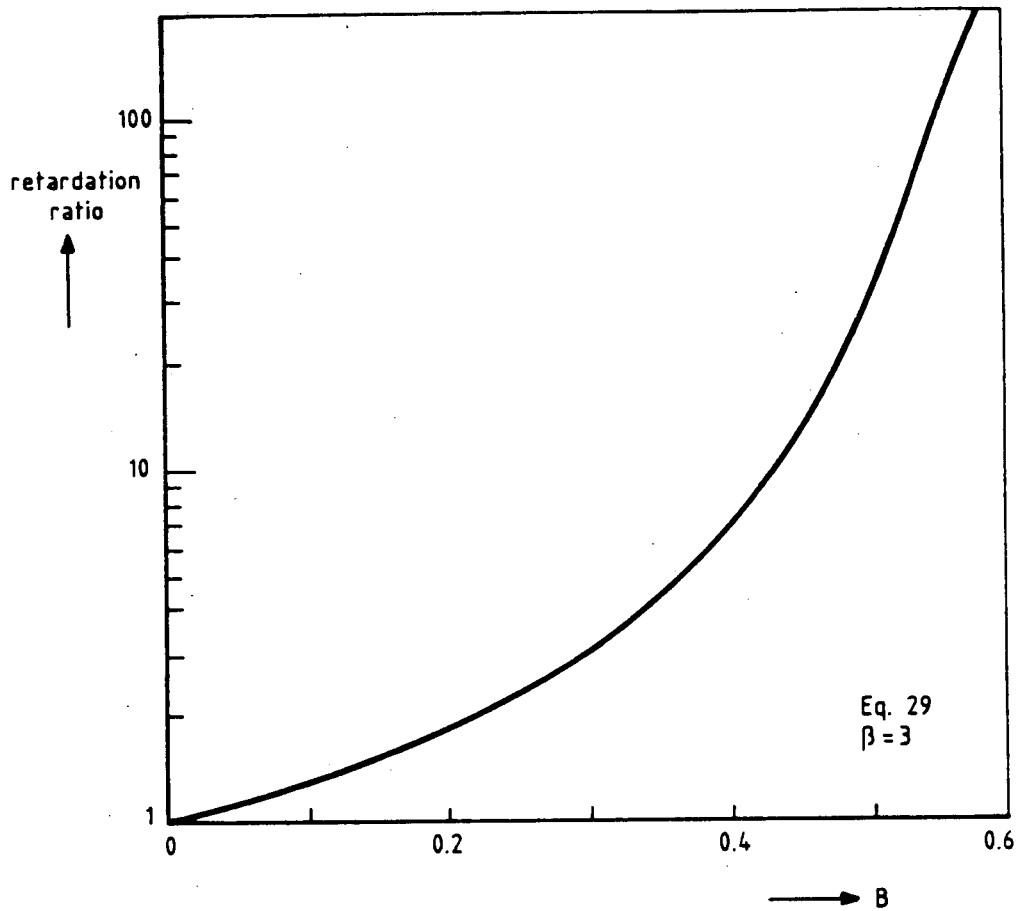Figure 8: Constant-amplitude tests with and without periodic overloads as a calibration for U = A + BR.

Figure 9: The retardation factor in constant-amplitude tests (R = 0.1) with periodic overload cycles (every 1000 cycle, $S_{OL} = 1.7 \, S_{max}$) according to the PREFFAS model.
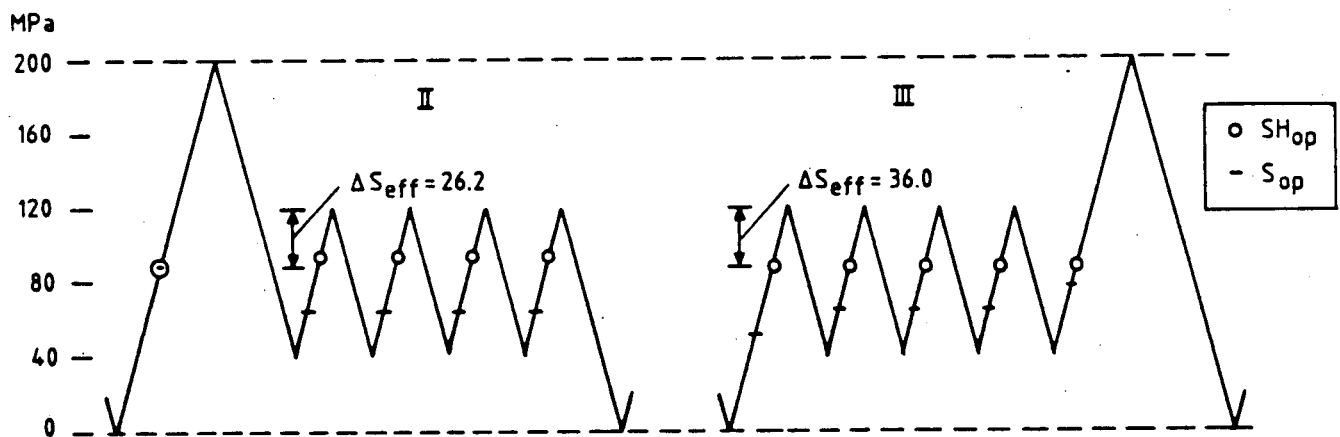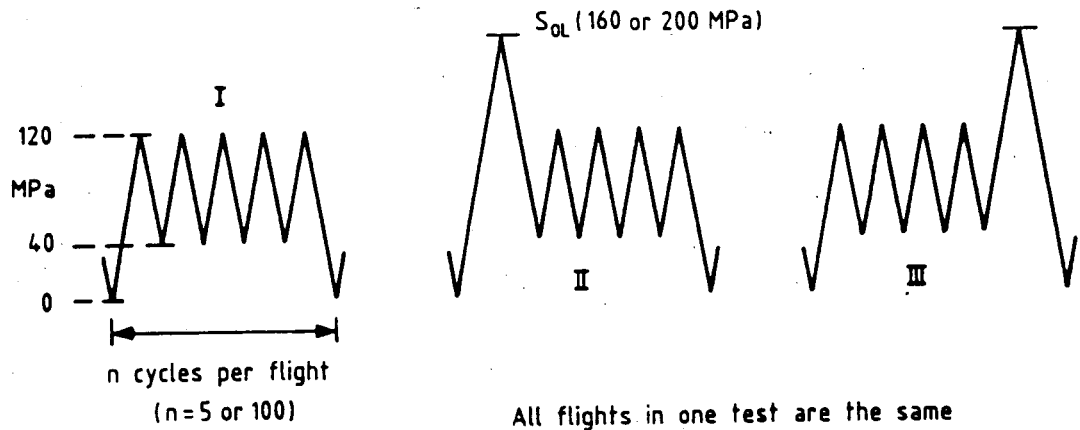

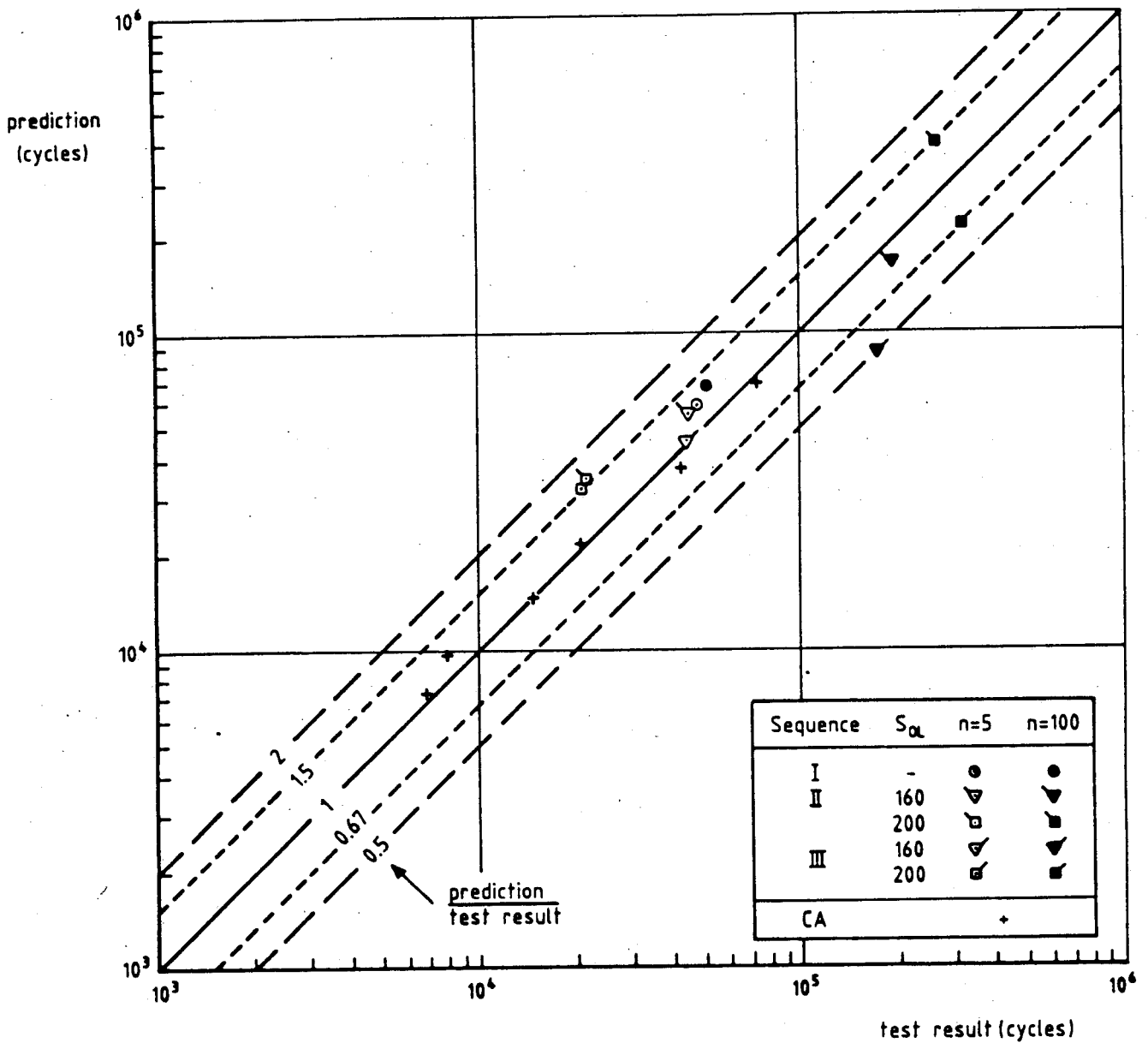Figure 11: Comparison between the opening stress according to PREFFAS (SH_{op}) and non-interaction (S_{op}).

Figure 10: Comparison between PREFFAS-predictions and test results [4] for central cracked sheet specimens of 2024-T3 material (t = 2 mm).
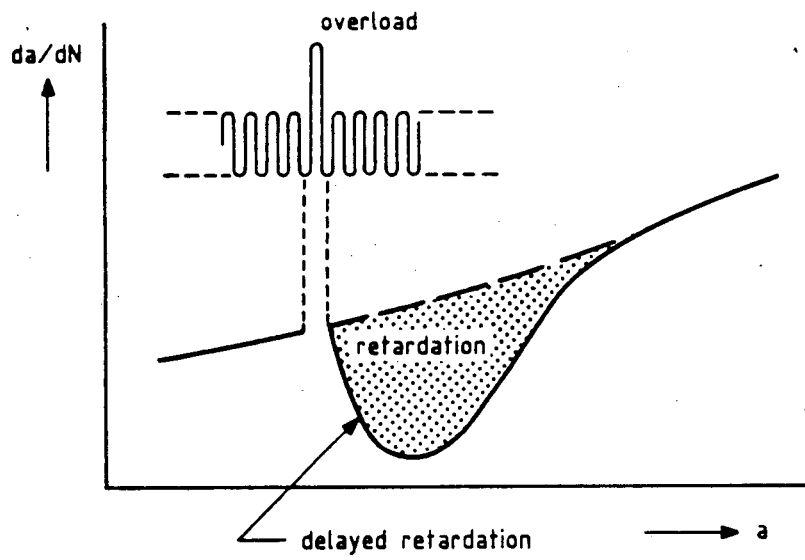
Figure 12: Delayed retardation after an overload (OL).

APPENDIX

CALCULATION OF THE SEQUENCE EFFICIENCY EF
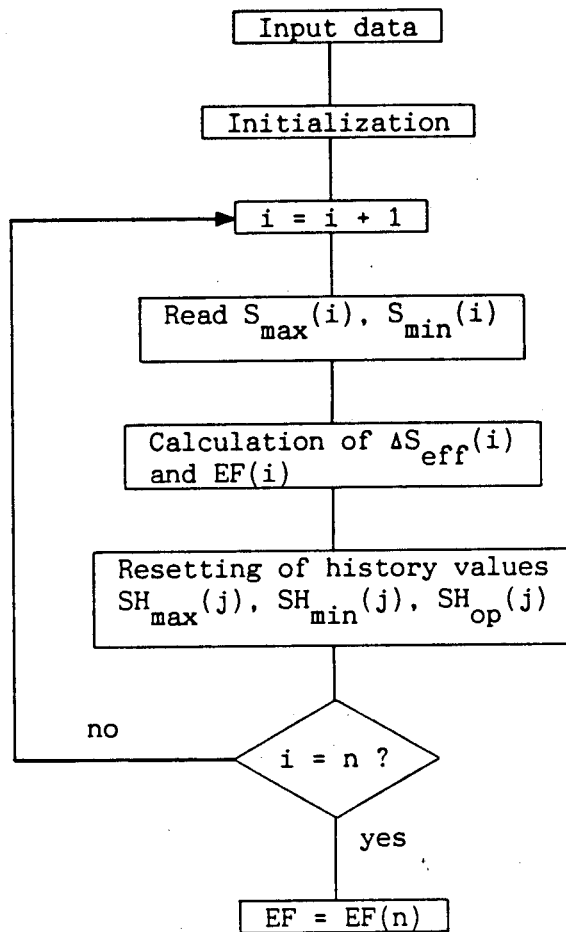
According to eq. (16) the definition of EF is:

$$EF = \sum_{i=1}^{n} \Delta S_{eff}^{\beta}$$

with   n = total number of cycles in one load spectrum period.

The load history is repeated periodically. It is supposed to start with a minimum and to end with the $\underline{same}$ minimum (as required by the transition to the next period).

Including those 2 minima the total number of peak values is (2n + 1), which implies that the first minimum is followed by n cycles consisting of a maximum, $S_{max}(i)$, followed by minimum $S_{min}(i)$ (i = 1 to n). It should be noted that the cycle definition differs from the definition in the paper, where a cycle was supposed to consist of a minimum followed by a maximum. However, the opposite sequence is more logical here in view of new S_{op}-levels and the possible re-assessment of history levels. The first minimum is $S_{min}(0)$ and the last one $S_{min}(n) = S_{min}(0)$.

The computation of EF occurs in the following steps:

```
                    ┌─────────────────┐
                    │   Input data    │
                    └────────┬────────┘
                             │
                    ┌────────┴────────┐
                    │ Initialization  │
                    └────────┬────────┘
                             │
      ┌──────────────►┌──────┴──────┐
      │               │  i = i + 1  │
      │               └──────┬──────┘
      │                      │
      │        ┌─────────────┴─────────────┐
      │        │ Read Smax(i), Smin(i)     │
      │        └─────────────┬─────────────┘
      │                      │
      │        ┌─────────────┴─────────────┐
      │        │ Calculation of ΔSeff(i)   │
      │        │ and EF(i)                 │
      │        └─────────────┬─────────────┘
      │                      │
      │        ┌─────────────┴─────────────┐
      │        │ Resetting of history values│
      │        │ SHmax(j), SHmin(j), SHop(j)│
      │        └─────────────┬─────────────┘
      │                      │
      │    no          ╱─────┴─────╲
      └───────────────┤   i = n ?   │
                       ╲─────┬─────╱
                             │ yes
                    ┌────────┴────────┐
                    │   EF = EF(n)    │
                    └─────────────────┘
```

A more detailed flow diagram is presented in figure A1.

The input data consist of files with all $S_{max}(i)$ and $S_{min}(i)$-values, and β.

The initialization process includes:

$i = 0$

Calculation of $S_{op}(1)$

$SH_{max}(1) = S_{max}(1)$

$SH_{min}(1) = S_{max}(1)$

$SH_{op}(1) = S_{op}(1)$

$NH = 1$

(NH = number of each of the 3 history levels SH_{max}, SH_{min} and SH_{op}).

After reading $S_{max}(i)$ and $S_{min}(i)$ and the calculation of $S_{op}(i+1)$, $\Delta S_{eff}(i)$ can be calculated, but first it has to be checked whether the current cycle (i) may lead to new history stress levels. In agreement with equations (19) the conditions to be checked are (see fig. A2).

$$S_{max}(i) > SH_{max}(NH) \qquad \text{(case I)}$$

$$S_{min}(i) < SH_{min}(NH) \qquad \text{(case II)}$$

$$S_{op}(o) > SH_{op}(NH) \qquad \text{(case III)}$$

If none of these three cases does apply the situation is referred to as case IV. Now $\Delta S_{eff}(i)$ can be calculated, keeping in mind that the rain-flow effect should be taken into account in agreement with eq. (12), which leads to:

$$[\Delta S_{eff}(i)]^{\beta} = [S_{max}(i) - SH_{op}(NH-NHE1)]^{\beta} + \sum_{j=1}^{NHE1} [SH_{max}(j^*) - SH_{op}(j^*)]^{\beta}$$

$$- \sum_{j=1}^{NHE1} [SH_{max}(j^*) - SH_{op}(j^* - 1)]^{\beta} \qquad \text{(A1)}$$

with $j^* = NH - NHE1 + j$,

where NHE1 is the number of SH_{max} values exceeded by $S_{max}(i)$, see figure A1.

For the other 3 cases (recall that NH is also the rank number of the last SH-levels):

$$\Delta S_{eff}(i) = S_{max}(i) - SH_{op}(NH) \qquad \text{(A2)}$$

unless it would be negative. In that case it is equal to zero.
After the calculation of $\Delta S_{eff}(i)$ resetting of the SH-levels is necessary for cases I, II and III.
For case I (see fig. A2): NHE1 sets of SH-levels are overruled and one new set is added. As a result:

$$NH \qquad = NH - NHE1 + 1$$

$$SH_{max}(NH) = S_{max}(i)$$

$$SH_{min}(NH) = S_{min}(i)$$

$$SH_{op}(NH) = S_{op}(i+1)$$

For case II (see fig. A2): NHE2 values of SH_{min} and NHE2-1 values of SH_{max} are overruled, and:

$$NH = NH - NHE2 + 1$$

$$SH_{max}(NH) = S_{max}(NH)$$

$$SH_{min}(NH) = S_{min}(i)$$

$$SH_{op}(NH) \text{ to be calculated from } SH_{max}(NH) \text{ and } SH_{min}(NH)$$
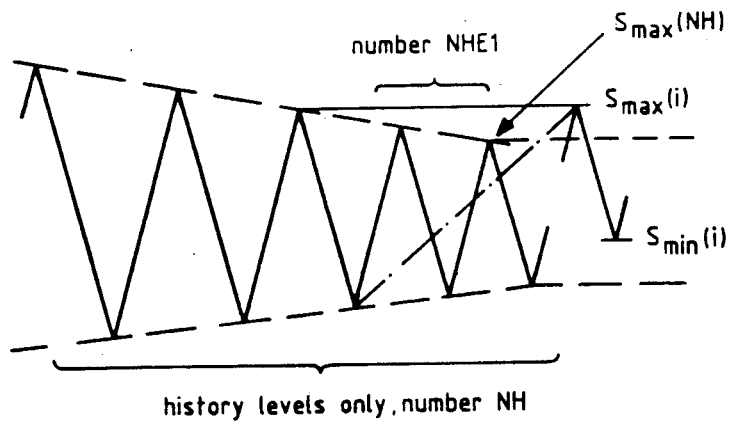
For case III (see fig. A2): one set of SH-levels is added.
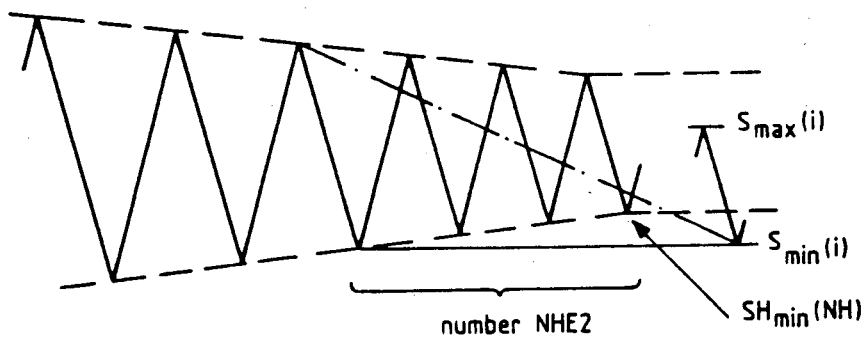
$$NH = NH + 1$$

$$SH_{max}(NH) = S_{max}(i)$$

$$SH_{min}(NH) = S_{min}(i)$$

$$SH_{op}(NH) = S_{op}(i+1)$$

After resetting $\overset{i}{\Sigma} \Delta S_{eff}^{\beta}$ is calculated. The loop is restarted with a new cycle (i = i + 1) until all cycles of one period (number n) has passed. Then EF = EF(i) = EF(n).
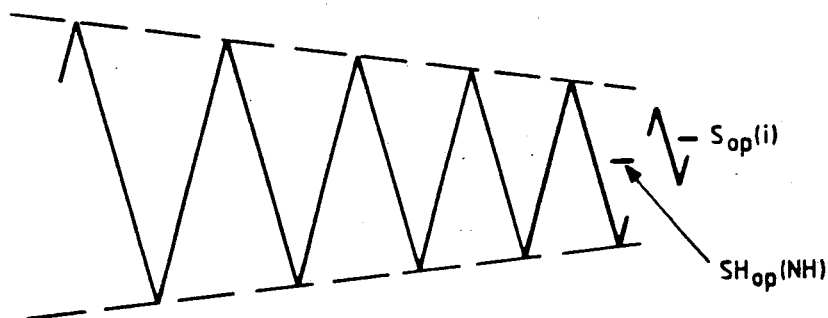
Figure A1: Three cases which modify history stress levels.

Figure A2: Flow diagram.