Effects of Uncertainties on Extreme Wave Heights

By Marshall D. Earle[1] and Ledolph Baer[2]

Abstract: The relative effects of uncertainties due to record lengths, natural variability, and measurement or hindcast errors on estimated extreme significant wave heights has been assessed objectively through simulation. The probability distributions describing extreme heights at a given return period have confidence intervals broader than those computed from typically used techniques and show substantial positive biases. Both the widths of the confidence intervals and the biases increase with the size of the total uncertainty which represents the combined effects of measurement or hindcast errors and natural variability. Tables and nomograms are provided for practical assessment of record lengths, natural variability and errors on extreme wave estimates.

Introduction

Design wave criteria for coastal and offshore structures are based on occurrence probabilities of extreme wave conditions. Wave heights as a function of return period are usually of interest. Other types of statistical descriptions such as encounter or nonencounter probabilities (3) also are used. Spectra, periods, and steepness may also be considered for determination of design criteria. Goda (8) has reviewed statistical aspects of wave parameters that may be of interest for particular applications. The return period, or recurrence interval, associated with a wave height, is the average time between occurrences equal to or greater than this wave height. For costly structures, such as breakwaters and offshore oil production platforms, designs are typically based on return periods of approximately 100 years. Designs may be based on shorter return periods for less expensive structures. Risk analysis may be performed by comparison of construction costs to probable cost associated with loss or damage for various wave heights as a function of occurrence probability.

Extreme even analyses for rainfall, river height, and wind speed are similar to those for waves. Although this paper is concerned with wave height, it also applies to the determination of extreme events for other phenomena.

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Ideally, extreme wave height probabilities could be determined by statistical analysis of a very long series of measured wave data. However, measured data usually cover time periods much shorter than the long return periods of primary interest so that this approach is seldom practical. An alternative technique is to construct a long series using hindcasts. For hindcasts, a model is used with past meteorological conditions as input to compute high wave conditions over a reasonably long time period, typically twenty to forty years. Although hindcasts can cover longer time periods than measurements, even they are generally shorter than return periods needed for design. The practical length of hindcasts is limited since the climate varies and there are potentially serious inadequacies in the meteorological input conditions of many years ago. In addition to the random hindcast uncertainties considered here, poor meteorolgical input to hindcast models could result in biases. Longer term hindcasts may also be affected by climatic trends which would decrease their useful accuracy for future statistical predictions.

The standard approach is to fit a probability distribution to measurements or hindcast results and to use this distribution for extrapolation to long return periods associated with wave height that have low exceedance probabilities. For statistical estimation based on either measurements or hindcasts, several factors cause uncertainties for wave heights as a function of return period. This study is concerned primarily with extreme wave height uncertainties that are caused by measurement or hindcast uncertainties, finite record lengths, and environmental variability. Lack of knowledge of the "true" distribution of extreme wave heights and of bias type uncertainties in measurements or hindcasts are the primary sources of uncertainty not addressed by this study.

Approach and Theory

Uncertainties of wave heights for selected return periods were estimated by computer simulation of wave records consisting of annual highest significant wave heights, H. These wave heights were generated by random selection of values from probability distributions representative of three wave height regions: low, high, and very high. Such a record of annual highest significant wave heights constitutes an annual series. It is common in wave statistical analysis to utilize a data set consisting of the highest significant wave heights above a selected minimum value during individual storm events which is called a partial series. Since extreme event statistical theory has been developed more extensively for annual series than for partial series, annual series were used for this study. Use of either annual or partial series for very rare events such as hurricanes requires special analysis which is not covered in this study. Extreme wave heights may also be limited for physical reasons such as bottom effects or fetch lengths which cause statistical analysis for such extremes to be inappropriate.

In oceanography, meteorology, and hydrology, a variety of probability distributions are used for extreme event studies. Sampling variability and measurement or hindcasting uncertainties mask the underlying distribution. In addition, several distributions may pass statistical "goodness of fit" tests when fit to a particular sample. When return periods of interest are not too long compared with the period of record, there are sometimes acceptably small differences between several distributions (20). Although there is no consensus on the "best" distribution, within recent years there has been increased interest in distributions with several parameters, for example the Wakeby distribution with five parameters (14,15). Statistical analysis of extreme events is most advanced in hydrology where use of distributions for extrapolation to long return period events has been a common practice for many years. Nevertheless, there is a wide range of opinion on the "best" distribution and numerous studies, such as several noted in Ref. 13, either provide inconsistent results or show that several distributions work well for particular situations. As an example of the controversy, the U.S. Water Resources Council has recommended (10) use of the log-Pearson type III distribution for hydrologic studies while some hydrologists (14) have found limitations for this distribution. The World Meteorological Organization suggests (9) use of the extremal type I (Gumbel) distribution for hydrologic studies. It is common practice in oceanography for wave heights to be fit by log-normal (7,12,23), extremal type I (2,4,20) and Weibull (20) distributions. The extremal type II distribution has also been used (21).

Since comparison of different probability distributions is not the purpose of this study, the log-normal distribution for extreme significant wave heights was selected for use. It has been used frequently, provides a reasonable description of extreme wave probabilities, and is computationally simple. In addition, log-normal and normal probability paper is widely available so that users of this work easily can employ graphical procedures to determine log-normal distributions for their measured data or hindcast results. The log-normal distribution for annual maximum significant wave height, H, is given by

$$p(H) = \frac{1}{H\sigma(2\pi)^{1/2}} \exp\left[\frac{-(\ln H - \mu)^2}{2\sigma^2}\right] \qquad (1)$$

in which $p(H)dH$ = the probability for H to be in the small interval of width, dH, given by $(H, H + dH)$. This distribution applies if logarithms of significant wave heights are normally distributed with mean, μ, and standard deviation, σ. It is often more convenient to work with the normal distribution of $y = \ln H$. This distribution is given by

$$p(y) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[\frac{-(y - \mu)^2}{2\sigma^2}\right] \qquad (2)$$

in which μ = the mean of y; and σ = the standard deviation of y. For the computer simulations, $\log H = \log_{10} H$ was used in place of the natural logarithm, $\ln H$, so that logarithms could be more easily interpreted in terms of significant wave heights. Consistent use of base 10 logarithms instead of natural logarithms has no affect on results. Since $\log H = 0.4343 \ln H$, such use is equivalent to a scale change for logarithms of significant wave heights.

Measurement and hindcast uncertainties in this study are considered to be unbiased and caused by sampling variability, model limitations, wind field specifications, nonstationary conditions, the utilized wave sensor, finite record lengths, data analysis procedures, and calibration procedures. The distribution for the net effect of several independent uncertainties approaches normal in accordance with the central limit theorem. Ninety percent confidence intervals, representing uncertainties of 0%, ±10%, ±20%, and ±30% of H, define e_t, the magnitude of uncertainty for each case. Standard deviations of the uncertainty distributions are 0%, ±6%, ±12%, and ±18% of H since the 90% confidence interval for a normal distribution is $\pm 1.645\sigma$. Values from normal distributions specified by these standard deviations were randomly selected and added algebraically to significant wave heights from the log-normal distributions. For applications, e_t can be estimated from

$$e_t^2 \simeq \sum_i e_i^2 \qquad (3)$$

in which each e_i represents 90% confidence intervals for individual contributing uncertainties.

Sampling variability occurs because a significant wave height is a sample estimate of the actual significant wave height (18). Confidence intervals for significant wave height depend on the total degrees-of-freedom, ν. The total degrees-of-freedom for a given wave record depends on the bandwidth of the frequency spectrum of the record. When wave variance spectra are calculated by fast Fourier transform methods, μ values can be estimated from

$$\nu = \frac{2\left(\sum_n E_n\right)^2}{\sum_n E_n^2} \qquad (4)$$

in which E_n = wave variance at the nth Fourier frequency; and the summations are over all Fourier frequencies. With modification of Eq. 4, ν values can also be estimated from band-averaged wave spectra. There is α percent confidence that the true significant wave height is within the following interval:

$$\left(\frac{\nu}{\chi^2_{\nu:100-\alpha/2}}\right)^{1/2} H \le H' < \left(\frac{\nu}{\chi^2_{\nu:100+\alpha/2}}\right)^{1/2} H \qquad (5)$$

in which H = estimated significant wave height; H' = true significant wave height; and χ^2 values are obtained from a chi-square distribution. The square root appears in Eq. 5 because the significant wave height is proportional to the square root of the record variance and the variance is chi-square distributed with ν degrees-of-freedom. Through Eq. 4, confidence intervals for H depend on the spectral distribution of wave energy. Although significant wave heights can be computed by either zero crossing or wave variance techniques with some differences between these methods due to finite bandwidths of wave spectra (17), confidence intervals for significant wave height depend on spectral bandwidths and require calculation of wave spectra. Ninety percent confidence intervals typically are ±10% to 15% about calculated H values as shown in Table 1.

Simulated records of highest annual significant wave heights (with uncertainty contamination) were analyzed following commonly used procedures of ranking and regression fitting to the assumed log normal distribution for estimation of extreme wave heights. For each given underlying distribution, record length, and uncertainty level, these procedures were used to generate 1.000 sets of extreme significant wave heights. The mean value, standard deviation, and probability

TABLE 1.—Examples of Ninety Percent Confidence Intervals for Significant Wave Heights (6)[a]

Significant wave height, H, in meters (1)	Percent deviation of lower limit of 90% confidence interval from H (2)	Percent deviation of upper limit of 90% confidence interval from H (3)
3.44	8.7	10.5
4.31	9.3	11.6
5.26	12.2	16.7
4.51	12.2	16.4
3.01	8.6	10.6
3.60	9.2	11.7
3.94	10.9	14.2
4.53	11.0	14.6
3.64	10.4	13.7
3.67	9.0	10.9
3.94	10.2	12.9
3.88	10.6	13.9
3.25	8.6	10.5
4.85	10.5	13.4
5.20	12.1	16.3
5.83	10.3	13.0
4.94	11.3	15.0
3.75	9.9	12.3
4.51	10.9	14.2
3.18	9.4	11.6
3.58	10.6	13.7
3.50	12.0	14.3
3.34	10.5	13.5
3.98	12.8	17.6
3.69	9.5	12.5

[a]These results were calculated from spectra of seventeen-minute Waverider buoy records collected by the National Oceanic and Atmospheric Administration in the Gulf of Alaska at 50.00° N, 149.97° W between 7/15/78 and 10/9/78.

distribution of the extreme significant wave heights at each return period were calculated. These results were compared with correct results based on the underlying distributions and the variation of these results with uncertainty levels and with record lengths was analyzed. Other analysis techniques such as the maximum likelihood method and the method of moments would provide similar results.

Underlying Probability Distributions

Fig. 1 was used to select representative distributions for study. It shows the approximate cumulative probability distributions of extreme significant wave heights at thirteen locations as estimated from the sources listed in Appendix I.

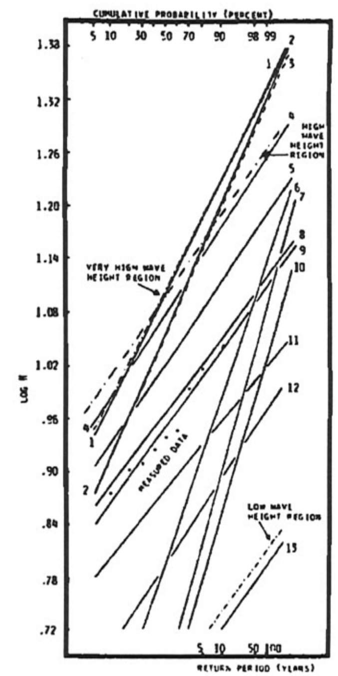

FIG. 1.—Approximate Log-Normal Distributions for Locations Listed in Appendix I

Only one of these distributions, that for Seven Stones Light Vessel, is estimated from actual measurements of annual highest significant wave heights. The other distributions were estimated from hindcasts. In preparing Fig. 1, return periods were converted to cumulative probabilities normalized to one event per year. For example, a fifty-year return period corresponds to a cumulative probability of 0.98. Log-normal lines were fit visually to plotted values of log H versus cumulative probability on plots many times larger than the reduced plot in Fig. 1. Only the individual points for the measurements at Seven Stones Light Vessel are shown. Mean values at the fifty percentile point increase moving vertically and standard deviations, which are a measure of each distribution's width, increase as the slopes of the lines increase. Fig. 1 is not comprehensive; it was used to estimate general variation of extreme significant wave heights. Although not important for the purposes of this study, there are uncertainties and considerations for the information used to prepare Fig. 1 that would be important for a particular application. For some regions, proprietary studies, particularly with advanced wave hindcast models, may provide more accurate information than the information available in open literature.

Based on Fig. 1, log-normal distributions representing a low wave height region, a high wave height region, and a very high wave height region, were defined. Values of μ and σ, the mean and standard deviation of log H, respectively, for these distributions, are listed in Table 2 along with extreme significant

TABLE 2.—Extreme Significant Wave Heights as Functions of Return Periods for Probability Distributions Used in Simulation Studies

Probability distribution (1)	Return period, in years 5 (2)	10 (3)	50 (4)	100 (5)
Low wave height region $\mu = 0.65$ $\sigma = 0.07$	5.12 m	5.49 m	6.22 m	6.50 m
High wave height region $\mu = 1.10$ $\sigma = 0.07$	14.42 m	15.48 m	17.53 m	18.32 m
Very high wave height region $\mu = 1.10$ $\sigma = 0.10$	15.28 m	16.91 m	20.20 m	21.51 m

wave heights for return periods of 5, 10, 50, and 100 years. The distribution for the high wave height region used μ for the very high wave height region and σ for the low wave height region so that relative effects of changing μ or σ can be observed. The low wave height distribution approximately represents wave conditions (without infrequently occurring tropical storms) off the California-Mexico border, and the very high wave height distribution approximately represents wave conditions in the Gulf of Alaska or the northeast Atlantic Ocean.

Simulations

Logarithms of annual highest significant wave heights were simulated for record lengths of 5, 10, 20, and 40 years. The shorter record lengths, 5 and 10 years, are representative of record lengths potentially available from wave measurements at the present time and in the relatively near-term future. The longer record lengths, 20 and 40 years, are representative of time periods over which wave hindcasts are frequently made.

Values of log H were randomly selected from the underlying probability distribution by use of Eq. 6 given in Ref. 1:

$$y_n = \mu + \sigma(-2 \ln u_{1n})^{1/2} \cos (2 \pi u_{2n}) \quad (6)$$

in which y_n = log H, μ = the mean that specifies the distribution of log H; σ = the standard deviation that specifies the distribution of log H; u_{1n} and u_{2n} are a pair of independent random numbers uniformly distributed between 0 and 1; ln = the natural logarithm, n ranges from 1 to N; and N = 5, 10, 20, and 40. Values of u_{1n} and u_{2n} were generated by the RANF random number function available on Control Data Corporation (CDC) Cyber 175 computer systems. This function uses the multiplicative congruential method with modulo 2^{48} and random numbers pass auto-correlation tests with lag ± 100 and pair-triplet tests for randomness. Random selection of u_{1n} and u_{2n} was initiated by specification of a seed parameter which was independently defined as a function of computer processing time and program index parameters for each of the 1,000 simulations. Thus, new sets of random numbers were generated for each simulation.

Several computer runs were made to test the simulation procedures. Numerous

sets of random numbers were printed and correlation coefficients between sets were computed to verify randomness in a simple manner. Probability distributions of y_n for several simulations with n ranging to 500 were computed and plotted to verify that the y_n were normally distributed with sample means and standard deviations corresponding to input means and standard deviations.

When a decision is made to determine wave conditions by either measurements or hindcasts for N_2 years instead of N_1 years, where $N_2 > N_1$, the annual highest significant wave heights contained in the time period, N_1 years, usually will be contained in the time period, N_2 years. For a given underlying probability distribution and uncertainty, the simulations considered this characteristic by addition of five years of simulated data to a 5-year record to yield a 10-year record, addition of 10 years of simulated data to this record to yield a 20-year record, and addition of 20 years of simulated data to the 20-year record to provide a 40-year record. However, each of the 1,000 simulations for a given record length was independent.

Measurement of hindcast uncertainties were combined with significant wave heights by Eq. 7:

$$H(\text{with error}) = H(\text{true}) + \left(\frac{\beta}{100}\right) H(\text{true}) \quad (7)$$

Percentage uncertainties, β, were selected randomly from normal distributions with zero means and standard deviations corresponding to 90% confidence limits of 0%, 10%, 20%, and 30% by means of Eq. 6 with β in place of y_n. Values of u_{1n} and u_{2n} were independently selected for uncertainty calculations and for significant wave height calculations so that uncertainties and significant wave heights were uncorrelated. Error contaminated wave heights were not permitted to be negative. For these simulations, the probability of obtaining an uncertainty greater than −100% which could cause a negative wave height, is negligible. Normal probability tables provide the uncertainty distribution for a specified input uncertainty. As an example, the following results occur for an input uncertainty of 20%: 50% of uncertainties between ±8%, 75% of uncertainties between ±14%, 90% of uncertainties between ±20%, and 95% of uncertainties between ±24%.

Error contaminated values of log H were ranked from lowest to highest and assigned cumulative probabilities, f_n, in which n ranges from one to the record length, N. The following plotting position equation was used:

$$f_n = \frac{n}{N + 1} \quad (8)$$

Eq. 8 is a widely used method (2,7,13,16,23) for assignment of cumulative probabilities to extreme values. For each f_n, the corresponding standard normal deviate, z_n, was calculated. For variables randomly selected from a normal distribution, a graph of the ranked values, y_n, versus the values, z_n, associated with the corresponding cumulative probabilities, f_n, approximates a line characterized by a mean and standard deviation close to the mean and standard deviation of the underlying normal distribution. Ranked values of log H deviate from a straight line as a result of sampling variability related to finite values of N and the introduced uncertainties.

Values of z_n corresponding to known cumulative probabilities, f_n, can be obtained from cumulative normal probability distribution tables. However, for the computer simulations, z_n values were calculated from the following equations (1,13):

$$z_n \simeq -w_n + \frac{c_0 + c_1 w_n + c_2 w_n^2}{1 + d_1 w_n + d_2 w_n^2 + d_3 w_n^3}, \quad \text{for} \quad f_n \le 0.5 \quad \text{(9)}$$

$$z_n \simeq w_n - \frac{c_0 + c_1 w_n + c_2 w_n^2}{1 + d_1 w_n + d_2 w_n^2 + d_3 w_n^3}, \quad \text{for} \quad f_n \ge 0.5 \quad \text{(10)}$$

in which $w_n = \left[\ln\left(\frac{1}{g_n^2}\right)\right]^{1/2}$; $g_n = f_n$, for $f_n \le 0.5$;

$$g_n = 1 - f_n, \quad \text{for} \quad f_n \ge 0.5 \quad \text{(11)}$$

and c_0 = 2.515517; c_1 = 0.802853; c_2 = 0.010328; d_1 = 1.432788; d_2 = 0.189269; and d_3 = 0.001308. These equations result in a maximum uncertainty of 4.5×10^{-4} for z_n values.

Least-squares lines were ealculated for each simulated record of N values of log H and corresponding standard normal deviates by standard linear regression procedures (11,22). For each of the 1,000 records based on specified input conditions, values of log H and H were computed from the least-squares line at return periods of 5, 10, 50, and 100 years. These return periods correspond to percentage cumulative probabilities of 80%, 90%, 98%, and 99%. The shorter return periods, 5 and 10 years, are representative of return periods which apply to design of relatively inexpensive or expendable structures. The longer return periods, 50 and 100 years, are representative of return periods which apply to the design of expensive structurses or those which must protect people or costly equipment.

Fig. 2 is an example showing results of these procedures and effects of sampling variability. For this example, N = 10 and sampling variability effects may be relatively large due to selection of small samples from naturally varying annual highest significant wave heights. Two data sets are shown to illustrate that the least-squares line may fit the actual underlying distribution very well or may deviate very much from it. In addition, substantial differences would occur if the least-squares regression were performed only for the higher wave heights. Significant wave heights at long return periods for the two data sets are considerably different. The samples shown in Fig. 2 are not contaminated by uncertainties.

One thousand computer simulations were made for each combination of the three underlying probability distributions, four record lengths, four uncertainty levels, and four return periods. The resulting 192 sets of 1,000 significant wave heights were analyzed to determine the mean value, standard deviation from the mean, probability distribution represented by a histogram, and the cumulative probability distribution computed by summing contributions to the histogram class intervals. Probability distributions and cumulative probability distributions were computed with a resolution of 0.1 standard deviation over a range of ±3 standard deviations. Extreme values outside of this range were printed for in-

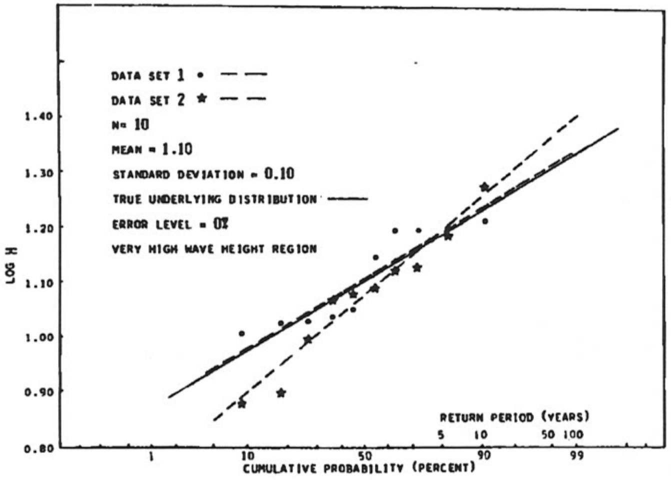

FIG. 2.—Effects of Sampling Variability Illustrated for Two Simulated Significant Wave Height Records

spection. In total, statistical results were based on 192,000 significant wavc heights as functions of return period which in turn were based on 900,000 annual highest significant wave heights. Approximately 100,000 additional annual highest significant wave heights were simulated to verify the procedures and the statistical reliability of results based on sets of 1,000 simulated records.

Results

Results for the low wave height, high wave height, and very high wave height regions are summarized in Tables 3, 4, and 5 as a function of region, record length, error level, and return period. Bias is defined as the difference between mean wave height and actual wave height for a given return period. The fact that biases are always positive is a result of uncertainties introduced by errors, statistical variability for finite records of N samples, and the ranking procedure. Since the standard deviation of the error contaminated record is larger than the standard deviation of the underlying true distribution while the mean remains the same, the contaminated distribution has a more positive slope than the true distribution, resulting in higher extreme values. Standard deviations about the means for each of the 1,000 H values for a given region, record length, error level, and return period also listed in Tables 3–5. Ninety percent confidence limits, which are listed in standard deviation units in Tables 3–5, were read from significant wave height histograms with a resolution of 0.1 standard dcviation.

Four histograms are plotted in Fig. 3 to demonstrate relative effects of rccord lengths and uncertainties. While histogram shapes are similar, histogram widths, as indicated by larger standard deviations, increase with increasing uncertainties and decreasing record lengths. Some variability shown for different class intervals in Fig. 3 is due to the use of a relatively narrow resolution. In addition,

TABLE 3.—Significant Wave Height Statistical Summaries for Low Wave Height Region[a]

Uncertainty, e_t	0%				10%				20%				30%			
Return period, in years,	5	10	50	100	5	10	50	100	5	10	50	100	5	10	50	100
with "true" mean (1)	5.12 (2)	5.49 (3)	6.22 (4)	6.50 (5)	5.12 (6)	5.49 (7)	6.22 (8)	6.50 (9)	5.12 (10)	5.49 (11)	6.22 (12)	6.50 (13)	5.12 (14)	5.49 (15)	6.22 (16)	6.50 (17)
(a) N = 5																
Mean	5.26	5.74	6.71	7.10	5.32	5.80	6.89	7.31	5.48	6.12	7.46	8.01	5.73	6.58	8.44	9.23
Bias	0.14	0.25	0.49	0.60	0.20	0.31	0.67	0.81	0.36	0.63	1.24	1.51	0.61	1.09	2.22	2.73
Standard deviation, σ	0.49	0.65	1.07	1.25	0.54	0.73	1.22	1.45	0.64	0.89	1.57	1.88	0.81	1.17	2.19	2.70
90% Confidence +	1.6	1.7	1.8	1.8	1.8	1.9	1.9	1.8	1.8	1.8	1.7	1.7	1.8	1.9	1.9	2.0
90% Confidence −	1.6	1.5	1.4	1.4	1.5	1.5	1.4	1.4	1.5	1.5	1.3	1.3	1.5	1.4	1.3	1.3
(b) N = 10																
Mean	5.21	5.64	6.50	6.84	5.26	5.74	6.69	7.06	5.39	5.97	7.16	7.63	5.60	6.35	7.94	8.60
Bias	0.09	0.15	0.28	0.34	0.14	0.25	0.47	0.57	0.27	0.48	0.94	1.13	0.48	0.86	1.72	2.10
Standard deviation, σ	0.32	0.41	0.65	0.75	0.35	0.47	0.74	0.87	0.42	0.56	0.93	1.10	0.51	0.70	1.22	1.47
90% Confidence +	1.7	1.7	1.7	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.8	1.8	1.7	1.8	1.8	1.9
90% Confidence −	1.5	1.5	1.5	1.5	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.6	1.5	1.5	1.5
(c) N = 20																
Mean	5.17	5.58	6.39	6.71	5.22	5.67	6.56	6.91	5.34	5.90	7.01	7.45	5.52	6.23	7.68	8.28
Bias	0.05	0.09	0.17	0.21	0.10	0.18	0.34	0.41	0.22	0.41	0.79	0.95	0.40	0.74	1.46	1.78
Standard deviation, σ	0.22	0.28	0.43	0.50	0.24	0.31	0.47	0.54	0.28	0.37	0.59	0.69	0.35	0.47	0.79	0.94
90% Confidence +	1.6	1.7	1.7	1.7	1.6	1.7	1.7	1.7	1.8	1.7	1.7	1.7	1.7	1.8	1.8	1.8
90% Confidence −	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
(d) N = 40																
Mean	5.15	5.54	6.32	6.61	5.20	5.63	6.48	6.81	5.31	5.84	6.90	7.32	5.48	6.15	7.53	8.09
Bias	0.03	0.05	0.10	0.11	0.08	0.14	0.26	0.31	0.19	0.35	0.68	0.82	0.36	0.66	1.31	1.59
Standard deviation, σ	0.15	0.20	0.30	0.39	0.17	0.21	0.32	0.37	0.19	0.25	0.39	0.46	0.25	0.32	0.54	0.64
90% Confidence +	1.7	1.7	1.7	1.8	1.7	1.7	1.6	1.6	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7
90% Confidence −	1.6	1.6	1.6	1.6	1.6	1.6	1.5	1.6	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.5

[a]Each result is based on 1,000 simulations of significant wave height. All units are in meters except for 90% confidence intervals which are in standard deviation, σ, units.

TABLE 4.—Significant Wave Height Statistical Summaries for High Wave Height Region[a]

Uncertainty, e_t	0%				10%				20%				30%			
Return period, in years,	5	10	50	100	5	10	50	100	5	10	50	100	5	10	50	100
with "true" mean (1)	14.42 (2)	15.48 (3)	17.53 (4)	18.32 (5)	14.42 (6)	15.48 (7)	17.53 (8)	18.32 (9)	14.42 (10)	15.48 (11)	17.53 (12)	18.32 (13)	14.42 (14)	15.48 (15)	17.53 (16)	18.32 (17)
(a) N = 5																
Mean	14.87	16.22	18.95	20.04	15.05	16.53	19.56	20.77	15.50	17.32	21.13	22.70	15.98	18.31	23.40	25.57
Bias	0.45	0.74	1.42	1.72	0.63	1.05	2.03	2.45	1.08	1.84	3.60	4.38	1.56	2.83	5.87	7.25
Standard deviation, σ	1.42	1.95	3.26	3.84	1.48	2.04	3.47	4.16	1.73	2.41	4.24	5.10	2.09	3.05	5.89	7.31
90% Confidence +	1.8	1.8	1.8	1.8	1.7	1.7	1.8	1.8	1.7	1.7	1.8	1.8	1.7	1.8	1.9	1.9
90% Confidence −	1.6	1.5	1.4	1.4	1.5	1.4	1.4	1.3	1.6	1.5	1.4	1.3	1.5	1.4	1.3	1.2
(b) N = 10																
Mean	14.70	15.94	18.39	19.34	14.83	16.18	18.88	19.94	15.23	16.88	20.26	21.61	15.72	17.82	22.28	24.13
Bias	0.28	0.46	0.86	1.02	0.41	0.70	1.35	1.62	0.81	1.40	2.73	3.29	1.30	2.34	4.75	5.81
Standard deviation, σ	0.94	1.23	1.93	2.24	1.00	1.32	2.10	2.44	1.17	1.56	2.59	3.07	1.39	1.95	3.47	4.20
90% Confidence +	1.7	1.7	1.8	1.8	1.7	1.7	1.8	1.8	1.8	1.7	1.7	1.8	1.7	1.7	1.8	1.8
90% Confidence −	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.5	1.6	1.6	1.6	1.5
(c) N = 20																
Mean	14.59	15.76	18.05	18.73	14.70	15.76	18.44	19.41	15.10	16.66	19.80	21.04	15.53	17.50	21.61	23.29
Bias	0.17	0.28	0.52	0.61	0.28	0.48	0.91	1.09	0.68	1.18	2.27	2.72	1.11	2.02	4.08	4.77
Standard deviation, σ	0.65	0.84	1.29	1.48	0.68	0.87	1.35	1.56	0.82	1.07	1.73	2.03	0.95	1.27	2.16	2.59
90% Confidence +	1.7	1.7	1.7	1.7	1.7	1.8	1.8	1.8	1.8	1.7	1.8	1.8	1.7	1.7	1.7	1.6
90% Confidence −	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.6
(d) N = 40																
Mean	14.52	15.65	17.86	18.72	14.63	15.85	18.23	19.16	14.97	16.45	19.41	20.58	15.44	17.34	21.27	22.87
Bias	0.10	0.17	0.33	0.40	0.19	0.37	0.70	0.84	0.55	0.97	1.88	2.26	1.02	1.86	3.74	4.55
Standard deviation, σ	0.47	0.59	0.89	1.02	0.47	0.59	0.90	1.04	0.55	0.71	1.12	1.30	0.67	0.89	1.48	1.76
90% Confidence +	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.7	1.7	1.7	1.7	1.7	1.7	1.7
90% Confidence −	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.6	1.6	1.5	1.5

[a]Each result is based on 1,000 simulations of significant wave height. All units are in meters except for 90% confidence intervals which are in standard deviation, σ, units.

TABLE 5.—Significant Wave Height Statistical Summaries for Very High Wave Height Region*

Uncertainty, ϵ_r	0%				10%				20%				30%			
Return period, in years, with "true" mean (1)	5 15.28 (2)	10 16.91 (3)	50 20.20 (4)	100 21.51 (5)	5 15.28 (6)	10 16.91 (7)	50 20.20 (8)	100 21.51 (9)	5 15.28 (10)	10 16.91 (11)	50 20.20 (12)	100 21.51 (13)	5 15.28 (14)	10 16.91 (15)	50 20.20 (16)	100 21.51 (17)
(a) N = 5																
Mean	16.04	18.19	22.79	24.73	16.07	18.31	23.14	25.17	16.42	18.95	24.51	26.89	16.93	19.96	26.89	29.95
Bias	0.76	1.28	2.59	3.22	0.79	1.40	2.94	3.66	1.14	2.04	4.31	5.38	1.65	3.05	6.69	8.44
Standard deviation, σ	2.23	3.20	5.84	7.12	2.36	3.23	5.86	7.12	2.41	3.49	6.49	7.97	2.84	4.24	8.44	10.63
90% Confidence +	1.7	1.8	1.9	1.9	1.9	1.9	2.0	2.0	1.8	1.9	2.0	2.0	1.8	1.9	1.9	1.9
90% Confidence −	1.5	1.4	1.3	1.2	1.5	1.4	1.3	1.3	1.5	1.5	1.3	1.3	1.5	1.4	1.3	1.2
(b) N = 10																
Mean	15.75	17.68	21.70	23.34	15.80	17.82	22.06	23.80	16.03	18.29	23.09	25.09	16.52	19.23	25.17	27.72
Bias	0.47	0.77	1.50	1.83	0.52	0.91	1.86	2.29	0.75	1.38	2.89	3.58	1.24	2.32	4.97	6.21
Standard deviation, σ	1.50	2.04	3.44	4.09	1.49	2.03	3.43	4.09	1.61	2.25	3.95	4.76	1.86	2.66	4.90	6.00
90% Confidence +	1.8	1.8	1.9	1.9	1.7	1.8	1.8	1.8	1.7	1.7	1.8	1.8	1.7	1.8	1.9	1.9
90% Confidence −	1.5	1.5	1.4	1.4	1.5	1.5	1.4	1.4	1.6	1.5	1.4	1.4	1.6	1.5	1.4	1.4
(c) N = 20																
Mean	15.57	17.40	21.16	22.67	15.65	17.55	21.49	23.09	15.84	17.95	22.38	24.20	16.32	18.85	24.32	26.62
Bias	0.29	0.49	0.96	1.16	0.37	0.64	1.29	1.58	0.56	1.04	2.18	2.69	1.04	1.94	4.12	5.11
Standard deviation, σ	1.01	1.34	2.17	2.56	1.01	1.35	2.23	2.63	1.10	1.49	2.53	3.02	1.29	1.79	3.16	3.82
90% Confidence +	1.8	1.8	1.8	1.9	1.6	1.6	1.7	1.7	1.7	1.7	1.8	1.8	1.8	1.8	1.8	1.9
90% Confidence −	1.5	1.5	1.5	1.5	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
(d) N = 40																
Mean	15.47	17.23	20.81	22.25	15.51	17.33	21.04	22.54	15.73	17.77	22.00	23.72	16.15	18.55	23.68	25.81
Bias	0.19	0.31	0.61	0.74	0.23	0.42	0.84	1.03	0.45	0.86	1.80	2.21	0.87	1.64	3.48	4.30
Standard deviation, σ	0.70	0.91	1.45	1.70	0.70	0.91	1.46	1.71	0.76	1.01	1.69	2.00	0.87	1.17	2.01	2.41
90% Confidence +	1.7	1.8	1.8	1.8	1.7	1.6	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.8	1.8
90% Confidence −	1.6	1.6	1.5	1.5	1.6	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.5

*Each result is based on 1,000 simulations of significant wave height. All units are in meters except for 90% confidence intervals which are in standard deviation, σ, units.

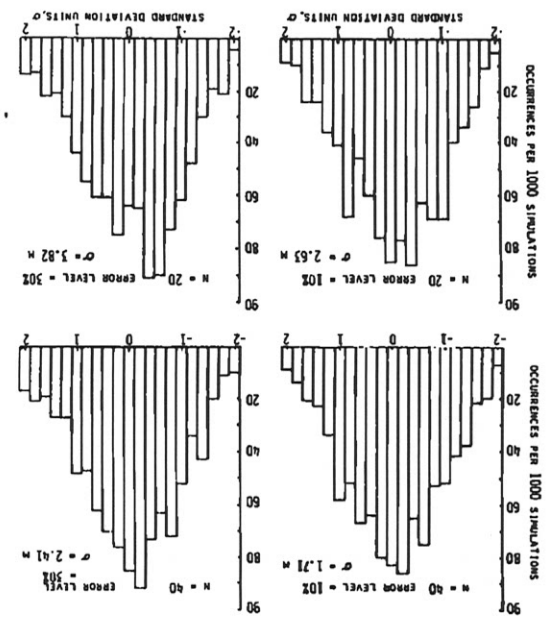

FIG. 3.—Histograms of 100-Year Return Period Significant Wave Heights in Class Intervals of 0.2 Standard Deviations about Mean Value for Very High Wave Height Region

duplicate simulations with 1,000 records showed that there are small statistical variability effects for short record lengths combined with high uncertainties. Upper 90% confidence limits are greater than lower 90% confidence limits with decreasing asymmetry for longer record lengths. Asymmetry of confidence limits is due to the same factors that produce positive biases and the use of logarithms of significant wave height.

Although extreme significant wave height histograms are not normal, a normal distribution defined by the sample mean and standard deviation for 1,000 simulations provides reasonable estimates of confidence intervals, especially for long record lengths, short return periods, and low uncertainties. For a normal distribution, a 90% confidence interval is given by ±1.645 standard deviations from the mean. Deviations of the confidence limits from consistent patterns in Tables 3–5 are due to histogram resolution and statistical variations expected for 1,000 simulations. Examination of variability of occurrences between histogram class intervals showed that confidence intervals in Table 3–5 are accurate to approximately ±0.1 standard deviation.

For the low wave height region and very high wave height region, biases and standard deviations shown in Tables 3–5 are plotted as functions of return period, record length, and error level in Figs. 4–11. Results for the high wave height region would be similar if plotted. These figures are drawn to the same scale for comparison and illustrate the following behavior: increasing bias and standard deviation with increasing error, decreasing bias and standard deviation with increasing record length, and decreasing bias and standard deviation with decreas-

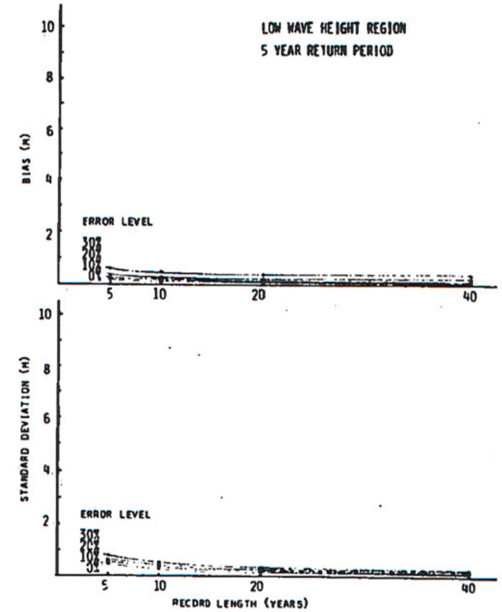

FIG. 4.—Bias and Standard Deviation of Significant Wave Height for Low Wave Height Region at 5-Year Return Period

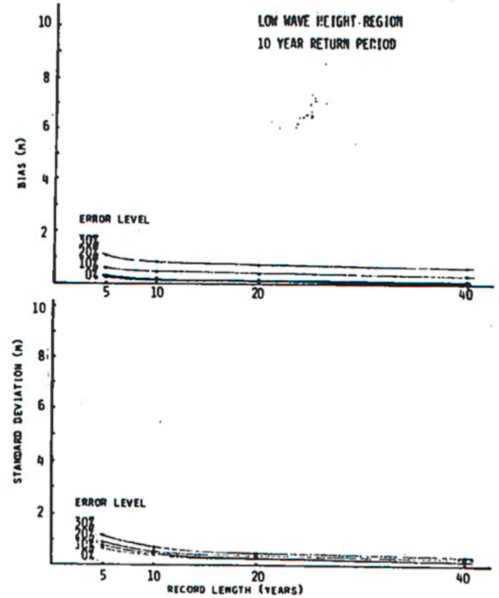

FIG. 5.—Bias and Standard Deviation of Significant Wave Height for Low Wave Height Region at 10-Year Return Period

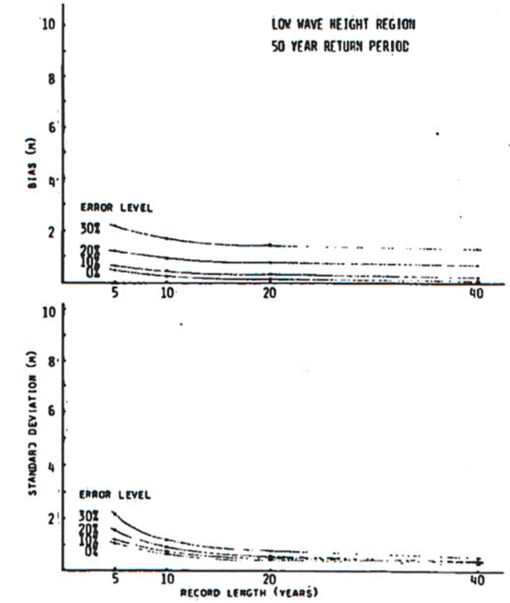

FIG. 6.—Bias and Standard Deviation of Significant Wave Height for Low Wave Height Region at 50-Year Return Period

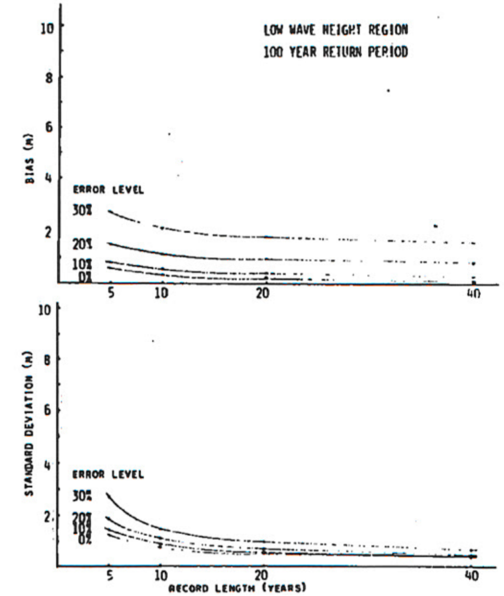

FIG. 7.—Bias and Standard Deviation of Significant Wave Height for Low Wave Height Region at 100-Year Return Period

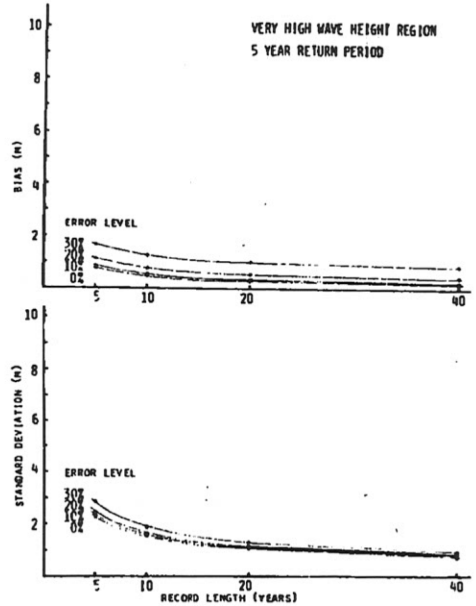

FIG. 8.—Bias and Standard Deviation of Significant Wave Height for the Very High Wave Height Region at 5-Year Return Period

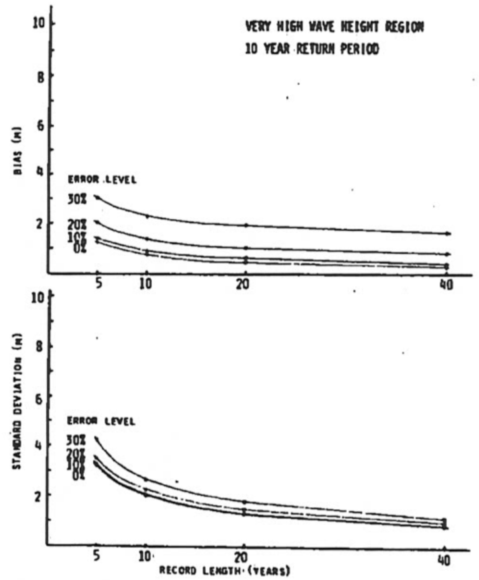

FIG. 9.—Bias and Standard Deviation of Significant Wave Height for Very High Wave Height Region at 10-Year Return Period

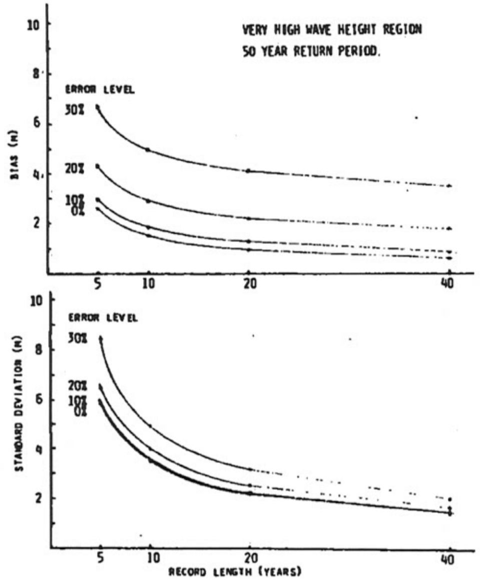

FIG. 10.—Bias and Standard Deviation of Significant Wave Height for Very High Wave Height Region at 50-Year Return Period

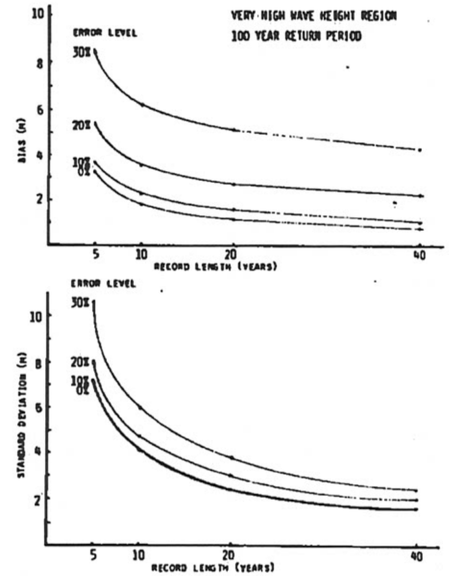

FIG. 11.—Bias and Standard Deviation of Significant Wave Height for Very High Wave Height Region at 100-Year Return Period

ing return period. There are smaller differences between results for 0% and 10% uncertainties than between results for larger uncertainties. Relative slopes of the curves in these figures indicate that 100 year return period significant wave heights should not be based on records shorter than approximately 30 years for the very high wave height region or shorter than approximately 20 years for the low wave height region.

For planning purposes, Figs. 4–11 or Tables 3–5 may be used to estimate the relative effects of record lengths and uncertainties. If uncertainties associated with wave measurements or hindcasts are known, the bias and/or standard deviation of significant wave heights for a given return period can be estimated as a function of record length. In practice, measurement uncertainties will usually not be less than approximately 10% due to short-term sampling variability alone. Uncertainties of 20% to 30% (at 90% confidence) are probably more typical.

Confidence intervals can be calculated from standard deviations by the factors in Tables 3–5. These can be considered with damage or loss consequences for the project of concern to perform risk analyses and establish safety factors. The value of longer record lengths can be weighted against the cost of obtaining longer record lengths. Likewise, the practical value of increasing measurement or hindcast accuracies for a given record length or a range of record lengths can be examined. One planning application is calculation of error and record length combinations that provide the same approximate confidence intervals for a selected return period. For example, interpolation within Table 5 shows that 40 years of H values with 30% error is roughly equivalent in terms of confidence interval width to 31 years with 20% error and 23 years with 10% error for estimation of 100 year return period significant wave heights. This table also indicates that 5 and 10 years of measured data would be nearly useless for reasonable determination of 100 year wave heights for the very high wave height region. Similar results for even shorter record lengths have been found using visual wave observations at an ocean weather station (19).

These results show that biases can be removed if the distribution of uncertainties in measurements or hindcasts is known. Only knowing maximum values for uncertainties is not adequate. Similar knowledge is needed to determine confidence intervals.

Effects of measurement or hindcast model uncertainties and record lengths for distributions that are not log-normal can be inferred from results of this study. In general, for wider distributions with longer tails, there are higher probabilities for extremes, resulting in biases and confidence intervals that are wider than for more narrow distributions. The width of a probability distribution depends on both the functional form of the distribution and values of its parameters. Inferences for other distributions can be made by selection of log-normal parameters, μ and σ, so that the two distributions match as closely as possible. From the relative widths and tail shapes of the distributions, it can be estimated whether this study's results are conservative or nonconservative. For particularly critical applications, the procedures used for this study could be repeated for a specific probability distribution.

Confidence Interval Comparisons

Comparisons were made between confidence intervals estimated by computer

simulations and confidence intervals calculated by other procedures (11,13,9) that are sometimes used. For these procedures, it is assumed that extreme events, here log H, for a given return period are distributed normally about the value estimated by extrapolation from finite records. If $(\log H)_R$ is the estimated value of log H at return period R, confidence limits of α percent are given by

$$(\log H)_R - z_{(\alpha+100)/2}S_{ER} \leq (\log H)'_R < (\log H)_R + z_{(\alpha+100)/2}S_{ER} \qquad (12)$$

in which $(\log H)'_R$ = the true value of $(\log H)_R$; z_γ = the standard normal deviate for cumulative probability, γ; and S_{ER} = a standard error for a normal distribution given by

$$S_{ER} = \left(1 + \frac{z_R^2}{2}\right)^{1/2} \frac{\sigma}{N^{1/2}} \qquad (13)$$

in which z_R = the standard normal deviate for the cumulative probability associated with R; σ = the sample standard deviation; and N = the number of sample data points. For return periods of 5, 10, 50, and 100 years, z_R has values of 0.842, 1.282, 2.054, and 2.326. For the frequently used 100-year return period and the very high wave height region, Table 6 compares confidence intervals estimated from the simulations and from Eq. 12. The standard deviations, σ, used in Eq. 13 were calculated from the log-normal distribution lines fitting the mean values of H as a function of return period that are given in Table 5. Confidence intervals for logarithms were converted to significant wave heights for the comparison. There are wider confidence intervals for the simulation method than for the theoretical method given by Eqs. 12 and 13.

While standard deviations from individual records may be greater than or less than the σ values used, the most probable standard deviations are near these values. Of practical consequence, an abnormally low σ for a single record may result in severe underestimation of confidence intervals by the theoretical method. Similarly, an abnormally high σ for a single record could cause overestimation of the upper confidence limit by the theoretical method. Incorporation of measurement or hindcast uncertainties increases potential differences between the theoretical method and the simulation method when the theoretical method is applied to an individual record.

TABLE 6.—Comparison of Confidence Interval Computations

	Record length, in years (1)	90% Confidence Interval, in meters: Equation 12 (2)	90% Confidence Interval, in meters: Simulation method (3)	Ratio of width of confidence interval by simulation method to width of confidence interval from Equation 12 (4)
0% Error level	5	−6.61, +9.40	−8.54, +13.53	1.38
	10	−4.91, +6.29	−5.73, +7.77	1.21
	20	−3.59, +4.28	−3.89, +4.86	1.11
	40	−2.60, +2.94	−2.55, +3.06	1.01
30% Error level	5	−8.92, +13.63	−12.76, +20.20	1.46
	10	−6.69, +9.03	−8.40, +11.40	1.26
	20	−4.92, +6.09	−5.73, +7.26	1.18
	40	−3.59, +4.17	−3.62, +4.34	1.03

Another method (11,22) for estimation of confidence intervals which is often used is one based on the fit of a regression line to data. This method estimates confidence intervals from the "*t*" distribution and the calculated standard deviation of the ranked events from the least-squares line which represents a particular cumulative probability distribution. Unfortunately, as shown by the the examples in Fig. 2 and by examples from other simulation results for wave heights (20), deviations of sample values from a least-squares line for a single record are not necessarily good indications of how well the least-squares line matches an underlying distribution. Calculations of the variances of log H values from the corresponding least-squares lines for the simulations showed that a confidence interval estimated by the the fit of a single record to a least-squares line generally underestimates the actual confidence interval.

Conclusions and Applications

Results presented here are useful for planning wave measurement or hindcast programs and for estimating uncertainties of significant wave heights as a function of return period. Relative uncertainties due to record lengths and uncertainties for measurements or hindcasts can be assessed objectively. Of importance is the fact that measurement or hindcast uncertainties and sampling variability cause a positive bias so that calculated extreme wave heights are, on the average, considerably greater than actual extreme wave heights. Potential effects of such positive bias have not previously been considered in studies of extreme wave conditions. Both positive biases and confidence intervals increase with greater measurement or hindcast uncertainties and decrease with longer record lengths. Computed biases and confidence intervals demonstrate the potentially large uncertainties associated with extrapolation to long return periods from relatively short records or records with large uncertainties. Some applications might warrant repetition of the procedures described here with different probability distributions, different means and standard deviations to specify log-normal distributions, or different error distributions. Because partial series often have practical advantages over annual series for extreme wave height estimates, future extension of this study to consider partial series would be useful. Consideration should also be given to studies which might allow a parameterization of the bias and confidence intervals such that individual simulations would not be needed for each case. Additional simulation studies to investigate effects of climatic trends, to consider other wave parameters than wave height, and to consider statistical procedures used in hurricane regions would also be valuable.

Acknowledgment

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Appendix I.—Sources of Information for Preparation of Fig. 1

1. Ocean weather station I. northeast Atlantic Ocean (59° N, 19° W), annual highest significant height from visual observations for 17 years.
2. Ocean weather station P. Gulf of Alaska (50° N. 145° W). annual highest significant height from visual observations for 16 years.
3. Ocean weather station K. northeast Atlantic Ocean (45° N. 16° W). annual highest significant height from visual observations for 15 years.
4. Gulf of Alaska (59° N, 146° W). directional spectra hindcasts (2) for 13 years.
5. Ocean weather station B. Labrador Sea (56°30′ N, 51° W). annual highest significant height from visual observations for 13 years.
6. Near Baltimore Canyon (U.S. east coast), significant height hindcasts (9) for numerous selected hurricanes and severe extratropical storms. Line in Fig. 1 estimated from only three points so that line accuracy may be poor.
7. Near Georges Bank (U.S. east coast). significant height hindcasts (9) for numerous selected hurricanes and severe extratropical storms. Line in Fig. 1 estimated from only three points so that line accuracy may be poor.
8. Ocean weather station E, central north Atlantic Ocean (35° N. 48° W). annual highest significant height from visual observations for 13 years.
9. Seven Stones Light Vessel, northeast Atlantic Ocean (50° N, 6° W), annual highest significant height from shipborne wave recorder (4) for nine years.
10. Deep water off Georgia, significant height hindcasts (23) for numerous selected hurricanes and severe extratropical storms. Line in Fig. 1 estimated from only three points so that line accuracy may be poor.
11. Deep water off California-Oregon border. significant height hindcasts (7) for 24 years.
12. Ocean weather station N. east central North Pacific Ocean (30° N. 140° W). annual highest significant height from visual observations for 13 years.
13. Deep water off California-Mexico border, significant height hindcasts (7) for 24 years.

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