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ANALYSIS OF TAXIING INDUCED VIBRATIONS IN AIRCRAFT BY THE POWER SPECTRAL DENSITY METHOD

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SUMMARY

The r.m.s. centre of gravity accelerations and undercarriage forces are determined for a KC-135 tanker aircraft taxiing on a randomly rough runway surface at various speeds up to 260 ft/sec. The maximum r.m.s. acceleration was found to be 0.32g at a taxiing speed of 210 ft/sec. Tyre deformations were found to be of the order of 0.6 in r.m.s. and strut displacements were about 0.34 in r.m.s. The maximum forces in the tyres and struts were found to be almost identical and equal about ten per cent of the static load on the main undercarriage.

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CONTENTS

Abstract

The Power Spectral Density Method is used to determine the r.m.s. values of centre of gravity acceleration, landing gear load, stroking displacement, and tyre displacement for a flexible taxiing aircraft. The landing gear is assumed to have a linear airspring but damping is of both the hydraulic and Coulomb type. Linearisation of the damping is made by choosing an equivalent linear damping coefficient which dissipates the same average energy as the non-linear damping. It is assumed that the process is normally distributed and an iterative method is used to obtain the final value of the damping coefficient. Linear transfer functions are then obtained from which the r.m.s. response of the aircraft is determined. The aircraft is considered to respond only to forces transmitted through the main landing gear therefore only translational modes are considered and the basic model takes account of coupling between the rigid body mode and the 1st elastic mode.

The results show that for a fully loaded KC-135 tanker aircraft taxiing on the specified runway, r.m.s. c.g. accelerations reach a maximum of 0.32g at about 210 ft/sec, assuming isentropic air compression in the struts. The assumption of an isothermal process reduces this value to 0.25g. Maximum values of the strut force are about 10 per cent of the static load. Tyre deformations have a maximum r.m.s. value of about 0.6 in. whereas strut displacements are of the order of 0.34 in. The r.m.s. tyre forces were found to be almost identical to the strut forces.

Notation

A, B, C	constants in runway roughness p.s.d. function
$a_r(t)$	generalised coordinate of mode r
AxL	volume of fully extended struts
C	runway unevenness constant
C_F, C_S	viscous damping coefficient
c_e	equivalent linearised damping coefficient
C_N	total hydraulic damping coefficient at strut static equilibrium position
E_{D_V}, $E_{D_{V\|V\|}}$, E_{D_F}	average energy dissipated in time t by viscous, hydraulic and friction damping, respectively.
F	Coulomb friction force in struts
F_S	total strut force transmitted to airframe
G_{ij}	terms in frequency response functions
G	symmetric 3 x 3 matrix of the G_{ij} terms
$H(\omega)$	linearised frequency response functions
K	total linearised strut air spring stiffness
K_r	generalised stiffness in rth mode, $M_r\omega_r^2$.
k_e	stiffness of tyre springs and strut air springs in series.
K_T	total tyre stiffness
M_F, M_S, M_W, M_O	Mass
M_r	generalised mass in rth mode
n	isentropic index for air spring
V	taxiing speed
V_{FW}	strut sliding velocity
W	weight of aircraft carried by main landing gear
$W_r(x,y,t)$	deflection of airframe in mode r at point (x,y)
Z_S, Z_F, Z_W, Z_T	deflections of equivalent three mass system
β_r	rth mode structural damping ratio
ξ_r	rth mode displacement at main landing gear location

Notation ctd

σ	r.m.s. values of response
$\Phi_{FS}(\omega)$	p.s.d. of strut force
$\Phi_{ZT}(\Omega)$, $\Phi_{ZT}(\omega)$	p.s.d. of runway unevenness
Ω, ω	spatial and circular frequencies respectively
$\Omega_1, \Omega_2, \Omega_3$	p.s.d. curve matching points
ω_r	rth mode natural circular frequency
ω_b	bounce frequency of rigid aircraft
λ	wavelength of runway roughness
γ	a constant

1. INTRODUCTION

Taxiing induced vibrations in large aircraft due to runway and taxiway unevenness have been recognised as a significant factor in causing airframe metal fatigue damage and dynamic stressing, as well as discomfort for the crew and passengers. Vibration of the landing gear also causes seal wear with subsequent leakage of air and hydraulic fluid. There is thus a need to establish a reliable method of predicting aircraft dynamic response during taxiing and take-off operations. The purpose of this report is to present an analytical method of determining the random vibration response of a flexible aircraft caused by runway unevenness transmitted through the main landing gear struts. The method adopted herein was originally reported by the author in ref.1 and for the sake of completeness the details are outlined below. Pitching motion is ignored in this report but its effect has been analysed by the author in ref.2. The aircraft used in the computation of vibration response is the Boeing KC-135A (Stratotanker) in the fully loaded configuration (324,000 lb) (146,963 kg).

The method assumes that the runway unevenness profile can be represented as a stationary,Gaussian random process. Any runway or taxiway that has been surveyed to determine its profile, can be defined as a deterministic form of displacement excitation for a taxiing aircraft. In that case a deterministic solution of the equations of motion will yield time histories of the response quantities required. In general however it is not possible to relate each part of the response time history to a particular part of the runway profile. An exception to this occurs when a runway has a section which is considerably more uneven than the rest of the runway length. In that event it is usually obvious which part of the response was caused by the patch of excessive roughness.

When a runway profile has the same degree of unevenness at all points along its length (i.e. no sections of outstanding roughness) it can be considered as a stationary random process. This means that its statistical properties such as the probability distribution are independent of the position along the length of the runway. If an aircraft is expected to use a large number of unsurveyed runways and taxiways the assumption that their profiles are random and therefore similar to known surveyed runways can enable estimates of taxiing induced random vibrations to be made.

While accepting the fact that the assumption of stationary,Gaussian random process is convenient for the purposes of analysis it must be borne in mind that on real runways this assumption is seldom completely justified. This is because unevenness usually occurs in patches rather than in a continuous manner. Thus the major failing of the statistical or power spectral approach is that it does not enable the aircraft response to be determined at any particular section of runway but only yields an 'average' or root mean square value of response. In spite of this disadvantage the power spectral approach is considered to be of some considerable usefulness in estimating fatigue effects in airframes and landing gear units. The method is also of value in investigating the effect of parameter variations such as strut damping, Coulomb friction and airspring stiffness, on taxiing induced vibrations, in the r.m.s. sense.

II. ANALYTICAL METHOD

1. Power Spectral Density of Runway Unevenness

The power spectral density of runway unevenness represents a continuous variation of runway profile amplitude as a function of wavelength λ. A simple analytical form of the power spectral density (P.S.D.) is given by

$$\Phi(\Omega) = \frac{C}{\Omega^2} \text{ ft}^2/\text{rad/ft. } (\text{m}^2/\text{rad/m}) \quad \dots (1)$$

where C = an unevenness constant, $\Omega = 2\pi/\lambda$, λ being the wavelength.

At a constant taxiing speed V, eq.1 can be written

$$\Phi(\omega) = \Phi(\Omega)/V = CV/\omega^2 \cdot ft^2/rad/sec. \quad (m^2/rad/sec) \qquad \ldots (2)$$

where ω is the circular frequency in rad/sec. Eq.2 shows that the input P.S.D. is proportional to V. In the present report an actual P.S.D. curve is used (supplied by A.F.F.D.L.) in place of eq.2. $\Phi(\Omega)$ is shown in figure 1. It is necessary in the analytical work to use a continuous analytical expression for $\Phi(\omega)$. Table 1 gives values of $\Phi(\Omega)$ measured for various values of Ω from figure 1, using the centre line P.S.D. curve. Thus although the wheel track of the KC-135A is 22 ft (6.6 m) it is assumed that the P.S.D. of the runway input to each strut is the same at all times. This implies that the aircraft performs no rolling motion about the longitudinal axis but only vertical heaving motion. The P.S.D. function is described by the expression

$$\Phi(\Omega) = \frac{1}{A\Omega^3 + B\Omega^2 + C\Omega} \qquad \ldots (3)$$

where A, B, C are constants which are determined by matching the function at three values of Ω, i.e. Ω_1, Ω_2, Ω_3. This procedure yields the three simultaneous equations

$$\begin{aligned} A\Omega_1^3 + B\Omega_1^2 + C\Omega_1 &= 1/\Phi(\Omega_1) \\ A\Omega_2^3 + B\Omega_2^2 + C\Omega_2 &= 1/\Phi(\Omega_2) \qquad \ldots (4) \\ A\Omega_3^3 + B\Omega_3^2 + C\Omega_3 &= 1/\Phi(\Omega_3) \end{aligned}$$

The values of A, B and C are found by solving eqs.4 by elimination. Due to the nature of the actual P.S.D. curve it is not possible to represent it accurately over the full range of Ω. Thus a different function of the form of eq.3 is determined for each taxiing speed V. Figure 2 illustrates $\Phi(\Omega)$ with three matching points. It is shown in reference 1 that most of the response of the fully loaded aircraft occurs in the region of the frequency ω_n = 11.2 rad/sec which corresponds to the major peak in the frequency response function of the heaving aircraft. Thus for a given speed V, Ω_2 is obtained from

$$\Omega_2 = \omega_n/V \qquad \ldots (5)$$

Table 2 gives the curve matching points for various values of V and Table 3 gives the corresponding values of A, B and C. Comparing eqs. 1 and 2 it can be seen that eq.3 may be written

$$\Phi_{ZT}(\omega) = \frac{1}{V\left[A(\omega/V)^3 + B(\omega/V)^2 + C(\omega/V)\right]} \quad \ldots (6)$$

which is the form used in the subsequent analysis.

2. Power Spectral Density Relationships

When a linear system is excited by a random disturbance having a P.S.D. $\Phi_y(\omega)$ applied to coordinate x of the system, the P.S.D. of the response at coordinate y is given by

$$\Phi_y(\omega) = \left|H(\omega)_{y/x}\right|^2 \Phi_x(\omega) \quad \ldots (7)$$

where $\left|H(\omega)_{y/x}\right|$ denotes the absolute value of the complex frequency response function of the system which gives the relationship between y and x for a sinusoidal input at coordinate x. The P.S.D. for acceleration is similarly given by

$$\Phi_{\ddot{y}}(\omega) = \omega^4 \Phi_y(\omega) \quad \ldots (8)$$

The r.m.s. value of y is given by

$$\left[y^2\right]^{1/2} = \left[\int_0^\infty \Phi_y(\omega)\, d\omega\right]^{1/2} \quad \ldots (9)$$

In practice finite values of the integration limits in eq.9 are used. These are given by the values of ω_1 and ω_3 in table 2.

3. Theoretical Representation of Aircraft

The P.S.D. relationships given in eqs. 7 and 8 apply only to linear systems, that is to say systems with linear stiffness and linear (or viscous) damping. In ref.1 it is shown that the dominant factors to be considered in a theoretical model of the aircraft are $(\text{velocity})^2$ damping in the strut, Coulomb friction in the strut seals, and the structural elastic modes coupled with the rigid body heaving mode. Due to the small amplitudes of stroking motion of the struts, the non-linear stiffness characteristic of the air-spring can be considered to be linear over the strut amplitude range considered.

It is thus seen that the landing gear has two main nonlinearities in the form of $(\text{velocity})^2$ orifice or hydraulic damping and Coulomb friction. In order to apply the power spectral approach it is necessary to obtain a quasi-linearised frequency response function. The method of achieving this is described further on. It is first necessary to set up a lumped-mass and spring system to represent the rigid body - first elastic mode interaction effect. Since all modes are excited by the strut force and since the strut force is determined by the resultant motion of the airframe, all the modes are essentially coupled together. However, most of the input displacement from the runway is of low frequency content, consequently the greatest part of the response occurs in the rigid body heaving mode and the first elastic mode. If the oleo force resulting from the interaction between the rigid body mode and the first elastic mode is determined, it can then be used to find the response in the individual higher modes assuming them to be uncoupled from each other and from the rigid body mode.

Let us now consider the coupling between the rigid body mode and the first elastic mode of the airframe. It is assumed that only symmetric wing bending occurs and that identical P.S.D. inputs exist at each strut. For the rigid mode the equation of motion is

$$M_o \ddot{a}_o = -F_s \qquad \ldots (10)$$

For the first elastic mode

$$M_1 \ddot{a}_1 + c_1 \dot{a}_1 + M_1 \omega_1^2 a_1 = -F_s \xi_1 \qquad \ldots (11)$$

where ξ_1 is the modal displacement at the attachment point of the strut and the airframe, M_o is the total mass of the aircraft supported by the main landing gear, M_1 is the generalised mass of the first mode, ω_1 the first mode natural frequency and c_1 its linear structural damping coefficient. F_S denotes the force transmitted to the airframe through the two struts. The next step is to determine an equivalent two mass system consisting of masses M_F and M_S connected by a spring of stiffness K_S and a linear damper with coefficient C_S, which represents the required two-mode interaction. Figure 3 shows the equivalent system. The representation of the strut and wheel shown below mass M_F is independent of the system resting on the strut. The equations of motion of M_F and M_S are

$$M_F\ddot{Z}_F + C_S(\dot{Z}_F-\dot{Z}_S) + K_S(Z_F-Z_S) = -F_S \qquad \text{... (12)}$$

and

$$M_S\ddot{Z}_S + C_S(\dot{Z}_S-\dot{Z}_F) + K_S(Z_S-Z_F) = 0 \qquad \text{... (13)}$$

Adding eqs.12 and 13 gives the equilibrium condition

$$M_F\ddot{Z}_F + M_S\ddot{Z}_S = -F_S \qquad \text{... (14)}$$

It is seen that the motion of Z_S and Z_F will contain components of the rigid body mode and the first elastic mode. We thus write

$$Z_S = a_o + a_1\gamma \quad \text{and} \quad Z_F = a_o + a_1\,\xi_1 \qquad \text{... (15)}$$

where γ is a constant to be determined.

Inserting eqs.15 into eqs.12 and 14, it is found by comparing coefficients of the various terms that

$$M_F = \frac{M_1 M_o}{\left[M_1+M_o\xi_1^{\,2}\right]}, \quad M_S = \frac{M_o^{\,2}\xi_1^{\,2}}{\left[M_1+M_o\xi_1^{\,2}\right]}, \quad M_O = M_F+M_S$$

$$C_S = \frac{C_1 \cdot M_o^{\,2}\xi_1^{\,2}}{\left[M_1+M_o\xi_1^{\,2}\right]^2}, \quad K_S = M_1\omega_1^{\,2}\cdot\frac{M_o^{\,2}\xi_1^{\,2}}{\left[M_1+M_o\xi_1^{\,2}\right]^2}, \quad \gamma = \frac{-M_1}{\xi_1 M_o}$$

Assuming a static force on the two main struts of 296,000 lbf (1.316×10^6 N) and taking the modal data given for the Boeing 707 in ref.3 as M_1 = 370 lb.sec.^{2} ft. (501.3 kg m^2), ξ_1 = -0.122 ft (0.037 m), ω_1 = 7.22 rad/sec,

c_1 = 133 lbf sec/ft (1940 N sec/m), the following values are obtained : M_S = 2480 lb sec^2/ft (36.19 x 10^3 kg), M_F = 6720 lb sec^2/ft (98.07 x 10^3 kg), C_S = 655 lbf sec/ft (9559 N sec/m), K_S = 94500 lbf/ft (1.379 x 10^6 N/m).

The mass of the wheel, tyre and lower part of the strut for both units is taken to be M_W = 105 lb sec^2/ft (1532 kg). c_1 was determined by assuming a structural damping ratio of 0.025 of critical. The total tyre stiffness for both wheel sets is taken as K_T = 1.16 x 10^6 lbf/ft (16.92 x 10^6 N/m).

4. Frequency Response Functions for Three-Degree-of-Freedom System

The linear frequency response fuctions of the system shown in figure 3 are obtained by neglecting the non-linear elements and assuming that the strut damping is viscous and that Coulomb friction is absent. The equations of motion in Laplace transform notation are

$$(M_S s^2 + C_S s + K_S)Z_S - (C_S s + K_S)Z_F = 0$$

$$-(C_S s + K_S)Z_S + (M_F s^2 + C_S s + C_V s + K_S + K)Z_F - (C_V s + K)Z_W = 0$$

$$-(C_V s + K)Z_F + (M_W s^2 + C_V s + K + K_T)Z_W = K_T Z_T \qquad \ldots (17)$$

where C_V is the viscous damping coefficient.

The complex frequency response function is obtained by writing s = iω in eqs.17 which are then written in matrix form thus

$$\begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & G_{23} \\ 0 & G_{32} & G_{33} \end{bmatrix} \begin{Bmatrix} Z_S \\ Z_F \\ Z_W \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ K_T Z_T \end{Bmatrix} \qquad \ldots (18)$$

or $$[G]\{Z\} = \{K_T Z_T\} \qquad \ldots (19)$$

where, writing $C_V = C$,

$$G_{11} = -M_S\omega^2 + C_S i\omega + K_S , \; G_{12} = G_{21} = -(C_S i\omega + K_S),$$

$$G_{22} = -M_S\omega^2 + C_S i\omega + Ci\omega + K_S + K, \; G_{23} = G_{32} = -(Ci\omega + K)$$

$$G_{33} = -M_W\omega^2 + Ci\omega + K + K_T \qquad \ldots (20$$

From eq.19

$$\{Z\} = [G]^{-1}\{K_T Z_T\}$$

or

$$\begin{Bmatrix} Z_S \\ Z_F \\ Z_W \end{Bmatrix} = \frac{1}{\text{det.G}} \begin{bmatrix} (G_{22}G_{33}-G_{23}^2), & (-G_{12}G_{33}), & (G_{12}G_{32}) \\ (-G_{12}G_{33}), & (G_{11}G_{33}), & (-G_{11}G_{23}) \\ (G_{12}G_{23}), & (-G_{11}G_{23}), & (G_{11}G_{22}-G_{12}^2) \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ K_T Z_T \end{Bmatrix} \quad \ldots (21)$$

where

$$\text{det.G} = G_{33}(G_{11}-G_{22}-G_{12}^2) - G_{23}^2 G_{11} \quad \ldots (22)$$

From eq.21 by equating the terms in each of the three rows on both sides, we find the following frequency response functions,

$$\frac{Z_S(\omega)}{Z_T(\omega)} = \frac{G_{12}G_{32}}{\text{det.G}} \cdot K_T \quad \ldots (23)$$

$$\frac{Z_F(\omega)}{Z_T(\omega)} = \frac{-G_{11}G_{23}}{\text{det.G}} \cdot K_T \quad \ldots (24)$$

$$\frac{Z_W(\omega)}{Z_T(\omega)} = \frac{(G_{11}G_{22}-G_{12}^2)}{\text{det.G}} \cdot K_T \quad \ldots (25)$$

$$\frac{Z_W(\omega)-Z_T(\omega)}{Z_T(\omega)} = \frac{Z_W(\omega)}{Z_T(\omega)} - 1 \quad \ldots (26)$$

$$\frac{Z_F(\omega)-Z_W(\omega)}{Z_T(\omega)} = \frac{\left[G_{12}^2-G_{11}(G_{22}+G_{23})\right]}{\text{det.G}} \cdot K_T \quad \ldots (27)$$

5. Approximate P.S.D. Analysis of Three-Degree-of-Freedom System with (velocity)^2 Damping and Coulomb Friction

In ref.1 it is shown that for a single-degree-of freedom system containing (velocity)^2 damping, the approximate frequency response function at any frequency of excitation is not a constant, but depends on the magnitude of the input. This behaviour is characteristic of systems with non-linear damping. In order to use the power spectral approach, making use of the linear frequency response functions given in eqs.23 through 27, it is necessary to linearise the frequency response function by obtaining an equivalent linear damping coefficient c_e. The method involves choosing a value of c_e so that the energy dissipated by the non-linear damping during random vibration is the same as that dissipated by the equivalent viscous damping.

In time t the energy dissipated by the linear damping is

$$E_{Dv} = c_e \int_0^t V_{FW}^2 \, dt. \qquad \ldots (28)$$

where V_{FW} denotes the strut stroking velocity $(\dot{Z}_F - \dot{Z}_W)$.

The mean square value of V_{FW} is

$$\sigma_{V_{FW}}^2 = \frac{1}{t} \int_0^t V_{FW}^2 . dt \qquad \ldots (29)$$

therefore the average energy dissipated in time t is

$$E_{D_V} = c_e . \sigma_{V_{FW}}^2 . t \qquad \ldots (30)$$

Assuming that for small strut amplitudes the $(\text{velocity})^2$ damping coefficient C_N remains constant at the strut static equilibrium value, the average energy dissipated during time t by the $(\text{velocity})^2$ damping is

$$E_{D_{V|V|}} = C_N \int_0^t V_{FW}^2 \left| V_{FW} \right| . \, dt \qquad \ldots (31)$$

Now

$$\frac{1}{t} \int_0^t V_{FW}^2 \left| V_{FW} \right| dt = \int_{-\infty}^{\infty} V_{FW}^2 . \left| V_{FW} \right| . \; p(V_{FW}) . dV_{FW} \qquad \ldots (32)$$

where $p(V_{FW})$ denotes the probability density function for the strut sliding velocity. If a Gaussian probability distribution is assumed the integrand of the right hand side of eq.32 is an even function, hence eq.32 becomes

$$\frac{1}{t}\int_0^t V_{FW}^2 \left|V_{FW}\right| dt = 2\int_0^\infty V_{FW}^3 . p(V_{FW}).dV_{FW}$$

... (33)

The Gaussian probability density function is given by

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-x^2/2\sigma^2)$$

The integral in eq.33 is given in ref.5 in the form

$$\int_0^\infty \frac{x^3 \exp(-x^2/2\sigma^2)dx}{\sigma\sqrt{2\pi}} = \sigma^3 \sqrt{2/\pi} \qquad \text{... (34)}$$

Making use of this result in eq.33 and substituting in eq.31 it is found that the average energy dissipated by $(\text{velocity})^2$ damping is

$$E_{D_{V|V|}} = 2\sqrt{2/\pi}\ \sigma_{V_{FW}}^3 . C_N.t \qquad \text{... (35)}$$

The value of C_N was taken from ref.3 as 21600 lbf/(ft/sec)^2 (103 x 10^4N/(m/sec))^2.

Similarly for the Coulomb friction force F.signV_{FW},

$$\frac{E_{D_F}}{t} = \frac{F}{t}\int_0^t V_{FW}(\text{sign}.V_{FW})dt = F\int_{-\infty}^{\infty} V_{FW}.p(V_{FW})\text{sign}V_{FW}.dV_{FW}$$

$$= 2F\int_0^\infty V_{FW}.p(V_{FW}).dV_{FW} \qquad \text{... (36)}$$

Making use of the integrals in ref. 5 to evaluate eq.36 it is found that the energy dissipated during random vibration by friction is

$$E_{D_F} = F\sqrt{2/\pi}\ \sigma_{V_{FW}}.t \qquad \text{... (37)}$$

where F is the friction force for both main struts, assumed in this report to be equal to 2000 lbf (8896N).

The total energy dissipated during random vibration in time t by the nonlinear damping is

$$E_D = \left(2\sqrt{2/\pi}\, C_N \cdot \sigma_{V_{FW}}^3 + F\sqrt{2/\pi}\, \sigma_{V_{FW}} \right) t \qquad \ldots (38)$$

Equating eqs. 30 and 38 the equivalent viscous damping is obtained as a function of the r.m.s. value of the stroking velocity of the strut, thus

$$c_e = 2\sqrt{2/\pi}\, C_N \cdot \sigma_{V_{FW}} + \left(F\sqrt{2/\pi} \Big/ \sigma_{V_{FW}} \right) \qquad \ldots (39)$$

This value of c_e may be inserted into any frequency response function in place of the actual viscous damping coefficient.

It should be noted that in computing the energy loss due to Coulomb friction it has been assumed that the strut is in continuous motion and is not caused to stick due to friction. This assumption is justifiable if the r.m.s. inertia force of the centre of gravity of the aircraft is considerably greater than the friction force in the struts. The results of this report show that this is indeed the case. The r.m.s. oleo stroking velocity in eq.39 is found from the relationship

$$\sigma_{V_{FW}} = \left[\int_{\omega_1}^{\omega_2} \omega^2 \Phi_{ZT}(\omega) \left| H_{(ZF-ZW)/ZT}(\omega) \right|^2 d\omega \right]^{1/2} \qquad \ldots (40)$$

where $H_{(ZF-ZW)/ZT}(\omega)$ is obtained from eq. 27.

The flow diagram of figure 4 illustrates the method used to compute the equivalent damping coefficient which was performed by means of an ICL 1905 digital computer.

The digital computer program was written in Fortran 4A and a typical program is shown in the Appendix.

A starting value of c_e equal to C_{max} was chosen being equivalent to a damping ratio of 0.415, being much higher than the expected final damping ratio. With this initial value of C, the various G_{ij} terms of eq.20 were computed for values of ω in the range of the integration limits of eq.40. The interval between the successive incremental values of ω depended on the accuracy specified in the integration by Simpson's rule. The next step is to evaluate eq.40 and then compute c_e from eq.39, this value being compared with the starting value. A cyclic process was then carried out until convergence to a final value of c_e was achieved, after which the r.m.s. values of the various quantities were obtained by using eq.6 with eqs.23 through 27. It can be seen that the r.m.s. quantities available at this point are $\ddot{Z}_F$ (the acceleration at the centre of gravity), $Z_F - Z_W$ (the stroke of the strut), and $Z_W - Z_T$ (the tyre compression).

A further quantity of interest is F_s the r.m.s. strut force transmitted to the airframe. This force is equal to the difference between the r.m.s. tyre force and the r.m.s. wheel inertia force, thus deterministically

$$F_s = K_T(Z_W - Z_T) - M_W\ddot{Z}_W \qquad \ldots (41)$$

In the subsequent calculations it was shown that the wheel inertia force was negligible compared with the tyre force and can therefore be ignored. Thus the P.S.D. of the strut force and tyre force is given by

$$\Phi_{F_s}(\omega) = K_T^2 \, \Phi_{ZW-ZT}(\omega) \qquad \ldots (42)$$

It should be recalled that all calculations carried out so far have been based upon frequency response functions which take account of rigid body-first symmetric elastic mode interaction effects. Contributions to the acceleration at the centre of gravity from higher elastic modes are obtained by assuming that each mode is excited independently by the strut force F_s. The assumption that no coupling occurs between the elastic modes can only be justified if their natural frequencies are well separated, as in the case of the KC-135A.

To determine the r.m.s. generalised acceleration $\ddot{a}_r$ in the rth elastic mode, for $r \geqslant 2$, we use the following expression

$$\left| H(\omega)_{\ddot{a}_r/F_s} \right|^2 = \frac{\omega^4(\xi_r/K_r)^2}{\left[(1-\omega^2/\omega_r^2)^2 + (2\beta_r\ \omega/\omega_r)^2\right]} \quad \ldots (43)$$

where ξ_r denotes the rth modal displacement at the strut-airframe attachment point.

The P.S.D. of $\ddot{a}_r$ is then found from eqs.42 and 43 which give

$$\Phi_{\ddot{a}_r}(\omega) = K_T^2 \left| H(\omega)_{\ddot{a}_r/F_s} \right|^2 \Phi_{ZW-ZT}(\omega) \quad \ldots (44)$$

The mean square value of $\ddot{a}_r$ is given by

$$\overline{\ddot{a}_r^2} = \int_{\omega_1}^{\omega_2} \Phi_{\ddot{a}_r}(\omega)\, d\omega \quad \ldots (45)$$

$$r = 2, 3, \ldots n.$$

The deterministic acceleration at any point on the airframe in mode r is given by

$$\ddot{W}_r(x,y,t) = \ddot{a}_r(t).\phi_r(x,y) \quad \ldots (46)$$

where $\phi_r(x,y)$ denotes the rth normal mode of the airframe. Hence the P.S.D. of the acceleration of the airframe is given by

$$\Phi_{\ddot{W}_r}(\omega) = \Phi_{\ddot{a}_r}(\omega).\ \phi_r^2(x,y) \quad \ldots (47)$$

Since it has been assumed that all the higher modes are uncoupled from each other, the random vibration response in each mode will be uncorrelated with that in any other mode.

Thus the P.S.D. of the total acceleration is obtained by merely adding the individual P.S.D. from each mode as follows

$$\Phi_{\ddot{W}}(\omega) = \sum_{r=0}^{n} \Phi_{\ddot{a}_r}(\omega) \,.\, \phi_r^{\,2}(x,y) \qquad \ldots (48)$$

The total r.m.s. acceleration is then found from

$$\ddot{W}(x,y)_{r.m.s.} = \left[\int_{\omega_1}^{\omega_2} \Phi_{\ddot{W}}(\omega)\,d\omega\right]^{1/2}$$

$$= \left[\sum_{r=0}^{n} \overline{\ddot{a}_r^{\,2}} \,.\, \phi_r^{\,2}(x,y)\right]^{1/2} \ldots (49)$$

For example the r.m.s. acceleration at the centre of gravity, taking into account the rigid body mode and the first two elastic modes is

$$(\ddot{W}_g)_{r.m.s.} = \left[\overline{\ddot{Z}_F^{\,2}} + \overline{\ddot{a}_2^{\,2}}\ \xi_2^{\,2}\right]^{1/2} \qquad \ldots (50)$$

A useful expression regarding the r.m.s. modal accelerations is found by reference to eqs.44 and 45 from which is obtained

$$(\ddot{a}_r)_{r.m.s.}\ \alpha (K_T\ \xi_r/K_r) \qquad \ldots (51)$$

where α denotes the proportionality sign.

6. Natural Frequency of Aircraft on Tyres and Struts

In eqs.17, K denotes the linearised stiffness of the two strut airsprings. Justification for assuming that the airspring stiffness can be linearised is based on the small stroking r.m.s. displacements in the struts. The stiffness of the struts is taken to be equal to the slope of the spring force-stroke characteristic at the static equilibrium position. The strut springs and tyre springs are in series and the total stiffness is shown in ref.1 to be given by

$$k_e = \frac{K_T nW^2}{K_T . P_0 AL + nW^2} \qquad \ldots (52)$$

where K_T is the tyre stiffness, n the polytropic compression index, W the weight of the aircraft carried on the main struts, P_0 the initial air pressure in the fully extended strut, L the total stroke of the strut. The stiffness of the airspring alone is given by

$$K = \frac{nW^2}{P_o AL} \qquad \ldots (53)$$

For a rigid aircraft the bounce frequency can be calculated from

$$\omega_b = (k_e/M)^{1/2} \qquad \ldots (54)$$

For the KC-135A, assuming $K_T = 1.16 \times 10^6$, (16.92 $\times 10^6$ N/m), W = 296,000 lb (134,262 kg), P_oA = 29,600 lbf (129,000 N), n = 1.3, it is found that K = 2.03 $\times 10^6$ lbf/ft (29.6 $\times 10^6$ N/m) and the bounce frequency is, for the rigid aircraft, ω_b = 9 rad/sec. If n is taken as unity (isothermal compression) K is reduced to 1.5615 $\times 10^6$ lbf/ft (22.77 $\times 10^6$ N/m).

In the present report the coupling between the rigid body mode and the first elastic mode leads to the dynamic system shown in figure 3. In ref. 1 the frequency response function is shown to have two main peaks, the smallest being at about 6 rad/sec and the greatest at about 11.0 rad/sec. Most of the response to random inputs occurs at the 11.0 rad/sec peak where the response in the first elastic mode is predominantly 180^o out of phase with the centre of gravity motion. Thus, due to elastic coupling the main bounce of the aircraft is no longer at 9 rad/sec but at 11.0 rad/sec.

In the computer program a value of n = 1 was first assumed, being the value used in ref.3. Experimental work at Cranfield on a small strut indicates that a value of n = 1.2 is an average value for random inputs. The value of the airspring stiffness, K, was therefore varied and the effect on the r.m.s. values of response determined. Clearly a higher value of airspring stiffness will lead to higher values of r.m.s. response. It is considered that n = 1 will give a lower bound to r.m.s. values of response and that n = 1.3 will give an upper bound.

7. Discussion of Results

Figures 5 and 6 show curves of r.m.s. acceleration and r.m.s. strut force for a wide range of taxiing speeds. The Coulomb friction force F was taken as 1000 lbf (4448N) in each strut. Two values of the linearised airspring stiffness, K, were used corresponding to isothermal compression (n = 1) and isentropic compression (n = 1.3). It can be seen that the stiffer airspring yields r.m.s. responses that are on average about 25 per cent greater than the softer airspring. Numerical values of response are given in table 4 and include

$\ddot{Z}_F/g$ - r.m.s. acceleration at centre of gravity

F_s - r.m.s. force in a single strut

Z_{FW} - r.m.s. stroke of strut

Z_{WT} - r.m.s. tyre compression

$\beta_{eq.}$ - equivalent damping ratio

It can be seen that the choice of the airspring stiffness has little influence on Z_{FW}, but that the tyre compression is increased with the stiffer airspring. The highest value of $\beta_{eq.}$ observed at the lowest taxiing speed is due to the low stroking velocity of the strut (see eq.39). Thus when the $(\text{velocity})^2$ damping is small the equivalent viscous damping due to Coulomb friction is high. This of course requires the strut to remain unlocked during taxiing. At the lowest value of taxiing speed the value of F_s is about five times the friction force in the strut, and it is considered that this is sufficient to cause the strut to remain unlocked. Higher values of F might invalidate this argument at low speed but it would become more valid as the value of V was increased. It is clear that a certain amount of Coulomb friction is beneficial in increasing strut damping at low taxiing speeds. High friction forces can however lead to increased wear of the strut seals. The low values of $\beta_{eq.}$ demonstrate the ineffectiveness of the strut in providing adequate damping during taxiing operations.

The maximum values of c.g. acceleration occur at about V = 210 ft/sec (64 m/sec) and are 0.25g and 0.32g respectively. It must be emphasised that these and all the other r.m.s. values were computed on the assumption that the runway profile is a stationary random process with a Gaussian distribution. In practice these assumptions are never completely justified and experimentally determined r.m.s. values are invariably lower than the theoretical ones. It is only by comparing experiment with theory that the degree of error can be ascertained. It should also be noted that the airspring stiffness has a maximum value when the aircraft has its greatest weight. Consequently r.m.s. values will be greatest when the aircraft is taxiing at maximum weight.

With regard to the fatigue effects of the strut force it can be seen that the maximum value of F_s is about 10 per cent of the static load carried by the strut.

8. Conclusions

This report has shown that by obtaining a quasi-linearised damping coefficient for a landing gear strut, the r.m.s. response of a taxiing aeroplane can be determined by means of the power spectral density method. The necessary assumptions in the analysis are that runway roughness can be represented as a stationary random normal process. The analysis is not restricted to the assumption of a Gaussian response and probability distributions determined from experiments could easily be used in the method.

To facilitate the analysis it was assumed that the response of the aircraft is Gaussian which is only true for a linear system. It has been shown by T.K. Caughey and discussed by the present author (ref.4), that for a single-degree-of-freedom system with $(\text{velocity})^2$ damping, the r.m.s. response is overestimated by about 22%. On that basis the results given in this report could be expected to overestimate the response to a similar extent.

As discussed on page 12 of the report, the r.m.s. tyre forces were found to be almost identical to the r.m.s. strut forces and are therefore not included separately.

Appendix 1
Computer Program

```
      MASTER RMS SOLUTION
      REALKS,KT,K,MF,MW,KR
      REALMSZWDD1,MSZWDD2,MSZWT,MSZWDD,MSFS,MSZGDD
      REALMSZFWD1,MSZFWD2,MSZF1,MSZF2,MSZFDD1,MSZFDD2,MSZWT1,MSZWT2,MSZF
     1W1,MSZFW2,MSZT1,MSZT2
      REAL MSZFWD,MSZF,MSZFDD,MSZWT,MSZFW,MSZT
      REALMSARDD1,MSARDD2,MSARDD
      REALMSRITDD1,MSRITDD2,MSROTDD1,MSROTDD2,MSROTDD,MSRITDD
      COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
      COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
     1AZW,AZWDD,MW,G23,G12,G11,DENOM,A3,B3,C3
      WRITE(2,1)
    1 FORMAT(10X,8HC L KIRK//15X,10HINPUT DATA//)
      READ(1,2)V,CN,A3,B3,C3
    2 FORMAT(5F0.0)
      WRITE(2,3)V,CN,A3,B3,C3
    3 FORMAT(10X,2HV=,F8.2,5X,3HCN=,F10.2,3HA3=,F10.3,3HB3=,F10.3,3HC3=,
     1F10.3//)
      F=2000.0
      ZETA=0.415
      ICOUNT=0
      AINC=0.1
   12 C=272000.0*ZETA
      E=0.001
      IND=5
      A=8.0
      B=10.2
      EXTERNALAZFWD2PHI
      CALLF4INTSMP(A,B,AZFWD2PHI,E,IND,MSZFWD1)
      IND=5
      A2=10.2
      B2=12.0
      EXTERNALAZFWD2PHI
      CALLF4INTSMP(A2,B2,AZFWD2PHI,E,IND,MSZFWD2)
      MSZFWD=MSZFWD1+MSZFWD2
      RMSZFWD=SQRT(MSZFWD)
      CEQ=2.0*CN*(SQRT(2.0/3.1416))*RMSZFWD+(SQRT(2.0/3.1416))*F/RMSZFWD
     1+CF
      IF(CEQ-C)10,11,11
   10 ZETA=ZETA-AINC
      GO TO 12
   11 ICOUNT=ICOUNT+1
      GO TO(21,22,23,24),ICOUNT
   21 ZETA=ZETA+0.09
      AINC=0.01
      GO TO 12
   22 ZETA=ZETA+0.009
      AINC=0.001
      GO TO 12
   23 ZETA=ZETA+0.0009
      AINC=0.0001
      GO TO 12
   24 EXTERNALAZF2PHI
      CALLF4INTSMP(A,B,AZF2PHI,E,IND,MSZF1)
      IND=5
      EXTERNALAZF2PHI
      CALLF4INTSMP(A2,B2,AZF2PHI,E,IND,MSZF2)
      MSZF=MSZF1+MSZF2
```

```
RMSZF=SQRT(MSZF)
IND=5
EXTERNALAZFDD2PHI
CALLF4INTSMP(A,B,AZFDD2PHI,E,IND,MSZFDD1)
IND=5
EXTERNALAZFDD2PHI
CALLF4INTSMP(A2,B2,AZFDD2PHI,E,IND,MSZFDD2)
MSZFDD=MSZFDD1+MSZFDD2
RMSZFDD=(SQRT(MSZFDD))/32.2
IND=5
EXTERNALAZWT2PHI
CALLF4INTSMP(A,B,AZWT2PHI,E,IND,MSZWT1)
IND=5
EXTERNALAZWT2PHI
CALLF4INTSMP(A2,B2,AZWT2PHI,E,IND,MSZWT2)
MSZWT=MSZWT1+MSZWT2
RMSZWT=SQRT(MSZWT)
IND=5
EXTERNALAZFW2PHI
CALLF4INTSMP(A,B,AZFW2PHI,E,IND,MSZFW1)
IND=5
EXTERNALAZFW2PHI
CALLF4INTSMP(A2,B2,AZFW2PHI,E,IND,MSZFW2)
MSZFW=MSZFW1+MSZFW2
RMSZFW=SQRT(MSZFW)
IND=5
EXTERNALPHIW
CALLF4INTSMP(A,B,PHIW,E,IND,MSZT1)
IND=5
EXTERNALPHIW
CALLF4INTSMP(A2,B2,PHIW,E,IND,MSZT2)
MSZT=MSZT1+MSZT2
RMSZT=SQRT(MSZT)
E=0.001
EXTERNALPHIARDD
CALLF4INTSMP(A,B,PHIARDD,E,IND,MSARDD1)
IND=5
EXTERNALPHIARDD
CALLF4INTSMP(A2,B2,PHIARDD,E,IND,MSARDD2)
MSARDD=MSARDD1+MSARDD2
RMSARDD=SQRT(MSARDD)
IND=5
EXTERNALRITDD2PHI
CALLF4INTSMP(A,B,RITDD2PHI,E,IND,MSRITDD1)
IND=5
EXTERNALRITDD2PHI
CALLF4INTSMP(A2,B2,RITDD2PHI,E,IND,MSRITDD2)
MSRITDD=MSRITDD1+MSRITDD2
RMSRITDD=SQRT(MSRITDD)
IND=5
EXTERNALROTDD2PHI
CALLF4INTSMP(A,B,ROTDD2PHI,E,IND,MSROTDD1)
IND=5
EXTERNALROTDD2PHI
CALLF4INTSMP(A2,B2,ROTDD2PHI,E,IND,MSROTDD2)
MSROTDD=MSROTDD1+MSROTDD2
RMSROTDD=SQRT(MSROTDD)
IND=5
EXTERNALAZWDD2PHI
```

```
      CALLF4INTSMP(A,B,AZWDD2PHI,E,IND,MSZWDD1)
      IND=5
      EXTERNALAZWDD2PHI
      CALLF4INTSMP(A2,B2,AZWDD2PHI,E,IND,MSZWDD2)
      MSZWDD=MSZWDD1+MSZWDD2
      RMSZWDD=SQRT(MSZWDD)
      MSFS=((KT**2)*MSZWT)-(MW**2)*(MSZWDD)
      RMSFS=SQRT(MSFS)
      WRITE(2,40)ZETA
   40 FORMAT(11X,5HZETA=,F6.4//)
      WRITE(2,51)MSZWDD1,MSZWDD2,RMSZWDD
   51 FORMAT(5X,4HZWDD,7X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,41)
   41 FORMAT(11X,15HM S 2.0 TO 15.0,5X,17HM S 15.0 TO 157.0,9X,3HRMS//)
      WRITE(2,42)MSZT1,MSZT2,RMSZT
   42 FORMAT(5X,1HQ,8X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,43)MSZF1,MSZF2,RMSZF
   43 FORMAT(5X,2HZF,7X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,44)MSZFDD1,MSZFDD2,RMSZFDD
   44 FORMAT(5X,4HZFDD,5X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,45)MSZWT1,MSZWT2,RMSZWT
   45 FORMAT(5X,3HZWT,6X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,46)MSZFW1,MSZFW2,RMSZFW
   46 FORMAT(5X,3HZFW,6X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,47)MSZFWD1,MSZFWD2,RMSZFWD
   47 FORMAT(5X,4HZFWD,5X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,48)MSARDD1,MSARDD2,RMSARDD
   48 FORMAT(5X,4HARDD,7X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,49)MSRITDD1,MSRITDD2,RMSRITDD
   49 FORMAT(5X,5HRITDD,7X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,50)MSROTDD1,MSROTDD2,RMSROTDD
   50 FORMAT(5X,5HROTDD,7X,F10.8,10X,F10.8,8X,F10.8//)
      WRITE(2,39)RMSFS
   39 FORMAT(5X,6HRMSFS=,F10.8//)
      MSZGDD=MSZFDD+(MSARDD)*0.037*0.037 (mean square c.g.acc.see eq.50)
      RMSZGDD=(SQRT(MSZGDD))/32.2
      WRITE(2,52)RMSZGDD
   52 FORMAT(5X,8HRMSZGDD=,F10.8)
      WRITE(2,100)
  100 FORMAT(1H1)
      STOP
      END

       SEGMENT   LENGTH   502, NAME   RMSSOLUTION
```

```
      SUBROUTINE SOLVE(W111)
      REALMS,KS,KT,K,MF,MW,KR
      COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
      COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
     1AZW,AZWDD,MW,G23,G12,G11,DENOM,A3,B3,C3
      MS=2480.0
      MF=6720.0
      MW=155.0
      CS=655.0
      KS=94500.0
      K=2030000.0
      KT=1160000.0
      CF=1632.0
      KR=90202.0
      ER=0.037
      BR=0.025
      WR=18.0
      EI=0.122
      GAM=-0.329
      RG11=KS-MS*W111*W111
      AG11=CS*W111
      RG12=-KS
      AG12=-CS*W111
      RG22=KS+K-MF*W111*W111
      AG22=(CS+C)*W111
      AG23=-C*W111
      RG23=-K
      RG33=K+KT-MW*W111*W111
      AG33=C*W111
      G11=CMPLX(RG11,AG11)
      G12=CMPLX(RG12,AG12)
      G22=CMPLX(RG22,AG22)
      G23=CMPLX(RG23,AG23)
      G33=CMPLX(RG33,AG33)
      DENOM=G33*(G11*G22-G12*G12)-G23*G23*G11
      ZF=-G11*G23*KT/DENOM
      ZW=(G11*G22-G12*G12)*KT/DENOM
      AZW=CABS(ZW)
      AZWDD=(W111)*(W111)*(W111)*(W111)*AZW
      ZFW=ZF-ZW
      ZWT=ZW-1.0
      AZF=CABS(ZF)
      AZFW=CABS(ZFW)
      AZWT=CABS(ZWT)
      AZFDD=AZF*W111*W111
      AZFWD=AZFW*W111
      PHIT=0.00694/(V*(A3*(W111/V)*(W111/V)*(W111/V)+B3*(W111/V)*(W111/
     1V)+C3*(W111/V)))
      RETURN
      END

 SEGMENT  LENGTH   324, NAME  SOLVE
```

Interpretation

SOLVE(W111) computes the modulii of the frequency response functions in terms of the circular frequency W111,see eqs.17 through 27.

```
FUNCTION AZFDD2PHI(W111)
COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
1AZW,AZWDD,MW
CALL SOLVE(W111)
AZFDD2PHI=AZFDD*AZFDD*PHIT
RETURN
END
```

SEGMENT LENGTH 20, NAME AZFDD2PHI

Interpretation

$$AZFDD2PHI = \left|H(\omega)\right|^2_{\frac{\ddot{Z}_F}{Z_T}} \times \Phi_{z_T}(\omega) = \text{P.S.D. of c.g. acceleration}$$

```
FUNCTION AZF2PHI(W111)
COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
1AZW,AZWDD,MW
CALL SOLVE(W111)
AZF2PHI=AZF*AZF*PHIT
RETURN
END
```

SEGMENT LENGTH 20, NAME AZF2PHI

Interpretation

$$AZF2PHI = \left|H(\omega)\right|^2_{\frac{Z_F}{Z_T}} \times \Phi_{z_T}(\omega) = \text{P.S.D. of c.g. displacement}$$

```
FUNCTION AZWT2PHI(W111)
COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
1AZW,AZWDD,MW
AZWT2PHI=AZWT*AZWT*PHIT
CALL SOLVE(W111)
RETURN
END
```

SEGMENT LENGTH 20, NAME AZWT2PHI

Interpretation

$$AZWT2PHI = \left|H(\omega)\right|^2_{\frac{Z_W-Z_T}{Z_T}} \times \Phi_{z_T}(\omega) = \text{P.S.D. of tyre deformation}$$

```
      FUNCTIONPHIW(W111)
      REALMS,KS,KT,K,MF,MW,KR
      COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
      COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
     1AZW,AZWDD,MW,G23,G12,G11,DENOM,A3,B3,C3
      PHIW=0.00694/(V*(A3*(W111/V)*(W111/V)*(W111/V)+B3*(W111/V)*(W111/
     1V)+C3*(W111/V)))
      RETURN
      END

  SEGMENT   LENGTH     55, NAME  PHIW
```

Interpretation

PHIW = P.S.D.of runway displacement input to tyres.

```
      FUNCTIONAZFW2PHI(W111)
      COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
      COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
     1AZW,AZWDD,MW
      CALL SOLVE(W111)
      AZFW2PHI=AZFW*AZFW*PHIT
      RETURN
      END

  SEGMENT   LENGTH     20, NAME  AZFW2PHI
```

Interpretation

AZFW2PHI = $\left|H(\omega)\right|^2_{\frac{Z_F-Z_W}{Z_T}} \times \Phi_{ZT}(\omega)$ = P.S.D.of strut relative displacement

```
      FUNCTION AZFWD2PHI(W111)
      COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
      COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
     1AZW,AZWDD,MW
      CALL SOLVE(W111)
      AZFWD2PHI=AZFWD*AZFWD*PHIT
      RETURN
      END

  SEGMENT   LENGTH     20, NAME  AZFWD2PHI
```

Interpretation

AZFWD2PHI = $\left|H(\omega)\right|^2_{\frac{\dot{Z}_F-\dot{Z}_W}{Z_T}} \times \Phi_{ZT}(\omega)$ = P.S.D.of strut relative velocity

```
      FUNCTIONPHIARDD(W111)
      REALMS,KS,KT,K,MF,MW,KR
      COMPLEXG11,G12,G13,G22,G23,G33,DENOM,ZF,ZW,ZFW,ZWT
      COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
     1AZW,AZWDD,MW
      CALL SOLVE(W111)
      AZWT2PHI=AZWT*AZWT*PHIT
      PHIARDD=(KT*ER/KR)**2*((W111)**4*AZWT2PHI)/((1-(W111/WR)**2)**2+
     1(2*BR*W111/WR)**2)
      RETURN
      END

       SEGMENT   LENGTH     61, NAME  PHIARDD
```

Interpretation

PHIARDD $= \Phi_{\ddot{q}_r}(\omega)$ = P.S.D.function of generalised acceleration in rth.elastic mode(see eq.44)

```
      FUNCTIONAZWDD2PHI(W111)
      COMPLEXG11,G12,G13,G22,G23,DENOM,ZF,ZW,ZFW,ZWT,AIT,AOT,G33
      COMMONC,AZF,AZFDD,AZWT,AZFW,AZFWD,PHIT,CF,V,KR,ER,KT,WR,BR,EI,GAM,
     1AZW,AZWDD,MW
      CALL SOLVE(W111)
      AZWDD2PHI=AZWDD*AZWDD*PHIT
      RETURN
      END

       SEGMENT   LENGTH     20, NAME  AZWDD2PHI
```

Interpretation

AZWDD2PHI= $\left|H(\omega)\right|^2_{\frac{\ddot{Z}_W}{Z_T}} \times \Phi_{Z_T}(\omega)$ = P.S.D.of wheel acceleration

D01 , NORM[D01 : LP00 ON 11/01/72 AT 02.31
C L KIRK

INPUT DATA

V= 40.00 CN= 21600.00A3=-32485.000B3= 18062.000C3= -2213.000

ZETA=0.0953 (equivalent damping ratio)

ZWDD 0.00002668 0.00014147 0.01296698
(r.m.s.acceleration of wheel)

	M S 2.0 TO 15.0	M S 15.0 TO 157.0	RMS
Q	0.00000902	0.00000386	0.00358891
(r.m.s.runway input)			
ZF	0.00004105	0.00032748	0.01919705
(r.m.s.c.g.displacement-rigid body and first elastic mode)			
ZFDD	0.31195636	5.31520974	0.07366972
(r.m.s.c.g.displacement-rigid body and first elastic mode)			
ZWT	0.00000640	0.00013020	0.01168790
(r.m.s.relative displacement of tyre)			
ZFW	0.00000173	0.00004053	0.00650066
(r.m.s.relative displacement of strut)			
ZFWD	0.00015899	0.00521721	0.07332255
(r.m.s.relative velocity of strut)			
ARDD	0.02169523	1.37105558	1.18014864
(r.m.s.generalised acceleration in second elastic mode)			

RMSFS=*13557.96241700 (r.m.s.force in two struts)

RMSZGDD=0.07368220 (r.m.s. c.g.acceleration - rigid body mode and first two elastic modes,see eq.50)

APPENDIX 2

FIGURES

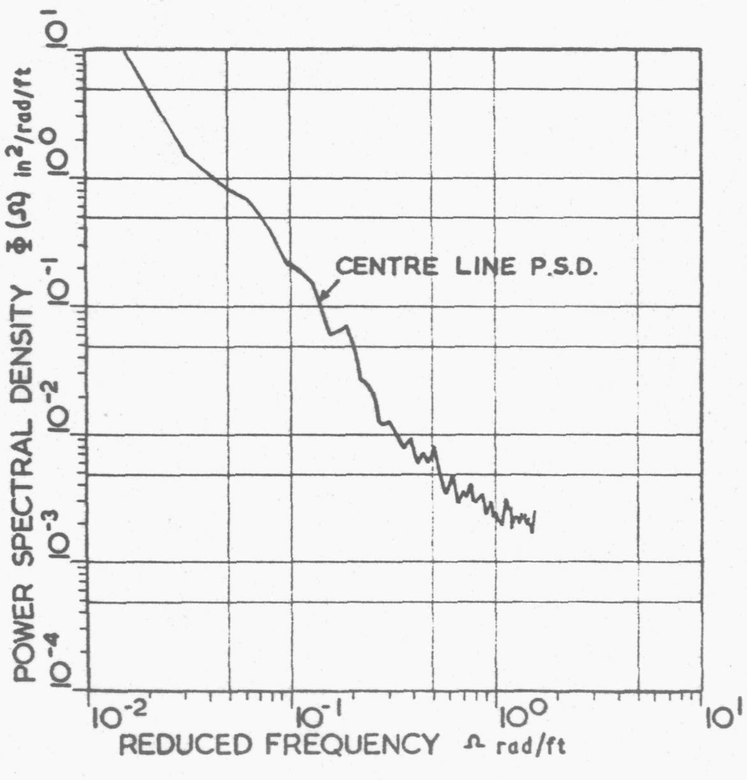

FIGURE 1.

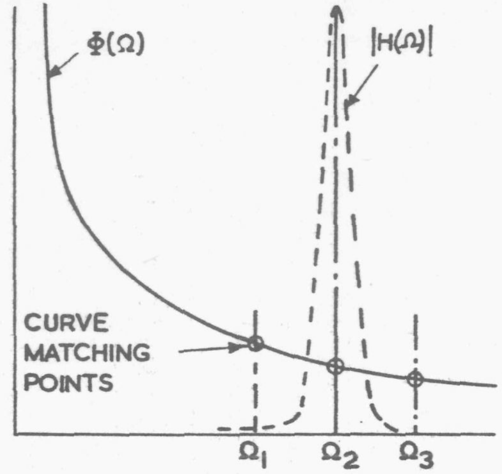

FIGURE 2. P.S.D. FUNCTION CURVE MATCHING POINTS

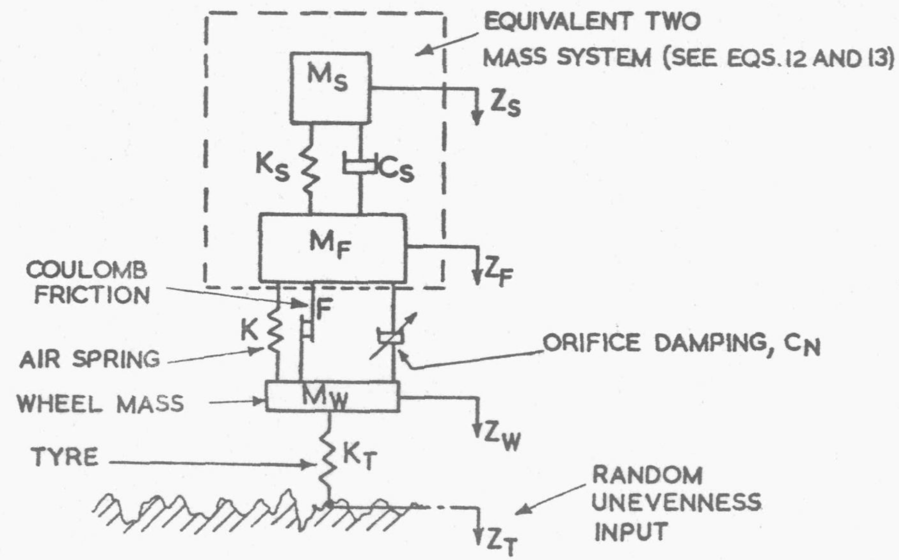

FIGURE 3. THREE MASS SYSTEM REPRESENTING INTERACTION BETWEEN RIGID HEAVING MODE, FIRST ELASTIC MODE, AND WHEEL UNIT

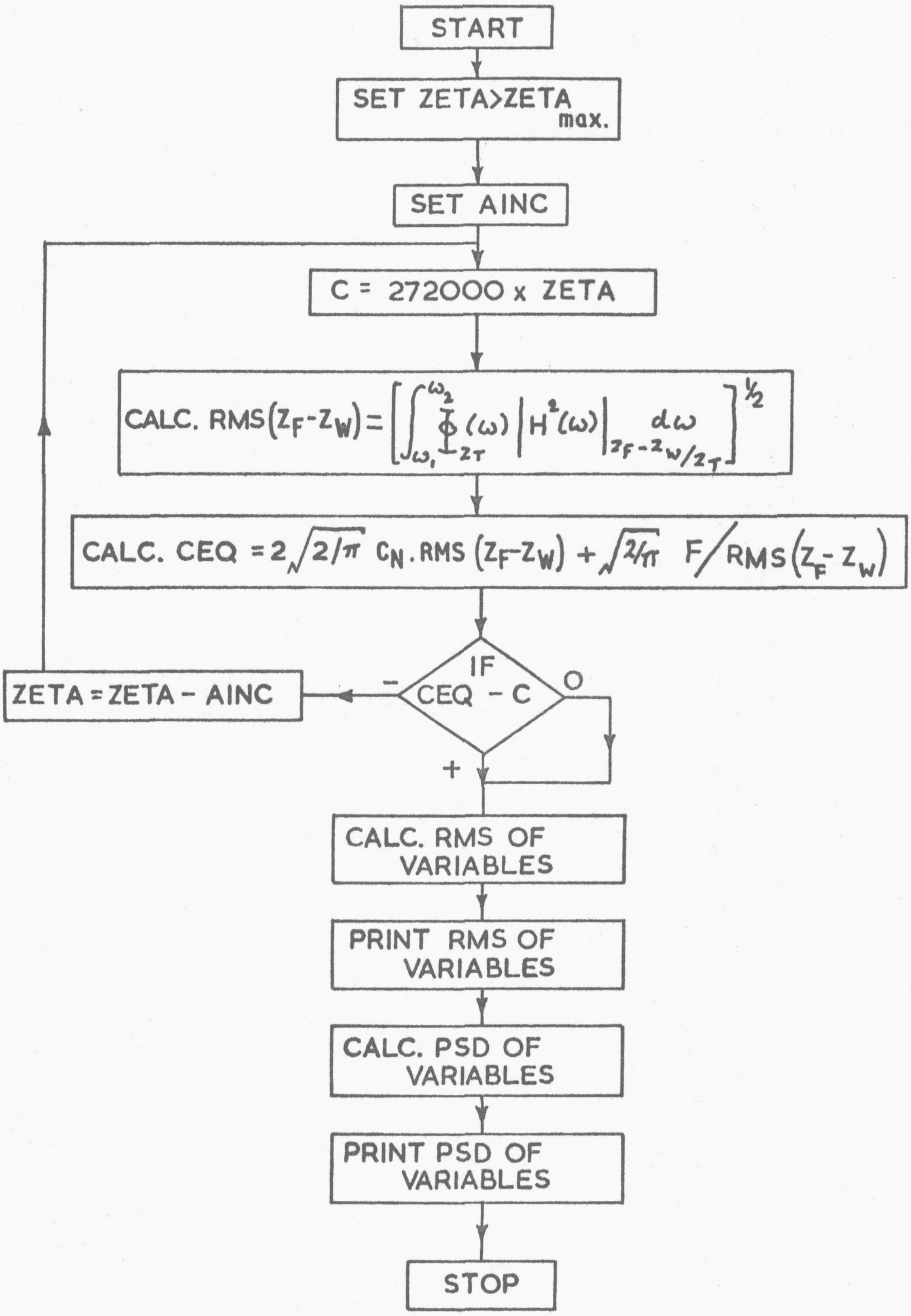

FIGURE 4. FLOW CHART FOR COMPUTER PROGRAM

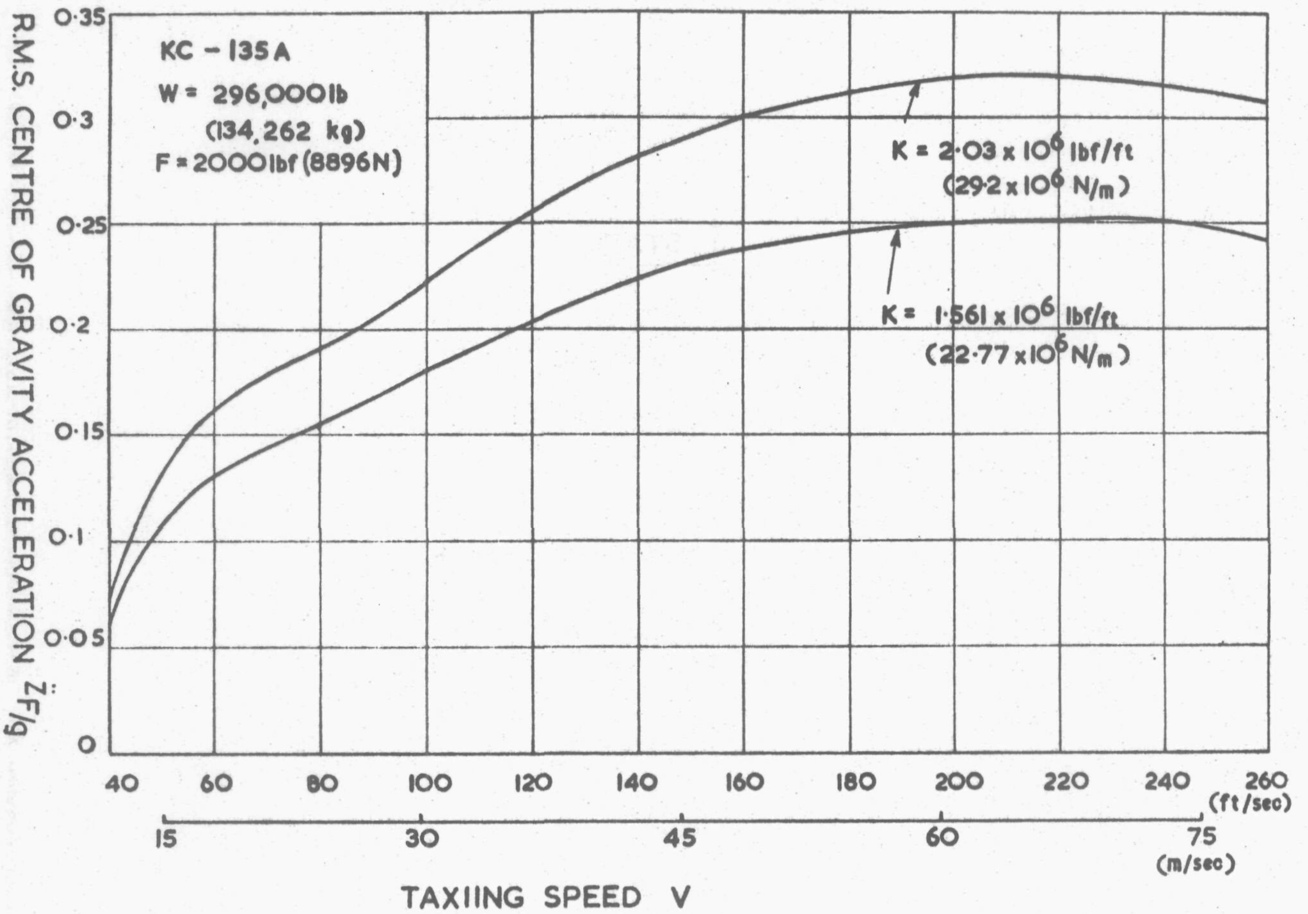

FIGURE 5. CENTRE OF GRAVITY ACCELERATION $\ddot{Z}F/g$

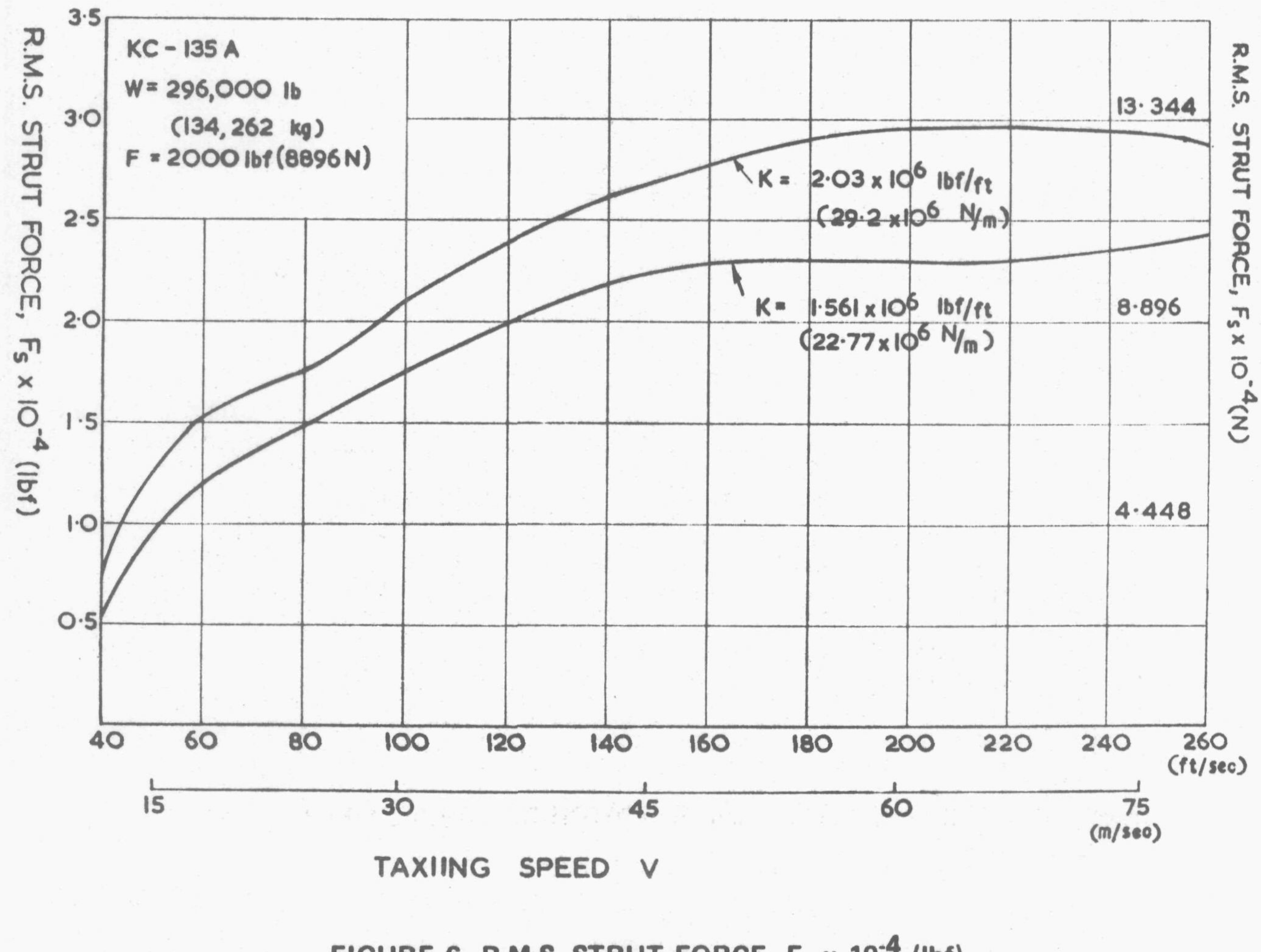

FIGURE 6. R.M.S. STRUT FORCE, $F_s \times 10^{-4}$ (lbf)

Appendix III - TABLES

Ω rad/ft	$\Phi(\Omega)$ in^2/rad/ft
0.015	10.0
0.02	5.0
0.03	1.5
0.04	1.05
0.05	0.80
0.06	0.71
0.07	0.54
0.08	0.37
0.09	0.25
0.10	0.21
0.12	0.16
0.14	0.11
0.16	0.060
0.18	0.070
0.20	0.050
0.30	0.012
0.40	0.008
0.50	0.008
0.60	0.0048
0.70	0.0034
0.80	0.00035
0.90	0.00025
1.00	0.00021

TABLE 1. VALUES OF P.S.D. FUNCTION MEASURED FROM FIGURE 1

V ft/sec	Ω_1	Ω_2	Ω_3	ω_1	ω_2	ω_3
40	0.2	0.28	0.3	8	10.2	12
60	0.16	0.186	0.2	9.6	11.16	12
80	0.10	0.140	0.16	8	11.2	12.8
100	0.075	0.112	0.15	7.5	11.2	15
150	0.05	0.074	0.1	7.5	11.1	15
200	0.03	0.056	0.075	6	11.2	15
250	0.03	0.045	0.075	7.5	11.2	18

units of ω, rad/sec

TABLE 2. P.S.D. FUNCTION CURVE MATCHING VALUES OF Ω

V ft/sec	A	B	C
40	-32485.0	18062.0	-2213.0
60	67560.0	-24433.0	2284.0
80	24969.0	-5550.0	353.0
100	4386.0	-263.0	24.06
150	11722.0	-1306.0	61.0
200	4076.0	-278.0	26.9
250	-2197.0	383.56	12.58

TABLE 3. VALUES OF COEFFICIENTS IN P.S.D. FUNCTION (EQ.6)

	V (ft/sec)	$\ddot{Z}_F/g$ r.m.s.	F_S(lbf) r.m.s.	Z_{FW}(in) r.m.s.	Z_{WT}(in) r.m.s.	$\beta_{eq.}$
n = 1	40	0.0576	5511.0	0.0768	0.1140	0.0988
	60	0.1312	11916.0	0.1791	0.2464	0.0626
	80	0.1555	14698.0	0.2101	0.3040	0.0608
	100	0.1800	17598.0	0.2380	0.3640	0.0606
	150	0.2310	22409.0	0.3065	0.4630	0.0630
	200	0.2500	22950.0	0.3230	0.4747	0.0647
	250	0.2467	23944.0	0.3289	0.4954	0.0648
n = 1.3	40	0.0736	6778.0	0.0780	0.1400	0.0953
	60	0.1637	15258.0	0.1750	0.3160	0.0624
	80	0.1890	17513.0	0.2020	0.3600	0.0609
	100	0.2238	20936.0	0.2400	0.4340	0.0606
	150	0.2896	26953.0	0.3110	0.5576	0.0634
	200	0.3168	29587.0	0.3420	0.6110	0.0653
	250	0.3127	29274.0	0.3370	0.6000	0.0654

TABLE 4. RESULTS FROM COMPUTER PROGRAM

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5. Crandall, S.H. and Mark, W.D. 'Random Vibration in Mechanical Systems', Academic Press (1963)