Headway variability in public transport

A review of metrics, determinants, effects for quality of service and control strategies

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Headway variability in public transport: a review of metrics, determinants, effects for quality of service and control strategies

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ABSTRACT

The most relevant issues related to headway variability in public transport planning, operations and quality of service are reviewed in this paper. We discuss the causes and consequences of headway variability, the alternative metrics that have been proposed to measure it, the preventive and reactive strategies to control headway variability in both research and practice, including the role of drivers and of present and future technology, and how service provision contracts might deal with headway variability through metrics and financial incentives. The most influential elements that explain headway variability along a route are the irregularity at which vehicles are dispatched, the scheduled frequency, the distance travelled or route length, the passenger demand and associated dwell times, and the number of stops. We conclude that there is a large gap between the state-of-the-art and the state-of-practice in terms of identification of headway variability issues, as well as in the development of mitigation and control measures. It is therefore paramount that future research will contribute to closing this gap by addressing organisational, contractual and technological barriers in the implementation of measures aimed at mitigating headway variability in public transport services.

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1. Introduction

In public transport operations, headway is defined as the time interval between two consecutive vehicles that belong to the same public transport line. Keeping regular headways and consistent travel times have been identified as key attributes from a public transport system to offer a reliable service (Durán-Hormazábal & Tirachini, 2016; El-Geneidy, Horning, & Krizek, 2011; Muñoz, Soza-Parra, & Raveau, 2020; TRB, 2020b). Travel times

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are inherently random in public transport operations and any delay has the potential to be exacerbated down the road, because a delayed bus or train is likely to encounter an increased number of passengers at downstream stops and stations, which increases the time at those stops due to longer passenger boarding and alighting processes, leading to even further delays. Conversely, the bus or train that comes next has a reduced number of passengers to pick up in the next stops and therefore runs faster than planned. Thus, operating vehicles under even headways is a perfect example of unstable equilibrium. If the operation is not actively controlled, vehicle bunching may occur, as analytically described by Newell and Potts (1964) and empirically shown in several studies (e.g. Byon et al., 2018; Cats, 2014; El-Geneidy et al., 2011; Feng & Figliozzi, 2015; Hammerle, Haynes, & McNeil, 2005; Strathman et al., 1999).

The negative effects of headway variability, which leads to vehicle bunching, are multifold. Headway variability leads to increases in expected waiting times (Osuna & Newell, 1972) and to the unpredictability of waiting times (Durán-Hormazábal & Tirachini, 2016). Since more passengers board a vehicle that arrives after a long headway, bunching causes an additional discomfort due to induced overcrowding of passengers (Tirachini, Hensher, & Rose, 2013) and decreases the chances of obtaining a seat when travelling (Babaei, Schmöcker, & Shariat-Mohaymany, 2014; Cats, West, & Eliasson, 2016). Bunching has also been shown to increase dwell times and running times (Verbich, Diab, & El-Geneidy, 2016). The economic analysis of measures to increase headway reliability can thus yield large social benefits, as the value of waiting time savings is generally larger than the value of in-vehicle time savings (Wardman, 2004), and travelling standing and in crowded conditions has also a larger value of time savings than travelling seated without crowding, as has been empirically estimated using smart card data (Hörcher, Graham, & Anderson, 2017; Tirachini, Sun, Erath, & Chakirov, 2016; Yap, Cats, & van Arem, 2020).

Because of its relevance for users and operators alike, headway variability in public transport is a problem that has been widely studied during the past decades. Several factors have been found to worsen bunching particularly in the case of bus services, such as service frequency, passenger demand, traffic congestion, number of bus stops, number of traffic signals and route length (Arriagada, Gschwender, Munizaga, & Trépanier, 2019; Figliozzi, Feng, Lafferriere, & Feng, 2012; Moreira-Matias, Ferreira, Gama, Mendes-Moreira, & de Sousa, 2012; Soza-Parra, Muñoz, & Raveau, 2021). Irregularity of bus dispatching at the beginning of a route is a key variable that increases bus bunching along a route (Arriagada et al., 2019; Hammerle et al., 2005; Soza-Parra et al., 2021). Fleet control strategies are usually proposed to deal with the problem of headway unreliability, such as bus holding, station skipping, deadheading, speed control, boarding limits and traffic signal priority (e.g. Andres & Nair, 2017; Barnett, 1974; Daganzo, 2009; Eberlein, Wilson, & Bernstein, 1999; Furth & Muller, 2000; Hickman, 2001; Koehler, Seman, Kraus, & Camponogara, 2019; Muñoz et al., 2013; Strathman et al., 2001). These studies usually make numerical applications over simulated or stylised public transport lines.

Despite a large number of research efforts, we find that in actual bus operations, the benefits of applying real-time headway controls have been quantified in very few case studies, such as bus lines in Stockholm (Cats, 2014; Fadaei & Cats, 2016), Washington DC (Soza-Parra, Cats, Carney, & Vanderwaart, 2019), Atlanta and San Antonio (Berrebi, Crudden, & Watkins, 2018) and Santiago (Lizana, Muñoz, Giesen, & Delgado, 2014), which might be attributed to the fact that most unscheduled bus services are not equipped with any kind of strategy to control for headway regularity. Even in systems that are equipped with Automated Vehicle Location (AVL) and passenger counting devices, we observe that planners sometimes struggle to use the vast amount of real-time information generated in a meaningful way (TRB, 2020a). Thus, bunched buses are easy to observe worldwide since this phenomenon, unfortunately, happens quite fast unless mitigated proactively and systematically.

In the case of rail systems, unscheduled train services, such as high-frequency metro lines, have a safety control scheme that prevents trains from getting too close. Thus, two metro trains next to another cannot be seen under normal operation. However, a poorly controlled metro service would still suffer from long headways getting longer along the route. And since the train behind will need to be held due to safety concerns in case of high-frequency services, headway variability may end up affecting the frequency of an entire line. Thus, headway control systems for metro lines addressing passenger demand variability have been proposed by several authors (e.g. Bueno-Cadena, Muñoz, & Tirachini, 2020; Carrel, Mishalani, Wilson, Attanucci, & Rahbee, 2010; Farhi, 2021; Meng & Zhou, 2011).

All-in-all, headway variability in public transport services is a problem that has been widely studied in the academic literature, although gaps in scientific efforts and real-world application remain. In this paper, we provide a comprehensive overview of the multiple dimensions around headway variability in public transport, including (i) symptoms, (ii) causes, (iii) measurement, (iv) treatment and (v) prevention measures. We start by discussing the consequences of headway variability for passengers, operators and society at large (Section 2). The determinants of headway variability upon dispatching and along the route are then reviewed (Section 3). We provide an overview of metrics used for measuring the extent of headway variability (Section 4), followed by measures aimed at treating and mitigating service irregularity, including the effects of automated vehicles (Section 5) and reviewing the potential integration of related incentives into contracts for provisioning public transport service (Section 6). We conclude with a reflection on the apparent gap between research and practice and other relevant lessons of this review (Section 7).

2. Consequences of headway variability

2.1. Increase in waiting time

An uncontrolled public transport service will inevitably exhibit an amplified degree of headway variability, because the service is subject to an inherent positive feedback loop when left unattended. The interested reader is referred to the Appendix, where a mathematical proof of this statement is provided. Even if the average headway does not change as vehicles move along a route, the variance of headways grows. Since the average number of users arriving at a stop during long headway is larger than during short ones, the average waiting time experienced by users grows. The expected waiting of the passenger is (Osuna & Newell, 1972):

$$E(w) = \frac{E(h)}{2}\left(1 + \left(\frac{\sigma(h)}{E(h)}\right)^2\right) \tag{1}$$

where $\sigma(h)$ corresponds to the standard deviation of headways at the stop, and thus $\frac{\sigma(h)}{E(h)}$

correspond to the coefficient of variation of the headways observed at the stop. As can be seen in Equation (1), the average waiting time is half of the headway only with perfectly regular headways ($\sigma(h) = 0$). In extremely unreliable service, the average waiting time can be as long as the full headway or longer. Moreover, when long headways become a recurrent possibility, users needing to reach a destination by a given time must plan accordingly assuming some of the worst cases. Since most of the time waiting time is not so long, passengers arrive several times early to the destination, triggering a hidden waiting time there. A detailed analysis of waiting time components for high- and low-frequency services is presented by Furth and Muller (2006).

2.2. Increase in the variability of waiting time

Headway variability leads not only to increases in expected waiting times but also to less predictable waiting times. This is a relevant issue since the variability of travel times is an extra source of disutility (Li, Hensher, & Rose, 2010). As for the increase in the average value of waiting time, Benezech and Coulombel (2013) provide a formula for the relationship between the standard deviation of waiting time and headways' distribution, which shows that waiting time variability increases with the average headway, standard deviation of headways and skewness of the headway distribution.

In practice, two sources of unpredictability in waiting times might be at play due to headway variability: there is an inherent correlation between the mean and standard deviation of waiting times for trips repeated day after day, as shown for both buses and metro trains by Durán-Hormazábal and Tirachini (2016). Second, for systems running with demand levels nearing capacity, headway variability will increase the probability of some vehicles getting full (even if the total capacity of the route is more than enough to satisfy demand), forcing denied boardings at subsequent stops or stations and therefore increasing waiting time. Because the state of each vehicle (either full or with available capacity) is random, depending on the randomness of headways and demand, waiting times become less predictable in such instances. What is worse, extra waiting times due to denied boarding are more annoying to users, increasing the value of waiting time savings in these delayed situations.

2.3. Induced crowding

Buses trailing a long headway will receive more passengers than those trailing a short one, therefore inducing passenger crowding in some vehicles. Feng and Figliozzi (2015) show that, when buses bunch, the leading vehicle tends to have an occupancy level larger than average, while the following vehicle has an occupancy level lower than average. A large occupancy of vehicles is associated with several negative effects from crowding on passengers' wellbeing, including lack of privacy, stress, decreased personal security, decreased comfort and increased anxiety (Tirachini et al., 2013). This is especially relevant during peak periods if the public transport system is designed for vehicles to approach the desired occupancy, which may even correspond to the available capacity at some point along their route. Although uneven loads do not affect the average number of passengers across buses, it increases the average crowdedness experienced by users since many users suffer the crowded bus, while few enjoy the emptier one. Furthermore, passengers at stops may be prevented from boarding these crowded vehicles, forcing them to keep waiting for the next vehicle, as discussed in Section 2.2. Notice that both groups of passengers (those on a crowded bus and those prevented from boarding) are the ones that experience longer waiting times. The crowding discomfort as experienced by users has been increased in the current COVID-19 situation, because virus spreading requires physical proximity to an infected passenger, which makes headway control strategies and other crowding management tools more relevant than ever (Gkiotsalitis & Cats, 2021b; Hörcher, Singh, & Graham, 2021; Tirachini & Cats, 2020).

An overcrowded vehicle will also experience longer dwell times than if all vehicles had an even load, therefore inducing longer in-vehicle times as well. When these vehicles arrive at a stop, more passengers than average will alight from them and more passengers than average will try to board. This creates very high friction between passengers trying to move in opposite directions within a very dense small area, potentially further contributing to a prolonged dwell time and hence exacerbating the bunching problem (West & Cats, 2017).

2.4. Increased travel time and induced operator costs

On the operational side, vehicles arriving together at stops or stations create queues that may have unintentional consequences for the rest of the traffic. When bunching occurs at bus stops, there might be increases in dwell time and running time, as found by Verbich et al. (2016). At the end of routes, bunched vehicles may require a larger terminal area where buses can queue before being dispatched to the first stop, which is often unavailable, especially in dense urban areas. Reductions in the operational speed due to bunching are also possible from crowding effects that increase dwell times of bunched vehicles, and therefore total travel times are affected. By increasing the average travel time, bunching may reduce the productivity of the line given by the maximum frequency to be offered. Moreover, in unreliable systems recovery times usually need to be increased, in order to comply with dispatching requirements (Furth & Muller, 2000), which is a second source of reduced productivity for a given fleet size.

2.5. Reduced user satisfaction, financial effects and social issues

Headway variability, with all its detrimental consequences for the quality of service, may influence the decision of users to leave the public transport system and resort to other modes of transport, or to switch between from a public transport mode or route that is more reliable to a public transport mode or route that is less reliable. In fact, Soza-Parra, Raveau, Muñoz, and Cats (2019) show that headway variability has a set of indirect effects, such as the possibility of denied boardings and the increase of passenger density, which reduces travel satisfaction significantly for both buses and metro. If passengers leave the public transport system altogether, there are consequences also for regulators, if the system is subsidised and the amount of subsidy depends on the number of passengers carried and the cost efficiency of the system. Bunching has also been reported to increase fare evasion by inducing passengers to enter through the back door of crowded buses, without paying the fare in systems in which fare validation is only through the front door (Lizana et al., 2014). Bunching may even create a bad reputation of the public transport system to non-users, since having lots of passengers waiting for a very crowded arriving vehicle with a second vehicle trailing behind is easily noticeable.

Regarding social issues, the induction of unnecessary long headways that increase waiting times may have disproportionate effects on some users. For instance, Fan, Guthrie, and Levinson (2016), based on public transport user surveys in Minnesota, find that women waiting for more than 10 min in environments that they feel as insecure report perceived waiting times much longer than what they really are, and longer than the perceived waiting time of men in the same situation. Older passengers may also have a larger perceived waiting time, as quantified in Athens (Psarros, Kepaptsoglou, & Karlaftis, 2011), which are, therefore, amplified by unreliable services. The lack of basic amenities in stops, such as benches and shelters, significantly increases the perception of waiting time (Fan et al., 2016), which is likely to have a larger impact on people with reduced mobility and elderly passengers, particularly on long waiting times.

3. Determinants of headway variability

Headway variability is caused by several factors, which to different extents contribute to the deterioration of public transport service reliability by inducing headway variability, particularly in uncontrolled or poorly controlled public transport services. The model presented in the Appendix suggests that chief causes of headway variability are (a) an initial headway discrepancy and three amplifying factors: (b) the number of passengers getting on and off vehicles, (c) the boarding and alighting times per passenger, and (d) the number of intermediate stops. Therefore, any factor that affects variables (a) to (d) has the potential to increase headway variability.

Table 1 presents a summary of empirical studies in which statistical models relating the evolution of headway variability along a route to a number of explanatory variables are estimated. When possible, based on information provided in each study, we indicate the strength of the reported influences, i.e. +, ++ and +++ stand for low, middle and large influences in explaining headway variability, respectively. NS means that the variable was tried but it was not statistically significant. Furthermore, the bottom row in Table 1 shows a summary of the likely overall strength of the influence, based on our assessment of the literature and the underlying physical process being analysed. For instance, the variables right-of-way, congestion and traffic signals are all related and the influence of each of those depends on their interplay. In particular, congestion, which in urban traffic is mostly produced as delays at traffic signals, increases the variability of travel times. In other words, without congestion, the influence of an exclusive or preferential right-of-way for public transport is low or negligible. We, therefore, assign "+/++" to these three variables, as their influence might be weak in off-peak periods but noticeable in peak periods. Empirically, the most influential variables on headway variability along routes are found to be the headway variability at the beginning of the route (when dispatching vehicles), the scheduled frequency, the distance travelled, the passenger demand and the number of stops. These and other variables are discussed in detail next.

(i) *Headway variability at the beginning of the route:* it is one of the chief variables that influence headway regularity of buses along a route, as shown by several

Table 1. Explanatory variables in headway variability and bus bunching studies.

	Explanatory variables for headway variability along routes											
Study	Headway variability at the beginning of the route	Scheduled frequency	Distance travelled	Passenger demand	Number of stops	Off-board payment stops	Right of way	Congestion	Traffic signals	Incidents	Driver behaviour or experience	Type of fleet
Hammerle et al. (2005)	+							+				
El-Geneidy et al. (2011)	+++	+	+	+	+						+	
Albright and Figliozzi (2012)	++			+	+		+					
Figliozzi et al. (2012)	+	+	+	+								
Feng and Figliozzi (2015)	+	++	+									
Diab et al. (2016)	+++	++	+	++			+		+			
Rashidi, Ranjitkar, Csaba, and Hooper (2017)		+	+	+	+		NS		NS			
Arriagada et al. (2019)	+++	+++	+++	++	+++	NS	+	NS	+	+		+
Soza-Parra et al. (2021)	+++	+	+	+		++	++		+			
Likely overall influence	+++	+++	++/+++	++	++	++	+/++	+/++	+/++	+/++	+	+

empirical studies using automatic vehicle location data (Arriagada et al., 2019; Diab, Bertini, & El-Geneidy, 2016; El-Geneidy et al., 2011; Hammerle et al., 2005; Soza-Parra et al., 2021). Godachevich and Tirachini (2020) identify three groups of variables that are significant in influencing headway variability at vehicle dispatching: first, operation and design of the network, as variables such as length of the route, the average passenger demand, the travel speed and the scheduled frequency affect the reliability of travel times and therefore the arrival of vehicles at terminals; second, operation and infrastructure of terminals and depots, including variables such as the complexity of rolling stock circulation associated with a given terminal and the distance between a depot and the first stop of the service; and third, the performance of the incumbent public transport service provider, as different operators take different actions to control headways at dispatching. The unavailability of drivers (or driver re-assignment owing to absenteeism) is also a relevant factor that causes problems for regular vehicle dispatching (Cham, 2006).

(ii) *Scheduled frequency*: The higher the scheduled service frequency, the more likely that vehicles bunch together (Arriagada et al., 2019; Diab et al., 2016; Figliozzi et al., 2012). However, *ceteris paribus*, an increase in frequency implies a reduction in passengers' waiting time, therefore, the analysis of an optimal service frequency must consider its full influence on waiting times (Gkiotsalitis & Cats, 2018; Tirachini, Hensher, & Bliemer, 2014).

(iii) *Distance travelled from the beginning of the route*: As the vehicles progress along a route, they face different sources of uncertainty on demand, traffic flow, incidents, etc., making it difficult for buses to keep regular headways, i.e. without proper control, the probability of bunching increases as buses move forward along routes (Chen, Yu, Zhang, & Guo, 2009; Figliozzi et al., 2012; Sáez et al., 2012; Soza-Parra et al., 2021).

(iv) *Passenger demand and dwell time variability*: As suggested by the model presented in the Appendix, the larger the passenger demand and the boarding and alighting times per passenger, the larger the increase of headway discrepancies along bus routes. These results are aligned with the insights from empirical models. The number of passengers that board and alight buses have been found as significant variables that increase bus bunching (Albright & Figliozzi, 2012; Arriagada et al., 2019; El-Geneidy et al., 2011). Furthermore, a large demand generally increases boarding and alighting times per passenger due to crowding effects (e.g. Milkovits, 2008), increasing the headway between two consecutive vehicles. Larger boarding and alighting times per passenger also increase the variability of dwell times (Sun, Tirachini, Axhausen, Erath, & Lee, 2014), which further leads to increases in headway variability. Therefore, upgrading from slow to quick fare collection systems and passenger boarding rules (such as the implementation of off-board payment stops) has the potential to significantly reduce headway variability (Soza-Parra et al., 2021; TRB, 2020a). Finally, it has also been found that infrequent stop activity such as the operation of lifts increases headway variability as well (Albright & Figliozzi, 2012; El-Geneidy et al., 2011).

(v) *Traffic conditions and right of way*: Congestion and incidents are one of the sources of travel time variability (Comi, Nuzzolo, Brinchi, & Verghini, 2017) and bunching (Byon et al., 2018). As travel time variability increases, headway variability is expected to increase as well. Dedicated right of way for buses reduces not only the average travel time but also the travel time variability (Durán-Hormazábal & Tirachini, 2016). Moreover, segregated busways have been shown to reduce headway variability, but bus lanes that can also be used by right-turning cars do not necessarily improve headway regularity (Arriagada et al., 2019). All-in-all, traffic congestion and incidents are expected to be strong drivers of headway variability; however, their influence has been tested in only a couple of studies, likely due to lack of proper data. This topic is a relevant venue for further research, with clear implications for the cost–benefit analysis of investment decisions on alternative right-of-way options for public transport.

(vi) *Number of traffic signals downstream of a bus stop:* The number of traffic signals is known to increase both the mean and the variance of travel times in public transport (Abkowitz & Engelstein, 1983; Cats, 2019). Similarly, it has been empirically found that traffic signals increase the headway variability in bus lines (Arriagada et al., 2019; Soza-Parra et al., 2021). However, traffic signals can also reduce headway variability if transit signal priority is applied considering headway variations from the schedule (Albright & Figliozzi, 2012; Furth & Muller, 2000; TRB, 2020a).

(vii) *Driver behaviour*: Drivers are critical to service reliability. If drivers feel that their work is appreciated, if vehicles are in good shape and drivers have good working conditions, it is more likely that measures to improve headway reliability will be successful (TRB, 2020b). Strathman, Kimpel, Dueker, Gerhart, and Callas (2002), Cats (2019) and Martinez, Munoz, and Delgado (2018) found driver-related differences in bus travel times and El-Geneidy et al. (2011) found that drivers with more years of working experience have lower levels of headway variability and travel time variability.

4. Measurement of headway variability

In the literature, there is no agreement on one single way to measure bus headway variability and several indicators have been proposed (Cats, 2014; Saberi, Zockaie, Feng, & El-Geneidy, 2013; TRB, 2020b). Because it is not clear which indicator is the most appropriate to analyse variability issues in any particular situation, researchers usually define and apply two or more headway variability metrics to the same dataset of public transport headways (Arriagada et al., 2019; Byon et al., 2018; Saberi et al., 2013). In this section, we review common headway variability indicators that have been proposed. Other metrics based on the shape of the probability distribution of headways (such as the width index) are shown in Saberi et al. (2013).

- *Standard deviation of observed headways, $\sigma(h^{obs})$*: a symmetrical measure of dispersion that is simple to compute. The standard deviation of headways is instrumental in calculating the average waiting times (Equation (1)). As a limitation, any symmetrical indicator is not able to properly represent the inclination that the probability distribution of headways may have.

- An alternative to this indicator is to compute the standard deviation of the difference between the observed headway and the scheduled or planned headway h^{sch}, $\sigma(h^{obs} - h^{sch})$.
- *Coefficient of variation of observed headways, $CV(h^{obs})$:* the ratio between the standard deviation of observed headways and their mean value. Its simplicity is an upside as it can be easily used for comparability of different settings.
- *Close headways*: Measured as two buses running within a certain threshold of each other at a certain point. For instance, in Chicago when the headway is 60 s or shorter, it is considered that buses have bunched (TRB, 2020a).
- *Excess waiting time, EWT:* When service frequency is high (e.g. average headway shorter than 10 min) and there are uniform arrivals of vehicles at stops, the average waiting time with regular headways is usually approximated as half of the interval $\frac{E(h^{obs})}{2}$, which also assumes that passengers arrive randomly at a constant rate and that the capacity constraint of vehicles is not binding (i.e. that passengers are able to board the first vehicle that they want to use). Osuna and Newell (1972) showed that when headways are subject to variability, the expected waiting time increases over $\frac{E(h^{obs})}{2}$, as a linear function of the headway variance (see Equation (1)). Any average waiting time over $\frac{E(h^{obs})}{2}$ is known as *excess waiting time*, which is used in cities such as London and Singapore to provide incentives to bus operators to improve service reliability (see Section 6).

$$\mathrm{EWT} = E(w) - \frac{E(h^{\mathrm{obs}})}{2} = \frac{\sigma(h^{\mathrm{obs}})^2}{2 \cdot E(h^{\mathrm{obs}})} \tag{2}$$

- *Ratio between observed waiting time and ideal waiting time (with uniform headways), WTR_j*: for bus stop j, the ratio between observed waiting time (Equation (1)) and the ideal waiting time as if all vehicles keep regular headways, is computed by Byon et al. (2018). A modified version of this indicator is simply computing the headway ratio HR_i, which is the rate between observed headway and scheduled headway (Strathman et al., 1999).
- *Indicator of regularity compliance based on headways out of range*, IR: a measure used to control the regularity performance of bus operators for high-frequency services in some settings (e.g. Santiago). First, an acceptable headway (TA) is defined for a bus route, as a summation of the scheduled or planned headway and an acceptable gap.

  The acceptable gap (g^{Inc}) can, in turn, be an increasing function of the scheduled headway (h^{sch}), with lower and upper bounds that can be exogenously defined. An example of the definition of the acceptable gap is $g^{\mathrm{Inc}} = \max\{a;\ \min\{(c \cdot h^{sch}); b\}\}$, where a and b are the lower and upper bounds, respectively, and c is the slope of a linear relationship between g^{Inc} and h^{sch}, within bounds a and b. In the case of high-frequency services in Santiago, g^{Inc} has minimum and maximum values defined as $a = 3$ and $b = 10$ min, while slope c is defined as 0.4 (MTT, 2012). Then, when the observed headway is larger than the summation of the scheduled headway and the acceptable gap, it is assumed that a major deviation from the scheduled headway has occurred, and corrective measures or penalties can be applied.

A potential downside of a headway variability indicator that only accounts for those surpassing a predetermined high limit, is that it implicitly considers that short headways are acceptable as passengers experience short waiting times. However, the number of passengers experiencing those short headways is proportional to the length of the headway, so a short headway is enjoyed by very few passengers, and the transport capacity is misused due to uneven loads between vehicles.

- *On-time performance*: it is a measure that addresses punctuality, commonly used in schedule-based bus operations. A service is considered "on time" if it runs between *d* minutes early and *e* minutes late. For instance, in the United States, it is common to define *d* as 1 min or 30 s and *e* as 5 min of a given schedule per bus stop (TRB, 2020a). In actual operations, even though arriving 30 s too early is considered on time, bus drivers might be instructed not to depart ahead of schedule.

5. Treatment and prevention

Measures to mitigate headway variability focus on one or more of the variables identified as contributors to variability (see Section 2 and the Appendix). Mitigation measures are usually aimed at reducing the extent to which an existing service irregularity is otherwise likely to escalate. For example, in the case of buses, the boarding time per passenger can be reduced by means of allowing boarding from all doors (Jara-Díaz & Tirachini, 2013; West & Cats, 2017) or pre-boarding validation as in the case of bus rapid transit (Delgado, Muñoz, & Giesen, 2016; Ishaq & Cats, 2020). For various public transport modes, the number of intermediate stops can be reduced by implementing stop skipping operations at the tactical (Wu, Liu, Jin, & Ma, 2019) or operational level (Liu, Yan, Qu, & Zhang, 2013), or alternatively by aborting an on-going trip and performing a short-turning or an interlining (Gkiotsalitis, Wu, & Cats, 2019). Similarly, the number of boarding passengers can be controlled by enforcing a limited boarding policy (Bueno-Cadena & Munoz, 2017; Delgado, Munoz, & Giesen, 2012). While potentially beneficial for service performance, measures to reduce the number of intermediate stops served or the number of passengers served at those stops are often objected to by service providers and not well-received among service users, therefore a comprehensive assessment that includes access, waiting and in-vehicle times in decisions concerning changes in bus stop settings must be made.

Most of the preventive measures are directed at reducing the initial headway discrepancy at upstream stops. Headway regularity upon dispatching from the original terminal is of prime importance because it is consequential for the offset throughout the trip (see Section 3) and because if directly targeted will reduce the need to apply corrective measures further downstream the route. Dispatching measures that effectively involve rescheduling are devised to reduce the variability in headway upon departure from the origin terminal (Gkiotsalitis & van Berkum, 2020). This is of special importance in the event of a disturbance downstream along the route, where introducing small modifications to metro dispatching times has been shown to greatly benefit service regularity and hence experienced level-of-service (Gkiotsalitis & Cats, 2020).

Measures to reduce variability en-route can be categorised into at-stop and between stops measures. Headway-based holding control strategies have been extensively studied in the literature (for a review of at-stop control measures, see Gkiotsalitis & Cats, 2021a). In the case of bus services, allowing for the bus trailing behind the bus with a long forward headway to overtake will alleviate some of the negative consequences induced by holding. The implementation of holding strategies involves determining the number and position of control locations as well as specifying the control logic. Analytical optimality formulations solved in the form of heuristics or exact solutions are either aimed at maximising headway regularity (Bartholdi & Eisenstein, 2012) or minimising passenger times (Sáez et al., 2012). The implications of rule-based strategies for equalising headways have been assessed analytically (Daganzo, 2009), using a stochastic and dynamic agent-based simulation model (Cats, Larijani, Koutsopoulos, & Burghout, 2011) and a numerical simulation (Andres & Nair, 2017). Recently, machine learning techniques have been applied for determining holding times to aid the mitigation of bus bunching (Moreira-Matias, Cats, Gama, Mendes-Moreira, & de Sousa, 2016; Wang & Sun, 2020). A comparison of several holding strategies over a specific bus route in Portland is performed by Berrebi, Hans, et al. (2018), which shows the advantages of prediction-based holding methods that dispatch buses according to the predicted arrival times of following buses at a control point. Many of these studies have demonstrated that even though holding control in itself prolongs trip time, its joint effect is the reduction of vehicle travel time variations and consequently reducing the fleet size required for offering a certain service frequency. This can be further stimulated by introducing measures such as speed adjustment (Muñoz et al., 2013) and conditional traffic signal priority (Koehler & Kraus, 2010). Bueno-Cadena and Munoz (2017) show that well-designed control strategies can simultaneously reduce user costs (travel time) and operating costs.

The abovementioned studies have analysed holding while considering each line in isolation. In reality service providers may want to consider the regularity of several lines simultaneously, either due to their inter-dependency due to running along a common corridor (Diab et al., 2016; Hernández, Muñoz, Giesen, & Delgado, 2015; Laskaris, Cats, Jenelius, Rinaldi, & Viti, 2019) or due to the need to reconcile the synchronisation between intersecting lines for transferring passengers in addition to single-line regularity considerations (Gavriilidou & Cats, 2019). In cases in which several private operators provide services and their revenue depends on how many passengers are served, Hernández et al. (2015) show that headway control is much more effective if it is handled by a central authority, because operator-specific incentives for keeping even headways may conflict with their objective to capture more demand if services from different companies operate in parallel. A company may choose to dispatch its buses to maximise the demand being captured (for instance, immediately ahead of the buses of the competition) instead of keeping regular headways.

As discussed in Section 3, differences in drivers' behaviour are related to increased headway variability. In turn, since in most systems the actions to prevent bunching must be implemented by drivers, their participation becomes key to mitigate bunching. Phillips, del Rio, Muñoz, Delgado, and Giesen (2015) show that a small fraction of non-complying drivers is enough to drastically harm the performance of headway control tools. Based on a drivers' survey, Martinez, Munoz, Delgado, and Watkins (2020) show that senior drivers are more reluctant to use these tools, while young ones recognise their value. Furthermore, Martinez et al. (2018) illustrate the potential of reducing bunching by simply allocating drivers to routes according to their operational speed. Drivers claim that a monetary incentive and formal instruction would foster its use. Indeed, provisioning headway-based information for drivers to continuously monitor their position in relation to the preceding and proceeding buses proved key in the success of implementing regularity-oriented operations in Stockholm (Cats, 2014). Fadaei and Cats (2016) quantified and monetised the passenger and operational benefits associated with implementing such interventions. Local service providers have chosen to maintain the even-headway control strategy even before this has been contractually required due to the associated operational benefits. In a related analysis of this Stockholm case study, Hlotova, Cats, and Meijer (2014) report that the introduction of even-headway control has even resulted in lower stress levels among bus drivers, therefore service reliability is also a health and safety issue for drivers (TRB, 2020b).

Finally, regarding technological innovations, connected and automated vehicles (CAVs) have the potential to influence the road-based public transport industry in several ways. Automation technologies that can be introduced in public transport services include automated collision avoidance, bus platooning, lane-keeping, bus precision docking, automated emergency braking and cooperative adaptive cruise control. It is fair to mention that CAV technologies are is still under development and uncertainties remain, therefore the true implications of CAV technologies for public transport are insofar largely unknown. Based on existing projections, these technologies could provide greater flexibility if vehicle capacity is dynamically adapted by assembling or disassembling multiple automated minibuses (Dai, Liu, Chen, & Ma, 2020). Regarding effects on planning, if a reduction of driver cost materialises, savings should be transferred to larger service frequencies (Fielbaum, 2020; Hatzenbühler, Cats, & Jenelius, 2020; Tirachini & Antoniou, 2020; Zhang, Jenelius, & Badia, 2019) and more direct lines with fewer transfers (Fielbaum, 2020).

One of the most relevant features of vehicle automation in public transport is the introduction of trajectory control, i.e. running times of automated vehicles can be centrally adjusted, taking into account forward and backward headways, for all vehicles at the same time (Dai et al., 2020). This poses a potential revolution for the deployment of headway control strategies, in which the key to success and improvement in service levels will be the objective function used and the specific control strategies to be implemented. As speed can be smoothly adjusted, holding strategies that affect on-board users' experience can be avoided. The meeting of vehicles at transfer stops or stations can be optimised to minimise transfer burden on passengers (Cao, Ceder, & Zhang, 2019). Research on these topics is still limited and authors usually resort to simulation to estimate reliability improvements in the form of schedule adherence and reduced headway variability and waiting times that are reachable with automated buses with or without varying capacity (Cao et al., 2019; Dai et al., 2020). More research efforts are expected as the technology of connected and automated vehicles evolves for its use in public transport.

6. Effects on contracting

Public transport performance metrics only focusing on the adherence of the average frequency and speed are not enough to control headway variability, and therefore, are incomplete incentives if the objective is improving users' satisfaction. As discussed earlier, effectively tackling headway variability and bunching most likely requires knowledge, data, technology and driver commitment. Thus, because efforts to address headway variability issues are usually costly, it is common that operators will implement preventive or corrective measures only if incentivised or demanded by their service provision contract. Therefore, strong performance indicators that specifically address headway variability are needed. In the case of private operators regulated by a public agency, the regulator must align the interest of the company with the interest of the users through the contract. Adding an incentive to provide reliable headways along public transport routes is tricky since not only other service dimensions might get affected by the incentive, but also because headway reliability must be addressed over multiple periods facing time-varying demand and frequency.

Examples of cities that explicitly embed headway variability considerations in the performance assessment of their public transport operations are London, Singapore, and Santiago (Leong, Goh, Hess, & Murphy, 2016; MTT, 2012; TfL, 2015). For the case of North American cities, see Kittelson & Associates, Brinckerhoff, KFH Group, Institute, and Arup (2013) and Diab, Badami, and El-Geneidy (2015). In London, bus services are clustered into two frequency categories: timetabled and non-timetabled (TfL, 2015), which is a relevant distinction as different waiting time components are attached to short and long headway services (Furth & Muller, 2006). For high-frequency services, the most relevant measure is the excess waiting time, as described in Equation (19). The share of long headways is also measured, as it might indicate the prevalence of other problems that lead to bus bunching. In the case of Singapore (Leong et al., 2016), until 2016 the Public Transport Council managed the Quality of Service Standards (QoS) for bus and rail operation by contractors. These standards considered that service regularity was achieved if 85% of the headways were no longer than 5 min of their scheduled headway each day. A drawback of this type of performance index (i.e. those that account for the number of headways exceeding a certain threshold) is that there is an incentive to drop excessive large headways, and even to increase it, in order to reduce those at risk of exceeding the threshold value. The excess waiting time was introduced in 2014 (Leong et al., 2016) and since 2016, different excess waiting time standards are defined for each bus service, and thus, operators must consider those values when planning their operation.

As noted by Cats (2014), none of these citywide public transport systems applies real-time regularity control, but they control for even headway upon dispatching or at different specific places along the route. There are many different bonus and penalty schemes that can be applied under these circumstances. Figure 1 presents a continuous penalty or bonus function that could be considered, depending on the expected waiting time of a user arriving randomly to a bus stop. In this example, a maximum bonus is given to the operator if the expected waiting time is kept below X_1. At an expected waiting of X_2 the bonus vanishes, while if the waiting time exceeds X_2 the operator receives penalties. If the expected waiting reaches X_3 where the maximum penalty is applied to the operator. The idea behind this scheme is to incentivise the operation to be as close as possible to a desired level of performance.

Of course, the definition of X_1, X_2 and X_3 would depend on the frequency being requested to the operator. Notice that the scheme is stop-based. Agencies usually

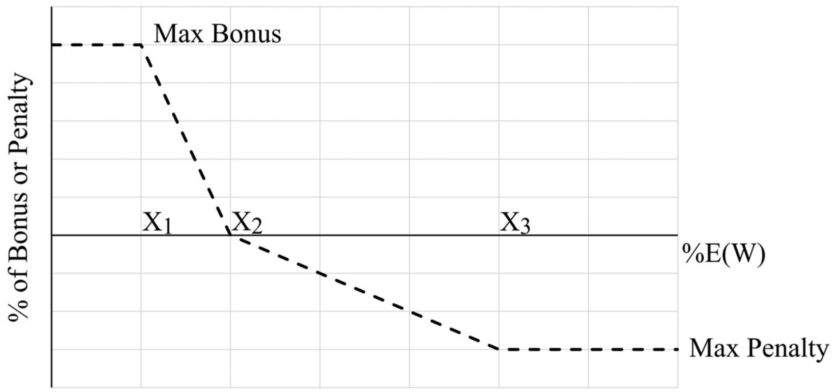

Figure 1. General representation of a bonus/penalty function.

apply the same incentive to a service in any of its stops. However, as headway variability tends to worsen downstream, regular intervals are more important to encourage at the beginning rather than at the end of the route. Thus, one possible way to increase the effectiveness of the incentives is to enlarge the weight of these indicators at the dispatch or first stops of the service in comparison to those at the last part of the route.

7. Concluding remarks: reflections on current research and practice

The present review allows us to conclude that there is a remarkably large gap between the state-of-the-art on mitigation and control measures for managing service regularity and the state-of-the-practice, where services are subject to schedule-based control or sometimes not subject to any proactive and/or corrective control measure at all. As discussed in Sections 2 and 3, the causes and symptoms of headway variability have been repeatedly identified and reported in detail. While strategic measures such as improving public transport right-of-way and removing the need to validate tickets on-board have been adopted by many systems worldwide, operational measures are still needed to correct in real-time for service variations. A summary of measures to reduce headway variability is presented in Table 2. Based on our assessment of the extant literature, we have added (+++), (++) and (+) to treatments with strong, medium and weak influences on headway variability, respectively, and (?) to the case of vehicle automation, whose real impact on service reliability is not yet known.

A large number of analytical and simulation studies have demonstrated the benefits of control measures aimed at preventing and mitigating bunching under various circumstances (Section 5). Notwithstanding, and despite few notable expectations, there is evidently a significant gap between the recommendations made by the state-of-the-art and the state-of-the-practice. In many cases, the lack of suitable and reliable technology and software that facilitate the continuous communication of control instructions to individual drivers still form considerable barriers. The reliance on street supervisors and control centre dispatchers to constantly monitor and instruct drivers has proven in a series of field experiments to be inefficient and ineffective (Section 5).

Table 2. Interventions to reduce headway variability in public transport.

Treatment	Placement of intervention			Type of intervention			Variable affected						
	At stops	In-between stops	Others	Operational	Physical	Policy	Initial headway discrepancy	Frequency	Demand variability	Boarding/alighting times	Travel time variability	Driver performance	Number of stops
Regularity at dispatching (+++)	x	X		x			x		x				
Bus holding (+++)	x			x			x		x		x		
Frequency optimisation (+++)			x			x		x					
Reliability incentives in contracts (++/+++)			x			x	x	x			x	x	
Traffic signal priority (++)		x		x							x		
Speed adjustment (++)		x		x							x		
Off-board fare payment (++)	x				x				x	x			
Limited-stop services (+/++)	x			x					x		x		x
Bus lanes and corridors (+/++)	x	x			x			x			x		
Driver training (+/++)			x			x						x	
Boarding limits (+)	x			x					x				
Vehicle automation (?)			x	x	x		x	x	x	x	x	x	

While suitable technology is a necessary condition, it is not a sufficient one. The availability of data devices to locate vehicles and passengers in real time is somewhat recent in several agencies that are in a transitional period, trying to translate the large amounts of data into meaningful metrics to monitor and improve service reliability (TRB, 2020a). In this respect, the building of technical capabilities within public transport agencies in order to take advantage of automated data is crucial for monitoring and improving service reliability. As pointed out by Cats (2014), service operations that are geared towards service regularity involve "a paradigm shift in production planning, operations, control centre and performance monitoring". In particular, it involves re-defining how performance is measured (Section 4) and re-designing the incentives scheme in the contracting of service provision (Section 6). The latter is often subject to inertia and the need to undergo negotiations and legal procedures.

Results from several small-scale and labour-intense headway control field experiments, such as those in Santiago (Lizana et al., 2014), Stockholm (Cats, 2014), Portland (Strathman et al., 2002), San Antonio and Atlanta (Berrebi, Crudden, et al., 2018) and Washington D.C. (Soza-Parra, Cats, et al., 2019) highlight the importance of the quality and reliability of the technical solution and driver compliance. The experience gained from the few successful implementations points to the importance of having all relevant stakeholders involved throughout the process, carefully assessing and refining the proposed measures in simulation models, testing software reliability prior to field trials, and using the field experiments to demonstrate the operational benefits as well as quantify and monetise the societal benefits for convincing policymakers.

Finally, on service reliability incentives in contracts, some contracting guidelines (e.g. Kittelson & Associates et al., 2013 in the United States), consider vehicle bunching effects on waiting time with metrics such as the coefficient of variation of headways and the excess waiting time. However, such metrics are not complete to account for other negative consequences of headway variability, as described in this article, including effects on reduced users' comfort (Muñoz et al., 2020). In bunched operations, vehicles running following longer headways tend to have a higher number of passengers on-board, that have waited longer and are travelling under less comfortable conditions. If the performance indicators do not consider the difference between the number of passengers on-board each vehicle (i.e. if every vehicle is equally weighted), there will be an overestimation of the quality of service provided, as such metrics do not correctly reflect the passengers' experience (Soza-Parra, Raveau, et al., 2019). With the increasing availability of passenger-level data, for instance, through automated fare collection techniques or mobile phone location data, fine-grained quality-of-service indicators that include headway variability, comfort and passenger' satisfaction outcomes should be developed in the future.

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Appendix. A mathematical formulation on public transport service bunching

Inherent sources of uncertainty that affect public transport operations include dispatching time from the origin terminal, traffic conditions, delays at intersections and stops, driver behaviour, travel demand and dwell time at stops. These stochastic factors are interrelated through the positive feedback loop amongst (i) headway between consecutive buses; (ii) the number of waiting passengers and bus loads, and; (iii) passenger service times at stops. These interrelations induce a self-reinforcing effect so that a late vehicle will have to pick up more passengers causing it to be further late while the succeeding vehicle increasingly catches up. This process leads to the well-known bunching phenomenon, the worst manifestation of which are pairs of buses platooning.

In this Appendix, we show how this problem occurs and its underlying mechanism. Let us consider the case of a service that has either a planned headway such that travellers arrive spontaneously at stops, or a timetable that is not sufficiently reliable for passengers to attempt to coordinate their arrival (Frumin & Zhao, 2012). The departure time from any stop s could be decomposed into the summations of dwell times at stops and riding times between stops as follows:

$$\tau_{k,s} = \sum_{i=s_0}^{s} d_{k,i} + \sum_{a=(s_0,s_0+1)}^{(s-1,s)} t_{k,a} \qquad (A1)$$

where $d_{k,i}$ and $t_{k,a}$ are the dwell time at upstream stop i and running time on upstream arc a, respectively. s_0 is the first stop on trip k.

The headway at any stop s can be represented as a function of the headway at a certain upstream stop j (e.g. s_0) which precedes stop s and vehicles progress (dwell times and running times) between stops j and s as follows:

$$h_{k,s} = h_{k,j} + \sum_{i=j}^{s} d_{k,i} + \sum_{a=(j,j+1)}^{(s-1,s)} t_{k,a} - \sum_{i=j}^{s} d_{k-1,i} - \sum_{a=(j,j+1)}^{(s-1,s)} t_{k-1,a} \tag{A2}$$

$h_{k,j}$ could be represented as a function of the planned headway: $h_{k,j} = \alpha_{k,j} h^0$ where $\alpha_{k,j} \geq 0$. The latter assumes that passing is not allowed or that vehicles are dynamically re-ordered. As discussed earlier, both supply and demand are subject to stochastic discrepancies. For example, an exogenous factor could lead to irregular dispatching from the first stop and result with $h_{k,s_0} \neq h^0$. Moreover, it could be that travel conditions, driver behaviour or irregular passenger activity at stops led to an either shorter or longer than usual travel time between stops s_0 and j. Such discrepancies may result with $h_{k,j}$ that is longer or shorter than the planned headway and thus $\alpha_{k,j} \neq 1$.

Dwell times at stops are composed of fixed times related to door opening and closing times, and variable times primarily stemming from passenger service time. The former can be in this context treated as part of the riding time leading to that stop while the latter is a function of the numbers of passengers boarding and alighting. Boarding volumes often dominate the service time function due to the more onerous boarding process, if boarding is restricted to a limited number of doors or in case tickets have to be purchased and/or validated upon boarding. Without loss of generality, let us consider the case where the passenger service time at bus stops could be approximated by a linear function of the number of boarding passengers, $q_{k,j}$

$$d_{k,j} = \beta \cdot q_{k,j} \tag{A3}$$

where $\beta > 0$ is a coefficient representing the boarding time per passenger. Passengers' arrival rate at stop j is denoted by λ_j, expressed for example in passengers per hour terms. Following the Poisson arrival process assumption, the expected number of passengers that wait at stop j for vehicle trip k is then:

$$q_{k,j} = \lambda_j h_{k,j} \tag{A4}$$

The Poisson distribution represents the situation in which passengers arrive randomly at stops, without information about the departure of vehicles (in practice, when scheduled departure times are known, some passengers adjust their arrival time to the departure time of vehicles, even for headways shorter than 10 min, see Ingvardson, Nielsen, Raveau, & Nielsen, 2018). Assuming that vehicle capacity is not binding, all waiting passengers can board the first arriving vehicle. Without loss of generality let us denote arrival rates at all stops by λ. If travel times between stops are assumed to be independent of headways and $\alpha_{k-1,i} = 1 \ \forall i = j, \ldots, s$ then the headway at stop $j+1$ can be written as:

$$h_{k,j+1} = h_{k,j} + (d_{k,j} - d_{k-1,j}) = \alpha_{k,j} h^0 + (\alpha_{k,j} - 1) h^0 \beta \lambda \tag{A5}$$

The deviation of the headway at stop $j+1$ from the planned headway h^0 can be thus expressed as:

$$\alpha_{k,j+1} = \frac{h_{k,j+1}}{h^0} = \max\{\alpha_{k,j} + \beta\lambda(\alpha_{k,j} - 1), 0\} \tag{A6}$$

Similarly, the headway at stop s could be formulated as a recursive expression in relation to any upstream stop j as follows:

$$h_{k,s} = h_{k,j} + \sum_{i=j}^{s} d_{k,i} - \sum_{i=j}^{s} d_{k-1,i} = \alpha_{k,j} h^0 + \sum_{i=j}^{s} \alpha_{k,i} h^0 \beta\lambda - \sum_{i=j}^{s} h^0 \beta\lambda \tag{A7}$$

The general term for headway ratio at stop s as a function of the headway ratio at an upstream stop j is hence:

$$\alpha_{k,j} = \frac{h_{k,s}}{h^0} = \max\{1 + (\alpha_{k,j} - 1)(\beta\lambda + 1)^{s-j}, 0\} \tag{A8}$$

Proof: This expression is proven by induction. In the basic case that $s = j$ the above expression amounts to $\alpha_{k,j} = 1 + (\alpha_{k,j} - 1)(\beta\lambda + 1)^{j-j}$. The two sides of the equation are equal hence the expression holds true for the base case. Next, we show that if Equation (A8) holds true for s then it holds also for $s + 1$. The expression for $s + 1$ is as follows:

$$\alpha_{k,s+1} = 1 + (\alpha_{k,j} - 1)(\beta\lambda + 1)^{s+1-j} \tag{A9}$$

Based on the recursive relationship (Equation (A6)) and after algebraic rearrangement:

$$\alpha_{k,s}(1 + \beta\lambda) = (1 - \beta\lambda) + (\alpha_{k,j} - 1)(\beta\lambda + 1)^{s+1-j} \tag{A10}$$

By dividing this equation by $(1 + \beta\lambda)$ we yield the expression for s (Equation (A8)). It is now proven by mathematical induction that it holds true for any natural $s > j$.

Equation (A8) implies that the headway at any stop could be formulated as the headway at any selected upstream stop. Note that if $\alpha_{k,j} = 1$ then also $\alpha_{k,s} = 1$ and the system is stable and the service is regular. If however $\alpha_{k,j} > 1$ then $\alpha_{k,s} > \alpha_{k,j}$ and $h_{k,s} > h_{k,j}$, whereas if $\alpha_{k,j} < 1$; then $\alpha_{k,s} < \alpha_{k,j}$ and $h_{k,s} < h_{k,j}$.

The extent to which the headway deviation escalates between any pair of stops along the route depends hence on the following: (i) initial headway ratio, $\alpha_{k,j}$; (ii) the boarding time per passenger, $q_{k,j}$; (iii) the number of intermediate stops, and; (iv) the corresponding passenger arrival rates, $(\lambda_j \ \ldots \ \lambda_s)$. Hence, services operating without on-board validation and allowing for boarding and alighting from a larger number of doors are thus less susceptible to headway irregularity and bunching. These relationships are also supported by empirical studies that analysed the propagation of service unreliability along public transport lines (e.g. West & Cats, 2017). Note that the more frequent the service, the more easily it can get bunched due to small perturbations in its running times (e.g. traffic signal cycle; see Table 1).

Equation (A8) could be reorganised to reflect the ratio of headway ratios along trip k:

$$\frac{h_{k,s} - h^0}{h_{k,j} - h^0} = \frac{\alpha_{k,s} - 1}{\alpha_{k,j} - 1} = (\beta\lambda + 1)^{s-j} \tag{A11}$$

In Equation (A11), the escalation rate is independent of the planned headway but increases as a function of the average passenger service time and passenger arrival rate. Moreover, each additional intermediate segment has a cumulative impact on the escalation rate.