MODULATION OF FINE-SCALE VELOCITY GRADIENT PHENOMENA BY CONCURRENT LARGE-SCALE VELOCITY FLUCTUATIONS IN A DEVELOPED SHEAR FLOW

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Abstract The interaction between the large-scale velocity fluctuations (u_L) and the small-scale velocity gradient phenomena, such as dissipation and the vortex stretching term, are examined in a turbulent free-shear flow. The difference between the probability density functions of these small-scale quantities conditioned on the sign of u_L is quantified by means of the Kullback-Leibler divergence. It is observed that the interaction between u_L and the velocity gradient phenomena is maximised at a filter length of 4λ, where λ is the Taylor length-scale. It is postulated that this is consistent with a mean shear mechanism.

INTRODUCTION AND NUMERICAL DATA

In the Richardson-Kolmogorov phenomenology of turbulence the fine-scales are considered to be "universal" and thus independent of the larger, energy-containing scales. However, starting with the work of [1], closer attention has been paid to the interaction between the large- and small-scales in shear flows, including wall bounded flows. By drawing an analogy to a large eddy simulation (LES), in which the largest scales present in the flow are resolved whilst the sub-grid scales (SGS) are modelled, [5] used their highly resolved experimental data to show a clear interaction between the large-scale and SGS fluctuations. It was observed that large-scale organised structures in a turbulent free shear flow have a significant impact on the statistical distribution of the SGS dissipation, even with filter lengths (separating the large-scales from the SGS) well into the inertial range. Additionally [6] showed that the SGS stresses had a significant effect on the evolution of the filtered velocity gradients. More recently [2] has shown that a concurrent interaction exists between the large- and small-scales in the far-field of a turbulent mixing layer such that low momentum fluctuations amplify the small-scale activity, with this effect being observed despite a significant separation in spectral space between the large- and small-scales. This manuscript examines the concurrent interaction between the large-scale velocity fluctuations and the fine-scale velocity gradient phenomena, such as dissipation (ϵ) and the vortex stretching term ($\Omega = \omega_i s_{ij} \omega_j$) in which $\boldsymbol{\omega}$ is the vorticity vector and s_{ij} is the fluctuating strain-rate tensor. Three independent snap shots of a direct numerical simulation (DNS) of a planar turbulent mixing layer are used as the basis for this abstract. A region of the domain, in which the flow is observed to be self-preserving, is isolated and a threshold based on enstrophy (ω^2) is used to discriminate between the turbulent and non-turbulent regions of the flow. The Reynolds number based on the Taylor length scale of $Re_\lambda = 220$ is observed for the entirety of this region of the flow. More details on the simulation can be found in [3].

METHODOLOGY AND RESULTS

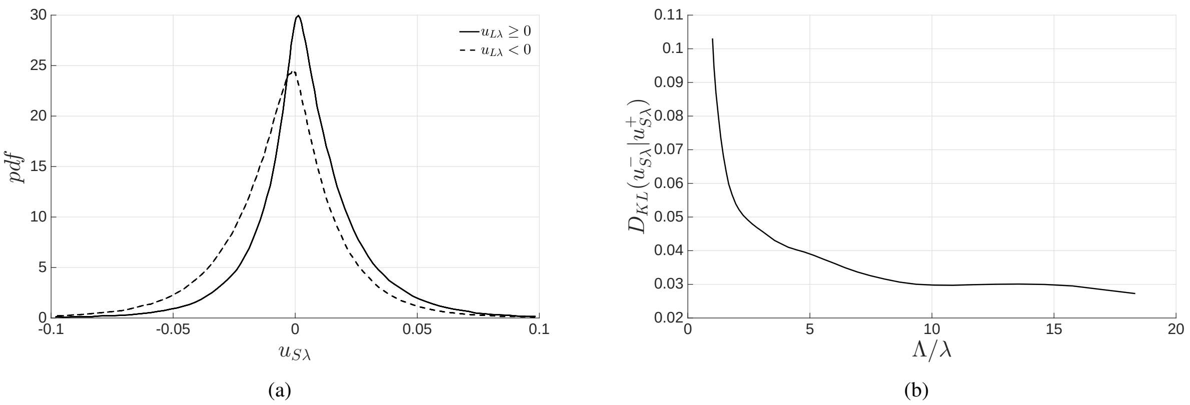

Figure 1. (a) pdfs of small-scale velocity fluctuations, $u_{S\lambda}$, conditioned on the sign of the concurrent large-scale fluctuation, $u_{L\lambda}$. (b) Kullback-Leibler divergence (KLD) of the pdfs of $u_{S\lambda}$ conditioned on $u_{L\Lambda} < 0$ and $u_{L\Lambda} \geq 0$ concurrently for filter length Λ.

The data was initially filtered at a cut-off length scale of λ with a fully three-dimensional sharp spectral filter. The low wavenumber content, corresponding to length scales greater than λ is denoted $u_{L\lambda}$ and is used to condition probability density functions (pdfs) of the concurrent high wavenumber content ($u_{S\lambda}$), corresponding to length scales less than λ. [2] observed an amplification of the "roughness" of the small-scale turbulent fluctuations concurrent with large-scale negative $u_{L\lambda}$ fluctuations in a region to the high speed side of the location of peak Reynolds stresses in a turbulent mixing layer

with comparable Reynolds number to this study. In this way they were able to postulate a convective mechanism, in which a negative $u_{L\lambda}$ fluctuation (coupled to a positive $v_{L\lambda}$ fluctuations due to the positivity of the turbulence production term) to attempt to explain this scale interaction. For consistency with this previous study, we present results in the high speed side of the mixing layer ($y > 0$). Figure 1(a) shows pdfs of the small-scale fluctuations ($u_{S\lambda}$) conditioned on the sign of the large-scale fluctuations ($u_{L\lambda}$). The distribution is less "peaky" for $u_{L\lambda} < 0$ verifying the increased small-scale activity for negative velocity fluctuations for the high speed side of the mixing layer. This result is, however, asymmetric due to the adjacency of $u_{L\lambda}$ and $u_{S\lambda}$ in spectral space, but the scale modulation is present nevertheless.
We subsequently investigate the modulation of $u_{S\lambda}$, which remains filtered at length-scale λ, by large-scale velocity fluctuations filtered at increasing filter lengths Λ, such that there is a separation in wavenumber space between $u_{S\lambda}$ and $u_{L\Lambda}$. We may quantify the difference between the pdfs of $u_{S\lambda}$ conditioned on the sign of $u_{L\Lambda}$ by means of the Kullback-Leibler divergence (KLD) defined as $D_{KL}(P\|Q) = \int_{-\infty}^{\infty} \ln\left[p(x)/q(x)\right]\ p(x)\mathrm{d}x$, where $p(x)$ and $q(x)$ are the pdfs of fluctuating variables P and Q [4]. It originates from information theory and physically it represents the information loss in modelling distribution $p(x)$ by $q(x)$ and is non-commutative. Figure 1(b) shows D_{KL} (between the pdfs of $u_{S\lambda}$ conditioned on $u_{L\Lambda} < 0$ and $u_{L\Lambda} \geq 0$) as a function of filter length Λ. It can be seen that there is a significant modulation of the finest-scales by the large-scales, despite the separation in wavenumber space between them, up until $\Lambda \approx 10\lambda$.

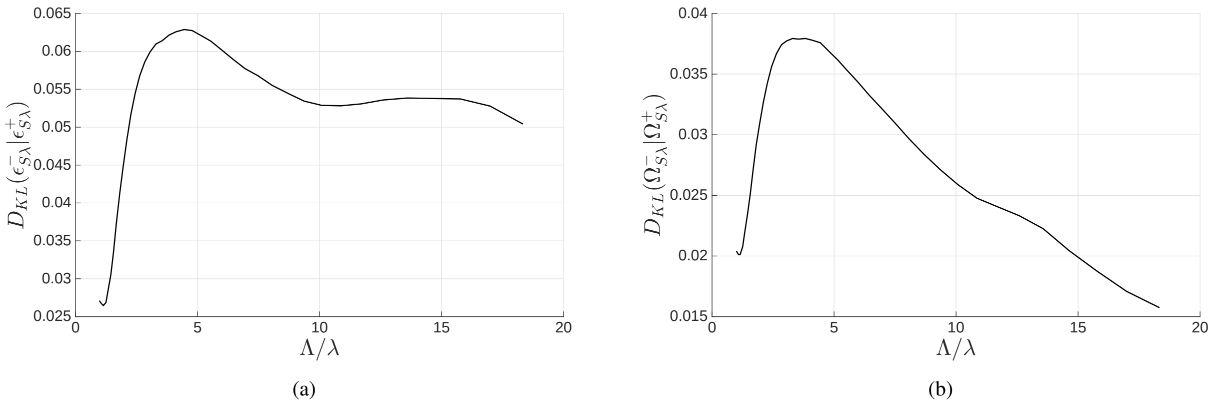

Figure 2. KLD for the pdfs of the small-scale dissipation rate, ϵ, (a) and the small-scale vortex stretching term, Ω, (b) conditioned on $u_{L\Lambda} < 0$ and $u_{L\Lambda} \geq 0$ concurrently for filter length Λ. N.B. to respect the Nyquist sampling theorem no data is presented for $\Lambda > 18\lambda$ after which the filter is constrained in the y direction.

Figures 2(a) and (b) illustrate D_{KL} for the pdfs of ϵ and Ω conditioned on $u_{L\Lambda} < 0$ and $u_{L\Lambda} \geq 0$. For both figures it is observed that there is an initial sharp increase in the KLD up to a maximum which coincides with $\Lambda \approx 4\lambda$, which is a peculiar length scale with no obvious physical interpretation. 4λ roughly corresponds to $L_{11}/4$, where L_{11} is the integral length scale for the flow at this Reynolds number. This is also observed for the amplitude modulation of the small-scale enstrophy by the large-scale velocity fluctuations, which follows a quantitatively very similar trend to that for dissipation but is not shown in this abstract for brevity. It is thus postulated that the mechanism behind this "peak interaction length scale" is driven by the mean shear of the flow. The final paper will further investigate this hypothesis. Whilst it can be seen that figure 2(a) plateaus from $\Lambda > 10\lambda$ the scale modulation of Ω decreases until $D_{KL}(\Omega^-_{S\lambda}|\Omega^+_{S\lambda})$ becomes smaller than the value for $\Lambda = \lambda$, in which there is no separation in spectral space, at $\Lambda \approx 15\lambda$. The final paper will also explore the origins of the seemingly linear slope of $D_{KL}(\Omega^-_{S\lambda}|\Omega^+_{S\lambda})$ with Λ as opposed to the plateau for $D_{KL}(\epsilon^-_{S\lambda}|\epsilon^+_{S\lambda})$. This tells us that dissipation is always "aware" of the concurrent large-scale velocity fluctuation whereas this is not the case for the vortex stretching term. This can potentially be explained by the scale dependency of the alignment tendencies of the eigenframe of the fluctuating strain-rate tensor with the vorticity vector, which is crucial in determining Ω.

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