



Heel-sway-yaw coupling hydrodynamic loads on a high speed vessel

M. Bonci

Delft University of Technology

M. Renilson

Australian Maritime College, University of Tasmania

P. de Jong & F. van Walree

MARIN, Wageningen

A.J. Keuning & R.H.M. Huijsmans

Delft University of Technology

ABSTRACT: The manoeuvring characteristics of high speed crafts are greatly influenced by the hydrodynamic loads generated by the asymmetrical underwater hull shape caused when the vessel heels. In order to provide an insight into this aspect of the manoeuvring of high speed crafts, an experimental study has been undertaken, considering a rescue vessel of the Royal Netherlands Sea Rescue Institution (KNRM). Captive model experiments were conducted in the model towing tank at the Delft University of Technology.

The experiments were divided in two main phases. In the first phase, the heel-sway, heel-yaw coupled linear coefficients and hydrodynamic heel moment were measured using static heeled model measurements over a range of speeds. The speed range investigated was between Froude number 0.4 – 1.3. The model's vertical position was constrained to its equilibrium running condition at speed. The second stage of the experiments examined the influence of different running trim attitudes on the manoeuvring coefficients values. The results from three running trim conditions were compared. These tests were carried out over a lower speed range (0.4 – 0.7 Froude number). Model tests are presented and discussed further, along with a description of the experimental set-up.

NOMENCLATURE

Roman

B_R	Total bias	[-]
f	Non-linear hydrodynamic loads	[N, Nm]
Fr	Froude number U/\sqrt{gL}	[-]
g	Gravity acceleration	[m/s ²]
GM	Metacentric height	[m]
I	Mass moment of inertia	[kgm ²]
J	Added mass moment of inertia	[kgm ²]
K_ϕ	Roll coefficient due to heel	[Nm/rad]
K_r	Roll coefficient due to yaw rate	[Nms/rad]
K_v	Roll coefficient due to sway	[Ns]
L	Ship length	[m]
m	Ship mass	[kg]
m_x	Added mass in x-direction	[kg]
m_y	Added mass in y-direction	[kg]
N_ϕ	Yaw coefficient due to heel	[Nm/rad]
N_r	Yaw coefficient due to yaw rate	[Nms/rad]
N_v	Yaw coefficient due to sway	[Ns]
P_R	Total precision limits	[-]
r	Yaw rate	[rad/s]
R_0	Resistance in upright conditions	[N]
T_m	Ship mean draft at zero speed	[m]

u	Advance speed	[m/s]
U	Total ship speed	[m/s]
U_R	Total uncertainty	[-]
v	Sway speed	[m/s]
Y_ϕ	Sway coefficient due to heel	[N/rad]
Y_r	Sway coefficient due to yaw rate	[Ns/rad]
Y_v	Sway coefficient due to sway	[Ns/m]
Z_0^G	Rise of centre of gravity	[m]

Greek

α_x	x-location of added mass centre	[m]
α_z	z-location of added mass centre	[m]
β	Drift angle	[deg]
δ	Control devices deflection	[deg]
Δ	Ship displacement	[N]
ϕ	Heel angle	[deg]
θ	Pitch angle	[deg]
θ_x	Sensitivity coefficients	-
ρ	Water density	[kg/m ³]

An over-dot denotes a time derivative. Primed symbols denote non-dimensional quantities.

1 INTRODUCTION

Traditionally the manoeuvring of surface ships has been examined by considering only the equations in: surge, sway and yaw. Motions in: heave, pitch and roll are usually neglected. However, it is well known that for those ships which experience significant heel motions when turning the coupling between heel and yaw/sway must be considered. See for example (Tuite & Renilson 1995, and Renilson & Tuite 1996, 1997). This is because the asymmetric underwater shape of the hull when the vessel is heeled will result in additional sway force and yaw moment.

The coupling between heeling and the manoeuvring on the horizontal plane is of particular importance for small high speed crafts, for which the heel angle in a turn can be substantial (Renilson 2007, Renilson & Manwarring 2000). This happens in calm water in tight and fast turns, but also in waves. Unlike large displacement ships, planing and semi-planing small hulls are more subjected to relatively large transverse motions when sailing in a seaway which could affect the manoeuvrability of the vessel. In extremely adverse cases this can compromise the course stability of the vessel. Past research studies examined the effect of large heel angles on manoeuvrability in order to obtain a quantitative prediction of broaching and capsize phenomena (Hashimoto et al. 2011, Umeda 1999).

It is then of fundamental importance to investigate the manoeuvring features of high speed craft to be able to understand their behaviour during a manoeuvre. Heel-sway-yaw coupling is still a not well understood phenomenon in ship manoeuvrability, especially for fast and small vessels. This coupling effect has been noted on displacement and semi-displacement vessels, such as naval ships or container ships – see for example (Tuite and Renilson 1995, Oltmann 1993), but not yet on small high speed crafts. Further studies on this special type of ships are then needed.

For high speed craft the equations of motion must be extended to include the equation for heeling (Yasukawa & Yoshimura 2014, Yasukawa 2010), together with the relevant coupling terms, as shown in Equations 1-4 below. The equations are given in non-dimensional form; heel angle is given in radians. The axis system is given in Figure 1.

One of the main difficulties when attempting to study the manoeuvring of high speed crafts is in the determination of the values of Y'_ϕ , and N'_ϕ which relate to the linear coupling between heel and sway force, and yaw moment respectively, as well as the value of the hydrodynamic heel moment, K'_ϕ . In the

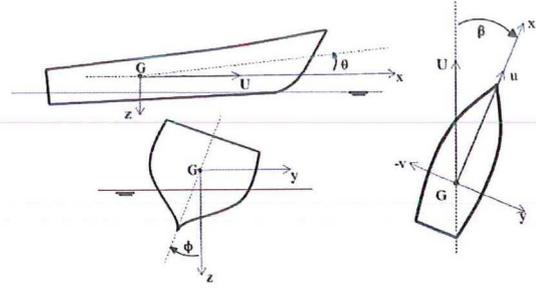


Figure 1 Coordinate system

following only these hydrodynamic linear components of the motion equations will be analysed (right hand of the equations). The linear coefficients due to sway velocity and yaw rate, surge dynamics as well as non-linear terms (denoted with f') are not considered in the present work as these are covered extensively by other researchers (Ommani et al. 2012), (Ommani & Faltinsen 2014).

$$(m' + m'_x)\dot{u}' - (m' + m'_y)v'r' = -f'_x(u') + f'_x(v', r', \delta, \phi, \dot{\phi}) \quad (1)$$

$$(m' + m'_y)\dot{v}' - (m' + m'_x)u'r' = Y'_v v' + Y'_r r' + Y'_\phi \phi + f'_x(v', r', \delta, \phi, \dot{\phi}) \quad (2)$$

$$(I'_x + J'_x)\ddot{\phi} - m'_y \alpha'_z \dot{v}' = -\Delta' GM' \phi + K'_v v' + K'_r r' + Y'_\phi \phi + f'_K(v', r', \delta, \phi, \dot{\phi}) \quad (3)$$

$$(I'_z + J'_z)\dot{r}' - m'_y \alpha'_x \dot{v}' = N'_v v' + N'_r r' + N'_\phi \phi + f'_N(v', r', \delta, \phi, \dot{\phi}) \quad (4)$$

This paper describes the investigation on heel-sway, heel-yaw and sway-heel coupling using a rescue craft of the Royal Netherlands Sea Rescue Institution (KNRM). The vessel is 18 meters long and has a maximum speed of 35 knots. It mainly operates in the pre-planing and planing regime. The vessel is the result of a joint project started in 2009 between KNRM, the Faculty of Marine Technology of TU Delft, Damen Shipyards and Willem de Vries Lentsch Yacht Designers & Naval Architects, addressing the development of a new type of life-boats capable of dealing with any adverse sea conditions and meeting the future regulations. NH-1816 has evolved from the previous concept of the Arie Visser life-boat. The vessel is based on the axe-bow concept, conceived by Keuning at TU Delft (Keuning 2006). The concept successfully demonstrates significant enhancements in the seakeeping capabilities in harsh sea states, reducing the vertical accelerations at the bow.

The coupled heel-sway, heel-yaw and heel moment hydrodynamic linear coefficients are determined by means of captive model experiments in calm water. The investigation covers the range of speeds at which the life-boat can operate, between 10 to 35 knots. The focus is only on the linear loads: this is made for the purpose of the initial course stability assessment, which could involve also the roll dynamics. This aspect is not directly discussed in the present paper, but it will be addressed in future works.

Particular interest was directed into the study of the heeling effects at different running attitudes of the ship. The vertical position of the vessel is in fact relevant for the total computation of the manoeuvring loads, as already discussed by many researchers (Yasukawa et al. 2016). Three different trim conditions were investigated: equilibrium planing attitude (with most of the bow region out of the water), and two bow down conditions, where a consistent part of the bow is immersed into the water. The planing running attitude is the most relevant scenario for high speed crafts, because they operate most of the time in this condition. The unrealistic position of the bow immersed in the water resembles what can happen to a small craft when sailing in rough sea. In those situations, the vessel can be affected by large and sudden motions which can have serious repercussions on its safety.

2 MODEL EXPERIMENTS

2.1 The model

The experimental campaign was executed using a model of the rescue boat SAR NH-1816 of the Royal Netherlands Sea Rescue Institution (KNRM). The hull lines and the model particulars are showed in Figure 2 and Table 1 respectively.

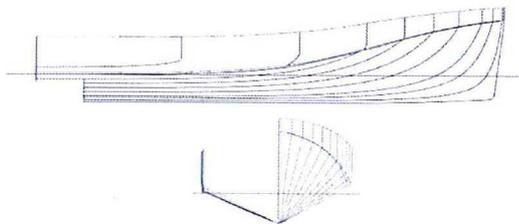


Figure 2 NH 1816 Hull lines

Table 1 NH1816 particulars

<i>NH-1816 model properties</i>	<i>SU</i>	<i>Values</i>
Model scale	[-]	10
Overall Length	[m]	1.93
Length between Perpendiculars	[m]	1.84
Overall Breadth	[m]	0.56
Draft	[m]	0.11
Weight	[kg]	26.28
Longitudinal Centre of Gravity	[m]	0.6
Wetted Surface at Zero Speed	[m ²]	0.78

This vessel was chosen for the present work for two main reasons. First, a great deal of experimental results, full scale direct observations and numerical data are available thanks to the long and detailed past research studies of the SAR NH-1816 project (de Jong et al. 2013). Second, further investigations on the manoeuvrability of axe-bow high speed crafts are still needed.

2.2 Experimental set-up

Captive model experiments were conducted in the model towing tank at the Delft University of Technology. The experiments were divided in two main phases. In the first phase, the heel-sway, heel-yaw coupled linear coefficients (Y'_ϕ , N'_ϕ) and hydrodynamic heel moment (K'_ϕ) were obtained using static heeled model measurements over a range of speeds. The speed range investigated was between Froude number 0.4 – 1.3, corresponding to 10 – 35 knots full scale. The tests were split in three stages, whose particulars are summarized in Table 2.

In Stage A of the experiments the model's rise and trim are constrained to the vessel equilibrium (planing or semi-planing) running conditions at speed. The second and third stages investigate the influence of different running trim attitudes on the manoeuvring coefficients values. The results of stage A are compared with other two different bow down positions, correspondent to Stage B1 and B2, respectively to -0.5 and -3.0 degrees. The heave of the model at these two additional trim conditions is constant at +5mm under the water surface (increased immersion). The effect of trim is examined over a lower range of speeds (0.4 – 0.7 Froude number).

The model was towed in static conditions at different speeds: the rise of the model, trim and heel angles were fixed during each run. The rotations were set around the centre of gravity of the model. The rise and orientations were set by a 6DOF oscillator which is based on the principle of the Stewart platform (Stewart 1965), also known as

Hexapod. The Hexapod can move in six degrees of freedom using six electrical actuators.

Table 2 Experimental tests summary

Stage	Fr [-]	Z_0^G [mm]	θ [deg]	ϕ [deg]
A	0.38	8.80	0.82	0 to 12
	0.48	13.0	2.63	
	0.57	7.40	3.38	
	0.67	-1.40	3.67	
	0.77	-11.0	3.82	
	0.86	-19.8	4.07	
	0.96	-28.8	4.27	
	1.05	-35.4	4.23	
	1.15	-39.8	4.08	
	1.25	-43.1	3.87	
	1.34	-45.8	3.65	
B1	0.38	5.00	-0.50	0 to 16
	0.48			
	0.57			
	0.67			
B2	0.38	5.00	-3.00	0 to 16
	0.48			
	0.57			
	0.67			

When the vessel is heeled, the trim may change with respect to straight upward forward conditions, due to asymmetries in the submerged body, in the motion or in the disturbance loads. Likewise, the vessel can rise or sink, depending on the pressure distribution on the hull. These effects are not considered in the present study: this change in running attitude is assumed to be small, and so the effect of heel and drift on the vertical motions is neglected.

Figure 3 shows an example run of the experiments.

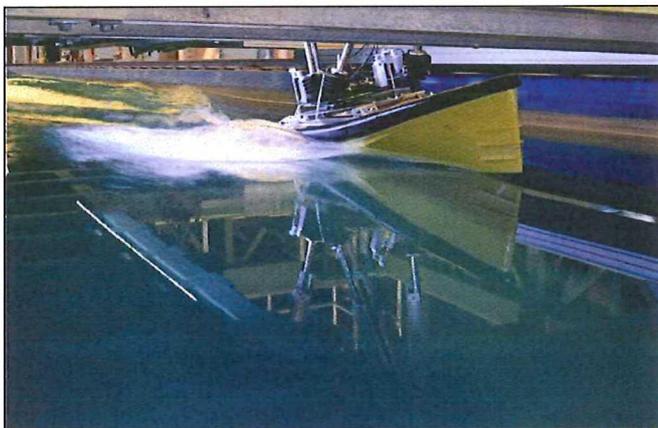


Figure 3 Model test run (Stage A, $Fr = 1.05$, $\phi = 12$ deg)

The model was mechanically attached underneath the moving platform of the hexapod by a

frame made up of two plates rigidly connected to each other by six strain gauges. The structure of the frame permitted the calculation of forces and moments in all directions.

2.3 Data post-processing

The objective of the experiments was to derive the hydrodynamic coefficients (for sway, roll and yaw) when the ship is statically heeled at one side. Hydrodynamic forces and moments were evaluated in two steps. Before each measurement at speed, the static loads were measured on the model at zero velocity in the same positions and orientations of the upcoming run. At this point, only buoyancy and gravity loads were measured. These static forces and moments were subtracted from the total ones evaluated during each run at speed, resulting in the only hydrodynamic components. The resulting values comprise hydrodynamic pressure, viscous and disturbed wave loads. During the runs at speed, the calm waterline was deformed because of the disturbance of the moving vessel, and as a consequence the buoyancy changes. This is considered to be a hydrodynamic effect due to speed.

At each speed, the hydrodynamic loads were measured at different heel angles, for example 0, 4, 8 and 12 degrees (see Figure 6). The curves obtained as functions of heel angle were fitted with a linear polynomial regression. The linear coefficients Y'_ϕ , K'_ϕ and N'_ϕ of each polynomial were calculated using the Least-Square fitting method. Forces and moments are expressed in non-dimensional form; these values are derived in Equations 5 and 6 (forces and moments respectively), according to the usual nomenclature of ship manoeuvrability.

$$F' = \frac{F}{0.5\rho L^2 U^2} \quad (5)$$

$$M' = \frac{M}{0.5\rho L^3 U^2} \quad (6)$$

In general, all the other quantities are non-dimensionalized in the same way, i.e. with the proper combination of ship total speed U , water density ρ and ship length L .

2.4 Uncertainty analysis

The uncertainty of the measured experimental values was estimated following the ITTC Recommended Procedures and Guidelines no. 7.5-02 06-04 (*Force and Moments Uncertainty Analysis, Example for Planar Motion Mechanism Test*). The uncertainty

estimation was made up of six elemental bias (B_x) and of the precision limits (P_x). The elemental bias considered were related to inaccuracies in the determination of model length L , model draft at zero speed T_m , water density ρ , carriage speed U , model alignment with respect to the towing tank and strain gauge calibration factors. The precision limits were estimated thanks to $N = 10$ repeated runs carried out on different days of tests.

The uncertainty results are presented in Figures 4 and 5. Precision limits (P_R) and the bias of draft (B_{T_m}), carriage speed (B_U) and model alignment bias are expressed in terms of percentage of the total uncertainty. Model length, water density and sensors calibration factors bias are omitted in the figure because their contributions to the total U_R are negligible with respect to the other components. The values are presented for sway, roll and yaw hydrodynamic loads for a single run used as example.

The model alignment is the largest bias component for the sway force and the roll hydrodynamic moment. The mean draft is the largest error of the yaw moment, even if the model alignment component is of the same entity. The precision limit is a quite large source of uncertainty of the measurements especially for the sway force, whose value is around the 20% of the total uncertainty.

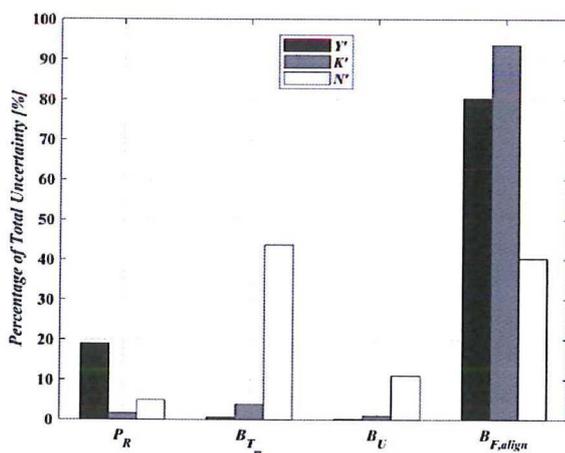


Figure 4 Percentage contributions over total uncertainty (Stage A, Fr = 1.05, φ = 12 deg)

In Figure 5 U_R is plotted as a function of speed using results from Stage A (see Section 2.2) at the same heeling angle (φ = 12deg), for the three loads considered. Side force uncertainty is the largest compared to the roll and yaw moments. Especially at low speeds, the difference is significant. For all the three loads, the uncertainty decreases with increase

of speed. Since the model alignment is the largest bias component, it means that alignment is of particular importance at lower speeds, when the loads measured are smaller and the ship is more immersed in the water.

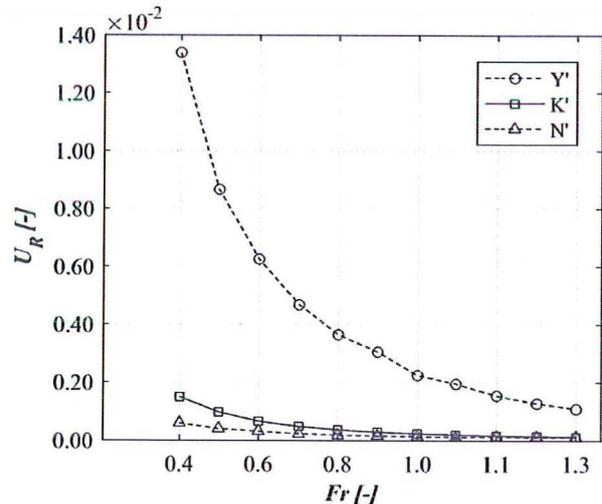


Figure 5 Total uncertainty of the measured hydrodynamic loads (Stage A)

3 RESULTS

In this section, the results of the experimental campaign are presented. In Section 3.1 and 3.2 sway-heel, yaw-heel coupling and roll hydrodynamic moment are shown for the model in vertical equilibrium (experimental Stage A). In Section 3.3 the effect of different running trims is discussed (Stage B1 and B2).

3.1 Sway-heel and yaw-heel coupling

The coupling between heel and yaw is quantified in Equations 2 and 4 by the terms $Y'_\phi\phi$ and $N'_\phi\phi$ which represent the linear sway force in due to heel angle and the linear yaw moment due to heel angle respectively. Typical results showing the measured non-dimensional side force and non-dimensional yaw moment as functions of heel angle are given in Figure 6. In this case the model was towed at the heave and pitch value corresponding to the values obtained when the model is upright – i.e. Stage A.

As can be seen, the relationships between the sway force, yaw moment and the heel angles are quite linear. This is the case for all the speeds tested. Note also that the measurements at zero heel angle don't correspond exactly to zero sway force or zero yaw moment. This is assumed to be due to a small asymmetry in the model and experimental installation.

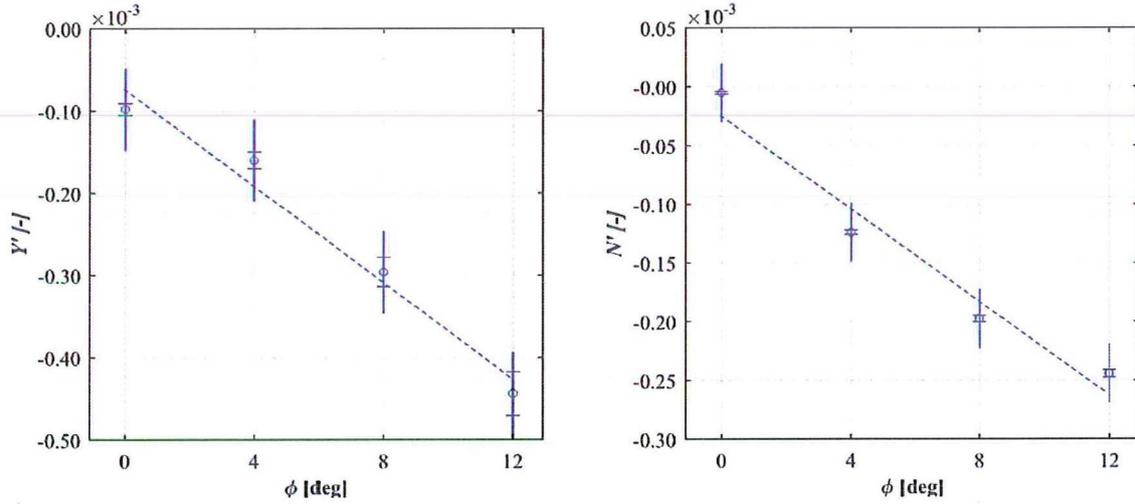


Figure 6 Non-dimensional sway force (left) and yaw moment (right) (Stage A, $Fr = 1.05$)

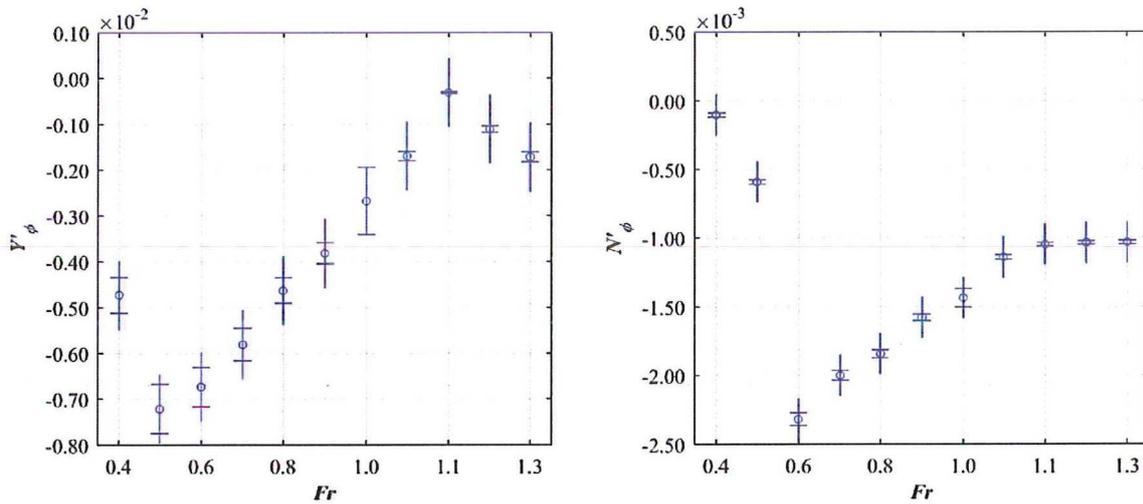


Figure 7 Hydrodynamic linear coefficient of sway force (left) and yaw moment (right) (Stage A)

The values of the non-dimensional linear coefficients Y'_ϕ and N'_ϕ are obtained from linear polynomial curve fitting. The values of the non-dimensional coefficients Y'_ϕ , and N'_ϕ are given in Figure 7 as functions of Froude number. For all the speeds, the values of the hydrodynamic coefficients are both negative. This means that when heeled to starboard, a hydrodynamic side force to port is generated by the heel angle. Also, the yaw moment turns the bow to port (the coordinate system is shown in Figure 1). The absolute values of both sway and yaw hydrodynamic coefficients are maximum around Froude number 0.5 – 0.6. Then the magnitude of the coefficients drops at higher speeds. One of the possible explanation of the smaller coefficients values at high speeds is that at high speed the lift on the hull bottom increases and raises the vessel out of the water. Being less submerged, heeling does not have such a great effect on the side

force and yaw turning moment, resulting in low hydrodynamic coefficients.

Uncertainty bars on the coefficients values are calculated averaging the uncertainty of each measured values which determine the slope of the polynomial fitting. The errors bars on coefficient Y'_ϕ are larger at lower speed, because of the contribution of the model alignment bias on the total uncertainty computation.

3.2 Roll moment

The roll hydrodynamic moment shows a linear slope with respect to heel (see Figure 8), as with the sway and yaw loads. This is the case for all the speeds tested.

The roll hydrodynamic coefficients at the speeds investigated are shown in Figure 9 as functions of Froude number. The hydrodynamic coefficients are compared with the linearized

restoring moment coefficients obtained in Equation 15. The static restoring moment was calculated numerically using the vessel geometry, i.e. the immersed volume for each speed at the calm waterline taking into account the rise and trim of the vessel.

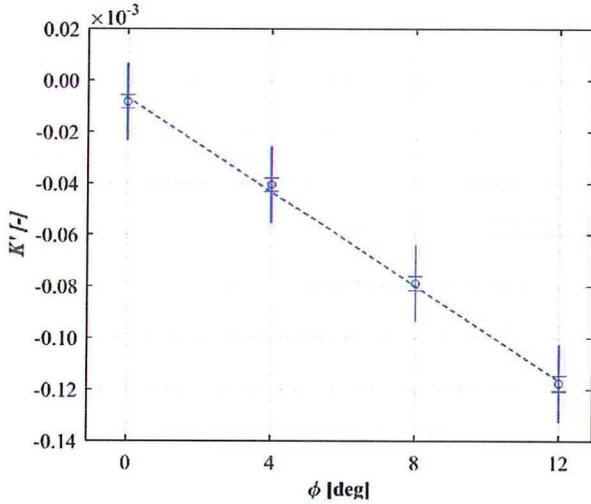


Figure 8 Non -dimensional hydrodynamic roll moment (Stage A, $Fr = 1.05$)

$$K'_{\phi|STAT} = GM'\Delta' \tag{15}$$

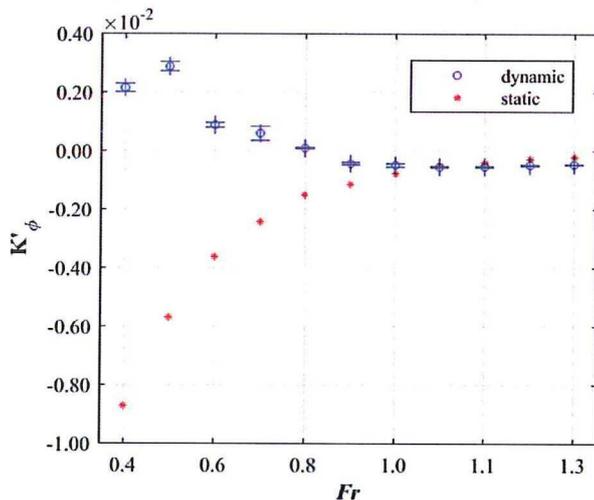


Figure 9 Hydrodynamic (blue) and hydrostatic (red) coefficients (Stage A)

At lower speeds hydrodynamic component of the total roll moment is positive. This may be explained by the hydrodynamic lift generated on the heeled side, especially at the chine and by the bow wave, are smaller than the remaining pressure forces developed on the bottom. This means that the hydrodynamic pressure distribution developed on the heeled hull creates a roll moment in the same direction as the vessel is heeled. This could

consistently decrease the effect of the restoring static moment. At higher speed, the bow wave builds up significantly, as well as the separation from the chine of the heeled side: this generates a negative roll moment which counteracts the heeling of the vessel.

3.3 Effect of vertical position

The same captive model tests were repeated in the other two stages: Stage B1 and B2. The model was towed in non-natural positions with the bow more immersed in the water. In Figure the vertical configuration for Stage A, in which the model is in its vertical equilibrium running attitude, is compared with the other two situations. The resulting hydrodynamic coefficients are shown in Figure 10.

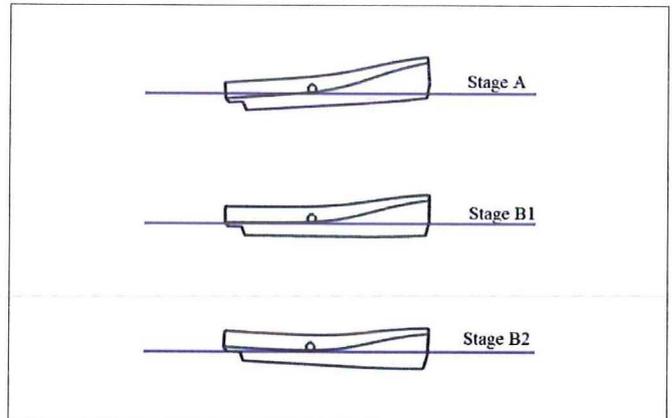


Figure 10 Different vertical running attitude of the vessel. Top: Stage A, $\theta = +3.38$ deg. Centre: Stage B1, $\theta = -0.5$ deg. Bottom: Stage B2, $\theta = -3.00$ deg

As can be seen, the hydrodynamic coefficient for sway is greater for both cases with trim by the bow compared to the equilibrium case. The differences between condition B1 and B2 are negligible.

The roll and yaw hydrodynamic coefficients do not show significantly different results in the three different conditions. The value of K'_{ϕ} and N'_{ϕ} present the same trend with respect to Froude number in all the three conditions (they decrease in absolute value with speed). The values of K'_{ϕ} are positive for all the three conditions. This means that the hydrodynamic roll moment is in the same direction as the vessel heels, tending to increase the heel angle. However, the static restoring roll moment is predominant in the speed range investigated (see Section 3.2). The same happens to the yaw hydrodynamic coefficients. The largest absolute values of N'_{ϕ} are the ones in Stage B2, especially at $Fr < 0.5$.

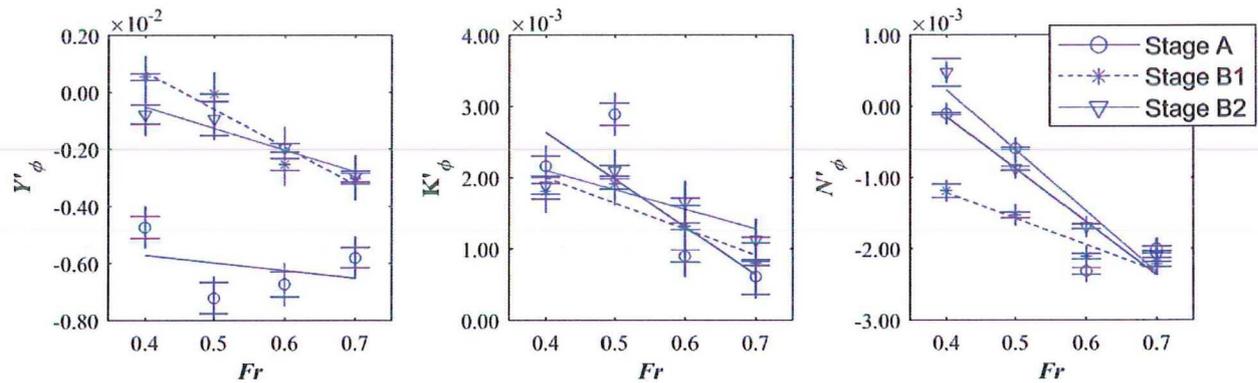


Figure 11 Comparison of sway (left), roll (centre) and yaw (right) hydrodynamic coefficients among the three different running trim conditions (Stage A, B1 and B2)

4 CONCLUDING REMARKS

Heel-sway, heel-yaw coupling and roll moment for a small high speed craft were investigated. The study was conducted by means of experimental captive model tests. The model was towed in static conditions. For each speed, the heel angle, pitch angle and heave position were statically fixed during the runs. The objective of the study was the estimation of the hydrodynamic coefficients in sway Y'_ϕ , roll K'_ϕ and yaw N'_ϕ .

In the first part of the experiments, the ship model was towed at its equilibrium vertical running condition for each different speed. Results showed a strong dependency of the non-dimensional coefficients on the speed, over the range investigated between Froude 0.30 to 1.30. Both the sway and yaw coefficients showed a maximum value around $Fr = 0.40 - 0.60$. This is of interest for the phenomena of non-oscillating dynamic instability in waves, typically occurring in the pre-planing regime, between Froude number 0.30 – 0.70. Future work will include an investigation into the manoeuvring aspects of the loss of dynamic stability in waves. At those speeds, the hydrodynamic roll moment is positive, i.e. to increase the heel angle. However, this effect is small when compared with the static restoring moment.

In the second part of the experimental campaign, the effect on the heel-sway-yaw coupling of bow down trim was examined. This test condition was carried out to simulate behaviour in rough sea situations, i.e. when the ship is exposed to large pitching motions. The results showed that heel-yaw, heel-sway coupling and roll hydrodynamic moment are not greatly affected by the change in trim. On the other hand, the sway force is substantially less when the bow is more immersed in the water.

The results of this research can be used in several ways. Firstly, the experimental results can be used to validate numerical methods, such as panel codes or CFD (see Bonci et al. 2017). Secondly, the measured hydrodynamic coefficients can be used in a state-of-art practical mathematical model, to provide a quick analysis of the manoeuvrability of high speed crafts.

Usually the main objective when performing manoeuvring captive model tests is to assess the course stability of the vessel. The effect of heel is not often considered in the common ship course keeping ability estimation. Further work will include the heel angle in the assessment of the stability of high speed craft in the near future.

As emerged from this study, the effect of heel on the manoeuvrability of high speed craft is valuable and interesting, and it deserves more and deeper research efforts.

REFERENCES

- Bonci, M., Renilson, M.R., de Jong, P., van Walree, F., Keuning, A.J. and Huijsmans, R.H.M. (2017) *Experimental and numerical investigation on the heel and drift induced hydrodynamic loads of a high speed vessel*, 14th Conference on Fast Sea Transportation, Nantes, 27-29 September
- De Jong, P., van Walree, F. and Renilson, M.R. (2013) *The broaching of a fast rescue craft in following seas*, 12th International Conference on Fast Sea Transportation, Amsterdam 2013.
- Hashimoto, H., Umeda, N., and Matsuda, A. (2011) *Model Experiment on Heel-Induced Hydrodynamic Forces in Waves for Realising Quantitative Prediction of Broaching*. Contemporary Ideas on Ship Stability and Capsizing in Waves, Fluid Mechanics and Its Applications 96.
- Keuning, J.A. (2006) *Grinding the bow*. International Shipbuilding Progress, 2006
- Oltmann, P. (1993) *Roll – An Often Neglected Element of Manoeuvring*. MARSIM '93, International Conference on Marine Simulation and Ship Manoeuvrability; 26 Sept – 2 Oct 1993, St. John's, Newfoundland, Canada. Procs. Vol 2, p 463.

- Ommani, B., Faltinsen, O.M. and Lugni, C. (2012)** *Hydrodynamic force on a semi-displacement vessel on a straight course with drift angle, 10th International Conference on Hydrodynamics (ICHHD)*
- Ommani, B. and Faltinsen, O.M. (2014)** *Cross-flow transverse force and yaw moment on a semi-displacement vessel with forward speed and drift angle, 11th International Conference on Hydrodynamics (ICHHD)*
- Renilson, M.R. and Tuite, A.J. (1996)** *The Effect of GM and Loss of Control of a Planing Mono-Hull in Following Seas. High Speed Marine Craft, Safe Design and Safe Operation, Bergen, Norway, September 1996.*
- Renilson, M.R., and Tuite, A. (1997)** *The Effect of GM on Broaching and Capsizing of Small Fishing Vessels in Following Seas. Proceedings of the 6th International Conference on Stability of Ships and Ocean Vehicles, (STAB'97), Vol. II, pp 149-161, Varna, Bulgaria, September 1997.*
- Renilson, M.R., and Manwarring, T. (2000)** *An Investigation into Roll/Yaw Coupling and its Effect of Vessel Motions in Following and Quartering Seas. Proceedings of 7th International Conference on Stability of Ships and Ocean Vehicles, Launceston, Tasmania, Australia, February 2000, pp 452 - 459.*
- Renilson, M.R. (2007)** *Predicting the Hydrodynamic Performance of Very High Speed Craft – A Note on Some of the Problems. International Journal of Small Craft Technology, Vol 149, Part B1 2007, pp 13-22.*
- Stewart, D. (1965–1966).** *A Platform with Six Degrees of Freedom, Proc. Institution of Mechanical Engineers (UK), 180 (Pt 1, No 15).*
- Tuite, A.J. and Renilson, M.R. (1995)** *The effect of GM on Manoeuvring of High Speed Container Ship. 11th International Maritime and Shipping Symposium: 21st Century Shipping, Sidney, November 1995, pp. 186-199.*
- Umeda, N. (1999)** *Nonlinear dynamics of ship capsizing due to broaching in following and quartering seas. Journal of Marine Science and Technology 4: 16-26.*
- Yasukawa, H. (2010)** *A consideration of roll-coupling effect on ship maneuverability. Japan Institute of Navigation, 123:153-158.*
- Yasukawa, H. and Yoshimura, Y. (2014)** *Roll-Coupling Effect on Ship Maneuverability. Ship Technology Research, 61:1, 16-32*
- Yasukawa, H., Hirata, N. and Nakayama, Y. (2016)** *High Speed Ship Maneuverability. Journal of Ship Research, Vol.60 No. 4, December 2016*

