Department of Precision and Microsystems Engineering

Improving the Correction Quality of Deformable Mirrors with In-Plane Boundary Actuation

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IMPROVING THE CORRECTION QUALITY OF DEFORMABLE MIRRORS WITH IN-PLANE BOUNDARY ACTUATION

by

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ABSTRACT

When a picture is to be made of an object in space the light that comes from the object starts as an unaberrated wavefront. As the light enters and travels through the earth's atmosphere the wavefront becomes aberrated. To obtain a high-resolution picture these aberrations need to be corrected. The wavefront aberrations are measured with a wavefront sensor and the obtained information is sent to a control system that drives a deformable mirror. The deformable mirror is deformed into the desired shape and the wavefront aberrations are corrected as good as possible with the actuators used on the mirror.

In current designs the focus is on out-of-plane surface actuators. These actuators are placed on the back of the mirror and perform out-of-plane actuation. Boundary actuators have not been researched extensively yet. The goal of this study is to find the effects of combining in-plane boundary actuators with the conventional out-of-plane surface actuators. A positive effect will be that the deformed mirror shape better resembles the desired shape. This results in a higher resolution.

The problem is solved with a finite element approach. The calculations are performed on 3 mirror configurations: they are modelled as plates which have different out-of-plane and in-plane actuator patterns. The desired shape of the plate is described in terms of the displacement of the nodes of the mesh. The desired shape is in general not perfectly obtained with the actuators used. The error between the desired shape and the shape that can be achieved with the actuators is described as a weighted sum of the errors of the nodes. The error is minimized to obtain the shape that resembles the desired shape the best.

It is found that using in-plane boundary actuation can reduce the error between the desired shape and the shape that is feasible with the out-of-plane actuators. Error reductions have been found ranging from 0.209% to 66.6%, depending on the mirror configuration used, the ultimate tensile strength of the material and the aperture size.

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1

INTRODUCTION

Deformable mirrors are used in adaptive optics systems to correct wavefront errors. The mirror is deformed into the desired correction shape. This is commonly achieved with out-of-plane surface actuators. In-plane boundary actuation has not been studied extensively yet.

The goal of this research is to find the effects of using in-plane boundary actuation in combination with the commonly used out-of-plane surface actuation. The hypothesis is that there is a positive effect on the performance of the deformable mirror. It might offer the possibility to improve the correction quality or to remove out-of-plane surface actuators while maintaining the same correction quality.

In Chapter 2 background information is provided. Pay special attention to the section on Zernike polynomials, because this will be used throughout this whole document. In Chapter 3 the framework that is used to quanitfy the wavefront error is provided. The minimization of this error is also covered here. This leads us to the model implementation in Chapter 4. Different mirror configurations are implemented and studied. The results obtained with the finite element model are discussed in Chapter 5. We will draw conclusions in Chapter 6 and finally some recommendations will be given in Chapter 7.

2

ADAPTIVE OPTICS

Deformable mirrors are widely used in adaptive optics. In layman terms you could say that adaptive optics is a method of automatically keeping the light focused when it gets out of focus [1]. The light is kept in focus by correcting for wavefront aberrations, which improves the image quality. Common fields of application are astronomy and vision science [2].

So how does adaptive optics work? An example of an adaptive optics system can be found in Figure 2.1. Suppose we want to make an image of a star. The incoming wavefront from this star is aberrated due to density differences in the earth's atmosphere. This makes the quality of the image deteriorate, which means that the resolution becomes lower. Other causes of image quality deterioration can be imperfect optical components and misalignment [3]. The wavefront error is measured with a wavefront sensor and the information obtained is sent to a control system. The deformations that are needed to correct the wavefront error are then applied to a wavefront corrector. A widely used type of wavefront corrector is the deformable mirror. A deformable mirror is basically a reflective facesheet with actuators attached to the back side or the boundary. The actuators effectuate the deformation of the mirror that is needed for the wavefront correction. This finally results in an image with a higher resolution [2, 3].



Figure 2.1: A systematic overview of an adaptive optics system [4]. The incoming light from, for example, a star is aberrated by the earth's atmosphere. The light reflects on the deformable mirror and then passes a beam splitter. With part of the light a high-resolution image is created and the remainder of the light is sent to a wavefront sensor. With the information obtained from the wavefront sensor the required mirror deformations are calculated.

2.1. CALCULATION OF THE CORRECTION QUALITY

The error that is still present after the wavefront correction is used to determine the quality of this correction. In [5] the error is calculated with a least squares problem. The deflection that is obtained with the actuators is Uc, where U is called the influence matrix and c is the control vector. The desired deflection is described by the vector \mathbf{u}_{des} . The root-mean-square surface error is calculated with $e_{RMS} = \sqrt{(\mathbf{Uc} - \mathbf{u}_{des})^T \mathbf{W} (\mathbf{Uc} - \mathbf{u}_{des})}$, where W is a matrix with wheighting factors. The optimal mirror shape for a certain udes can be determined by minimizing e_{RMS} . A similar approach has been used in [6] and [7]. A similar approach is used in this research and will be further elaborated in Chapter 3.

2.2. WAVEFRONT RECONSTRUCTION

In the previous paragraphs the desired deformation, or desired shape, is mentioned several times. This deformation is determined by reconstructing the wavefront. The desired shape of the mirror is the shape that is opposite to the disturbed wavefront [3].

How is this wavefront reconstructed? The reconstruction principle used for deformable mirrors is based on either the zonal or the modal approach. In the zonal approach the mirror is divided into separate zones. The correction needed for each zone is calculated in terms of a slope. The actuator configuration that results in the best approximation of the desired slopes is calculated using least squares techniques. [8]

In the modal approach the desired shape of the mirror is seen as a whole. The wavefront aberrations, and the corresponding mirror deformations, are described as a summation of weighted polynomials. Zernike polynomials are often used as a basis for these polynomials [9]. Because this approach will be used in this research, we will further elaborate on this in the remainder of this section.

2.2.1. ZERNIKE POLYNOMIALS

The following text on Zernike polynomials is based on [10] and [11]. Zernike polynomials are a set of functions that are orthogonal over a circle with unit radius. They can be described with polar coordinates as a function of a radial polynomial and an angular polynomial

$$Z_n^m(\rho,\theta) = R_n^m(\rho)\sin\theta \qquad \text{for } n - m \text{ odd,}$$

$$Z_n^m(\rho,\theta) = R_n^m(\rho)\cos\theta \qquad \text{for } n - m \text{ even,}$$
(2.1)

where R_n^m is the radial polynomial, ρ is the radial distance, θ is the in-plane angle, n is the degree of the polynomial, and *m* the angular dependence parameter. The parameters *n* and *m* are both even or both odd, and $n - 2m \ge 0$. The radial distance ρ is normalized, so $0 \le \rho \le 1$. The radial polynomials are described by

$$R_n^m(\rho) = \sum_{s=0}^{(n-m)/2} (-1)^s \frac{(n-s)!}{s! [\frac{1}{2}(n+m)-s]! [\frac{1}{2}(n-m)-s]!} \rho^{n-2s}.$$
 (2.2)

The radial polynomials have the following property

$$R_n^m = R_n^{-m} = R_n^{|m|}.$$
(2.3)

With Equation (2.2) we can calculate the first few radial polynomials. They are

$$\begin{split} R_0^0(\rho) &= 1, \\ R_1^1(\rho) &= \rho, \\ R_2^0(\rho) &= 2\rho^2 - 1, \\ R_2^2(\rho) &= \rho^2, \\ R_3^1(\rho) &= 3\rho^2 - 2\rho, \\ R_3^3(\rho) &= \rho^3, \\ R_4^0(\rho) &= 6\rho^4 - 6\rho^2 + 1, \\ R_4^2(\rho) &= 4\rho^4 - 3\rho^2, \\ R_4^4(\rho) &= \rho^4. \end{split}$$

Zernike polynomials can be added as a weighted sum to describe a function (wavefront). For the implementation of Zernike polynomials the reader is referred to Chapter 3. The first 15 Zernike polynomials are displayed in Table 2.1.

Table 2.1: The first 15 Zernike polynomials. The amplitude is equal to 1 for each polynomial. The figure is based on [12].



2.3. DEFORMABLE MIRROR DESIGNS

Deformable mirrors come in different designs. They can roughly be categorised by the type of actuation that is used to obtain the desired shape. The most common type is actuation with actuators that are placed on the backside of the mirror surface. This surface actuation can be performed with out-of-plane or in-plane actuators. A different class of deformable mirrors are microelectromechanical deformable mirrors. In this report we focus on mirrors in the macroscopic scale so we will not elaborate on microelectromechanical deformable mirrors. The interested reader is referred to [13].

2.3.1. OUT-OF-PLANE SURFACE ACTUATION

Out-of-plane actuation is performed by means of push-pull actuators attached to the back of the mirror. There are different types of actuators: those that impose out-of-plane displacements on the mirror and those that impose out-of-plane loads. In this subsection both principles are discussed.

DISPLACEMENT BASED ACTUATION

The most common type of out-of-plane actuators are stacked array ferroelectric actuators. An example of stacked array actuators glued on a base plate can be found in Figure 2.2. The reflective surface is positioned on top of the stacked array actuators. The actuators consist of stacked ferroelectric plates. These ferroelectric plates are either piezoelectric or electrostrictive. Lead zirconate titanate (PZT), a poled ceramic, is the most commonly used piezoelectric. Applying an electric field to a ferroelectric plate results in a change in its dimensions. Stacking PZT plates results in a bigger stroke. For example, PZT actuators consisting of 40 stacked

plates can deliver a $\pm 5 \mu m$ stroke for a voltage of ± 400 V. Changes in temperature have almost no effect on PZT actuators [13].



Figure 2.2: Stacked array actuators glued on a base plate [13].

Lead Magnesium Niobate (PMN), a non-poled ceramic, is the most commonly used electrostrictive. PMN actuators can deliver a bigger stroke (up to 10 μ m for a control voltage between 0 and 150 V) than PZT actuators, but they are more sensitive to temperature changes [13].

Stacked array deformable mirrors offer many possibilities: a large number of actuators is possible, both rectangular and hexagonal actuator patterns can be made and a small actuator pitch (the distance between neighboring actuators) is possible. Other advantages are the high stiffness, the high reliability and the large stroke. For these reasons stacked array deformable mirrors are used in many different adaptive optics systems. Some of the main disadvantages of stacked array deformable mirrors are: the large amount of cables needed for the many actuators, the high driving voltages and the high cost. Nevertheless, it is the most frequently used wavefront corrector [13].

LOAD BASED ACTUATION

Another type of out-of-plane actuation is voice-coil actuation. An example of a voice coil actuated deformable mirror is given in Figure 2.3. Voice coils are placed on a thick metallic plate. A reflective plate with magnets attached to the back floats on the magnetic field created by the voice coils. If the current in a voice coil is changed, the magnetic field changes and as a result the mirror becomes deformed. Voice-coil actuation is used in large secondary deformable mirrors.



Figure 2.3: Example of a voice-coil actuated deformable mirror [14]. The reflective plate is lying on its supporting tool.

A stroke up to 50 μ m can be reached. This makes it possible to perform tip-tilt correction on top of high order correction. Another positive quality of voice-coil actuated deformable mirrors is that if one of the actuators

fails, the surrounding actuators can compensate for this failure without introducing actuator print-through. When actuator print-through occurs one can point out on the final images where an actuator is positioned on the deformable mirror. Voice-coil actuators have very good accuracy and a short response time (around 1 ms).

Some of the main disadvantages of voice-coil actuated deformable mirrors are the complexity, the high power consumption and the operation risks caused by the shell brittleness. Nevertheless, this technique is often used in large secondary deformable mirrors [13].

2.3.2. IN-PLANE SURFACE ACTUATION

In-plane actuation is based on in-plane expansion/contraction of the mirror. In-plane actuation with piezoelectric actuation and thermal actuation will be discussed below.

PIEZOELECTRIC BIMORPH ACTUATION

Piezoelectric bimorph actuation is based on the expansion of piezoelectric materials when a voltage is applied. In [6] a thin, stiff optical quality substrate with a layer of piezoelectric material bonded to its back-face is used. An electrode pattern is coated to one face of the piezoelectric layer and a continuous ground layer is deposited on the other face. An exploded view of this deformable mirror is given in Figure 2.4. Applying a voltage to an electrode will cause a difference in strain in the plate, which results in deformation of the plate. Classical layouts for the electrode pattern are keystone patterns, honeycomb patterns and lattice patterns (see Figure 2.5) [6].



Figure 2.4: Exploded view of the bimorph deformable mirror studied in [6].

The manufacturing of bimorph deformable mirrors is much simpler than the manufacturing of stacked array DMs, because the electrode pattern simply has to be deposited on a PZT layer. However, attaching wires to the electrodes is a difficult task. Some of the advantages of pieozelectric bimorph actuation are the high reliability, the large stroke and the moderate costs. Piezoelectric bimorph deformable mirrors are often used in telescopes with a diameter of up to 10 m. Some disadvantages are the high driving voltages, the creep of the ferroelectric material and the low resonance frequencies.



Figure 2.5: Classical layouts for the electrode pattern of piezoelectric bimorph deformable mirrors [6].

THERMAL ACTUATION

In-plane actuation can also be established by applying temperature differences to the mirror. This technique is suitable for correcting slowly varying wavefront aberrations. Thermally actuated deformable mirrors have high position resolution and high reproducibility. Another advantage is that they cost less than other types of deformable mirrors [15]. A disadvantage of this technique is that it is not suitable for correction of rapidly varying wavefront aberrations. This is due to the long time scale inherent to changes in the temperature.

2.3.3. BOUNDARY ACTUATION

A type of actuation that has not been studied extensively yet is boundary actuation. Sometimes tension is applied to the mirror boundary to remove wrinkles from the membrane [3]. This has to be done before usage and is not changed during operation, that is to say, it is not active.

Actuation of the mirror boundary, which is active, is not very common. In [7] and [16] 12 boundary actuators (and no surface actuators) are used. These actuators can carry out normal, radial and tangential displacements of the boundary. Using no surface actuators limits the deformations that can be achieved. No research on combining in-plane boundary actuation with out-of-plane surface actuation has been found.

3

THEORETICAL FRAMEWORK REGARDING THE MODEL

In this chapter the theoretical framework used for the model will be discussed. The goal of this framework is to determine the actuator configuration for which a certain desired shape can be made with the smallest error possible. Some examples will be given to further clarify the framework. A linear beam will be used as the basis for these examples. The calculations are performed with *Matlab R2014b*.

3.1. QUANTIFICATION OF THE WAVEFRONT ERROR

The first step in quantifying the wavefront error is discretizing the problem: the plate is divided into smaller elements and each element shares nodes with other elements (this is called the mesh). The nodes and the individual degrees of freedom of the nodes define the number of degrees of freedom N of the system. The next step is describing the displacements and rotations of the nodes of the mirror, also called the shape of the mirror. This shape can be described in two different ways: with a vector representing the desired shape \mathbf{u}_{des} and with a vector representing the feasible shape \mathbf{u}_{FEM} . The feasible shape is calculated with a finite element analysis (FEA). The length of \mathbf{u}_{des} and \mathbf{u}_{FEM} is equal to the number of degrees of freedom N. The desired shape is the shape that is needed to correct for the wavefront aberrations. The desired shape is described as

$$\mathbf{u}_{\mathbf{des}} = \mathbf{Z}\mathbf{m}.\tag{3.1}$$

Z is a matrix that contains a number of mode shapes. In this research Zernike polynomials are used for the mode shapes in **Z** (refer to Chapter 1 for information on Zernike polynomials). This matrix is multiplied by the desired amplitude vector **m**. The vector **m** determines the contribution of each Zernike mode. Hence, each desired shape is described as a weighted sum of Zernike polynomials. The size of **Z** is ($N \times n$) and the size of **m** is ($n \times 1$), where *n* is the number of modes used. Vector **m** is scaled to a unit vector.

Example: mode shape matrix ${f Z}$ and amplitude vector ${f m}$

A schematic representation of the linear beam used in the examples is given in Figure 3.1 and the constants used are listed in Table 3.1. The beam is divided into ten equal elements, so we have 11 nodes. The free nodes can move in x- and y-directions and they are free to rotate. Node 1 and 11 (the first and last nodes) are fixed in x- and y-directions but are free to rotate. This results in $N = (3 \cdot 11) - 4 = 29$ degrees of freedom. For the mode shape matrix **Z** two mode shapes based on the radial functions R_1^1 and R_2^0 of the Zernike polynomials (see Chapter 1) are used, so n = 2. We call these mode shapes **Z**₁ and **Z**₂ respectively. These desired shape is imposed on node 2 to 10, so the first and last nodes are not included. The displacements in y-direction corresponding to these mode shapes are displayed in Figure 3.2. For both mode shapes an absolute amplitude in y-direction of 0.01 m is used.



Figure 3.1: The linear beam used in the examples in this chapter. The beam is constrained in x- and y-directions at the first and the last nodes.

Constant	Value
Beam length	1 m
Number of elements	10
Young's modulus	$70 \cdot 10^9 \text{ N/m}^2$
Cross-sectional area	$2 \cdot 10^{-5} \text{ m}^2$
Area moment of inertia	$3 \cdot 10^{-11} \text{ m}$

Table 3.1: Constants used in linear beam examples.



Figure 3.2: Displacements in y-direction corresponding to the mode shapes used in the beam example. The absolute amplitude in y-direction is equal to 0.01 m for both mode shapes. The first and last nodes are not included in the mode shapes, but they are forced to be at y = 0 due to the imposed boundary conditions.

Using these modeshapes results in the following mode shape matrix Z:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{1} \\ \mathbf{Z}_{2} \end{bmatrix}^{T}$$
$$= \begin{bmatrix} \phi_{1,1} & x_{1,2} & -0.01 & \phi_{1,2} & x_{1,3} & -7.5e - 3 & \cdots & 7.5e - 3 & \phi_{1,9} & x_{1,10} & 0.01 & \phi_{1,10} & \phi_{1,11} \\ \phi_{2,1} & x_{2,2} & 0.01 & \phi_{2,2} & x_{2,3} & 1.25e - 3 & \cdots & 1.25e - 3 & \phi_{2,9} & x_{2,10} & 0.01 & \phi_{2,10} & \phi_{2,11} \end{bmatrix}^{T},$$
(3.2)

where $x_{i,j}$ is the desired displacement in y-direction defined by mode shape i for node j. The same definitions are used for the rotations $\phi_{i,j}$. Note that only the desired displacements in y-direction are described. The other degrees of freedom are not described by the (radial) Zernike polynomials. This will be dealt with in the third example in this chapter.

The mode shape matrix **Z** is multiplied with desired amplitude vector **m** according to Equation (3.1). If we, for example, desire a contribution of $0.8 \cdot Z_1$ and a contribution of $0.6 \cdot Z_2$ we use a desired amplitude vector

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix}. \tag{3.3}$$

The shape that is feasible with the out-of-plane actuators is, in general, not equal to the desired shape:

$$\mathbf{u}_{\text{FEM}} \neq \mathbf{u}_{\text{des}}.$$
 (3.4)

The feasible shape vector \mathbf{u}_{FEM} is defined by

$$\mathbf{K}\mathbf{u}_{\mathbf{FEM}} = \mathbf{f},\tag{3.5}$$

where **K** is the stiffness matrix and **f** a vector that contains the loads (forces and moments) exerted on the nodes. **K** is an $(N \times N)$ matrix and **f** is an $(N \times 1)$ vector. Load vector **f** can be described by a matrix with load intensities **F** and a corresponding actuator amplitude vector **c** as

$$\mathbf{f} = \mathbf{F}\mathbf{c}.\tag{3.6}$$

The size of **c** is $(A \times 1)$, where *A* is the number of actuators used. The size of **F** is $(N \times A)$.

EXAMPLE: LOAD VECTOR **f**, LOAD INTENSITY MATRIX **F** AND ACTUATOR AMPLITUDE VECTOR **c** Actuators that actuate in the y-direction are placed on our beam at nodes 2, 3, 8 and 9, so A = 4. This is illustrated in Figure 3.3. These actuators are the 2D equivalent of the out-of-plane actuators used for the mirror (3D).

The force intensity in the y-direction for each of the actuators is 1. All the other entries of the (29×4) force intensity matrix **F** are 0. This results in a matrix with the following structure:



Figure 3.3: Beam constrained in x- and y-directions at the first and the last nodes. Forces in the y-direction are imposed on nodes 2, 3, 8 and 9.

In this case **c** is a (4×1) vector. Suppose we use the following actuator amplitude vector

$$\mathbf{c} = \begin{bmatrix} 1\\ 0.5\\ 0.7\\ 0.8 \end{bmatrix}.$$
 (3.8)

This would result in the following load vector **f**:

$$\mathbf{f} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & \cdots & 0 & 0.7 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \end{bmatrix}^{T}.$$
 (3.9)

Substituting Equation (3.6) in Equation (3.5) and rewriting this results in the feasible shape vector:

$$\mathbf{u}_{\text{FEM}} = \mathbf{K}^{-1} \mathbf{F} \mathbf{c}. \tag{3.10}$$

We now introduce the intensity matrix U:

$$\mathbf{U} = \mathbf{K}^{-1}\mathbf{F}.\tag{3.11}$$

The size of **U** is $(N \times A)$. Substituting this in Equation (3.10) results in

$$\mathbf{u}_{\text{FEM}} = \mathbf{U}\mathbf{c}.\tag{3.12}$$

As stated in Equation (3.4) the feasible shape is in general not equal to the desired shape. The error e between \mathbf{u}_{des} and \mathbf{u}_{FEM} is described as:

$$e = (\mathbf{u}_{des} - \mathbf{u}_{FEM})^T \mathbf{W} (\mathbf{u}_{des} - \mathbf{u}_{FEM}) = (\mathbf{Zm} - \mathbf{Uc})^T \mathbf{W} (\mathbf{Zm} - \mathbf{Uc}),$$
(3.13)

where **W** is a matrix with weighting factors that define the importance of the correctness of the DOF at the nodes. **W** is a diagonal $(N \times N)$ matrix. In the case of a deformable mirror **W** can be used to define the size of the aperture. The aperture diameter is in general smaller than the mirror diameter. The nodes that fall outside of the aperture have a weighting factor equal to 0 and the nodes inside the aperture have a weighting factor equal to 1.

EXAMPLE: DIAGONAL MATRIX W

Since we have 29 degrees of freedom in the beam example, \mathbf{W} is a (29 × 29) (diagonal) matrix. It is described as

$$\mathbf{W} = \text{diag}(0, 0, 1, 0, 0, 1, \dots, 0, 0, 1, 0, 0). \tag{3.14}$$

The ones in this matrix belong to the displacement in y-direction of nodes 2 to 10. The zeros belong to the other degrees of freedom. So the errors in the y-direction of nodes 2 to 10 are of equal importance and the errors for the other degrees of freedom do not contribute to the final error e. This is also why it does not matter what we use for $x_{i,j}$ and $\phi_{i,j}$ in Equation (3.2): their value does not influence the calculated error e.

3.2. MINIMIZATION OF THE ERROR

The next step is to minimize the error e. This is done by taking the first derivative of e with respect to the vector **c** and equating this to zero:

$$\frac{de}{d\mathbf{c}} = -2\mathbf{U}^T \mathbf{W} (\mathbf{Z}\mathbf{m} - \mathbf{U}\mathbf{c}) = \mathbf{0}.$$
(3.15)

Rewriting this with Equation (3.16) to Equation (3.18) results in Equation (3.19).

$$\mathbf{U}^T \mathbf{W} \mathbf{U} \mathbf{c} = \mathbf{U}^T \mathbf{W} \mathbf{Z} \mathbf{m}. \tag{3.16}$$

$$\mathbf{A} = \mathbf{U}^T \mathbf{W} \mathbf{U}. \tag{3.17}$$

$$\mathbf{B} = \mathbf{U}^T \mathbf{W}.$$
 (3.18)

$$\mathbf{c} = \mathbf{A}^{-1} \mathbf{B} \mathbf{Z} \mathbf{m}. \tag{3.19}$$

The amplitude vector **c** that is found by solving Equation (3.19) is the amplitude vector for which the desired shape (defined by **Zm**) is obtained with the smallest error possible (i.e. the error is minimized). To verify if this error *e* indeed is the minimum, we calculate the second derivative of *e* with respect to **c**:

$$\frac{d^2 e}{d\mathbf{c}^2} = 2\mathbf{U}^T \mathbf{W} \mathbf{U}. \tag{3.20}$$

Since **W** is a diagonal matrix with only values ≥ 0 and **U** is a matrix with only real values, we know that the outcome of Equation (3.20) is always ≥ 0 and thus the global minimum. So we can conclude that Equation (3.19) always gives the amplitude vector *c* that corresponds to the minimal error. Substituting Equation (3.19) in Equation (3.13) results in this minimal error:

$$e_{\min} = \mathbf{m}^T (\mathbf{Z} - \mathbf{U}\mathbf{A}^{-1}\mathbf{B}\mathbf{Z})^T \mathbf{W} (\mathbf{Z} - \mathbf{U}\mathbf{A}^{-1}\mathbf{B}\mathbf{Z})\mathbf{m} = \mathbf{m}^T \mathbf{Y}\mathbf{m},$$
(3.21)

where **Y** is an $(n \times n)$ matrix. So, for each desired-amplitude vector **m**, that describes a certain **u**_{des}, the minimal error e_{\min} and its corresponding actuator configuration defined by **c** can be calculated with Equation (3.21) and Equation (3.19) respectively.

The unit of e_{\min} is m² and e_{\min} is dependent of the number of nodes used in the finite element mesh. This results in values of e_{\min} that are not useful for comparison. To solve this problem the root mean square (RMS) error can be used:

$$e_{\rm RMS} = \sqrt{\frac{e_{\rm min}}{N_{\rm nz}}},\tag{3.22}$$

where N_{nz} is the number of degrees of freedom for which the weighting factor in matrix **W** is nonzero, i.e. the number of nodes that can account for an error. The dimension of e_{RMS} is m. Finally the error can be made dimensionless by dividing by the amplitude *h* used for the Zernike polynomials:

$$\tilde{e}_{\rm RMS} = \frac{e_{\rm RMS}}{h}.$$
(3.23)

EXAMPLE: CALCULATION OF $e_{\rm RMS}$

Suppose we use the values in Equation (3.3) for the desired-amplitude vector. The matrices that are used for **Z**, **F** and **W** were discussed in the previous examples. The inverse of the stiffness matrix \mathbf{K}^{-1} is calculated for the constants given in Table 3.1. This gives us all the ingredients needed to find e_{RMS} , the corresponding actuator amplitude vector **c** and feasible shape \mathbf{u}_{FEM} . The actuator amplitude vector that results in the minimal error is calculated with Equation (3.19). This results in

$$\mathbf{c} = \begin{bmatrix} -7.54\\ 6.39\\ -40.7\\ 53.1 \end{bmatrix}.$$
 (3.24)

The minimal error e_{min} is calculated with Equation (3.21) and is substituted in Equation (3.22). When we use $N_{nz} = 9$, we obtain from Equation (3.22) that

$$e_{\rm RMS} = 2.3 \cdot 10^{-3} \,\mathrm{m.}$$
 (3.25)

For the Zernike polynomials an amplitude of h = 0.01 m was used, so the normalized error is

$$\tilde{e}_{\rm RMS} = 2.3 \cdot 10^{-1} \ (-). \tag{3.26}$$

The corresponding \mathbf{u}_{des} and \mathbf{u}_{FEM} are given in Figure 3.4.



Figure 3.4: The desired shape \mathbf{u}_{des} and feasible shape \mathbf{u}_{FEM} for the vector \mathbf{m} as given by Equation (3.3). The feasible shape displayed is the shape that results in the smallest error possible with the used actuators. The beam is constrained in x- and y-directions at the first and the last nodes. Forces in the y-direction are imposed on nodes 2, 3, 8 and 9.

3.3. FINDING THE EXTREME VALUES OF THE MINIMIZED ERROR

For the next step we will keep using e_{\min} instead of e_{RMS} or \tilde{e}_{RMS} , because this makes the calculations easier to carry out and to follow. In the end we will convert to the RMS error.

We can imagine that the mimimal error e_{\min} can be different for different desired amplitude vectors. We now want to find the desired amplitude vectors **m** which correspond to the minima and maxima of the error e_{\min} . These extreme values tell us which desired shape can be made the easiest (smallest e_{\min}) and which desired shape is the most difficult to make (largest e_{\min}). They are found by using the following optimization problem

Maximize
$$e_{\min} = \mathbf{m}^T \mathbf{Y} \mathbf{m}$$
,
Subject to $\mathbf{m}^T \mathbf{I} \mathbf{m} = 1$, (3.27)

where I is the identity matrix. The corresponding Lagrangian is

$$\mathscr{L} = \mathbf{m}^T \mathbf{Y} \mathbf{m} + \lambda (\mathbf{m}^T \mathbf{I} \mathbf{m} - 1), \qquad (3.28)$$

where the coefficients λ are the Lagrange multipliers [11]. The number of Lagrange multipliers is equal to *n* (the number of polynomials used). The next step is taking the first derivative of \mathcal{L} with respect to **m** and the first derivative of \mathcal{L} with repect to λ and equate these two derivatives to zero:

$$\mathbf{Y}\mathbf{m} + \lambda \mathbf{I}\mathbf{m} = \mathbf{0},$$

$$\mathbf{m}^T \mathbf{I}\mathbf{m} - 1 = 0.$$
 (3.29)

Solving this results in the extreme values of e_{\min} (represented by the found values for λ) and the corresponding desired amplitude vectors **m**. The extreme values of e_{\min} are found by calculation the eigenvalues of **Y**.

EXAMPLE: SOLVING THE OPTIMIZATION PROBLEM - SEVERAL APPROACHES

To solve the optimization problem for our beam we first need the matrix Y. This matrix is

$$\mathbf{Y} = 10^{-5} \cdot \begin{bmatrix} 1.4344 & 2.2617 \\ 2.2617 & 4.5518 \end{bmatrix}.$$
 (3.30)

The eigenvalues of matrix **Y** are $\lambda_1 = 2.4632 \cdot 10^{-6}$ and $\lambda_2 = 5.7399 \cdot 10^{-5}$. This results in the following root mean square errors

$$e_{\text{RMS1}} = 5.2315 \cdot 10^{-4} \text{ m},$$

 $e_{\text{RMS2}} = 2.5254 \cdot 10^{-3} \text{ m}.$
(3.31)

The eigenvectors give us the corresponding desired amplitude vectors:

$$\mathbf{m}_{1} = \begin{bmatrix} -0.8853\\ 0.4650 \end{bmatrix},$$

$$\mathbf{m}_{2} = \begin{bmatrix} 0.4650\\ 0.8853 \end{bmatrix}.$$
(3.32)

The desired shape and feasible shape are plotted for the smallest error e_{RMS1} (Figure 3.5a) and the largest error e_{RMS2} (Figure 3.5b).



Figure 3.5: Plots of the achieved displacements and desired displacements of the linear beam with uncoupled actuators. On the left the results for the smallest error $e_{\text{RMS1}} = 5.2315 \cdot 10^{-4}$ m are plotted and on the right the results for the largest error $e_{\text{RMS2}} = 2.5254 \cdot 10^{-3}$ m are plotted.

We now want to check whether the eigenvectors in Equation (3.32) indeed correspond to the smallest error and the largest error. We do this by writing the error e_{\min} as a function of the angle α between the entries m_1 and m_2 of the vector **m**. The amplitude vector is calculated with

$$\mathbf{m}(\alpha) = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix},\tag{3.33}$$

where α is in radians. According to Equation (3.21) the minimal error is now calculated with

$$e_{\min}(\alpha) = (\mathbf{m}(\alpha))^T \mathbf{Y}(\mathbf{m}(\alpha)). \tag{3.34}$$

This results in the plot of $e_{\min}(\alpha)$ in Figure 3.6. It can be seen that the error is at its minimum for $\alpha = 2.658$ rad and at its maximum for $\alpha = 1.087$ rad. These values indeed correspond to the previously found eigenvectors \mathbf{m}_1 and \mathbf{m}_2 .



Figure 3.6: e_{\min} as a function of the angle ϕ .

When the size of the actuators is larger than the size of the mesh, they will perform a load on several nodes. The effect can be illustrated by coupling some of the nodes in our example. Actuator 2 and 3 are coupled and actuator 8 and 9 are coupled. This means that if a load in y-direction is applied to actuator 2, the same load is applied to actuator 3 and vice versa. The same holds for actuator 8 and 9. This results in new root mean square errors

$$e_{\text{RMS1}} = 2.7940 \cdot 10^{-3} \text{ m},$$

 $e_{\text{RMS2}} = 6.3779 \cdot 10^{-3} \text{ m}.$
(3.35)

These eigenvalues are significantly higher than for the problem with the uncoupled actuators. The eigenvectors give us the corresponding desired-amplitude vectors:

$$\mathbf{m}_{1} = \begin{bmatrix} -0.9974\\ 0.0723 \end{bmatrix},$$

$$\mathbf{m}_{2} = \begin{bmatrix} 0.0723\\ 0.9974 \end{bmatrix}.$$
(3.36)

It can be concluded that this actuator configuration is better able to produce mode shape R_1^1 than mode shape R_2^0 . The desired displacements and achieved displacements for the problem with the coupled actuators are plotted for λ_1 (Figure 3.7a) and λ_2 (Figure 3.7b).



Figure 3.7: Plots of the achieved displacements and desired displacements of the linear beam with coupled actuators. Actuator 2 and 3 are coupled and actuator 8 and 9 are coupled. On the left the results for the smallest error $e_{\text{RMS1}} = 2.7940 \cdot 10^{-3}$ m are plotted and on the right the results for the largest error $e_{\text{RMS2}} = 6.3779 \cdot 10^{-3}$ m.

To show the strength of the method used a more complex example is studied. We still use the beam as defined in Figure 3.1, with the actuators uncoupled. The first 9 radial functions of the Zernike polynomials are used for the mode shape matrix \mathbf{Z} , instead of only 2 mode shapes. This results in the following RMS errors

$$e_{\text{RMS1}} \approx 0 \dots e_{\text{RMS6}} \approx 0 \text{ m},$$

 $e_{\text{RMS7}} = 3.8498 \cdot 10^{-4} \text{ m},$
 $e_{\text{RMS8}} = 2.4686 \cdot 10^{-3} \text{ m},$
 $e_{\text{RMS9}} = 8.8264 \cdot 10^{-3} \text{ m}.$
(3.37)

This shows the advantage of the method used: it can be seen clearly where the biggest improvements can be made. First of all, the beam can be deformed into the shapes corresponding to $e_{RMS1} \dots e_{RMS6}$ very well (since the error is close to 0). If we are looking for an improvement of the system behaviour, we should try to start with decreasing e_{RMS9} . The next step is decreasing e_{RMS8} and then e_{RMS7} . For this simple beam problem applying tension to the boundaries would not have any useful effect. It would only make the beam more stiff, so it would be harder to obtain a certain shape with a minimal error, but this error would stay the same. But for a plate applying tension to the boundary does affect the eigenvalues, i.e. it does effect the extreme values of e_{RMS} . This will be discussed in the remainder of this chapter.

Up until now we have only discussed the situation without the tension induced by boundary actuators. We describe this tension as a stress vector σ caused by in-plane loads **T** applied to the rim of the mirror:

$$\sigma = \Sigma \mathbf{T},\tag{3.38}$$

where Σ is the stress tensor of order 2. The stress vector σ is linearly dependent on the in-plane loads:

 σ

The stiffness matrix is dependent on the stress vector, and thus on the in-plane loads as follows

$$\mathbf{K} = \mathbf{K}_{\mathbf{el}} + \mathbf{G}[\boldsymbol{\sigma}[\mathbf{T}]], \tag{3.40}$$

where \mathbf{K}_{el} is the elastic stiffness matrix and \mathbf{G} the geometric stiffness matrix. \mathbf{G} is approximately linearly dependent on \mathbf{T} . Using Equation (3.11) we can say that \mathbf{U} is a function of the in-plane loads:

$$\mathbf{U} = (\mathbf{K}_{\mathbf{el}} + \mathbf{G}[\boldsymbol{\sigma}[\mathbf{T}]]^{-1})\mathbf{F},$$

$$\mathbf{U} = \mathbf{U}[\mathbf{T}].$$
 (3.41)

Using Equation (3.1) to Equation (3.21) we can finally conclude that e_{\min} and thus e_{RMS} and \tilde{e}_{RMS} are a function of the in-plane loads:

$$e_{\min} = e_{\min}[\mathbf{T}],$$

$$e_{RMS} = e_{RMS}[\mathbf{T}],$$

$$\tilde{e}_{RMS} = \tilde{e}_{RMS}[\mathbf{T}].$$
(3.42)

Note that the error is nonlinearly dependent on **T**. The main idea is to minimize the maximum value for e_{\min} as found with Equation (3.29) and thus improve the behaviour of the mirror.

4

PLATE MODEL FOR THE DEFORMABLE MIRROR

This chapter covers the implementation of the model as defined in Chapter 3. We start with an explanation and argumentation on the mirror design and the different plates studied. Once the chosen specifications are clear, the data processing is discussed.

4.1. MIRROR DESIGN

4.1.1. MIRROR MATERIAL

Common optical glasses are Ultra Low Expansion glass (ULE), fused silica and Zerodur [17]. These are materials with a similar Young's modulus and a similar ultimate tensile strength. In this research we chose to use Zerodur. This material is used frequently in astronomy, because of its extremely high thermal and mechanical stability [17]. The material constants of Zerodur can be found in Table 4.1. Note that there is a range for the ultimate tensile strength. The exact value depends on the way the material is processed. This will be taken into account when discussing the results.

Table 4.1: Constants used for the material of the hexadecagonal mirror. The material is Zerodur. *The correct value for the Young's modulus actually is 90.3 GPa, but in this research 91 GPa was used. ** The value depends on how the material has been processed.

Constant	Value
Young's modulus	91 GPa [<mark>18</mark>]*
Ultimate tensile strength	10 – 50 MPa ^{**} [18]
Poisson's ratio	0.24 [18]
Density	2530 kg/m ³ [18]

4.1.2. MIRROR GEOMETRY

The geometry that is used to model the mirror is as a thin hexadecagonal plate (a plate with 16 sides). The details of the geometry can be found in Figure 4.1. The mirror geometry is usually circular. There are two reasons to choose the mirror geometry to be hexadecagonal in this research:

- 1. The pretensions that will be applied to the sides of the mirror are equally distributed loads. In practice it is easier to achieve this when the mirror sides are straight instead of curved.
- 2. It is easier to model the plate in a Cartesian coordinate system. This has to do with the limitations of the used Finite Element program (more information about this can be found in the next section). It is not very appealing to model a circular plate in a Cartesian coordinate system. A hexadecagonal plate is a good approximation of a circular plate.

The characteristic length *L* of the plate is 0.2 m and the thickness *t* is $2 \cdot 10^{-4}$ m. These dimensions result in a thin plate ($t/L \ll 1$) [19]. It is desirable to have a thin plate, because applying pretension has a larger effect on the stiffness for a thin plate than for a thick plate.

The boundary condition on side s_1 and s_3 (see Figure 4.1) are as follows: constrained in x-direction and zdirection, free in y-direction and for all rotations. The boundary conditions on side s_2 and s_4 are: constrained in y-direction and z-direction, free in x-direction and for all rotations. All the other sides are free to translate and rotate.



Figure 4.1: Geometry of the plate used to model the mirror. The characteristic length L is 0.2 m and the thickness t of the plate is $2 \cdot 10^{-4}$ m. The boundary condition on side s_1 and s_3 are: constrained in x-direction and z-direction, free in y-direction and for all rotations. The boundary conditions on side s_2 and s_4 are: constrained in y-direction and z-direction, free in x-direction and for all rotations. The blue arrows indicate in which direction the sides are free to translate. All other sides are free to translate and rotate.

4.1.3. ACTUATOR CONFIGURATIONS

Three different plate configurations are studied. They are presented in Figure 4.2, Figure 4.3 and Figure 4.4. The corresponding constants are given in Table 4.2. For the out-of-plane surface actuation nine actuators are used (called a_i , where i = 1, ..., 9). They are modelled as a small area on which a pressure (N/m²) can be applied. What we see often in deformable mirrors is that the whole mirror surface is covered with actuators placed in a hexagonal pattern. For the sake of simplicity and maintaining a good overview of what is happening it is decided to use only 9 actuators in a star-like configuration. It is easier from the modelling viewpoint as well (see the next section for more information).

Pretensions are applied to boundaries of the plate. In the first and second design pretension T_1 is applied to sides s_1 and s_3 , pretension T_2 is applied to sides s_2 and s_4 . In the third design two extra sets of pretension are added. The pretensions are modelled as equally distributed loads (N/m), which can vary in magnitude. In practice it may be useful to use even more pretensions. In this report we use a maximum of four pretensions to maintain a good overview.

Even though the results may not completely reflect the behaviour of a circular mirror with a hexagonal actuator pattern and more sets of pretension, they do give a good impression of the general effect of in-plane boundary actuation. This is why all the simplifications are justified.



Figure 4.2: Actuator configuration 1. The out-of-plane actuators are rotated 45° with respect to the in-plane actuators.



Figure 4.3: Actuator configuration 2. The out-of-plane actuators are in line with the in-plane actuators.



Figure 4.4: Actuator configuration 3. This is the similar to configuration 1, but now two extra sets of pretension are added.

Table 4.2: Constants used for the geometry of the hexadecagonal mirror.

Constant	Value
Characteristic length L	0.2 m
Actuator size a	$1 \cdot 10^{-2} \text{ m}$
Actuator distance d_1	$4 \cdot 10^{-2} \text{ m}$
Actuator distance d_2	$8 \cdot 10^{-2} \text{ m}$
Plate thickness t	$2 \cdot 10^{-4} \text{ m}$
Number of nodes	748

4.2. DATA PROCESSING

4.2.1. FINITE ELEMENT PROGRAM Charles

The finite element calculations are performed in the finite element program *Charles*. This program is developed for academical purposes and it is particularly good at solving plate and shell problems. The advantage of *Charles* over commercially available finite element programs, like *COMSOL* and *ANSYS*, is that it gives more insight in what the program is doing and it offers more possibilities to control this.

The main disadvantage of *Charles* lies in the limitations of geometries that can be modelled. As one surface cannot be modelled twice, each actuator has to be modelled separately with the plate modelled around them. This is time consuming and prone to errors. But once the model has been properly prepared, *Charles* shows its utility as a very strong finite element program.

Triangular elements are used for the finite element mesh. The used elements are appropriate for large rotations and small deformations. The meshes that are used for the calculations in *Charles* are presented in Figure 4.5.



Figure 4.5: Meshes used for configuration 1 and 3 (left) and configuration 2 (right).

4.2.2. DATA GATHERING AND FINAL CALCULATIONS

The method by which the data is gathered is first explained for plate configuration 1. The method is the same for the other plate configurations. The first 15 Zernike polynomials are used for modeshape matrix **Z**, so matrix **Y** is a 15 × 15 matrix with 15 corresponding RMS errors e_{RMS1} to e_{RMS15} . e_{RMS1} gives the smallest error and e_{RMS15} gives the biggest error. To find e_{RMS1} for each combination of T_1 and T_2 , we need to find the lowest eigenvalue of **Y** (see Equation (3.21)) and substitute this into Equation (3.22) for these pretension combinations.

Y will be calculated with **U**, **Z** and **W**. First the matrix **U** is constructed for different combinations of pretensions T_1 and T_2 . According to Equation (3.10) the first column of **U** corresponds to the displacements achieved when only out-of-plane actuator 1 is in operation, the second column of **U** corresponds to the displacements achieved when only out-of-plane actuator 2 is in operation, etc. These columns are constructed by calculating the displacement field for each actuator separately when a pressure of 1 N/m^2 is applied. These displacement fields are calculated in *Charles* and saved using a *Bash*-script. The positions of the nodes in the undeformed shape are used to construct matrix **Z** and matrix **W**. For matrix **W** two different aperture sizes D are used: $D = 1 \cdot L$ and $D = 0.9 \cdot L$. Now Equation (3.21) can be solved and \tilde{e}_{RMS} can be found for the used combinations of T_1 and T_2 .

The minimum value used for the sets of pretension is limited by the buckling load. The maximum value is limited by the ultimate tensile strength *S* of the material. For plate configurations 1 and 2 the pretensions that are used for T_1 and T_2 are:

$$\mathbf{T}_{set1} = \begin{bmatrix} -70.71 & 0 & 643.5 & 1358 & 2072 & 2786 & 3500 & 4214 & 4929 & 5643 & 6357 & 7071 \end{bmatrix}$$
 N/m. (4.1)

For the minimum value -70.71 N/m we are already close to the buckling load. When the ultimate tensile strength is S = 10 MPa, the last value in \mathbf{T}_{set1} for which the plate does not plastically deform is $T_1 = T_2 = 1358$ N/m. For an ultimate tensile strength of S = 50 MPa this is $T_1 = T_2 = 7071$ N/m. Since we have two sets of pretension of length 12 (defined by \mathbf{T}_{set1}) and 9 actuators, $12 \times 12 \times 9 = 1296$ displacement fields have to be calculated for both plate configuration 1 and 2. The results are processed for the two different aperture sizes and for the ultimate tensile strengths S = 10 MPa and S = 50 MPa. The same is done for plate configuration 2. For plate configuration 3 we use another set of pretensions for T_1 , T_2 , T_3 and T_4 :

$$\mathbf{T}_{set2} = \begin{bmatrix} -100 & 0 & 1500 & 7000 \end{bmatrix} \text{N/m.}$$
 (4.2)

For the minimum value -100 N/m we moved a little closer to the buckling load, to have a bigger range. When the ultimate tensile strength is 10 MPa the last value in \mathbf{T}_{set1} for which the plate does not plastically deform is $T_1 = T_2 = T_3 = T_4 = 1500 \text{ N/m}$. This is a little higher than in \mathbf{T}_{set1} , because we wanted to move a little closer

to the ultimate tensile strength. For an ultimate tensile strength of 50 MPa this value is $T_1 = T_2 = T_3 = T_4 =$ 7000 N/m. This value is a little lower than in \mathbf{T}_{set1} , because we preferred using a round number. Since we have four sets of pretension of length 4 (defined by \mathbf{T}_{set2}) and 9 actuators, $4 \times 4 \times 4 \times 4 \times 9 = 2304$ displacement fields have to be calculated for plate configuration 3.

When the displacement fields are found, they are used for the final calculations in *Matlab*. With the DOF we can construct **Z** and **W** as well. Now Equation (3.21) can be solved for the used pretension combinations. Again, the results are processed for the two different aperture sizes $D = 1 \cdot L$ and $D = 0.9 \cdot L$ and for the ultimate tensile strengths S = 10 MPa and S = 50 MPa.

5

RESULTS

In this chapter the results obtained with the method explained in Chapter 4 will be discussed. This will first be done for the different plate configurations separately. After that the optimal plate configuration will be discussed in more detail.

5.1. PLATE CONFIGURATION 1

For each combination of pretension T_1 and T_2 there is a desired shape that can be reached with the smallest error possible. This error is \tilde{e}_{RMS1} . For an aperture of size $D = 1 \cdot L$ and an ultimate tensile strength S = 50 MPa, plotting \tilde{e}_{RMS1} for each pretension combination results in the plot in Figure 5.1a. The largest error possible, \tilde{e}_{RMS15} , is plotted in Figure 5.1b for the same conditions. It can be seen that \tilde{e}_{RMS1} and \tilde{e}_{RMS15} are both at a minimum when T_1 and T_2 are at their minimum value of -70.71 N/m.

For each ultimate tensile strength and aperture size the minimal value of \tilde{e}_{RMS1} can be found. The result is given in Figure 5.2a. The minimum value of \tilde{e}_{RMS1} occurs at $T_1 = -70.71$ N/m and $T_2 = -70.71$ N/m for all the used aperture sizes and ultimate tensile strength values. The same happens for \tilde{e}_{RMS15} (see Figure 5.2b). Hence, an overall system improvement occurs when we apply a constant pretension of $T_1 = -70.71$ N/m and $T_2 = -70.71$ N/m. \tilde{e}_{RMS1} is reduced by 30.3% for an aperture of $1 \cdot L$ and by 42.0% for an aperture of $0.9 \cdot L$. \tilde{e}_{RMS15} is reduced by 0.595% for an aperture of $1 \cdot L$ and by 0.628% for an aperture of $0.9 \cdot L$. We see that for an aperture of $0.9 \cdot L$ the error without pretension is smaller than for an aperture $1 \cdot L$ and the error reduction is larger.



Figure 5.1: \tilde{e}_{RMS1} and \tilde{e}_{RMS15} plotted in 3D for all combinations of T_1 and T_2 . It can be seen that both \tilde{e}_{RMS1} and \tilde{e}_{RMS15} are at a minimum when $T_1 = T_2 = -70.71$ N/m. The aperture used is $D = 1 \cdot L$ and the ultimate tensile strength is S = 50 MPa.



(b) \tilde{e}_{RMS15}

Figure 5.2: Results for \tilde{e}_{RMS1} and \tilde{e}_{RMS15} of plate configuration 1. The errors are given for the case where no pretension is applied and for the pretension that results in the minimal error. The minimum values are indicated with a plus sign. Both \tilde{e}_{RMS1} and \tilde{e}_{RMS15} are at a minimum when $T_1 = -70.71$ N/m and $T_2 = -70.71$ N/m. The best result is obtained when $D = 0.9 \cdot L$. The ultimate tensile strength does not influence the minimum.

5.2. PLATE CONFIGURATION 2

The overall results for configuration 2 are given in Figure 5.3. Just like in configuration 1 the minimal value of both \tilde{e}_{RMS1} and \tilde{e}_{RMS15} occur when $T_1 = -70.71$ N/m and $T_2 = -70.71$ N/m for all the used aperture sizes and ultimate tensile strength values. \tilde{e}_{RMS1} is reduced by 24.1% for an aperture of $1 \cdot L$ and by 66.6% for an aperture of $0.9 \cdot L$. \tilde{e}_{RMS15} is reduced by 1.63% for an aperture of $1 \cdot L$ and by 0.883% for an aperture of $0.9 \cdot L$. We can see that the errors \tilde{e}_{RMS1} and \tilde{e}_{RMS15} are smaller in plate configuration 2 than in plate configuration 1. We also see that the error reduction is larger in plate configuration 2 (except for \tilde{e}_{RMS1} when S = 50 MPa and $D = 1 \cdot L$). Overall, the behaviour of configuration 2 is better than that of configuration 1.

Please note that some of the calculations performed for plate configuration 2 are left out. The reason for this is that they were obvious outliers. They showed large peaks in the results. If these outliers would have been in the trend of the other calculated points, they would not have been a maximum or a minimum. This is why it is expected that leaving these points out does not influence the final results. The calculations for the following pretension combinations are left out: $T_1 = -70.71$ N/m with $T_2 = 6357$ N/m, $T_1 = 0$ N/m with $T_2 = 6357$ N/m, $T_1 = 6357$ N/m with $T_2 = -70.71$ N/m, and $T_1 = 6357$ N/m with $T_2 = 0$ N/m







(b) \tilde{e}_{RMS15}

Figure 5.3: Results for \tilde{e}_{RMS1} and \tilde{e}_{RMS15} of plate configuration 2. The errors are given for the case where no pretension is applied and for the pretension that results in the minimal error. The minimum values are indicated with a plus sign. Both \tilde{e}_{RMS1} and \tilde{e}_{RMS15} are at a minimum when $T_1 = -70.71$ N/m and $T_2 = -70.71$ N/m. The best result is obtained when $D = 0.9 \cdot L$. The ultimate tensile strength does not influence the minimum.

5.3. PLATE CONFIGURATION 3

The overall results for configuration 3 with an ultimate tensile strength of 50 MPa are given in Figure 5.4. For both aperture $1 \cdot L$ and $0.9 \cdot L$, the minimal value for \tilde{e}_{RMS1} occurs when $T_1 = -100$ N/m, $T_2 = 0$ N/m, $T_3 = -100$ N/m and $T_4 = 0$ N/m. By using this pretension, the error \tilde{e}_{RMS1} is reduced by 36.7% when the aperture is $1 \cdot L$ and by 56.3% when the aperture is $0.9 \cdot L$. For this same pretension \tilde{e}_{RMS15} is reduced with 0.992% when the aperture is $1 \cdot L$ and with 0.209% when the aperture is $0.9 \cdot L$.

If we want to further reduce \tilde{e}_{RMS15} a different pretension is needed (refer to Figure 5.4 for the pretension values). For this pretension the error \tilde{e}_{RMS15} is reduced by 6.15% when the aperture is $1 \cdot L$ and by 8.37% when the aperture is $0.9 \cdot L$. But the error \tilde{e}_{RMS1} will be increased by 130% and 118% respectively. This may not seem to be a fair exchange, but it does improve the overall system, since the maximum possible error is reduced. Also, the absolute value by which \tilde{e}_{RMS15} is reduced is an order bigger than the value by which \tilde{e}_{RMS1} is increased.



UTS = 50, Aperture = 1L UTS = 50, Aperture = 0.9L Ultimate tensile strength (MPa), Aperture size (m)

(a) $\tilde{e}_{\rm RMS1}$



Ultimate tensile strength (MPa), Aperture size (m)

(b) \tilde{e}_{RMS15}

Figure 5.4: Results for \tilde{e}_{RMS1} and \tilde{e}_{RMS15} of plate configuration 3 when the ultimate tensile strength is S = 50 MPa. The errors are given for the case where no pretension is applied and for the pretensions that results in the minimal error for either \tilde{e}_{RMS1} or \tilde{e}_{RMS15} . The minimum values are indicated with a plus sign. Error \tilde{e}_{RMS1} is at a minimum when $T_1 = -100$ N/m, $T_2 = 0$ N/m, $T_3 = -100$ N/m and $T_4 = 0$ N/m. Error \tilde{e}_{RMS15} is at a minimum when either $T_1 = -100$ N/m, $T_2 = -100$ N/m and $T_4 = 7000$ N/m or $T_1 = -100$ N/m, $T_2 = 7000$ N/m, $T_3 = -100$ N/m and $T_4 = -100$ N/m. The best results are obtained when $D = 0.9 \cdot L$.

The overall results for configuration 3 when the ultimate tensile strength is 10 MPa are given in Figure 5.5. The occurrence of the minimal value for \tilde{e}_{RMS1} and the corresponding values for \tilde{e}_{RMS15} are the same as with UTS = 50 MPa.

For the reduction of \tilde{e}_{RMS15} less high percentages can be reached when S = 10 MPa is used. The error reduction of \tilde{e}_{RMS15} is 3.37% when the aperture is $1 \cdot L$ and 6.49% when the aperture is $0.9 \cdot L$. The corresponding error \tilde{e}_{RMS1} is increased with 32.6% when the aperture is $1 \cdot L$ and it is reduced with 31.0% when the aperture is $1 \cdot L$.

Of the different plate configurations, aperture sizes and ultimate tensile strengths, the smallest \tilde{e}_{RMS15} can be obtained with configuration 3, when an aperture size of $0.9 \cdot L$ and an ultimate tensile strength of 50 MPa are used. Even though this results in a higher value for \tilde{e}_{RMS1} , it results in a larger reduction of the maximum possible error \tilde{e}_{RMS1} . So, if we want to improve the overall system by applying one constant pretension, we should use plate configuration 3 with $D = 0.9 \cdot L$, S = 50 MPa and the pretensions resulting in the minimal error \tilde{e}_{RMS15} as given in Figure 5.4b.

5.3.1. Ultimate tensile strength S = 50 MPa, aperture size $D = 0.9 \cdot L$

Now that we have found that the best overall improvement is obtained for plate configuration 3 with $D = 0.9 \cdot L$ and S = 50 MPa, we want to take a closer look at this configuration under these conditions. More detailed results are presented in Figure 5.6 (Figure A.10 in Appendix A) and Figure 5.7.

In Figure 5.6 the errors \tilde{e}_{RMS} with which the Zernike shapes can be made are plotted. This gives us insight into which Zernike polynomials can be made with a small error. We can see that Zernike polynomial Z_2^{-2} can always be made with a relatively small error (ranging from 0.073 to 0.209) and Zernike polynomial Z_2^2 is always made with a relatively large error (ranging from 0.340 to 0.388). Also Z_4^0 always has a relatively large error (ranging from 0.310 to 0.450). Zernike polynomials Z_0^0 , Z_1^{-1} , Z_1^1 and Z_2^{-2} can all be made with an error smaller than 0.116. This may still seem quite large, but keep in mind that we are only using 9 out-of-plane actuators. This limits the deformations that are possible. For the other Zernike polynomials the minimal error is even larger. But if we calculate the percentage by which the error is improved, this gives a different picture (refer to Figure 5.7). For example for Zernike polynomial Z_3^3 , the error remains quite large, but the error reduction is als quite large (14.8%). The largest error reduction occurs for Z_0^0 : 41.8%. The smallest error reduction occurs for Z_2^{-2} : 2.67%.

Figures similar to Figure 5.6 were made for the other configurations and with different combinations for the ultimate tensile strength and the aperture size. These figures can be found in Appendix A. This is left for the reader to look at.



(b) \tilde{e}_{RMS15}

Figure 5.5: Results for \tilde{e}_{RMS1} and \tilde{e}_{RMS15} of plate configuration 3 when the ultimate tensile strength is S = 10 MPa. The errors are given for the case where no pretension is applied and for the pretensions that results in the minimal error for either \tilde{e}_{RMS1} or \tilde{e}_{RMS15} . The minimum values are indicated with a plus sign. Error \tilde{e}_{RMS1} is at a minimum when $T_1 = -100$ N/m, $T_2 = 0$ N/m, $T_3 = -100$ N/m and $T_4 = 0$ N/m. This is the same as in the case with an ultimate tensile strength S = 50 MPa. Error \tilde{e}_{RMS15} is at a minimum when $T_1 = -100$ N/m, $T_2 = 1500$ N/m, $T_3 = -100$ N/m and $T_4 = 1500$ N/m. The best results are obtained when $D = 0.9 \cdot L$.

Ultimate tensile strength (MPa), Aperture size (m)

5.3.2. Ultimate tensile strength S = 10 MPa, aperture size $D = 0.9 \cdot L$

It may occur that the ultimate tensile strength is limiting the behaviour of the mirror. If S = 10 MPa the best overall improvement is still obtained for plate configuration 3 with $D = 0.9 \cdot L$. Taking a closer look at this plate, we see something similar happening as with S = 50 MPa, but with less promising numbers (refer to Figure 5.8 and Figure 5.9).

From Figure 5.8 we see that Zernike polynomial Z_2^{-2} still can always be made with a relatively small error (ranging from 0.073 to 0.095) and Zernike polynomial Z_2^2 still is always made with a relatively large error (ranging from 0.359 to 0.388). Note that the maximum error for Z_2^{-2} has grown even smaller and the minimal error for Z_2^2 has grown even bigger. The latter definitely is unwanted behaviour. Also Z_4^0 always has a relatively large error (ranging from 0.310 to 0.450). This is the same as with S = 50 MPa. The only Zernike polynomial that can be made with an error smaller than 0.116 is Z_2^{-2} . The largest error reduction now occurs for Z_1^{-1} and Z_1^1 : 28.3%. Note that this is much smaller than the largest error reduction for S = 50 MPa. The smallest error reduction still occurs for Z_2^{-2} : 2.67%.

5.3. PLATE CONFIGURATION 3

0.311 0.316 0.312 0.248 0. 0.316 0.315 0.312 0.248 0 0.346 0.173 0.302 0.306 0.255 0 0.346 0.173 0.302 0.306 0.255 0 0 0.46 0.174 0.316 0.306 0.255 0 0 0.346 0.174 0.316 0.306 0.253 0 0 0.346 0.174 0.316 0.311 0.316 0.311 0 0 0.346 0.179 0.316 0.311 0.316 0.311 0.316 0 0 0.346 0.179 0.301 0.300 0.299 0 0 0 0.381 0.221 0.301 0.300 0.299 0 0 0 0.385 0.229 0.301 0.300 0.299 0.399 0 0 0 0.386 0.221 0.301 0.203	z_{3}^{3} z_{4}^{0} z_{4}^{2} z_{4}^{2} z_{4}^{4} z_{4}^{4}
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Figure 5.6: Results for the dimensionless error \tilde{e}_{RMS} of the Zernike polynomials for plate configuration 3. The ulitmate tensile strength is S = 50 MPa and the aperture size is $D = 0.9 \cdot L$. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the largest error possible. Zernike polynomial Z_2^2 can always be made with a relatively small error and Zernike polynomial Z_2^{-1} can always be made with a relatively small error and Zernike polynomial Z_2^{-1} can always be made with a relatively small error and Zernike polynomial Z_2^{-1} can all be made with a relatively small error and Zernike polynomial Z_2^{-1} can always be made with a relatively small error and Zernike polynomial Z_2^{-1} can always be made with a relatively small error and Zernike polynomial Z_2^{-1} can always made with a relatively large error. Zernike polynomials Z_0^0 , Z_1^{-1} , Z_1^1 and Z_2^{-2} can all be made with a error smaller than 0.116.

Figure 5.7: Results for the improvement of the dimensionless error \tilde{e}_{RMS} of the Zernike polynomials for plate configuration 3. The ulitmate tensile strength is S = 50 MPa and the aperture size is $D = 0.9 \cdot L$. The improvement is calculated for each polynomial with respect to its error \tilde{e}_{RMS} when not pretensions are applied. A positive improvement is a reduction in the error. A negative improvement is an increase in the error. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the largest error possible. The largest error reduction occurs for Z_0^2 : 41.8%. The smallest error reduction occurs for Z_0^2 : 2.67%.

40		- 20		0 +		-20		-40		(%) tue	rovemo	ő ő	Err	-100	-		-170		-140				
0.000	6.129	-2.810	-2.811	-9.811	-4.554	-25.275	6.037	-19.786	-19.782	-20.249	-20.260	-20.253	-20.259	-12.011	-11.984	4.189	4.127	22.756	5.278	3.221	-22.519	-22.519	Z ⁴ 4
0.000	-0.001	1.948	1.948	1.798	0.742	0.145	-0.094	3.854	3.855	3.458	4.678	3.458	4.678	0.172	0.172	1.195	0.112	0.114	-0.095	1.097	0.828	0.828	Z ⁻⁴ 4
0.000	0.021	4.414	4.415	-0.003	0.172	-0.082	0.012	4.596	4.585	4.319	10.413	4.330	10.413	-0.046	-0.046	-0.026	-0.038	0.039	0.007	0.173	1.671	1.671	Z_4^2
0.000	+ 208.6	1.770	1.770	1.064	1.067	-32.315	-2.001	-25.804	-25.808	-28.038	-14.498	-28.051	-14.496	-30.155	-30.014	-37.616	-39.621	-28.404	-5.497	-38.873	-33.088	-33.088	Z_4
0.000	3.489	-7.632	-7.634	-40.018	-38.937	-15.633	-7.796	-18.573	-18.677	-19.845	-17.099	-19.745	-17.099	-11.608	-12.064	-15.238	-14.738	-14.766	-8.941	-14.332	-19.706	-19.713	Z ⁰ 4
0.000	4.826	11.661	-16.661	-7.043	-9.050	-20.694	-0.809	-21.661	6.215	6.259	-20.976	-21.675	14.758	-16.598	-14.767	-8.980	-16.057	-17.009	-0.916	-18.394	-0.699	-22.777	Z ³ 3
0.000	4.826	-16.661	11.661	-8.423	-8.571	-20.694	-0.809	6.215	-21.662	-21.676	14.758	6.259	-20.976	-14.750	-16.609	-8.702	-21.741	-18.408	-0.916	-16.919	-22.77	-0.699	Z ₃ mial
0.000	-5.080	-0.223	11.767	-0.834	-1.887	-3.952	1.404	-7.891	-34.707	-32.390	-28.970	-9.481	-19.251	-41.783	-35.570	-35.287	-43.203	-55.333	0.907	-28.215	-13.188	-5.553	Z ¹ Z3
0.000	-5.080	11.767	-0.223	-1.788	-2.581	-3.952	1.404	-34.698	-7.893	-9.483	-19.249	-32.382	-28.966	-35.521	-41.858	-32.181	-42.531	-28.255	0.907	-55.299	-5.553	-13.188	Z ₃ Zarnil
0.000	-0.001	7.394	7.394	2.624	3.785	-0.017	-0.043	9.908	9.908	9.839	12.315	9.839	12.314	-0.005	-0.005	0.244	-0.025	0.255	-0.043	0.249	7.376	7.376	Z_2^2
0.000	2.670	-15.062	-15.062	-10.098	-10.605	-44.950	-39.629	-47.884	-47.870	-40.794	-33.756	-40.781	-33.755	-40.165	-40.555	-177.30	-133.862	-97.697	-37.082	-133.182	-43.665	-43.665	Z_{2}^{-2}
0.000	-4.286	-13.964	-13.964	2.735	4.459	17.838	-13.965	-13.127	-13.124	-13.651	-22.114	-13.655	-22.113	19.698	19.643	-40.806	-44.524	-45.101	-13.223	-44.105	-9.898	-9.897	Z ⁰ 2
0.000	3.407	-35.369	19.525	8.086	6.131	32.991	-32.934	39.288	-41.089	-40.299	21.444	39.196	-63.454	24.042	21.039	0.166	14.889	15.547	-32.397	2.878	-26.323	38.658	Z ₁ ¹
0.000	3.407	19.525	-35.369	6.152	6.513	32.991	-32.934	-41.085	39.288	39.196	-63.452	-40.296	21.446	21.051	24.039	-5.601	-0.811	3.122	-32.397	15.506	38.658	-26.323	Z ₁ ⁻¹
0.000	4.445	-27.645	-27.645	11.004	12.993	41.772	-42.085	-19.961	-19.981	-19.584	-34.157	-19.562	-34.152	31.607	31.650	34.400	31.334	29.363	-41.580	31.514	-9.519	-9.519	Z ⁰
$T_1 = 0, T_2 = 0, T_3 = 0, T_4 = 0$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = -100$	$T_1 = 1500, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = 7000, T_3 = -100, T_4 = 7000$	$T_1 = 7000, T_2 = -100, T_3 = 7000, T_4 = -100$	$T_1 = 7000, T_2 = 7000, T_3 = -100, T_4 = -100$	E $T_1 = -100, T_2 = -100, T_3 = 7000, T_4 = 7000$	$\overleftarrow{\mathbf{z}}$ $\mathbf{T}_1 = -100, \mathbf{T}_2 = 0, \mathbf{T}_3 = 7000, \mathbf{T}_4 = 7000$	$\overrightarrow{3}$ T ₁ = 7000, T ₂ = -100, T ₃ = -100, T ₄ = -100 .	$T_1 = 7000, T_2 = 7000, T_3 = -100, T_4 = 0$	$\begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$ T ₁ = -100, T ₂ = -100, T ₃ = 7000, T ₄ = -100.	$T_1 = -100, T_2 = 7000, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 7000$	$T_1 = 0, T_2 = -100, T_3 = -100, T_4 = 7000$	$T_1 = -100, T_2 = 7000, T_3 = 0, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 0, T_4 = 7000$	$T_1 = 7000, T_2 = 0, T_3 = 7000, T_4 = 0$	$T_1 = 0, T_2 = 7000, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = 7000, T_3 = 7000, T_4 = 7000$	$T_1 = 7000, T_2 = 7000, T_3 = -100, T_4 = 7000$	

Figure 5.8: Results for the dimensionless error $\tilde{e}_{\rm RMS}$ of the Zernike polynomials for plate configuration 3. The ulitmate tensile strength is S = 10 MPa and the aperture size is $D = 0.9 \cdot L$. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the smallest error possible. Zernike polynomial is imposed on the plate with the smallest error possible. polynomial Z_2^{-2} can always be made with a relatively small error and Zernike polynomial Z_2^2 is always made with a relatively large error. Zernike polynomial Z_2^{-2} can be made with an error smaller than 0.116.



Figure 5.9: Results for the improvement of the dimensionless error \tilde{e}_{RMS} of the Zernike polynomials for plate configuration 3. The ulitmate tensile strength is S = 10 MPa and the aperture size is $D = 0.9 \cdot L$. The improvement is calculated for each polynomial with respect to its error \tilde{e}_{RMS} when not pretensions are applied. A positive improvement is a reduction in the error. A negative improvement is an increase in the error. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the largest error possible. The largest error reduction occurs for Z_1^{-1} and Z_1^{-1} : 28.3%. The smallest error reduction occurs for Z_2^{-2} : 2.67%.

	20		10		0	(%) tuəmə		Er	00-		ç		40	
0.000	6.129	-2.810	-2.811	-9.811	-4.554	• -19.850	5.757	-17.752	-17.752	-11.049	-11.054	3.508	-18.370	Z 4 4
0.000	-0.001	+ 1.948	+ 1.948	1.798	0.742	0.092	-0.063	0.627	0.627	1.294	1.294	-0.064	0.071	Z-4 4
0.000	0.021	+ 4.414	4.415	-0.003	0.172	-0.058	0.014	1.358	1.358	2.594	2.592	0.002	-0.057	Z ² 4
0.000	9.802	1.770	1.770	1.064	1.067	-22.202	4.328	-23.304	-23.303	-14.125	-14.072	-2.551	-22.238	Z_4
0.000	3.489	-7.632	-7.634	-40.018	-38.937	-10.197	-1.235	-13.593	-13.596	-9.822	-9.853	-3.927	-10.332	Z 4
0.000	4.826	11.661	-16.661	-7.043	-9.050	-15.598	0.380	-0.005	-20.614	-18.053	5.355	-0.348	-14.224	Z ³ 3
0.000	4.826	-16.661	11.661	-8.423	-8.571	-15.598	0.379	-20.614	-0.005	5.355	-18.053	-0.348	-14.224	z ₃ mial
0.000	-5.080	-0.223	11.767	-0.834	-1.887	1.346	4.819	-5.011	2.260	5.631	• -7.922	3.919	0.764	z ¹ ke polyno
0.000	-5.080	+	-0.223	-1.788	-2.581	1.346	4.819	2.260	-5.011	• -7.923	5.631	3.919	0.764	z ₃ Zernil
0.000	-0.001	7.394	7.394	2.624	3.785	-0.001	• 0.026	4.702	4.702	5.755	5.755	-0.025	-0.004	Z ² 2
0.000	2.670	-15.062	-15.062	-10.098	-10.605	-22.601	-17.898	O -25.857	-25.857	-22.178	-22.178	-15.629	-22.832	Z ⁻² 2
0.000	-4.286	-13.964	• -13.964	2.735	4.459	+ 17.747	-11.154	-2.823	-2.823	-6.074	-6.073	-9.036	15.272	Z ⁰ 2
0.000	3.407	-35.369	19.525	8.086	6.131	22.053	-24.905	-19.471	28.287	25.175	-26.234	-23.507	16.764	Z ¹
0.000	3.407	19.525	-35.369	6.152	6.513	22.053	-24.905	+ 28.287	-19.471	-26.234	25.175	-23.507	16.764	Z ⁻¹
0.000	4.445	-27.645	-27.645	11.004	12.993	22.707	-28.624	- 9.048	-9.048	-16.839	-16.838	-27.437	17.380	20 Z
$T_1 = 0, T_2 = 0, T_3 = 0, T_4 = 0$	T ₁ = -100, T ₂ = -100, T ₃ = -100, T ₄ = -100	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = -100$	T ₁ = 1500, T ₂ = -100, T ₃ = -100, T ₄ = -100	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = -100, T_4 = -100$	Z/F (2) 11 = -100, Т ₂ = 1500, Т ₃ = -100, Т ₄ = 1500	$\frac{1}{1} = 1500, T_{2} = -100, T_{3} = 1500, T_{4} = -100$	$\sum_{n=1}^{\infty} T_1 = -100, T_2 = 1500, T_3 = 1500, T_4 = 1500$	$T_{1} = 1500, T_{2} = 1500, T_{3} = -100, T_{4} = 1500$	$T_1 = 1500, T_2 = 1500, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = 1500$	$T_1 = 1500, T_2 = 0, T_3 = 1500, T_4 = 0$	$T_1 = 0, T_2 = 1500, T_3 = 0, T_4 = 1500$	

6

CONCLUSIONS

In this research 3 plate configurations were used to study the effects of applying in-plane boundary actuation. All plates showed that using in-plane boundary actuation can be used to reduce the errors \tilde{e}_{RMS1} and \tilde{e}_{RMS15} . By using an aperture of $D = 0.9 \cdot L$ these errors could be even further reduced.

For plate configuration 1 with an aperture $D = 1 \cdot L$ the error \tilde{e}_{RMS1} can be reduced by 30.3% and \tilde{e}_{RMS15} can be reduced with 0.595%. When an aperture of $D = 0.9 \cdot L$ is used the error \tilde{e}_{RMS1} can be reduced by 42.0% and \tilde{e}_{RMS15} can be reduced by 0.628%. The reduction percentages do not change when the ultimate tensile strength is changed.

For plate configuration 2 with an aperture $D = 1 \cdot L$ the error \tilde{e}_{RMS1} can be reduced by 24.1% and \tilde{e}_{RMS15} can be reduced by 1.63%. When the aperture is changed to $D = 0.9 \cdot L$, the error \tilde{e}_{RMS1} can be reduced by 66.6% and \tilde{e}_{RMS15} can be reduced by 0.883%. As with plate configuration 1, these reduction percentages do not change when the ultimate tensile strength is changed. The minimal values for the errors \tilde{e}_{RMS1} and \tilde{e}_{RMS15} that are obtained with configuration 2 are smaller than those obtained with configuration 1. Hence, configuration 2 performs better than configuration 2.

Note that 4 measurement points of configuration 2 were not used in the processing of the results. It is expected that these points do not change the final results.

For plate configuration 3 the best results were obtained for an ultimate tensile strength S = 50 MPa. When an aperture $D = 1 \cdot L$ is used, the error \tilde{e}_{RMS1} can be reduced by 36.7% and the error \tilde{e}_{RMS15} can be reduced by 6.15%. These maximum error reductions occur for different sets of pretension. For an aperture of $D = 1 \cdot L$, the error \tilde{e}_{RMS1} can be reduced with 56.3% and the error \tilde{e}_{RMS15} can be reduced with 8.37%. Again, these maximum error reductions occur for different sets of pretension.

The results for plate configuration 3 change when an ultimate tensile strength of S = 10 MPa is used. For an aperture $D = 1 \cdot L$, the error \tilde{e}_{RMS1} can be reduced by 36.7% and the error \tilde{e}_{RMS15} can be reduced by 3.37%. When an aperture $D = 1 \cdot L$ is used, the error \tilde{e}_{RMS1} can be reduced with 56.3% and the error \tilde{e}_{RMS15} can be reduced with 6.49%. Again, for both aperture sizes the maximum error reductions of \tilde{e}_{RMS1} and \tilde{e}_{RMS15} occur for different sets of pretension.

Since the best values for error \tilde{e}_{RMS15} occur in plate configuration 3 with $D = 0.9 \cdot L$, this is the best plate configuration. Using Zerodur with ultimate tensile strength S = 50 MPa will give the best overall results. If this is not possible, and S = 10 MPa is used, then still plate configuration 3 with $D = 0.9 \cdot L$ will give the best results.

The Zernike polynomial that can be imposed the best on plate configuration 3 with $D = 0.9 \cdot L$ is Z_2^{-2} . The smallest error with which this shape can be made is $\tilde{e}_{\text{RMS}} = 0.073$. This is the same for both used ultimate tensile strengths. The Zernike polynomial that can be imposed the most difficult on plate configuration 3 with $D = 0.9 \cdot L$ is Z_2^2 . The smallest error with which this shape can be made is $\tilde{e}_{\text{RMS}} = 0.340$ for S = 50 MPa and $\tilde{e}_{\text{RMS}} = 0.359$ for S = 10 MPa.

7

RECOMMENDATIONS

The first recommendation is to recalculate and include the measurement points that were left out for plate configuration 2. Even though it is expected that these points do not change the final results, to be certain that the results are correct these points should be correctly calculated as well.

Once it is made sure that all the used data is correct, the results can be processed further. For example, a comparison could be made between plate configuration 2 with not pretension applied on the one hand and configuration 2 with four out-of-plane actuators removed, but with pretension, on the other hand. If we do so, the total amount of actuators (in-plane or out-of-plane) remains the same. This allows us to make a more fair comparison between using out-of-plane and in-plane actuators. Other combinations of the out-of-plane and in-plane actuators could be studied as well. This could be done for all three plate configurations.

We saw that plate configuration 3 was similar to plate configuration 1, with extra in-plane actuators added. Configuration 3 performed best of all plate configurations. But plate configuration 2 also performed better than plate configuration 1. There is a chance that plate configuration 2 will perform even better than configuration 3 when extra in-plane actuators are added. This is why we recommend to perform calculations with plate configuration 2 with a total of 4 sets of pretension. This will result in a fair comparison with plate configuration 3.

A next step would be trying to find an optimal actuator pattern for both the in-plane and the out-of-plane actuators. The actuator pattern that is used in this study gives the plate some deformation preferences. This is especially the case for the out-of-plane actuators. It is recommended to first find an optimal out-of-plane actuator pattern. After that the optimal configuration for the in-plane actuators can be studied.

A final recommendation is to look into the effect of changing the boundary conditions. The boundary conditions influence which shapes can be imposed well on the plate. Changing the boundary conditions may improve the behaviour of the plate.

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Appendices

DETAILED RESULTS OF THE PLATE CONFIGURATIONS USED

A.1. PLATE CONFIGURATION 1



	10			Error (-)		10			
	0.3	0.3			0.2	- 0.1		0.1	
0.248	0.250	0.262	0.262	+ 0.235	0.299	0.299	0.291	0.291	Z 4 4
0.312	0.312	0.307	0.307	0.312	+ 0.299	+	0.300	0.300	Z 4 4
0.316	0.316	0.305	0.305	0.316	+ 0.285	0.285	0.290	0.290	Z ²
0.176	0.175	0.184	0.184	0.186	0.209	0.209	0.201	0.201	Z ⁻² 4
0.321	+ 0.321	0.349	0.349	0.350	0.383	0.383		0.376	Z 4
0.280	0.279	0.253	0.326	0.282	0.339	+ 0.241	0 0.341	0.243	Z ³
0.280	0.279	0.326	0.253	0.282	+ 0.241	0.339	0.243	0 0.341	z ₃ mial
0.218	0.221	0.219	0.196	0.216	0.274	0.258	0.231	0.240	z ₃ ike polyno
0.218	0.221	+ 0.196	0.219	0.216	0.258	0.274	0.240	0.231	z ₃ Zern
		0.363	0.363	0.388	+ 0.342	+ 0.342	0.347	0.347	Z ² 2
0.075	0.074	0.085	0.085	0.103	660.0	660'0	0.096	0.096	Z ⁻² 2
0.285	0.281	0.318	0.318	0.323	0.350	0.350	0.335	0.335	Z ⁰ 2
0.190	0.182	0.252	0.155	0.252	0.143	0.308	0.134	0.290	z11
0.190	0.182	0.155	0.252	0.252	0.308	0.143	0.290	+ 0.134	z_1 ¹
0.210	+ 0.201	0.262	0.262	0.297	0.281	0.281	0.279	0.279	Z ⁰
$T_1 = 0, T_2 = 0$	T ₁ = -70.71, T ₂ = -70.71	$T_1 = -70.71, T_2 = 1358$	E T ₁ = 1358, T ₂ = -70.71	1) sevice $T_1 = 7071, T_2 = 7071$	Pretens T ₁ = 7071, T ₂ = -70.71	$T_1 = -70.71, T_2 = 7071$	T ₁ = 4929, T ₂ = -70.71	$T_1 = -70.71, T_2 = 4929$	

Figure A.4: Results for the dimensionless error \hat{e}_{RMS} of the Zernike polynomials for plate configuration 1. The ulitmate tensile strength is S = 10 MPa and the aperture size is $D = 0.9 \cdot L$. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the smallest error possible.

A.2. PLATE CONFIGURATION 2

	55	L	D	45		4 Error (-)	35		m	25		2	
	0			ö		Ö				-0 -		.0	I
0.301	0.299	0.284	0.284	0.275	0.266	0.266	0.274	0.274	0.267	0.267	0.278	0.288	Z ⁴ 4
0.314	0.314	0.314	0.314	0.314	0.314	0.314	0.314	0.314	0.314	0.314	0.314	0.314	Z_4
0.216	0.216	0.228	0.228	0.211	0 0.243	0 0.243	0.232	0.233	0.243	0.243	0.210	+ 0.208	Z ² 4
0.299	0.299	0.299	0.299	O 0.299	0.299	0.299	0.299	0.299	0.299	0.299	0.299	0.299	Z ⁻² 4
0.335	0.339	0.311	0.311	0.286	0.310	0.310	0.298	0.297	0.309	0.309	0.286	0.292	Z40
0.329	0.327	0.318	0.334	0.324	0.350	+ 0.289	0.339	0.307	0.350	0.290	0.325	0.328	Z ³
0.329	0.327	0.334	0.318	0.324	0.289	0.350	0.307	0.339	0.290	0.350	0.325	0.328	Z ₃ nial
0.256	0.261	0.227	0.244	0.235	0.289	0.279	+ 0.224	0.261	0.276	0.278	0.234	0.233	z ¹ ke polynor
0.256	0.261	0.244	0.227	0.235	0.279	0.288	0.261	+ 0.224	0.278	0.276	0.234	0.233	Z ₃ Zerni
0.207	0.203	0.265	0.265	0.307	0.333	0.333	0.311	0.311	0.331	0.331	0.299	0.276	Z ² 2
0.393	0.393	0.394	0.394	0.394	0.394	0.394	0.394	0.394	0.394	0.394	0.394	0.394	Z ⁻² 2
0.294	+ 0.290	0.308	0.308	0.390	0.303	0.303	0.344	0.344	0.305	0.305	0.384	0.362	Z ⁰
0.186	0.175	0.361	0.174	0.379	0.210	0.466	0.249	0.440	0.207	0.465	0.368	0.329	z11
0.186	0.175	0.174 +	0.361	0.379	0.466	0.210	0.440	0.249	0.465	0.206	0.368	0.329	z_1^1
0.230	+ 0.216	0.330	0.330	0.595	0.406	0.406	0.491	0.491	0.410	0.410	0.575	0.495	0Z
$T_1 = 0, T_2 = 0$	T ₁ = -70.71, T ₂ = -70.71	$T_1 = -70.71, T_2 = 1358$	T ₁ = 1358, T ₂ = -70.71	$T_1 = 7071, T_2 = 7071$	$T_1 = 7071, T_2 = -70.71$	$T_1 = -70.71, T_2 = 7071$	$T_1 = 7071, T_2 = 1358$	$T_1 = 1358, T_2 = 7071$	$T_1 = 7071, T_2 = 0$	$T_1 = 0, T_2 = 7071$	$T_1 = 4929, T_2 = 4929$	$T_1 = 2072, T_2 = 2072$	•

(m/N) səulsv noisnəfərq

Figure A.5: Results for the dimensionless error \bar{e}_{RMS} of the Zernike polynomials for plate configuration 2. The ulitmate tensile strength is S = 50 MPa and the aperture size is $D = 1 \cdot L$. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the smallest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the largest error possible.

Figure A.6: Results for the dimensionless error \tilde{e}_{RMS} of the Zernike polynomials for plate configuration 2. The ulitmate tensile strength is S = 50 MPa and the aperture size is $D = 0.9 \cdot L$. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the largest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the largest error possible.

0.4		0.35			0.3	Error (-)	0.25		- 0.2		- 0.15		1
0.295	0.295	0.279	0.279	0.269	0.257	0.257	0.264	0.264	0.267	0.267	+ 0.258	0.258	Z ⁴ 4
0.312	+ 0.312	0.312	0.312	• 0.313	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	Z ⁻⁴ 4
0.215	+ 0.215	0.229	0.229	0.235	0.253	0.253	0.247	0.247	0.238	0.238	0.249	0.249	Z 4
0.316	+ 0.316	0.316	0.316	0.316	0.316	0.316	0.316	0.316	0.316	0.316	0.316	0.316	Z-2 4
0.356	0.358	0.348	0.348	0.353	0.358	0.357	0.355	0.354	+ 0.347	+ 0.347	0.352	0.351	Z ⁰ 4
0.317	0.316	0.308	0.319	0.314	0 0.347	0.272	0.323	0.294	0.324	0.291	0.338	0.275	Z ³ 3
0.317	0.316	0.319	0.308	0.314	0.272	0 0.347	0.294	0.323	0.291	0.324	0.275	0.338	z ₃ nial
0.238	0.241	0.227	0.230	0.246	0.314	0.295	+ 0.224	0.274	0.225	0.246	0.256	0.281	z ₃ ke polynor
0.238	0.241	0.230	0.227	0.246	0.295	0.314	0.274	+ 0.224	0.246	0.225	0.281	0.256	z ₃ Zerni
0.123	0.120	0.184	0.184	0.228	0.273	0.273	0.240	0.240	0.221	0.221	0.258	0.258	Z ² 2
0.387	+ 0.387	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388	Z ⁻² 2
0.233	+ 0.228	0.259	0.259	0.356	0.277	0.277	0.310	0.310	0.269	0.269	0.275	0.275	Z ⁰ 2
0.122	0.112	0.286	+ 0.111	0.294	0.136	0 0.414	0.170	0.377	0.112	0.352	0.122	0.402	z11
0.122	0.112	+ 0.111	0.286	0.294	0.414	0.136	0.377	0.170	0.352	0.112	0.402	0.122	z_1^1
0.122	0.110 +	0.215	0.215	0.418	0.299	0.299	0.348	0.347	0.258	0.258	0.291	0.291	20 2
$T_1 = 0, T_2 = 0$	$T_1 = -70.71, T_2 = -70.71$	$T_1 = -70.71, T_2 = 1358$	$T_1 = 1358, T_2 = -70.71$	$T_1 = 7071, T_2 = 7071$	$E T_1 = 7071, T_2 = -70.71$	even values T 1 = -70.71, T 2 = 7071 -	Pretensio T ₁ = 7071, T ₂ = 1358	$T_1 = 1358, T_2 = 7071$	$T_1 = 2786, T_2 = -70.71$	$T_1 = -70.71, T_2 = 2786$	$T_1 = 5643, T_2 = -70.71$	$T_1 = -70.71, T_2 = 5643$	-

Zernike polynomial

A.3. PLATE CONFIGURATION 3

	0.4							0.3		(-) ,	Erroi	- 0.25				0.7			0.15				0.1	l		
0.242	0.232	0.245	0.245	0.259	0.249	0.298	0.236	0.283	0.283	0.285	0.278	0.285	0.278	0.268	0.268	0.256	0.255	0.214	0.283	0.283	0.281	0.281	0.250	0.289	0.264	Z ⁴
0.314	0.314	0.311	0.311	0.313	0.311	0.314	0.314	0.309	0.309	0.309	0.308	0.309	0.308	0.314	0.314	0.309	0.312	0.314	0.308	0.308	0.307	0.307	0.314	0.314	0.314	Z ⁻⁴ 4
0.299	0.299	0.280	0.280	0.298	0.295	0.299	0.299	0.285	0.285	0.285	0.266	0.285	0.266	0.299	0.299	0.299	0.299	0.299	0.268	0.268	0.285	0.285	0.299	0.299	0.299	Z_4^2
0.185	0.166	0.174	0.174	0.180	0.181	0.241	0.184	0.222	0.222	0.227	0.196	0.227	0.196	0.225	0.225	0.230	0.221	0.222	0.207	0.207	0.224	0.224	0.224	0.244	0.239	Z_4^{-2}
0.305	0.289	0.335	0.335	0.421	0.412	0.340	0.340	0.357	0.358	0.361	0.352	0.361	0.352	0.331	0.333	0.322	0.324	0.328	0.360	0.360	0.352	0.352	0.353	0.339	0.352	Z ⁰ 4
0.282	0.274	0.262	0.315	0.298	0.297	0.333	0.285	0.335	0.276	0.275	0.341	0.336	0.257	0.322	0.322	0.308	0.310	0.316	0.340	0.258	0.273	0.335	0.291	0.324	0.300	Z ³
0.282	0.274	0.315	0.262	0.296	0.300	0.333	0.285	0.276	0.335	0.336	0.257	0.275	0.341	0.322	0.322	0.300	0.328	0.317	0.258	0.340	0.335	0.273	0.291	0.323	0.300	z ₃ mial
0.204	0.204	0.196	0.188	0.220	0.222	0.245	0.201	0.245	0.278	0.275	0.271	0.249	0.241	0.314	0.298	0.269	0.277	0.296	0.269	0.242	0.278	0.243	0.226	0.258	0.242	z ₃ te polynoi
0.204	0.204	0.188	0.196	0.222	0.220	0.245	0.201	0.278	0.245	0.249	0.241	0.275	0.271	0.298	0.314	0.271	0.270	0.256	0.242	0.269	0.243	0.278	0.226	0.254	0.242	Z ₃ Zernik
0.394	0.394	0.376	0.376	0.387	0.386	0.394	0.394	0.370	0.370	0.370	0.367	0.370	0.367	0.394	0.394	0.392	0.393	0.394	0.367	0.367	0.370	0.370	0.394	0.394	0.394	Z_2^2
0.097	0.094	0.103	0.103	0.103	0.104	0.143	0.115	0.133	0.133	0.129	0.110	0.129	0.110	0.130	0.131	0.220	0.189	0.153	0.112	0.112	0.136	0.136	0.125	0.136	0.137	Z ⁻² 2
0.297	0.303	0.306	0.306	0.292	0.287	0.300	0.307	0.330	0.330	0.333	0.324	0.333	0.324	0.280	0.280	0.439	0.41	0.416	0.333	0.333	0.331	0.331	0.319	0.291	0.326	Z ⁰ 2
0.234	0.229	0.283	0.207	0.224	0.225	0.188	0.278	0.179	0.290	0.288	0.212	0.179	0.318	0.203	0.209	0.237	0.216	0.217	0.204	0.316	0.290	0.180	0.272	0.190	0.259	z ¹
0.234	0.229	0.207	0.283	0.225	0.225	0.188	0.278	0.290	0.179	0.179	0.318	0.288	0.212	0.208	0.203	0.247	0.239	0.228	0.316	0.204	0.180	0.290	0.272	0.190	0.259	z_1^1
0.300	0.293	0.340	0.340	0.280	0.275	0.228	0.369	0.325	0.325	0.324	0.349	0.324	0.349	0.244	0.244	0.250	0.246	0.254	0.348	0.348	0.328	0.328	0.357	0.237	0.334	0 Z
$T_1 = 0, T_2 = 0, T_3 = 0, T_4 = 0$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = -100$	$T_1 = 1500, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = 7000, T_3 = -100, T_4 = 7000$	$T_1 = 7000, T_2 = -100, T_3 = 7000, T_4 = -100$	$T_1 = 7000, T_2 = 7000, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 7000, T_4 = 7000$	$\prod_{1} = -100, T_2 = 0, T_3 = 7000, T_4 = 7000$	\overrightarrow{z} T ₁ = 7000, T ₂ = -100, T ₃ = -100, T ₄ = -100	$\frac{1}{100}$ T ₁ = 7000, T ₂ = 7000, T ₃ = -100, T ₄ = 0	$\int_{0}^{\infty} T_{1} = -100, T_{2} = -100, T_{3} = 7000, T_{4} = -100$	$\overrightarrow{1}_{1}$ T ₁ = -100, T ₂ = 7000, T ₃ = -100, T ₄ = -100	Δ T ₁ = -100, T ₂ = -100, T ₃ = -100, T ₄ = 7000	$T_1 = 0, T_2 = -100, T_3 = -100, T_4 = 7000$	$T_1 = -100, T_2 = 7000, T_3 = 0, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 0, T_4 = 7000$	$T_1 = 7000, T_2 = 0, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = 7000, T_4 = 0$	$T_1 = -100, T_2 = 7000, T_3 = 7000, T_4 = -100$	$T_1 = 7000, T_2 = -100, T_3 = -100, T_4 = 7000$	$T_1 = 7000, T_2 = 1500, T_3 = 7000, T_4 = 1500$	$T_1 = -100, T_2 = 7000, T_3 = -100, T_4 = 1500$	$T_1 = 7000, T_2 = 7000, T_3 = 7000, T_4 = 7000$	

Figure A.10: Results for the dimensionless error \tilde{e}_{RMS} of the Zernike polynomials for plate configuration 3. The ulitmate tensile strength is S = 50 MPa and the aperture size is $D = 0.9 \cdot L$. A plus sign indicates that the considered Zernike polynomial is imposed on the plate with the largest error possible. A circle indicates that the Zernike polynomial is imposed on the plate with the largest error possible.

			0.35					Error (-)					0.2										
0.248	0.233	0.255	0.255	0.273	0.260	0.3110	0.233	0.298	0.298	0.299	0.299	0.299	0.299	0.278	0.278	0.238	0.238	0.192	0.235	0.240	0.304	0.304	Z ⁴ 4
0.312	0.312	0.306	0.306	0.306	0.310	0.311	0.312	0.300	0.300	0.301	0.297	0.301	0.297	0.311	0.311	0.308	0.312	0.312	0.312	0.308	0.309	0.309	Z_4^{-4}
0.316	0.316	0.302	0.302	0.316	0.315	0.316	0.316	0.301	0.301	0.302	0.283	0.302	0.283	0.316	0.316	0.316	0.316	0.316	0.316	0.315	0.311	0.311	Z_4^2
0.176	0.159	0.173	0.173	0.174	0.174	0.233	0.179	0.221	0.221	0.225	0.201	0.225	0.201	0.229	0.229	0.242	0.246	0.226	0.186	0.244	0.234	0.234	Z_4^{-2}
0.321	0.310	0.346	0.346	0.450	0.446	0.371	0.346	0.381	0.381	0.385	0.376	0.385	0.376	0.358	0.360	0.370	0.368	0.369	0.350	0.367	0.384	0.384	Z ⁰ ₄
0.280	0.266	0.247	0.326	0.299	0.305	0.338	0.282	0.340	0.262	0.262	0.338	0.340	0.238	0.326	0.321	0.305	0.325	0.327	0.282	0.331	0.282	0.343	Z ³
0.280	0.266	0.326	0.247	0.303	0.304	0.338	0.282	0.262	0.340	0.340	0.238	0.262	0.338	0.321	0.326	0.304	0.340	0.331	0.282	0.327	0.343	0.282	z ₃ mial
0.218	0.229	0.218	0.192	0.220	0.222	0.226	0.215	0.235	0.293	0.288	0.281	0.238	0.260	0.309	0.295	0.295	0.312	0.338	0.216	0.279	0.247	0.230	z ₃ ke polyno
0.218	0.229	0.192	0.218	0.222	0.223	0.226	0.215	0.293	0.235	0.238	0.260	0.288	0.281	0.295	0.309	0.288	0.310	0.279	0.216	0.338	0.230	0.247	z ₃ Zerni
0.388	0.388	0.359	0.359	0.377	0.373	0.388	0.388	0.349	0.349	0.350	0.340	0.350	0.340	0.388	0.388	0.387	0.388	0.387	0.388	0.387	0.359	0.359	Z_2^2
0.075	+ 0.073	0.087	0.087	0.083	0.083	0.109	0.105	0.111	0.111	0.106	0.101	0.106	0.101	0.106	0.106	0.209	0.176	0.149	0.103	0.176	0.108	0.108	Z ⁻² 2
0.285	0.297	0.325	0.325	0.277	0.272	0.234	0.325	0.323	0.323	0.324	0.348	0.324	0.348	0.229	0.229	0.401	0.412	0.414	0.323	0.411	0.313	0.313	Z ⁰ 2
0.190	0.184	0.258	0.153	0.175	0.179	0.128	0.253	0.116 +	0.269	0.267	0.150	0.116 +	0.311	0.145	0.150	0.190	0.162	0.161	0.252	0.185	0.240	0.117	Z ¹
0.190	0.184	0.153	0.258	0.179	0.178	0.128	0.253	0.269	0.116	0.116	0.311	0.267	0.150	0.150	0.145	0.201	0.192	0.184	0.252	0.161	0.117	0.240	Z ⁻¹
0.210	0.201	0.268	0.268	0.187	0.183	0.122	0.298	0.252	0.252	0.251	0.282	0.251	0.282	0.144	0.144	0.138	0.144	0.148	0.297	0.144	0.230	0.230	Z ⁰ 0
$T_1 = 0, T_2 = 0, T_3 = 0, T_4 = 0$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = -100$	$T_1 = 1500, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = 7000, T_3 = -100, T_4 = 7000$	$T_1 = 7000, T_2 = -100, T_3 = 7000, T_4 = -100$	$T_1 = 7000, T_2 = 7000, T_3 = -100, T_4 = -100$	\widehat{E} T ₁ = -100, T ₂ = -100, T ₃ = 7000, T ₄ = 7000	\overrightarrow{z} T ₁ = -100, T ₂ = 0, T ₃ = 7000, T ₄ = 7000	$\frac{3}{5}$ T ₁ = 7000, T ₂ = -100, T ₃ = -100, T ₄ = -100	$T_1 = 7000, T_2 = 7000, T_3 = -100, T_4 = 0$	$\begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$ T ₁ = -100, T ₂ = -100, T ₃ = 7000, T ₄ = -100	$T_1 = -100, T_2 = 7000, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 7000$	$T_1 = 0, T_2 = -100, T_3 = -100, T_4 = 7000$	$T_1 = -100, T_2 = 7000, T_3 = 0, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 0, T_4 = 7000$	$T_1 = 7000, T_2 = 0, T_3 = 7000, T_4 = 0$	$T_1 = 0, T_{2} = 7000, T_{3} = -100, T_{4} = -100$	$T_1 = -100, T_2 = 7000, T_3 = 7000, T_4 = 7000$	$T_1 = 7000, T_2 = 7000, T_3 = -100, T_4 = 7000$	

0.4			0.3					0.2		0.15		1			
0.242	+ 0.232	0.245	0.245	0.259	0.249	0.284	0.234	0.280	0.280	0.264	0.264	0.249	0.249	0.261	Z ⁴
0.314	0.314	0.311	0.311	0.313	0.311	0.314	0.314	0.313	0.313	0.314	0.314	+ 0.311	+ 0.311	0.314	Z ⁻⁴ 4
0.299	0.299	0.280	0.280	0.298	0.295	0.299	0.299	0.293	0.293	0.299	0.299	0.281	0.281	0.299	Z ² 4
0.185	0.166	0.174	0.174	0.180	0.181	0.228	0.173	• 0.229	0.229	0.213	0.213	0.182	0.182	0.227	Z ⁻² 4
0.305	+ 0.289	0.335	0.335	0.421	0.412	0.333	0.318	0.347	0.347	0.319	0.319	0.338	0.338	0.343	Z ⁰ 4
0.282	0.274	0.262	0.315	0.298	0.297	0.318	0.283	0.285	0.331	0.305	0.303	0.318	0.263	0.297	Z ³
0.282	0.274	0.315	0.262	0.296	0.300	0.318	0.283	0.331	0.285	0.303	0.305	0.263	0.318	0.297	z ₃ mial
0.204	0.204	0.196	0.188	0.220	0.222	0.230	0.192	0.231	0.231	0.226	0.229	0.193	0.200	0.229	z ₃ te polynoi
0.204	0.204	+ 0.188	0.196	0.222	0.220	0.230	0.192	0.231	0.231	0.229	0.226	0.200	0.193	0.229	z ₃ Zernik
0.394	0.394	+ 0.376	+ 0.376	0.387	0.386	0.394	0.394	0.382	0.382	0.394	0.394	0.377	0.377	0.394	Z ² 2
0.097	+ 0.094	0.103	0.103	0.103	0.104	0.124	0.105	0.125	0.125	0.113	0.113	0.104	0.104	0.124	Z ⁻² 2
0.297	0.303	0.306	0.306	0.292	0.287	0.289	0.306	0.317	0.317	0.283	0.283	0.307	0.307	0.313	Z ⁰ 2
0.234	0.229	0.283	0.207	0.224	0.225	0.203	0.268	0.262	0.195	0.210	0.210	0.206	0.282	0.252	z11
0.234	0.229	0.207	0.283	0.225	0.225	0.203	0.268	+ 0.195	0.262	0.210	0.210	0.282	0.206	0.252	Z ⁻¹
0.300	0.293	0.340	0.340	0.280	0.275	+ 0.261	0.347	0.310	0.310	0.272	0.272	0.339	0.339	0.325	0Z
$T_{1} = 0, T_{2} = 0, T_{3} = 0, T_{4} = 0$	T ₁ =-100,T ₂ =-100,T ₃ =-100,T ₄ =-100	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = -100$	$T_1 = 1500, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = -100, T_4 = -100$	$\frac{1}{2} T_1 = -100, T_2 = 1500, T_3 = -100, T_4 = 1500$	ا 1500, T ₂ =-100, T ₃ = 1500, T ₄ =-100	0 6 7 1 = -100, T ₂ = 1500, T ₃ = 1500, T ₄ = 1500	а. Т ₁ = 1500, Т ₂ = 1500, Т ₃ = -100, Т ₄ = 1500	$T_1 = -100, T_2 = 0, T_3 = -100, T_4 = 1500$	$T_1 = -100, T_2 = 1500, T_3 = -100, T_4 = 0$	$T_1 = 1500, T_2 = -100, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = 1500, T_4 = -100$	$T_1 = 1500, T_2 = 1500, T_3 = 1500, T_4 = 1500$	

	4 0	t,	0.35		0.3	(-)	0.25 Error		0.2		0.15		0.1	
														I
0.248	0.233	0.255	0.255	0.273	0.260	0.298	0.234	0.293	0.293	0.276	0.276	0.240	0.294	Z ⁴ 4
0.312	0.312	0.306	0.306	0.306	0.310	0.312	0.312	0.310	0.310	0.308	0.308	0.312	0.312	Z ⁻⁴ 4
0.316	0.316	0.302	0.302	0.316	0.315	0.316	0.316	0.312	0.312	0.308	0.308	0.316	0.316	Z ² 4
0.176	+ 0.159	0.173	0.173	0.174	0.174	0.215	0.168	0.217	0.217	0.201	0.201	0.180	0.215	Z ⁻² 4
0.321	+ 0.310	0.346	0.346	0.450	0.446	0.354	0.325	0.365	0.365	0.353	0.353	0.334	0.354	Z ⁰ 4
0.280	0.266	+ 0.247	0.326	0.299	0.305	0.323	0.279	0.280	0.337	0.330	0.265	0.281	0.319	Z ³
0.280	0.266	0.326	0.247	0.303	0.304	0.323	0.279	0.337	0.280	0.265	0.330	0.281	0.319	z ₃ mial
0.218	0.229	0.218	0.192	0.220	0.222	0.215	0.207	0.229	0.213	0.206	0.235	0.209	0.216	z ₃ ke polynoi
0.218	0.229	+ 0.192	0.218	0.222	0.223	0.215	0.207	0.213	0.229	0.235	0.206	0.209	0.216	Z ₃ Zernik
0.388	0.388	+ 0.359	0.359	0.377	0.373	0.388	0.388	0.369	0.369	0.365	0.365	0.388	0.388	Z ² 2
0.075	+ 0.073	0.087	0.087	0.083	0.083	0.092	0.089	0 0.095	0.095	0.092	0.092	0.087	0.093	Z ⁻² 2
0.285	0.297	0.325	0.325	0.277	0.272	+ 0.235	0.317	0.293	0.293	0.302	0.302	0.311	0.242	Z ⁰ 2
0.190	0.184	0.258	0.153	0.175	0.179	0.148	0.238	0.227	+ 0.137	0.142	0.240	0.235	0.158	z11
0.190	0.184	0.153	0.258	0.179	0.178	0.148	0.238	0.137	0.227	0.240	0.142	0.235	0.158	Z ₁ ¹
0.210	0.201	0.268	0.268	0.187	0.183	0.162	0.270	0.229	0.229	0.245	0.245	0.268	0.174	0Z
$T_1 = 0, T_2 = 0, T_3 = 0, T_4 = 0$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = -100$	$T_1 = 1500, T_2 = -100, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = -100, T_4 = 0$	$T_1 = -100, T_2 = 0, T_3 = -100, T_4 = -100$	V/T (3 T ₁ = -100,T ₂ = 1500,T ₃ = -100,T ₄ = 1500	$\frac{3}{10}$	$\frac{100}{1}$ T ₁ = -100, T ₂ = 1500, T ₃ = 1500, T ₄ = 1500	$T_1 = 1500, T_2 = 1500, T_3 = -100, T_4 = 1500$	$T_1 = 1500, T_2 = 1500, T_3 = -100, T_4 = -100$	$T_1 = -100, T_2 = -100, T_3 = 1500, T_4 = 1500$	$T_1 = 1500, T_2 = 0, T_3 = 1500, T_4 = 0$	$T_1 = 0, T_2 = 1500, T_3 = 0, T_4 = 1500$	