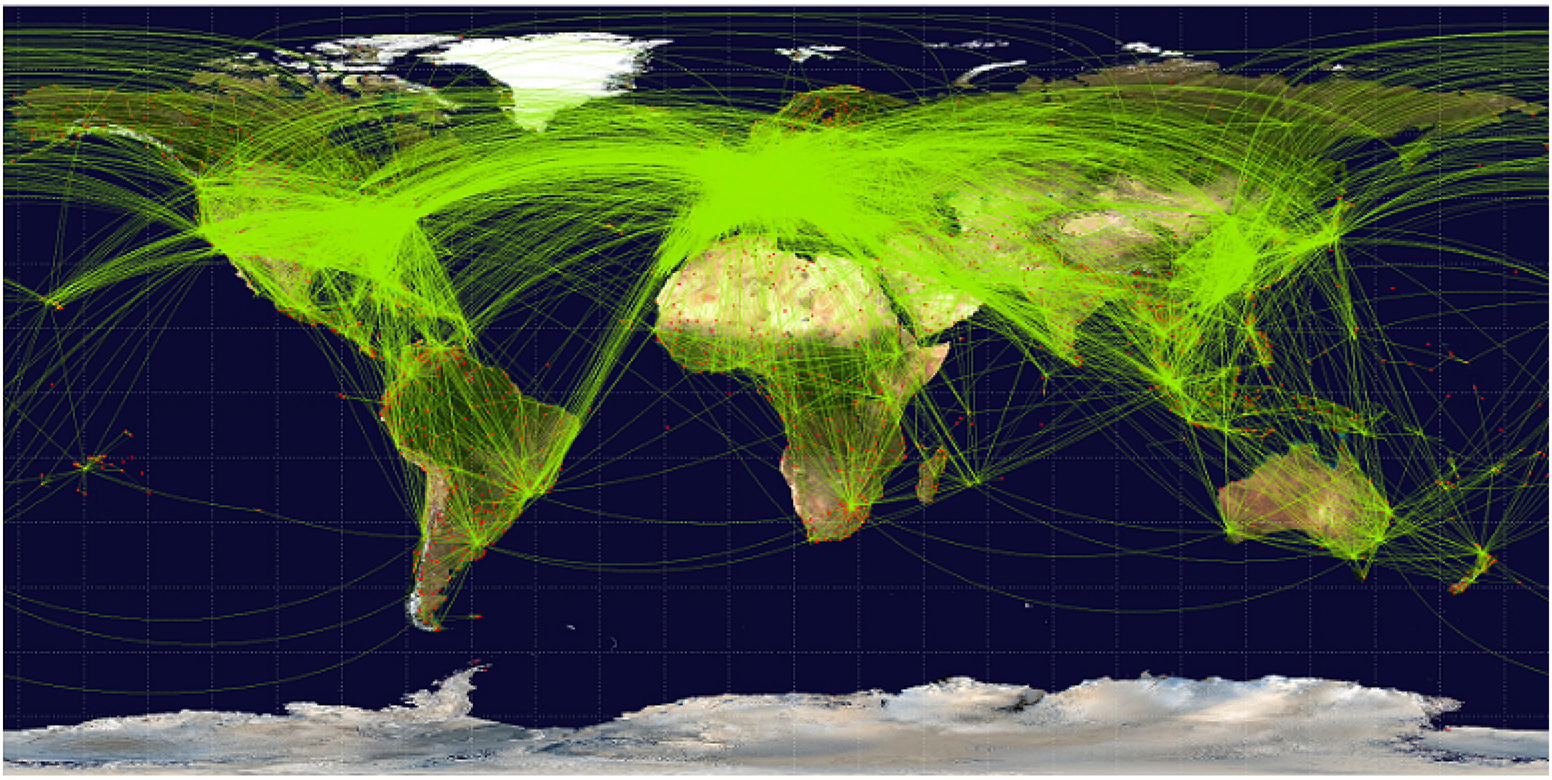
Master of Science Thesis

Airline and Alliance Networks

Topology and Robustness from a Complex Network Approach

S.J. Wijdeveld BSc

December 4th 2015



Faculty of Aerospace Engineering · Delft University of Technology

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Master of Science Thesis

For obtaining the degree of Master of Science in Aerospace Engineering at Delft University of Technology

S.J. Wijdeveld BSc

December 4th 2015

Faculty of Aerospace Engineering · Delft University of Technology

Delft University Of Technology
Department Of
Air Transport and Operations

The undersigned hereby certify that they have read and recommend to the Faculty of Aerospace Engineering for acceptance a thesis entitled **"Airline and Alliance Networks"** by **S.J. Wijdeveld BSc** in partial fulfillment of the requirements for the degree of **Master of Science**.

Dated: December 4$^{\text{th}}$ 2015

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Summary

Increased market deregulation and the accompanied rise of LCCs over the last decades has put profits for many of the old FSCs under pressure forcing them to merge and form alliances. This sparked the research into finding the most efficient structure for a single airline network in terms of profit and passenger mobility. Along with this, the social and economical dependancy on air transport grew and hence the need to assess the robustness of the network rose.

Complex network theory offers a way to assess the efficiency of the networks using amongst other the degree (distribution), the betweenness, the average path length and the clustering coefficient. The main focus of current literature is on the analysis of global and regional airport networks, with limited coverage of separate airline networks and codeshare and alliance formation. Furthermore current research uses very standardized methods of assessing robustness and more realistic assumptions are needed. The first aim of this study is to get an insight into the differences in structure of FSCs and LCCs by analyze the topology and robustness of 17 European separate airline networks, using complex network theory. The second aim is to investigate the influence of codeshare and alliance formation on the topology and robustness of ATNs. Finally the third aim is to improve the methods used to analyze the robustness of ATNs.

First the topology of the seperate airline networks of both FSCs and LCCs is analyzed in order to distinguish between the business solutions used by the airlines (PPs and HSs). Additionaly the influence of using codeshares and an alliances on the topology of the airline networks is investigated. This is performed by using complex network indicators to compare the seperate and combined (both codeshare and alliance) network layouts. The analysis confirms literature regarding FSCs, which turn out to use SFN associated with HS. LCCs however, are found not found to have RN associated with PP as suggested in literature, but a SFN with multiple interconnected hubs. The most important difference found between FSCs and LCCs is that LCCs tend to focus on diversity of destinations over frequency, whilst FSCs tend to focus on frequency over diversity. Combining networks into codeshare networks or the Skyteam alliance, increases the diversity of the network, the size and number of hubs and brings the behaviour closer to LCCs, however still with a focus on frequency over diversity.

After this the synthetic static robustness of the seperate airline, codeshare and the Skyteam alliance networks is investigated in order to distinguish between the robustness behaviour of both FSCs and LCCs. The link between the complex network indicators and the synthetic static robustness of the ATN is also explored. This is performed by simulating error and attack on the separate airline networks and the codeshare and Skyteam alliance networks. Error is based on the random removal of airports from the network, while attack is based on the consecutive removal of nodes based on the heights of the degree, seat strength and (weighted) betweenness of the airports. The analysis confirms literature regarding FSCs, which shows low robustness against attack and high robustness against error. LCCs again show similar behaviour as the FSCs, contradicting literature, but confirming the results from Chapter 3. The shape of the curve of the cumulative degree distribution can be directly linked to the robustness independant of the size of the network. The higher the amount of hubs (with relative high degree), the higher robustness against attack. The robustness against error is much higher and similar for all networks. Combining networks into codeshare netrworks or the Skyteam alliance, will thus increase the robustness against attack.

Finally the robustness analysis of ATNs is improved by introducing new methods of simulating more realistic error and attack scenarios. The link between the realistic robustness analysis and the synthetic robustness analysis is also investigated Three different phenomena are simulated: weather, strikes and volcano eruptions. Weather and volcano eruptions are simulated using the introduced geographic attack. Geographic attack is based on starting at an initiation airport and removing the other airports using geographic radial spreading. Strikes are simulated using the geographic degree, which groups the airport into FIR. The analysis puts the synthetic robustness in perspective. It shows that not only the number of hubs is important in order to improve the robustness of an ATN, but also the geographic spreading of the hubs.

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List of Abbreviations

ATN	Air Transport Network
FIR	Flight Information Region
FSC	FlagShip Carrier or Full-Service Carrier
HS	Hub-and-Spoke Network
LCC	Low-Cost Carrier
PP	Point-to-Point Network
RN	Random Network
SFN	Scale-Free Network
SWN	Small-World Network

List of Symbols

A	Adjacency matrix
b_i	Betweenness of node i
b^s	Betweenness of node (seats)
c_i	Local clustering coefficient
C	(Average) clustering coefficient
C^f	(Average) weighted clustering coefficient (flights)
C^s	(Average) weighted clustering coefficient (seats)
$c_{\mathbf{GEO}_i}$	Local geographic clustering coefficient
c_i^w	Local weighted clustering coefficient
C^w	(Average) weighted clustering coefficient
d_{ij}	Shortest path or geodesic between node i and j through the network
$d_{\mathbf{GEO}_{ij}}$	Geographic distance between node i and j
d	Radius from the origin
E	Efficiency of the network
f	Fraction of airports removed
γ	Degree exponent
γ^f	Strength exponent (flights)
γ^s	Strength exponent (seats)
γ^{in}	Ingoing degree exponent
γ^{out}	Outgoing degree exponent
$\langle k \rangle$	Average undirected (node) degree
K	Total number of directed links
k_i	Undirected degree of node i
k_i^{in}	Ingoing degree of node i
k_i^{out}	Outgoing degree of node i
$k_{\mathbf{GEO}_k}$	Geographic (node) degree
L	Average shortest path length
l_{ij}	Link between node i and j
L^w	Average weighted shortest path length
λ	Algebraic connectivity (eigenvalue)
N	Total number of nodes

$n_{jk}(i)$	Number of shortest paths connecting j and k running through i
n_{jk}	Total number of shortest paths connecting j and k
$P(k)$	Degree distribution
$R(s)$	Undirected strength distribution
s_i	Undirected strength of node i
$\langle s^f \rangle$	Average undirected (node) strength (flights)
$\langle s^s \rangle$	Undirected (node) strength (seats)
s^s	Undirected (node) strength (seats)
s_i^{in}	Ingoing strength of node i
s_i^{out}	Outgoing strength of node i
w_{ij}	Weighth of a link between node i and j

Part I

Introduction

Chapter 1

Introduction

Increased market deregulation and the accompanied rise of LCCs over the last decades has put profits for many of the old FSCs under pressure forcing them to merge and form alliances. This sparked the research into finding the most efficient structure for a single airline network in terms of profit and passenger mobility. Along with this, the social and economical dependancy on air transport grew and hence the need to assess the robustness of the network rose.

Complex network theory offers a framework to model and analyze both efficiency and robustness of ATNs. Although the theory is rather young, the application of complex networks has grown significantly over the last decades. It offers a great variety of quantities to analyze the topology and robustness of many real networks (Boccaletti et al., 2006). The first aim of this study is get an insight into the differences in structure of FSCs and LCCs by analyze the topology and robustness of 17 European separate airline networks, using complex network theory. The second aim is to investigate the influence of codeshare and alliance formation on the topology and robustness of ATNs. Finally the third aim is to improve the methods used to analyze the robustness of ATNs.

The thesis is structured as follows. Chapter 2 introdoces the area of research. Reviewing the history of the air transport industry and presenting the network types used by airlines and the general levels of study in current research, highlighting the relevance of this thesis. Chapter 3 uses complex network theory to analyze the topology of seperate airline networks of both FSCs and LCCs in order to distinguish between the business solutions used by the airlines (PPs and HSs). Furthermore the influence of using codeshares and an alliances on the topology of the airline networks is explored. Chapter 4 analyzes the synthetic static robustness of the seperate airline, codeshare and the Skyteam alliance networks. It distinguished between the robustness of both FSCs and LCCs. Additionaly the link between the complex network indicators from Chapter 3 and the synthetic static robustness of the ATN is investigated. Chapter 4 improves the robustness analysis of ATNs by introducing new methods of simulating more realistic error and attack scenarios, giving perspective to the synthetic robustness scenario. Finally in chapter 6 and 7 the conclusions and recommendations of the thesis are presented.

Chapter 2

Literature Study: Air Transport Network Topology and Robustness

2.1 Abstract

This chapter serves three purposes. First of all it introduces the area of research. Reviewing the history of the air transport industry and presenting the network types used by airlines and the general levels of study in current research. Secondly, using the levels of research as a framework, current literature is reviewed. Thirdly it serves as the theoretic framework. Describing the indicators and methods used to analyze the topology and robustness of air transport networks. The study shows that the main reason for the analysis of the topology and robustness of air transport networks has been the increased competition due to deregulation and the increased dependance on the network. Complex network theory offers a way to assess the efficiency of the networks using amongst other the degree (distribution), the betweenness, the average path length and the clustering coefficient. The main focus of current literature is on the analysis of global and regional airport networks and future emphasis should be on the analysis of alliance and airline networks. Furthermore additional research should be conducted into including more realistic assumptions into the analyses.

2.2 Introduction

The air transport network (ATN) has become one of the fundamental elements of our modern societies and economies, guaranteeing a high level of mobility and contributing to our everyday welfare. It has added to globalization and the formation of a world city network (Taylor et al., 2013), in which the travel between the cities is mainly operated by airlines (Derudder and Witlox, 2008). The increasing dependancy on this network makes it important to study its ability to cope with disruptions (robustness) such as intentional (terrorist) attacks and events as the recent eruption of the Eyjafjallajökull on Iceland. Due to this eruption air travel throughout most of Europe was suspended for several days highlighting the vulnerability of the European ATN (Wilkinson et al., 2011). This and other events have raised research into proactively detecting critical elements in ATNs (Lordan, Simo, and Gonzalez-Prieto, 2014).

Single airline networks experience simular robustness considerations, however there is more to consider. Recovering from the deepest worldwide economic recession since the 1930s, overall demand is picking up at the cost of lower fare prices (Pearce, 2012). Furthermore airlines have to deal with a lack of competition in the aircraft production chain and an ever growing competition on the market due to deregulation. Particularly in recent years the rise of low-cost carriers has put profits under pressure, forcing flagship carriers to form alliances or low-cost offshoots to further decrease operating costs (Morrell, 2005). Another way for airlines to cope with these developments is organizing their route network in a more cost effective way (H. Liu et al., 2011).

Complex network theory offers a framework to model and analyze the performance and robustness of the route network of an airline. Although the theory is rather young, the application of complex networks has grown significantly over the last decades. It offers a great variety of quantities to analyze the topology and robustness of many real networks (Boccaletti et al., 2006) and has been used for transportation networks from the Internet (Pastor-Satorras et al., 2001) and the neural network (Bullmore and Sporns, 2009) up to railways (Seaton and Hackett, 2004) and the power grid (Petreska et al., 2010). The theory has also been applied to analyze the topology ATNs on a high level (Bagler, 2008) and on an airline level (Han et al., 2009) and has been used to gain insight into the robustness of such networks (Wilkinson et al., 2011). Combining this model with an optimization algorithm gives airlines an ability to analyze their network not only for performance (i.e. operating cost) but also for robustness, for example by centralising flows in the network using a hub-and-spoke network or by including redundancy by joining an alliance.

The aim of this literature study is to provide a theoretical basis for this thesis treating the topology and robustness of ATNs. Furthermore the study provides an introduction into the area of research.

The study is structured as follows. Section 2.3 describes the main aspects of the air transport industry; the different network types and ends with the multi-layer build up of ATNs. Section 2.4 and 2.5 serve as theoretical basis for this thesis. Section 2.4 covers a review of the study of topology of ATNs. Starting with a historic survey of the field of study, introducing complex network theory and its basic quantities and ending with a thorough review of the most recent ATN topology analyses. Section 2.5 correspondingly reviews the study of robustness of ATNs. Section **??** resumes and discusses the main findings of the literature review and present some future lines of research. Finally in section 2.6 the most important conclusions are listed, highlighting the relevance of this thesis.

2.3 Air Transport Industry

This section describes the main aspects of the air transport industry offering a background to the literature study. First a short historic review of the industry is presented, followed by the different network types connected to the different airline concepts. Finally the air transport network (ATN) levels are presented, which is used as a framework for the literature study.

2.3.1 Air Transport Short History

The airline industry came to existence in the late 1910s during a time of regulation and subsidy (Cook, 1996). The first European airlines favoured comfort of passengers over speed and efficiency, offering spacious and luxurious cabins. In contrary, the first U.S. airlines initially focussed on mail services, sporadically offering low comfort passenger services. This changed with the introduction of DC-2 and later DC-3 in the 1930s, enabling U.S. airlines to offer reliable passenger service with an acceptable level of comfort, increasing worldwide traffic volume. At this point the industry was heavily subsidised and regulated, consisting of a checkerboard of individual protected companies in protected markets and airspaces. Even the aircraft were often produced domestically. This remained unchanged until the Chicago Convention in 1944 (ICAO, 1944), where the legal frame work was set for post-war international civil aviation and the first five 'freedoms of the air' (ICAO, 2004) were introduced. The five freedoms were not fully included in the convention, however freedom of flight through foreign airspace was included and signified a considerable increase in freedom for airlines. In practice the inability to include the five freedoms into the Chicago Convention resulted in many seperate bilateral agreements to regulate international flight.

After the second world war the industry continued to experience a steady grow in traffic volume. In the 1970s however, the heavy regulations and limited entrance possibilities for new carriers, resulting in higher fares in the U.S, raised a call for change (Cook, 1996). This started with the liberalization of the U.S. domestic market in 1979, followed by the liberalization of the European market, which was not completed until 1997 (Lordan, Sallan, and Simo, 2014). Recently further deregulation has taken form with the Open Skies Agreements (U.S. General Services Administration, consulted 2015). The first open skies agreement, signed in 2007, permits unrestricted flights from European and American airlines to any destination within the E.U. and the U.S, including all 'freedoms of the air'. Simular open skies agreements have been signed in the year following between the U.S. and Australia, the U.S. and Switzerland and the U.S. and Japan. As a results of these deregulations new airlines emerged, lowering ticket prices and pressuring profits, forcing many of the older airlines to merge or form alliances. Alliances were formed on such a scale that in 2012 the three major alliances (Star Alliance, SkyTeam and Oneworld) accounted for 60% of the worlds available seat-kilometers for total scheduled passengers (ICAO, 2012). Furthermore many airlines started using hub-and-spoke networks to consolidate air traffic to central airports and lower costs even more (Anderson et al., 2005), leading to a new field of flight and schedule coordination within and between airlines. Contra to this, many of the new airlines, having low marginal cost per passenger, continued to operate point-to-point networks (Williams, 2001).

2.3.2 ATN Types

Deregulation in the air transport market resulted in two different business models for scheduled airlines: the Low-Cost Carrier (LCC) and the FlagShip Carrier or Full-Service Carrier (FSC).

LCCs are carriers founded after the deregulation of the air transport market, seeking a competitive advantage based on a business model of low-cost and basic service with additional payed services. Part of the strategy of these airlines is using a single fleet, reducing the maintenance costs and operating at secondary airports, reducing landing and parking fees. LCCs operate a point-to-point network, without intercontinental flight, based on the principle that every airport is connected by a direct flight and no transfers are needed. The absence of transfers simplifies scheduling adding to the low-cost nature of the carriers. Nevertheless it should be noted that in recent years LCCs increasingly choose to operate at primary airport.

FSCs, as the name already suggest, are the former national flag carriers, founded during the period the industry was still heavily regulated and subsidised. These carriers seek a competitive advantage based on a business model of full-service, vertical pricing (i.e. economy and business class) and a great variety of destinations, both continental and intercontinental. FSCs operate a hub-and-spoke network based on the principle that passenger flows are consolidated at hub airports and all destinations are rechable through the hub(s). This implies passenger transfers at the hub airports and thus costly schedule and fleet coordination. After the deregulation the FSCs formed alliances to reduce cost with the same or better destination variety. The formation of these alliances however, did create a need for cross airline schedule coordination further increasing complexity and thus cost of schedule and fleet coordination.

It is important to note that these two business models refer to scheduled airlines, there are other business models for unscheduled airlines, such as the charter carrier or holiday carrier. Due to the irregular nature of the networks of these unscheduled airlines, it is however not possible to analyze the network in an effective way or link the business model to a single network type.

2.3.3 ATN Levels

The global ATN can be represented as a network composed of interconnected hub-and-spoke and point-to-point networks operated by different airlines in constant dynamic interaction with eachother. Within this global ATN the single airline networks can be organized into separate layers of networks (Cardillo et al., 2013). This layering can be used to categorize the literature on ATNs. Lordan, Simo, and Gonzalez-Prieto (2014) recognizes three levels of study in literature : the global / regional airport network (L1), the (airline) alliance network (L2) and the seperate single airline network (L3) (figure 2.1)

The first level (L1) is the global / regional airport level and reviews the industry and general network developments. Instead of airport networks, also city networks are reviewed, combining airports close to one city. Due to the networks size and its geopolitical contraints, the network is reviewed both globally (Guimera et al., 2005) and regionally (Bagler, 2008; Guida and Maria, 2007). The airport network is in reality a multilayer

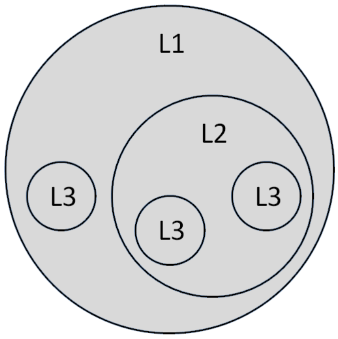


Figure 2.1: Schematic overview of the layering of the study levels (Lordan, Sallan, and Simo, 2014)

network of both L2 and L3 (see figure 2.1), but does not need to be represented in that way necessarily. This level of research also includes the detection of critical locations in the airport network in order to prevent network collapses such as with the eruption of the Eyjafjallajökull on Iceland in 2010 (Wilkinson et al., 2011).

The second level (L2) is the airline alliance level and studies the properties of alliances and the effect of alliance formation. Formation of alliances can have substantial benefits, not only regarding operating cost, but also regarding redundancy and diversity of the network. This level however has had limited coverage in the research up till now (partially covered by for example Reggiani et al. (2009)) as can be seen in table 2.1.

The third level (L3) is the single airline level and contains airline specific properties (Reggiani et al., 2009; Han et al., 2009; Reggiani et al., 2010). Analysis on this level is of interest for airline management and can increase the reliability and security of the network (H. Liu et al., 2011). This can decrease the number of delays and cancellations and result in higher profitability on the long run (increase in company reputation). The network setup used in recent studies is reviewed in table 2.1. This table serves as starting point for section 2.4 and 2.5, in which the papers are reviewed in more depth.

2.4 ATN Topology Analysis

The air transport industry is an increasingly competitive industry, forcing airlines to have insight into the performance of their network. Topology of physical layout analysis offers a way to classify and analyze ATNs. This section describes the study of the topology of ATNs. First a short historic overview of ATN topology analysis is presented, after which complex network theory is introduced and the basic quantities offered by the theory are treated. Finally the results of the study of ATNs on all three levels is discussed.

2.4.1 ATN Topology Analysis History

The general study of networks started with graph theory and the Koningsberg bridge problem found to be infeasable by Leonard Euler in the 18^{th} century, where he attempted to find a round trip crossing each of the main bridges of Koningsberg exactly once (see figure 2.2).

Table 2.1: Literature review of ATNs, expansion of the review conducted by Lordan, Sallan, and Simo (2014)

Year	Authors	Level of Research
1985	Toh and Higgins	L1*
1998	Reynolds-Feighan	L1*
2001	Reynolds-Feighan	L1*
2002	Bowen	L1
2004	Barrat et al.	L1
	Guimera and Amaral	L1
	Li and Cai	L1*
	Chi and Cai	L1*
2005	Guimera et al.	L1
2007	Guida and Maria	L1*
2008	Malighetti et al.	L1*
	Bagler	L1*
	Hu and di Paolo	L3
2009	Reggiani et al.	L2/L3
	Han et al.	L3
	da Rocha	L1*
	Zanin et al.	L1*
	Lacasa et al.	L1*
2010	H.-K. Liu et al.	L1*
	Reggiani et al.	L2/L3
	J. Zhang et al.	L1*
2011	Wilkinson et al.	L1*
	Wang et al.	L1*
	H. Liu et al.	L3
2012	Cai et al.	L1*
	Sawai	L1
	Jia and Jiang	L1*
2013	Zanin and Lillo	L1
	Cardillo et al.	L1*
2014	H. Zhang et al.	L1
	Lordan	L3
	Wei et al.	L1*
	Lordan, Simo, and Gonzalez-Prieto	L1
2015	Lordan, Sallan, et al.	L2
	Lordan, Florido, et al.	L3

* Regional networks.

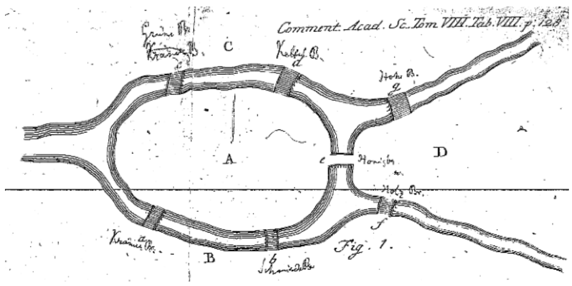


Figure 2.2: The Koningsberg bridge problem and the start of graph theory

This was followed by more research on graph theory eventually complemented by the analysis of social networks in the early 1920s. The aim of defining the most efficient structure for a single airline network in terms of profit and passenger mobility, initiated the analysis of the structure of ATNs years before the formulation of complex network theory (Bania et al., 1998). In the past, much attention has been paid to finding the shortest route algorithm (for example, the travelling salesman problem), in which the geographical configuration of the network was the center of investigation (Reggiani et al., 2010). Integer programming, linear and nonlinear programming turned out to offer a appropriate toolbox for topology analysis.

Liberalization of the market, however lead to the emergence of hub-and-spoke network and the formation of alliances, significantly increasing the complexity of ATNs. During the 80s and 90s a focus on two distinct themes is found in ATN topology analysis literature focussing: firstly on the consequences of the deregulation of the air transport market on the FSCs (Toh and Higgins (1985); Reynolds-Feighan (1998); Reynolds-Feighan (2001) and Bowen (2002)); and secondly on the emergence of LCCs (Reynolds-Feighan (2001). Different studies were conducted to describe and classify ATNs using concentration indices, such as the Shimbel index (Bowen, 2002), GINI index (Reynolds-Feighan, 1998), Freeman index (Reggiani et al., 2010) etc. These studies suggested two distinct network solutions: point-to-point networks and hub-and-spoke networks, respectively linked to LCCs and FSCs. However although concentration indices are proper measures of frequency or traffic concentration in a simple, well-organized networks, in more complex real ATNs the indices may yield high values for all types of network structures, failing to clearly discriminate between different network structures. (Alderighi et al., 2007). Complex network theory offers new quantities to analyze real networks, which are applicable to every ATN and most importantly are better able to distinguish between different ATN structures.

2.4.2 Complex Network Theory

Most recent studies of ATNs have focussed on classifying the topology through complex network analysis. Under the influence of increasing computing power and data collection of real networks this field of research flourished over the last decades resulting in a series of principles and properties of real networks. The analysis of complex networks was sparked by two important papers by Watts and Strogatz (1998) on small-world networks and by Barabasi and Albert (1999) introducing scale-free networks. The aim of the study has since focussed on the one hand on increasing the complexity of the network model by incorporating spatiality (Barthelemy, 2003) and weighted links (Barrat et al., 2004) and on the other hand on incorporating dynamics into the model such as vulnerability and robustness (Chi and Cai, 2004), which is elaborated in Section 2.5. A thorough review of complex network theory has been conducted by Boccaletti et al. (2006). Recent review papers regarding the use of complex network theory in the field of ATNs were written by Zanin and Lillo (2013) and Lordan, Sallan, and Simo (2014). All three reviews were used throughout the remainder of this literature study. An overview of the levels and type of studies conducted using complex networks is depicted in table 2.2. The following sections elaborates on the topology analysis of ATNs as complex networks.

Topological Quantities

Complex network theory offers several basic quantities to help characterize the topology of a network as extensively described by Boccaletti et al. (2006). Before reviewing the quantities used throughout literature to analyze ATNs first some definitions and notations.

Networks consist of two elements, N number of nodes $[n_1, n_2, ..., n_N]$ and K number of links $[l_1, l_2, ..., l_K]$, in which l_{ij} corresponds to a link between node i and node j. In directed networks $l_{ij} \neq l_{ji}$ and in weighted networks there is also a weight associated to every link w_{ij}. This weight can for instance refer to the importance of each link, its

Table 2.2: Literature review of levels and types of studies conduced of ATNs as complex networks

Year	Authors	Level	Type Topology	Robustness
2004	Barrat et al.	L1	✓	
	Guimera and Amaral	L1	✓	
	Li and Cai	L1*	✓	
	Chi and Cai	L1*		✓
2005	Guimera et al.	L1	✓	
2007	Guida and Maria	L1*	✓	
2008	Malighetti et al.	L1*	✓	
	Bagler	L1*	✓	
	Hu and di Paolo	L3	✓	✓
2009	Reggiani et al.	L2/L3	✓	
	Han et al.	L3	✓	
	da Rocha	L1*	✓	
	Zanin et al.	L1*	✓	✓
	Lacasa et al.	L1*	✓	✓
2010	H.-K. Liu et al.	L1*	✓	
	Reggiani et al.	L2/L3	✓	
	J. Zhang et al.	L1*	✓	
2011	Wilkinson et al.	L1*		✓
	Wang et al.	L1*	✓	
	H. Liu et al.	L3	✓	✓
2012	Cai et al.	L1*	✓	
	Sawai	L1	✓	
	Jia and Jiang	L1*	✓	
2013	Cardillo et al.	L1*	✓	
2014	H. Zhang et al.	L1	✓	
	Lordan	L3	✓	✓
	Wei et al.	L1*		✓
	Lordan, Simo, and Gonzalez-Prieto	L1		✓
2015	Lordan, Sallan, et al.	L2	✓	✓
	Lordan, Florido, et al.	L1*	✓	✓

* Regional networks.

capacity or the observed flow. A network is defined to be connected if there is a geodesic between all nodes in the network, otherwise it is defined as disconnected. Furthermore the connectivity or adjacency matrix A is a square NxN matrix containing either a 1 or a 0 depending on whether there is exists a link between two nodes.

With these definitions and notations the basic quantities used to analyze complex networks can be constructed:

k_i **Degree**

The degree is the number of links connected to node i and is defined in terms of the adjacency matrix A with elements a_{ij} as:

$$k_i = \sum_{j \in N} a_{ij} \tag{2.1}$$

In a directed network an ingoing k_i^{in} and outgoing k_i^{out} node degree can be defined.

$P(k)$ **Degree Distribution**

The degree distribution is defined as *the probability that a node chosen uniformly at random has degree k*. In other words, the fraction of the total nodes having the degree k. The degree distribution is the most important measure of network topology. For most ATNs the degree distribution turns out to be shaped by a power law $P(k) \sim Ak^{-\gamma}$ with exponent of degree distribution γ in the range of $2 \leq \gamma < 3$ for hub-and-spoke networks (SFN) and $\gamma > 3$ for point-to-point networks

(RN) (Barabasi and Oltvai, 2004), see table 2.3. In analogy with the degree in a directed network an ingoing $P(k^{in})$ and outgoing $P(k^{out})$ degree distribution with corresponding coefficient γ^{in} and γ^{out} can be defined as well.

L **Average Shortest Path Length / Diameter**
A shortest path or geodesic d_{ij} between node i and j is defined as the shortest path from node i to node j through the network. The average shortest path length and diameter are connected to the fact that transport networks are spatial networks (Barthelemy, 2003). The diameter is defined as the maximum geodesic distance d_{ij} in the network. The average shortest path length or characteristic path length L is defined as the mean geodesic length over all couple of nodes:

$$L = \frac{1}{N(N-1)} \sum_{i,j \in N, i \neq j} d_{ij} \tag{2.2}$$

b_i **Betweenness**
The betweenness is a centrality measure representing the importance of a node or link for movements inside a network. Node betweenness is defined as:

$$b_i = \sum_{j,k \in N, j \neq k} \frac{n_{jk}(i)}{n_{jk}} \tag{2.3}$$

where $n_{jk}(i)$ is the number of shortest paths connecting j and k running through node i and n_{jk} is the total number of shortest paths between j and k. Link betweenness is similarly defined as the number of shortest paths between pairs of nodes that run through that link.

C **Clustering Coefficient**
The clustering or transivity, measures the number of traingles in the network. Connected to this is the local clustering coefficient c_i measuring the probability that two nodes j and m, connected to the same third node i, also share a connection (Watts and Strogatz, 1998). The local clustering coefficient is defined as:

$$c_i = \frac{\sum_{j,m} a_{ij} a_{jm} a_{mi}}{k_i(k_i - 1)} \tag{2.4}$$

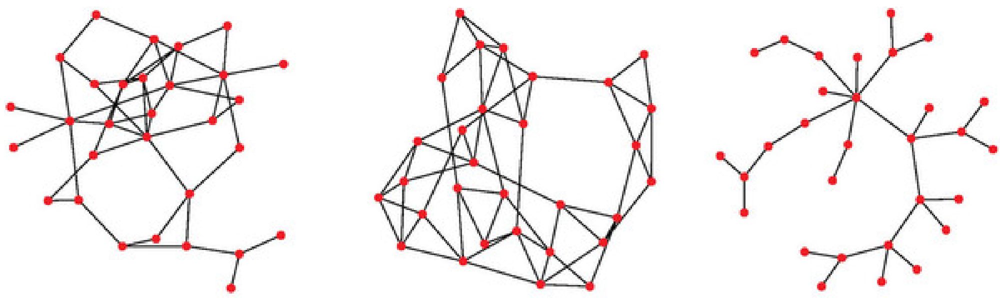
The total clustering coefficient of the network is defined as:

$$C = \frac{1}{N} \sum_{i \in N} c_i \tag{2.5}$$

Complex network theory differentiates three types of networks: Random Networks, Small-World Networks and Scale-Free Networks. These network types turn out to have substantial coupling with the network solution proposed by previous research.

Random Networks

Random networks (RNs) were introduced by Erdös and Rényi in 1959. Starting with networks with N disconnected nodes the RN is generated by connecting random couples



From left to right: Random Network, Small-World Network and Scale-Free Network

Figure 2.3: Schematic overview of the different complex network types

of nodes and prohibiting multiple connections between nodes, until the number of links is equal to K. This leads to a homogenous dispersed network with limited clusters and a Poisson degree distribution (Erdos and Réyni, 1960). A RN is a pure theoretical network and cannot be considered real (Boccaletti et al., 2006). It does however provide a standard to which real networks can be compared (Lordan, Sallan, and Simo, 2014) and can be usefull to describe point-to-point networks, which is commonly associated with LCCs. An example of a RN can be found in figure 2.3

Small-World Networks

Small-world networks (SWNs) were introduced by Watts and Strogatz (1998), named after the small-world property. The small-world property is the characteristic of most real networks that the distance between any two nodes is still relatively short despite the size of the network. This property is mathematically characterized by L, the average geodesic length between nodes in the network, depending at most logarithmically on the size of the network N. Another property of the small-world network is a high C, a high number of triangles in the network (see figure 2.3). Several real ATNs analyses suggests a SWN structure, such as the work conducted by Guida and Maria (2007) and Bagler (2008). According to the research by Hu and di Paolo (2008) and H. Liu et al. (2011) also point-to-point networks can have the properties of a SWN, combining a RN structure with low average L and high C.

Scale-Free Networks

Scale-free networks (SFNs), or more precisely evolving scale-free networks, were introduced by Barabasi and Albert (1999). SFNs incorporate two mechanisms observed in many real networks: growth and preferential attachment. Growth incorporates the dynamics that networks grow by addition of nodes and preferential attachment incorportates the effects of new nodes entering the network, by connecting to those nodes with the most links (the hubs) (Lordan, Sallan, and Simo, 2014). SFNs have the property of a power shaped degree distribution $P(k) \sim Ak^{-\gamma}$. Analysis by Barabasi and Oltvai (2004) found that a SFN with $\gamma = 2$ corresponds to a (pure) hub-and-spoke network, $2 < \gamma \leq 3$ is indicative of a hierarchy of hubs, while $\gamma > 3$ indicates behaviour as a RN (point-to-point).

Multiple analyzes of real ATNs suggests a SFN structure (Guimera et al., 2005). An example of a SFN is depicted in figure 2.3.

Weighted Networks

Complex network theory can also be used to implement capacity and strength to the links of the network, information which, if left ingnored, can lead to a substantial loss of information on the complex network (Lordan, Sallan, and Simo, 2014). This is done using weighted networks. For weighted complex networks with weight w_{ij} similar quantities are used to analyze the topology of the network. Conventional weights used in ATNs are number of flights (Bagler, 2008) and number of available seats (Barrat et al., 2004). Using these weights automatically leads to the need for a time frame in the analysis. For airport networks the time frame is often limited to 1 year, for alliance or single airline networks most of the times the busiest week of operation is chosen as time frame for the analysis. The following quantities are used throughout literature (again for more extensive descriptions the reader is referred to the work by Boccaletti et al. (2006)):

s_i **Strength**
The node strength is the analog in a weighted network for the node degree in an unweighted network and is defined as:

$$s_i = \sum_{j \in N} w_{ij} \tag{2.6}$$

In analogy with the degree in a directed network an ingoing s_i^{in} and outgoing s_i^{out} strength degree can be defined. The relation between the strength and the degree of a node provides information on how capacities are distributed over the network and approximately follows a power law $s(k) \sim k^{\beta}$. In the same way, relating the strength and betweenness of a node provides information on the centrality of the airport $s(b) \sim b^{\beta_b}$. To assess the relation between the frequency of connections between two nodes and their connectivity, the weight of the link is related to the degrees of the connected nodes as $w_{ij} \sim (k_i k_j)^{\theta}$ (Zanin and Lillo, 2013).

$R(s)$ **Strength Distribution**
The strength distribution is defined as the probability that a node chosen uniformly at random has strength s (Barrat et al., 2004), and is the equavalent for a weighted network of the degree distribution in an unweighted network. In analogy with the strength in a directed network an ingoing $R(s^{in})$ and outgoing $R(s^{out})$ strength distribution can be defined as well. The strength distribution is not frequently used in literature.

L^w **Average Weighted Shortest Path Length**
The average weighted shortest path length is not necessarily equal to the average unweighted shortest path length. Depending on the nature of the weight the length of the link l_{ij} can be introduced for example equal to $l_{ij} = 1/w_{ij}$. The average weighted shortest path length is defined as the length over all couple of nodes:

$$L^w = \frac{1}{N(N-1)} \sum_{i,j \in N, i \neq j} l_{ij} \tag{2.7}$$

The algorithm by Newman (2001) can also be used to calculate the (average) weighted shortest path length. In unweighted networks $l_{ij} = 1$ for all links, thus the shortest path reduces to the minimum number of links traversed d_{ij}.

C^w **Weighted Clustering Coefficient**
The weighted clustering coefficient, defined by Barrat et al. (2004), includes the fact that in weighted networks some links or nodes are more important than other, the local weighted clustering coefficient c_i^w is defined as:

$$c_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,m} \frac{(w_{ij} + w_{jm})}{2} a_{ij} a_{jm} a_{mi} \tag{2.8}$$

The total weighted clustering coefficient of the network is defined as:

$$C^w = \frac{1}{N} \sum_{i \in N} c_i^w \tag{2.9}$$

Spatial Networks

Spatial networks are networks whose nodes occupy a precise position in two or three-dimensional Euclidean space, and whose edges are real physical connections (Boccaletti et al., 2006). Although a lot of research is conducted on spatial networks, including ATNs, the main focus is on the topological properties rather than the spatial aspects of the networks. However including spatiality into ATNs can lead to a more realistic network analysis. Some examples of spatial considerations in real ATNs are:

First of all in spatial ATNs the length of a link is limited to the range of the aircraft used or possibly to the length of a working day of the crew. This leads to distant nodes being less likely to be connected and a decreased average path length. Furthermore the maximum number of links is limited to the number of aircraft in the fleet or to total operating time of the fleet, again linked to the type and speed of the aircraft and the maximum working hours of the crew within the time frame of the analysis. Both limitations are closely related and part of the optimization program developed by H. Liu et al. (2011). Another limitation in spatial ATNs is the node degree, the number of aircraft landing on an airport is often limited within the chosen time frame.

2.4.3 Topology Analysis of ATNs as Complex Networks

Complex network theory offers a methodological way to characterize network structures such as ATNs, using several basic quantities, as presented in the previous sector. This subsection will review previous studies that have applied complex network theory to analyze ATNs. Reviewing the different layers of study and comparing the results.

Airport Networks (L1)

The first level (L1) is the global / regional airport level, which is in reality a multilayer network of both L2 and L3. On this level the general network characteristics are evaluated.

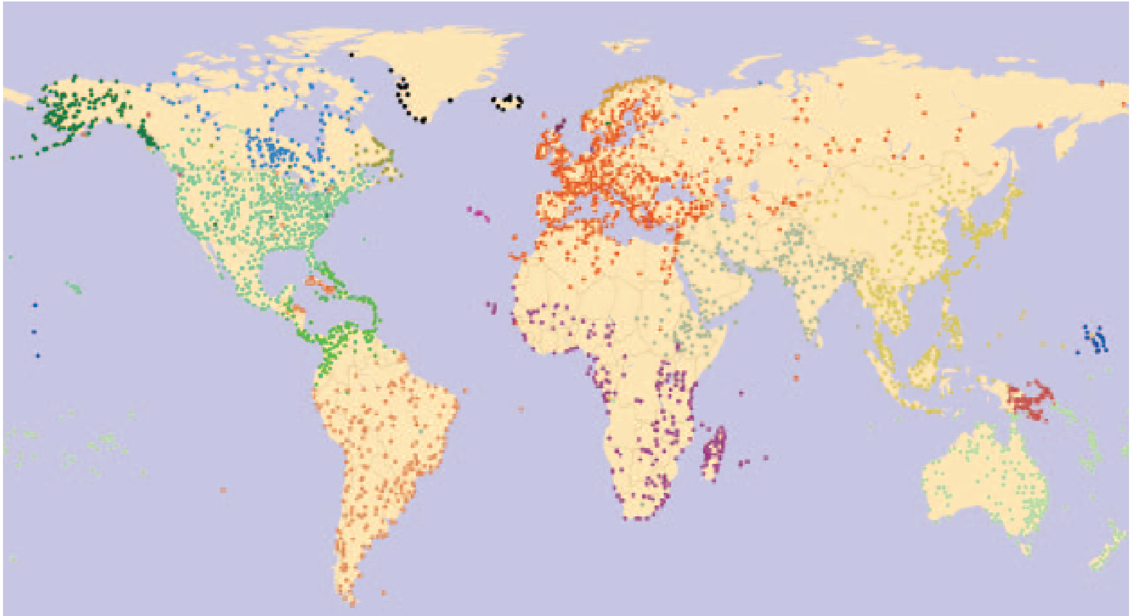


Figure 2.4: Communities in the global airport network as found by Guimera et al. (2005) Each color corresponds to a different community.

ATNs are usually modelled using airports as the nodes and flight connections as the links of the network. Although especially for the global airport network topology analysis sometimes cities are used as nodes (Guimera and Amaral, 2004). The global airport network is reviewed both unweighted (Guimera and Amaral (2004); Guimera et al. (2005)) and weighted (Barrat et al., 2004). All three studies find the global airport network to be a SFN with small-world properties. Barrat et al. (2004) includes a comparison of the unweighted and the weighted properties of the network, showing the added value of weights to the analysis of ATNs. Guimera et al. (2005) adds to this the community structure of the worldwide ATN. A community is defined as a group of nodes having higher density of edges within the groups than between the groups (Boccaletti et al., 2006). This highlights the importance of geo-political constraints in the global airport network (see figure 2.4).

Most recent studies on this level have focussed on regional airport networks, such as for example for: China (Li and Cai (2004); H.-K. Liu et al. (2010); J. Zhang et al. (2010); Wang et al. (2011) and Cai et al. (2012)), Italy (Guida and Maria, 2007), India (Bagler, 2008), Brazil (da Rocha, 2009), the U.S.A. (Jia and Jiang, 2012) and Europe (Lordan, Florido, et al., 2015). The airport network of Italy and India are found to be a SFN with small-world properties. In contrast the Chinese and Brazilian airport networks are found to be SWNs but not scale-free. da Rocha (2009) analyses the development of the Brazilian airport network over a decade and finds that the diversity of the network decreases and a gradual focus on the most profitable routes can be seen. H.-K. Liu et al. (2010) and J. Zhang et al. (2010) analyse the Chinese airport network over several years, both coupling the growth of the network to the economic growth of China. Jia and Jiang (2012) analyses the U.S. airport network, characterizing it as a SFN with small-world properties and uncovering a community structure within the domestic network.

The European airport network is also found to be a SFN with small-world properties by Lordan, Florido, et al. (2015).

Alliance Networks (L2)

The second level (L2) is the airline alliance level used to study the effect of alliance formation. This level has had very limited coverage in the research up till now. It is partially covered by (Reggiani et al., 2009) and (Reggiani et al., 2010) in which both the European and global Star Alliance network are analyzed. The authors report the networks to be SFNs. Another analysis is conducted by Lordan, Sallan, et al. (2015) analysing the three major alliances: Star Alliance, SkyTeam and oneworld over een period of 4 months. The study shows that all alliance networks are quite similar: SFNs, with small-world properties. No research has been found on the relation between L2 and L3 ATNs.

Single Airline Networks (L3)

The third level (L3) is the single airline level and contains airline specific properties. Analysis on this level is of interest for airline management. Only three papers and one thesis have been written on this level of study. Han et al. (2009) analyzes the daily network in the busiest week for four different European airlines (for full airline names see A.1 in appendix A), classifying them into two groups LH with KL/AF and BA with OS based on γ. All airline networks show scale-free properties as well as small-world properties, coupled to a hub-and-spoke network, which make sense all airlines analyzed are FSCs. The works of Reggiani et al. (2009) and Reggiani et al. (2010) also investigate LH, both the worldwide and European network, both turn out to be SFNs, just as in the paper by Han et al. (2009). Finally the thesis by Lordan (2014) analyzing multiple airlines in different alliances and parts of the world. Differentiating between LCCs (FR, WN and U2), intermediate airlines (AB, MU and CZ) and FSCs (LH, UA, US, AA, BA, AF and DL). The results of the analyses are in table 2.3.

Topological Results

Table 2.3 shows a comparison of topological quantities found in several complex network analyses of ATNs. Several observations can be made. At the global level (L1) only two studies are compared (Barrat et al. (2004), Guimera and Amaral (2004)), similar values are found for the L and γ. At the regional level (L1*) the values found for L and C are similar in all studies, and point to a SWN (low average path length and high C). As stated the analyses of the Chinese (Li and Cai (2004); H.-K. Liu et al. (2010); J. Zhang et al. (2010); Wang et al. (2011)) and Brazilian (da Rocha, 2009) airport network do not find the network to be scale-free, on the contrary the analyses of the Italian (Guida and Maria, 2007), Indian (Bagler, 2008), U.S. (Jia and Jiang, 2012) and European (Lordan, Florido, et al., 2015) airport networks do characterize the airport network as SFNs. It should be noted that the study by da Rocha (2009) and J. Zhang et al. (2010) analyse the development of the network over several years, so a range is shown, furthermore Li and Cai (2004), H.-K. Liu et al. (2010) and J. Zhang et al. (2010) use two values for γ, for the first (low values of k) and second part (high values of k) of the degree distribution, hence

Table 2.3: Comparison of topological quantities found in several complex network analyzes of ATNs

Year	Authors	Level	Nodes	Links	γ	γ_2	L	C
2004	Barrat et al.	L1	3880	18810	2.0	–	4.37	–
	Li and Cai	L1*	128	1165	0.428	4.161	2.067	0.733
2005	Guimera et al.	L1	3883	27051	2.0	–	4.4	0.62
2007	Guida and Maria	L1*	42	310	1.7	–	1.97	0.1
2008	Bagler	L1*	79	442	2.2	–	2.259	0.657
2009	Reggiani et al.	LH EU	111	522	2.0	–	–	–
		LH W	188	692	2.2	–	–	–
		Star Alliance EU	111	3230	2.5	–	–	–
		Star Alliance W	188	6084	2.5	–	–	–
	Han et al.	OS	130	721	2.428	–	2.31	0.122
		BA	320	2161	2.791	–	2.64	0.124
		KL/AF	348	3853	1.833	–	2.19	0.371
		LH	430	4643	1.755	–	2.90	0.376
	da Rocha	L1*	142 - 211	–	–	–	2.26 - 2.49	0.61 - 0.66
2010	H.-K. Liu et al.	L1*	121	1378	2.07	0.47	2.263	0.748
	J. Zhang et al.	L1*	120 - 144	–	2.52 - 2.79	0.41 - 0.49	2.21 - 2.29	0.70 - 0.81
2011	Wang et al.	L1*	144	1018	–	–	2.23	0.69
2012	Jia and Jiang	L1*	732	6086	–	–	2.61	0.58
2014	Lordan	LH	209	395	–	–	2.18	0.93
		UA	362	933	–	–	2.57	0.91
		US	203	408	–	–	2.26	0.96
		AB	119	361	–	–	2.31	0.51
		AA	272	523	–	–	2.3	0.94
		BA	186	223	–	–	2.87	0.15
		AF	178	258	–	–	2.42	0.46
		MU	182	571	–	–	2.5	0.55
		CZ	178	576	–	–	2.45	0.62
		DL	328	882	–	–	2.38	0.88
		FR	178	1396	–	–	2.16	0.44
		U2	131	601	–	–	2.19	0.39
		WN	86	507	–	–	1.97	0.72
2015	Lordan, Sallan, et al.	Star Alliance	1150	4240	–	–	3.24	0.77
		SkyTeam	896	3226	–	–	3.13	0.74
		oneworld	741	1670	–	–	3.28	0.71

* Regional networks.

no SFN. One exception with regards to C is the work by Guida and Maria (2007) on the Italian network. This is probably caused by the network being much smaller compared to the other regional networks and more approaching of a single airline network (L3), which the C value also confirms. Global (L1) and regional (L1*) overall show similar topological properties, with the exception of the L, being logically longer on a global scale.

Comparing the studies at an airline level (L3) of Reggiani et al. (2009), Han et al. (2009) and Lordan (2014), show a similarity in the topological properties of airlines and alliances. Just as at L1 and L1* L2 and L3 networks turn out to be scale-free networks, however on airline level the small-world property is less strong (i.e. the C is lower), Ls are similar. It is also noticable that some airlines have more clustered networks than other (KL/AF and LH in the analysis by Han et al. (2009)) which points to the ability of airlines to steer and optimize their network structure also with regards to robustness. The study by Reggiani et al. (2009) shows a glimpse of the influence of the formation of alliances, increasing the exponent γ, which can be explained with help of the theory on scale-free networks by the increase of the number of hubs in the network. Finally a great addition to this is the thesis paper by Lordan (2014) showing similar values for L and C, by further analysis even classifying airlines into LCCs and FSCs with help of their degree distribution curves.

Table 2.4 shows a comparison of topological quantities found in several weighted complex network analyses of ATNs. The number of studies is very limited as is the number of topological quantities and not a lot of conclusions can be drawn from the topological

Table 2.4: Comparison of topological quantities found in several weighted complex network analyzes of ATNs

Year	Authors	Level	Weight	β	β_b	θ
2004	Barrat et al.	L1	Available seats	1.5	0.8	0.5
2008	Bagler	L1*	Number of flights	1.43	–	–
2009	Han et al.	L3	Number of flights	1.06 - 1.08	–	–

* Regional networks.

properties found. The only observation that can be made in all studies is that there is a strong correlation between degree of the node and the quantity of passengers passing through that node. Increasing the number of destination at an aiport (higher degree) wil have a honeypot effect attracting disproportionaly more passengers.

Dynamics

Several dynamic effects also influence real air route networks (Zanin and Lillo, 2013), which can be modeled on L1, L2 and L3. For example passenger dynamics: not all flight legs are used as a point-to-point connection, especially in hub-and-spoke networks several flight legs are used as feeder flights for other (intercontinental) flight legs. The standard network model does not incorporate the effects of connecting flight and transfer passengers. This can be accomplished by including time into the model and analyzing the network in terms of minimum traveltime, incorporating the effect of waiting time into L (Malighetti et al., 2008). Another example is air traffic jamming, which was described by Lacasa et al. (2009). In this study the air network is modeled with the airports as nodes and flight legs as links adding a weight corresponding to the travel time over that link. Using simple queuing rules and limitations for every airports capacity to describe the interplay between traffic in and out of the airports, the jamming in the air network was modeled using a Monte Carlo method.

Optimization

Optimization is used in only a few ATN studies. The studies by Hu and di Paolo (2008) and H. Liu et al. (2011) optimize a single airline network for operating cost (shortest path length) and robustness using genetic optimization. Sawai (2012) uses a ant-colony optimization to optimize the global air network to flight path length. These approaches could in future be used to optimize airline or alliances networks not only for operating cost but also for robustness.

2.5 ATN Robustness Analysis

The ATN is a dynamic network, where airports and airspaces can be closed temporarily for various reasons, such as terrorist attack, weather, environmental accidents, strikes etc. Temporal closure of airspace or airports is both costly for the industry (e.g. grounded aircraft / damage to reputation) and the economy (e.g. reduced mobility of the public) (Guimera and Amaral, 2004). Three different concepts are (sometimes) interchangebly used throughout literature to describe a similar phenomenon: vulnerability, robustness and resillience. These three concepts are first elaborated, after which a short history of ATN robustness analysis is presented along with the current developments. Finally the most recent ATN robustness analysis studies are thoroughly reviewed.

2.5.1 Vulnerability, Robustness and Resillience

In order to prevent any confusion, this small section presents the definition of the three terms conceptualized in a static and dynamic manner by Scholz et al. (2012) and used as base for the definitions in this literature study. Static vulnerability is the same as risk, the weak spots within a network. Static robustness is the ability of networks to avoid malfunction when damaged. Dynamic vulnerability is the antonym of static robusteness. Evidently the term vulnerability is used in two different ways in literature. Firstly for finding weak spots in a network; and secondly for finding the inability to avoid malfuntions when damaged (for example this term is used by Wilkinson et al. (2011)). Finally resilience is the ability for a ATN to recover from malfunctions when damaged and is also called dynamic robustness.

This literature study uses the following three definitions:

- (Static) Robustness Analysis – The ability of an ATN to **avoid** malfuntion when damaged.
- Vulnerability Analysis – The **opposite** of (static) robustness.
- Dynamic Robustness Analysis – The ability of an ATN to **recover** from malfunctions when damaged.

2.5.2 ATN Robustness Analysis History

Robustness analysis can be divided into static robustness analysis and dynamic robustness analysis. Static robustness deals with the removal of nodes or links, without incorporating the redistribution of the flow over the newly formed network, while dynamic robustness deals with the removal of nodes and links, with incorporation of the redistribution of the flow. This latter is a rather complex (usually) numerical simulation for example studied by Moreno et al. (2002).

Most static robustness analyses of ATNs consider two different damages to the ATNs. Firstly error or random failures; and secondly premeditated attacks. Error or random failure is based on the consecutive random removal of nodes from the ATN, simulating random events such as for example the closure of an airport due to bad weather. Attack is based on consecutive removal of nodes from the ATN starting with the most important node moving down to least important node, based on some (complex network) property of the node, simulating for example a terrorist attack. To assess the robustness of the ATN the percentage or number of airports removed is compared to the percentage or number of links (flights or passengers) removed (Wilkinson et al., 2011), another way is to compare the percentage or number of airports removed to the size of the giant component of the ATN (Lordan, Simo, and Gonzalez-Prieto, 2014). The size of the giant component of the network is equal to the number or percentage of airports still connected in the network after the removal of nodes from the network. The percentage of airports removed is often called the f-value, a removal of 2.5% for example is equal to an f of 0.025.

The study by Albert et al. (2000) shows that SFN are extremely robust against error at the cost of being extremely vulnerable to targeted attacks. This can be explained by the fact that the SFN has a power law degree distribution. For error the chance is high that an unimportant node will fail and the effect on the rest of the network will be rather small. For targeted attack (removing nodes according to their importance in the network) however, the consequences will be disastrous, since they are connected to many other nodes in the network. For ATNs SFNs translate to hub-and-spoke networks, however hub-and-spoke networks usually also have the small-world property, meaning high clustering, which allows for alternative routing, increasing its dynamic robustness. RNs (point-to-point) have a similar reaction to attacks, but show a much higher vulnerability to error (Gorman et al., 2007).

The following criteria are used for the attack on the network (the sequence of removal of nodes) in literature on ATN robustness analysis:

k_i **Degree**
Used in the papers by: Chi and Cai (2004), Hu and di Paolo (2008), H. Liu et al. (2011), Lordan, Simo, and Gonzalez-Prieto (2014), Lordan, Florido, et al. (2015) and Lordan, Sallan, et al. (2015).

b_i **Betweenness**
Used in papers by: H. Liu et al. (2011), Lordan, Simo, and Gonzalez-Prieto (2014), Lordan (2014), Lordan, Florido, et al. (2015) and Lordan, Sallan, et al. (2015).

λ **Algebraic connectivity**
The algebraic connectivity is defined as the second smallest Laplacian eigenvalue λ of a graph by Fiedler (1973) and is used in the robustness optimization by (Wei et al., 2014).

Bonacich
The Bonacich power criterion introduced by Bonacich (1987) uses an iterative estimation approach which weights each nodes' degree by the degree of the nodes with which it is connected (Lordan, Simo, and Gonzalez-Prieto (2014) and Lordan, Sallan, et al. (2015)).

E **(Inverted) Efficiency**
The efficiency is introduced by Latora and Marchiori (2001) and is linked to the geodesic distance d_{ij} in the network. The efficiency E is defined as the harmonic mean geodesic length over all couple of nodes:

$$E = \frac{1}{N(N-1)} \sum_{i,j \in N, i \neq j} \frac{1}{d_{ij}} \tag{2.10}$$

The efficiency is used for an inverted robustness analysis by Lordan, Sallan, et al. (2015). Inverted robustness meaning that the network is built starting from the least important node up to the most important node.

Damage
The critical damage of a node is the reduction of the giant component when disconnecting the node (Latora and Marchiori, 2005) used in the analyses by Lordan, Simo, and Gonzalez-Prieto (2014) and Lordan, Sallan, et al. (2015).

w_{ij} **Modal Connectivity**
Modal analysis is based on the analysis of the eigenvalues of the Laplacian graph. The modal connectivity matrix Γ is defined by Petreska et al. (2010) as:

$$\Gamma = L'\Phi \tag{2.11}$$

The modal connectivity of node i is used in the studies by Lordan, Simo, and Gonzalez-Prieto (2014) and Lordan, Sallan, et al. (2015) and is defined as:

$$w_i = \sum_j |\gamma_{ij}| \tag{2.12}$$

in which γ_{ij} is the corresponding element in the modal connectivity matrix Γ.

2.5.3 Static Robustness Analysis of ATNs as Complex Networks

Static robustness analyzes the ability of ATN to avoid malfuntion when damaged. Only limited attention has been paid to the robustness analysis of ATNs as complex networks (see table 2.2 and the reviews written by Zanin and Lillo (2013) and Lordan, Sallan, and Simo (2014)) This subsection will review the robustness analysis of ATNs as complex networks, reviewing the different layers of study and comparing the results.

Airport Networks (L1)

The first level (L1) is the global / regional airport level. Most of the research on robustness analysis of ATNs as complex networks has been conducted on this level. Lordan, Simo, and Gonzalez-Prieto (2014) analyzes the global ATN using five different selection criteria to simulate an attack on the network: degree, betweenness, damage, modal analysis and the Bonacich power criterion (Bonacich, 1987). The study confirms the literature showing that the SFN is very vulnerable to attack and shows that the damage criterion (Latora and Marchiori, 2005) is most important for $f < 0.025$, and the betweenness criterion overcomes damage for higher values of f (see figure 2.5). The study also confirms the existance of communities in the global ATN, with the airport with the highest betweenness in each community being the most vulnerable airport (i.e. most important hubs for the FSCs.)

On a regional level (L1*) several studies focus on robustness of ATNs. For example analyzing the airport network of the U.S.A. (Chi and Cai, 2004) and Europe (Lordan, Florido, et al., 2015). Chi and Cai (2004) confirm that the U.S airport network is a SFN and is thereforee vulnerable against attack and robust against error, analyzing the ATN using node degree. Error is in this paper based on the consecutive removal of nodes from the ATN starting with the airport with the lowest degree up to the airport with the highest degree (reversed attack). It also studies the evolution of the complex

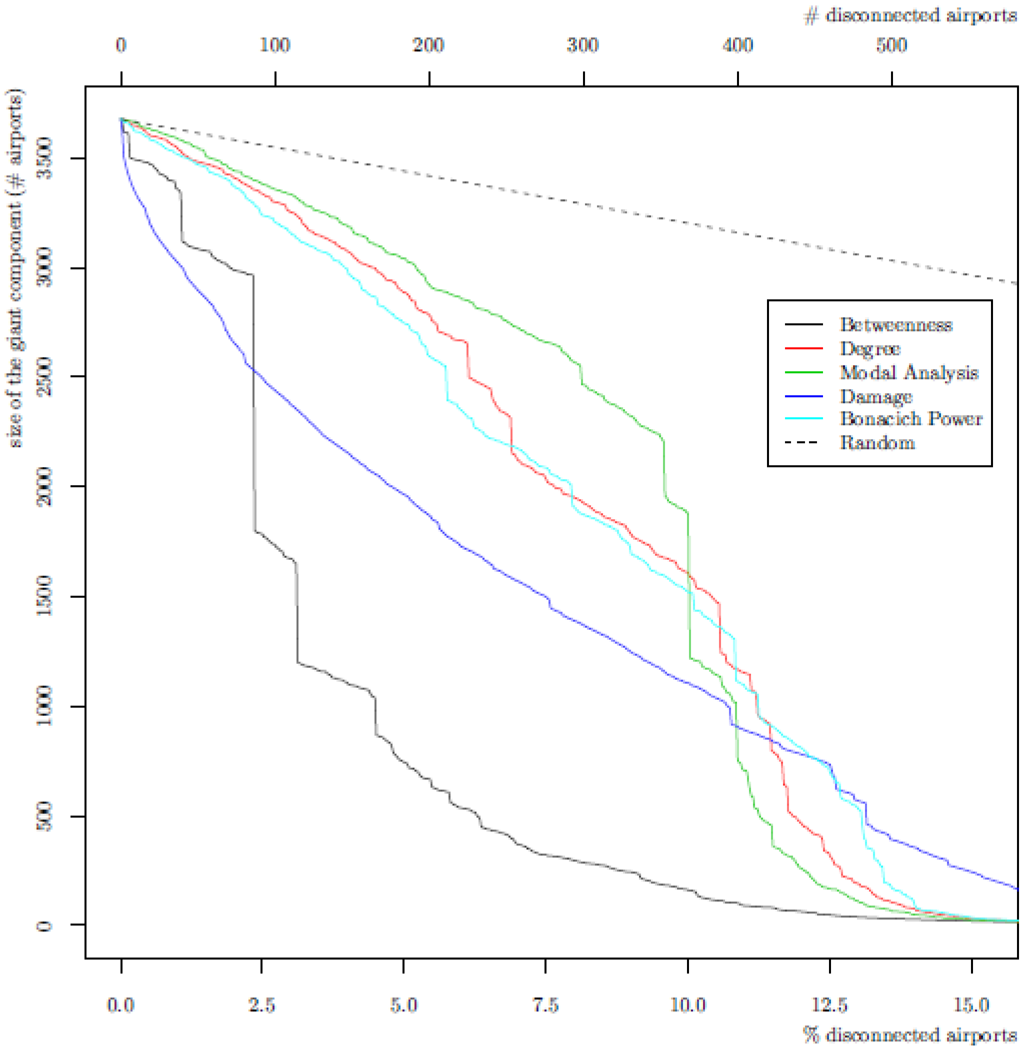


Figure 2.5: Robustness analysis of the global ATN as conducted by Lordan, Simo, and Gonzalez-Prieto (2014)

network properties during attack and error. Lordan, Florido, et al. (2015) analyze the robustness of the European airport network, which is found to be a SFN with small-world properties. It determines that the betweenness is the most important measure for robustness. The study by Wilkinson et al. (2011) includes spatiality, studying the effect of the vulcano eruption in 2010 on the European air traffic network. The regional airport networks is attacked by exposing the network to failure starting from a geographic point and expanding circular into the network. The study demonstrates that the effect on the European air traffic was disproportionately severe due to the network possessing a truncated, scale-free distribution and a spatial degree distribution that is uniform with distance from the centre of the network, resulting in a network that is vulnerable to spatial hazards. Finally Wei et al. (2014) analyses the robustness of an ATN using the maximization of the algebraic connectivity in order to improve the robustness of the network.

Alliance Networks (L2)

The second level (L2) is the airline alliance level and is only studied in a recent paper by Lordan, Sallan, et al. (2015) comparing the robustness of the three major airline alliance networks: Star Alliance, SkyTeam and oneworld in a similar way and with similar results as the study by (Lordan, Simo, and Gonzalez-Prieto, 2014), only including the inverted efficiency criterium, inverting the process by building a network and connecting the node with highest efficiency as late as possible. This criterium shows similar importance as the betweenness criterium for higher values of f. The study also uses the multi-scale analysis by normalization of the betweenness (Mishkovski et al., 2011) and comparing betweenness of all the alliance networks. The study shows that the Star Alliance network is the most robust airline alliance network of the three. No studies have been conducted investigating the advantages or disadvantages of using airline alliances with regards to robustness.

Single Airline Networks (L3)

The third level (L3) is the single airline level and is studied in two optimization papers by Hu and di Paolo (2008) and H. Liu et al. (2011). An analysis of the robustness of airline networks is conducted in the thesis by Lordan (2014) based on the betweenness. Confirming that FSCs are more vulnerable to attacks and less vulnerable to error than LCCs and suggesting several ways for airlines to improve their networks. For example for FSCs to incorporate flights between less central point and to use strategically spaced hubs in a multi-hub network in order to improve robustness of their network.

2.5.4 Dynamic Robustness Analysis of ATNs as Complex Networks

Dynamic robustness analyzes the ability of ATN to recover from malfuntions when damaged. Only the study by Lacasa et al. (2009) implements dynamic robustness into a ATN model. The paper presents the European airport network and uses a numerical simulation to diffuse a number of agents (aircraft) first over an efficient network (no jamming) and later introducing sudden bottlenecks in the network, leading to poor agent diffusion. It incorporates a redistribution of the flow of agents after an attack (introducing a sudden bottleneck) and is the only known clear example of dynamic robustness modelling of an ATN.

2.6 Conclusions and Future Work

Concluding the aim of this literature study was threefold. First of all introducing the area and background of this thesis; secondly, serving as theoretic framework for this thesis; thirdly, reviewing current literature and determining the current state of research, highlighting the relevance of this thesis. All three will be consecutively reviewed in this section.

Increased market deregulation and the accompanied rise of LCCs over the last decades has put profits for many of the old FSCs under pressure forcing them to merge and form alliances. This sparked the research into finding the most efficient structure for a single airline network in terms of profit and passenger mobility. Along with this, the social and economical dependancy on air transport grew and hence the need to assess the robustness of the network rose. Complex network theory offers a framework to model and analyze efficiency and robustness of ATNs.

The most important topological properties are the degree and degree distribution, the betweenness, the average path length, the clustering coefficient. The static robustness of ATNs is analyzed using error and attack. Error entails the consecutive removal of random nodes from the network, while attack entails the consecutive removal of the node with the highest criteria. Different criteria are used, but betweenness and degree are most common. Betweenness turns out to be the best predictor for robustness of ATNs.

Three different network types were characterized, with the following properties:

RN Random Networks are linked to point-to-point networks and LCCs. A higher robustness against attacks and a lower robustness against error in comparison to SFNs is found.

SWN Small-World Networks are linked to the small-world properties: high clustering and a low average path length.

SFN Scale-Free Networks are linked to hub-and-spoke networks and FSCs. A higher robustness against error and a lower robustness against attack in comparison to SFNs is found. Often ATN with a SFN also exhibit small-world properties.

Three levels of study are distinguished: the global airport network (L1), the airline alliance network (L2) and the single airline network (L3). Most research has been conducted on the global and regional level (L1) and has mainly focussed on topology analysis (see table 2.1). The study on alliances (L2) has not been fully developed yet (only covered by Reggiani et al. (2009) and Lordan, Sallan, et al. (2015)) and some research has been carried out on the level of the airlines (L3) by Reggiani et al. (2009), Reggiani et al. (2010), Han et al. (2009) and Lordan (2014). In a way analysis on the global level is finished and the focus should be on the analysis of alliances and airlines.

Current network models incorporate none to little of the real air transport industry aspects. Several weighted models incorporate available seats (Barrat et al., 2004) or number of flights (Bagler, 2008), but none of the models incorporate other fleet and crew aspects: such as operating time, fleet scheduling, crew scheduling etc. Future work should include the incorporations of these networks into a complex network model. Another aspect is passenger dynamics, which was modeled by Malighetti et al. (2008). Expanding on this research could lead to incorporating dynamics such as transfer passengers into the ATN model.

Part II

Topology Analysis

Chapter 3

Topology of European Airline Networks - Influence of Alliance Formation

3.1 Abstract

The first aim of this chapter is to analyze the topology of seperate airline networks of both FSCs and LCCs in order to distinguish between the business solutions used by the airlines (PPs and HSs). The second aim is to study the influence of using codeshares and an alliances on the topology of the airline networks. This is performed by using complex network indicators to compare the seperate and combined (both codeshare and alliance) network layouts. The analysis confirms literature regarding FSCs, which turn out to use SFN associated with HS. LCCs however, are found not found to have RN associated with PP as suggested in literature, but a SFN with multiple interconnected hubs. The most important difference found between FSCs and LCCs is that LCCs tend to focus on diversity of destinations over frequency, while FSCs tend to focus on frequency over diversity. Combining networks into codeshare networks or the Skyteam alliance, increases the diversity of the network, the size and number of hubs and brings the behaviour closer to LCCs, however still with a focus on frequency over diversity.

3.2 Introduction

The air transport industry is an increasingly competitive industry. The liberalization of the U.S. domestic market in the 70s, followed by the European market in the 80s and 90s, and the additional signing of several Open Skies Agreements (U.S. General Services Administration, consulted 2015), changed the industry from a closed subsidised business

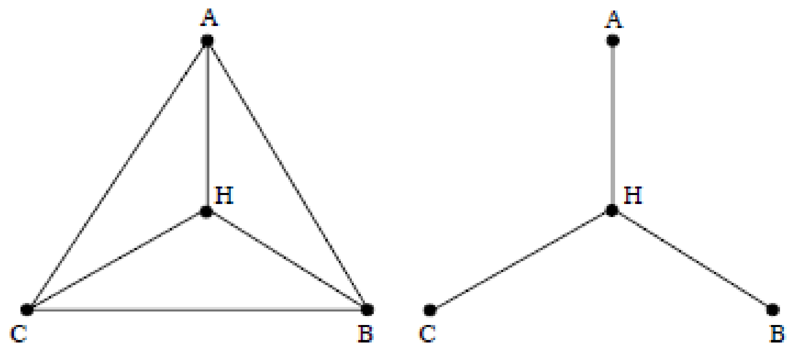


Figure 3.1: Two distinct networks solutions used by airlines (simplified): on the left side a point-to-point network on the right side a hub-and-spoke network using airport H as hub (Alderighi et al., 2007)

to an open market with corresponding competition. The subsequent emergence of LCCs pressuring profits, lead to a significant change in the operations of the ATNs of existing FSCs. Topology analysis, the analysis of a schematic representation of the airline network, offers a way to classify and analyze the difference in structure and performance of different ATNs.

LCCs (Reynolds-Feighan (2001)) and deregulation (Toh and Higgins (1985); Reynolds-Feighan (1998); Reynolds-Feighan (2001) and Bowen (2002)), are consequently the focus of the topology analysis literature during the 80s and 90s. Various studies were conducted to describe and classify ATNs using concentration indices, such as the Shimbel index (Bowen, 2002), GINI index (Reynolds-Feighan, 1998), Freeman index (Reggiani et al., 2010) etc. These studies suggested two distinct network (business) solutions: point-to-point networks (PPs), in which the airports are connected with direct flights and hub-and-spoke networks (HSs), in which airports are connected through a hub airport (see figure 3.1). In literature these types of networks have been respectively associated with FSCs and LCCs.

Nevertheless another consequence of the rise of LCCs: the formation of alliances and codeshares between the FSCs, further increased the complexity of the ATNs. Although the concentration indices used in the 80s and 90s are proper measures of frequency or traffic concentration in a simple, well-organized networks, in more complex ATNs the indices may yield high values for all types of network structures, failing to clearly discriminate between different network structures (Alderighi et al., 2007).

The emergence of complex network theory, sparked by Watts and Strogatz (1998) and Barabasi and Albert (1999) offered new quantities to analyze real networks, applicable to every ATN and most importantly better able to distinguish between different ATN structures. In the last decades most research has been conducted using complex network theory. The focus lies on global and regional airport networks. The study of alliances has not been fully developed yet (only covered by Reggiani et al. (2009) and Lordan, Sallan, et al. (2015)) and some research has been carried out on the level of separate airline networks by Reggiani et al. (2009), Reggiani et al. (2010), Han et al. (2009) and Lordan (2014).

Three different network types are characterized, with the following properties:

RN Random Networks, in which airports are randomly connected, and associated with PP and LCCs.

SWN Small-World Networks having the small-world properties: high clustering and a low average path length. This characteristic is found in both FSCs and LCCs.

SFN Scale-Free Networks, in which airports are linked to the airport with highest amount of existing links (hub), and associated with HS and FSCs.

This chapter adds to the limited research. The first aim of this chapter is to analyze the topology of seperate airline networks of both FSCs and LCCs in order to distinguish between the business solutions used by the airlines (PPs and HSs). The second aim is to study the influence of using codeshares and an alliances on the topology of the airline networks. This is performed by using complex network indicators to compare the seperate and combined (both codeshare and alliance) network layouts.

3.3 Methodology

3.3.1 Network Setup

In this chapter 17 European airline networks are analyzed; 15 FSCs belonging to the Skyteam alliance and 2 LCCs (see figure 3.2). The period of analysis is week 31 of 2015. This is one of the busiest weeks for European airlines, due to extensive holiday travel. The airline data is acquired from the respective websites of the airlines or other freely available internet sources.

The airline networks are modelled as simplified networks. The network consists of two elements: N number of nodes representing the airports $[n_1, n_2, ..., n_N]$ and K number of links representing the operated flight legs $[l_1, l_2, ..., l_K]$. In this context l_{ij} corresponds to an operated flight between airport i and airport j. For each airline three different networks are constructed: one unweighted network and two weighted networks using the number of flights and number of seats as weights w_{ij} for the flight link l_{ij}. The different networks are indicated by the superscripts f and s respectively, no superscript indicates the unweighted network. The connectivity or adjacency matrix A is defined as a square $N \times N$ matrix, with elements a_{ij}, containing either a 1 or a 0 depending on whether there exists a connection between two airports. Furthermore every network is undirected, meaning that $l_{ij} = l_{ji}$, unless stated otherwise. This since only a limited number of flights follow a circular path.

To illustrate the network quantities, figure 3.3 presents an example of an HS airline network represented as an undirected weighted complex network. The network consists of $N = 4$ airports $[A, B, C, H]$ and $K = 6$ flight legs $[l_{AB}, l_{AC}, l_{AH}, l_{BC}, l_{BH}, l_{CH}]$. Corresponding to these flights legs it contains the weights $[w_{AB}, w_{AC}, w_{AH}, w_{BC}, w_{BH}, w_{CH}]$ respectively equal to $[1, 2, 3, 1, 2, 5]$ representing the number of flights between the airports in the period of analysis.

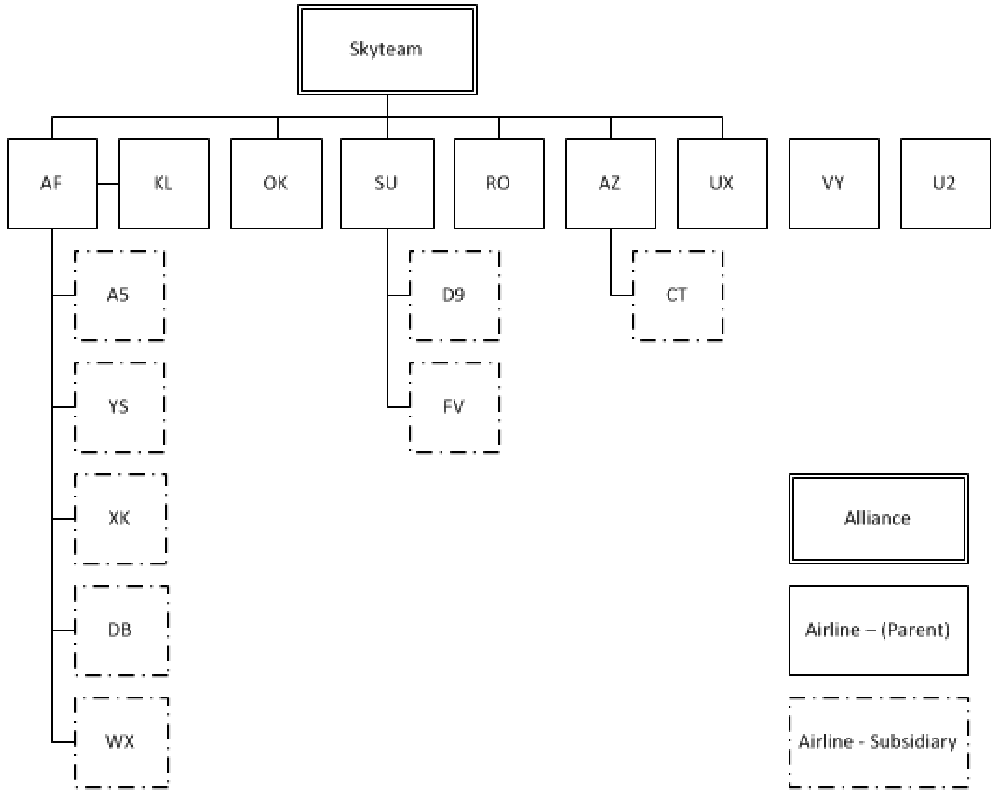


Figure 3.2: Relation between the airlines analyzed in this study, for full airline names see A.1 in appendix A

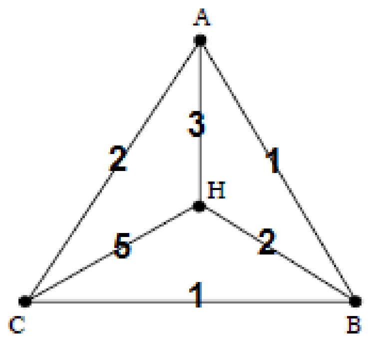


Figure 3.3: Airline network modelled as a weighted complex network, adapted from (Alderighi et al., 2007)

Next to the analysis of the topological properties of the independent airline networks, the European Skyteam alliance network as well as the combinations of parent and subsidiary airlines (see figure 3.2) are also analyzed. This in order to study the effects on the topological properties of alliance formation and codesharing. The network including all subsidiaries is refered to as the *total* network, for example the total Aeroflot network SU_{TOT} contains the networks of Aeroflot (SU), Donavia (D9) and Rossiya Airlines (FV).

3.3.2 Network Topology Indicators

Complex network theory offers several basic quantities to define the topology of a network as extensively described by Boccaletti et al. (2006). The following quantities are analyzed in this study using a Python program (see appendix B.1). The quantities are presented in the context of airline networks using the example of figure 3.3, with the weight being the number of flights per flight leg.

Nodes and Links

The number of nodes N represents the number of airports in the network. The number of links K represents the number of flights legs in the network. Both indicators are a measure of the size of the airline network. For the example $N = 4$ and $K = 6$.

Degree and Strength

The average degree $\langle k \rangle$ of an ATN represents the average number of flights legs connected to each airports, in other words the average number of destinations per airport. For an individual airport i the degree k_i is defined in terms of the adjacency matrix A with elements a_{ij} as:

$$k_i = \sum_{j \in N} a_{ij} \tag{3.1}$$

The degree distribution function $P(k)$ is defined as the fraction of the total airports having the degree k. For most ATNs the degree distribution turns out to be shaped by a power law $P(k) \sim Ak^{-\gamma}$ (see Section 2.4.3), with the exponent of degree distribution γ. In literature the value of γ is associated with the type of network. γ in the range of $2 \leq \gamma < 3$ is associated with SFN and HS, while $\gamma > 3$ associates with RN and PP. Using the example the degree for airport B k_B is equal to 3, the airport is connected to 3 airports with a direct flight (A,C and H).

The average flight strength $\left\langle s^f \right\rangle$ of the network represents the average number of direct flights connected to each airport. The average seat strength $\langle s^s \rangle$ represents the average number of direct seats connected to each airport. The strength s_i is the analog for the degree in a weighted network and it is defined for an individual airport i as, using the number of flights and number of seats as weight w_{ij} respectively:

$$s_i = \sum_{j \in N} w_{ij} \tag{3.2}$$

The strength distribution function $R(s)$ is correspondingly defined as the fraction of the total airports have strengtg s. It is also approximated by a power law $R(s) \sim As^{-\gamma}$ with exponents of strength distribution γ^f and γ^s. Using the example the flight strength of airport B s^f_B is equal to 4, one direct flight to both airport A and C and two direct flights to airport H.

Average Shortest Path Length

A shortest path or geodesic d_{ij} is defined as the shortest path from airport i to airport j through the network. The average shortest path length L of the network is equal to the average number of flights needed to travel between all couple of airports within the network. It is defined as:

$$L = \frac{1}{N(N-1)} \sum_{i,j \in N, i \neq j} d_{ij} \tag{3.3}$$

It represents a measure of the ability of direct travel between airport in the airline network. For this reason a pure PP (as shown in the example) will have $L = 1$, since all airports are connected with a direct flight, thus path. For a HS with a single hub the value will be smaller than 2 since all paths will pass through the one hub, however for a HS with multiple (interconnected) hubs the value will be generally higher than 2, since there will be shortest paths through multiple hubs.

Total Clustering Coefficient

The clustering or transivity c_i, is a measure of the number of traingles in the network. It measures the probability that two airports j and m, connected to the same third airport i, also share a mutual connection (Watts and Strogatz, 1998) and is defined as:

$$c_i = \frac{\sum_{j,m} a_{ij} a_{jm} a_{mi}}{k_i(k_i - 1)} \tag{3.4}$$

For example the clustering of airport B c_B is equal to 1 since it is only connected to other interconnected airports. The total clustering coefficient C of the airline network is a measure of transivity through the entire network. In other words it is a measure of the amount of alternative routes between airports in the airline network:

$$C = \frac{1}{N} \sum_{i \in N} c_i \tag{3.5}$$

Analogously for the weighted networks the weighted clustering coefficient is defined as:

$$c_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,m} \frac{(w_{ij} + w_{jm})}{2} a_{ij} a_{jm} a_{mi} \tag{3.6}$$

The total weighted clustering coefficient C^w is equal to:

$$C^w = \frac{1}{N} \sum_{i \in N} c_i^w \tag{3.7}$$

A pure PP (as shown in the example) will have a values of $C = 1$ since all airports will be connected, thus all alternative routes are possible. A HS with only one hub in contrary will have $C = 0$, since no alternative routes are present, the value will rise with increasing

Table 3.1: Topological properties of the separate airline (unweighted and weighted), Skyteam alliance and codeshare networks. Indicators analyzed: number of airports N, number of operated flight legs K, average degree $\langle k \rangle$ and strength $\langle s \rangle$, degree/strength exponent γ, average shortest path length L and total clustering coefficient C

	N	K	$\langle k \rangle$	$\langle s^f \rangle$	$\langle s^s \rangle$	γ	γ^f	γ^s	L	C	C^f	C^s
Separate parent airlines												
KL	74	73	1.97	62	8578	2.33	2.66	2.4	1.95	0	0	0
SU	72	74	2.06	66	9503	1.88	2.08	2.21	1.97	0.049	0.012	0.012
AZ	69	109	3.16	68	9859	2.24	2.21	2.14	2.09	0.317	0.067	0.066
AF	66	87	2.64	104	15415	2.14	2.51	2.34	2.24	0.152	0.024	0.023
RO	36	38	2.11	34	3792	2.2	2.21	2.49	2.13	0.115	0.025	0.02
UX	36	80	4.44	52	7008	1.58	1.78	2.63	2.07	0.252	0.049	0.041
OK	35	40	2.29	21	2714	1.99	1.9	2.69	1.93	0.267	0.024	0.014
VY	154	362	4.7	31	4578	1.9	1.94	1.96	2.07	0.394	0.038	0.038
U2	132	669	10.14	74	11057	2.18	1.68	1.7	2.17	0.33	0.055	0.055
Separate subsidiary airlines												
FV	33	33	2	27	3991	2.19	2.37	2.42	1.94	0.061	0.021	0.021
CT	30	33	2.2	36	3463	2.01	2.18	2.45	2.15	0.136	0.019	0.023
A5	29	54	3.72	40	2505	2.45	2.6	2.14	2.26	0.273	0.083	0.062
YS	24	22	1.83	54	5788	2.22	3.24	2.59	1.85	0	0	0
XK	12	21	3.5	49	4355	4.16	2.39	1.57	1.85	0	0	0
D9	10	12	2.4	32	4830	4.52	2.64	2.71	2.09	0	0	0
DB	9	8	1.78	27	2133	2.08	1.92	1.98	1.78	0	0	0
WX	7	5	1.43	60	4717	2.12	2.11	2.14	2.07	0	0	0
Skyteam alliance and codeshare networks												
Skyteam	191	617	6.46	161	21439	2.24	2.07	2.05	2.52	0.533	0.06	0.053
AF–KL$_{TOT}$	104	251	4.83	146	18917	1.91	2.29	2.28	2.13	0.6	0.159	0.144
AF$_{TOT}$	86	179	4.16	123	15495	1.79	2.3	2.36	2.16	0.271	0.033	0.024
SU$_{TOT}$	74	111	3	81	11678	1.86	2.19	2.13	1.97	0.419	0.071	0.07
AZ$_{TOT}$	73	119	3.26	78	10678	2.27	2.32	2.33	2.05	0.355	0.051	0.056

numbers of (interconnected) hubs, creating more alternative routes and thereby rising the value of C.

Combined the average shortest path length L and total clustering coefficient C are a measure for the efficiency of travel through the network. High efficiency is related to a relative low average shortest path length and high total clustering. This property of ATN is called the small-world property.

3.4 Results

The main topological indicators of the seperate airlines along with the indicators of the Skyteam alliance and codeshare networks are depicted in table 3.1. In the following three sections first the seperate parent and subsidiary airlines are analyzed, after which in Section 3.4.4 the Skyteam alliance and codeshare networks are investigated.

3.4.1 Degree and Strength

From the definition of the degree it logically follows that in general the higher the number of flights legs K compared to the number of airports N, the higher the average degree of the airline network. Three different sets of airlines are present: airlines with a low degree $\langle k \rangle \approx 2$ and $K \approx N$, airlines with intermediate degree $2.5 < \langle k \rangle < 4$ and $K > N$, airlines with high degree $\langle k \rangle > 4$ and $K >> N$.

The first set only contains FSCs with one main hub, connected to the other airports. On the one hand KL, DB, YS and WX only have flight legs between the hub and the other

Table 3.2: Top 10 airports ranked to their degree k for four representative airline networks

	KL	SU	AF	U2
1	AMS:73	SVO:71	CDG: 55	LGW: 105
2	AAL:1	LED:4	ORY:12	BRS:49
3	AES:1	FCO:2	MRS:10	MXP:48
4	ATH:1	MXP:2	NCE:7	BSL:47
5	ARN:1	PRG:2	TLS:7	LTN:47
6	BGO:1	AAQ:1	LYS:4	SXF:42
7	BHD:1	AER:1	NTE:4	CDG:39
8	BHX:1	AGP:1	AMS:3	MAN:39
9	BIO:1	AMS:1	BES:3	FCO:37
10	BLL:1	ATH:1	BOD:3	EDI:32

airports, for example in case of the KLM network all airports have a degree of 1 except for the hub AMS which has a degree of 73, see table 3.2. On the other hand SU, RO, OK, FV and CT have some interconnecting flights between the minor airports, exemplified with the top 10 airports of the Aeroflot parent network in the same table.

The second set also contains FSCs, however with multiple hubs (AZ, AF, A5, XK, D9), usually one major and a few minor hubs. Representative for this group is the Air France parent network, a top 10 of airports in the network ranked to their degree is presented in table 3.2.

The third set contains both LCCs VY and U2 and one of the Skyteam members FSC UX. These airlines contain multiple big hubs of similar size (for example EasyJet in table 3.2), offering on average more direct destinations per airport.

This result is expected. The FSCs have in general a lower average degree, since flights are concentrated through the hub(s). Also the increase in degree for increased hubs is expected. In general the higher the number of hubs the higher the average degree. Air Europa seems to be an exceptional FSC with a behaviour closer to the LCCs VY and U2.

The average flight strength and average seat strength gives a less clear division between the airlines. However comparing the strengths and degrees results in more useable information. Table 3.3 depicts the ratio of both average flight strength over average degree and average seat strength over average flight strength. It also shows the total capacity of the airline in flights per week.

The average flight strength over average degree is a measure of the average number of flights per destination per airport. It represents a strategic choice of an airline between offering diversity or frequency. Table 3.3 shows that the LCCs VY and U2 offer relatively low frequency of flights to a relatively high number of destinations, while the FSCs offer a high frequency of flights to a lower number of destinations, both off course related to the total capacity of the airline. Another observation is that especially the smaller subsidiary airlines (XK, D9, DB and WX) focus on a relative large frequency of flights on a limited number of routes. This also shows a strategic decision of the parent airlines to use subsidiary airlines for certain focus areas or airports.

The average seat strength over average flight strength is a measure of the average number of seats per flight. It represents the composition of the fleet of the airline networks. Table 3.3 shows that overall that the bigger airlines use similar fleet consisting of regular narrow body aircraft types. The subsidiary airlines of Air France (A5, YS, XK, DB and WX) and Alitalia (CT) mainly use regional aircraft types, signaling a possible strategic decision of these parent airlines to use low capacity aircraft for their subsidiaries to increase the frequency of flights (or cope with limited demand).

Table 3.3: Capacity of the airline in flights per week, average flights per destination ($\langle s^f \rangle$ over $\langle k \rangle$) and seats per flight ($\langle s^s \rangle$ over $\langle s^f \rangle$) of the separate airline, Skyteam alliance and codeshare networks

	Capacity	$\frac{\langle s^f \rangle}{\langle k \rangle}$	$\frac{\langle s^s \rangle}{\langle s^f \rangle}$
Separate parent airlines			
KL	4588	31	138
SU	4752	32	144
AZ	4692	22	145
AF	6864	39	148
RO	1224	16	112
UX	1872	12	135
OK	735	9	129
VY	4774	7	148
U2	9768	7	149
Separate subsidiary airlines			
FV	891	14	148
CT	1080	16	96
A5	1160	11	62
YS	1296	30	107
XK	588	14	89
D9	320	13	151
DB	243	15	79
WX	420	42	79
Skyteam alliance and codeshare networks			
Skyteam	30751	25	133
AF–KL$_{TOT}$	15184	30	130
AF$_{TOT}$	10578	30	126
SU$_{TOT}$	5994	27	144
AZ$_{TOT}$	5694	24	137

3.4.2 Small-World Property

The small-world property consists of both a low average shortest path length (L) and a high total clustering (C).

For all parent and subisidiary airlines $L \approx 2$ (table 3.1). All airline networks with one dominant hub (KL, SU, OK, FV, YS, XK, DB and WX) have a value of $L < 2$ as expected. The other (multi-hub) FSCs all have a value of $L > 2$ which was also expected for this type of HS. Both LCC have a value of L similar to the multi-hub FSCs, pointing to a HS network and not a pure PP, for which a lower value is expected. It should nevertheless be noted that the LCCs networks are a lot bigger than the FSC with similar L, pointing to an ability to connect airports more directly than the FSCs, which is a property of PP.

The clustering for all airline networks with only one dominant hub is equal or close to 0 as expected. There are however two additional airline networks without clustering: XK and D9, both networks have multiple disconnected hubs, resulting in $C = 0$. The other FSCs have a higher C with increasing number of interconnected hubs. The highest values of C are as expected from theory for the two LCCs VY and U2, however not close to a pure PP, in which case the value should be close to 1.

Overall the airline networks with multiple hubs have strong small-world properties, with low average path length and high clustering.

3.4.3 Degree and Strength Distribution

The degree distribution is the most important measure of network topology used in literature, linked to this is the degree exponent γ. Most airlines (including both LCCs) have a $\gamma \approx 2$, with several below 2, which is also seen in other research for example by (Han et al., 2009), associated with a SFN and HS. Two FSC airlines XK and D9 have a $\gamma > 3$ pointing to a RN and PP. Regarding both LCCs these results confirm the other results pointing to both networks having a multi-hub network rather than a PP. It should be noted that since some of the networks have only two different degrees the resolution of the graph and thus the power approximation is limited.

Figure 3.4 shows on top the cumulative degree $P(k)$cum and strength $R(s)$cum (both using flights and using seats as weights) distribution of the separate parent and subsidiary airlines on a double logarithmic scale, $P(k)$cum being the chance that an airport has degree k or higher (in the case of $R(s)$cum the chance that an airport has strength s or higher).

The cumulative degree distribution curves of parent and subsidiary airlines show four different types of behaviour. Curves with a downward concave shape, curves with an upward concave shape, curves with a near-linear shape and linear curves.

First the linear curves (KL, DB, YS and WX) are the airlines with one main hub resulting in a linear curve since only two different degrees are present in the network: $N - 1$ for the hub and 1 for the other airports.

The downward concave curves (U2, UX, XK and D9) have the lowest initial drop, meaning a relative high amount of airports with a higher degree associated with PP. The parent airline networks (U2 and UX) have multiple hubs resulting in this behaviour, for example for U2 more than 30% of the airports have a degree of 10 or higher. The behaviour of XK and D9 confirms the results of their value of γ, which also point to a PP.

The upward concave curves (SU, RO, OK, FV, CT) have the highest initial drop, meaning a relative high amount of airports with a low degree, pointing to a HS with one major hub and some interconnected flights.

The final set of curves (AZ, AF, VY and A5) show intermediate near linear behaviour. These are airline networks with multiple smaller hubs. It should be noted that, although Vueling shows near linear behaviour, more than 20% of the airports have a degree of 5 or higher.

Figure 3.4 shows in the middle and at the bottom respectively the cumulative strength distribution for flights and seats. Including the weights increases the resolution of the results, however as already stated in Section 3.4.1 the results are also diluted. The curves are less clear to differentiate. The curves have a similar shape in the $R(s)$cum as in the $P(k)$cum distribution. Since most of the airline networks with concave upward curves in $P(k)$cum have a high value of average flight strength over average degree (see table 3.3) the curves shift to the right in the $R(s^f)$cum distribution overlapping with the curves of U2 and UX. Since almost all parent airlines and subsidiary airlines have similar average seat strength over average flight strength values the $R(s^s)$cum distribution does not add any new information. Overall the results show that the flights (and seats) are proportionaly distributed over the flight legs, leaving the nature of the network unchanged.

3.4.4 Alliance and Codeshare Formation

The lower part of table 3.1 shows the results of the Skyteam and codeshare networks. What is the effect of using codeshares and a Skyteam alliance?

First of all evidently the use of codeshares and the Skyteam alliance increases the number of destinations N and number of direct connections K offered by the airlines. The number of hubs is increased as well as the size of existing hubs leading to a higher value of $\langle k \rangle$, this also leads to an inreased diversity of the network offering on average more direct destinations per airport. The average frequency per destination and seats per flight remain relatively unchanged for the parent networks.

Using codeshares and the Skyteam alliance also increases the small-world property of the network by increasing the total clustering coefficient C. Since the number of hubs is increased the average shortest path length L is slightly increased, but remains relatively low. The Skyteam alliance in particular has a very high clustering, making the network bahave more like a PP.

According to the degree and strength exponent γ the natures of the networks remain relatively unchanged ($\gamma \approx 2$) pointing to a HS. Figure 3.5 shows on top the cumulative degree distribution of both the codeshare and Skyteam networks and the separate parent airlines associated with these networks. Two different behaviours can be observed by the introduction of the codeshares and Skyteam alliance.

In the case of Alitalia the introduction of CT into the network does not change the curve of the cumulative degree distribution, the combined $\mathrm{AZ_{TOT}}$ network still has near linear behaviour pointing to a HS with multiple smaller hubs. Additionaly in the case of Aeroflot the combined $\mathrm{SU_{TOT}}$ network remains a network with one main hub and some very small hubs, still having an upward concave curve.

In the other cases ($\mathrm{AF_{TOT}}$, $\mathrm{AF\text{-}KL_{TOT}}$) combining the parent airlines with the subsidiary airlines and eventually into the total Skyteam network changes the curve of the cumulative degree distribution moving it to the right and decreasing the intial drop of the curve implying a network which is closer to PP. For the skyteam network almost 20% of the airports have a degree of 10 or higher, this mainly due to an increased size of the already existing airports when combining the networks.

The effects of including weights is similar to that in the separate airline networks. The effect of including airlines with on average smaller amount of seats per flight is too small to dilute the average number of seats per flight (see also table 3.3), hence the effect on the cumulative strength distribution is negligible.

3.5 Conclusions

In this chapter the topology of the European network of 17 seperate airlines including both FSCs and LCCs was analyzed using complex network indicators. This in order to distinguish between the different business models used by the airlines. Additionaly the influence of using codeshares and an alliances on the topology of the airline networks was analyzed.

The analysis confirms that FSCs use a HS network, however the networks are very diverse, from a HS with only one hub and no interconnected airports such as the network of KLM, up to a multi-hub HS network with several large interconnected hubs, such as the network of Air Europa. The degree distribution analysis confirms the results and characterizes the network as a SFN, in literature associated with HS. In general the FSCs tend to have a focus on frequency over diversity in their routing. Subsidiaries are used on one hand to service focus areas, such as Air Corsica and on the other hand to guarantee regular frequency of flights on less busy routes using a lower number of seats per flights (regional aircraft type).

The analysis does not irrefutably link LCCs and PP. Both LCCs U2 and VY are found to be HS with a high amount of big interconnected hubs. Although it can be concluded that the behaviour of LCCs is closer to PP, it is blunt to state that LCC have a RN as suggested in literature. The degree distribution analysis point to a SFN, with high small-world properties. LCCs in contrast to the FSC tend to have a focus on diversity over frequency in their routing, offering a variaty of destinations. The HS behaviour of LCCs can be explained by the fact that the carriers use hubs as operational bases, for example for maintenance. LCCs use a similar number of seats per flight as the parent FSCs, pointing to a similar aircraft type.

The codeshare and Skyteam network analyses yield several conclusions. First of all using codeshares and an alliance increases the diversity of the network, more direct connections between airports and more destinations serviced. Secondly it combines the hubs of the separate networks and in general the combined network contains bigger more interconnected hubs, increasing the small-world effect and bringing the behaviour closer to that of a LCC. Compared to LCCs however, the focus is still on frequency rather than diversity of the routing.

Some practical conclusions can be drawn from the analysis. The degree exponent γ does not clearly distinguish between the networks. Plotting the cumulative degree distribution gives a clearer image of the difference between the networks. Using the strength distribution function increases the resolution of the results, however the behaviour of the networks remain largerly the same and the networks are less clear to distinguish, so the degree distribution function is preferred.

Now that the topology of the airline networks is investigated, what are the effects of these properties on the robustness of the networks? This will be treated in the next chapter.

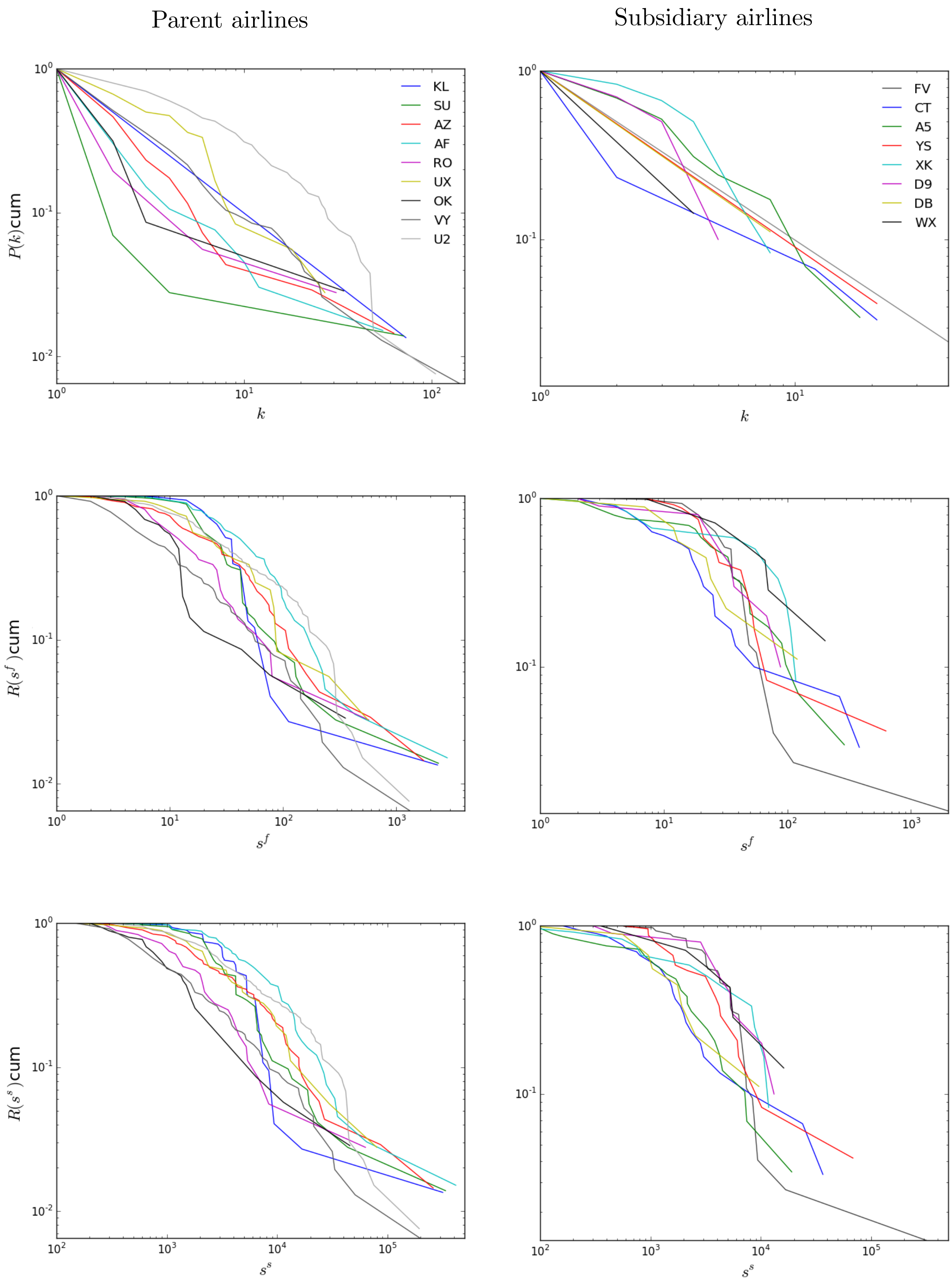


Figure 3.4: Cumulative degree distribution of the separate airline networks (unweighted and weighted). On the left side the parent airlines, on the right side the subsidiary airlines

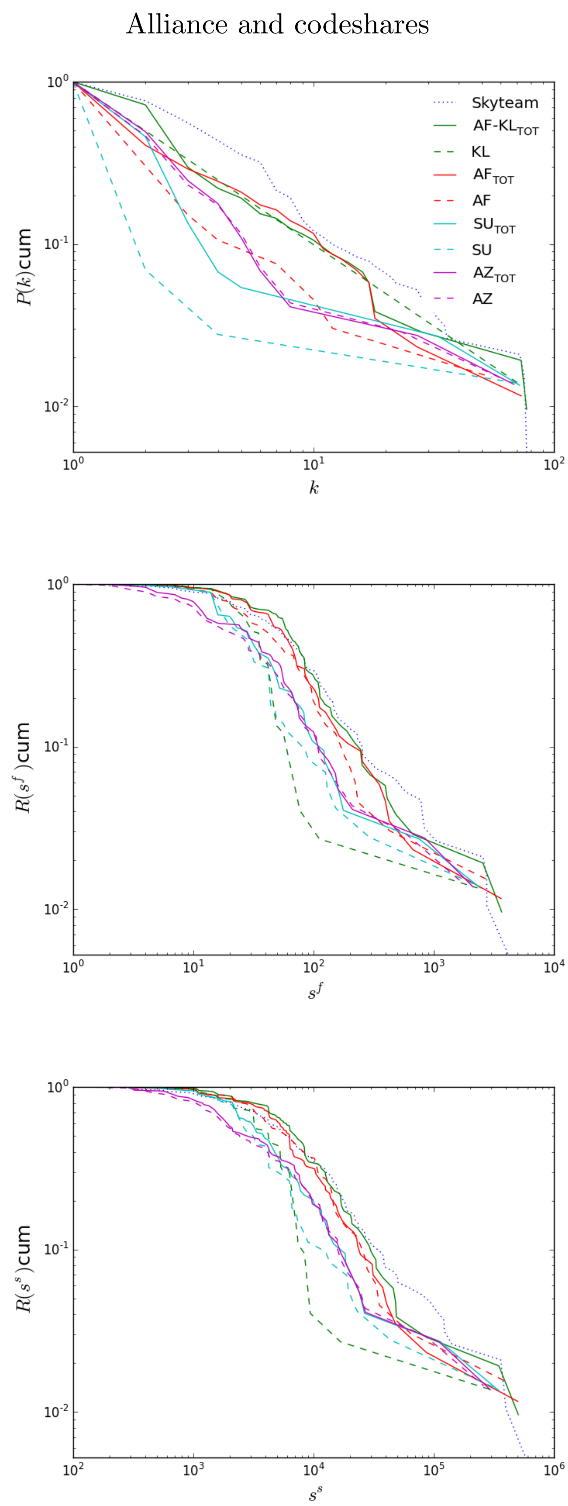


Figure 3.5: Cumulative degree distribution of the Skyteam alliance and codeshare networks compared to the separate parent airline networks

Part III

Robustness Analysis

Chapter 4

Synthetic Static Robustness of European Airline Networks

4.1 Abstract

The first aim of this chapter is to analyze the synthetic static robustness of the seperate airline, codeshare and the Skyteam alliance networks in order to distinguish between the robustness behaviour of both FSCs and LCCs. The second aim is to study the link between the complex network indicators investigated in Chapter 3 and the synthetic static robustness of the ATN. This is performed by simulating error and attack on the separate airline networks and the codeshare and Skyteam alliance networks. Error is based on the random removal of airports from the network, while attack is based on the consecutive removal of nodes based on the heights of the degree, seat strength and (weighted) betweenness of the airports. The analysis confirms literature regarding FSCs, which shows low robustness against attack and high robustness against error. LCCs again show similar behaviour as the FSCs, contradicting literature, but confirming the results from Chapter 3. The shape of the curve of the cumulative degree distribution can be directly linked to the robustness independant of the size of the network. The higher the amount of hubs (with relative high degree), the higher the robustness against attack. The robustness against error is much higher and similar for all networks. Combining networks into codeshare networks or the Skyteam alliance, will increase the robustness against attack.

4.2 Introduction

Airports and airspaces can be closed temporarily for various reasons, such as terrorist attack, weather, environmental accidents, strikes etc. Temporal closure of airspace or airports is both costly for the industry (e.g. grounded aircraft / damage to reputation) and the economy (e.g. reduced mobility of the public) (Guimera and Amaral, 2004). Robustness analysis deals with the consequences of these events on networks. It composes of the analysis and classification of networks into robust and vulnerable and tries to find solution to increase the robustness.

Several robustness studies using complex network theory have been conducted over the years, linking the topology of a network to its robustness. The main conclusions regarding ATNs, linked to the two business models (HS and PP) are:

HS Linked to SFN and a relatively higher robustness against error and a lower robustness against attack in comparison to RNs

LCC Linked to RN and a relatively higher robustness against attacks and a lower robustness against error in comparison to SFNs.

One of the few analysis of the robustness of airline networks is conducted in the thesis by Lordan (2014) based on the betweenness. Confirming that FSCs are more vulnerable to attacks and less vulnerable to error than LCCs and suggesting several ways for airlines to improve their networks.

The first aim of this chapter is to analyze the synthetic static robustness of the seperate airline, codeshare and the Skyteam alliance networks in order to distinguish between the robustness behaviour of both FSCs and LCCs. The second aim is to study the link between the complex network indicators investigated in Chapter 3 and the synthetic static robustness of the ATN. This is performed by analyzing the effects of errors and attack in the ATN.

4.3 Methodology

4.3.1 Synthetic Static Robustness Analysis

In this chapter a synthetic static robustness analysis is conducted of the 17 European airline networks introduced in Chapter 3. Static robustness refers to the removal of nodes or links, without incorporating the redistribution of the flow over the newly formed network. Synthetic refers to the standard way of assessing the robustness using complex network indicators as described in current literature (see also Section 2.5). Two different ways of damage are considered in this study; firstly error or random failures; and secondly premeditated attacks.

The airline networks are modelled as simplified networks as elaborately explained in Section 3.3. Next to the analysis of the robustness of the independent airline networks, the European Skyteam alliance network as well as the combinations of parent and subsidiary airlines are also analyzed.

Error

Error or random failure is based on the consecutive random removal of airports from the ATN, simulating random events such as for example the closure of an airport due to bad weather. In this chapter the average value over 100 iterations of random airport removal per network is used as error scenario.

Attack

Attack is based on consecutive removal of airports from the ATN. Starting with the removal of the most important airport, the complex network indicators are recalculated. After this the new most important airport is removed. This process is repeated until the network is completely disconnected (for the Python program see Appendix B.2). The following criteria are used for the attack on the networks (the sequence of removal of airports):

k **Degree**
The degree k_i of an ATN represents the number of flights legs connected to the airports, in other words the average number of destinations per airport. For an individual airport i the degree k_i is defined in terms of the adjacency matrix A with elements a_{ij} as:

$$k_i = \sum_{j \in N} a_{ij} \tag{4.1}$$

s^s **Seat Strength**
The seat strength s^s represents the number of direct seats connected to each airport. It is defined for an individual airport i as, using the number of seats as weight w_{ij} respectively:

$$s_i^s = \sum_{j \in N} w_{ij} \tag{4.2}$$

b **Betweenness**
The betweenness is a centrality measure representing the importance of an airport for movements inside the network. $n_{jk}(i)$ is the number of shortest paths connecting airport j and k running through airport i and n_{jk} is the total number of shortest paths between airport j and k.

$$b_i = \sum_{j,k \in N, j \neq k} \frac{n_{jk}(i)}{n_{jk}} \tag{4.3}$$

b^s **Seat Betweenness**
The seat betweenness is a centrality incorporating the weights (seats) per flight leg, thus putting more emphasis on important flight legs. $n_{jk}^s(i)$ is the number of weighted shortest paths connecting airport j and k running through airport i and n_{jk}^s is the total number of weighted shortest paths between airport j and k.

$$b_i^s = \sum_{j,k \in N, j \neq k} \frac{n_{jk}^s(i)}{n_{jk}^s} \tag{4.4}$$

4.3.2 Size of Giant Component

To assess the robustness of the ATN the percentage and number of airports removed is compared to the size of the giant component. The size of the giant component is the percentage of passengers still connected within the networks after removal of a percentage/number of airports. In case of the degree and betweenness passengers are represented by the amount of flight legs, while in flight strength and seat strength they are represented by the amount of flights and seats, respectively. In this way the direct impact of error or attack on the capactiy of the network is measured.

4.3.3 f-values

The percentage of airports removed is often called the f-value. Different f-values can be evaluated, for example the $f_{0.05}$-value is the percentage of airports removed in order to have 5% of the giant component left. This value represents an almost completely disconnected ATN and is used to measure the difference between ATNs under attack (higher meaning more robust). The value of 5% is chosen because it was used in one of the few studies of the robustness of airline networks by (Lordan, 2014) and makes the results in this chapter comparable to the existing literature. To measure the difference between ATNs under error the $f_{0.95}$-value is used.

4.4 Results

The resulting f-values of the robustness analysis of the different separate airline, codeshare and Skyteam alliance networks are depicted in table 4.1. For attack the $f_{0.05}$-values for the four different criteria are depicted, for error the $f_{0.95}$-values are shown. The evolution of the size of giant component (y) over the amount of airports removed (x) is also plotted. Figure 4.1 displays the plots for the separate parent and subisidiary airlines and figure 4.2 the plots for the codeshare and Skyteam alliance networks. The following sections first investigate the error and attack behaviours of the different networks. After which in Section 4.4.3 the outcome of the robustness analysis is compared to the outcome of the topology analysis in Chapter 3.

4.4.1 Error

First some general result observations. The results of the error and attack simulations confirm that in general ATNs are much more robust against error than against attack. This is illustrated by the comparison of the dashed lines and the solid lines in the plots in figure 4.1 and 4.2. Furthermore the differences in behaviour of networks under attack is more diverse, than under error. The $f_{0.95}$-values for all airlines is between 5 and 8%, with the three smallest airline networks D9, DB and WX as an exception. This mainly due to the size of the network and the according limited resolution of the plot. The LCCs VY and U2 show a value of 8 and 5% respectively, similar to the FSCs. This confirms that the LCCs do not have a RN, for which from literature a lower robustness against error is expected.

Table 4.1: Overview of the f-values of an attack on and error of the separate airline, Skyteam alliance and codeshare networks. Attack based on: Degree k, seat strength s^s and betweenness (unweighted b and weighted with seats b^s). Error over 100 runs. The table shows the $f_{0.05}$-values for attack and $f_{0.95}$-values for error

	k	s^s	b	b^s	error
Separate parent airlines					
KL	0.01	0.01	0.01	0.01	0.05
SU	0.01	0.01	0.01	0.01	0.06
AZ	0.12	0.04	0.10	0.04	0.07
AF	0.08	0.05	0.08	0.05	0.08
RO	0.17	0.06	1	0.06	0.06
UX	0.25	0.17	0.31	0.28	0.06
OK	0.09	0.03	0.09	0.03	0.06
VY	0.08	0.06	0.08	0.07	0.08
U2	0.16	0.14	0.16	0.16	0.05
Separate subsidiary airlines					
FV	0.03	0.03	0.03	0.03	0.06
CT	0.07	0.07	0.07	0.07	0.07
A5	0.28	0.21	0.28	0.31	0.03
YS	0.04	0.04	0.04	0.04	0.08
XK	0.33	0.33	0.33	0.5	0.08
D9	0.60	0.50	1	1	0.10
DB	0.11	0.11	0.11	0.11	0.11
WX	0.43	0.14	1	0.14	0.14
Skyteam alliance and codeshare networks					
Skyteam	0.17	0.06	0.15	0.07	0.06
$\text{AF–KL}_{\text{TOT}}$	0.10	0.04	0.09	0.09	0.07
AF_{TOT}	0.12	0.05	0.12	0.10	0.07
SU_{TOT}	0.05	0.03	0.04	0.03	0.07
AZ_{TOT}	0.11	0.03	0.10	0.03	0.08

4.4.2 Attack

The results of the attack simulations confirm literature (see Section 2.5.3) showing that the betweenness b is in general a better indicator of robustness against attack than the degree k. The size of the giant component is faster reduced to 5% by using the betweenness as criterium for the attack rather than by using the degree as criterium. Including the seats as weight into the analysis shows that in most cases the seat strength s^s is an even better indicator of robustness. The weighted betweenness b^s however, shows no improvement in the results and is not a better indicator of robustness. Overall the best indicator of robustness against attack in the study of these ATNs is the seat strength and therefore it will be used to compare the networks.

The smaller networks (subisidiary) in general have a higher value of $f_{0.05}$ than the larger parent airline, codeshare and Skyteam alliance networks. In percentual sence the small networks are more robust than the bigger networks. For example the HSs with only one pure hub KL and YS have a $f_{0.05}$-value of 1 and 4% respectively. If however the real number of airports removed is reviewed, rather than the percentage of airports both networks have only 1 airport removed. A more extreme example is the comparison between U2 and WX, both having a $f_{0.05}$-value of 14%. In case of EasyJet this is equal to 19 airports, while in case of CityJet this is equal to 1 airport. While comparing f-values the size of the network under review should therefore always be taken into account. For this reason first the results of the subsidiary networks are reviewed, after which the results of the parent and the codeshare and Skyteam alliance network will be analyzed.

Firstly the subsidiary airline networks. Three groups can be distinguished: one group with relative low robustness, one group with relative high robustness and one intermediate group. FV, YS, DB and WX have the relative lowest $f_{0.05}$-values, looking at figure

4.1, these networks lose more than 95% of their giant size component when removing 1 airport. This points to a HS with one major hub. CT has one major hub, which if removed reduces the giant size component by 70% and can also be added to this group. A5 has an intermediate value, losing 95% of the giant component after disconnecting 6 airports, this points to a multi-hub network, with several medium sized hubs. XK and D9 have a very high $f_{0.05}$ value, which is in case of XK losing one third of his network and in case of D9 losing half the network. This points to a relative high robustness against attack associated with RNs and PP.

Secondly the parent airline networks. Again the same three groups can be distinguished. KL, SU and OK having a low robustness against attack, only after removing 1 airport these networks lose more than 95% of their giant size component. Tarom can also be included in this group, since after removal of 1 airport it loses just a little under 95% of its giant component (see figure 4.1). The intermediate airlines include AZ, AF and having $f_{0.05} \approx 5\%$, pointing to a network with multiple medium hubs, a removal of three airports results in a breakdown of the network until only 5% of the giant component is left. The last group are the most robust airlines and include VY, UX and U2. Although the $f_{0.05}$-value of VY is similar to AZ and AF, the network is much larger and around 8 airports need to be removed to reach 5% of the giant size component. For Air Europa this value is 6 airport. Both networks seem to have a similar structure with several big hubs. EasyJet has the most robust network, almost 19 airports need to be removed before 5% of the giant size component is reached. This behaviour is associated with PP.

Thirdly the codeshare and Skyteam alliance networks. In general the $f_{0.05}$-value is slightly higher after formation of the codehare of Skyteam alliance. Nevertheless, since the size of the network has grown as well, the influence on the robustness is significant. The number of hubs is increased, leading to a more robust network against attack.

4.4.3 Linking Synthetic Robustness and Topology

Several qualitative links can be drawn between the synthetic robustness of an ATN and its (cumulative) degree distribution. The analysis in the previous section reviewed the attack on the network using seat strength. The compariosn of the cumulative strength and degree distribution showed the same sets of network types in chapter 3. Hence a qualitative comparison of the degree distribution and attack on network using seat strength is justified.

The degree exponent γ for most airlines (including both LCCs) was shown to have a $\gamma \approx 2$, with several below 2. Two FSC airlines XK and D9 have a $\gamma > 3$ pointing to a RN and PP. Both XK and D9 also have a relative high $f_{0.05}$ value. This points to a relative high robustness against attack associated with RNs and PP. The error simulations of these networks however, also show a higher $f_{0.95}$-value, which contradicts the results and is associated with SFNs and HS. It is therefore more likely that the divergent values have to do with the limited size and thus resolutions of the plots of these networks. Furthermore a network with relative high robustness UX, has a very low value of γ. Overall γ does not seem to be a good indicator for robustness of the ATN.

The cumulative degree distribution curves presented in chapter 3 were divided into four different network types, based on the shape of the curve. These curve shapes can be directly linked to the relative robustness of the networks, independant of the size of the network.

Firstly the linear curves (KL, DB, YS and WX) are the airline networks with one main hub and all show relative low robustness against attacks. Connected to this are the upward concave curves (SU, RO, OK, FV, CT) with one major hub and some interconnected flights, which also show low robustness against attack.

Secondly the near linear curves (AZ, AF, VY and A5) are airline networks with multiple smaller hubs and intermediate robustness against attack. It should be noted that, although Vueling shows near linear behaviour, more than 20% of the airports have a degree of 5 or higher. This is also confirmed by the robustness results showing that the robustness against attack of VY is much higher than for the other airlines in the group.

Thirdly the downward concave curves (U2, UX, XK and D9) are airline networks with a relative high amount of airports with a higher degree. These networks also show the highest robustness against attack. U2 and UX however still have a robustness against attack associated with HS, while XK and D9 show clear behaviour of a RN and PP.

Overall all networks (except XK and D9) have a low robustness against attack, which confirms the results of the previous chapter classifying these networks as SFNs and HSs. It therefore also confirms the result that LCCs do not have RNs.

4.5 Conclusions

In this chapter the robustness of the European network of 17 seperate airlines including both FSCs and LCCs was analyzed by simulating error and attack on the ATNs. Error was based on the random removal of airports from the network, while attack was based on the consecutive removal of nodes based on the heights of the degree, seat strength and (weighted) betweenness of the airports. The best criterium of attack turns out to be the seat strength and is therefore used to compare the results if the robustness analyses of the different ATNs. Additionaly the influence of using codeshares and an alliances on the robustness of the airline networks was analyzed. Finally a comparison was made between the topology analysis in Chapter 3 and the robustness analysis in this chapter.

The analysis confirms literature that FSCs have a low robustness againt premeditated attacks, such as terrorist attacks, but are very robust against error, such as a random airport closure due to bad weather. Just as the topology analysis showed a range of FSCs with corresponding indicators, there is also a range in the robustness against attack for these airlines. In general the bigger the network and the higher the number of (equally sized) hubs, the more robust the network against attack. The effect of codeshares and the Skyteam alliance is just the afforementioned and has therefore a positive effect on the robustness against attack. The robustness against error is relatively similar for all FSC and very high compared to the robustness against attack as expected for HS.

The analysis shows that the LCCs VY and U2 show similar behaviour as the FSCs. Being bigger of size and having more equally sized hubs, results in a relative higher robustness against attack. The typical PP behaviour of high robustness against attack and relative low robustness against error are not directly confirmed. One can say that although both networks are closer to PP than the FSC, they still show the characteristics regarding robustness of SFNs and thus HS. XK en D9 seem to show PP behaviour, with very high $f_{0.05}$-values against attack. Nevertheless their $f_{0.95}$-values against error are also relatively high, which is not typically associated with PP. These results could have to do with the small size of the networks and thus bad resolution of the results.

A clear link is established between the cumulative degree distribution curve and the robustness against attack behaviour of the network. The shape of the curve can be directly linked to the robustness independant of the size of the network. From linear and downward concave curves up to near linear and upward concave curve the robustness of the network generally increases. In other words a high amount of hubs (with relative high degree) leads to a higher robustness against attack. The degree exponent γ again shows not to be a good indicator, being incapable of distinguishing between the robustness of HSs.

Several recommendations can be made to help airlines increase their robustness. The most important recommendation is to increase the number of hubs in the network. One of the solutions for this is the formation of codeshares and alliances, which is shown to increase the robustness of the network. Also the division of passengers over the hubs should be homogenous further increasing the robustness of the network against attacks. Nevertheless synthetic static robustness analysis is a quite general way of analyzing networks. In real networks also dynamics play an important role. Increasing the total clustering of the network for example will result in increased alternative routes between airports and will improve the real dynamic robustness of the airline network. Although both static simulations can be linked to real phenomena the question remains are there other, better methods to simulate real attack and error scenarios of ATN? This will be the subject of the next chapter.

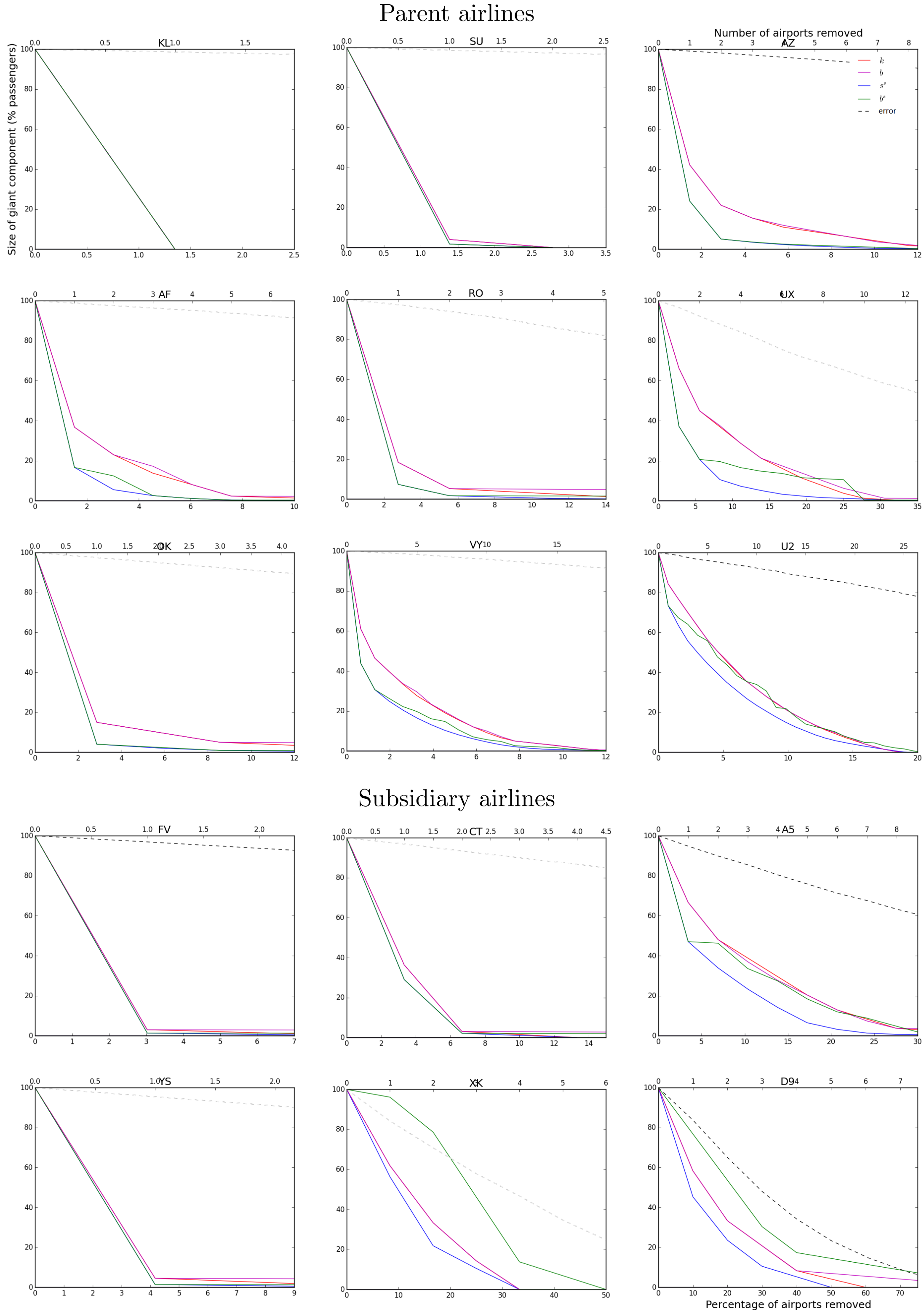


Figure 4.1: Robustness analysis of attack on and error of the separate airline networks. Attack based on: Degree k, seat strength s^s, betweenness (unweighted b and weighted with seats b^s). Error over 100 runs

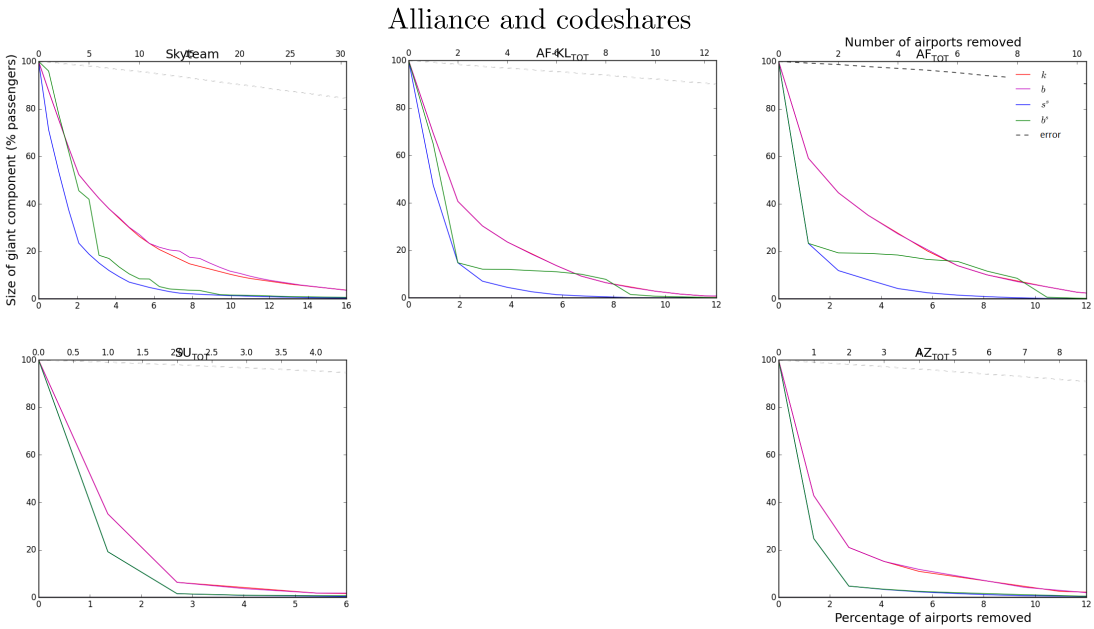


Figure 4.2: Robustness analysis of an attack on and error of the Skyteam alliance and codeshare networks. Attack based on: Degree b, seat strength s^s and betweenness (unweighted b and weighted with seats b^s). Error over 100 runs

Chapter 5

Realistic Static Robustness of European Airline Networks

5.1 Abstract

The first aim of this chapter is to improve the robustness analysis of ATNs by introducing new methods of simulating more realistic error and attack scenarios. The second aim is to study the link between the realistic robustness analysis and the synthetic robustness analysis in Chapter 4. Three different phenomena are simulated: weather, strikes and volcano eruptions. Weather and volcano eruptions are simulated using the introduced geographic attack. Geographic attack is based on starting at an initiation airport and removing the other airports using geographic radial spreading. Strikes are simulated using the geographic degree, which groups the airport into FIR. The analysis puts the synthetic robustness in perspective. It shows that not only the number of hubs is important in order to improve the robustness of an ATN, but also the geographic spreading of the hubs.

5.2 Introduction

The synthetic analysis of the robustness of ATNs in Chapter 4 uses two very general ways to simulate error and attack on networks. These methods do not necessarily represent realistic scenarios regarding the robustness of ATNs. This chapter focusses on including more realistic attack and error scenarios to evaluate the robustness of ATNs.

Only limited research into improving the reality of robustness analysis has been conducted. One of the few recent studies already using a more realistic approach is the study by Wilkinson et al. (2011). In his paper he studies the effect of the vulcano eruption in 2010 on the European air traffic network. The regional airport network is attacked by exposing the network to failure starting from the location of the volcano and expanding circular into the network, for the first time including spatiality into the simulation. The study demonstrates that the effect on the European air traffic was disproportionately severe due to the network possessing a truncated, scale-free distribution and a spatial degree distribution that is uniform with distance from the centre of the network, resulting in a network that is vulnerable to spatial hazards.

The first aim of this chapter is to improve the robustness analysis of ATNs by introducing new methods of simulating more realistic error and attack scenarios. The second aim is to study the link between the realistic robustness analysis and the synthetic robustness analysis in Chapter 4.

5.3 Methodology

5.3.1 Realistic Robustness Analysis

In this chapter a more realistic way of assessing robustness is proposed. Using these robustness analysis methods a selection of the 17 European airline networks introduced in Chapter 3 are analyzed. Next to the analysis of the independent airline networks, the European Skyteam alliance network as well as a selection of the codeshare networks are analyzed. The airline networks are again modelled as simplified networks as elaborately explaned in Section 3.3.

Geographic Error and Attack

Geographic error and attack refers to a different way of simulating the breakdown of the ATN. Many of the problems leading to network breakdown for airlines are related to the geographic location of an airport. For example strikes can hit an entire country or region. Also the spreading of some of these phenomena is geographic in nature. For example bad weather travels to another area (storms) or expands (mist) in a certain area. A volcano erupts and the clouds are blown by the wind, hence geographic spreading. Attacking a network geographically incorporates these geographic correlations between airports. In the following sections several real phenoma are simulated using geographic clustering (strikes) or spreading (weather and volcanoes).

5.3.2 Weather and the Geographic Concentration

In order to simulate weather phenomena such as storm or mist the network is attacked spreading radially from an initiation airport. In other words the initiation airport is first removed, after this the airport geographically closest to the initiation point is removed, then the second closest and so on, until the network is completely broken down (for the Python program see Appendix B.3). Four different scenarios are used to determine the initiation point of the simulation. Two represent more or less the worst case scenario (betweenness and geographic clustering) and the two others represent a basic scenario (random and probability):

b **Betweenness**
The betweenness is a centrality measure representing the importance of an airport for movements inside the network. $n_{jk}(i)$ is the number of shortest paths connecting airport j and k running through airport i and n_{jk} is the total number of shortest paths between airport j and k. It is used to simulate a worst case scenario, bad weather hitting the most important spot in the network regarding movement.

$$b_i = \sum_{j,k \in N, j \neq k} \frac{n_{jk}(i)}{n_{jk}} \tag{5.1}$$

c_{GEO_i} **Geographic Clustering**
The geographic clustering c_{GEO_i} is a proposed measure of the centrality of an airport in the network regarding its geographic location, independant of flight links. The local geographic clustering of airport i is equal to 1 minus the sum of all distances from airport i to all other airports in the network, divided by the sum of all distances between all airports in the networks. In which $d_{\text{GEO}_{ij}}$ is the geographic distance between airport i and airport j. This is order to simulate weather hitting the geographically most dense spot in the network:

$$c_{\text{GEO}_i} = 1 - \frac{\sum_{j \in N} d_{\text{GEO}_{ij}}}{\sum_{i,j \in N, i \neq j} d_{\text{GEO}_{ij}}} \tag{5.2}$$

Random
The initiation point is randomly chosen to simulate the uncertainty of the location of everyday bad weather. The average value over 100 iterations of random airport removal per network is used.

Probability
The initiation point is chosen based on the probability of bad weather at an airport. This probability is based on the historic weather at all airport in Europe over the last year. Of course this should be expanded to make it more representable, but the data over the last year was used as a starting point for this study. Bad weather was defined as either mist (visibility below 1000m), thunder or storm (windspeed higher than 75km/h). The data was obtained from an online weather database. The average value over 100 iterations of airport removal by probability per network is used.

5.3.3 Strikes and the Geographic Degree

Two types of strikes are considered: strikes at an airport and strikes at flight control. Strikes at an airport are already simulated using the synthetic attack scenario, assuming that employees planning a strike are likely to choose locations with high importance to the ATN. Strikes at flight control are simulated using the geographic degree k_{GEO_k}. The geographic degree basically reduced the airport network to a Flight Information Region (FIR) network, grouping airports belonging to one region. The geographic degree of FIR k is defined as the number of the airports within the boundary of that FIR:

$$k_{\text{GEO}_k} = \sum_{i \in M} i \tag{5.3}$$

With help of this indicator, the attack is simulated in a similar way as a synthetic attack. First the FIR with the highest geographic degree is removed after which the FIR with the second highest value is removed, etc. This indicator gives an idea of the robustness of the airline network against FIR strikes.

5.3.4 Volcanoes

Volcanoes are simulated in a similar way as the weather phenomena, with the difference that the initiation point is the known location of a volcano. Of course in reality the impact of a volcano ash cloud is very much dependant on the windspeed and direction, however as a starting point radial spreading is used. Two of the most active volcanoes in Europe are used as initiation point: The Vesuvius in Italy and the Eyjafjallajökull on Iceland. For this robustness analysis the $d_{0.05}$-values are used, representing the distance from the volcano, the origin of the simulation, at which 5% of the giant component is left.

5.4 Results

The resulting f and d-values of the robustness analysis of the different scenarios are depicted in table 5.1. For attack the $f_{0.05}$-values for the four different criteria are depicted, for error the $f_{0.95}$-values are shown. In case of the volcano eruptions the $d_{0.05}$-values are depicted. The evolution of the size of giant component (y) over the amount of airports removed (x) is plotted in Figure 5.1 for the weather and strike robustness analyses. The evolution of the size of giant component (y) over the radius from the origin (x) is plotted in figure 5.2 for the volcano robusteness analysis. The following sections first investigate the weather, strike and volcano robustness results. After which in Section 5.4.4 the outcome of the realistic robustness analysis is compared to the outcome of the synthetic robustness analysis in Chapter 4.

5.4.1 Robustness against Bad Weather

The first observation looking at figure 5.1 is that in genaral the ATNs are much more robust against random or probability chosen point of initiation (dashed and dotted line), compared to the initiation based on the betweenness and geographic degree. Furthermore the initial behaviour of the random and probability scenario are very similar, suggesting that including probabilities of bad weather into the simulation will not necessarily increase the reliability of the analysis. All networks seem equally robust against the random and probability weather scenario, however some of the smaller networks are less robust. This result makes sense, since the scenario spreads geographically, small concentrated networks like YS and XK will be less robust.

The scenario using betweenness as initiation is by far the best scenario to simulate a worst case scenario regarding weather. The least robust networks in this scenario are again the networks with a limited number of hubs (SU, RO, FV and YS). This scenario is also a measure for the spreading of vulnerable airports. The more robust the network is in the analysis the better the spread of vulnerable airports. It should be noted that for a

Table 5.1: Overview of the f-values and d-values of simulated bad weather, strikes and volcano eruptions on the separate airline, Skyteam alliance and codeshare networks. Geographic attack based on: Geographic degree k_{GEO_k} to simulate strikes, geographic clustering c_{GEO_i} (combined with betweenness b_i) to simulate weather and two of the most active volcanoes in Europe. Geographic error over 100 runs based on random based and probability based initiation also to simulate weather. The table shows the $f_{0.05}$-values for attack, $f_{0.95}$-values for error and the $d_{0.05}$-values for the volcano scenarios

	Weather				Strike	Volcano	
	c_{GEO_i}	b_i	Random	Prob.	k_{GEO_k}	Vesuvius	Eyjaf.
Separate parent airlines							
SU	0.74	0.01	0.01	0.03	0.29	3000	6425
RO	0.36	0.08	0.03	0.03	0.19	1400	5525
UX	0.53	0.53	0.03	0.03	0.53	2050	3475
OK	0.06	0.03	0.03	0.03	0.17	1050	4075
VY	0.46	0.34	0.03	0.03	0.41	1950	4300
U2	0.61	0.42	0.02	0.02	0.7	2275	3750
Separate subsidiary airlines							
FV	0.15	0.03	0.03	0.03	1	2650	5600
CT	0.33	0.43	0.03	0.03	0.5	800	4300
A5	0.52	0.52	0.03	0.03	0.83	1625	3400
YS	0.5	0.04	0.04	0.04	1	1625	2975
XK	0.67	0.5	0.08	0.08	0.67	700	3725
D9	0.5	0.7	0.1	0.1	0.5	3000	6850
Skyteam alliance and codeshare networks							
Skyteam	0.85	0.81	0.03	0.03	0.72	2650	6300
AF–KL$_{\mathrm{TOT}}$	0.27	0.28	0.03	0.02	0.77	1700	3025
AZ$_{\mathrm{TOT}}$	0.12	0.23	0.03	0.03	0.32	800	4300

weather scenario in some cases only the first part of the graph is important. Geographic spreading of for example mist is usually limited to a certain distance after which it will not spread.

The scenario using geographic clustering shows results similar to the results of betweenness. Two ATNs stand out: SU and YS are far more robust using geographic clustering than using betweenness. This can be explained by the fact that in both networks the main hub airport is located relatively to the edge of the network. An initial attack on the most geographically concentrated airport will leave the hub relatively unharmed.

5.4.2 Robustness against Strikes

The $f_{0.05}$-values resulting from the robustness against strikes scenarios shows that the small networks are very robust against such an attack. These values however are completely unrepresentable for the realistic behaviour of the networks under strike. A better measure would be the number of strikes needed to remove 95% of the giant size component. The plots in figure 5.1 give a better image as it shows the steps of the robustness analysis. For example RO, OK, YS, XK, D9 and AZ$_{\mathrm{TOT}}$ show that a single removal of the most important FIR results in the breakdown of the network (visible as the first line segment in the plot). These networks have their main hub(s) located within one FIR making them extremely vulnerable for strikes. ATNs having a bigger spread of their vulnerable airports (higher $f_{0.05}$-value for the betweenness weather scenario) also have a higher robustness against strikes, for example the LCCs VY and U2 and the FSC UX.

5.4.3 Robustness against Volcano Eruptions

The results of the robustness against volcano eruptions (table 5.1) show the distance from the origin after which the size of the giant component is reduced to 5%. This represents the distance a potential ash cloud needs to travel to break down the ATN. Figure 5.2 shows the plots of the robustness against volcano eruptions. The vertical dashed line represents the maximum distance the ash cloud of the Eyjafjallajökull reached during the last eruption (Wilkinson et al., 2011). The (quite obvious) results is that ATN located close to volcanoes (CT, XK, AZ_{TOT}) are very quickly disconnected after a volcano eruption. Furthermore the results show that an eruption of the Vesuvius with an ash cloud of the magnitude of the Eyjafjallajökull in 2010 would have disastrous consequences for air travel in Europe, disconnecting almost all networks.

5.4.4 Linking Realistic and Synthetic Robustness

Comparing the results from the realistic robustness scenarios to the synthetic robustness scenarios the first thing that stands out is that the general behaviour is quite similar. Synthetic error shows little difference in behaviour between the networks and the random and probability initiated weather scenarios also show little difference between the networks.

Synthetic attack shows that the more hubs a network has, the more robust the network is against attack. The realistic results put this into perspective. First of all not only the number of hubs is important but also the placement and spacing of the hubs is crucial to the robustness of the ATN. The attack results show that having multiple hubs in one FIR decreases the realistic robustness (AZ_{TOT}), making the network more vulnerable for strikes. Additionally the weather and volcano attack show that geographically concentrating hubs or the entire network (XK, CT) will also decrease the realistic robustness, making the network more vulnerable for geographic natural phenomena such as ash clouds or storms. The realistic results finally show that ATNs, despite having relative good synthetic robustness, are always prone to large unavoidable phenoma such as a large volcano eruption.

5.5 Conclusions

In this chapter a more realistic way of assessing robustness was proposed. Using these robustness analysis methods a selection of the 17 European airline networks introduced in Chapter 3 were analyzed. A new way of attacking networks was proposed using and initiation airport and geographic radial spreading. Three different phenomena were simulated: weather, strikes and volcano eruptions.

The weather robustness analysis included the introduction of geographic clustering as a measure of the closeness of airport within a network, regardless of connections between the networks. The weather robustness analysis shows that using probability or random initiation will not lead to a very different outcome. It also confirms the results from the synthetic error simulation, which showed that all networks have a similar robustness against random failure events, such as bad weather. Both worst case weather scenarios using the betweenness and geographic clustering show that ATNs are less robust against bad weather starting at their most important or central airport.

The analysis shows similar relative results as the synthetic attack simulation, however an important lesson can be learned from the geographically spreading attack scenarios. Not only the number and relative size of the hubs determines the robustnes of both FSCs and LCC, but also the geographic spreading of these hubs. The bigger the spread of the hubs, the more robust the network. (Also the bigger the entire network, the more robust.) A good example of this is AZ_{TOT}. Under the synthetic attack it shows similar robustness as the AF-KL_{TOT} network. The spreading of KLM-Air France's hubs over France and the Netherlands however makes the network in reality much more robust, compared to Alitalia with his major hubs located within (an area of) Italy. The realistic weather analysis therefore puts the synthetic results in a better perspective.

The strike robustness analysis adds to this that apart from the geographic spreading in general, also the spreading over FIRs, influence the real robustness of an ATN. Again especially AZ_{TOT} turns out to be relatively much less robust under the realistic scenario, compared to the synthetic attack scenario. LCCs have a big advantage regarding realistic robustness as usually they have a good geographic spread of their hubs, also with regards to FIRs. Small geographically concentrated networks again turn out relatively vulnerable. For example XK almost completely located within one FIR.

The volcano robustness analysis confirms the results from (Wilkinson et al., 2011) and shows that an eruption of the Vesuvius of a similar size of the Eyjafjallajökull will have disastrous consequences for air travel through Europe for all ATNs. This realistic scenario put the synthetic results in perspective, showing that some natural events such as volcano eruptions are unavoidable and designing a robust network against these phenomena is impossible.

Several additional recommendations can be made to help airlines increase their robustness. Apart from increasing the number of hubs, when designing a ATN also the spreading of the hubs should be taken into account. Again a way to cope with this is to form strategic partnerships with airlines from a diffrerent region of Europe for example AF-KL_{TOT}. Once more the importance of codeshares and alliances is underlined in improving the robustness of ATNs.

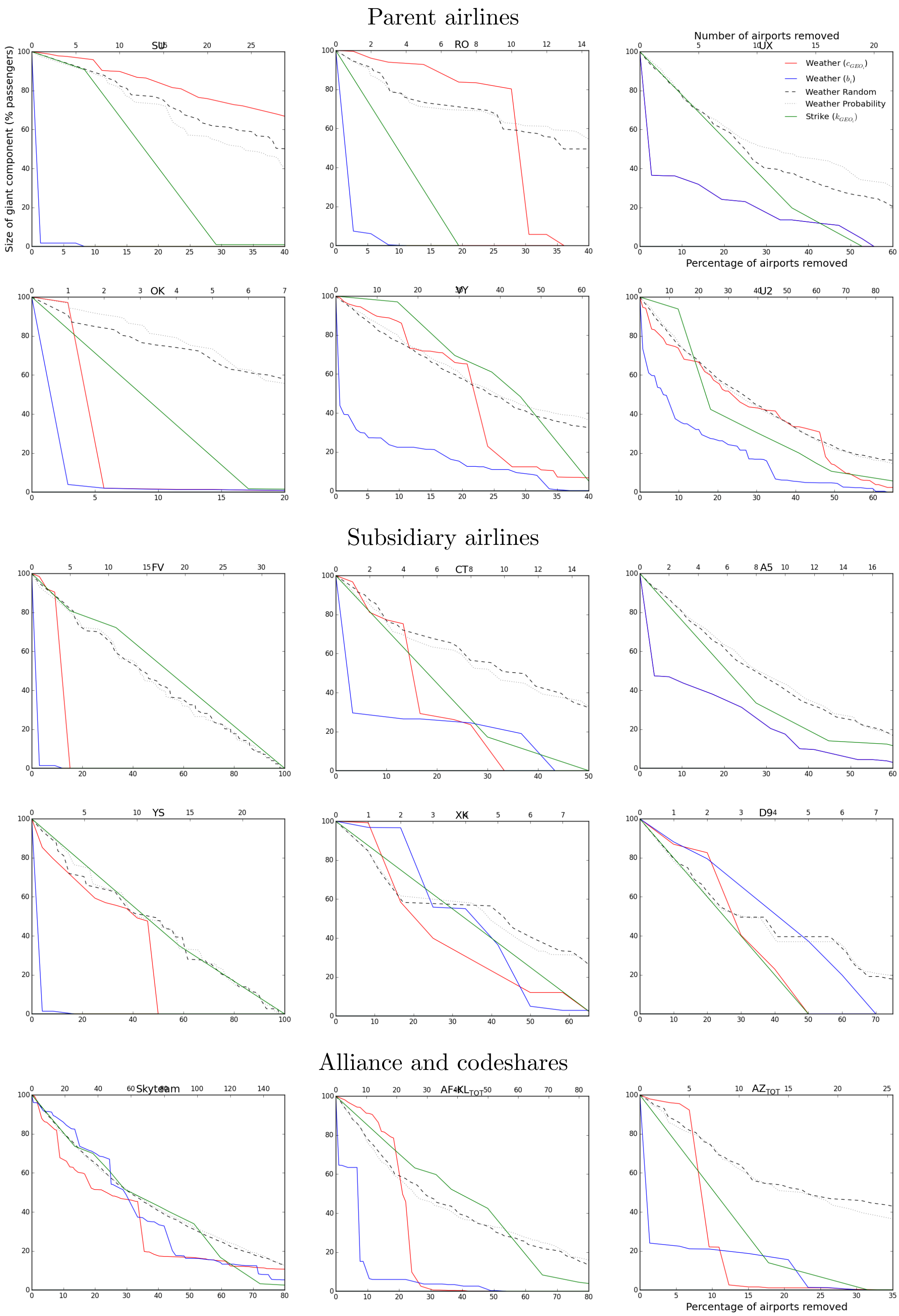


Figure 5.1: Robustness analysis simulating bad weather and strikes using geographic attack on and error of the separate airline, Skyteam alliance and codeshare networks. Geographic attack initiation based on: Geographic clustering c_{GEO_i}, betweenness b_i and geographic degree k_{GEO_k}. Error over 100 runs

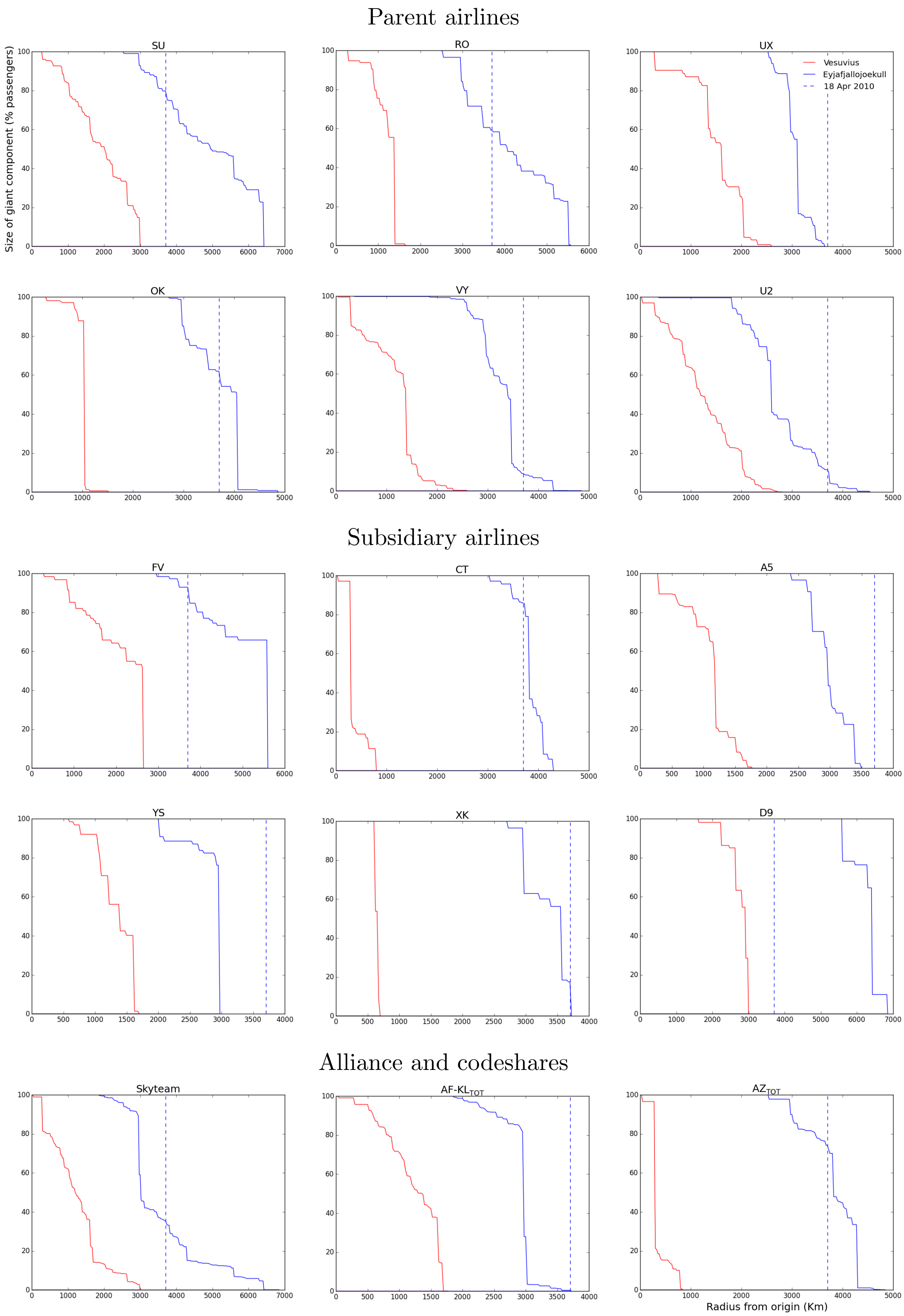


Figure 5.2: Robustness analysis simulating volcano eruptions using geographic attack on the separate airline, Skyteam alliance and codeshare networks. Geographic attack initiation based on the Vesuvius and Eyjafjallajökull

Part IV

Conclusions and Recommendations

Chapter 6

Conclusions

A reviews of current literature shows three levels of study: the global airport network (L1), the airline alliance network (L2) and the single airline network (L3). Most research has been conducted on the global and regional level (L1) and has mainly focussed on topology analysis (see table 2.1). The study on alliances (L2) has not been fully developed yet (only covered by Reggiani et al. (2009) and Lordan, Sallan, et al. (2015)) and some research has been carried out on the level of the airlines (L3) by Reggiani et al. (2009), Reggiani et al. (2010), Han et al. (2009) and Lordan (2014). In a way analysis on the global level is finished and the focus should be on the analysis of alliances and airlines.

Studies suggest two distinct network (business) solutions: point-to-point networks (PPs), in which the airports are connected with direct flights and hub-and-spoke networks (HSs), in which airports are connected through a hub airport (see figure 3.1).

Complex network theory offers indicators to distinguish between the different airline types. Using complex network theory three different network topology types were characterized linked to the respective business models, with the following properties:

RN RNs are linked to PP and LCCs. A higher robustness against attacks and a lower robustness against error in comparison to SFNs is found.

SWN SWNs are linked to the small-world properties: high clustering and a low average path length.

SFN SFNs are linked to hub-and-spoke networks and FSCs. A higher robustness against error and a lower robustness against attack in comparison to SFNs is found. Often ATN with a SFN also exhibit small-world properties.

The topology analysis of the European network of 17 seperate airlines including both FSCs and LCCs in Chapter 3 confirms that FSCs use a HS network, however the networks are very diverse, from a HS with only one hub and no interconnected airports such as the network of KLM, up to a multi-hub HS network with several large interconnected hubs, such as the network of Air Europa. The degree distribution analysis confirms the results and characterizes the network as a SFN, in literature associated with HS. In general the FSCs tend to have a focus on frequency over diversity in their routing. Subsidiaries are used on one hand to service focus areas, such as Air Corsica and on the other hand to guarantee regular frequency of flights on less busy routes using a lower number of seats per flights (regional aircraft type).

The analysis does not irrefutably link LCCs and PP. Both LCCs U2 and VY are found to be HS with a high amount of big interconnected hubs. Although it can be concluded that the behaviour of LCCs is closer to PP, it is blunt to state that LCC have a RN as suggested in literature. The degree distribution analysis point to a SFN, with high small-world properties. LCCs in contrast to the FSC tend to have a focus on diversity over frequency in their routing, offering a variaty of destinations. The HS behaviour of LCCs can be explained by the fact that the carriers use hubs as operational bases, for example for maintenance. LCCs use a similar number of seats per flight as the parent FSCs, pointing to a similar aircraft type.

The codeshare and Skyteam network analyses yield several conclusions. First of all using codeshares and an alliance increases the diversity of the network, more direct connections between airports and more destinations serviced. Secondly it combines the hubs of the separate networks and in general the combined network contains bigger more interconnected hubs, increasing the small-world effect and bringing the behaviour closer to that of a LCC. Compared to LCCs however, the focus is still on frequency rather than diversity of the routing.

The synthetic robustness analysis of these European network is conducted in Chapter 4. Static robustness refers to the removal of nodes or links, without incorporating the redistribution of the flow over the newly formed network. Synthetic refers to the standard way of assessing the robustness using complex network indicators as described in current literature (see also Section 2.5). Two different ways of damage are considered in this study; firstly error or random failures; and secondly premeditated attacks.

The study confirms literature that FSCs have a low robustness againt premeditated attacks, such as terrorist attacks, but are very robust against error, such as a random airport closure due to bad weather. Just as the topology analysis showed a range of FSCs with corresponding indicators, there is also a range in the robustness against attack of these airlines. In general the bigger the network and the higher the number of (equally sized) hubs, the more robust the network against attack. The effect of codeshares and the Skyteam alliance is just the afforementioned and has therefore a positive effect on the robustness against attack. The robustness against error is relatively similar for all FSC and very high compared to the robustness against attack as expected for HS.

The analysis shows that the LCCs VY and U2 show similar behaviour as the FSCs. Being bigger of size and having more equally sized hubs, results in a relative higher robustness against attack. The typical PP behaviour of high robustness against attack and relative low robustness against error are not directly confirmed. One can say that although both networks are closer to PP than the FSC, they still show the characteristics regarding robustness of SFNs and thus HS. XK en D9 seem to show PP behaviour, with very high $f_{0.05}$-values against attack. Nevertheless their $f_{0.95}$-values against error are also relatively high, which is not typically associated with PP. These results could have to do with the small size of the networks and thus bad resolution of the results.

A clear link is established between the cumulative degree distribution curve and the robustness against attack behaviour of the network. The shape of the curve can be directly linked to the robustness independant of the size of the network. From linear and downward concave curves up to near linear and upward concave curve the robustness of the network generally increases. In other words a high amount of hubs (with relative high degree) leads to a higher robustness against attack. The degree exponent γ again shows not to be a good indicator, being incapable of distinguishing between HSs.

Synthetic static robustness analysis however, is a quite general way of analyzing networks using complex network theory. In 5 a more realistic way of assessing robustness was proposed. Attacking networks using and initiation airport and geographic radial spreading. Three different phenomena were simulated: weather, strikes and volcano eruptions.

The weather robustness analysis included the introduction of geographic clustering as a measure of the closeness of airport within a network, regardless of connections between the networks. The weather robustness analysis shows that using probability or random initiation will not lead to a very different outcome. It also confirms the results from the synthetic error simulation, which showed that all networks have a similar robustness against random failure events, such as bad weather. Both worst case weather scenarios using the betweenness and geographic clustering show that ATNs are less robust against bad weather starting at their most important or central airport.

The analysis shows similar relative results as the synthetic attack simulation, however an important lesson can be learned from the geographically spreading attack scenarios. Not only the number and relative size of the hubs determines the robustnes of both FSCs and LCC, but also the geographic spreading of these hubs. The bigger the spread of the hubs, the more robust the network. (Also the bigger the entire network, the more robust.) A good example of this is $\mathrm{AZ}_{\mathrm{TOT}}$. Under the synthetic attack it shows similar robustness as the $\mathrm{AF\text{-}KL}_{\mathrm{TOT}}$ network. The spreading of KLM-Air France's hubs over France and the Netherlands however makes the network in reality much more robust, compared to Alitalia with his major hubs located within (an area of) Italy. The realistic weather analysis therefore puts the synthetic results in a better perspective.

The strike robustness analysis adds to this that apart from the geographic spreading in general, also the spreading over FIRs, influence the real robustness of an ATN. Again especially $\mathrm{AZ}_{\mathrm{TOT}}$ turns out to be relatively much less robust under the realistic scenario, compared to the synthetic attack scenario. LCCs have a big advantage regarding realistic robustness as usually they have a good geographic spread of their hubs, also with regards to FIRs. Small geographically concentrated networks again turn out relatively vulnerable. For example XK almost completely located within one FIR.

The volcano robustness analysis confirms the results from (Wilkinson et al., 2011) and shows that an eruption of the Vesuvius of a similar size of the Eyjafjallajökull will have disastrous consequences for air travel through Europe for all ATNs. This realistic scenario put the synthetic results in perspective, showing that some natural events such as volcano eruptions are unavoidable and designing a robust network against these phenomena is impossible.

Chapter 7

Recommendations

Several recommendations can be made to help airlines increase their robustness. The most important recommendation is to increase the number of hubs in the network. One of the solutions for this is the formation of codeshares and alliances, which is shown to increase the robustness of the network. Apart from increasing the number of hubs, when designing a ATN also the spreading of the hubs should be taken into account. Again a way to cope with this is to form strategic partnerships with airlines from a diffrerent region of Europe for example AF-$\mathrm{KL}_{\mathrm{TOT}}$.

Regarding the research some practical conclusions can be drawn from the analysis. The degree exponent γ does not clearly distinguish between the networks. Plotting the cumulative degree distribution gives a clearer image of the difference between the networks and should be preffered infuture research. Using the strength distribution function increases the resolution of the results, however the behaviour of the networks remain largerly the same and the networks are less clear to distinguish, so the degree distribution function is preferred. In the robustness analysis The best criterium of attack turns out to be the seat strength and is therefore used to compare the results if the robustness analyses of the different ATNs. The degree exponent γ again shows not to be a good indicator, being incapable of distinguishing between HSs.

Overall this thesis added to the limited literature reviewing the robustness of several airline networks as well as codeshare networks and the Skyteam alliance. Furthermore it introduced a different way of assessing the robustness of these network, using geographic spreading and clustering as the basis of the analysis. Future research should expand on implementing more reality into the robustness analysis of ATNs. The investigation of dynamic robustness could be a next step to improve the quality of the robustness analysis and could provide new insights into the influence of for example the clustering and shortest path length on the robustness of an ATN. Eventually the research could lead to an improved structure of the worldwide ATN, taking into account not only direct economic consideration for the airlines but also the greater economic value of having a robust network for the economy as a whole.

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Appendix A

IATA airline designators

Table A.1: IATA airline designators

IATA-code	Airline
A5	Hop!
AA	American Airlines
AB	Air Berlin
AF	Air France
AZ	Alitalia
BA	British Airways
CT	Alitalia Cityliner
CZ	China Southern Airlines
D9	Donavia
DB	Brit Air
DL	Delta Air Lines
FR	Ryanair
FV	Rossiya Airlines
KL	KLM Royal Dutch Airlines
LH	Lufthansa
MU	China Eastern Airlines
OK	Czech Airlines
OS	Austian Airlines
RO	Tarom
SU	Aeroflot Russian Airlines
U2	Easyjet
UA	United Airlines
US	US Airways
UX	Air Europa Lineas Aereas
VY	Vueling
WN	Southwest Airlines
WX	Cityjet
XK	Air Corsica
YS	Régional

Appendix B

Python Program

Appendix containing the Python programs used for the research

B.1 Topology Analysis

```
#import Excel file
import pandas as pd
import numpy as np
import networkx as nx
from scipy.misc import imread
import powerlaw

#Define Excel File (Include Airport Code)
airline_network = "D:\Dropbox\MSc Assignment\_Data\AZ-Europe.xlsx"
xls = pd.ExcelFile(airline_network)
airport_data = "D:\Dropbox\MSc Assignment\_Data\Raw\Airports.xlsx"
xls2 = pd.ExcelFile(airport_data)

#Day Monday = 1 - Sunday = 7 / Week = 8

day=8

#Import Adj and Weight for all days and convert to array
Adj = xls.parse(day,index_col=0,parse_cols=("A:C"))
W = xls.parse(day,index_col=0,parse_cols=("A,D,F"))
Adj = Adj.values
W = W.values

#Generate Undirected Weighted Network (flights) (also used for
   unweighted)
G_und = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G_und.add_edge(Adj[i][0],Adj[i][1],W=W[i][0])

#Generate Undirected Weighted Network (seats)
G2_und = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G2_und.add_edge(Adj[i][0],Adj[i][1],W=W[i][1])

#Number of Nodes and Number of Flight Legs / Connections
N = nx.number_of_nodes(G_und)
K = nx.number_of_edges(G_und)

#Degree
k_tot = G_und.degree()
k_tot_av = np.round(np.mean(list(k_tot.values())),2)

#Strength (flights)
s_tot=G_und.degree(weight='W')
s_tot_av = np.round(np.mean(list(s_tot.values())),2)

#Strength (seats)
seats_tot= G2_und.degree(weight='W')
seats_tot_av = np.round(np.mean(list(seats_tot.values())),0)

#Degree Distribution
```

```
dist = list(k_tot.values())
fit=powerlaw.Fit(dist, discrete=True)
gam=np.round(fit.power_law.alpha,2)

#Strength Distribution (flights)
dist = list(s_tot.values())
fit=powerlaw.Fit(dist, discrete=True)
gam_fl=np.round(fit.power_law.alpha,2)

#Strength Distribution (seats)
dist = list(seats_tot.values())
fit=powerlaw.Fit(dist, discrete=True)
gam_seats=np.round(fit.power_law.alpha,2)

#Average Shortest Path Length
try:
    L = np.round(nx.average_shortest_path_length(G),2)
except nx.NetworkXError:
    pass

#Betweenness
b_i = nx.betweenness_centrality(G)

#Betweenness (flights)
b_i = nx.betweenness_centrality(G,weight='W')

#Betweenness (seats)
b_i = nx.betweenness_centrality(G2,weight='W')

#Average Clustering Coefficient
c_i = nx.clustering(G_und)
c_i = nx.clustering(G_und, weight='W') #Flights
c_i = nx.clustering(G2_und, weight='W') #Seats
C = np.round(nx.average_clustering(G_und),3)
C_w = np.round(nx.average_clustering(G_und, weight='W'),3)
C_w2 = np.round(nx.average_clustering(G2_und, weight='W'),3)
```

B.2 Synthetic Robustness Analysis

```
#import Excel file
import pandas as pd
import numpy as np
import networkx as nx
from scipy.misc import imread
import operator
import random
import sys

#Define Excel File
airline_network = "D:\Dropbox\MSc Assignment\_Data\A5-Europe.xlsx"
xls = pd.ExcelFile(airline_network)
airport_data = "D:\Dropbox\MSc Assignment\_Data\Raw\Airports.xlsx"
xls2 = pd.ExcelFile(airport_data)

#Define Day Monday = 1 - Sunday = 7 / Week = 8
#Select Output Figures 1/0 (xlim %), Map 1/0, Simple 1/0
day = 8

#Import Adj and Weight for all days and convert to array
Adj = xls.parse(day,index_col=0,parse_cols=("A:C"))
W = xls.parse(day,index_col=0,parse_cols=("A,D:F"))
Adj = Adj.values
W = W.values

#robustness against Attack (Degree)
G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1])

pax_sum = np.zeros(len(Adj[:,1]))
pax_perc_deg = np.zeros(len(Adj[:,1]))
airp_sum = np.zeros(len(Adj[:,1]))
airp_deg = np.zeros(len(Adj[:,1]))
airp_perc_deg = np.zeros(len(Adj[:,1]))

for j in range(0, len(Adj[:,1])):
    #Degree/Strength
    s_tot=G.degree()
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
    pax_perc_deg[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_deg[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_deg[j] = np.round(((airp_sum[0]-airp_sum[j])/airp_sum
       [0])*100,2)
#    if pax_perc_deg[j] <= 5:  # Find f-values
#        print(airp_perc_deg[j])
#        break
```

```
    if sumpax == 0:
        break
    for name, s in s_tot.items():
        if s == max(pax):
            G.remove_node(name)

#robustness against Attack (Flights)
G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1],W=W[i][0])

pax_sum = np.zeros(len(Adj[:,1]))
pax_perc_fli = np.zeros(len(Adj[:,1]))
airp_sum = np.zeros(len(Adj[:,1]))
airp_fli = np.zeros(len(Adj[:,1]))
airp_perc_fli = np.zeros(len(Adj[:,1]))

for j in range(0, len(Adj[:,1])):
    #Degree/Strength
    s_tot=G.degree(weight='W')
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
    pax_perc_fli[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_fli[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_fli[j] = np.round(((airp_sum[0]-airp_sum[j])/airp_sum
       [0])*100,2)
    if sumpax == 0:
        break
    for name, s in s_tot.items():
        if s == max(pax):
            G.remove_node(name)

#robustness against Attack (Seats)
G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1],W=W[i][2])

pax_sum = np.zeros(len(Adj[:,1]))
pax_perc_seat = np.zeros(len(Adj[:,1]))
airp_sum = np.zeros(len(Adj[:,1]))
airp_seat = np.zeros(len(Adj[:,1]))
airp_perc_seat = np.zeros(len(Adj[:,1]))

for j in range(0, len(Adj[:,1])):
    #Degree/Strength
    s_tot=G.degree(weight='W')
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
```

```
    pax_perc_seat[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_seat[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_seat[j] = np.round(((airp_sum[0]-airp_sum[j])/
       airp_sum[0])*100,2)
#     if pax_perc_seat[j] <= 5:  # Find f-values
#         print(airp_perc_seat[j])
#         break
    if sumpax == 0:
        break
    for name, s in s_tot.items():
        if s == max(pax):
            G.remove_node(name)

#robustness against Attack (Betweenness)
G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1])

pax_sum = np.zeros(len(Adj[:,1]))
pax_perc_bet = np.zeros(len(Adj[:,1]))
airp_sum = np.zeros(len(Adj[:,1]))
airp_bet = np.zeros(len(Adj[:,1]))
airp_perc_bet = np.zeros(len(Adj[:,1]))

for j in range(0, len(Adj[:,1])):
    #Betweenness and Degree/Strength
    b_i=nx.betweenness_centrality(G)
    s_tot=G.degree()
    bet = list(b_i.values())
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
    pax_perc_bet[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_bet[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_bet[j] = np.round(((airp_sum[0]-airp_sum[j])/airp_sum
       [0])*100,2)
#     if pax_perc_bet[j] <= 5:  # Find f-values
#         print(airp_perc_bet[j])
#         break
    if sumpax == 0:
        break
    for name, b in b_i.items():
        if b == max(bet):
            G.remove_node(name)

#robustness against Random Failure 100 runs
pax_sum = np.zeros((100,len(Adj[:,1])))
pax_perc_rnd100 = np.zeros((100,len(Adj[:,1])))
airp_sum = np.zeros((100,len(Adj[:,1])))
```

```
airp_rnd100 = np.zeros((100,len(Adj[:,1])))
airp_perc_rnd100 = np.zeros((100,len(Adj[:,1])))
pax_perc_rnd = np.zeros(len(Adj[:,1]))
airp_rnd = np.zeros(len(Adj[:,1]))
airp_perc_rnd = np.zeros(len(Adj[:,1]))
G = nx.Graph()
for k in range(0, len(Adj[:,1])):
    G.add_edge(Adj[k][0],Adj[k][1],W=W[k][0])
N = nx.number_of_nodes(G)
airp_rnd100.fill(N)
airp_perc_rnd100.fill(100)

for i in range(0,100):
    G = nx.Graph()
    for k in range(0, len(Adj[:,1])):
        G.add_edge(Adj[k][0],Adj[k][1])

    print '{}\r'.format(i),
    for j in range(0, len(Adj[:,1])):
        #Degree/Strength
        s_tot=G.degree()
        pax = list(s_tot.values())
        sumpax = np.sum(pax)
        pax_sum[i,j] = np.round(np.sum(pax),2)
        pax_perc_rnd100[i,j] = np.round(100-(((pax_sum[i,0]-np.sum(
            pax))/pax_sum[i,0])*100),2)
        airp_sum[i,j] = len(pax)
        airp_rnd100[i,j] = np.round(airp_sum[i,0]-airp_sum[i,j],2)
        airp_perc_rnd100[i,j] = np.round(((airp_sum[i,0]-airp_sum[i
            ,j])/airp_sum[i,0])*100,2)
        if sumpax == 0:
            break
        result = False
        while result == False:
            try:
                for name, s in s_tot.items():
                    if s == random.choice(pax):
                        index = name
                G.remove_node(index)
            except nx.NetworkXError:
                result = False
            else:
                result = True

for k in range(0, len(Adj[:,1])):
    pax_perc_rnd[k] = np.mean(pax_perc_rnd100[:,k])
    airp_rnd[k] = np.mean(airp_rnd100[:,k])
    airp_perc_rnd[k] = np.mean(airp_perc_rnd100[:,k])
#    if pax_perc_rnd[k] <= 95:  # Find f-values
#        print(airp_perc_rnd[k])
#        break
```

B.3 Realistic Robustness Analysis

```
#import Excel file
import pandas as pd
import numpy as np
import networkx as nx
from scipy.misc import imread
import operator
import random
import sys

#Define Excel File
airline_network = "D:\Dropbox\MSc Assignment\_Data\*AIRLINE*-Europe
    .xlsx"
xls = pd.ExcelFile(airline_network)
airport_data = "D:\Dropbox\MSc Assignment\_Data\Raw\Airports.xlsx"
xls2 = pd.ExcelFile(airport_data)

#Define Day Monday = 1 - Sunday = 7 / Week = 8
#Select Output Figures 1/0 (xlim %), Map 1/0, Simple 1/0
#Define Weighted 1/0
#Define Weights 0 = flights, 2 = pax
day = 8

#Import Location and Distances and convert to array
loc = xls2.parse(0,header=0,index_col=0,parse_cols=("A,K:N"))
dis = xls.parse(9,parse_cols=("A:B"))
group = xls.parse(10,parse_cols=("A:B"))
volcano = xls.parse(11,parse_cols=("A:E"))
prob_all = xls.parse(12,parse_cols=("G"))
loc = loc.values
dis = dis.values
group = group.values
volcano = volcano.values
prob_all = prob_all.values
dis = xls.parse(9,parse_cols=(len(dis)))
dis = dis.values

Adj = xls.parse(day,index_col=0,parse_cols=("A:C"))
W = xls.parse(day,index_col=0,parse_cols=("A,D:F"))
Adj = Adj.values
W = W.values

prob = {}
for i in range (0, len(prob_all)):
    prob[i] = prob_all[i]

prob = list(prob.values())

G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1])
```

```
N = nx.number_of_nodes(G)
print(xlim)
print(N)
xlim2 = xlim*N/100
print(xlim2)

dist = np.zeros((len(dis[:,0])-2,len(dis[0,:])-3))
for i in range(0, len(dis[:,0])-2):
    for j in range(0, len(dis[0,:])-3):
        dist[i,j] = dis[i+2,j+3]

#list of airports
airport = range(len(dis[:,0])-2)
for i in range(0, len(dis[:,0])-2):
    airport[i] = dis[1,i+3]

#robustness against Attack (Weather)
#geographic distances in network

pax_sum = np.zeros(140)
pax_perc_weat = np.zeros(140)
airp_sum = np.zeros(140)
airp_weat = np.zeros(140)
airp_perc_weat = np.zeros(140)
airp_km_weat = np.zeros(140)

G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1],W=W[i][2])

#geographic clustering
c_geo = {}
for i in range (0, len(airport)):
    c_geo[airport[i]] = 1-(sum(dist[i,:])/np.sum(dist))

#max geographic clustering (initiation)
max_geo_air = max(c_geo.iteritems(), key=operator.itemgetter(1))[0]
max_geo_value = max(c_geo.values())
for k in range(0, len(airport)):
    if airport[k] == max_geo_air:
        number = k

#dictionary distances from max clustering airport
dist_dict = {}
for i in range(0, len(airport)):
    dist_dict[airport[i]] = dis[i+2,number+3]

for j in range(0, 140):
    #Degree/Strength
    s_tot=G.degree(weight='W')
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
```

```
    pax_perc_weat[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
        pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_weat[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_weat[j] = np.round(((airp_sum[0]-airp_sum[j])/
        airp_sum[0])*100,2)
    airp_km_weat[j] = j*25
    if pax_perc_weat[j] <= 5:  # Find f-values
        print(airp_perc_weat[j])
        break
    if sumpax == 0:
        break
    for name, dist in dist_dict.items():
        if dist <= j*25:
            G.remove_node(name)
            dist_dict[name] = 20000

#robustness against Attack (Weather and Betweenness)
#geographic distances in network
dist = np.zeros((len(dis[:,0])-2,len(dis[0,:])-3))
for i in range(0, len(dis[:,0])-2):
    for j in range(0, len(dis[0,:])-3):
        dist[i,j] = dis[i+2,j+3]

#list of airports
airport = range(len(dis[:,0])-2)
for i in range(0, len(dis[:,0])-2):
    airport[i] = dis[1,i+3]

pax_sum = np.zeros(180)
pax_perc_weat_b = np.zeros(180)
airp_sum = np.zeros(180)
airp_weat_b = np.zeros(180)
airp_perc_weat_b = np.zeros(180)
airp_km_weat_b = np.zeros(180)

G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1],W=W[i][2])

#geographic clustering and betweenness
b_i = nx.betweenness_centrality(G,weight='W')

c_geo = {}
for i in range (0, len(airport)):
    c_geo[airport[i]] = 1-(sum(dist[i,:])/np.sum(dist))

#combine dictionaries
ds = [b_i, c_geo]
d = {}
for k in b_i.iterkeys():
    d[k] = tuple(d[k] for d in ds)
```

```
#geographic clustering and betweenness combination
c_geo_b = {}
for name, (b, geo) in d.items():
    if combination == True:
        c_geo_b[name] = b*geo
    if combination == False:
        c_geo_b[name] = b

#max geographic clustering + betweenness (initiation)
max_geo_b_air = max(c_geo_b.iteritems(), key=operator.itemgetter(1)
   )[0]
max_geo_b_value = max(c_geo_b.values())
for k in range(0, len(airport)):
    if airport[k] == max_geo_b_air:
        number = k

#dictionary distances from max clustering airport
dist_dict = {}
for i in range(0, len(airport)):
    dist_dict[airport[i]] = dis[i+2,number+3]

for j in range(0, 180):
    #Degree/Strength
    s_tot=G.degree(weight='W')
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
    pax_perc_weat_b[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_weat_b[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_weat_b[j] = np.round(((airp_sum[0]-airp_sum[j])/
       airp_sum[0])*100,2)
    airp_km_weat_b[j] = j*25
    if pax_perc_weat_b[j] <= 5:  # Find f-values
        print(airp_perc_weat_b[j])
        break
    if sumpax == 0:
        break
    for name, dist in dist_dict.items():
        if dist <= j*25:
            G.remove_node(name)
            dist_dict[name] = 20000

#robustness against Attack (Weather Random)
#geographic distances in network
dist = np.zeros((len(dis[:,0])-2,len(dis[0,:])-3))
for i in range(0, len(dis[:,0])-2):
    for j in range(0, len(dis[0,:])-3):
        dist[i,j] = dis[i+2,j+3]

#list of airports
airport = range(len(dis[:,0])-2)
```

```
for i in range(0, len(dis[:,0])-2):
    airport[i] = dis[1,i+3]

pax_sum = np.zeros((100,250))
pax_perc_weat_rnd100 = np.zeros((100,250))
airp_sum = np.zeros((100,250))
airp_weat_rnd100 = np.zeros((100,250))
airp_perc_weat_rnd100 = np.zeros((100,250))
airp_km_weat_rnd100 = np.zeros((100,250))
pax_perc_weat_rnd = np.zeros(250)
airp_weat_rnd = np.zeros(250)
airp_perc_weat_rnd = np.zeros(250)
airp_km_weat_rnd = np.zeros(250)
G = nx.Graph()
for k in range(0, len(Adj[:,1])):
    G.add_edge(Adj[k][0],Adj[k][1],W=W[k][0])
N = nx.number_of_nodes(G)
airp_weat_rnd100.fill(N)
airp_perc_weat_rnd100.fill(100)

for i in range(0,100):
    G = nx.Graph()
    for k in range(0, len(Adj[:,1])):
        G.add_edge(Adj[k][0],Adj[k][1])

    #random intitiation point
    number = random.randint(0, len(airport)-1)

    #dictionary distances from max clustering airport
    dist_dict = {}
    for l in range(0, len(airport)):
        dist_dict[airport[l]] = dis[l+2,number+3]

    print '{}\r'.format(i),
    for j in range(0, 250):
        #Degree/Strength
        s_tot=G.degree(weight='W')
        pax = list(s_tot.values())
        sumpax = np.sum(pax)
        pax_sum[i,j] = np.round(np.sum(pax),2)
        pax_perc_weat_rnd100[i,j] = np.round(100-(((pax_sum[i,0]-np
            .sum(pax))/pax_sum[i,0])*100),2)
        airp_sum[i,j] = len(pax)
        airp_weat_rnd100[i,j] = np.round(airp_sum[i,0]-airp_sum[i,j
            ],2)
        airp_perc_weat_rnd100[i,j] = np.round(((airp_sum[i,0]-
            airp_sum[i,j])/airp_sum[i,0])*100,2)
        airp_km_weat_rnd100[i,j] = j*25
        if sumpax == 0:
            break
        for name, dist in dist_dict.items():
            if dist <= j*25:
                G.remove_node(name)
```

```
                dist_dict[name] = 20000

for k in range(0, 250):
    pax_perc_weat_rnd[k] = np.mean(pax_perc_weat_rnd100[:,k])
    airp_weat_rnd[k] = np.mean(airp_weat_rnd100[:,k])
    airp_perc_weat_rnd[k] = np.mean(airp_perc_weat_rnd100[:,k])
    airp_km_weat_rnd[k] = k*25
    if pax_perc_weat_rnd[k] <= 95:  # Find f-values
        print(airp_perc_weat_rnd[k])
        break

#robustness against Attack (Weather Probability)
#geographic distances in network
dist = np.zeros((len(dis[:,0])-2,len(dis[0,:])-3))
for i in range(0, len(dis[:,0])-2):
    for j in range(0, len(dis[0,:])-3):
        dist[i,j] = dis[i+2,j+3]

#list of airports
airport = range(len(dis[:,0])-2)
for i in range(0, len(dis[:,0])-2):
    airport[i] = dis[1,i+3]

pax_sum = np.zeros((100,250))
pax_perc_weat_prob100 = np.zeros((100,250))
airp_sum = np.zeros((100,250))
airp_weat_prob100 = np.zeros((100,250))
airp_perc_weat_prob100 = np.zeros((100,250))
airp_km_weat_prob100 = np.zeros((100,250))
pax_perc_weat_prob = np.zeros(250)
airp_weat_prob = np.zeros(250)
airp_perc_weat_prob = np.zeros(250)
airp_km_weat_prob = np.zeros(250)
G = nx.Graph()
for k in range(0, len(Adj[:,1])):
    G.add_edge(Adj[k][0],Adj[k][1],W=W[k][0])
N = nx.number_of_nodes(G)
airp_weat_prob100.fill(N)
airp_perc_weat_prob100.fill(100)

for i in range(0,100):
    G = nx.Graph()
    for k in range(0, len(Adj[:,1])):
        G.add_edge(Adj[k][0],Adj[k][1])

    #random intitiation point based on probability
    element = random.choice(prob)
    number = airport.index(element)

    #dictionary distances from max clustering airport
    dist_dict = {}
    for l in range(0, len(airport)):
        dist_dict[airport[l]] = dis[l+2,number+3]
```

```
    print '{}\r'.format(i),
    for j in range(0, 250):
        #Degree/Strength
        s_tot=G.degree(weight='W')
        pax = list(s_tot.values())
        sumpax = np.sum(pax)
        pax_sum[i,j] = np.round(np.sum(pax),2)
        pax_perc_weat_prob100[i,j] = np.round(100-(((pax_sum[i,0]-
            np.sum(pax))/pax_sum[i,0])*100),2)
        airp_sum[i,j] = len(pax)
        airp_weat_prob100[i,j] = np.round(airp_sum[i,0]-airp_sum[i,
            j],2)
        airp_perc_weat_prob100[i,j] = np.round(((airp_sum[i,0]-
            airp_sum[i,j])/airp_sum[i,0])*100,2)
        airp_km_weat_prob100[i,j] = j*25
        if sumpax == 0:
            break
        for name, dist in dist_dict.items():
            if dist <= j*25:
                G.remove_node(name)
                dist_dict[name] = 20000

for k in range(0, 250):
    pax_perc_weat_prob[k] = np.mean(pax_perc_weat_prob100[:,k])
    airp_weat_prob[k] = np.mean(airp_weat_prob100[:,k])
    airp_perc_weat_prob[k] = np.mean(airp_perc_weat_prob100[:,k])
    airp_km_weat_prob[k] = k*25
    if pax_perc_weat_prob[k] <= 95:  # Find f-values
        print(airp_perc_weat_prob[k])
        break

#robustness against Attack (Vesuvius)
pax_sum = np.zeros(140)
pax_perc_ves = np.zeros(140)
airp_sum = np.zeros(140)
airp_ves = np.zeros(140)
airp_perc_ves = np.zeros(140)
airp_km_ves = np.zeros(140)

G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1],W=W[i][2])

#dictionary distances from Vesuvius
dist_dict = {}
for i in range(0, len(airport)):
    dist_dict[volcano[i+2,2]] = volcano[i+2,3]

for j in range(0, 140):
    #Degree/Strength
    s_tot=G.degree(weight='W')
    pax = list(s_tot.values())
```

```
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
    pax_perc_ves[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_ves[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_ves[j] = np.round(((airp_sum[0]-airp_sum[j])/airp_sum
       [0])*100,2)
    airp_km_ves[j] = j*25
    if pax_perc_ves[j] <= 5:  # Find d-values
        print(airp_km_ves[j])
        break
    if sumpax == 0:
        break
    for name, dist in dist_dict.items():
        if dist <= j*25:
            G.remove_node(name)
            dist_dict[name] = 20000

#robustness against Attack (Eyjafjallojoekull)
pax_sum = np.zeros(320)
pax_perc_eyjaf = np.zeros(320)
airp_sum = np.zeros(320)
airp_eyjaf = np.zeros(320)
airp_perc_eyjaf = np.zeros(320)
airp_km_eyjaf = np.zeros(320)

G = nx.DiGraph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1],W=W[i][2])

#dictionary distances from Eyjafjallojoekull
dist_dict = {}
for i in range(0, len(airport)):
    dist_dict[volcano[i+2,2]] = volcano[i+2,4]

for j in range(0, 320):
    #Degree/Strength
    s_tot=G.degree(weight='W')
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
    pax_perc_eyjaf[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_eyjaf[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_eyjaf[j] = np.round(((airp_sum[0]-airp_sum[j])/
       airp_sum[0])*100,2)
    airp_km_eyjaf[j] = j*25
    if pax_perc_eyjaf[j] <= 5:  # Find d-values
        print(airp_km_eyjaf[j])
        break
    if sumpax == 0:
```

```
        break
    for name, dist in dist_dict.items():
        if dist <= j*25:
            G.remove_node(name)
            dist_dict[name] = 20000

#robustness against Attack (Geographic Segment Degree)
#geographic distances in network
#list of airports
airport = range(len(dis[:,0])-2)
for i in range(0, len(dis[:,0])-2):
    airport[i] = dis[1,i+3]

groups = np.unique(group[:,1])
group1 = group[:,1].tolist()

pax_sum = np.zeros(len(Adj[:,1]))
pax_perc_geodeg = np.zeros(len(Adj[:,1]))
airp_sum = np.zeros(len(Adj[:,1]))
airp_geodeg = np.zeros(len(Adj[:,1]))
airp_perc_geodeg = np.zeros(len(Adj[:,1]))

G = nx.Graph()
for i in range(0, len(Adj[:,1])):
    G.add_edge(Adj[i][0],Adj[i][1],W=W[i][2])

#airports with FIR
FIR_dict = {}
for i in range (0, len(group)):
    FIR_dict[group[i,0]] = group[i,1]

#geographic clustering
k_geo = {}
for i in range (0, len(groups)):
    k_geo[groups[i]] = group1.count(groups[i])

for j in range(0, len(Adj[:,1])):
    #Degree/Strength
    s_tot=G.degree(weight='W')
    pax = list(s_tot.values())
    sumpax = np.sum(pax)
    pax_sum[j] = np.round(np.sum(pax),2)
    pax_perc_geodeg[j] = np.round(100-(((pax_sum[0]-np.sum(pax))/
       pax_sum[0])*100),2)
    airp_sum[j] = len(pax)
    airp_geodeg[j] = np.round(airp_sum[0]-airp_sum[j],2)
    airp_perc_geodeg[j] = np.round(((airp_sum[0]-airp_sum[j])/
       airp_sum[0])*100,2)
    k_geo_max = max(k_geo.values())
    if pax_perc_geodeg[j] <= 5:  # Find f-values
        print(airp_perc_geodeg[j])
        break
    if sumpax == 0:
```

```
        break
    for name, k_geog in k_geo.items():
        if k_geog == k_geo_max:
            FIRseg = name
            k_geo[name] = 0
            for name, FIR in FIR_dict.items():
                if FIR == FIRseg:
                    G.remove_node(name)
```